

STUDIES ON PROTON EMISSION FROM MEDIUM, HEAVY AND SUPERHEAVY NUCLEI



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Submitted by

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(Ref. No. 06976/Ph. D-K3/Physics/PT/April 2019)

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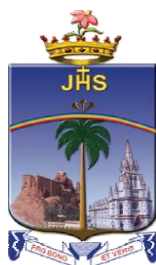
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DECLARATION

I hereby declare that the thesis entitled “**STUDIES ON PROTON EMISSION FROM MEDIUM, HEAVY AND SUPERHEAVY NUCLEI**” embodies the results of my research work carried out under the guidance and supervision of **Dr. S. Alfred Cecil Raj, M.Sc., M.Phil., Ph.D.** Supervisor, Associate Professor, Department of Physics, St. Joseph’s College (Autonomous), Tiruchirappalli & **Dr. H.C. Manjunatha, M.Sc., M.Phil., Ph.D.** Co-Supervisor, Associate Professor, Department of Physics, Govt. College for Women, Kolar. I have not submitted the above thesis to any University for any Degree, Diploma, Fellowship or any other similar titles previously.

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CONTENTS

LIST OF FIGURES	iii
LIST OF TABLES	vii
ABSTRACT	x
1 Introduction	1
1.1 Introduction	1
1.2 Proton emission	1
1.3 Theoretical studies	2
1.4 Experimental studies	7
1.5 Objectives of research work	25
2 Semi empirical formula for proton radioactivity half lives	27
2.1 Construction of Semi-empirical formula for one and two proton decay	27
3 Study of Proton radioactivity using Theoretical models	34
3.1 Theory	34
3.1.1 Modified Generalized Liquid Drop Model (MGLDM)	34
3.1.2 Coulomb and Proximity Potential Model (CPPM)	37
3.1.3 Effective Liquid Drop Model(ELDM)	39
3.2 Results	41
4 Competition between different decay modes	49
4.1 Method of Calculation of half-lives	49
4.2 Results	50
4.2.1 Proton radioactivity of Lanthanides	50
4.2.2 Proton radioactivity of heavy nuclei ($72 \leq Z \leq 88$)	62
4.2.3 Proton radioactivity of actinides ($89 \leq Z \leq 103$)	70
4.2.4 Systematics of actinides	78

4.2.5	Proton radioactivity of superheavy nuclei ($104 \leq Z \leq 126$)	83
5	Comparison of present work with Microscopic models	93
5.1	Theory of Macroscopic models	93
5.1.1	Generalized Liquid Drop Model (GLDM)	93
5.1.2	universal decay law for proton emission (UDLP)	94
5.1.3	Gamow-like model for Proton decay (GLM)	94
5.1.4	Unified Fission model for Proton decay (UFM)	95
5.2	Results	96
6	Summary	106
6.1	Semi-empirical formula for one and two proton radioactivity	106
6.2	Verification of Geiger-Nuttall law for Proton Radioactivity.	106
6.3	Studies on Proton radioactivity using Macroscopic models	108
6.4	Proton radioactivity in Lanthanides ($57 \leq Z \leq 71$)	108
6.5	Proton radioactivity of Heavy nuclei ($72 \leq Z \leq 88$)	109
6.6	Proton radioactivity of Actinide nuclei ($89 \leq Z \leq 103$)	110
6.7	Proton radioactivity of Superheavy nuclei ($104 \leq Z \leq 126$)	111
6.8	Scope of research work	112
	REFERENCES	112
	LIST OF PUBLICATIONS	154

List of Figures

2.1	(a-d): The variation of $\log T_{1/2}$ during one proton radioactivity as a function of a where $Z_D^x/\sqrt{Q}$. (e)-(i) : The variation of $\log T_{1/2}$ during one proton radioactivity as a function of b where $(Z_D^{0.7} + \ell^y)/\sqrt{Q}$	29
2.2	(a)-(e): The variation of $\log T_{1/2}$ during two proton radioactivity as a function of $a = Z_D^x/\sqrt{Q}$. (f)-(i) : The variation of $\log T_{1/2}$ during one two radioactivity as a function of $b = (Z_D^{0.7} + \ell^y)/\sqrt{Q}$	31
3.1	Schematic presentation of molecular phase of the di-nuclear system. [14, 176] . .	39
4.1	Variation of experimental and present formula half lives as a function of $Z_d/\sqrt{Q}$	51
4.2	Variation of potential energy as a function of R for different proximity potentials.	53
4.3	Competition between different decay modes for lanthanide nuclei.	56
4.4	Nuclide chart of Proton emitters in the lanthanide region.	57
4.5	The quantity of energy released during proton radioactivity and the mass number of Dysprosium parent nuclei.	59
4.6	Comparison of $\log T_{1/2}$ of different decay modes such as $\beta^\pm$, 1P, and α decay versus mass number of parent nuclei.	60
4.7	Decay chains in the isotopes of $^{133-135}\text{Dy}$	61
4.8	A variation of amount of energy released during the proton emission in the nuclei region $72 < Z < 88$ with the mass number of parent nuclei.	63
4.9	A variation of logarithmic half-lives for the proton emission in the nuclei region $72 < Z < 88$ with the mass number of parent nuclei.	63
4.10	A variation of logarithmic half-lives for the proton emission in the nuclei region $72 < Z < 88$ with the product of atomic number of daughter nuclei and amount of energy released during proton emission ($Z_d Q^{-1/2}$).	64
4.11	A variation of logarithmic half-lives for the proton activity, alpha decay, β^+ and β^- decay as a function of mass number of the parent nuclei (A)	65

4.12 A Comparison of logarithmic half -lives of proton radioactivity of present work with that of available experimental values	65
4.13 A plot of Q-values during 1P- decay with the mass number of parent nuclei for the $^{151-157}\text{Ta}$ nuclei.	67
4.14 Variation of total potential using three models such as CPPM, ELDM and MGLDM as function of separation distance in ^{151}Ta nuclei.	67
4.15 A comparison of proton-decay, alpha-decay and beta-decay half-lives using CPPM and semi-empirical relations with that of available experiments.	69
4.16 Variation of $\log(T_{1/2})$ of proton activity, alpha decay, spontaneous fission and beta decay as a function of mass number of the parent nuclei (A).	70
4.17 Variation of $\log(T_{1/2})$ of proton activity, alpha decay, spontaneous fission and beta decay as a function of mass number of the parent nuclei (A).	71
4.18 Variation of $\log(T_{1/2})$ of proton activity, alpha decay, spontaneous fission and beta decay as a function of mass number of the parent nuclei (A).	72
4.19 Variation of $\log(T_{1/2})$ of proton activity, alpha decay, spontaneous fission and beta decay as a function of mass number of the parent nuclei (A).	73
4.20 Variation of decay modes such as proton activity, alpha decay, spontaneous fission, beta plus decay and beta minus decay as a function of mass number of the parent nuclei (A).	75
4.21 A variation of branching ratios of proton decay to the alpha decay (NRDX), SF (Bao et al., [295]), beta (minus) decay and beta (plus) decay as a function mass number of the nuclei.	75
4.22 The variation of calculated logarithmic half-lives of proton decay of different mass number of proton emitters with the available experimental values [6, 34, 35, 115, 117, 198, 216, 234, 246–248, 296–303] in the actinide range.	76
4.23 A standard deviation of proton decay from the experimental values with that of mass number of parent nuclei.	77
4.24 The variation of amount of energy released during proton decay with the mass number of parent nuclei in the actinide region.	79
4.25 The variation of logarithmic half-lives of proton decay with the mass number of parent nuclei in the actinide region.	79

4.26	Variation of logarithmic half-lives of proton decay with the product of $Z_d Q^{-1/2}$ in the actinide region.	80
4.27	Competition between different decay modes such as proton decay, spontaneous fission and alpha decay for superheavy elements.	86
4.28	Variation of logarithmic proton decay half-lives versus $1/\sqrt{Q}$	87
4.29	Variation of logarithmic proton decay half-lives versus $Z_d/\sqrt{Q}$	88
4.30	Variation of $\sqrt{R}$ against mass number of the parent nuclei A	89
4.31	The variation of logarithmic half-lives of the proton decay, spontaneous fission and alpha decay with the mass number of parent nuclei	91
4.32	The variation of amount of energy released, penetration probability and normalization factor with the mass number of parent nuclei and the variation of logarithmic half-lives with the product of atomic number and energy released during proton decay.	92
5.1	Deviations between macroscopic models such as UFM, GLM, UDLP, GLDM, ELDM and CPPM with that of experimental logarithmic half-lives of proton radioactivity from ^{109}I to ^{209}Bi as a function of Z^2/A	99
5.2	Comparison of proton decay half-lives produced by macroscopic models such as CPPM, ELDM, GLDM, UDLP, GLM and UFM with that of experiments for various $Z/\sqrt{Q}$ values.	101
5.3	Deviation between microscopic models such as DDM3Y [94, 95], JLM [51], M3Y+EX [96], R3Y+EX [97], SLy4[98], SRG[100], Skc and SkD [93] with that of experimental logarithmic half-lives of proton radioactivity from The nuclei ^{109}I to ^{209}Bi for various Z^2/A values.	102
5.4	Comparison of proton decay half-lives produced by microscopic models such as DDM3Y, JLM, M3Y+EX, R3Y+EX, SLy4, SRG, Skc and SkD with that of experiments for various $Z/\sqrt{Q}$ values.	103

6.1	Variation of logarithmic half lives as function of $Z_d^n/Q^{1/2}$. (a) One proton decay - Yellow circles represents the data corresponds to experiments [271], the blue colour circles represents the values produced by previous formula [277] and con- tinues red line represents values obtained from the present formula and (b) Two proton decay- Yellow circles represents the two proton decay half-lives corre- sponds to experiments [271], blue colour circles represents the values produced by previous formula [329] and continues red line represents values obtained from the present formula.	107
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List of Tables

2.1	Comparison of calculated half-lives with the different models.	32
2.2	Comparison of 1P decay logarithmic half lives obtained using present formula (PF) with that of experiments[109].	33
2.3	Comparison of 2P decay logarithmic half lives obtained using present formula (PF) with that of experiments [145]	33
3.1	A tabulation of standard deviation obtained for one and two proton decay logarithmic half-lives using CPPM, ELDM and MGLDM with that of available experiments.	41
3.2	Comparison of 1P decay logarithmic half lives obtained using CPPM, ELDM, MGLDM and present formula (PF) with that of experiments.	42
3.3	Tabulation of one proton radioactivity in the atomic number range $4 \leq Z \leq 43$ using theoretical models such as CPPM, ELDM, MGLDM and semi-empirical formula (PF).	43
3.4	Tabulation of one proton radioactivity in the atomic number range $43 \leq Z \leq 105$ using theoretical models such as CPPM, ELDM, MGLDM and semi-empirical formula (PF). Table 3.3 continued.	44
3.5	Comparison of 2P decay logarithmic half lives obtained using CPPM, ELDM, MGLDM and present formula (PF) with that of experiments and theoretical data available in literature [160].	45
3.6	Tabulation of two proton radioactivity in the atomic number range $3 \leq Z \leq 65$ using theoretical models such as CPPM, ELDM, MGLDM and semi-empirical formula (PF).	46
4.1	The range of lanthanide isotopes having positive proton decay energy	52
4.2	Comparison of evaluated proton decay halflives using different proximity functions with that of the experiments	54
4.3	Mean square error with respect to experiments for different proximity functions and proposed present formula	55

4.4	Identification of decay modes of $^{133-138}\text{Dy}$ and $^{177-180}\text{Dy}$	62
4.5	Comparison of present work with experiments [5, 34, 35, 234] and available semi empirical formulae such as Hatsukawa et al.[294] and Gamow [10]	66
4.6	Tabulation of $\log T_{1/2}$ using three different models such as CPPM, ELDM and MGLDM for predicted proton emitters from $^{151-157}\text{Ta}$ is compared to available experiments.	68
4.7	Prediction of logarithmic half-lives of β^- -decay in the isotopes of heavy nuclei $^{220-244}\text{Bi}$	69
4.8	List of studied actinide nuclei for proton decay	72
4.9	Energy released, penetration factor and logarithmic half-lives for proton decay in actinide nuclei	74
4.10	The comparison of calculated half-lives with the experimental values [6, 34, 35, 115, 117, 198, 216, 234, 246–248, 296–303].	77
4.11	Standard deviation of proton decay half-lives in the nuclei region $Z=51-83$	78
4.12	Proton decay half-lives, penetration factor and amount of energy released during proton decay in actinides.	80
4.13	A comparison of logarithmic half-lives of proton decay with Royer, Univ, NRDX, Denisov and Bao.	81
4.14	The comparison of calculated half-lives with the experimental values [117, 246, 247],[6, 34, 35, 115, 198, 216, 234, 248, 296–303].	82
4.15	List of studied superheavy nuclei for proton decay	83
4.16	Energy released, penetration factor, normalisation factor and logarithmic half-lives for proton decay in superheavy nuclei	84
4.17	Energy released, penetration factor, normalisation factor and logarithmic half-lives for proton decay in superheavy nuclei	85
5.1	Tabulation of rms and average value ($\bar{\delta}$) for mass excess produced by different theoretical models.	97
5.2	The comparison logarithm half-lives of proton radioactivity produced by different macroscopic models CPPM, ELDM, UDLP, GLM and UFM with that of experiments [325] along with decay energies and angular momentum.	98

5.3	The root mean square error (RMSE) and root mean square deviations (σ) of the calculations from macroscopic models such as CPPM, ELDM, GLDM, UDLP, GLM and UFM with respect to the experimental data and microscopic models such as DDM3Y, JLM, M3Y+EX, R3Y+EX, SLy4, SRG, Skc and SkD with that of experiments.	104
6.1	The index “n” and fitting coefficients a , b and c of one and two proton decay. . .	108

ABSTRACT

A thorough study on one proton and two proton radioactivity is carried in the atomic number range $3 \leq Z \leq 126$ using well accepted Coulomb and proximity potential model (CPPM), effective liquid drop model(ELDM), generalized liquid drop model(GLDM) and modified generalised liquid drop model(MGLDM) models. The proton decay half-lives produced by these models are compared with the experiments. Predictive power of these models are assessed by evaluation of the mean squared error and it can be concluded that CPPM model produces proton decay half-lives close to experiments. Furthermore, we constructed semi empirical formula for one and two proton decay half-lives by including angular momentum term . The values produced by the present formula is also compared with experiments. Even though, identified one and two proton emitters along with half lives and decay energies are based on theory, further investigations requires comparison of these predicted half-lives with the other decay modes.

The competition between proton decay and other possible decay modes such as alpha decay, beta decay and spontaneous fission in Lanthanide, Actinide and Super heavy region to identify the dominant decay mode is studied. Also identified the proton emitters from medium, heavy and super heavy nuclei ie among Lanthanides, Actinides and super heavy region.

The proton decay half-lives in the lanthanide region ($Z=58$ to 71) have been systematically studied using different proximity functions such as Prox. 13, Prox. 77, MP 77, Ng 80 and Bass 80. Though the experimental values are found to be in good agreement with the proximity function of Ng 80, we have constructed an empirical formula to calculate the half-lives of such proton-emitting lanthanides. The half life values produced by the present formula is compared with that of NG 80.

The half-lives of one-proton emitters in the actinide region ($Z=89-103$) are theoretically presented using the coulomb and proximity potential method. In the present work the calculated one-proton decay half-live values compare fairly well with the available experimental values. To check the Geiger Nuttal law for proton decay in actinide nuclei, logarithmic proton decay half-lives are plotted against $1/\sqrt{Q}$. The competition of proton decay with different decay modes such as alpha decay and spontaneous fission are also studied. The one proton emitters in the actinide region are identified in the unexplored isotopes of actinide region which is not specified in the nuclear chart.

The proton radioactivity of heavy nuclei of atomic number range $72 < Z < 88$ is studied. The energy released during the proton decay (Q_P) and half-lives of proton decay are evaluated for heavy nuclei. The competition between different decay modes is studied by comparing the proton decay half lives with that of other decay modes such as alpha decay, β^+ and β^- decay. To check the Geiger-Nuttall law for proton decay, the logarithmic proton decay half-lives are plotted against $1/\sqrt{Q}$. The possible proton emitters of heavy nuclei corresponding in the atomic number range $72 < Z < 88$ are also highlighted.

The proton decay in almost all super heavy nuclei with atomic number $Z=104-126$ is studied by theoretically calculating the energy released during the proton decay (Q_P), penetration factor (P), normalisation factor (F) and half lives of proton decay, out of which proton decay is possible in few super heavy nuclei. The study of competition of proton decay with different decay modes of super heavy nuclei such as alpha decay and spontaneous fission reveals that proton decay is not dominant decay mode in the super heavy nuclei region. This means super heavy nuclei is stable against the proton decay.

The proton radioactivity half-lives in the atomic number range $53 \leq Z \leq 83$ is studied using macroscopic models such as Coulomb and Proximity potential model, effective liquid drop model (ELDM), Generalised liquid drop model (GLDM), Universal decay law for proton emission (UDLP), Gamow-like model (GLM) and Unified fission model (UFM). The proton decay half-lives produced by the macroscopic models are compared with that of microscopic models such as DDM3Y, JLM, M3Y+EX, R3Y+EX, SLy4, SRG, Skc and SkD. After detail analysis, it is found that among macroscopic models, UDLP and ELDM produces proton decay half-lives close to experiments. Furthermore, among microscopic models, DDM3Y produces proton decay half-lives close to the experiments. To study the proton decay process, microscopic approach is more appropriate than the macroscopic approach. We have also investigated the correct mass excess data which can be used in proton decay studies. Thus, Both UDLP and ELDM macroscopic models and DDM3Y microscopic model can be effectively used in the prediction of half-lives of unexplored proton emitters.

CHAPTER 1

Introduction

1.1 Introduction

Proton emission is a type of radioactive decay that occurs when a proton is released from a nucleus. After a beta decay, Proton emission can happen in a nucleus from highly excited states, known as beta-delayed proton emission, or from the ground state of highly proton-rich nuclei, which is comparable to alpha decay. During proton emission, a proton is emitted from the nucleus of an atom. An atom converts from one element to another when it loses a proton during proton emission. An atom of nitrogen (containing 7 protons) becomes an atom of carbon after undergoing proton emission (with 6 protons). The phenomenon of proton radioactivity has gained attention as an important tool for understanding the nuclear structure of nuclides far from the stability line[1]. Nuclear structure information such as shell structure and interaction among bound and unbound nuclear states may be extracted using proton radioactivity.[2]. Beyond beta stability, proton emission from the nuclear ground state is intended to yield information on nuclear masses and structure.[3].

1.2 Proton emission

Proton emission is one method through which unstable atoms might become more stable. A proton is simply released from a nucleus containing surplus protons in proton decay, a rare kind of

radioactive decay. Proton emission happens in the most proton-rich nuclides and after a positive beta decay from a nucleus's high-lying excited states. In proton decay, quantum tunnelling is also involved. Before a proton can be released, it must first pass through a potential barrier. The proton separation energy must be negative for a proton to exit a nucleus; as a result, the unbound proton tunnels out of the nucleus in a finite time. Some nuclei, such as ^{45}Fe , decay by double proton emission. When a nucleus undergoes proton decay, the atomic number and mass number of the daughter nucleus change by one, and it becomes a new element. The neutron drip line is distinguished by its location above the nuclei that may decay in this manner. In naturally occurring isotopes, proton emission is not seen. Nuclear reactions that use particle accelerators can produce proton radioactive isotopes. Protons are not produced by naturally occurring isotopes. The proton emission half-lives of spherical proton emitters were investigated using the unified fission model (UFM) in conjunction with the phenomenological attack frequency[4].

1.3 Theoretical studies

Nuclear structure and nuclear reaction research in unusual nuclei is now focusing on proton emission investigations. Theoretical ways for studying the characteristics of such nuclei utilising proton emission are discussed [5]. Delion et al., predicted a simple formula in proton decay processes that relates logarithmic half-life, modified by the centrifugal barrier along with the Coulomb parameter. [6]. Buck et al., investigated proton emission from heavy nuclei's ground states, employing a model of charged particle emission that has previously been shown to accurately describe unimpeded s-wave alpha and exotic decays in heavy nuclei [7]. Proton systematics were studied by Delion and Dumitrescu in terms of barrier penetration and formation likelihood. To do this, they used a parabolic dependency characterising the nuclear portion and a pure Coulomb potential to simulate the true proton-core interaction [8]. Sreeja and Balasubramaniam presented an em-

empirical formula for two-proton decay half-lives based on their publication of an empirical method for determining the logarithmic half-lives of one-proton emitters. For the two proton emitters, a four-parameter formula as a function of rotational momentum is provided and the findings of the effective liquid drop model were used to fit the formula's parameters (ELDM) [9].

Zdeb et al., explained proton emission using a model with basic phenomenological formalism, which is based on the Gamow theory for alpha decay and is expanded by incorporating the centrifugal factor [10]. Karny et al., used a unique approach of digital processing of overlapping recoil implantation and decay data to identify the fine structure in proton emission from ^{145}Tm [11]. Axelsson et al., investigated the two-proton emission in the decay of ^{31}Ar using the energy and angular distributions of the two protons [12]. Using an empirical method, Sreeja and Balasubramaniam revealed half-life estimates for one proton transition from ground state to isomeric states of 44 proton emitters [13]. Goncalves et al., calculated the half-lives for two-proton radioactivity of emitter nuclides using ELDM that has been successfully used to alpha decay, one-proton radioactivity, and cold fission processes. They used this method to calculate half-lives for many 2p-emitted nuclei and compared their findings to predictions from other models as well as existing data in the literature, focusing on the parent nuclei ^{16}Ne , ^{19}Mg , ^{45}Fe , ^{48}Ni , ^{54}Zn , and ^{67}Kr [14].

Ferreira et al., investigated the relationship between the half-lives for proton emission from deformed nuclei and the various single particle potentials that reflect typical nuclear structural attributes that have been published in the literature [15]. Using the effective liquid drop model of heavy-particle nuclei decay, Guzman et al., determined half-lives for proton emission from proton-rich nuclei [16]. Zhang Hong-Fei et al., computed the proton radioactivity half-lives of spherical proton emitters and suggested two formulae for the proton decay half-life of spherical proton emitters based on the experimental data available, and were able to successfully recreate the experimental half-lives [17].

Dong et al., [18] investigated theoretically the proton radioactivity half-lives of spherical proton emitters by determining the potential barriers preventing protons from being emitted in the quasi molecular shape path within a generalised liquid drop model. . Various theoretical methods to proton emission from spherical nuclei were examined by Sven Aberg et al., who derived decay widths that were found to be qualitatively insensitive to the parameters of the proton-nucleus potential [19]. Delion et al., explored proton decay from triaxially deformed nuclei ^{161}Re and ^{185}Bi by determining the decay width and angular distribution of the decaying particle, as well as their relationship to the triaxial deformation parameters [20].

Starting from a mean field HF potential and employing the Skyrme interaction, J. S. AlKhalili et al., outlined a two potential approach to one proton emission for the situation of spherical nuclei, resulting to an expanded 3D TPA model [21]. Bugrov and Kadmenskii derived an equation for the proton decay width of a deformed nucleus based on proton radioactivity theory, with many-particle effects taken into account [22]. Using the multiparticle theory of proton radioactivity, Kadmensky and Bugrov estimated the half-lives of ^{147}Tm , ^{147m}Tm , ^{150}Lu , and ^{151}Lu nuclei with regard to proton decay. In these computations, the deformation of decaying nuclei examined in the spherical model was taken into consideration [23]. Maglione et al.,[24] predicted precisely the proton decay half-lives $T_{1/2}$ from the nuclei $^{109}_{53}\text{I}$, $^{131}_{63}\text{Eu}$, and $^{141}_{67}\text{Ho}$ assuming that the released proton travels in a distorted single particle Nilsson level. Delsanto et al.,[25] evaluated the proton separation energies, half-lives, and excitation energies of ^{69}Kr and ^{68}Se from beta-delayed proton emission.

Alavi et al., [26] used the WKB Method to calculate the proton radioactivity half-lives of 45 proton emitters and noticed a decline in the values of estimated half-lives employing the orientation angle dependent formalism . Baye et al.,[27] calculated the decay probability per second for the one-neutron halo nuclei ^{11}Be , ^{19}C , and ^{31}Ne . Feix and Hilf [28] computed the Decay widths of

nuclear proton emission from neutron-deficient odd- Z nuclei, $51 \leq Z \leq 71$, by the quasi-stationary model. Giusti et al., [29] developed the theoretical frame work of emission of two protons in electron induced reactions. Arumugam et al., [30] investigated the proton emission, gamma deformation, and the spin of the isomeric state of ^{141}Ho and revealed the proton deformations and other structural properties of exotic nuclei beyond the proton drip-line. Duarte et al., [31] used the Effective Liquid Drop Model (ELDM) to investigate the half-lives for proton emission, alpha decay, cluster radioactivity, and cold fission processes employing a combination of variable mass asymmetry shape description for mass transfer with Werner-Wheeler's inertia coefficient. Ferreira et al., [32] investigated proton radioactivity from spherical nuclei by using a self-consistent computation. Maglione and Ferreira computed precisely the half-lives of proton emission from the drip line nucleus ^{131}Eu to the first excited 2^+ state of the daughter nucleus ^{130}Sm [33]. Santhosh and Indu Sukumaran used the Coulomb and proximity potential model for deformed nuclei (CPPMDN) to predict half-lives for proton emitters with $Z > 50$ in the ground state and isomeric state [34].

Blank and Borge discussed certain features of beta-delayed decay modes, one- and two-proton radioactivity, and the experimental procedures to get a deeper understanding of the atomic nucleus' organisation [35]. Manjunatha et al., [36] proposed a new empirical formula for the mass excess of heavy and superheavy nuclei in the $Z=96-129$ range. In non-minimal SUSY SU (5) GUTs, Borut Bajc et al., computed the high and low scale threshold corrections to dimension-six proton decay operators [37]. Mukha et al., [38] discovered one and two-proton radioactivity in ^{94}Ag isomers with more than 21 high-spin isomers. Rykaczewski examined experimental data on proton-radioactive nuclei ^{131}Eu , ^{145}Tm , and ^{146}Tm , which included the finding of fine structure in proton emission and analyses of excited states in proton-emitting nuclei [39]. Denisov and Khudenko developed set of relations for evaluating the half-lives of alpha emitters [40].

Chang Xu et al., [41] evaluated the α -decay half-lives for the nuclei range $Z \geq 90$ using the distorted version of the density-dependent cluster model. Yibin Qian et al., [42] evaluated proton decay half-lives using the modified two-potential method. Dongdong Ni et al., [43] developed, with some approximations, a generic formula of half-lives and decay energies for decay and cluster radioactivity directly determined from the WKB barrier penetration probability. Delion approximated the preformation amplitude for heavy cluster decays till ^{14}C emission by offering numerical findings for ^{14}C emission and outlining the fission-like theory of emission processes based on the Two Center Shell Model[44]. Horia Hulubei et al., [45] determined the proton emission half-lives for $Z \geq 51$ nuclei using a simple analytical model based on the WKB approximation for the barrier penetration probability, which incorporates centrifugal and overlapping effects in addition to electrostatic repulsion. Gerald Gilbert investigated the possibility of proton decay in the presence of topology-changing field configurations in euclidean quantum gravity[46]. Uusitalo et al., [47] discovered proton radioactivity from the closed neutron shell nucleus ^{155}Ta utilising the p4n fusion evaporation channel and a ^{58}Ni beam on a ^{102}Pd target.

Livingston et al., [48] discovered proton emission from the new isotope ^{146}Tm and identified two transitions at energies of 11195 keV and 11895 keV, corresponding to Q-values of 11275 keV and 11975 keV, respectively. Sonzogni et al., [49] detected fine structure in the ground-state proton radioactive decay of severely deformed ^{131}Eu . Dehghani and Alavi present two empirical formulae for proton decay half-lives which included nuclear deformation. [50]. On the basis of the real-energy continuum shell model, Rotureau et al., [51] developed a theory of two-proton radioactivity. Bogdanov et al., [52] investigated proton decay of the ground states (PDGS) of the nuclei ^{121}Pr and ^{117}La within the context of the many-particle proton radioactivity theory. Honkanen et al., [53] discovered beta-delayed emission of two protons in the decay of ^{26}P and estimated a two-proton summed energy group of 4.914 MeV to the decay to the ^{24}Mg ground

state after the superallowed beta-decay of ^{26}P . Fengzhu Xing et al., [54] extended the unified fission model to explore two-proton radioactivity of nuclei's ground states, as well as two-proton radioactivity of excited states of ^{14}O , $^{17,18}Ne$, ^{22}Mg , ^{29}S , and ^{94}Ag . Balasubramanyam and Arunachalam Calculated the half-lives of different spherical proton emitters using unified fission model [55]. Oudih et al., [56] investigated the proton decay of spherical proton emitters from the ground and isomeric states in the WKB approximation by using the unified fission model with a Modified-Woods–Saxon (MWS) nuclear potential. In grand unified theories Rubakov detailed the theoretical studies of proton decay monopole catalysis. [57].

1.4 Experimental studies

In an experiment conducted at ISAC-TRIUMF, Ayyad et al., [58] reported delayed proton decay in ^{11}Be by directly quantifying the released protons and their energy distribution for the first time with the prototype Active Target Time Projection Chamber. Delion et al., [59] investigated the two-proton decay process using a basic technique based on scattering theory, assuming that the decaying nucleus is in a pairing state and that the two-particle wave function on the nuclear surface corresponds to the two protons travelling in time-reversed states. Jinter et al., [60] investigated the Proton emission from ^{150}Lu using Recoil Mass Spectrometer at Holifield Radioactive Ion Beam Facility. M.G. Procter et al., [61] used gamma-ray coincidence techniques to investigate proton emission from an oblate ^{151}Lu nucleus and determined the lifetime of the first excited state above the proton-emitting ground state using the recoil-distance Doppler-shift approach coupled with recoil-decay tagging. C.R. Bain et al., [62] investigated two proton emission by bombarding a radioactive beam of ^{13}N ions with a $(CH_2)_n$ target to fill a narrow resonance in ^{14}O at 7.77 MeV.

^{54}Zn was discovered for the first time, and its decay via two-proton emission was seen for the first time. In an experiment at GANIL's SISSI/LISE3 facility, Blank et al., [63] made the first de-

tection of the nucleus ^{54}Zn in the quasi-fragmentation of a ^{58}Ni beam at 74.5 MeV/nucleon in a nat Ni target. Lund et al., [64] studied the beta-delayed proton emission from ^{20}Mg at ISOLDE, CERN. Particle radioactivity was discovered in the closed neutron shell nucleus ^{155}Ta by Uusitalo et al., [65] by utilising a ^{58}Ni beam on a ^{102}Pd target in the p4n fusion evaporation channel. Canchel et al., [66] in a GANIL LISE3 experiment, produced radioactive ^{27}S isotopes by projectile fragmentation of a 95 AMeV ^{36}Ar primary beam and measured the half life and main decay branches of the isotope of interest by implanting in a silicon-detector telescope after selection with the LISE3 separator.

Giovinazzo et al., [67] explored the decay of the proton drip line nucleus ^{45}Fe in an experiment at GANIL's SISSI-LISE3 facility. Bertram Blank and Marek Loszajczak looked into the experimental findings that led to the discovery of two-proton radioactivity, as well as experimental investigations of two-proton emission from excited states inhabited by nuclear decay or inelastic processes [68]. Chong Qi et al., [69] described the systematics of proton decay half-lives and found that the proton formation probability is a relevant parameter for determining the mother nucleus's deformation property.

Raciti et al., [70] examined the decay of ^{18}Ne excited states via the simultaneous emission of two protons. ^{18}Ne nuclei, produced at the FRIBs facility of the Laboratori Nazionali del Sud (LNS). Coniglione et al., [71] explored the generation mechanism of high energetic protons in heavy ion reactions close to the Fermi energy in an experiment done with the MEDEA detector. Ludewigt et al., [72] investigated proton emission in α -induced reactions at 43 MeV/nucleon. At energies $E = 100$ MeV and 172 MeV, inclusive proton spectra and proton-proton correlations were observed from $+^{58}\text{Ni}$ and $+^{197}\text{Au}$ reactions, respectively. Riisager et al., [73] experimentally studied beta -delayed proton emission from ^{11}Be nucleus using accelerator mass spectrometry on a sample taken at CERN's ISOLDE facility (AMS). Shi et al., [74] explored the β -delayed

two-proton ($\beta 2p$) decay of ^{27}S utilising a cutting-edge silicon array and Clover-type HPGe detectors. Robinson et al., [75] by blasting a ^{96}Rn target with a 297-MeV ^{58}Ni beam, ^{150}Lu was created through the $1p3n$ fusion-evaporation channel, and evaporation residues were examined using the fragment mass analyzer at Argonne National Laboratory. Batchelder et al., [76] witnessed the proton emission from ^{145}Tm for the first time through the $^{92}Mo (^{58}Ni, p 4n)$ reaction using Holifield Radioactive Ion Beam Facility Recoil Mass Spectrometer. Lis et al.,[77] explored the decay of ^{31}Ar , which was created by fragmentation of a ^{36}Ar beam at 880 MeV/nucleon.

Faux et al., [78] determined the half-life of ^{52}Ni and the energies of β -delayed protons released during the decay by fragmenting a ^{58}Ni beam at 68 MeV/nucleon on a nickel target using the LISE spectrometer at GANIL. Cable et al., [79] discovered beta-delayed two-proton radioactivity for the two nuclei ^{22}Al and ^{26}P , revealing that the major two-proton emission mechanism is a sequential process from Proton proton coincidence studies done at small and large angles. Sun Li-Jie et al., [80] examined beta-delayed proton decay mode by conducting a beta-delayed proton emission experiment using $^{36,37}Ca$ under a high-intensity continuous-beam mode provided by the Radioactive Ion Beam Line in Lanzhou.

Page et al., [81] detected proton radioactivity in ^{160}Re and ^{156}Ta by mass separating in flight evaporation residues from fusion reactions of 300 MeV ^{58}Ni ions with ^{106}Cd targets and implanting them onto a double-sided silicon strip detector. Ascher et al., [82] observed the two protons released in the decay of ^{54}Zn for the first time in a temporal projection chamber. Pomorski et al., [83] investigated the decay of the neutron-deficient ^{48}Ni using an image time-projection chamber that enabled the tracking of charged particle tracks. Azhari et al., [84] examined the proton-unbound nucleus ^{11}N using kinematic reconstruction of the released proton in conjunction with the remnant ^{10}C daughter nucleus.

The nuclei with decay energy greater than zero, that are above the drip line are proton unstable

and also they are proton rich nuclei [85]. Jackson et al., [86, 87] investigated proton radioactivity from the isomeric state of ^{53}Co . Till date, different nuclear decay modes such as α , $\beta^\pm$, electron capture, spontaneous fission, proton radioactivity, cluster radioactivity and many more have been identified both theoretically and experimentally. From past two decades an attention has also given for heavy particle radioactivity [88–90]. The two proton radioactivity was experimentally confirmed from the proton rich nuclei ^{48}Fe [91, 92].

Both microscopic approach and macroscopic studies have given insight into theoretical prediction of one and two proton-decay. The microscopic approaches such as two-potential approach with Skyrme-Hartree-Fock [93], DDM3Y and JLM [94, 95] real-energy continuum shell model (SMEC) [51], M3Y effective interaction [96], R3Y nucleon-nucleus interaction potential [97], Skyrme-Sly4 interaction [98], Hartree-Fock-Bogoliubov [99], similarity renormalization group (SRG) [100] and Skyrme force of SkC and SkD [93]. The skyrme interaction is an important phenomenon which explains the ground state properties of nuclei.

Further, the macroscopic models such as preformation cluster model by Gupta et al., [101], unified fission model [102], effective liquid drop model [14, 103], generalised liquid drop model [2], Coulomb and proximity potential model [104–108], Geiger Nuttal law [109] for proton decay and within WKB penetration probability [101, 110, 111] the proton half-lives were investigated. Routray et al., [112, 113] investigated proton decay half-lives in ^{113}Cs to ^{185}Bi nuclei. Chen et al., [109] proposed two-parameter formula for proton radioactivity using Geiger–Nuttall law. Using various proximity potentials earlier researchers [114] have studied proton decay within generalised liquid drop model.

The proton radioactivity has been experimentally observed in the fusion reaction of $^{58}\text{Ni} + ^{96}\text{Ru} \rightarrow ^{154}\text{Hf}$ [115]. Earlier researchers [116] have studied the formation probability in one proton radioactivity. Two proton nuclei ^{113}Cs and ^{109}I [117] is experimentally produced by bombarding

^{58}Ni and ^{54}Fe on ^{58}Ni beams. Mukha et al., [118] experimentally investigated one proton decay from the 21^+ isomer state of ^{94}Ag in to ^{93}Pd nuclei. Later Aggarwal [119] theoretically proved the proton radioactivity of ^{94}Ag using macroscopic–microscopic approach which is in good agreement with the experimental value of Mukha et al., [118]. Further, Roeckl et al., [120] studied one and two proton radioactivity in ^{94}Ag . The proton radioactivity half-life of 17 seconds were experimentally observed when ^{53m}Co decays to ^{52}Fe [121]. Proton decay half-lives are investigated both in ground and isomeric states of the nuclei [122]. Poli et al., [123] experimentally measured proton and alpha decay half-lives from ^{185}Bi . Later, using self-consistent relativistic density functionals [32] proton radioactivity of odd-odd nuclei were studied. Within covariant density functional (CDF) theory [124, 125], the properties of proton emitters were investigated. Previous works [126–129] shows investigation of proton decay in actinides.

The present work investigates the half-lives of proton radioactivity using various macroscopic models such as Coulomb and proximity potential model (CPPM), effective liquid drop model (ELDM), generalised liquid drop model (GLDM), universal decay law for proton emission, Gamow-like model and unified fission model for proton radioactivity. The values produced using macroscopic models is compared with the different microscopic models available in literature.

The study of the radioactive decay is crucial importance for the further development of nuclear physics. Proton radioactivity studies are becoming important tool for nuclear structure [130]. Two-proton radioactivity is the emission of a pair of protons from a nuclear ground state. Giovinazzo et al., [92] studied the two proton radioactivity of ^{45}Fe and fragment-implantation events that correlate with radioactive decay events. Anguiano et al., [131] investigated the photo-emission of two protons from ^{12}C , ^{16}O and ^{40}Ca nuclei for the study of short range correlations. Raciti et al., [132] measured the emission of two protons from the decay of ^{18}Ne . Goldanskii [133] experimentally proposed one and two proton radioactivity for both odd and even atomic number. Mukha et al.,

[38, 134] observed one and two-proton radioactivity of more than 21 high-spin isomers in ^{94}Ag . Previous workers [135, 136] experimentally studied one and two proton radioactivity of ^{45}Fe and ^{54}Zn . The two-proton radioactivity was experimentally observed in ^{45}Fe [67, 91] and later in ^{19}Mg [137, 138], ^{48}Ni [83] and ^{54}Zn [63]. Davids et al., [139] identified the proton radioactivity from highly deformed nuclei (^{141}Ho and ^{131}Eu). Goigoux et al., [140] observed two proton emission for ^{67}Kr . The two proton radioactivity was experimentally observed [91, 115, 141, 142] in ^{58}Fe to ^{151}Lu .

Theoretically, many models have been used to predict one and two proton decay. Giusti et al., [143] studied the two proton emission in electron induced reactions within the theoretical framework. Furthermore, the two proton decay was estimated in the region $18 < A < 68$ [144] by the quantum mechanical tunneling mechanism through a potential barrier. In addition, ELDM was also used to evaluate two proton radioactivity of nuclei of mass number $A < 70$ [145]. The proton decay process can be treated as quantum tunneling process which passes through a potential barrier like an α -decay, is evaluated using Wentzel–Kramers–Brillouin approximation method. Theoretical models of proton radioactivity from spherical nuclei can predict the systematics of proton decay and spectroscopic factors.

Dossat et al., [146] studied the decay of the two proton rich nuclei ^{45}Fe and ^{48}Ni . Cui [147] first extended the GLDM to study the two proton radioactivity half lives of the ground state of nuclei. Grigorenko et al., [148] studied the two proton radioactivity using the three body model. The investigation of proton radioactivity was carried out using several models such as density-dependent M3Y effective interaction [94, 149, 150], unified fission model [55, 102], the Jeukenne, Lejeune and Mahaux (JLM) interaction [94], the cluster model [17], the CPPM for deformed nuclei [34], the deformed density-dependent model [151, 152], the Gamow-like model [10], the covariant density functional theory [153], the analytic formula [154], the distorted-wave Born ap-

proximation [19], the two-potential approach [19], and so on [7, 24, 155].

Most of the previous studies were focused on range of nuclei $22 \leq Z \leq 30$ [156–159]. Earlier researchers [107, 126, 127] have studied the proton radioactivity in the actinide and pre-actinide region. Spherical proton emitters were studied using the GLDM [18], one and two proton emitters [160] were identified in the Lanthanides and pre-actinide region. Half-lives in the heavy and superheavy region were evaluated using different theoretical models such as Modified Generalised Liquid Drop Model, Coulomb and Proximity Potential Model, and Effective Liquid Drop Model [110, 111, 161–169].

Royer et al., [170, 171] presented the GLDM which includes quasi molecular shapes and nuclear proximity energy. By adding different proximity potentials, pre-formation factor with isospin parameter, size of cluster and daughter nucleus and the modified pre-formation factor GLDM is modified and is referred as MGLDM [172–174]. Goncalves and Duarte [175] introduced ELDM, a super asymmetric fission model, to analyse α -decay, proton emission, cluster radioactivity, and cold fission in a single framework. ELDM is validated by many experiments [31, 175–180].

Two proton radioactivity arises for element isotopes with substantial negative proton separation energies. Because these protons must tunnel through a combined Coulomb and orbital angular momentum barrier, their half-lives are very sensitive to proton energy and angular momentum. Proton emitters have shorter half-lives because they are far from stability. As a result, an effort was undertaken to investigate one and two proton radioactivity in the atomic number range $3 \leq Z \leq 126$ using well-known CPPM, ELDM, and MGLDM models. Furthermore, semi empirical formula was constructed for one and two proton decay half-lives.

Light and medium nuclei mostly show proton decay whereas lanthanides show the proton and β decay. Furthermore, heavy nuclei ($Z = 72 - 88$) show β^+ and β^- decay, actinides ($Z = 89 - 103$) and superheavy nuclei or transactinides decay through α particles. It is also predicted that

nuclei with $Z \geq 126$ may undergo cluster decay / exotic decay [181]. The competition between decay modes depends sensitively on the Coulomb and centrifugal barriers. Proton radioactivity studies provide a unique insight into the structure of nuclei beyond the drip line limit [39]. Proton emission from long-lived excited states has been investigated since the 1970's in ^{53m}Co [86]. Subsequent discoveries of proton decay from ^{151}Lu [115], ^{113}Cs , ^{109}I [117] and eventually other exotic heavy isotopes like ^{117}La [182] and ^{135}Tb additional measurements.

The Lanthanide series includes 14 elements having atomic numbers from 58 to 71. The lanthanides and their analogs find importance in radiotherapy due to their physical properties physical half-life, type(s) of decay emission(s), energy of the emission, cost and availability, and specific activity [183]. Few studies have been devoted towards different decay modes of actinides, lanthanides and transactinides [107, 184–188]. Sridhara et al., [163] studied the cluster radioactivity in actinide nuclei. Quadrelli et al., [189] analyzed the quadratic decay observed for Ln(III) ionic radii and calculated bond distances and lanthanide atomic orbital expectation values. Nitscke et al., [190] identified a total of 24 new β - delayed proton precursors and several new decay branches in the region of $56 < Z < 72$ and $N < 82$ using OASIS online mass separator facility. Davids et al., [139] identified proton decay from ^{141}Ho and ^{131}Eu . Sowmya et al., [110] studied the competition between different decay modes such as binary, ternary, cluster radioactivity and alpha decay of Darmstadtium. Although different decay modes are explained for few series of actinide and the heavy elements, the lanthanide series yet to be explored. The presence of high coulomb barrier for heavy nuclei ($Z > 52$), reduces the proton barrier penetration probability to the extent that proton decay taking place from the ground states of nuclei have measurable long half lives [107]. There are various methods to investigate the proton radioactivity such as the density – dependent M3Y effective interaction [94, 152], the Jeukenne-Lejeune-Mahaux (JLM) interaction [94], the unified fission model [55, 102], the generalized liquid drop model [18], the cluster

model [17], the deformed density model [10], the coulomb and proximity potential model [34], the covariant density functional theory [153] and so on. These nuclear proximity potentials provide the phenomenological potentials for nuclear reaction and structure including nuclear decay [191]. Santhosh et al., [34] explained the half life predictions for the proton emitters with $Z > 50$ in the ground state and isomeric state using Coulomb and proximity potential model for the deformed nuclei. Dong et al., [192] studied the α -decay using double-folding potentials from chiral effective field theory for the nuclei ^{104}Te .

The coulomb and nuclear proximity potential provides another simple and practical formalism to estimate the strength of the nuclear interactions during collision of heavy-ions. When two surfaces are approaching each other, approximately at a distance of 2–3 fm, an additional force due to the proximity of surfaces will appear which is called proximity potential [193]. There are adjustable parameters in various parts of the proximity formalism such as the radius parameter R , the surface energy coefficient and the universal function which lead to introduce different versions of the proximity potentials. Santhosh et al., [194] studied the Coulomb and proximity potential as interacting barrier for post-scission region and calculated half – life time for different modes of exotic decay treating parent and fragments as spheres and these values are compared with experimental data. Dutta et al., [195] performed a detailed comparative study of fusion barriers for asymmetric colliding nuclei using different versions of phenomenological proximity potential as well as other parameterizations within the proximity concept. From the detailed analysis of literature, it is found that, there is no systematic study of proton decay in the lanthanide region. The aim of the study is to predict the unexplored proton emitters in the lanthanide region.

Proton decay is rare type of decay which can exist in following ways, in high lying excited states of a nucleus after beta-decay or from ground state of very rich proton nuclei or nucleus with odd atomic numbers beyond proton drip line [196]. Buck et al., [7] investigated proton emission from

the ground state of heavy nuclei using charged particle emission model. Proton decay, half-life and branching ratio measurements aid in determining the angular momentum ℓ [197–199]. The spherical proton decay half-lives are studied using two potential approach, quasi-classical methods and distorted wave Born approximation (DWBA) [19]. Guzman et al., [16] investigated proton radioactivity in proton rich nuclei in heavy nuclei using effective liquid drop model(ELDM). Using CPPM (Coulomb and proximity model), Santhosh et al., [34] predicted proton decay half-lives in heavy nuclei $Z > 50$.

Under experimental approach Belli et al., [200] observed one proton decay and β^+ -decay in Dysprosium using γ detector. Schardt et al., [201] studied beta delayed proton emission in Dysprosium. A microscopical variational method is developed resulting in a Hamiltonian for the non-adiabatical particle rotator model, where numerical calculations have been performed for the positive parity bands in the $^{157,159,161}\text{Dy}$ [202]. The ft-values for the Gamov-Teller β^+ decay of $^{148,146}\text{Dy}$ is calculated in the random phase approximation taking into account of interactions in particle-hole and particle-particle channels [203]. β^+ -decay strength functions have been measured for the neutron-deficient isotopes of $^{149,151}\text{Dy}$ using self-consistent HF+RPA approach with Skyrme forces [204].

Earlier studies have shown detail analysis of proton radioactivity [107, 126, 127, 205]. Nuclear structural information can be extracted via proton radioactivity Dong et al., [18]. Many theoretical studies shows inclusion of different proximity potential and different theoretical models to evaluate half-lives [90, 111, 161, 173, 206, 207]. As a result of the thorough literature review, it is obvious that the sensitivity of experimental and theoretical approaches to proton decay in Dysprosium need improvement. As a result, we were inspired to investigate several decay modes in the current study such as β -decay, proton decay and α -decay half-lives of isotopes of Dysprosium in the mass number range $133 \leq A \leq 180$ and to determine the various decay modes of Dysprosium

nuclei.

In the energy domain of radioactivity, proton can be considered as a point charge having highest probability of being present in the parent nucleus. Gonclaves et al., [14] studied the two-proton radioactivity of nuclei of mass number $A < 70$ using the effective liquid drop model. Delion et al., [5] reviewed the theories of proton emission to analyse the properties of nuclear matter. Maglione et al., [24] analysed the proton emission half-lives from the deformed nuclei ^{109}I , ^{131}Eu and ^{141}Ho . Delsanto et al., [25] investigated the β -delayed proton emission of ^{69}Kr and ^{68}Se and extracted their proton separation energies, half-lives and excitation energies. Alavi et al., [26] calculated the proton radioactivity half-lives of 45 proton emitters by WKB Method and observed the decrease in values of calculated half-lives using the orientation angle dependent formalism. Raciti et al., [70] measured the emission of two protons from the decay of ^{18}Ne . Baye et al., [27] evaluated the decay probability per second for ^{11}Be , ^{19}C and ^{31}Ne one-neutron halo nuclei. Feix et al., [28] computed the decay widths of nuclear proton emission for $Z=51$ to 71 nuclei using Droplet Model. Anguiano et al., [208] investigated the photo-emission of two protons from the ^{12}C , ^{16}O and ^{40}Ca nuclei for the study of short range correlations. Coniglione et al., [71] explored high energy proton emission in heavy ion reactions close to the Fermi energy by investigating the production mechanism of energetic protons in an experiment performed with the MEDEA detector. Giusti et al., [29] developed the theoretical frame work of emission of two protons in electron induced reactions.

Ludewigt et al.,[72] studied the proton emission in α -induced reactions at 43 MeV nucleon. Guzman et al.,[16] analysed the proton emission from proton-rich nuclei and calculated the half-lives using the effective liquid drop model. Delion et al.,[6] proposed semi empirical formula for logarithmic half-lives of proton decay. Dong et al., [18] theoretically calculated the half-lives of proton emitters using generalized liquid drop model (GLDM) and WKB approximation. Maglione

et al., [209] studied the proton emission from ^{125}Pm and the behaviour of the half-lives were discussed as a function of deformation, spin of the decaying state, and energy of the emitted protons. Arumugam et al., [30] investigated the proton emission, gamma deformation, and the spin of the isomeric state of ^{141}Ho and revealed that proton deformations and other structural properties of exotic nuclei beyond the proton drip-line. Duarte et al., [31] explored the half-lives for proton emission, alpha decay, cluster radioactivity, and cold fission processes theoretically. Ferreira et al., [32] planned to study the proton radioactivity from spherical nuclei theoretically based on relativistic density functional derived from meson exchange and point coupling models. Ginter et al., [60] studied the proton emission from ^{150}Lu and new proton emitting state was observed. Delion et al., [20] investigated proton decay from tri-axially deformed nuclei ^{161}Re and ^{185}Bi and studied the dependence of angular distribution of decaying particle on triaxial deformation parameters. Earlier workers [15, 33, 67] studied one and two proton decay half-lives of ^{131}Eu , ^{45}Fe and also studied proton emission from the deformed nuclei. In the literature, different theoretical approaches are available [105–108, 161, 163–165, 169, 207, 210–215] to study different decay modes including proton decay.

Proton decay is one of the key predictions of various grand unified theories (GUTS) proposed in the 1970s, another major one being the existence of magnetic monopoles. Both concepts have been the focus of major experimental physics efforts since the early 1980s. The proton decay hypothesis was first formulated by Andrei Sakharov in 1967 [37]. During the year 1981 at GSI Darmstadt one proton (1P) ground decay was observed [216]. Half-lives of proton emission of nuclei such as ^{151}Lu , ^{53}Co and so on have been studied [196, 217]. Many theoretical models [7, 16, 19, 55, 150] have been made used to study 1P-decay. M.Pfutzner et al., [91] observed the decays of fine ^{45}Fe atoms at the fragment separator of GSI. Bajc et al., [218] systematically studied proton decay in the minimal super symmetric $\text{SU}(5)$ grand unified theory. Goldman and

Ross [219] predicted theoretical upper limit for proton decay. Two proton decay of ^{67}Kr is experimentally observed [140]. The life time of proton has been identified by earlier researchers [220]. Santosh and Indu sukumaran [34] theoretically predicted half-lives of proton emitters with the atomic number of $Z > 50$. The proton radioactivity has been studied using various proximity potentials [221]. Experimental evidence shows proton drip line of ^{45}Fe [67]. After bombardment of ^{92}Mo target nuclei with ^{50}C , Woods et al., [222] observed proton decay. Developmental theories of proton decay has been predicted by Maglione et al., [223]. Detail analysis of proton decay has been done by Rykaczewskiaet al.,[39]. Ferreira et al., [32] based on relativistic density functional theory, the proton radioactivity from spherical nuclei were studied.

Delion et al., [5] examined the characteristics of nuclear matter by reviewing proton emission hypotheses. Recent literature [107, 126, 129] also predicts proton emitters in the atomic number range $72 < Z < 88$ and actinides. Many theoretical studies shows the prediction of possible decay mode in the superheavy region [161, 164, 165, 213]. Hence, in the present work we made an attempt to study one proton radioactivity of Tantalum using different models such as Coulomb and proximity potential model(CPPM), effective liquid drop model (ELDM) and modified generalised liquid drop model (MGLDM).

During the last two decades, significant progress has been made in the experimental investigation of processes leading to superheavy nuclei, their decay properties and structure. The most stable superheavies are anticipated to be positioned along the β -stability line, which is unreachable by fusion reactions with stable beams. The literature studies shows the competition between different decay modes [173, 205]. The proton decay half-lives of lanthanides and actinides were studied [107, 111, 161, 224]. Qian et al., [225] systematically studied α -decay half-lives of heavy and superheavy elements. Tan et al., [226] investigated the β^+ decays of some medium-mass nuclei.

Many theoretical models have been proposed to explore the half-lives of spherical and deformed nuclei. Earlier workers [164] have studied different decay modes of superheavy nuclei. Hence, in the present work we have examined possible decay modes such as proton decay using Coulomb and Proximity potential Model (CPPM), $\beta^\pm$ -decay and an alpha decay are evaluated using semi-empirical relations in the isotopes of Bismuth.

Protons may be thought of as a point charge with the highest chance of being present in the parent nucleus in the energy domain of radioactivity. For both odd and even atomic numbers, Goldanskii [85] predicted one and two proton radioactivity in the mid-1960s. Mukha et al [38, 134] reported one- and two-proton radioactivity in ^{94}Ag in over 21 high-spin isomers. Routray et al., [112] used Yukawa effective interaction to calculate the half-lives of proton radioactivity. Deng et al., [221] investigated proton radio activity, α decay, and heavy particle radioactivity using various proximity potentials. At the RIKEN Nishina Center in 2015, prior researcher [227] evaluated proton radioactivity in ^{67}Kr . Two-proton radioactivity in ^{45}Fe was explored experimentally by Giovinazzo et al., [228] by the use of silicon detectors. Previous research [135, 136] looked at ^{45}Fe and ^{54}Zn radioactivity at one and two proton levels.

The systematics of proton decay and spectroscopic effects can be predicted using theoretical proton radioactivity models from spherical nuclei [229]. Santhosh et al., [34] used the Coulomb Proximity Potential Model for deformed nuclei to predict proton radioactivity in nuclei with $Z > 50$. Using the newly constructed velocity separator SHIP, which was created at GSI, Darmstadt, the researchers [230] identified proton active nucleus outside the proton drip line. The proton decay of ^{108}I was discovered experimentally by Auranen et al., [231]. Alavi et al., [26] used the WKB approach to calculate proton decay half-lives. Previously, researches [232, 233] revealed about proton decay emission in actinides and alkaline metals. Pfutzner et al., [234] investigated the proton emission phenomena of odd Z nuclei and discovered extensive structural data. The two-

proton radioactivity was experimentally observed in ^{45}Fe [67, 91], and later in ^{19}Mg [137, 138], ^{48}Ni [83], and ^{54}Zn [63]. The proton radioactivity was experimentally detected by Jackson [86] in 1970 by detecting proton emission from ^{53}Co to the ground state of ^{52}Fe . Many theoretical models for studying the half-lives of spherical and deformed nuclei have been proposed since then. [69, 122, 223, 235–239]. Previously, researchers [108, 161, 164, 165, 169] used various proximity potentials to evaluate the half-lives of superheavy nuclei. The search for proton emission nuclei will lead to the determination of nuclear stability in proton-rich nuclei. Although proton emission is a challenging process, the simplified way of one proton overcoming the coulomb barrier will illustrate the process quite well. Proton emissions have the lowest coulomb potential and the least decreased mass when compared to the other decay types. According to the literature, a comprehensive investigation of one-proton decay half-lives in the actinide area is necessary. The study of proton decay half-lives in the actinide area is the major goal of this research. Proton radioactivity was studied using the coulomb and proximity potential models, which have been used for alpha and cluster decay for many years. The study predicts that in future experiments, proton unstable nuclei close or outside the proton drip line would be detected. The half-lives of proton emitters in the actinide region that had not yet been observed experimentally are predicted.

The nuclei beyond the proton drip line with $Q_p > 0$ are the one with proton unstable and also exhibit exotic decay modes. The understanding of the proton decay is important to study the nuclear structure. The exotic nuclei exists away from the stability. The binding energy of protons above the drip line gradually decreases and hence one-proton and two proton decay is predicted. Brown [240] studied two proton decay in $Z=22-28$ in the ground state. Goldanskii [85] for the first time studied the one proton and two proton decay for odd and even atomic number. Janecke [241] studied the emission of protons from the light nuclei $^{12,13}\text{O}$, ^{21}Mg and $^{24,32}\text{Si}$. The spherical proton and deformed proton emitters were investigated in lanthanides and transition metals. Previous

workers [63, 67, 83, 86, 91, 135–138, 227, 228, 231] experimentally observed one and two proton decay in proton rich nuclei. There are several theoretical models [38, 112, 134], studied one proton and two proton activity in light nuclei.

Using different proximity potentials previous workers [34, 221, 229] studied proton activity in the light nuclei. The emission of heavy particles such as one proton, one neutron, two protons, two neutrons and alpha particle emission takes place when the nuclei are proton rich, neutron rich and very heavy nuclei. Successively many theoretical models [69, 122, 223, 235–239] were presented to study the half-lives of spherical and deformed nuclei. Dobaczewski and Nazarewicz [232] studied two-proton stability in doubly magic nuclei ^{100}Sn using self-consistent Skyrme-Hartree-Fock-Bogoliubov theory. Olsen et al., [233, 233] investigated two-proton decay in even-even nuclei and also studied competition between proton decay and alpha decay. Poenaru et al., [242] measured half-lives and branching ratios for ^{12}C , ^{16}O and ^{28}Si and proton and neutron rich nuclei with $Z=56-64$.

The observations of proton decay is quite recent, they are several approaches to study this proton decay process, such as distorted-wave Born approximation [19], the study of effective interaction by the density dependent M3Y (DDM3Y) [94, 150]. The construction of proton nucleus potential by Jeukenne, Lejeune and Mahaux (JLM) applied to finite nuclei in the Local Density Approximation [55], the unified fission model [5], the coupled-channels approach [16] and also generalized liquid drop models [18, 243, 244]. Earlier workers [108, 111, 161, 164, 165, 169, 210, 213, 245] studied half-lives of spontaneous fission, ternary fission, cluster decay and alpha decay in the superheavy region using different proximity potentials. Faestermann [117] experimentally observed proton decay half-lives and proton energies in ^{113}Cs and ^{109}I . Sellin [246] experimentally measured proton decay half-lives in ^{150}Lu , ^{151}Lu and ^{147}Tm . Page et al., [247] reported proton emitter ^{112}Cs with the half-life of $500\pm 100\mu\text{s}$. Livingston [248] experimentally ob-

served proton emission from the ^{146}Tm . The two proton radioactivity [63, 67, 83, 91, 137, 138] was experimentally observed ^{45}Fe , ^{19}Mg , ^{48}Ni and ^{54}Zn . In the year 1970, Jackson [86] confirmed proton radioactivity from the proton emitter ^{53}Co .

The proton radioactivity is applied for nuclear astrophysics. In the nuclear astrophysics, the process of two-proton radiation capture process is considered, which is important for extremely high densities and temperatures. The example of such an astrophysical environment is the sources of gamma bursts related with the explosive burning of deposited hydrogen on the surface of neutron stars. Previous workers [249–251] explained the astrophysical applications of the two-proton radioactivity.

From the available literature, the study on one proton emission in the actinide region is required. The study on the proton decay not only provides information about the drip line, but also provides spectroscopic information on the unpaired proton not substantial in its orbit. Hence, in the present work we want to emphasize on the possible proton emitters in the actinide region and also prediction of half-lives in the same region. The main objective is to systematically study one proton decay half-lives of spherical and deformed nuclei in the actinide region.

A Systematic study of proton decay in superheavy elements Goncalves et al., [145] studied two-proton radioactivity of nuclei of mass number $A < 70$ using the effective liquid drop model. Delion et al., [252] reviewed the theories of proton emission to analyse the properties of nuclear matter. Maglione et al., [253] analyzed the proton emission from the some deformed nuclei. Santo et al., [254] investigated the β -delayed proton emission of ^{69}Kr and ^{68}Se and extracted their proton separation energies, half lives and excitation energies. Alavi et al., [255] calculated the proton radioactivity half-lives of 45 proton emitters by WKB Method and observed the decrease in values of calculated half lives using the orientation angle dependent formalism. Raciti et al., [132] measured the emission of two protons from the decay of ^{18}Ne excited states. Baye and Tursuno [256]

studied that a proton is emitted during β decay of one neutron halo nuclei. Feix and Hilf [257] computed the decay widths of proton emission for $Z=51$ to 71 nuclei using droplet model potentials and spectroscopic data from shell model considerations. Anguiano et al., [131] investigated the photo-emission of two protons from the ^{12}C , ^{16}O and ^{40}Ca nuclei for the study of short range correlations. Coniglione et al., [258] explored high energy proton emission in heavy ion reactions close to the Fermi energy by investigating the production mechanism of energetic protons in an experiment performed with the MEDEA detector.

Giusti and Pacati [143] developed the theoretical frame work of emission of two protons in electron induced reactions. Ludewigt et al. [259] studied the proton emission in alpha induced reactions at 43 MeV nucleon. Guzman et.al. [260] analyzed the proton emission from proton-rich nuclei and calculated the half lives using the effective liquid drop model. Delion et al. [261] also studied the proton emission. Dong et al [2] theoretically calculated the half lives of proton emitters using generalized liquid drop model (GLDM) and WKB approximation. Maglione and Ferreira [262] studied the proton emission from ^{125}Pm and the behaviour of the half lives were discussed as a function of deformation, spin of the decaying state, and energy of the emitted protons. Arumugam et al., [263] investigated the proton emission , gamma deformation, and the spin of the isomeric state of ^{141}Ho and revealed that proton emission measurements could be a precise tool to probe triaxial deformations and other structural properties of exotic nuclei beyond the proton drip-line. DUARTE et al., [264] studied the half-lives for proton emission, alpha decay, cluster radioactivity, and cold fission processes theoretically. Ferreira et al., [265] also studied the proton radioactivity from spherical nuclei theoretically based on relativistic density functional derived from meson exchange and point coupling models. Also previous researchers studied on the decay modes[107, 111, 126, 129, 163, 181, 207, 207, 266–270]. From study of literature survey, it reveals that there is a lack of study on proton emission from superheavy nuclei. Superheavy

nuclei is unstable and it decay through different decay modes. Hence the present work studies the systematics of proton emission from superheavy nuclei.

In the line of stability, the excess protons still adequately bound to the nucleus with the nuclear forces, hence direct emission of proton is not possible. However, while beyond the line of stability the protons are no longer bound by the nuclear forces. In order to study the proton emission beyond the stability line Conclaves et al., [14] studied two-proton radioactivity in the mass number $A < 70$ using liquid drop model. Earlier [6, 24] studied proton emission from the deformed nuclei. One proton, two proton, β decay [25–28, 70] were studied using droplet model and WKB approximation. Giusti et al., [29] established theoretical frame work for the emission of two protons in electron induced reactions. Using generalized liquid drop model and WKB approximation, Dong et al., [18] theoretically studied proton decay half-lives of spherical proton emitters.

Previous workers [29, 72] theoretically studied half-lives of proton radioactivity. Earlier workers [108, 161, 164, 165, 169] were studied ternary fission, binary fission, cluster radioactivity and alpha decay in the superheavy region using different proximity functions. From the available literature, it is essential to study the proton radioactivity in the Dubnium. Hence, in the present work first attempt was made to study proton radioactivity in the isotopes of Dubnium.

1.5 Objectives of research work

- Construction of semi empirical formula for proton decay half-lives in the complete range of medium, heavy and superheavy nuclei region.
- A detail study of proton radioactivity using effective liquid drop model (ELDM), generalized liquid drop model (GLDM) and modified generalised liquid drop model (MGLDM) models.
- To identify the dominant decay mode by comparing proton decay with other decay modes such as alpha decay, beta decay, and spontaneous fission in Lanthanide, Actinide and Super-

heavy region.

- Identification of new proton emitters among the Lanthanides, Actinides, heavy and Super-heavy nuclei.
- The comparison of proton decay half-lives calculated using macroscopic models with that of the microscopic models.

CHAPTER 2

Semi empirical formula for proton radioactivity half lives

2.1 Construction of Semi-empirical formula for one and two proton decay

The knowledge of accurate Q-values helps to analyse the 1p and 2p radioactivity in the atomic number region $3 \leq Z \leq 126$. In order to evaluate the Q-values, the mass excess values available in the literature [271] are taken. The Q-values of 2p and 1p radioactivity are selected in such way that the $Q_{2p,1p} > 0$.

The selection of angular momentum and centrifugal barrier is more important in proton decay. The effect of reduced mass is smaller when compared to angular momentum and the proton, in most instances, has a non-vanishing angular momentum.. The values of angular momentum accompanying with the one and two proton radioactivity is deduced from the spin and conservation laws as described. In addition one and two proton decay half-lives are compared with that of $\beta^\pm$ -decay [107], α -decay [161] and spontaneous fission [107]. Among these different decay modes, the possible decay modes were compared and finally predicted pure one and two proton emitters.

However, the different microscopic and macroscopic models such as folding model analysis [96], relativistic mean field approach [96], modified preformed cluster model [272], effective liquid drop model [14] in which either preformation probability is taken as unity or majority of the cases in which the effect of spectroscopic factor were not considered. It is also evident from the literature that the MGLDM was used to evaluate cluster-decay [273], alpha-decay [274] and heavy

particle radioactivity [90] but not for proton-decay half-lives. Even though, spectroscopic factor is an important key factor and it is model dependent, but these macroscopic models also predicts half-lives close to experimental values. The significant correlation between $\log T_{1/2}$ values of α -decay and amount of energy released (Q) was successfully explained by Geiger and Nuttall [275]. Also tried to construct the semi empirical formula for one proton radio activities using the Geiger-Nuttall law.

Proton decay half lives are evaluated using well accepted models such as MGLDM [274], ELDM [239] and CPPM [276]. Even-though, all the three models have its own physical significance in reproducing the experimental half-lives more accurately and precisely but, CPPM half-lives are considered for construction of semi empirical formula. Since, the standard deviation obtained using CPPM model is smaller when compared to MGLDM and ELDM models i.e the half-lives predicted using CPPM are in close agreement with that of experiments. Hence, it is appropriate to consider the CPPM half-lives to fit a semi-empirical formulae. As a consequence, half-lives obtained using CPPM in an unexplored nuclei may also predict the half-lives more accurately. In this view, we have considered half-lives produced by CPPM. The logarithmic half-lives of 1P radioactivity as a function of atomic number of daughter nuclei, angular momentum and amount of energy released and it is as follows;

$$\log T_{1/2}(1P) \propto f(Z_D, \ell, Q) \quad (2.1)$$

The function $f(Z_D, \ell, Q)$ is evaluated by studying the variation of $\log T_{1/2}(1P)$ as a function of $f(Z_D, \ell, Q)$. To derive suitable empirical formula for logarithmic half-lives of 1P radioactivity, it is assumed that $\log T_{1/2}(1P)$ is directly proportional to Z_D^x and inversely proportional to $\sqrt{Q}$ and

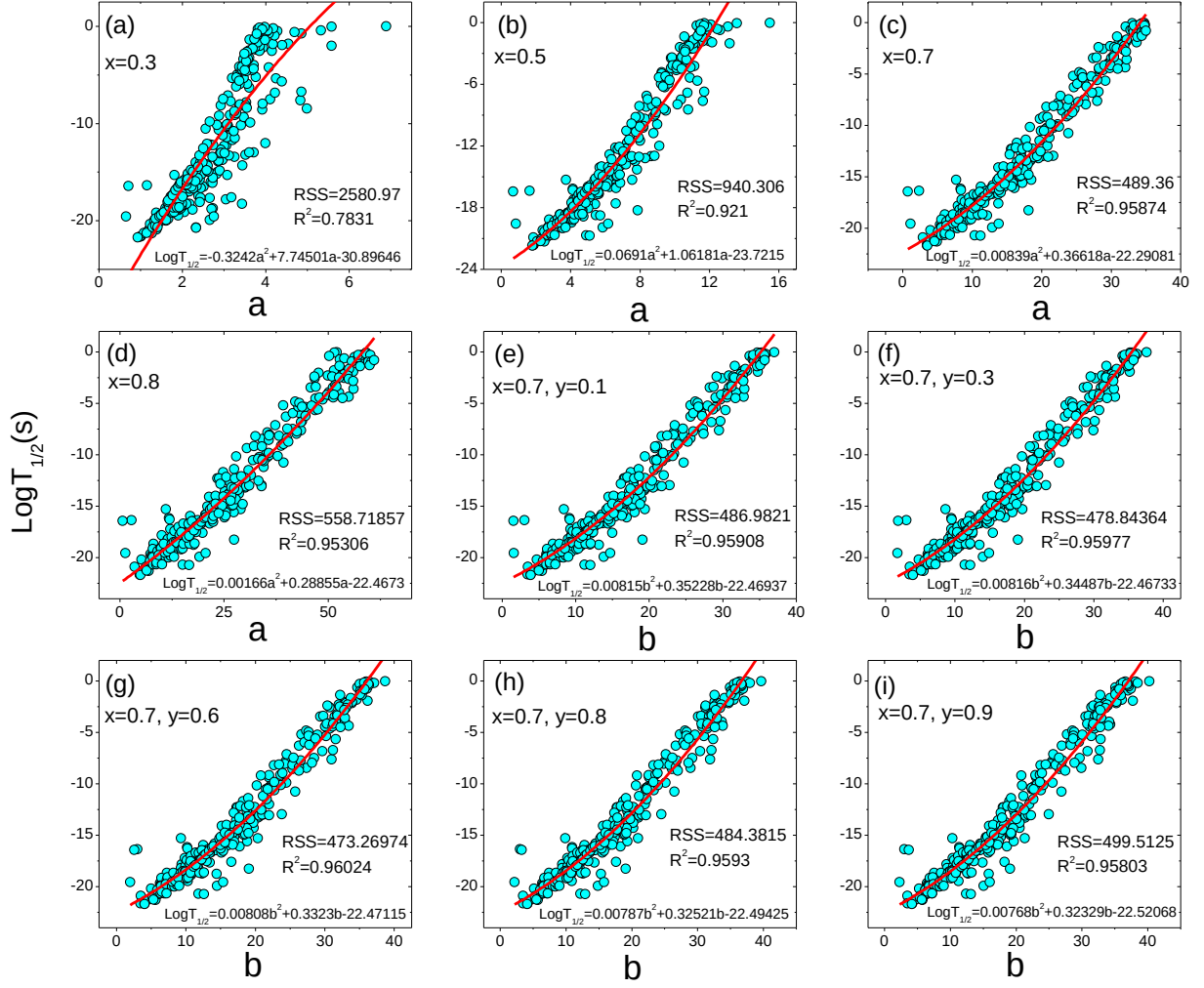


Fig. 2.1 (a-d): The variation of $\log T_{1/2}$ during one proton radioactivity as a function of a where $Z_D^x/\sqrt{Q}$. (e)-(i): The variation of $\log T_{1/2}$ during one proton radioactivity as a function of b where $(Z_D^{0.7} + \ell^y)/\sqrt{Q}$.

it is as follows;

$$\log T_{1/2}(1P) = f(Z_D^x/\sqrt{Q}) \quad (2.2)$$

We tried many functions such as $A_1a + A_2/a^2 + A_3$, $A_1a + A_2a/(a - A_3)$, $A_1/a + A_2 + A_3/a^2$, $A_1\ln(a - A_2) + A_3$, $A_1\exp^{A_2}/(a - A_3\ln a)$, $A_1\ln(a) + A_2$, $\frac{1}{(A_1\ln Z + A_2)}$ and polynomial functions such as $A_4a^4 + A_3a^3 + A_2a^2 + A_1a + A_0$. Among all the studied functions, we have considered polynomial function whose residual sum of squares (RSS) is minimum value and coefficient of determination is $R^2 \approx 1$. The figure 2.1(a) to (d) shows the variation of $\log T_{1/2}$ of the one

proton emission with $Z_D^x/\sqrt{Q}$ for different values of $x=0.3, 0.5, 0.7$ and 0.8 respectively. Among the studied functions, the function with $x = 0.7$ shows smaller RSS and larger R^2 value. The effect of angular momentum plays a major role in the proton decay and from the literature [13, 109, 180] many empirical relations included the effect of angular momentum. The inclusion of angular momentum significantly effect the RSS and R^2 value. Dehghani and Alavi [277] has also shown that the inclusion of deformation, angular momentum and Q-values reduces the rms value. Hence, in order to obtain accurate half-lives, the angular momentum is included in the fitting of the semi-empirical formulae. Later by keeping $X = 0.7$, the effect of angular momentum included by considering the $(Z_D^{0.7} + \ell^y)/\sqrt{Q}$. The maximum value of coefficient of determination $R^2 = 0.96024$ is observed when $y = 0.6$. The final constructed semi empirical formula for one proton radioactivity is as follows;

$$\log T_{1/2}(1P) = 0.00808 \left(\frac{Z_D^{0.7} + \ell^{0.6}}{\sqrt{Q}} \right)^2 + 0.3323 \left(\frac{Z_D^{0.7} + \ell^{0.6}}{\sqrt{Q}} \right) - 22.47115 \quad (2.3)$$

where Z_D , ℓ and Q are the atomic number of daughter nuclei, angular momentum and amount of energy released during one proton decay respectively.

To obtain further insight into systematics of two proton radioactivity, an attempt was made to construct semi empirical formula similar to one proton radioactivity by including the effects of atomic number, Q-values and angular momentum. The figure 2.2 (a) to (i) shows the variation of $\log T_{1/2}$ of two proton radioactivity with $a = Z_D^x/\sqrt{Q}$. The remaining layers of the same figure 2.2(g) to (i) depicts the variation of $\log T_{1/2}$ with $b = (Z_D^x + \ell^y)/\sqrt{Q}$. From this study, it is observed that the variation of $\log T_{1/2}$ with $(Z_D^x + \ell^y)/\sqrt{Q}$ is found to be more systematic at $x=0.7$ and $y=0.01$ and it is shown in the figure 2.2(g). The constructed semi empirical formula having

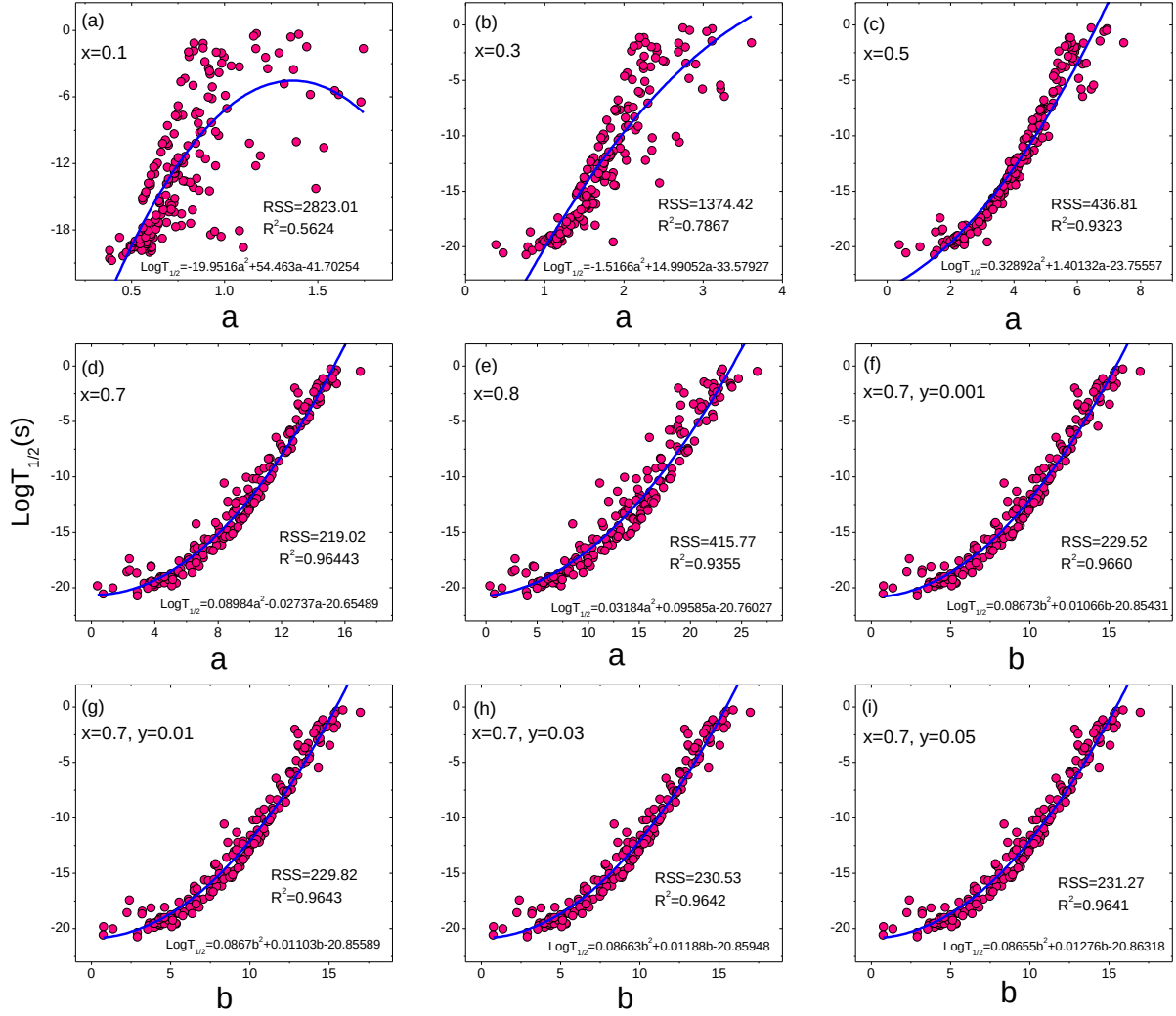


Fig. 2.2 (a)-(e): The variation of $\log T_{1/2}$ during two proton radioactivity as a function of $a = Z_D^x / \sqrt{Q}$. (f)-(i): The variation of $\log T_{1/2}$ during one two radioactivity as a function of $b = (Z_D^{0.7} + \ell^y) / \sqrt{Q}$

maximum R^2 and minimum RSS for the two proton radioactivity is expressed as;

$$\log T_{1/2}(2P) = 0.08673 \left(\frac{Z_D^{0.7} + \ell^{0.001}}{\sqrt{Q}} \right)^2 + 0.01066 \left(\frac{Z_D^{0.7} + \ell^{0.001}}{\sqrt{Q}} \right) - 20.85431 \quad (2.4)$$

where Z_D , ℓ and Q are the atomic number of daughter nuclei, angular momentum and amount of energy released during two proton decay respectively.

1p and 2p radioactivity half-lives are evaluated for the nuclei whose experimental values are accessible using the proposed semi empirical formulae defined in the equations 2.3 and 2.4.

Table 2.1 Comparison of calculated half-lives with the different models.

Reaction	Q(MeV)	l_{min}	$LogT_{1/2}(s)$	
			PF	Models
${}^6Be \rightarrow {}^4He$	1.372	0	-20.67	-19.97 [145]
${}^7B \rightarrow {}^5Li$	1.42	0	-20.55	-19.55[145]
${}^8C \rightarrow {}^6Be$	2.111	0	-20.55	-19.62[145]
${}^{10}N \rightarrow {}^8B$	1.3	1	-19.70	-17.64[145]
${}^{12}O \rightarrow {}^{10}C$	1.638	0	-20.17	-18.27[145]
${}^{16}Ne \rightarrow {}^{14}O$	1.401	0	-19.68	-16.60[145]
${}^{12}O \rightarrow {}^{10}C$	1.82	0	-20.24	-19.46[147]
${}^{12}O \rightarrow {}^{10}C$	1.79	0	-20.23	-19.43[147]
${}^{12}O \rightarrow {}^{10}C$	1.8	0	-20.23	-19.44[147]

To test the predictive power of the constructed semi-empirical formula, we have evaluated the mean squared error. The evaluated mean squared error in predicting the half-lives corresponds to 1p and 2p radio activities are 0.75 and 0.53 respectively. Further, the values obtained from the present work is compared with that of two proton radioactivity available in literature for which semi-empirical fit is adopted and it is tabulated in table 2.1. The half-lives obtained from present formulae are nearly equal to the values proposed by the previous models. However, a change in the magnitude of two to three order has been observed from the present semi-empirical formulae. The values produced by the present semi-empirical formula is compared with that of experiments.

The table 2.2 shows the comparison of one proton radioactivity half lives produced by the present formula with that of experiments. From this comparison it is observed that the present formula sucessfully produces experimental half lives.

The table 2.3 shows the comparison of two proton radioactivity half lives produced by the present formula with that of experiments. The present formula also sucessfully produces two proton radioactivity half lives which are close to the experiments.

Table 2.2 Comparison of 1P decay logarithmic half lives obtained using present formula (PF) with that of experiments[109].

Reaction	Q_{1P}	$J_i \rightarrow J_f$	ℓ	$\log T_{1/2}(s)$	
				Exp	PF
$^{146}\text{Tm} \rightarrow ^{145}\text{Er}$	0.891	$1^+ \rightarrow 1/2^+ \#$	0	-0.81	-0.30 ± 0.63
$^{146}\text{Tm}^m \rightarrow ^{145}\text{Er}^m$	1.001	$5^- \rightarrow 11/2^- \#$	0	-1.12	-1.60 ± 0.42
$^{150}\text{Lu}^m \rightarrow ^{149}\text{Yb}$	1.291	$1^+, 2^+ \rightarrow 1/2^+$	0	-4.4	-3.47 ± 0.21
$^{157}\text{Ta} \rightarrow ^{156}\text{Hf}$	0.941	$1/2^+ \rightarrow 0^+$	0	-0.53	-0.32 ± 0.39
$^{160}\text{Re} \rightarrow ^{159}\text{W}$	1.271	$4^- \rightarrow 7/2^- \#$	0	-3.16	-2.73 ± 0.13
$^{161}\text{Re} \rightarrow ^{160}\text{W}$	1.201	$1/2^+ \rightarrow 0^+$	0	-3.36	-2.22 ± 0.34
$^{167}\text{Ir} \rightarrow ^{166}\text{Os}$	1.071	$1/2^+ \rightarrow 0^+$	0	-1.13	-0.64 ± 0.43
$^{171}\text{Au} \rightarrow ^{170}\text{Pt}$	1.448	$1/2^+ \rightarrow 0^+$	0	-4.65	-3.12 ± 0.32
$^{176}\text{Tl} \rightarrow ^{175}\text{Hg}$	1.261	$3^-, 4^-, 5^- \rightarrow 7/2^-$	0	-2.21	-1.64 ± 0.25
$^{177}\text{Tl} \rightarrow ^{176}\text{Hg}$	1.155	$1/2^+ \rightarrow 0^+$	0	-1.18	-0.68 ± 0.42
$^{185}\text{Bi}^m \rightarrow ^{184}\text{Pb}$	1.607	$1/2^+ \rightarrow 0^+$	0	-4.19	-3.45 ± 0.17
$^{147}\text{Tm}^m \rightarrow ^{146}\text{Er}$	1.12	$3/2^+ \rightarrow 0^+$	2	-3.44	-2.70 ± 0.21
$^{151}\text{Lu}^m \rightarrow ^{150}\text{Yb}$	1.291	$3/2^+ \rightarrow 0^+$	2	-4.78	-2.67 ± 0.44
$^{156}\text{Ta} \rightarrow ^{155}\text{Hf}$	1.021	$2^- \rightarrow 7/2^- \#$	2	-0.83	-0.68 ± 0.18
$^{166}\text{Ir} \rightarrow ^{165}\text{Os}$	1.161	$2^- \rightarrow 7/2^-$	2	-0.84	-0.48 ± 0.42
$^{170}\text{Au} \rightarrow ^{169}\text{Pt}$	1.471	$2^- \rightarrow 7/2^-$	2	-3.49	-2.55 ± 0.26
$^{145}\text{Tm} \rightarrow ^{144}\text{Er}$	1.741	$11/2^- \rightarrow 0^+$	5	-5.5	-5.0 ± 0.09
$^{147}\text{Tm} \rightarrow ^{146}\text{Er}$	1.059	$11/2^- \rightarrow 0^+$	5	0.57	0.28 ± 50
$^{155}\text{Ta} \rightarrow ^{154}\text{Hf}$	1.451	$11/2^- \rightarrow 0^+$	5	-2.49	-3.30 ± 0.32
$^{156}\text{Ta}^m \rightarrow ^{155}\text{Hf}$	1.111	$9^+ \rightarrow 7/2^- \#$	5	0.92	0.81 ± 0.11
$^{161}\text{Re}^m \rightarrow ^{160}\text{W}$	1.321	$11/2^- \rightarrow 0^+$	5	-0.68	-0.26 ± 0.61
$^{165}\text{Ir}^m \rightarrow ^{164}\text{Os}$	1.721	$11/2^- \rightarrow 0^+$	5	-3.43	-4.00 ± 0.16
$^{166}\text{Ir}^m \rightarrow ^{165}\text{Os}$	1.331	$9^+ \rightarrow 7/2^-$	5	-0.09	-1.94 ± 2.0
$^{167}\text{Ir}^m \rightarrow ^{166}\text{Os}$	1.246	$11/2^- \rightarrow 0^+$	5	0.78	0.54 ± 0.3
$^{170}\text{Au}^m \rightarrow ^{169}\text{Pt}$	1.751	$9^+ \rightarrow 7/2^-$	5	-2.97	-3.88 ± 0.30
$^{171}\text{Au}^m \rightarrow ^{170}\text{Pt}$	1.702	$11/2^- \rightarrow 0^+$	5	-2.59	-3.70 ± 0.42
$^{177}\text{Tl}^m \rightarrow ^{176}\text{Hg}$	1.962	$11/2^- \rightarrow 0^+$	5	-3.46	-4.3 ± 0.24

Table 2.3 Comparison of 2P decay logarithmic half lives obtained using present formula (PF) with that of experiments [145]

Reaction	Q_{2P}	$J_i \rightarrow J_f$	ℓ	$\log T_{1/2}(s)$	
				Exp	PF
$^6\text{Be} \rightarrow ^4\text{He}$	1.371	$0^+ \rightarrow 0^+$	0	-19.51	-20.48 ± 0.04
$^{12}\text{O} \rightarrow ^{10}\text{C}$	1.79	$0^+ \rightarrow 0^+$	0	-20.31	-20.04 ± 0.01
$^{16}\text{Ne} \rightarrow ^{14}\text{O}$	1.4	$0^+ \rightarrow 0^+$	0	-19.58	-19.82 ± 0.01
$^{19}\text{Mg} \rightarrow ^{17}\text{Ne}$	0.75	$1/2^- \# \rightarrow 1/2^-$	0	-11.4	-13.42 ± 0.17
$^{45}\text{Fe} \rightarrow ^{43}\text{Cr}$	1.14	$3/2^+ \# \rightarrow 3/2^+$	0	-2.07	-3.82 ± 0.84
$^{48}\text{Ni} \rightarrow ^{46}\text{Fe}$	1.29	$0^+ \rightarrow 0^+$	0	-2.52	-4.06 ± 0.61
$^{54}\text{Zn} \rightarrow ^{52}\text{Ni}$	1.48	$0^+ \rightarrow 0^+$	0	-2.43	-4.16 ± 0.71
$^{67}\text{Kr} \rightarrow ^{65}\text{Se}$	1.69	$3/2^- \rightarrow 3/2^- \#$	0	-1.7	-2.18 ± 0.28

CHAPTER 3

Study of Proton radioactivity using Theoretical models

3.1 Theory

A detail study is carried on one and two proton radioactivity using well accepted theoretical models such as MGLDM, CPPM and ELDM.

3.1.1 Modified Generalized Liquid Drop Model (MGLDM)

For a deformed nucleus, total energy is the sum of the volume energy E_v , surface energy E_s , coulomb energy E_C , proximity energy E_P and centrifugal energy E_ℓ and it is given by;

$$E = E_v + E_s + E_c + E_P + E_\ell \quad (3.1)$$

For the deformed nuclei, the volume E_v , surface E_s and coulomb E_C energies are given by;

$$E_v = -15.494(1 - 1.8I^2)AMeV \quad (3.2)$$

$$E_s = 17.9439[(1 - 2.6I_1^2)A_1^{2/3} + (1 - 2.6I_2^2)A_2^{2/3}]MeV \quad (3.3)$$

$$E_c = 0.6e^2(Z^2/R_0) \times 0.5 \int (V(\theta)/V_0)(R(\theta)/R_0)^3 \sin(\theta) d\theta \text{ MeV} \quad (3.4)$$

where I is the relative neutron excess, $R(0)$ is the effective sharp radius, $V(\theta)$ is the electrostatic potential at the surface and V_0 is the surface potential of the sphere. When the nuclei are far apart then above equations are written as;

$$E_v = -15.494(1 - 1.8I_1^2)A_1 + (1 - 1.8I_2^2)A_2 \text{ MeV} \quad (3.5)$$

here A_1 is the daughter nuclei and A_2 is the one/two proton mass number.

$$E_s = 17.9439(1 - 2.6I_1^2)A_1^{2/3} + (1 - 2.6I_2^2)A_2^{2/3} \text{ MeV} \quad (3.6)$$

$$E_c = 0.6e^2(Z_1^2/R_1) + 0.6e^2(Z_2^2/R_2) + e^2 Z_1 Z_2 / r \text{ MeV} \quad (3.7)$$

Here A_i , Z_i , R_i , $I_i(1,2)$ and r are mass number, atomic number, radii of daughter nuclei and one/two protons relative neutron excess of the daughter nuclei and distance between the mass centres respectively. The centrifugal energy E_ℓ of the emitted proton is expressed as;

$$E_\ell(r) = \frac{\hbar^2 \ell(\ell + 1)}{2\mu r^2} \text{ MeV} \quad (3.8)$$

where $\mu = \frac{A_1 A_2}{A_1 + A_2}$, r and ℓ are the reduced mass, separation distance between two nuclei and angular momentum, respectively. The selection rule for proton decay [109] is as follows;

$$J_p = J_d + J_{p^\ell} \quad (3.9)$$

$$\pi_p = \pi_d \pi_{p^\ell} (-1)^\ell \quad (3.10)$$

where J_p , π_p , J_d , π_d , J_{p^ℓ} and π_{p^ℓ} are spin and parity values of the parent, daughter and outgoing one/two proton respectively. The proton has a non zero value of spin and positive parity, therefore the minimal value of angular momentum at the proton transition is expressed as

$$\ell_{min} = \begin{cases} \Delta_j & \text{for even } \Delta_j \text{ and } \pi_p = \pi_d \\ \Delta_j + 1 & \text{for even } \Delta_j \text{ and } \pi_p \neq \pi_d \\ \Delta_j & \text{for odd } \Delta_j \text{ and } \pi_p \neq \pi_d \\ \Delta_j + 1 & \text{for odd } \Delta_j \text{ and } \pi_p = \pi_d \end{cases} \quad (3.11)$$

where $\Delta_j = |J_p - J_d - J_{p^\ell}|$. The proximity function is evaluated as described in the literature[278] and it is expressed as;

$$\Phi(\epsilon) = \begin{cases} -1.7817 + 0.9270\epsilon + 0.143\epsilon^2 & \text{for } \epsilon \leq 0.0; \\ -1.7817 + 0.9270\epsilon + 0.0169\epsilon^2 - 0.05148\epsilon^3 & \text{for } 0 \leq \epsilon \leq 1.9475; \\ -4.41 \exp\left(\frac{-\epsilon}{0.7176}\right) & \text{for } \epsilon \geq 1.9475 \end{cases} \quad (3.12)$$

The proton decay half-lives is evaluated using the probability of penetration and it is evaluated using the WKB integration;

$$P = \exp \left[-\frac{2}{\hbar} \int_{R_{in}}^{R_{out}} \sqrt{2B(r)E(r) - E(sphere)} \right] \quad (3.13)$$

here $E(r)$ is evaluated as explained in equation (3.1) and $E(sphere) = Q$ is the amount of energy released during one and two proton radioactivity. R_{in} and R_{out} are the classical turning points

and are evaluated using the following conditions $V(r = R_{in}) = V(r = R_{out}) = Q$ and μ is the reduced mass of the daughter and one or two protons. Both one and two proton decay half-lives [34] were evaluated as follows;

$$T_{1/2} = \frac{\ln 2}{\nu P} \quad (3.14)$$

where ν is the assault frequency of proton against potential energy barrier. $\nu = \frac{41}{hA^{1/3}}$.

3.1.2 Coulomb and Proximity Potential Model (CPPM)

The total interacting potential is the sum of Coulomb potential, proximity potential and centrifugal potential and it is evaluated as;

$$V(R) = V_N(R) + V_c(R) + \frac{\ell(\ell + 1)}{2\mu \times R^2} \quad (3.15)$$

Here ℓ and μ are the angular momentum and reduced mass of the emitted one or two proton and daughter nuclei respectively. The Coulomb potential is evaluated using the following expression;

$$V_c(R) = \frac{Z_1 Z_2 e^2}{r} \quad (3.16)$$

In above equation, Z_1 and Z_2 are the atomic numbers of daughter and emitted proton. The nuclear potential [279] is given by;

$$V_N(R) = 4\pi\gamma\bar{R}\Phi(s) \quad (3.17)$$

here the mean curvature radius is $\bar{R} = \frac{C_1 C_2}{C_1 + C_2}$ and C_i is the centre of matter radii and it is evaluated as follows;

$$C_i = c_i + \frac{N_i}{A_i} t_i (i = 1, 2) \quad (3.18)$$

here c_i is the half density radius of charge distribution is evaluated as follows;

$$c_i = R_{00i} \left(1 - \frac{7b^2}{R_{00i}^2} - \frac{49b^4}{8R_{00i}^4} \right) (i = 1, 2) \quad (3.19)$$

where R_{00i} is the nuclear charge radius;

$$R_{00i} = 1.256 A_i^{1/3} \left(1 - 0.202 \left(\frac{A_i - 2Z_i}{A_i} \right) \right) \quad (3.20)$$

and neutron skin t_i of nucleus is evaluated as follows;

$$t_i = \frac{3}{2} r_o \left[\frac{J I_i - \frac{1}{12} g Z_i A_i^{-1/3}}{Q + \frac{9}{4} A_i^{-1/3}} \right] (i = 1, 2) \quad (3.21)$$

with $r_0 = 1.14\text{fm}$, $J = 32.65$ MeV is the nuclear symmetric energy coefficient, $g = 0.757895$ MeV, $Q = 35.4$ MeV is the neutron skin stiffness coefficient and surface energy co-efficient γ is evaluated as follows;

$$\gamma = \frac{1}{4\pi r_o^2} \left[18.63 - Q \frac{(t_1^2 + t_2^2)}{2r_o^2} \right] \quad (3.22)$$

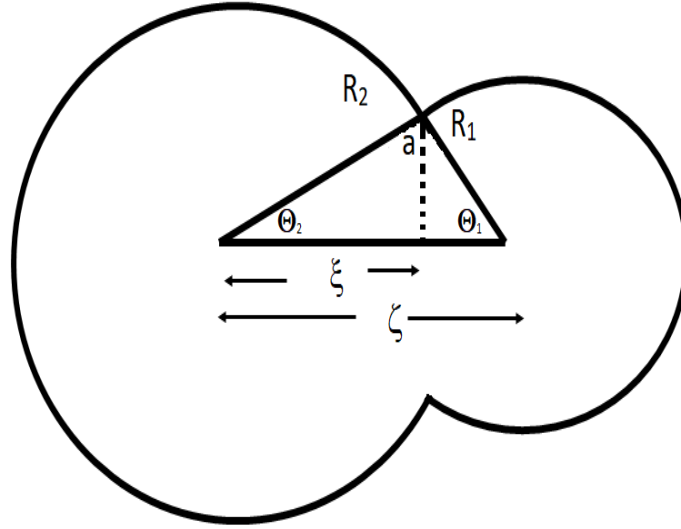


Fig. 3.1 Schematic presentation of molecular phase of the di-nuclear system. [14, 176]

The universal function in nuclear potential is given by

$$\Phi(\xi) = \begin{cases} -0.1353 + \sum_{n=0}^5 \left[\frac{c_n}{(n+1)} \right] (2.5 - \xi)^{n+1} \\ -0.0955 \exp \left(\frac{2.75 - \xi}{0.7176} \right) \end{cases}$$

Here $\chi = (r - C_1 - C_2)/b$ and the width parameter $b \approx 1$. The different values of c_n constant are $c_0 = -0.1886$, $c_1 = 0.2628$, $c_2 = -0.15216$, $c_3 = -0.04562$, $c_4 = 0.069136$ and $c_5 = -0.011454$. The penetration probability P and half-lives are evaluated as explained in section 3.1.1.

3.1.3 Effective Liquid Drop Model(ELDM)

The ELDM [14, 176] is based on a calculation of Coulomb and surface energies. The electrostatic energy is expressed as;

$$V_c = \frac{8\pi}{9} a^5 \epsilon(\theta_1, \theta_2) \rho_c \quad (3.23)$$

where ρ_c is the initial charge density, $\epsilon(\theta_1, \theta_2)$ is a function of the angular variables, and a is the

radius of the sharp neck. The surface potential energy reads as;

$$V_s = \sigma_{eff}(S_{1p/2P} + S_D) \quad (3.24)$$

where $S_{1p/2P}$ is the surface area of the one and two proton nuclei and S_D denote the area of the surface of the daughter nuclei as follows;

$$S_{1p/2p} = 2\pi R_{1p/2p}(R_{1p/2p} + \zeta + \xi) \quad (3.25)$$

$$S_D = 2\pi R_D(R_D + \xi) \quad (3.26)$$

The centrifugal potential energy V_ℓ is written as;

$$V_\ell = \frac{\ell(\ell + 1)\hbar^2}{2\mu\zeta^2} \quad (3.27)$$

here $\mu = \frac{A_1 A_2}{A_1 + A_2}$ where A_1 is the mass number of daughter nuclei and A_2 is the mass number of one or two proton. ℓ is the angular momentum and ζ is the distance between geometrical centers. The term $\hbar$ is the reduced planck's constant. The effective total potential energy is calculated as follows;

$$V = V_c + V_s + V_\ell \quad (3.28)$$

The penetrability factor P is expressed as;

$$P = \exp \left[-\frac{2}{\hbar} \int_{\zeta_0}^{\zeta_c} \sqrt{2\mu[V(\zeta) - Q]} d\zeta \right] \quad (3.29)$$

The limit of integration are $\zeta_0 = R_P - R_{1p/2p}$ and $\zeta_c = Z_{1p/2p}Z_De^2/Q$. The half-life of one and two proton radioactivity is evaluated as follows;

$$T_{1/2} = \frac{\ln 2}{\lambda_0 P} \quad (3.30)$$

where, $\lambda_0 = 4.96 \times 10^{19}$ [145]

3.2 Results

The half-lives of one and two proton radioactivity are studied using the CPPM, ELDM, MGLDM and semi-empirical theoretical formalism for macroscopic models is explained in the theory section and semi-empirical formula construction is explained in chapter 2. The half-lives calculated using macroscopic models such as CPPM, MGLDM and ELDM are compared with the experiments. Standard deviation produced by each model is also tabulated in table 3.1. The Q-values are evaluated using the mass excess data available in the literature [36, 270, 271, 280]. The one and two proton radioactivity is energetically feasible when the Q-values are positive and it is expressed as;

Table 3.1 A tabulation of standard deviation obtained for one and two proton decay logarithmic half-lives using CPPM, ELDM and MGLDM with that of available experiments.

Type of Decay	N	Standard deviation		
		CPPM	ELDM	MGLDM
1P	27	1.40	1.77	1.80
2P	08	0.42	0.76	1.24

$$Q_{1p} = \Delta M_P - \Delta M(A - 1, Z - 1) - m_p \quad (3.31)$$

$$Q_{2p} = \Delta M_P - \Delta M(A - 2, Z - 2) - m_{2p} \quad (3.32)$$

here ΔM_P is the mass excess of parent nuclei, $\Delta M(A - 1, Z - 1)$ and $\Delta M(A - 2, Z - 2)$

Table 3.2 Comparison of 1P decay logarithmic half lives obtained using CPPM, ELDM, MGLDM and present formula (PF) with that of experiments.

Reaction	Q_{1P}	$J_i \rightarrow J_f$	ℓ	$\log T_{1/2}(s)$				
				Exp	CPPM	ELDM	MGLDM	PF
$^{146}Tm \rightarrow ^{145}Er$	0.891	$1^+ \rightarrow 1/2^+ \#$	0	-0.81	-0.25 ± 0.67	-0.71 ± 0.12	-1.55 ± 0.91	-0.30 ± 0.63
$^{146}Tm^m \rightarrow ^{145}Er^m$	1.001	$5^- \rightarrow 11/2^- \#$	0	-1.12	-1.41 ± 0.26	-1.34 ± 0.19	-2.34 ± 1.08	-1.60 ± 0.42
$^{150}Lu^m \rightarrow ^{149}Yb$	1.291	$1^+, 2^+ \rightarrow 1/2^+$	0	-4.4	-4.4 ± 0	-4.25 ± 0.03	-4.91 ± 0.11	-3.47 ± 0.21
$^{157}Ta \rightarrow ^{156}Hf$	0.941	$1/2^+ \rightarrow 0^+$	0	-0.53	-0.35 ± 0.34	-0.58 ± 0.09	-1.06 ± 1	-0.32 ± 0.39
$^{160}Re \rightarrow ^{159}W$	1.271	$4^- \rightarrow 7/2^- \#$	0	-3.16	-3.05 ± 0.03	-3.1 ± 0.01	-3.24 ± 0.02	-2.73 ± 0.13
$^{161}Re \rightarrow ^{160}W$	1.201	$1/2^+ \rightarrow 0^+$	0	-3.36	-3.48 ± 0.03	-3.55 ± 0.05	-3.07 ± 0.08	-2.22 ± 0.34
$^{167}Ir \rightarrow ^{166}Os$	1.071	$1/2^+ \rightarrow 0^+$	0	-1.13	-1.12 ± 0	-1.18 ± 0.04	-1.29 ± 0.14	-0.64 ± 0.43
$^{171}Au \rightarrow ^{170}Pt$	1.448	$1/2^+ \rightarrow 0^+$	0	-4.65	-4.48 ± 0.03	-4.25 ± 0.08	-4.15 ± 0.10	-3.12 ± 0.32
$^{176}Tl \rightarrow ^{175}Hg$	1.261	$3^-, 4^-, 5^- \rightarrow 7/2^-$	0	-2.21	-2.5 ± 0.13	-2.52 ± 0.14	-2.01 ± 0.09	-1.64 ± 0.25
$^{177}Tl \rightarrow ^{176}Hg$	1.155	$1/2^+ \rightarrow 0^+$	0	-1.18	-1.11 ± 0.05	-2.38 ± 1.01	-1.07 ± 0.09	-0.68 ± 0.42
$^{185}Bi^m \rightarrow ^{184}Pb$	1.607	$1/2^+ \rightarrow 0^+$	0	-4.19	-4.13 ± 0.01	-4.23 ± 0.02	-4.45 ± 0.06	-3.45 ± 0.17
$^{147}Tm^m \rightarrow ^{146}Er$	1.12	$3/2^+ \rightarrow 0^+$	2	-3.44	-3.17 ± 0.07	-3.55 ± 0.03	-3.35 ± 0.02	-2.70 ± 0.21
$^{151}Lu^m \rightarrow ^{150}Yb$	1.291	$3/2^+ \rightarrow 0^+$	2	-4.78	-4.55 ± 0.04	-4.21 ± 0.11	-4.44 ± 0.07	-2.67 ± 0.44
$^{156}Ta \rightarrow ^{155}Hf$	1.021	$2^- \rightarrow 7/2^- \#$	2	-0.83	-0.55 ± 0.33	-0.5 ± 0.39	-0.85 ± 0.02	-0.68 ± 0.18
$^{166}Ir \rightarrow ^{165}Os$	1.161	$2^- \rightarrow 7/2^-$	2	-0.84	-0.62 ± 0.26	-0.71 ± 0.15	-0.87 ± 0.03	-0.48 ± 0.42
$^{170}Au \rightarrow ^{169}Pt$	1.471	$2^- \rightarrow 7/2^-$	2	-3.49	-3.55 ± 0.01	-3.21 ± 0.08	-3.4 ± 0.02	-2.55 ± 0.26
$^{145}Tm \rightarrow ^{144}Er$	1.741	$11/2^- \rightarrow 0^+$	5	-5.5	-5.55 ± 0.01	-5.58 ± 0.01	-5.25 ± 0.04	-5.0 ± 0.09
$^{147}Tm \rightarrow ^{146}Er$	1.059	$11/2^- \rightarrow 0^+$	5	0.57	0.12 ± 0.79	0.36 ± 0.36	0.56 ± 0.01	0.28 ± 50
$^{155}Ta \rightarrow ^{154}Hf$	1.451	$11/2^- \rightarrow 0^+$	5	-2.49	-2.4 ± 0.03	-2.68 ± 0.07	-2.12 ± 0.14	-3.30 ± 0.32
$^{156}Ta^m \rightarrow ^{155}Hf$	1.111	$9^+ \rightarrow 7/2^- \#$	5	0.92	0.57 ± 0.38	1.01 ± 0.09	0.93 ± 0.01	0.81 ± 0.11
$^{161}Re^m \rightarrow ^{160}W$	1.321	$11/2^- \rightarrow 0^+$	5	-0.68	-0.62 ± 0.08	-0.25 ± 0.63	-0.52 ± 0.23	-0.26 ± 0.61
$^{165}Ir^m \rightarrow ^{164}Os$	1.721	$11/2^- \rightarrow 0^+$	5	-3.43	-3.74 ± 0.09	-3.36 ± 0.02	-3.49 ± 0.01	-4.00 ± 0.16
$^{166}Ir^m \rightarrow ^{165}Os$	1.331	$9^+ \rightarrow 7/2^-$	5	-0.09	-0.21 ± 1.3	-0.23 ± 1.5	-0.21 ± 1.3	-1.94 ± 2.0
$^{167}Ir^m \rightarrow ^{166}Os$	1.246	$11/2^- \rightarrow 0^+$	5	0.78	0.76 ± 0.02	0.46 ± 0.41	0.72 ± 0.07	0.54 ± 0.3
$^{170}Au^m \rightarrow ^{169}Pt$	1.751	$9^+ \rightarrow 7/2^-$	5	-2.97	-2.65 ± 0.10	-2.26 ± 0.23	-2.97 ± 0	-3.88 ± 0.30
$^{171}Au^m \rightarrow ^{170}Pt$	1.702	$11/2^- \rightarrow 0^+$	5	-2.59	-2.28 ± 0.11	-2.18 ± 0.15	-2.6 ± 0.01	-3.70 ± 0.42
$^{177}Tl^m \rightarrow ^{176}Hg$	1.962	$11/2^- \rightarrow 0^+$	5	-3.46	-3.73 ± 0.07	-3.44 ± 0.01	-3.75 ± 0.08	-4.3 ± 0.24

are the mass excess of daughter nuclei during one and two proton, m_p and m_{2p} are masses of one and two proton emission respectively. From this analysis, it is observed that around 306 nuclei are energetically feasible for one proton radioactivity in the atomic number region $3 \leq Z \leq 126$. Among this, 29 one proton emitters are experimentally observed and these are listed in table 3.2. Remaining 277 one proton emitters are newly identified and it is tabulated along with their half-lives and decay energy in table 3.3 and 3.4. Even-though, around 277 proton emitters with positive Q-value were identified but comparison of logarithmic half-lives with other decay modes such as β^+ -decay, β^- -decay, electron capture and alpha-decay may result in most dominant decay mode. Hence, table 3.3 and 3.4 only gives the prediction of one proton logarithmic half-lives.

Furthermore, around 182 nuclei are energetically feasible for two proton radioactivity in the

Table 3.3 Tabulation of one proton radioactivity in the atomic number range $4 \leq Z \leq 43$ using theoretical models such as CPPM, ELDM, MGLDM and semi-empirical formula (PF).

Reaction	Q_{1p} (MeV)	l_{min}	$LogT_{1/2}(s)$				Reaction	Q_{1p} (MeV)	l_{min}	$LogT_{1/2}(s)$			
			CPPM	ELDM	MGLDM	PF				CPPM	ELDM	MGLDM	PF
${}^5Be \rightarrow {}^4Li$	5.384	3	-19.57	-19.85	-18.85	-21.78	${}^{51}Zn \rightarrow {}^{50}Cu$	2.621	3	-17.04	-16.39	-16.51	-17.96
${}^7B \rightarrow {}^6Be$	2.204	1	-16.37	-19.85	-20.46	-17.38	${}^{52}Zn \rightarrow {}^{51}Cu$	1.951	3	-20.73	-16.06	-15.12	-19.98
${}^{10}N \rightarrow {}^9C$	2.601	1	-20.90	-19.85	-20.53	-21.20	${}^{53}Zn \rightarrow {}^{52}Cu$	1.518	3	-19.92	-15.82	-15.20	-18.97
${}^{12}O \rightarrow {}^{11}N$	0.455	1	-17.82	-17.49	-17.23	-18.62	${}^{53}Ga \rightarrow {}^{52}Zn$	5.431	1	-20.66	-17.56	-20.85	-19.63
${}^{14}F \rightarrow {}^{13}O$	2.257	2	-20.21	-19.55	-18.85	-20.60	${}^{54}Ga \rightarrow {}^{53}Zn$	4.171	1	-19.99	-18.78	-20.15	-19.12
${}^{19}Mg \rightarrow {}^{18}Na$	1.561	2	-18.66	-17.44	-18.11	-19.62	${}^{55}Ga \rightarrow {}^{54}Zn$	3.958	1	-19.83	-18.67	-20.01	-18.96
${}^{23}P \rightarrow {}^{22}Si$	6.207	0	-21.63	-19.85	-20.90	-21.21	${}^{56}Ga \rightarrow {}^{55}Zn$	2.893	1	-18.75	-18.36	-18.87	-18.18
${}^{24}S \rightarrow {}^{23}P$	2.791	2	-19.67	-17.00	-19.37	-20.06	${}^{57}Ga \rightarrow {}^{56}Zn$	2.538	1	-18.25	-18.22	-18.36	-17.76
${}^{25}S \rightarrow {}^{24}P$	4.114	2	-20.91	-19.34	-20.38	-20.47	${}^{58}Ga \rightarrow {}^{57}Zn$	1.525	1	-15.83	-15.45	-15.86	-15.93
${}^{25}Cl \rightarrow {}^{24}S$	6.991	2	-21.66	-19.85	-20.96	-20.98	${}^{58}Ge \rightarrow {}^{57}Ga$	2.541	3	-16.69	-18.29	-17.02	-17.60
${}^{26}Cl \rightarrow {}^{25}S$	3.621	2	-20.90	-18.87	-19.91	-20.27	${}^{56}Ge \rightarrow {}^{55}Ga$	1.381	3	-15.02	-15.44	-13.77	-15.25
${}^{27}Cl \rightarrow {}^{26}S$	6.031	2	-21.27	-19.85	-20.89	-20.77	${}^{57}Ge \rightarrow {}^{56}Ga$	1.472	3	-19.56	-17.93	-14.17	-18.44
${}^{28}Cl \rightarrow {}^{27}S$	1.725	2	-19.11	-17.84	-17.80	-18.92	${}^{57}As \rightarrow {}^{56}Ge$	5.351	3	-19.54	-18.91	-19.23	-19.26
${}^{27}Ar \rightarrow {}^{26}Cl$	3.111	2	-19.79	-16.66	-19.37	-20.01	${}^{58}As \rightarrow {}^{57}Ge$	4.161	3	-18.70	-18.54	-18.27	-18.70
${}^{29}Ar \rightarrow {}^{28}Cl$	1.564	2	-18.25	-17.62	-17.49	-18.60	${}^{59}As \rightarrow {}^{58}Ge$	3.945	3	-18.49	-18.43	-18.07	-18.53
${}^{29}K \rightarrow {}^{28}Ar$	8.531	2	-21.71	-19.85	-21.04	-20.99	${}^{60}As \rightarrow {}^{59}Ge$	3.312	3	-17.81	-18.22	-17.35	-18.03
${}^{30}K \rightarrow {}^{29}Ar$	5.381	2	-21.05	-19.57	-20.84	-20.52	${}^{61}As \rightarrow {}^{60}Ge$	2.427	3	-16.40	-17.94	-17.19	-17.04
${}^{31}K \rightarrow {}^{30}Ar$	4.788	2	-21.31	-17.91	-20.60	-20.34	${}^{62}As \rightarrow {}^{61}Ge$	1.476	1	-15.20	-15.03	-15.24	-15.40
${}^{32}K \rightarrow {}^{31}Ar$	1.836	2	-18.21	-17.49	-17.81	-18.68	${}^{59}Se \rightarrow {}^{58}As$	2.321	3	-15.98	-15.47	-16.09	-17.03
${}^{30}Ca \rightarrow {}^{29}K$	4.021	2	-20.29	-16.82	-19.97	-20.17	${}^{60}Se \rightarrow {}^{59}As$	1.241	3	-12.29	-12.26	-13.17	-13.29
${}^{31}Ca \rightarrow {}^{30}K$	3.391	2	-19.50	-18.18	-19.53	-19.88	${}^{61}Se \rightarrow {}^{60}As$	1.14	3	-11.71	-11.14	-13.30	-13.70
${}^{32}Ca \rightarrow {}^{31}K$	1.381	2	-17.44	-15.19	-16.44	-17.98	${}^{62}Se \rightarrow {}^{61}As$	0.993	3	-14.30	-11.99	-12.32	-12.74
${}^{33}Ca \rightarrow {}^{32}K$	1.073	2	-18.72	-17.01	-15.41	-17.11	${}^{63}Se \rightarrow {}^{62}As$	0.745	1	-12.01	-10.79	-11.54	-11.26
${}^{32}Sc \rightarrow {}^{31}Ca$	4.701	3	-20.73	-17.12	-19.36	-20.18	${}^{61}Br \rightarrow {}^{60}Se$	5.281	3	-19.42	-18.66	-19.08	-19.06
${}^{33}Sc \rightarrow {}^{32}Ca$	6.111	3	-21.26	-19.66	-20.03	-20.45	${}^{62}Br \rightarrow {}^{61}Se$	4.871	3	-19.13	-18.50	-18.78	-18.86
${}^{34}Sc \rightarrow {}^{33}Ca$	4.301	3	-19.17	-16.71	-19.17	-19.96	${}^{63}Br \rightarrow {}^{62}Se$	4.061	3	-18.45	-18.22	-18.05	-18.39
${}^{35}Sc \rightarrow {}^{34}Ca$	4.608	3	-19.89	-18.62	-19.27	-20.01	${}^{64}Br \rightarrow {}^{63}Se$	3.041	1	-18.48	-17.90	-18.61	-17.83
${}^{36}Sc \rightarrow {}^{35}Ca$	2.007	3	-17.24	-17.67	-17.09	-18.40	${}^{65}Br \rightarrow {}^{64}Se$	2.651	3	-16.57	-17.74	-18.01	-17.06
${}^{34}Ti \rightarrow {}^{33}Sc$	1.931	5	-16.18	-17.15	-18.02	-18.23	${}^{66}Br \rightarrow {}^{65}Se$	2.05	1	-16.65	-17.55	-17.95	-16.41
${}^{35}Ti \rightarrow {}^{34}Sc$	1.981	5	-16.31	-17.65	-18.14	-18.21	${}^{63}Kr \rightarrow {}^{62}Br$	2.361	3	-16.46	-17.74	-17.30	-16.81
${}^{36}Ti \rightarrow {}^{35}Sc$	1.191	5	-17.65	-16.71	-15.83	-16.54	${}^{64}Kr \rightarrow {}^{63}Br$	1.331	3	-12.31	-14.59	-13.93	-17.12
${}^{37}Ti \rightarrow {}^{36}Sc$	1.443	3	-17.63	-17.34	-18.49	-17.40	${}^{65}Kr \rightarrow {}^{64}Br$	1.031	1	-18.27	-14.41	-13.50	-13.18
${}^{36}V \rightarrow {}^{35}Ti$	5.611	3	-20.33	-18.90	-19.70	-20.22	${}^{66}Rb \rightarrow {}^{65}Kr$	4.691	1	-19.93	-18.20	-20.12	-18.80
${}^{37}V \rightarrow {}^{36}Ti$	4.891	3	-19.98	-17.58	-19.31	-20.00	${}^{67}Rb \rightarrow {}^{66}Kr$	4.331	3	-18.55	-18.05	-18.18	-18.36
${}^{38}V \rightarrow {}^{37}Ti$	3.921	3	-19.36	-17.81	-18.64	-19.61	${}^{68}Rb \rightarrow {}^{67}Kr$	3.381	1	-18.70	-17.73	-18.84	-17.90
${}^{39}V \rightarrow {}^{38}Ti$	3.47	3	-19.05	-15.30	-18.26	-19.34	${}^{69}Rb \rightarrow {}^{68}Kr$	3.181	3	-18.24	-17.64	-18.63	-17.40
${}^{40}V \rightarrow {}^{39}Ti$	1.541	1	-18.08	-16.54	-17.54	-17.69	${}^{70}Rb \rightarrow {}^{69}Kr$	1.766	3	-16.00	-17.21	-15.61	-15.06
${}^{38}Cr \rightarrow {}^{37}V$	2.651	5	-15.31	-15.83	-18.79	-16.67	${}^{68}Sr \rightarrow {}^{67}Rb$	1.071	3	-10.18	-14.06	-11.86	-12.45
${}^{39}Cr \rightarrow {}^{38}V$	2.181	3	-18.01	-16.78	-19.47	-18.36	${}^{69}Sr \rightarrow {}^{68}Rb$	0.911	1	-12.99	-13.95	-10.28	-11.83
${}^{40}Cr \rightarrow {}^{39}V$	0.891	5	-17.59	-13.16	-13.44	-16.54	${}^{70}Y \rightarrow {}^{69}Sr$	4.041	1	-19.20	-17.75	-19.35	-18.27
${}^{40}Mn \rightarrow {}^{39}Cr$	5.461	3	-20.65	-16.56	-19.57	-20.01	${}^{71}Y \rightarrow {}^{70}Sr$	3.941	1	-19.12	-17.68	-19.26	-18.16
${}^{41}Mn \rightarrow {}^{40}Cr$	4.791	3	-20.68	-17.93	-19.18	-19.78	${}^{72}Y \rightarrow {}^{71}Sr$	2.781	1	-16.97	-17.30	-17.65	-17.08
${}^{42}Mn \rightarrow {}^{41}Cr$	3.851	1	-19.10	-17.85	-20.35	-19.58	${}^{73}Y \rightarrow {}^{72}Sr$	2.771	1	-17.01	-17.26	-17.64	-17.00
${}^{43}Mn \rightarrow {}^{42}Cr$	1.741	3	-20.70	-16.25	-18.59	-19.75	${}^{74}Y \rightarrow {}^{73}Sr$	1.06	1	-13.21	-13.60	-11.31	-12.23
${}^{42}Fe \rightarrow {}^{41}Mn$	1.991	5	-16.42	-13.38	-17.43	-15.85	${}^{72}Zr \rightarrow {}^{71}Y$	1.541	1	-14.83	-16.94	-13.85	-14.65
${}^{43}Fe \rightarrow {}^{42}Mn$	1.861	1	-18.88	-16.22	-17.71	-17.96	${}^{73}Zr \rightarrow {}^{72}Y$	1.471	1	-13.56	-16.86	-13.54	-14.31
${}^{44}Fe \rightarrow {}^{43}Mn$	1.311	5	-18.78	-16.59	-15.72	-16.95	${}^{74}Zr \rightarrow {}^{73}Y$	0.421	1	-2.03	-2.03	-4.37	-3.05
${}^{44}Co \rightarrow {}^{43}Fe$	5.231	1	-20.49	-17.80	-20.83	-19.96	${}^{74}Nb \rightarrow {}^{73}Zr$	4.121	2	-18.67	-17.51	-18.60	-18.00
${}^{45}Co \rightarrow {}^{44}Fe$	3.811	3	-18.68	-17.49	-18.29	-19.18	${}^{75}Nb \rightarrow {}^{74}Zr$	4.481	4	-17.53	-17.58	-16.99	-17.99
${}^{46}Co \rightarrow {}^{45}Fe$	2.462	1	-18.55	-16.22	-18.71	-18.44	${}^{76}Nb \rightarrow {}^{75}Zr$	3.581	2	-18.07	-17.27	-17.96	-17.50
${}^{47}Co \rightarrow {}^{46}Fe$	2.659	3	-17.43	-17.47	-19.88	-18.27	${}^{77}Nb \rightarrow {}^{76}Zr$	3.321	4	-16.86	-17.13	-17.05	-16.98
${}^{46}Ni \rightarrow {}^{45}Co$	2.721	5	-19.46	-16.23	-18.61	-18.19	${}^{78}Nb \rightarrow {}^{77}Zr$	2.461	2	-16.45	-16.84	-16.39	-16.06
${}^{47}Ni \rightarrow {}^{46}Co$	1.581	1	-16.51	-16.37	-16.58	-17.11	${}^{77}Mo \rightarrow {}^{76}Nb$	1.271	2	-13.45	-10.01	-11.10	-13.76
${}^{48}Ni \rightarrow {}^{47}Co$	0.405	5	-8.45	-7.84	-8.04	-7.30	${}^{78}Mo \rightarrow {}^{77}Nb$	1.121	4	-9.36	-9.87	-10.28	-11.23
${}^{48}Cu \rightarrow {}^{47}Ni$	5.481	3	-19.77	-18.00	-19.46	-19.68	${}^{79}Mo \rightarrow {}^{78}Nb$	1.551	2	-13.77	-16.50	-12.69	-13.76
${}^{49}Cu \rightarrow {}^{48}Ni$	3.994	1	-19.97	-17.20	-20.15	-19.28	${}^{82}Mo \rightarrow {}^{81}Nb$	0.599	4	-7.62	-8.14	-5.27	-4.42
${}^{50}Cu \rightarrow {}^{49}Ni$	2.843	1	-18.89	-16.63	-19.00	-18.51	${}^{79}Tc \rightarrow {}^{78}Mo$	3.901	4	-16.72	-17.12	-18.38	-16.54
${}^{51}Cu \rightarrow {}^{50}Ni$	2.442	1	-18.32	-16.41	-18.44	-18.07	${}^{80}Tc \rightarrow {}^{79}Mo$	2.881	2	-16.73	-16.77	-16.54	-16.54

Table 3.4 Tabulation of one proton radioactivity in the atomic number range $43 \leq Z \leq 105$ using theoretical models such as CPPM, ELDM, MGLDM and semi-empirical formula (PF). Table 3.3 continued.

Reaction	Q_{1p} (MeV)	l_{min}	$LogT_{1/2}(s)$				Reaction	Q_{1p} (MeV)	l_{min}	$LogT_{1/2}(s)$			
			CPPM	ELDM	MGLDM	PF				CPPM	ELDM	MGLDM	PF
$^{81}Tc \rightarrow ^{80}Mo$	3.961	4	-18.67	-17.06	-16.24	-17.34	$^{115}Ce \rightarrow ^{114}La$	0.891	4	-5.67	-3.44	-4.21	-3.90
$^{82}Tc \rightarrow ^{81}Mo$	3.221	4	-16.37	-16.78	-17.45	-16.59	$^{115}Pr \rightarrow ^{114}Ce$	3.861	3	-15.95	-15.17	-17.70	-15.67
$^{83}Tc \rightarrow ^{82}Mo$	1.651	4	-12.69	-16.27	-13.23	-13.36	$^{116}Pr \rightarrow ^{115}Ce$	3.061	3	-15.62	-14.19	-16.20	-14.52
$^{84}Tc \rightarrow ^{83}Mo$	3.509	4	-17.52	-16.79	-17.91	-16.78	$^{117}Pr \rightarrow ^{116}Ce$	2.811	3	-13.99	-14.06	-15.61	-14.01
$^{81}Ru \rightarrow ^{80}Tc$	1.471	2	-11.41	-12.89	-13.76	-13.26	$^{118}Pr \rightarrow ^{117}Ce$	1.821	3	-10.23	-10.57	-12.00	-11.10
$^{82}Ru \rightarrow ^{81}Tc$	0.591	4	-6.74	-6.91	-4.75	-4.25	$^{119}Pr \rightarrow ^{118}Ce$	1.411	3	-8.15	-9.04	-9.46	-8.83
$^{83}Ru \rightarrow ^{82}Tc$	1.721	4	-13.90	-16.25	-13.35	-13.61	$^{118}Nd \rightarrow ^{117}Pr$	1.131	3	-7.96	-4.56	-6.66	-6.68
$^{84}Ru \rightarrow ^{83}Tc$	0.921	4	-8.57	-9.09	-7.92	-8.84	$^{120}Pm \rightarrow ^{119}Nd$	3.461	3	-14.96	-14.01	-14.65	-14.91
$^{86}Ru \rightarrow ^{85}Tc$	1.936	4	-16.10	-16.16	-14.23	-14.01	$^{121}Pm \rightarrow ^{120}Nd$	3.301	3	-15.94	-14.59	-14.31	-14.62
$^{83}Rh \rightarrow ^{82}Ru$	3.861	4	-16.47	-16.82	-18.17	-17.16	$^{122}Pm \rightarrow ^{121}Nd$	2.361	3	-12.25	-13.52	-11.62	-12.63
$^{84}Rh \rightarrow ^{83}Ru$	2.571	4	-14.09	-16.39	-15.93	-15.56	$^{123}Pm \rightarrow ^{122}Nd$	1.981	3	-10.41	-12.60	-10.03	-11.32
$^{85}Rh \rightarrow ^{84}Ru$	2.441	4	-15.76	-16.30	-15.61	-15.26	$^{123}Sm \rightarrow ^{122}Pm$	1.371	3	-6.30	-5.11	-5.91	-8.13
$^{86}Rh \rightarrow ^{85}Ru$	1.701	4	-13.18	-16.06	-13.09	-13.27	$^{125}Eu \rightarrow ^{124}Sm$	3.981	3	-15.71	-15.48	-15.43	-15.31
$^{87}Rh \rightarrow ^{86}Ru$	1.491	4	-13.30	-15.93	-12.07	-12.33	$^{126}Eu \rightarrow ^{125}Sm$	3.191	3	-14.45	-14.21	-15.32	-14.16
$^{88}Rh \rightarrow ^{87}Ru$	3.94	4	-18.62	-16.63	-18.32	-16.98	$^{127}Eu \rightarrow ^{126}Sm$	2.701	3	-12.10	-13.26	-14.02	-13.14
$^{90}Pd \rightarrow ^{89}Rh$	1.139	4	-12.97	-12.01	-11.17	-10.53	$^{128}Eu \rightarrow ^{127}Sm$	2.011	2	-13.06	-12.97	-10.63	-11.24
$^{89}Ag \rightarrow ^{88}Pd$	2.381	4	-14.30	-15.99	-16.72	-14.86	$^{129}Eu \rightarrow ^{128}Sm$	1.459	3	-8.39	-8.05	-6.42	-8.12
$^{90}Ag \rightarrow ^{89}Pd$	1.561	4	-15.30	-15.71	-13.68	-13.35	$^{128}Gd \rightarrow ^{127}Eu$	1.061	3	-2.55	-4.31	-2.43	-4.65
$^{91}Ag \rightarrow ^{90}Pd$	1.401	4	-11.51	-11.97	-12.80	-11.48	$^{130}Tb \rightarrow ^{129}Gd$	3.171	2	-13.29	-13.14	-14.27	-14.05
$^{92}Ag \rightarrow ^{91}Pd$	2.914	4	-16.69	-16.01	-17.95	-15.60	$^{131}Tb \rightarrow ^{130}Gd$	2.771	2	-14.80	-12.96	-13.18	-13.21
$^{92}In \rightarrow ^{91}Cd$	2.331	4	-12.84	-15.73	-14.77	-13.54	$^{132}Tb \rightarrow ^{131}Gd$	2.071	2	-10.39	-12.65	-10.52	-11.15
$^{93}In \rightarrow ^{92}Cd$	2.121	4	-12.15	-15.63	-14.10	-13.96	$^{133}Tb \rightarrow ^{132}Gd$	1.971	2	-10.23	-12.56	-10.04	-10.68
$^{94}In \rightarrow ^{93}Cd$	1.331	4	-8.21	-11.54	-10.23	-9.77	$^{136}Ho \rightarrow ^{135}Dy$	2.151	0	-11.21	-12.34	-11.36	-11.84
$^{95}In \rightarrow ^{94}Cd$	1.161	4	-9.90	-11.42	-8.92	-9.51	$^{137}Ho \rightarrow ^{136}Dy$	2.021	2	-10.08	-12.25	-9.90	-10.56
$^{97}Sb \rightarrow ^{96}Sn$	5.731	4	-18.06	-16.52	-19.62	-17.75	$^{141}Tm \rightarrow ^{140}Er$	2.331	0	-9.21	-12.00	-11.77	-10.04
$^{98}Sb \rightarrow ^{97}Sn$	5.011	4	-17.39	-16.21	-19.05	-17.31	$^{142}Tm \rightarrow ^{141}Er$	1.831	0	-9.13	-11.77	-9.28	-10.12
$^{99}Sb \rightarrow ^{98}Sn$	5.001	4	-17.39	-16.17	-19.05	-17.26	$^{143}Tm \rightarrow ^{142}Er$	1.801	0	-8.96	-10.43	-9.11	-9.89
$^{100}Sb \rightarrow ^{99}Sn$	2.58	4	-15.20	-15.30	-15.25	-14.51	$^{146}Lu \rightarrow ^{145}Yb$	2.401	5	-10.64	-11.65	-10.10	-10.74
$^{101}Sb \rightarrow ^{100}Sn$	2.891	4	-14.13	-15.35	-16.03	-15.01	$^{147}Lu \rightarrow ^{146}Yb$	2.081	5	-7.14	-10.16	-8.61	-9.46
$^{99}Te \rightarrow ^{98}Sb$	2.911	4	-14.01	-15.40	-15.93	-15.16	$^{148}Lu \rightarrow ^{147}Yb$	1.581	5	-5.94	-6.05	-5.42	-6.63
$^{100}Te \rightarrow ^{99}Sb$	1.911	4	-13.81	-15.03	-14.47	-12.78	$^{149}Lu \rightarrow ^{148}Yb$	1.919	5	-10.78	-9.99	-7.73	-8.55
$^{101}Te \rightarrow ^{100}Sb$	2.121	4	-15.60	-15.05	-15.28	-13.35	$^{151}Ta \rightarrow ^{150}Hf$	2.361	5	-8.14	-11.21	-7.55	-10.25
$^{102}Te \rightarrow ^{101}Sb$	1.281	4	-7.83	-10.75	-10.92	-9.56	$^{152}Ta \rightarrow ^{151}Hf$	1.781	5	-5.97	-8.67	-6.46	-7.54
$^{103}Te \rightarrow ^{102}Sb$	0.951	4	-7.51	-8.00	-7.79	-6.45	$^{153}Ta \rightarrow ^{152}Hf$	1.691	5	-5.34	-5.60	-5.84	-6.89
$^{101}I \rightarrow ^{100}Te$	5.731	4	-17.91	-16.22	-16.55	-17.60	$^{154}Ta \rightarrow ^{153}Hf$	1.233	5	-4.23	-5.28	-4.03	-2.85
$^{102}I \rightarrow ^{101}Te$	4.801	4	-17.01	-16.37	-19.93	-17.00	$^{155}Ta \rightarrow ^{154}Hf$	1.451	5	-2.4	-2.68	-2.12	-4.90
$^{103}I \rightarrow ^{102}Te$	5.141	4	-17.38	-16.45	-20.20	-17.19	$^{156}Re \rightarrow ^{155}W$	1.951	5	-8.74	-9.27	-7.17	-8.12
$^{104}I \rightarrow ^{103}Te$	3.901	4	-15.82	-16.00	-19.01	-16.15	$^{157}Re \rightarrow ^{156}W$	1.821	5	-4.83	-9.17	-6.35	-7.63
$^{105}I \rightarrow ^{104}Te$	3.601	4	-15.34	-15.28	-18.60	-15.77	$^{164}Ir \rightarrow ^{163}Os$	1.57	5	-4.52	-7.53	-4.06	-4.96
$^{106}I \rightarrow ^{105}Te$	2.76	4	-14.05	-14.95	-17.02	-14.47	$^{169}Au \rightarrow ^{168}Pt$	1.961	5	-4.98	-9.70	-6.53	-7.09
$^{107}I \rightarrow ^{106}Te$	2.405	4	-14.96	-14.78	-16.10	-13.65	$^{170}Au \rightarrow ^{169}Pt$	1.474	5	-2.29	-3.77	-2.75	-3.53
$^{103}Xe \rightarrow ^{102}I$	3.251	4	-16.52	-15.19	-17.91	-15.43	$^{173}Tl \rightarrow ^{172}Hg$	1.798	5	-3.45	-7.77	-5.03	-5.76
$^{104}Xe \rightarrow ^{103}I$	2.441	4	-14.45	-14.89	-16.04	-13.94	$^{175}Tl \rightarrow ^{174}Hg$	1.478	5	-2.39	-2.68	-2.36	-3.04
$^{105}Xe \rightarrow ^{104}I$	2.421	4	-14.22	-14.84	-15.99	-13.82	$^{178}Bi \rightarrow ^{177}Pb$	3.571	3	-12.54	-11.39	-13.94	-12.03
$^{106}Xe \rightarrow ^{105}I$	1.651	4	-9.15	-10.46	-12.91	-11.23	$^{179}Bi \rightarrow ^{178}Pb$	3.413	5	-10.53	-10.40	-12.05	-11.40
$^{107}Xe \rightarrow ^{106}I$	0.921	4	-6.84	-4.67	-7.07	-5.50	$^{180}Bi \rightarrow ^{179}Pb$	2.808	3	-10.14	-9.17	-11.59	-10.06
$^{106}Cs \rightarrow ^{105}Xe$	4.821	4	-16.86	-16.11	-19.85	-16.84	$^{181}Bi \rightarrow ^{180}Pb$	3.18	5	-9.82	-10.21	-11.37	-10.74
$^{107}Cs \rightarrow ^{106}Xe$	4.541	4	-16.53	-16.78	-19.60	-16.59	$^{182}Bi \rightarrow ^{181}Pb$	2.266	3	-7.73	-8.83	-9.22	-7.94
$^{108}Cs \rightarrow ^{107}Xe$	3.621	4	-15.15	-15.04	-18.46	-15.64	$^{183}Bi \rightarrow ^{182}Pb$	2.557	5	-7.47	-8.91	-9.04	-8.72
$^{109}Cs \rightarrow ^{108}Xe$	3.401	4	-14.76	-16.38	-18.11	-15.30	$^{184}At \rightarrow ^{183}Po$	2.101	3	-6.22	-8.49	-7.91	-6.99
$^{110}Cs \rightarrow ^{109}Xe$	1.981	4	-13.10	-14.39	-14.32	-12.20	$^{185}At \rightarrow ^{184}Po$	1.901	5	-3.40	-4.12	-5.01	-5.37
$^{111}Cs \rightarrow ^{110}Xe$	2.426	4	-12.30	-14.49	-15.91	-13.42	$^{189}Fr \rightarrow ^{188}Rn$	2.271	5	-5.33	-8.14	-6.95	-7.08
$^{108}Ba \rightarrow ^{107}Cs$	2.391	4	-14.91	-14.55	-15.64	-13.54	$^{190}Fr \rightarrow ^{189}Rn$	1.851	5	-4.62	-6.34	-4.25	-4.60
$^{109}Ba \rightarrow ^{108}Cs$	2.231	4	-13.43	-14.45	-15.11	-13.05	$^{191}Fr \rightarrow ^{190}Rn$	1.761	5	-3.92	-3.48	-3.56	-3.86
$^{110}Ba \rightarrow ^{109}Cs$	1.181	4	-5.21	-5.74	-9.13	-7.86	$^{195}Ac \rightarrow ^{194}Ra$	2.161	5	-5.24	-7.63	-5.97	-6.07
$^{111}Ba \rightarrow ^{110}Cs$	0.931	4	-5.37	-4.06	-6.74	-5.05	$^{200}Pa \rightarrow ^{199}Th$	2.111	1	-5.61	-7.22	-7.77	-6.29
$^{110}La \rightarrow ^{109}Ba$	4.421	4	-16.18	-16.54	-19.35	-16.35	$^{201}Pa \rightarrow ^{200}Th$	2.091	5	-4.43	-7.18	-5.18	-5.20
$^{111}La \rightarrow ^{110}Ba$	4.321	4	-16.05	-15.61	-19.25	-16.22	$^{213}Am \rightarrow ^{212}Pu$	1.951	3	-3.35	-3.46	-5.00	-3.72
$^{112}La \rightarrow ^{111}Ba$	3.791	4	-16.29	-15.39	-18.56	-15.63	$^{219}Bk \rightarrow ^{218}Cm$	2.231	3	-4.17	-6.05	-6.53	-4.97
$^{113}La \rightarrow ^{112}Ba$	3.071	4	-14.78	-14.48	-17.30	-14.59	$^{220}Bk \rightarrow ^{219}Cm$	1.841	3	-4.36	-4.71	-3.73	-3.30
$^{114}La \rightarrow ^{113}Ba$	1.891	4	-10.54	-14.02	-13.61	-11.54	$^{224}Es \rightarrow ^{223}Cf$	2.181	3	-4.44	-5.67	-5.84	-4.29
$^{115}La \rightarrow ^{114}Ba$	2.517	4	-14.34	-14.18	-15.92	-13.36	$^{225}Es \rightarrow ^{224}Cf$	2.181	3	-4.17	-5.65	-5.85	-4.22
$^{113}Ce \rightarrow ^{112}La$	1.971	4	-13.01	-14.04	-13.78	-11.92	$^{235}Lr \rightarrow ^{234}No$	2.161	3	-4.50	-4.95	-4.95	-3.30
$^{114}Ce \rightarrow ^{113}La$	1.491	4	-8.05	-9.48	-11.16	-9.63	$^{241}Db \rightarrow ^{240}Rf$	2.131	3	-2.63	-4.57	-2.43	-2.63

Table 3.5 Comparison of 2P decay logarithmic half lives obtained using CPPM, ELDM, MGLDM and present formula (PF) with that of experiments and theoretical data available in literature [160].

Reaction	Q_{2p}	$J_i \rightarrow J_f$	ℓ	$LogT_{1/2}$						
				Exp	CPPM	ELDM	MGLDM	PF	Direct [160]	Diproton [160]
${}^6Be \rightarrow {}^4He$	1.371	$0^+ \rightarrow 0^+$	0	-19.51	-19.573 $\pm$ 0.003	-19.51 $\pm$ 0	-20.135 $\pm$ 0.3	-20.48 $\pm$ 0.04	-	-
${}^{12}O \rightarrow {}^{10}C$	1.79	$0^+ \rightarrow 0^+$	0	-20.31	-20.035 $\pm$ 0.01	-19.855 $\pm$ 0.02	-19.861 $\pm$ 0.02	-20.04 $\pm$ 0.01	-	-
${}^{16}Ne \rightarrow {}^{14}O$	1.4	$0^+ \rightarrow 0^+$	0	-19.58	-19.171 $\pm$ 0.02	-18.365 $\pm$ 0.06	-18.205 $\pm$ 0.07	-19.82 $\pm$ 0.01	-	-
${}^{19}Mg \rightarrow {}^{17}Ne$	0.75	$1/2^- \# \rightarrow 1/2^-$	0	-11.4	-10.58 $\pm$ 0.07	-12.889 $\pm$ 0.13	-13.286 $\pm$ 0.16	-13.42 $\pm$ 0.17	-11.21	-10.91
${}^{45}Fe \rightarrow {}^{43}Cr$	1.14	$3/2^+ \# \rightarrow 3/2^+$	0	-2.07	-1.971 $\pm$ 0.04	-2.195 $\pm$ 0.06	-4.687 $\pm$ 1.26	-3.82 $\pm$ 0.84	-2.96	-2.06
${}^{48}Ni \rightarrow {}^{46}Fe$	1.29	$0^+ \rightarrow 0^+$	0	-2.52	-2.018 $\pm$ 0.19	-2.235 $\pm$ 0.11	-4.649 $\pm$ 0.84	-4.06 $\pm$ 0.61	-2.17	-2.28
${}^{54}Zn \rightarrow {}^{52}Ni$	1.48	$0^+ \rightarrow 0^+$	0	-2.43	-2.452 $\pm$ 0.01	-2.588 $\pm$ 0.06	-5.19 $\pm$ 1.13	-4.16 $\pm$ 0.71	-3.00	-3.10
${}^{67}Kr \rightarrow {}^{65}Se$	1.69	$3/2^- \rightarrow 3/2^- \#$	0	-1.7	-1.659 $\pm$ 0.02	-1.671 $\pm$ 0.01	-2.877 $\pm$ 0.69	-2.18 $\pm$ 0.28	-	-

atomic number region $3 \leq Z \leq 65$. Eventually it is also observed that two proton radioactivity is not energetically feasible when $Z > 65$. Among these nuclei 8 two proton emitters are experimentally observed and it is tabulated in the table 3.5. We have also compared $\log T_{1/2}$ predicted from earlier researchers [160] for two proton decay using direct and diproton models. Remaining 174 two proton emitters are newly identified and tabulated along with their half-lives and decay energies in the table 3.6. Even-though, around 174 two proton emitters with positive Q-value were identified but comparison of logarithmic half-lives with other decay modes may result in most dominant decay mode. Hence, table 3.6 only gives the prediction of two-proton logarithmic half-lives. Even though, identified one and two proton emitters along with half lives and decay energies are based on theory, but it gives blueprint in the experiments of proton radioactivity and also comparison with other possible decay modes.

It can also be noticed from the experiment that the nuclei ${}^{45}Fe$ shows the half-life of $3.2^{+2.6}_{-1.0}$ ms for the Q-value of 1.1 ± 0.1 MeV[91]. Giovinazzo et al.,[67] experimentally observed decay energy spectrum at 1.14 ± 0.04 MeV with the half-life of $4.7^{+3.4}_{-1.4}$ ms. These experimentally observed 2p radioactivity was good agreement with the theoretically predicted half-lives [85, 281]. From these experimental values, The half-lives are obviously influenced by the decay energy. A small change in the value of decay energy results in the measurable change in the half-lives.

The one and two proton radioactivity half-lives studied using the well accepted theoretical models such as CPPM, ELDM and MGLDM are compared with that of available experiments.

Table 3.6 Tabulation of two proton radioactivity in the atomic number range $3 \leq Z \leq 65$ using theoretical models such as CPPM, ELDM, MGLDM and semi-empirical formula (PF).

Reaction	Q_{2p} (MeV)	l_{min}	$LogT_{1/2}(s)$				Reaction	Q_{2p} (MeV)	l_{min}	$LogT_{1/2}(s)$			
			CPPM	ELDM	MGLDM	PF				CPPM	ELDM	MGLDM	PF
${}^3Li \rightarrow {}^1H$	6.8	1	-19.85	-19.85	-19.85	-20.80	${}^{62}Se \rightarrow {}^{60}Ge$	3.42	0	-13.71	-15.38	-13.14	-12.92
${}^5Be \rightarrow {}^3He$	8.48	0	-20.58	-19.85	-20.92	-20.81	${}^{63}Se \rightarrow {}^{61}Ge$	2.22	0	-7.07	-7.91	-8.13	-8.38
${}^6Be \rightarrow {}^4He$	1.37	0	-18.59	-18.23	-20.13	-20.38	${}^{61}Br \rightarrow {}^{59}As$	6.52	0	-18.07	-16.43	-18.38	-16.77
${}^6B \rightarrow {}^4Li$	3.7	0	-20.02	-19.85	-19.87	-20.67	${}^{63}Br \rightarrow {}^{61}As$	5.05	0	-16.18	-17.01	-16.43	-15.54
${}^7B \rightarrow {}^5Li$	1.61	1	-18.44	-18.36	-19.9	-19.92	${}^{64}Br \rightarrow {}^{62}As$	3.78	0	-13.57	-15.28	-13.79	-13.34
${}^8C \rightarrow {}^6Be$	2.14	0	-17.41	-18.19	-20.28	-18.33	${}^{65}Br \rightarrow {}^{63}As$	2.53	0	-9.15	-9.36	-9.29	-9.40
${}^{10}N \rightarrow {}^8B$	1.3	2	-18.08	-18.11	-18.33	-18.93	${}^{63}Kr \rightarrow {}^{61}Se$	7.23	0	-17.79	-17.04	-18.96	-17.00
${}^{12}O \rightarrow {}^{10}C$	1.77	0	-18.13	-18.62	-19.83	-19.58	${}^{64}Kr \rightarrow {}^{62}Se$	5.39	0	-15.57	-17.01	-16.67	-15.57
${}^{14}F \rightarrow {}^{12}N$	0.74	3	-14.25	-15.98	-14.93	-15.51	${}^{65}Kr \rightarrow {}^{63}Se$	4.07	0	-13.04	-15.24	-14.14	-13.71
${}^{16}Ne \rightarrow {}^{14}O$	1.40	0	-19.58	-19.17	-18.36	-18.30	${}^{66}Kr \rightarrow {}^{64}Se$	2.66	0	-8.32	-9.17	-9.33	-9.72
${}^{19}Mg \rightarrow {}^{17}Ne$	0.75	0	-10.58	-12.89	-13.28	-12.65	${}^{67}Kr \rightarrow {}^{65}Se$	1.69	0	-1.66	-1.67	-2.87	-2.98
${}^{23}P \rightarrow {}^{21}Al$	4.96	2	-19.49	-19.5	-20.24	-19.25	${}^{66}Rb \rightarrow {}^{64}Br$	5.72	0	-16.67	-16.91	-16.91	-15.65
${}^{24}S \rightarrow {}^{22}Si$	8.99	0	-20.36	-19.85	-20.95	-20.09	${}^{67}Rb \rightarrow {}^{65}Br$	4.34	0	-13.33	-15.1	-14.43	-13.86
${}^{25}S \rightarrow {}^{23}Si$	5.05	0	-19.9	-19.38	-20.7	-19.42	${}^{68}Rb \rightarrow {}^{66}Br$	3.23	0	-10.19	-10.5	-11.26	-11.27
${}^{26}S \rightarrow {}^{24}Si$	0.63	0	-6.46	-8.19	-7.47	-8.95	${}^{69}Rb \rightarrow {}^{67}Br$	2.3	0	-5.88	-4.92	-6.87	-7.13
${}^{25}Cl \rightarrow {}^{23}P$	9.78	2	-18.69	-19.85	-20.98	-19.93	${}^{68}Sr \rightarrow {}^{66}Kr$	5.4	0	-15.05	-15.33	-16.15	-15.10
${}^{26}Cl \rightarrow {}^{24}P$	7.73	2	-19.61	-18.25	-20.88	-19.63	${}^{69}Sr \rightarrow {}^{67}Kr$	4.29	0	-12.88	-14.88	-13.99	-13.47
${}^{27}Cl \rightarrow {}^{25}P$	5.84	2	-19.72	-18.21	-20.58	-19.16	${}^{70}Sr \rightarrow {}^{68}Kr$	3.09	0	-9.23	-10.11	-10.26	-10.41
${}^{28}Cl \rightarrow {}^{26}P$	1	2	-10.06	-11.04	-10.14	-10.65	${}^{71}Sr \rightarrow {}^{69}Kr$	2	0	-3.3	-3.18	-4.27	-4.43
${}^{27}Ar \rightarrow {}^{25}S$	6.73	0	-19.52	-18.54	-20.82	-19.64	${}^{70}Y \rightarrow {}^{68}Rb$	4.95	0	-14.86	-14.94	-15.07	-14.31
${}^{28}Ar \rightarrow {}^{26}S$	5.26	0	-19.61	-19.19	-20.65	-19.23	${}^{71}Y \rightarrow {}^{69}Rb$	3.85	2	-11.98	-13.71	-11.79	-11.39
${}^{29}Ar \rightarrow {}^{27}S$	3.28	0	-17.62	-18.47	-18.64	-18.12	${}^{72}Y \rightarrow {}^{70}Rb$	3.02	2	-9.11	-8.27	-8.87	-8.57
${}^{30}Ar \rightarrow {}^{28}S$	1.43	0	-12.22	-13.72	-13.25	-14.32	${}^{73}Y \rightarrow {}^{71}Rb$	1.71	2	-0.29	-0.16	-0.42	-0.10
${}^{29}K \rightarrow {}^{27}Cl$	7.76	0	-19.77	-18.27	-20.88	-19.68	${}^{72}Zr \rightarrow {}^{70}Sr$	5.48	0	-15.56	-14.89	-15.75	-14.70
${}^{30}K \rightarrow {}^{28}Cl$	6.94	0	-19.55	-18.31	-20.84	-19.48	${}^{73}Zr \rightarrow {}^{71}Sr$	4.25	0	-12.11	-13.6	-13.19	-12.78
${}^{31}K \rightarrow {}^{29}Cl$	4.43	0	-18.76	-18.69	-19.81	-18.61	${}^{74}Zr \rightarrow {}^{72}Sr$	3.19	2	-8.38	-9.49	-9.14	-8.78
${}^{32}K \rightarrow {}^{30}Cl$	1.39	0	-11.32	-11.08	-12.36	-13.45	${}^{74}Nb \rightarrow {}^{72}Y$	5.59	1	-15.38	-14.71	-15.5	-13.95
${}^{30}Ca \rightarrow {}^{28}Ar$	12.55	0	-20.76	-19.85	-21.09	-20.08	${}^{75}Nb \rightarrow {}^{73}Y$	4.9	5	-12.36	-14.38	-11.51	-12.83
${}^{31}Ca \rightarrow {}^{29}Ar$	8.77	0	-20.06	-19.85	-20.94	-19.71	${}^{76}Nb \rightarrow {}^{74}Y$	3.8	1	-11.37	-9.44	-11.32	-10.33
${}^{32}Ca \rightarrow {}^{30}Ar$	6.16	0	-19.03	-19.17	-20.78	-19.16	${}^{77}Nb \rightarrow {}^{75}Y$	2.73	5	-6.16	-7.29	-4.14	-5.97
${}^{33}Ca \rightarrow {}^{31}Ar$	2.9	0	-16.2	-18.02	-17.3	-17.12	${}^{77}Mo \rightarrow {}^{75}Zr$	4.85	0	-13.8	-14.14	-13.99	-13.21
${}^{34}Ca \rightarrow {}^{32}Ar$	0.77	0	-5.8	-5.92	-6.03	-6.30	${}^{78}Mo \rightarrow {}^{76}Zr$	4.44	0	-12.88	-13.9	-12.99	-12.35
${}^{32}Sc \rightarrow {}^{30}K$	8.09	5	-19.41	-19.68	-19.93	-19.31	${}^{79}Mo \rightarrow {}^{77}Zr$	4.01	0	-11.73	-12.85	-11.82	-11.27
${}^{33}Sc \rightarrow {}^{31}K$	7.49	3	-19.54	-19.48	-20.37	-19.12	${}^{79}Tc \rightarrow {}^{77}Nb$	5.02	0	-13.85	-13.94	-14.02	-13.19
${}^{34}Sc \rightarrow {}^{32}K$	5.37	5	-19.55	-18.74	-19.07	-18.34	${}^{80}Tc \rightarrow {}^{78}Nb$	4.43	0	-12.51	-12.83	-12.6	-12.02
${}^{35}Sc \rightarrow {}^{33}K$	3.7	3	-17.71	-18.11	-17.6	-17.08	${}^{81}Tc \rightarrow {}^{79}Nb$	3.71	0	-10.42	-8.63	-10.45	-10.13
${}^{36}Sc \rightarrow {}^{34}K$	0.8	3	-5.44	-5.31	-4.26	-3.50	${}^{81}Tc \rightarrow {}^{79}Nb$	2.88	0	-6.03	-6.56	-7.0	-7.06
${}^{34}Ti \rightarrow {}^{32}Ca$	8.04	0	-19.55	-19.52	-20.9	-19.43	${}^{83}Tc \rightarrow {}^{81}Nb$	2.25	0	-3.22	-2.07	-3.16	-2.60
${}^{35}Ti \rightarrow {}^{33}Ca$	6.28	0	-19.56	-18.92	-20.63	-18.95	${}^{81}Ru \rightarrow {}^{79}Mo$	4.35	0	-11.96	-12.64	-11.99	-11.70
${}^{36}Ti \rightarrow {}^{34}Ca$	5.79	0	-19.25	-18.71	-20.34	-18.71	${}^{82}Ru \rightarrow {}^{80}Mo$	4.55	0	-12.49	-12.6	-12.56	-11.94
${}^{37}Ti \rightarrow {}^{35}Ca$	3.45	0	-16.57	-17.85	-17.65	-17.13	${}^{83}Ru \rightarrow {}^{81}Mo$	4.94	0	-13.42	-13.51	-13.58	-12.50
${}^{38}Ti \rightarrow {}^{36}Ca$	0.96	0	-5.81	-7.01	-6.04	-7.08	${}^{84}Ru \rightarrow {}^{82}Mo$	2.57	0	-3.81	-3.1	-4.75	-4.58
${}^{36}V \rightarrow {}^{34}Sc$	7.59	0	-19.21	-19.22	-20.87	-19.21	${}^{85}Ru \rightarrow {}^{83}Mo$	4.25	0	-11.75	-13.12	-11.83	-10.82
${}^{37}V \rightarrow {}^{35}Sc$	6.08	0	-19.25	-18.68	-20.35	-18.73	${}^{86}Ru \rightarrow {}^{84}Mo$	2.78	0	-5.11	-5.94	-6.06	-5.29
${}^{38}V \rightarrow {}^{36}Sc$	5.36	0	-18.7	-18.37	-19.77	-18.35	${}^{83}Rh \rightarrow {}^{81}Tc$	4.45	0	-11.89	-11.93	-11.91	-11.58
${}^{39}V \rightarrow {}^{37}Sc$	2.44	0	-14.48	-17.36	-14.72	-15.19	${}^{84}Rh \rightarrow {}^{82}Tc$	4.29	0	-11.47	-11.79	-11.49	-11.08
${}^{40}V \rightarrow {}^{38}Sc$	0.68	0	-1.64	-1.18	-0.8	-0.70	${}^{85}Rh \rightarrow {}^{83}Tc$	3.36	0	-7.3	-7.79	-8.27	-8.17
${}^{38}Cr \rightarrow {}^{36}Ti$	7.54	0	-19.07	-19.04	-20.87	-19.07	${}^{86}Rh \rightarrow {}^{84}Tc$	2.44	0	-2.42	-1.38	-3.36	-3.13
${}^{39}Cr \rightarrow {}^{37}Ti$	6.1	0	-19.09	-18.5	-20.17	-18.57	${}^{87}Rh \rightarrow {}^{85}Tc$	3.42	0	-7.62	-7.64	-8.59	-7.98
${}^{40}Cr \rightarrow {}^{38}Ti$	4.36	0	-17.34	-17.86	-18.43	-17.55	${}^{88}Rh \rightarrow {}^{86}Tc$	2.51	0	-2.98	-2.45	-3.93	-3.05
${}^{41}Cr \rightarrow {}^{39}Ti$	1.62	0	-10.19	-11.94	-10.4	-11.70	${}^{86}Pd \rightarrow {}^{84}Ru$	2.9	0	-3.44	-5.63	-5.58	-4.93
${}^{40}Mn \rightarrow {}^{38}V$	7.64	0	-19.01	-18.89	-20.88	-18.95	${}^{89}Pd \rightarrow {}^{87}Ru$	3.25	0	-7.44	-5.53	-7.39	-6.86
${}^{41}Mn \rightarrow {}^{39}V$	5.68	0	-18.56	-18.19	-19.62	-18.21	${}^{90}Pd \rightarrow {}^{88}Ru$	1.83	0	-0.51	-0.72	-0.65	-0.34
${}^{42}Mn \rightarrow {}^{40}V$	3.93	0	-16.41	-17.54	-17.46	-16.92	${}^{88}Ag \rightarrow {}^{86}Rh$	2.86	0	-3.95	-3.45	-4.87	-4.90
${}^{42}Fe \rightarrow {}^{40}Cr$	6.78	0	-19.85	-18.42	-20.3	-18.56	${}^{91}Ag \rightarrow {}^{89}Rh$	2.54	0	-2.03	-2.35	-2.97	-2.39
${}^{43}Fe \rightarrow {}^{41}Cr$	5.71	0	-18.37	-18	-19.44	-18.04	${}^{90}Cd \rightarrow {}^{88}Pd$	2.9	0	-3.68	-4.9	-4.61	-4.93
${}^{44}Fe \rightarrow {}^{42}Cr$	3.05	0	-14.15	-14.08	-15.19	-15.44	${}^{92}In \rightarrow {}^{90}Ag$	2.7	0	-1.98	-2.53	-2.91	-3.21
${}^{45}Fe \rightarrow {}^{43}Cr$	1.13	0	-3.554	-4.42	-4.61	-5.85	${}^{97}Sb \rightarrow {}^{95}In$	5.1	2	-10.37	-11.84	-11.18	-9.90
${}^{44}Co \rightarrow {}^{42}Mn$	7.09	0	-19.26	-18.35	-20.38	-18.50	${}^{99}Te \rightarrow {}^{97}Sn$	7.92	0	-15.37	-14.35	-16.53	-14.41
${}^{45}Co \rightarrow {}^{43}Mn$	5.12	0	-17.53	-17.62	-18.61	-17.50	${}^{100}Te \rightarrow {}^{98}Sn$	6.91	0	-13.99	-13.52	-15.09	-13.05
${}^{46}Co \rightarrow {}^{44}Mn$	2.35	0	-12.22	-11.68	-12.42	-13.37	${}^{101}Te \rightarrow {}^{99}Sn$	4.7	0	-9.33	-8.12	-10.28	-9.25
${}^{47}Co \rightarrow {}^{45}Mn$	1.23	0	-4.83	-5.26	-5.0	-6.19	${}^{102}Te \rightarrow {}^{100}Sn$	4.17	0	-7.66	-6.67	-8.6	-7.60
${}^{46}Ni \rightarrow {}^{44}Fe$	6.53	0	-18.65	-17.95	-19.76	-18.14	${}^{101}I \rightarrow {}^{99}Sb$	7.64	0	-14.8	-14.06	-15.96	-13.70
${}^{47}Ni \rightarrow {}^{45}Fe$	4.04	0	-15.6	-17.09	-16.65	-16.34	${}^{102}I \rightarrow {}^{100}Sb$	6.92	0	-13.76	-13.74	-14.84	-12.84
${}^{48}Ni \rightarrow {}^{46}Fe$	1.29	0	-2.02	-2.23	-4.65	-2.32	${}^{103}I \rightarrow {}^{101}Sb$	6.42	0	-12.94	-12.62	-13.98	-12.10
${}^{49}Ni \rightarrow {}^{47}Fe$	1.04	0	-1.47	-1.02	-1.57	-2.40	${}^{105}I \rightarrow {}^{103}Sb$	3.16	0	-2.84	-3.29	-3.76	-2.65
${}^{48}Cu \rightarrow {}^{46}Co$	7.06	2	-18.53	-17.93	-19.51	-17.80	${}^{103}Xe \rightarrow {}^{101}Te$	8.05	0	-15.15	-13.52	-16.27	-13.86
${}^{49}Cu \rightarrow {}^{47}Co$	4.39	2	-16.37	-17	-16.35	-15.83	${}^{104}Xe \rightarrow {}^{102}Te$	7.58	0	-14.55	-13.29	-15.68	-13.33
${}^{50}Cu \rightarrow {}^{48}Co$	2.91	2	-14.43	-14.46	-13.13	-13.09	${}^{105}Xe \rightarrow {}^{103}Te$	6.32	2	-12.21	-11.35	-13.05	-11.12
${}^{51}Zn \rightarrow {}^{49}Ni$	5.46	0	-17.94	-17.15	-18.28	-17.09	${}^{106}Xe \rightarrow {}^{104}Te$	5.25	0	-10.18	-10.9	-11.15	-9.72
${}^{52}Zn \rightarrow {}^{50}Ni$	4.39	0	-15.67	-16.71	-16.71	-16.05	${}^{107}Xe \rightarrow {}^{105}Te$	3.68	0	-4.96	-4.46	-5.88	-4.78
${}^{53}Zn \rightarrow {}^{51}Ni$	3.04	0	-13.35	-16.19	-13.57	-13.74	${}^{106}Cs \rightarrow {}^{104}I$	7.24	0	-13.78	-11.54	-14.84	-12.76
${}^{54}Zn \rightarrow {}^{52}Ni$	1.48	0	-2.45	-2.58	-5.19	-3.68	${}^{107}Cs \rightarrow {}^{105}I$	6.19	0	-11.99	-11.06	-12.97	-11.27
${}^{53}Ga \rightarrow {}^{51}Cu$	7.38	0	-18.85	-17.58	-19.94	-17.90	${}^{108}Cs \rightarrow {}^{106}I$	4.54	0	-7.81	-8.73	-8.75	-7.64
${}^{54}Ga \rightarrow {}^{52}Cu$	5.68	0	-17.23	-16.93	-18.41	-16.92	${}^{109}Cs \rightarrow {}^{107}I$	3.25	0	-1.17	-1.95	-3.31	-2.18
${}^{55}Ga \$													

Table 3.2 and 3.5 summarises the half-lives obtained using the three models with available experimental values for one proton decay mode. From this observation, it is clear that the values obtained using the CPPM model are close to the experiments than that of the other two models. However, quite discrepancy has seen in case of the parent nuclei ^{146}Tm , ^{147}Tm , ^{156}Ta and ^{166}Ir . Similarly, the half-lives obtained using ELDM and MGLDM models are also producing experimental values successfully. Moreover, for the nuclei ^{177}Tl , ^{161}Re and ^{167}Ir the deviation produced by the ELDM is larger when compared to CPPM and MGLDM. ^{146}Tm and ^{157}Ta will have change in the order of one magnitude when compared to experimental values in case of MGLDM. The overall deviation may vary between one or two magnitude of order using the three models, which are within the limit of experimental error. Similarly, comparison is extended to two proton decay and the same is listed in table 3.5. From this comparison, it is noticed that although the three models used in the present work are quite good in agreement with that of available experiments. The semi-empirical formulae for one and two proton decay half-lives were constructed based on the half-lives produced by present work. Among these three models, the proton decay half-lives produced by the CPPM are closer to the experiments than that of other two models. Eventually, we have used the proton decay half-lives produced by CPPM for the construction of semi-empirical formulae. The uncertainty of the model is also included in the error associated with this calculation. From the table it is inferred that the standard deviation is smaller for 1P and 2P in CPPM when compared to ELDM and MGLDM.

Eventually, to know the predictive power of CPPM, ELDM and MGLDM in producing the proton decay logarithmic half-lives, we have also evaluated the mean squared error. The sum of squared residuals ($\text{SSR} = \sum_{i=1}^n e_i^2$) are calculated. Further, the mean squared errors ($\sigma^2 = \frac{\text{SSR}}{n-2}$) is evaluated. The evaluated mean squared error for CPPM, ELDM and MGLDM in predicting the one proton decay logarithmic half lives are 0.06, 0.13 and 0.14 respectively. Similarly, the mean

squared error for CPPM, ELDM and MGLDM in predicting the two proton decay logarithmic half-lives are 0.19, 0.67 and 1.1 respectively. From this analysis, it can be concluded that CPPM model produces one proton and two proton decay half-lives values close to experiments than the other two models.

CHAPTER 4

Competition between different decay modes

New isotopes were explored by studying the competition between different decay modes such as alpha-decay, beta-decay, spontaneous fission and proton-decay. Thus the proton decay half-lives evaluated using the present work is compared with that of other decay modes. The decay mode which is having smaller half-life than the other decay modes is identified as dominant decay mode. Eventually by studying the competition between different decay modes new proton emitters were explored in the different regions such as lanthanides, heavy nuclei, actinides and superheavy nuclei.

4.1 Method of Calculation of half-lives

According to WKB approximation (Wentzel-Kramers-Brillouin) of the penetration probability P through the potential barrier were studied for the cluster and alpha decay by the following equation;

$$P = \exp \left\{ -\frac{2}{\hbar} \int_{R_a}^{R_b} \sqrt{2\mu(V - Q)} dr \right\} \quad (4.1)$$

Where V is the potential and it is calculated using the procedure explained in the chapter 3. where μ is the reduced mass of proton decay system, R_a and R_b are the inner and outer turning points

and these turning points were evaluated using following conditions;

$$V_T(R_a) = Q = V_T(R_b) \quad (4.2)$$

The alpha decay half-life is studied using following equation ;

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{vPS_p} \quad (4.3)$$

Where λ is decay constant and v is the assault frequency. S_p is the spectroscopic factor and it is model dependent and very sensitive to decay energy. It is also evident from the literature [282, 283] that the spectroscopic factors are assumed as one in proton decay half-life calculation while using the WKB approximation. In the present work, we have used WKB approximation and accurate recent mass excess values in the calculation of decay energies. Thus the spectroscopic factors are assumed to be one. E_v is the empirical vibration energy and it is evaluated using the following equations;

$$v = \frac{\omega}{2\pi} = \frac{2E_v}{h} \quad (4.4)$$

$$E_v = Q \left\{ 0.056 + 0.039 \exp \left[\frac{4 - A_2}{2.5} \right] \right\} \text{ for } A_2 \geq 4 \quad (4.5)$$

4.2 Results

4.2.1 Proton radioactivity of Lanthanides

4.2.1.1 Empirical formula

The variation of experimental $\log(T_{1/2})$ of proton decay in the lanthanide region as a function of $Z_d/\sqrt{Q}$ is shown in the Fig.4.1. We have fitted empirical relation for experimental $\log(T_{1/2})$

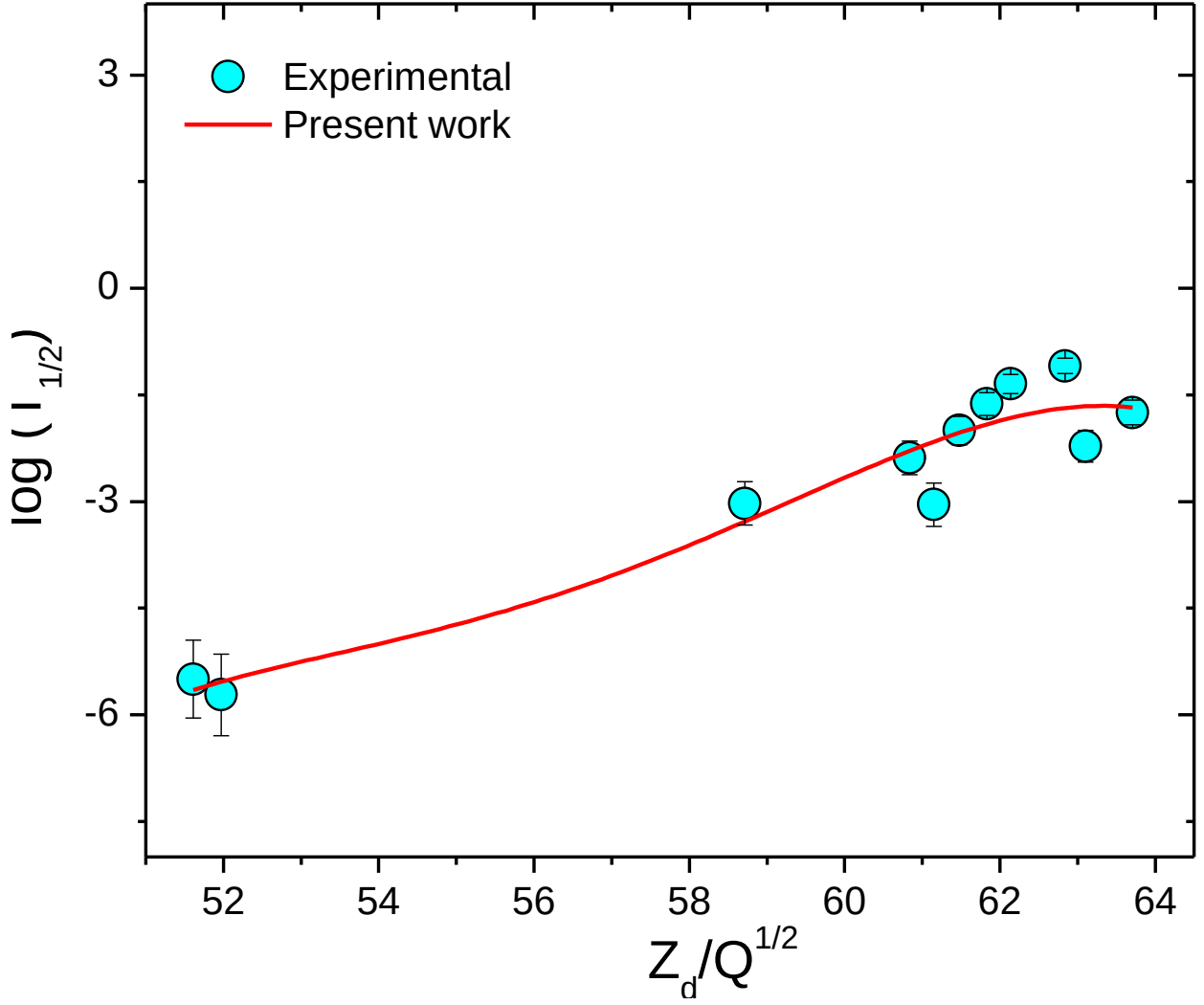


Fig. 4.1 Variation of experimental and present formula half lives as a function of $Z_d/\sqrt{Q}$.

in such a way that it should have maximum R^2 and minimum residual sum of squares. Hence, proposed empirical formula for $\log(T_{1/2})$ of proton decay is given below; the half-lives for proton decay in the lanthanide region as a function of fissility parameter $Z_d/\sqrt{Q}$ is given by:

$$\log(T_{1/2}) = \sum_{i=0}^{i=4} A_i \left(\frac{Z_d}{\sqrt{Q}} \right)^i \quad (4.6)$$

Where Z_d is the atomic number of the daughter nuclei and Q is the decay energy. The fitting parameters A_0 , A_1 , A_2 , A_3 and A_4 are having the values -1.61, -20.82×10^{-2} , 71×10^{-4} , -8.18×10^{-5} and $-3.11 \times 10^{-7} \text{ MeV}^{1/2}\text{s}$ respectively.

Table 4.1 The range of lanthanide isotopes having positive proton decay energy

Z	Range of mass number studied
57	$110 \leq A \leq 119$
58	$113 \leq A \leq 115$
59	$115 \leq A \leq 123$
60	$118 \leq A \leq 119$
61	$120 \leq A \leq 128$
62	$123 \leq A \leq 125$
63	$125 \leq A \leq 135$
64	$128 \leq A \leq 130$
65	$130 \leq A \leq 139$
66	$133 \leq A \leq 135$
67	$136 \leq A \leq 143$
68	$138 \leq A \leq 139$
69	$141 \leq A \leq 149$
70	$143 \leq A \leq 147$
71	$146 \leq A \leq 155$

4.2.1.2 Results on proton radioactivity of Lanthanides

The phenomenon of proton decay is treated as the transmission of the proton across a potential barrier developed due to combined effect of Coulombic and nuclear potential [236]. Experimentally 11 proton emitters were identified in the lanthanide region. We have studied the proton decay for lanthanide nuclei in which its decay energy (Q_p) is positive. In the present work, it is of first kind where we systematically explored the unexplored 24 proton decay emitters in the lanthanide region. These proton emitters having half-lives in terms of $1s - 1\mu s$. Generally, the half-lives of proton emitters nuclei have been determined by quantum-mechanical tunneling calculation through a potential barrier [284].

The universal function proposed by five different versions of Coulomb and nuclear proximity potentials such as Prox. 13, Prox. 77, MP 77, Ng 80 and Bass 80 are used to calculate the half lives of proton emitters in the lanthanide region for different isotopes of lanthanides. Table 4.1 gives the range of studied lanthanide isotopes having positive proton decay energy. In order to study

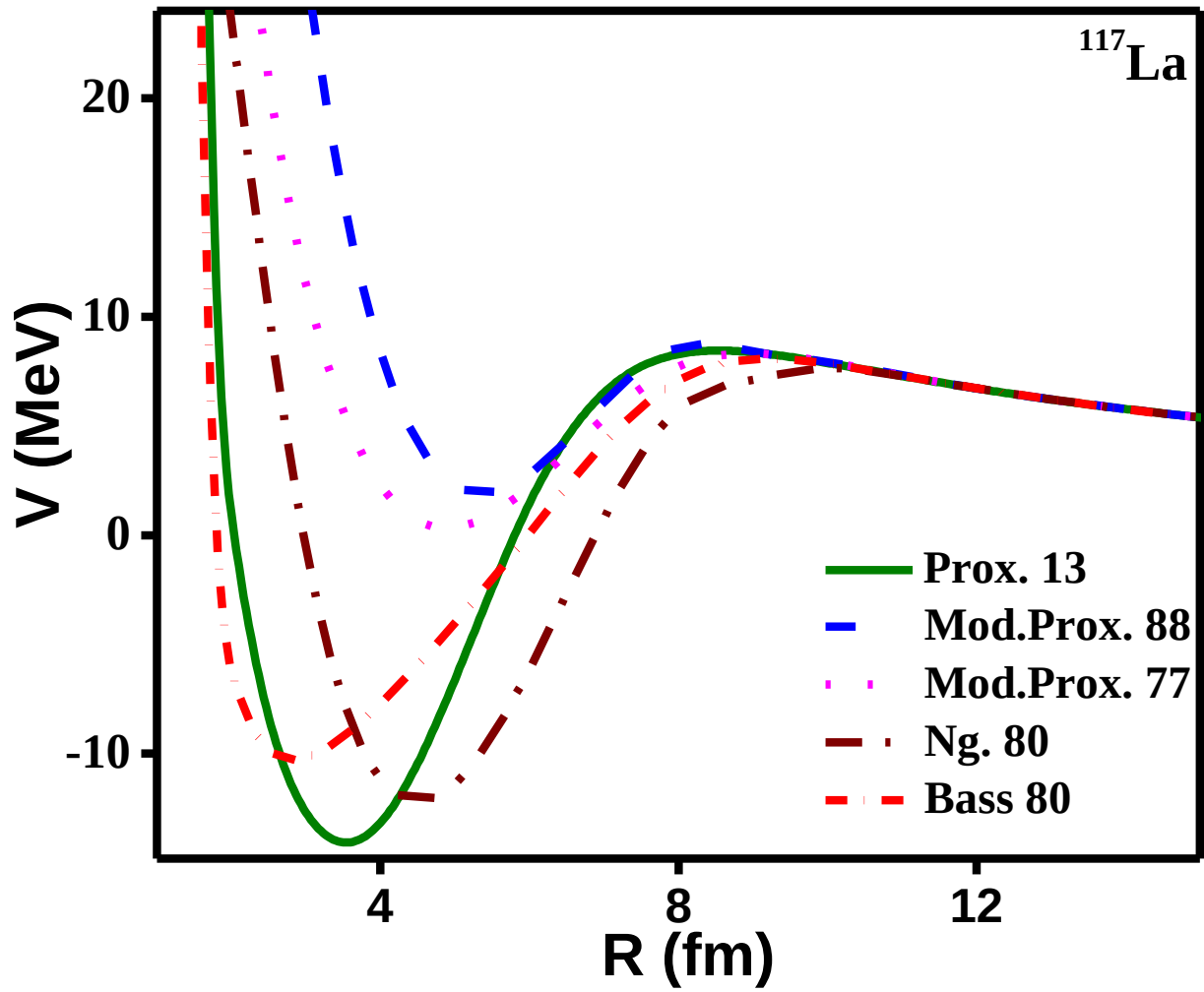


Fig. 4.2 Variation of potential energy as a function of R for different proximity potentials.

whether the shape of the potential leads to different half lives, the different proximity potentials are plotted as a function of R as shown in Fig. 4.2. X- axis corresponds to the distance between interacting nuclei and Y -axis corresponds to interacting potential. The area under the potential curve is directly proportional to the penetration probability. If the area under the potential curve is more, the probability of penetration is more which clearly indicates the short half life of the decay particle and vice versa. In the present study, from the Fig. 4.2, it is observed that, the area under the curve is found to be maximum for Bass 80 and then follows the order Prox. 13, Mod. Prox. 77, Prox. 77 and Ng 80.

The calculated proton decay half lives are compared with the experiments. The calculated

Table 4.2 Comparison of evaluated proton decay halflives using different proximity functions with that of the experiments

Proton emitter	Q_p MeV	$\log T_{1/2}(s)$							Present formula
		Expt	Ng. 80	Mp. 88	Mp. 77	Bass. 80	Prox. 13		
$^{111}La \rightarrow ^{110}Ba$	4.321	-	-3.80	-6.96	-6.82	-17.94	-17.76	-3.51	
$^{112}La \rightarrow ^{111}Ba$	3.791	-	-3.62	-6.62	-6.46	-17.18	-17.03	-3.45	
$^{113}La \rightarrow ^{112}Ba$	3.071	-	-3.38	-5.85	-5.67	-15.86	-15.51	-3.35	
$^{114}La \rightarrow ^{113}Ba$	1.891	-	-3.13	-4.25	-4.00	-12.21	-12.07	-2.98	
$^{115}La \rightarrow ^{114}Ba$	2.517	-	-3.01	-5.19	-4.99	-14.51	-14.13	-3.21	
$^{116}La \rightarrow ^{115}Ba$	1.206	-	-2.77	-2.18	-1.89	-7.89	-7.76	-2.51	
$^{117}La \rightarrow ^{116}Ba$	0.803	-1.63	-2.58	-2.11	-2.43	-2.99	-2.92	-2.12	
$^{118}La \rightarrow ^{117}Ba$	0.378	-	-2.36	-2.33	-3.68	-2.00	-6.22	-2.13	
$^{113}Ce \rightarrow ^{112}La$	1.971	-	-3.45	-4.15	-3.88	-12.32	-11.94	-2.98	
$^{114}Ce \rightarrow ^{113}La$	1.491	-	-3.26	-2.91	-2.61	-9.76	-9.37	-2.71	
$^{115}Ce \rightarrow ^{114}La$	0.891	-	-3.06	-0.15	-0.79	-3.94	-3.55	-2.17	
$^{115}Pr \rightarrow ^{114}Ce$	3.861	-	-3.44	-6.58	-6.41	-17.03	-16.91	-3.43	
$^{117}Pr \rightarrow ^{116}Ce$	2.811	-	-3.00	-5.43	-5.24	-14.95	-14.61	-3.25	
$^{119}Pr \rightarrow ^{118}Ce$	1.411	-	-2.59	-2.74	-2.43	-8.98	-8.88	-2.61	
$^{121}Pr \rightarrow ^{120}Ce$	0.837	-2	-2.24	0.37	0.02	-2.80	-2.43	-2.11	
$^{122}Pr \rightarrow ^{121}Ce$	0.526	-	-2.05	3.44	3.79	1.11	4.00	-2.01	
$^{123}Pr \rightarrow ^{123}Ce$	0.209	-	-0.56	7.73	7.58	3.93	9.76	-0.46	
$^{118}Nd \rightarrow ^{117}Pr$	1.131	-	-2.89	-1.44	-1.11	-6.21	-6.15	-2.33	
$^{119}Nd \rightarrow ^{118}Pr$	0.741	-	-2.71	1.36	0.50	-0.93	-0.31	-2.01	
$^{121}Pm \rightarrow ^{120}Nd$	3.301	-	-2.70	-6.01	-5.82	-15.77	-15.69	-3.31	
$^{123}Pm \rightarrow ^{122}Nd$	1.981	-	-2.28	-4.09	-3.81	-11.78	-11.44	-2.89	
$^{125}Pm \rightarrow ^{124}Nd$	0.438	-	-1.85	5.48	6.15	2.47	6.45	-2.16	
$^{127}Pm \rightarrow ^{126}Nd$	0.545	-	-1.56	3.58	3.94	1.44	4.63	-2.01	
$^{124}Sm \rightarrow ^{123}Pm$	0.481	-	-2.16	5.08	5.12	2.35	6.02	-2.11	
$^{129}Eu \rightarrow ^{128}Sm$	1.459	-	-1.61	-2.47	-2.14	-8.31	-7.98	-2.49	
$^{130}Eu \rightarrow ^{129}Sm$	1.028	-3.05	-2.43	-0.31	-1.69	-4.14	-4.15	-2.15	
$^{131}Eu \rightarrow ^{130}Sm$	0.939	-1.75	-1.28	0.04	0.40	-2.94	-2.64	-2.10	
$^{133}Eu \rightarrow ^{132}Sm$	0.675	-	-0.96	2.53	3.81	0.63	1.84	-1.98	
$^{135}Tb \rightarrow ^{134}Gd$	0.524	-3.03	-2.91	-4.75	5.18	2.66	6.43	-2.11	
$^{140}Ho \rightarrow ^{139}Dy$	1.094	-2.23	-2.50	-0.36	0.06	-3.69	-3.44	-2.11	
$^{141}Ho \rightarrow ^{140}Dy$	1.176	-2.39	-2.35	-0.61	-2.01	-4.65	-4.72	-2.16	
$^{144}Tm \rightarrow ^{143}Er$	1.712	-5.73	-4.12	-6.34	-6.83	-4.11	-8.66	-5.53	
$^{145}Tm \rightarrow ^{144}Er$	1.736	-5.49	-5.07	-2.59	-1.24	-8.79	-8.84	-5.63	
$^{150}Lu \rightarrow ^{149}Yb$	1.27	-1.35	-1.10	-0.58	-2.28	-4.49	-4.61	-2.13	
$^{151}Lu \rightarrow ^{150}Yb$	1.241	-1.09	-1.78	-0.64	-0.22	-4.19	-3.99	-2.11	

Table 4.3 Mean square error with respect to experiments for different proximity functions and proposed present formula

Proximity function	Ng. 80	MP. 88	MP. 77	Bass. 80	Prox. 13	Present formula
σ	1.23	1.52	1.64	1.82	2.12	1.60

Mean square error of different proximity functions with respect to experiments is shown in Table.

4.3. The sum of the squared residuals between the $\log(T_{1/2})$ of experimental and different proximity potentials ($SSR = \sum_{i=1}^n e_i^2$), where e_i is the i^{th} residual or difference and n is the number of data points. Mean square error with respect to experiments for different proximity functions and proposed present formula ($\sigma_\epsilon^2 = \frac{SSR}{n-2}$) are shown in Table. 4.3. From Table. 4.3, it is observed that the mean square error was found to be less for Ng 80 compared to other proximity potentials. The experimental values are found to be agree well with Ng 80 among the studied proximity functions. Thus, Ng 80 proximity potential was used to study the competition between different decay modes in the lanthanide region.

We have constructed new simple empirical relation to calculate the half life of proton emitters in the lanthanide region for different isotopes of lanthanides other than the above mentioned models. The constructed empirical formula is given in Eq. 4.6. The half-lives values produced with proximity function NG80 is close to the experiment. From the comparison of mean square error it shows that MP88 is better than the present empirical formula. Mean square error difference between Mp88 and Present Formula is 0.08 and it is almost negligible, means both methods used to calculate half lives will produce the almost same deviation. But, to calculate half lives using the MP88 proximity function involves many physical quantities. Whereas, the present formula produces the half lives with simple inputs of Z_d and Q values and this we may call pocket formula. So that the present formula is more advantageous than the MP88. The evaluated proton decay half-lives using present formula and different proximity functions along with the experiments are presented in the table 4.2. From this table, it is found that the present formula produces proton

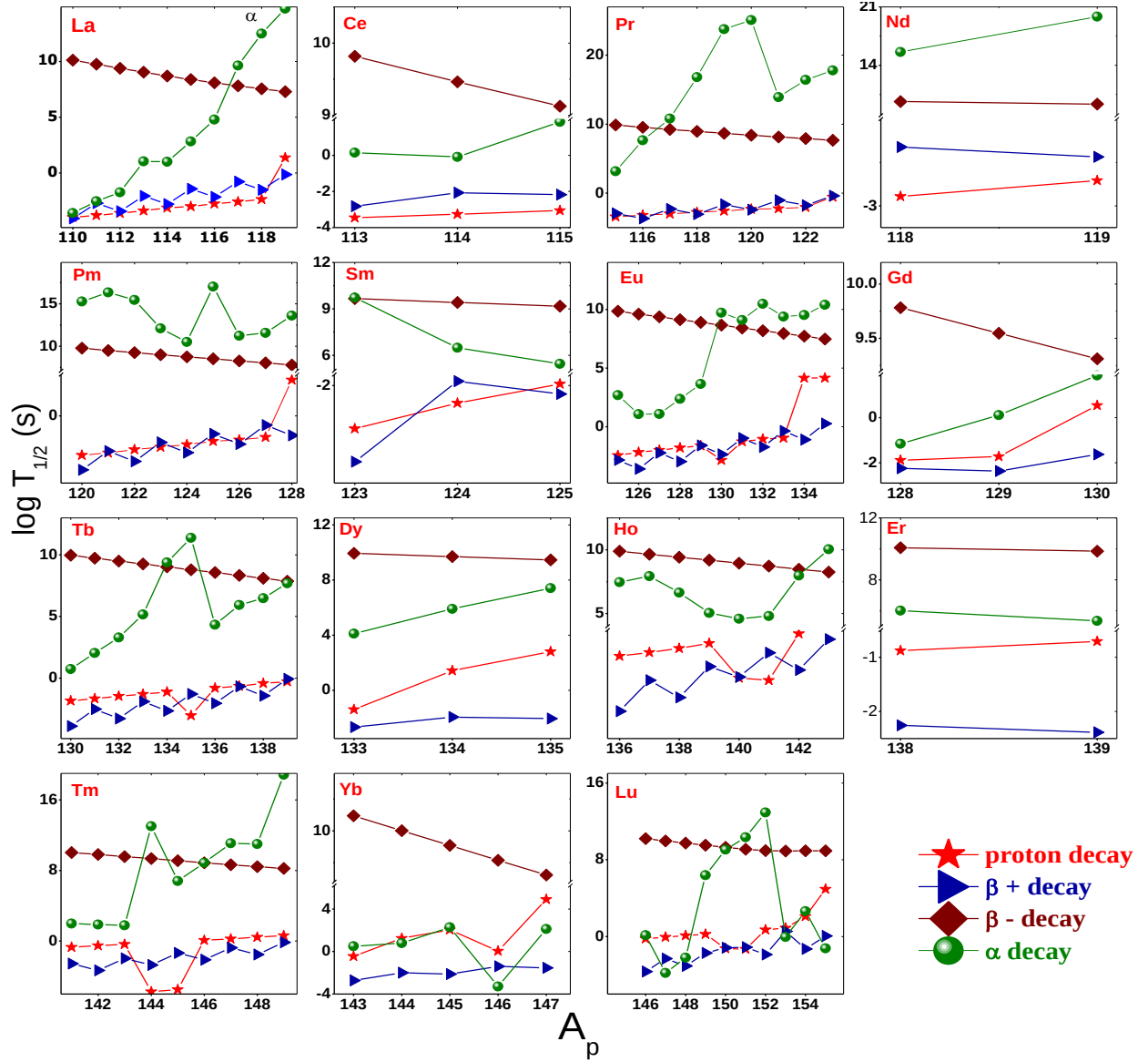
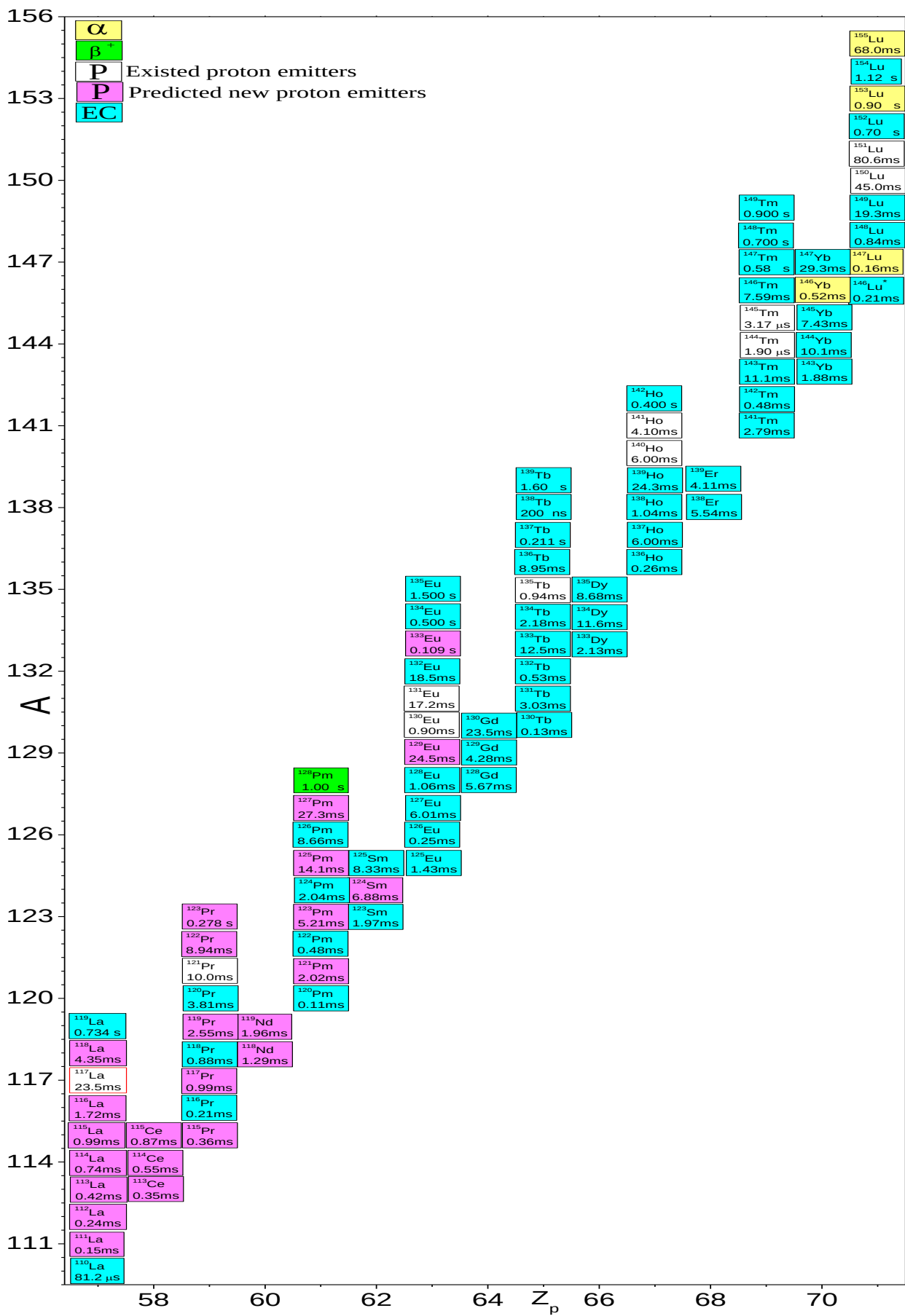


Fig. 4.3 Competition between different decay modes for lanthanide nuclei.

decay half lives close to the experiments. Proton decay energies are also presented in this table 4.2.

Dominant decay mode can be identified by studying the competition between the different possible decay modes such as alpha, β^+ , β^- , Spontaneous fission (SF) and proton decay. We have also calculated the half lives of possible decay modes using the well established formulae available in the literature [alpha[285], β^+ [286], β^- [286] and SF[287]. The competition between different decay modes in the studied lanthanide region is shown in Fig. 4.3. The decay mode which is



having shorter half life among the possible decay modes will be identified as the dominant decay mode. The observation of Fig. 4.3 clearly indicates that some isotopes of lanthanides with atomic number ranging between 57 - 63 (La, Ce, Pr, Nd, Pm, Sm and Eu) are newly identified as proton emitters in the lanthanide region whereas the Gadollinium, Dysprosium and Erbium shows β^+ decay as a dominant decay mode. In Terbium, Holmium, Thulium, even though maximum isotopes are β^+ decay emitters, few of them are proton decay emitters. In Ytterbium and Lutetium, few isotopes are β^+ decay emitters and few of them are α decay emitters. The newly identified 24 proton emitters in the lanthanide region are ^{111}La , ^{112}La , ^{113}La , ^{114}La , ^{115}La , ^{116}La , ^{118}La , ^{113}Ce , ^{114}Ce , ^{115}Ce , ^{115}Pr , ^{117}Pr , ^{119}Pr , ^{122}Pr , ^{123}Pr , ^{118}Nd , ^{119}Nd , ^{121}Pm , ^{123}Pm , ^{125}Pm , ^{127}Pm , ^{124}Sm , ^{129}Eu , ^{133}Eu . The different α , β^+ , existing proton emitters, nuclei with electron capture decay mode and the formula predicted new proton emitters are shown in the Nuclide chart (Fig. 4.4). The predicted new 24 proton emitters are highlighted in pink color, whereas α , β^+ , electron capture and existing proton emitters are highlighted in yellow, green, aqua blue and brick red respectively.

4.2.1.3 Systematics of proton radioactivity in Dysprosium

The competing decay modes such as proton-radioactivity, β^+ -decay, β^- -decay and alpha decay were studied in the Dysprosium of mass number range $133 \leq A \leq 180$. The proton decay selection rule [109] is as follows;

$$J_p = J_d + J_{p^\ell} \quad (4.7)$$

$$\pi_p = \pi_d \pi_{p^\ell} (-1)^\ell \quad (4.8)$$

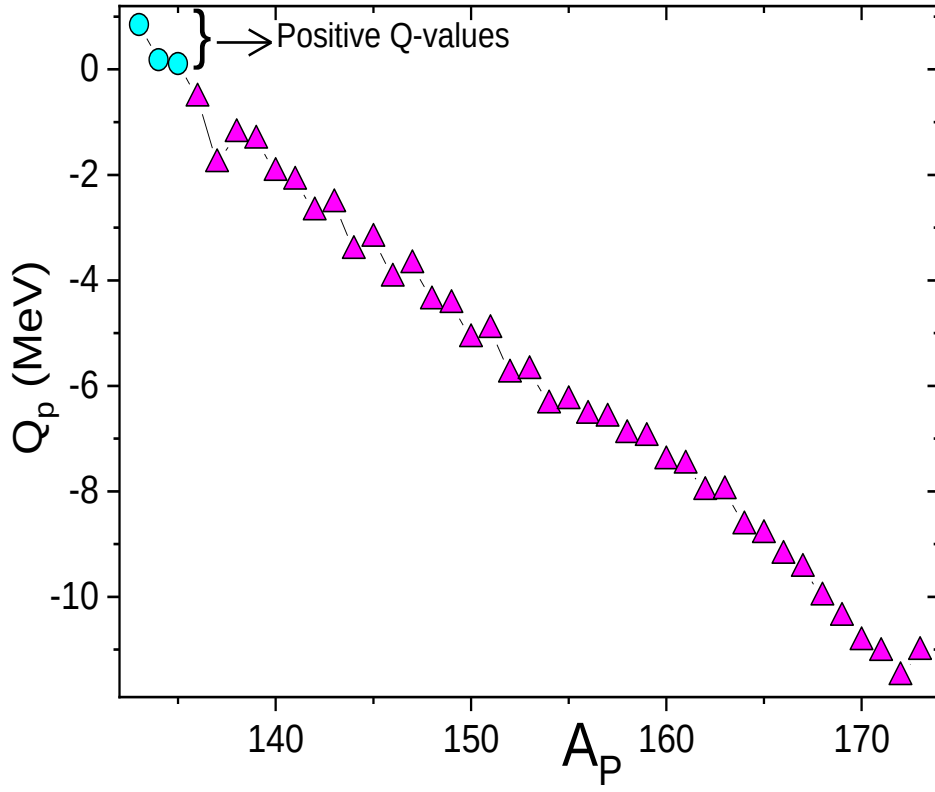


Fig. 4.5 The quantity of energy released during proton radioactivity and the mass number of Dysprosium parent nuclei.

where J_p , J_d and J_{p^ℓ} are the spin of parent, daughter and outgoing proton nuclei respectively. π_p , π_d and π_{p^ℓ} are parity of parent, daughter and outgoing proton respectively. The angular momentum for proton transition is evaluated using the ℓ_{min} as explained in literature [109]. Using recent mass excess values, the amount of energy released during proton radioactivity is calculated [288]. Wherever, recent mass excess values are not available, the mass excess values have been taken from the mass excess data [280]. The Q-value of one proton radioactivity can be calculated using the mass excess [288] and [280] values. When the Q-value is positive, i.e. $Q > 0$, proton radioactivity is energetically possible. The graph 4.5 illustrates the relationship between the quantity of energy released during proton radioactivity and the mass number of parent nuclei. The amount of

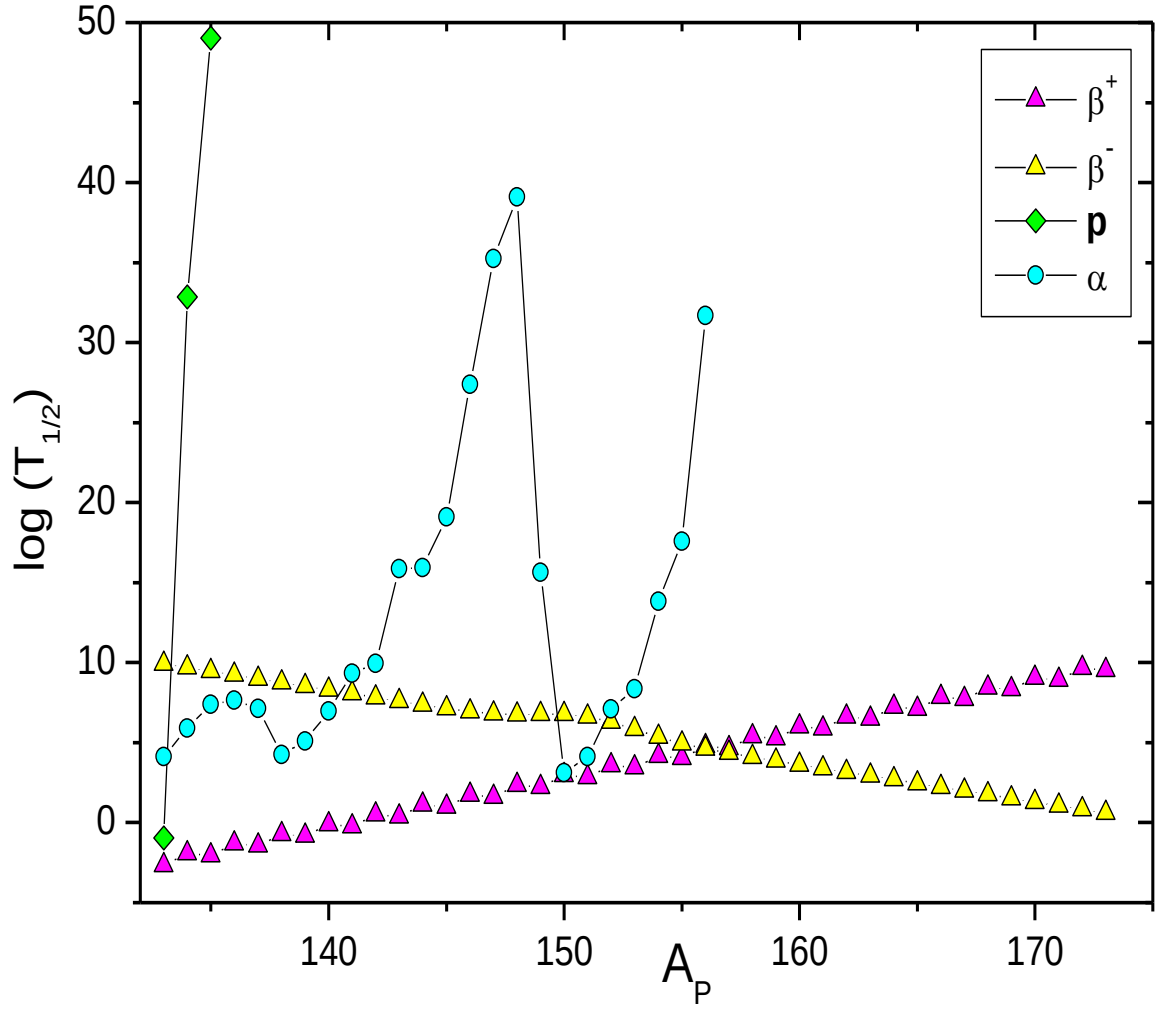


Fig. 4.6 Comparison of $\log T_{1/2}$ of different decay modes such as $\beta^\pm$, 1P, and α decay versus mass number of parent nuclei.

energy released gradually decreases as the mass number of parent nuclei increases. The Q-value is positive and proton radioactivity is energetically feasible for the isotope of $^{133-135}\text{Dy}$. Since, in addition to proton radioactivity, the competing decay modes such as β^- -decay and β^+ -decay and alpha decay have been evaluated as explained in theory section. The comparison of different competing decay modes such as β^- , β^+ , and α -decay with that of proton decay is studied and it is shown in figure 4.6. From the figure it is clear that β^+ -decay is dominant in the isotopes of $^{133-149}\text{Dy}$, $^{151-154}\text{Dy}$, alpha decay is dominant in ^{150}Dy and again in $^{155-173}\text{Dy}$, β^- -decay is dominant. Even though, there is less probability of proton radioactivity in isotopes of Dy but the

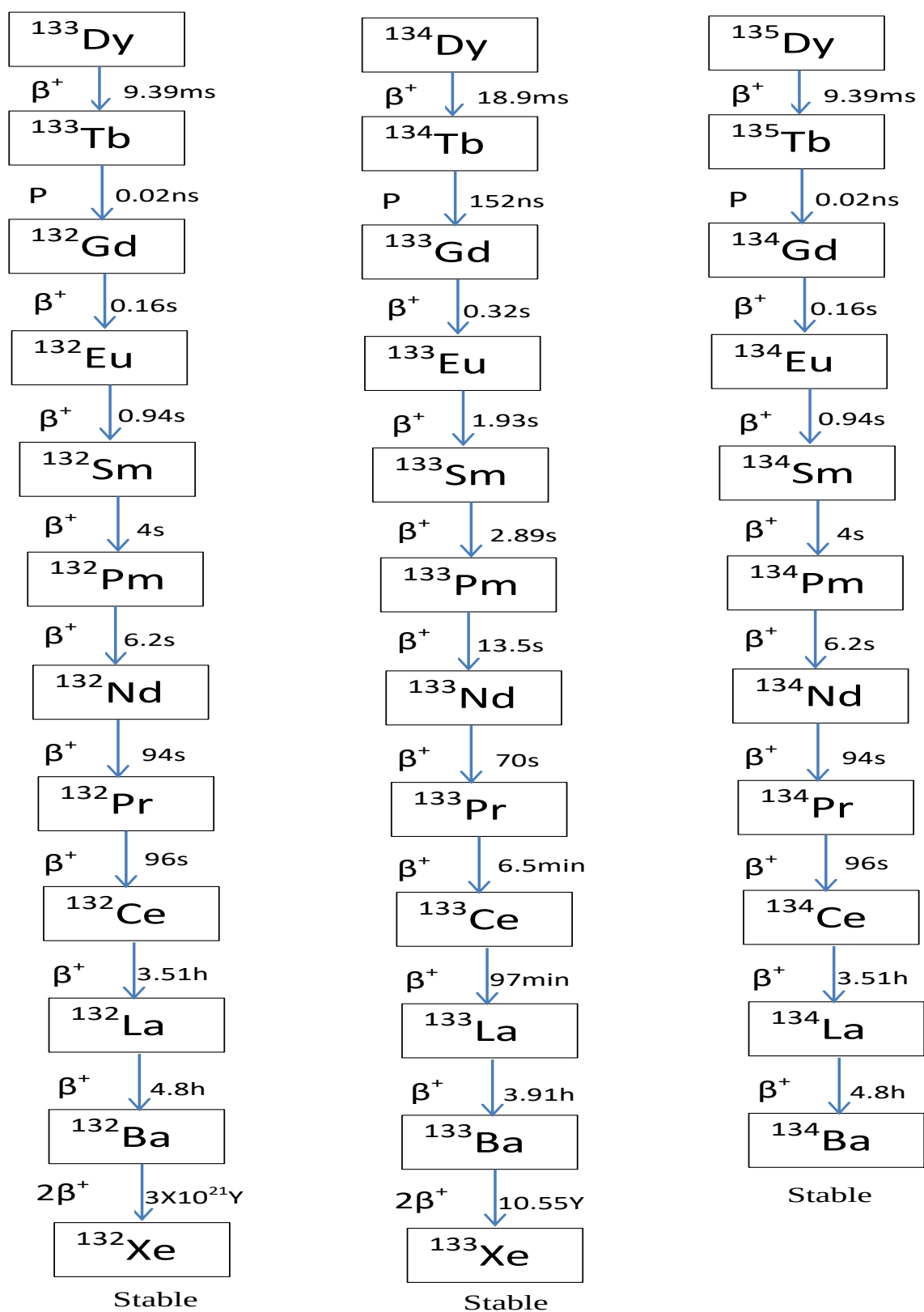


Fig. 4.7 Decay chains in the isotopes of $^{133-135}\text{Dy}$

Table 4.4 Identification of decay modes of $^{133-138}\text{Dy}$ and $^{177-180}\text{Dy}$

Nuclei	Q(MeV)	$\log T_{1/2}$	Decay mode
^{133}Dy	13.48	-2.03	β^+
^{134}Dy	10.67	-1.72	β^+
^{135}Dy	12.03	-1.42	β^+
^{136}Dy	7.93	-1.11	β^+
^{137}Dy	9.50	-0.81	β^+
^{138}Dy	7.67	-0.50	β^+
^{177}Dy	7.17	-0.35	β^-
^{178}Dy	6.05	-0.58	β^-
^{179}Dy	7.78	-0.82	β^-
^{180}Dy	6.73	-1.05	β^-

positive Q-value in isotopes of $^{133-135}\text{Dy}$ triggered us to analyse the decay chains of the same. The figure 4.7 shows the proton radioactivity of $^{133-135}\text{Tb}$. An isotope of ^{133}Dy doesn't sustain β^+ -decay and it decays to ^{133}Tb within the half-life of 9.39ms, again ^{133}Tb undergoes proton decay within 0.02ns and converts to ^{132}Gd . Later, the ^{132}Gd follows series of β^+ -decay up to ^{132}Xe and then it becomes stable. Similarly, the decay chains of ^{134}Dy and ^{135}Dy is also shown with the consisted decay chains until it reaches stable nuclei ^{133}Cs and ^{135}Ba respectively. The newly found isotopes of Dysprosium are shown in table 4.4. The decay modes of the newly identified isotopes of Dysprosium along with their decay energy and half-lives are also included. The newly discovered isotopes have decay energies ranging from 7MeV to 14MeV and half-lives ranging from milliseconds to seconds. These identified isotopes are first of its kind and may be useful in radiation physics.

4.2.2 Proton radioactivity of heavy nuclei ($72 \leq Z \leq 88$)

The amount of energy released during proton decay are studied using mass excess values available in the reference [36, 270, 289–291]. We have studied driving potential, penetration factor and half-lives of proton emission in the nuclei region $72 < Z < 88$ as explained in the theory section. The variation of amount of energy released during proton decay with the mass number of

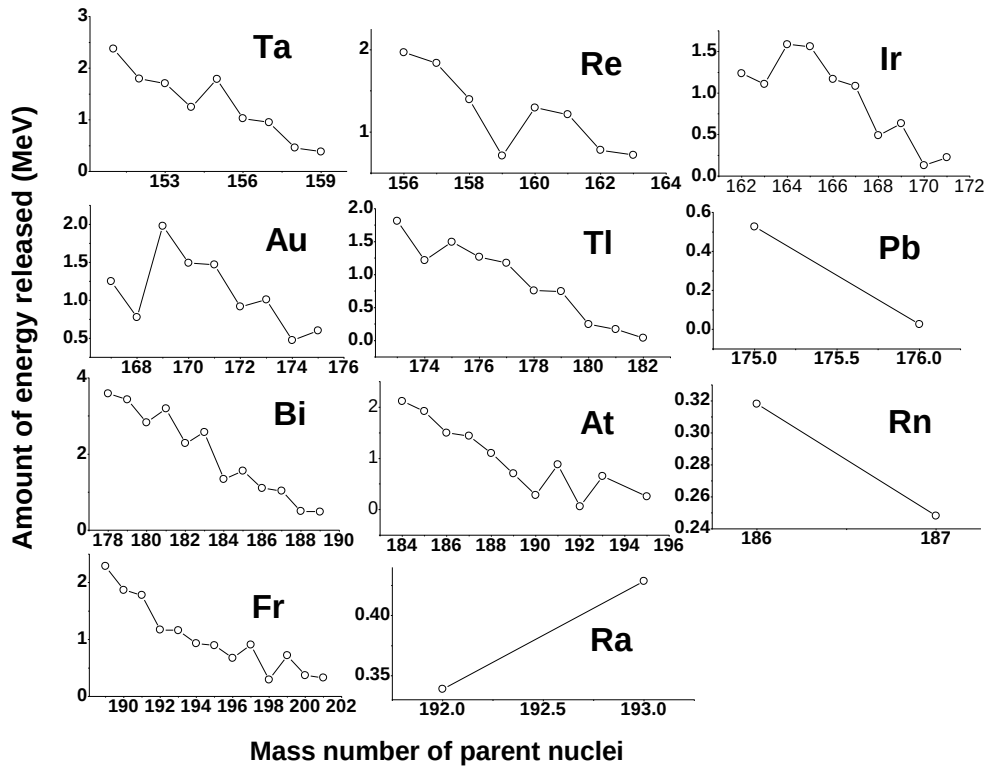


Fig. 4.8 A variation of amount of energy released during the proton emission in the nuclei region $72 < Z < 88$ with the mass number of parent nuclei.

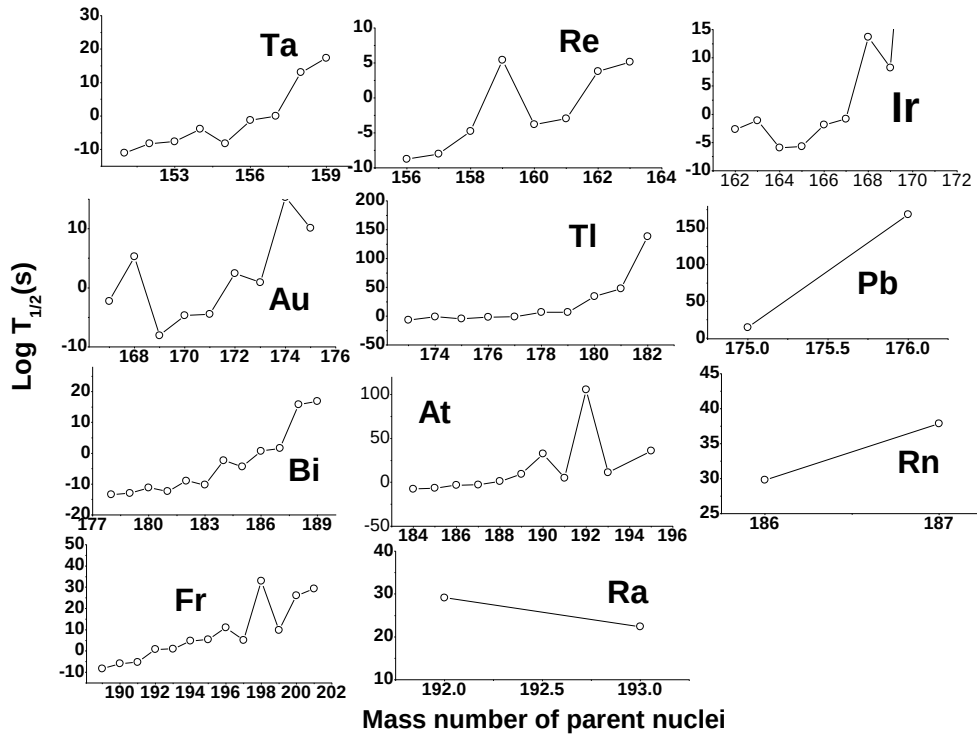


Fig. 4.9 A variation of logarithmic half-lives for the proton emission in the nuclei region $72 < Z < 88$ with the mass number of parent nuclei.

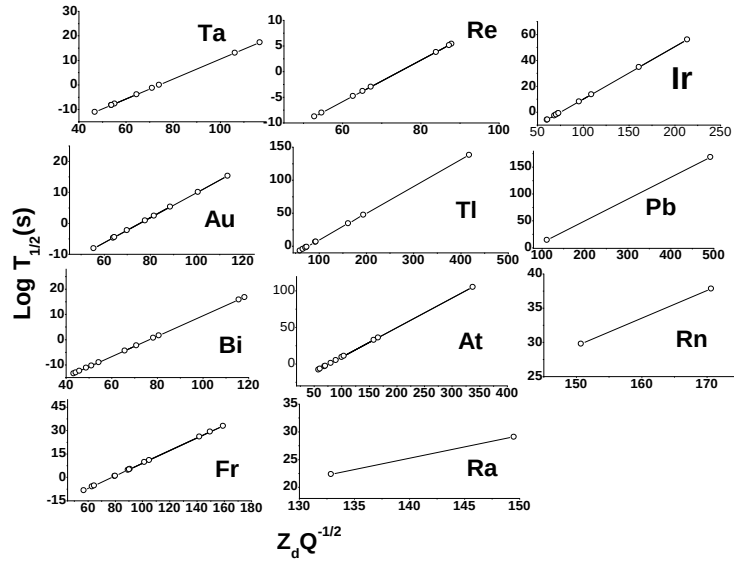


Fig. 4.10 A variation of logarithmic half-lives for the proton emission in the nuclei region $72 < Z < 88$ with the product of atomic number of daughter nuclei and amount of energy released during proton emission ($Z_d Q^{-1/2}$).

parent nuclei is as shown in figure 4.8. For the heavy nuclei Ta, Re, Ir, Au, Tl, Bi, At and Fr, the trend in the amount of energy released during the proton radioactivity is not unique, there is both increase and decrease in the decay energy and hence their half-lives. In case of Pb, Rn and Ra only two nuclei undergo proton radioactivity. In case of Pb and Rn, the decay energy of first nuclei is greater than that of second nuclei. Hence decay energy decreases. $T_{1/2}$ of first nuclei is smaller than $T_{1/2}$ of second nuclei. Hence half lives increases. In case of Ra the decay of first nuclei is less than that of second nuclei. Hence decay energy increases. $T_{1/2}$ of first nuclei is greater than of $T_{1/2}$ of second nuclei. Hence half lives decreases. Since only two proton emitters are identified in those elements, the trend of variation is difficult to predict. From the figure 4.10 we have observed linear variation of logarithmic half-lives with the $Z_d Q^{-1/2}$. We have also studied the competition between different decay modes such as alpha decay, β^+ , β^- decay and proton decay. The half-lives corresponding to β^+ -decay and β^- -decay are evaluated using the semi empirical formula available in the literature [292, 293]. Alpha decay half-lives are evaluated using the procedure explained in the previous work [15]. The plot of different decay modes are as shown in figure 4.11

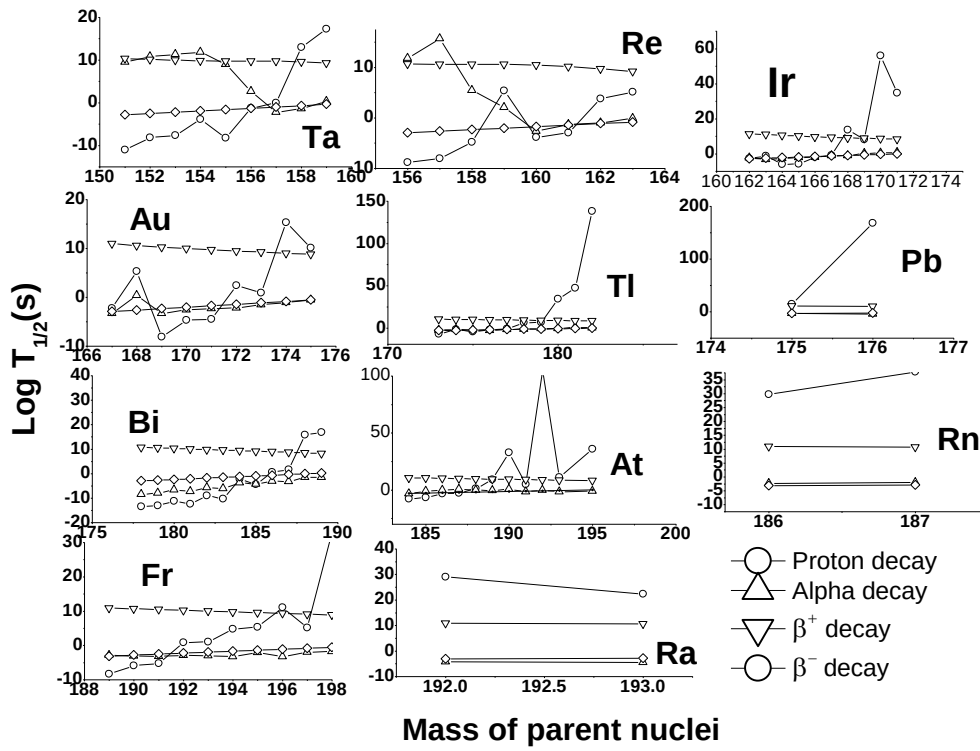


Fig. 4.11 A variation of logarithmic half-lives for the proton activity, alpha decay, β^+ and β^- decay as a function of mass number of the parent nuclei (A) .

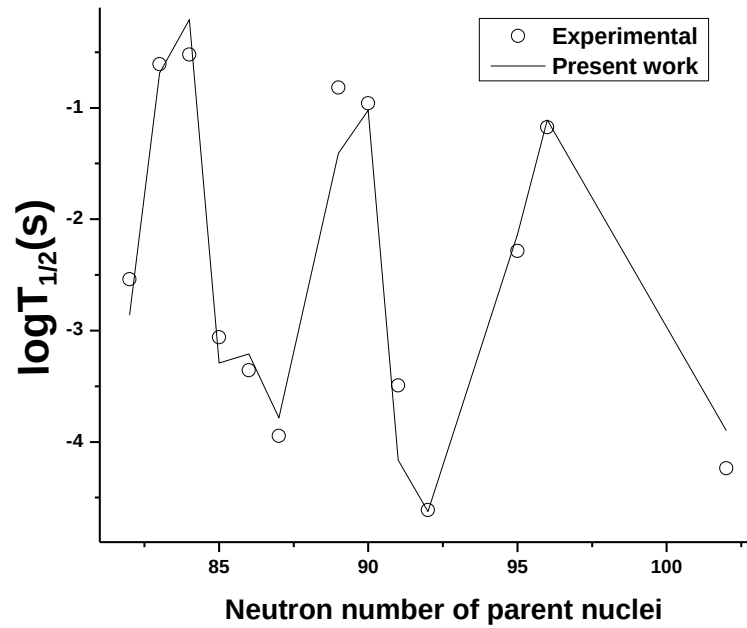


Fig. 4.12 A Comparison of logarithmic half -lives of proton radioactivity of present work with that of available experimental values

Table 4.5 Comparison of present work with experiments [5, 34, 35, 234] and available semi empirical formulae such as Hatsukawa et al.[294] and Gamow [10]

Parent nuclei	$Q_{exp.}$ (MeV)	$\log T_{1/2}$ [5, 34, 35, 234]	Q_{PW} (MeV)	ℓ	$\log T_{1/2}$		
					PW	Hatsukawa [294]	Gamow [10]
^{155}Ta	1.45	-2.54	1.79	5	-2.84	-2.38	-2.84
^{156}Ta	1.02	-0.61	1.03	2	-0.62	-6.74	-0.62
^{157}Ta	0.93	-0.52	0.95	0	-0.49	-7.28	-0.41
^{160}Re	1.27	-3.06	1.29	2	-2.29	-4.56	-3.36
^{161}Re	1.20	-3.36	1.21	0	-3.61	-5.05	-4.61
^{164}Ir	1.54	-3.95	1.58	5	-3.60	-2.29	2.40
^{166}Ir	1.15	-0.82	1.17	2	-0.87	-4.83	-3.40
^{167}Ir	1.07	-0.96	1.08	0	-1.02	-5.37	-1.32
^{170}Au	1.47	-3.49	1.49	2	-3.50	-2.24	1.15
^{171}Au	1.45	-4.61	1.47	0	-4.59	-2.35	2.19
^{176}Tl	1.27	-2.28	1.27	0	-2.36	-3.11	-2.36
^{177}Tl	1.16	-1.17	1.18	0	-1.18	-3.70	-2.84
^{185}Bi	1.53	-4.24	1.56	4	-4.12	-3.51	-3.93

and also highlighted possible proton emitters with the corresponding energies and half-lives in the atomic number range $72 < Z < 88$. To validate the present work, the proton emission half-lives produced by the present work are compared with that of experiments and available semi empirical formulae such as Hatsukawa et al. [294] and Gamow [10]. It is tabulated in table 4.5. We have also compared proton radioactivity logarithmic half-lives of present work with that of available experimental values and it is depicted in figure 4.12. From the figure 4.12 and the table 4.5, it is clearly observed that the present work is in close agreement with the experimental values.

4.2.2.1 Proton radioactivity of Tantalum

Using three models such as CPPM, ELDM and MGLDM, proton decay from the proton rich emitter Tantalum is studied. The 1P-decay is energetically possible only when Q-value of the reaction is positive. The decay energy is evaluated using the following equation;

$$Q = \delta M_P - (\delta M_d + \delta M_Z) + k(Z_P^\epsilon - Z_d^\epsilon) \quad (4.9)$$

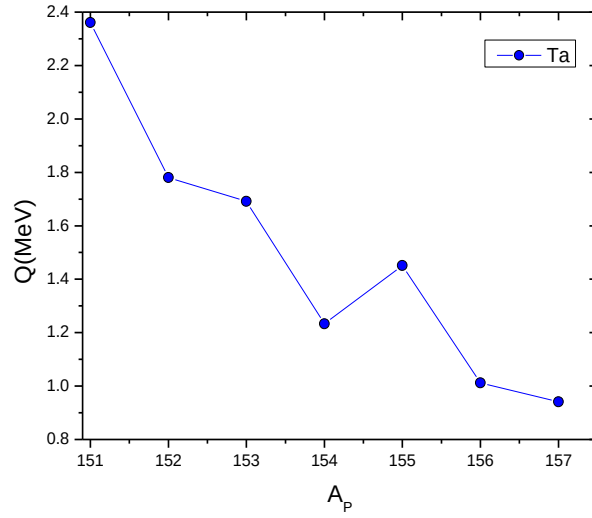


Fig. 4.13 A plot of Q-values during 1P- decay with the mass number of parent nuclei for the $^{151-157}\text{Ta}$ nuclei.

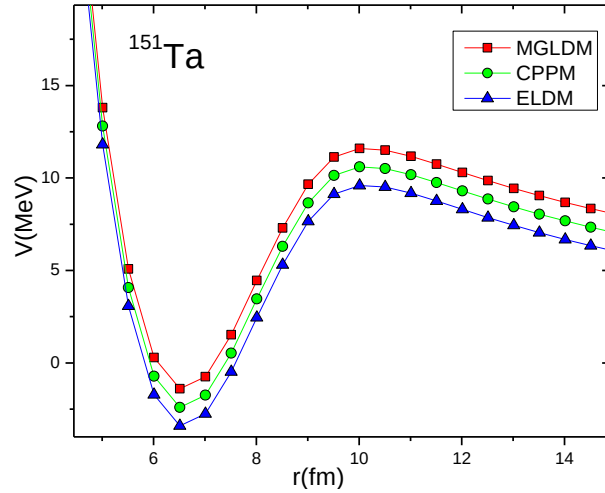


Fig. 4.14 Variation of total potential using three models such as CPPM, ELDM and MGLDM as function of separation distance in ^{151}Ta nuclei.

where δM_P is the mass excess of the parent nuclei, δM_d is the mass excess of the daughter nuclei and δM_Z is the mass excess of the emitted proton. The term $kZ_{P(d)}^\epsilon$ is the total binding energy of electrons in the parent or daughter nuclei. The value of $k=13.6$ eV and $\epsilon = 2.408$ for the nuclei $Z \leq 60$ and $k=8.7$ eV and $\epsilon = 2.517$ for the nuclei $Z > 60$. Figure 4.13 shows a plot of Q-values during 1P-decay with the mass number of parent nuclei. The minimum Q-value is observed in case of ^{157}Ta with 0.941 MeV and maximum is observed for ^{151}Ta with 2.361 MeV when compared to their neighboring one. Then, we have calculated total potential using

Table 4.6 Tabulation of $\log T_{1/2}$ using three different models such as CPPM, ELDM and MGLDM for predicted proton emitters from $^{151-157}\text{Ta}$ is compared to available experiments.

PN	DN	Q (MeV)	ℓ	exp.[34]	CPPM	ELDM	MGLDM
^{151}Ta	^{150}Hf	2.361	5	-	-11.21	-10.55	-10.18
^{152}Ta	^{151}Hf	1.781	5	-	-8.67	-7.46	-7.9
^{153}Ta	^{152}Hf	1.691	5	-	-5.6	-5.84	-7.43
^{154}Ta	^{153}Hf	1.233	5	-	-5.28	-4.03	-4.1
^{155}Ta	^{154}Hf	1.451	5	-2.49	-2.68	-2.12	-2.51
^{156}Ta	^{155}Hf	1.012	2	-0.83	-0.55	-0.5	-0.85
^{157}Ta	^{156}Hf	0.941	0	-0.53	-0.35	-0.58	-0.51

three models in nuclei $^{151-157}\text{Ta}$, the studied potential as function of separation distance is shown in figure 4.14. From the figure, the minimum potential is observed when the separation energy is 6.5fm. Then the potential gradually increases and area below the curve gives information on penetration probability. Later, the evaluated penetration probability and 1P-decay half-lives in $^{151-157}\text{Ta}$ using three models and were tabulated in table 4.6. The evaluated $\log T_{1/2}$ value varies between -11.21s to -0.35s in case of CPPM. However, in case of ELDM it varies between -10.55s to -0.58s and in case of MGLDM the $\log T_{1/2}$ varies between -10.18s to -0.51s for the nuclei $^{151-157}\text{Ta}$. The values obtained using present work is compared with the available experimental value. The studied $\log T_{1/2}$ corresponding to $^{155-157}\text{Ta}$ shows close agreement with the available experimental values. However, the value obtained using MGLDM produces experimental half-lives more accurately.

4.2.2.2 Competition between different decay modes in Bismuth nuclei

The proton decay half-lives are studied in the isotopes of heavy nuclei Bismuth (Bi) using CPPM with harmonic oscillator frequency. However, alpha-decay and $\beta^\pm$ -decay half-lives are evaluated using semi-empirical relations. If the Q-value of the reaction in proton decay is positive, then the proton radioactivity is energetically feasible [224]. The mass excess values in order to evaluate Q-value of the reaction is taken by recent mass excess data available in literature [288].

Table 4.7 Prediction of logarithmic half-lives of β^- -decay in the isotopes of heavy nuclei $^{220-244}\text{Bi}$

Parent Nuclei	$T_{1/2}$	Parent Nuclei	$T_{1/2}$
^{220}Bi	0.01	^{233}Bi	-0.55
^{221}Bi	0.79	^{234}Bi	-0.9
^{222}Bi	0.04	^{235}Bi	-0.71
^{223}Bi	0.42	^{236}Bi	-1.08
^{224}Bi	-0.14	^{237}Bi	-0.85
^{225}Bi	0.15	^{238}Bi	-1.16
^{226}Bi	-0.35	^{239}Bi	-0.94
^{227}Bi	-0.01	^{240}Bi	-1.22
^{228}Bi	-0.49	^{241}Bi	-1.01
^{229}Bi	-0.22	^{242}Bi	-1.28
^{230}Bi	-0.62	^{243}Bi	-1.33
^{231}Bi	-0.4	^{244}Bi	-1.59
^{232}Bi	-0.77		

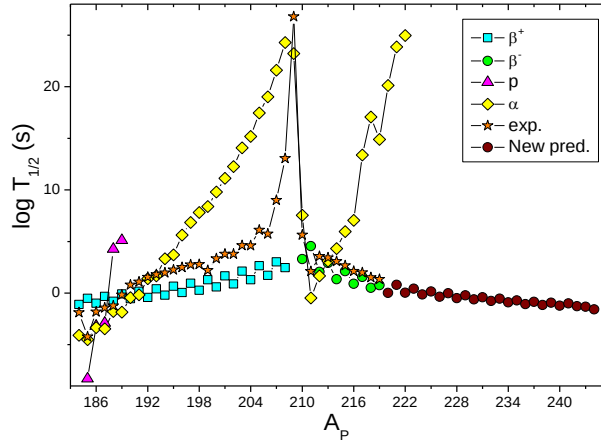


Fig. 4.15 A comparison of proton-decay, alpha-decay and beta-decay half-lives using CPPM and semi-empirical relations with that of available experiments.

The proton decay, alpha-decay and beta-decay half-lives obtained from the present work are compared with available experiments. The figure 4.15 shows comparison of proton, alpha and beta-decay half-lives using CPPM and semi-empirical relations with that of available experiments.

From this comparison it is observed that the nuclei $^{184,186-189}\text{Bi}$ and $^{191,209,211-212}\text{Bi}$ which possess alpha decay half-lives are in good agreement with the available experimental alpha decay half-lives. Similarly, the nuclei $^{190,192-208}\text{Bi}$, $^{210,213-244}\text{Bi}$ and ^{185}Bi are having β^+ , β^- and proton decay half-lives respectively are in close agreement with the available experimental values. the β^- -decay in the isotopes of heavy nuclei $^{220-244}\text{Bi}$ shows shorter half-lives when compared to other

decay modes. Hence, the possible decay mode in heavy nuclei $^{220-244}\text{Bi}$ is β^- -decay only. The table 4.7 shows the predicted β^- -decay half-lives in the heavy nuclei $^{220-244}\text{Bi}$. These predicted half-lives are in seconds to ms.

4.2.3 Proton radioactivity of actinides ($89 \leq Z \leq 103$)

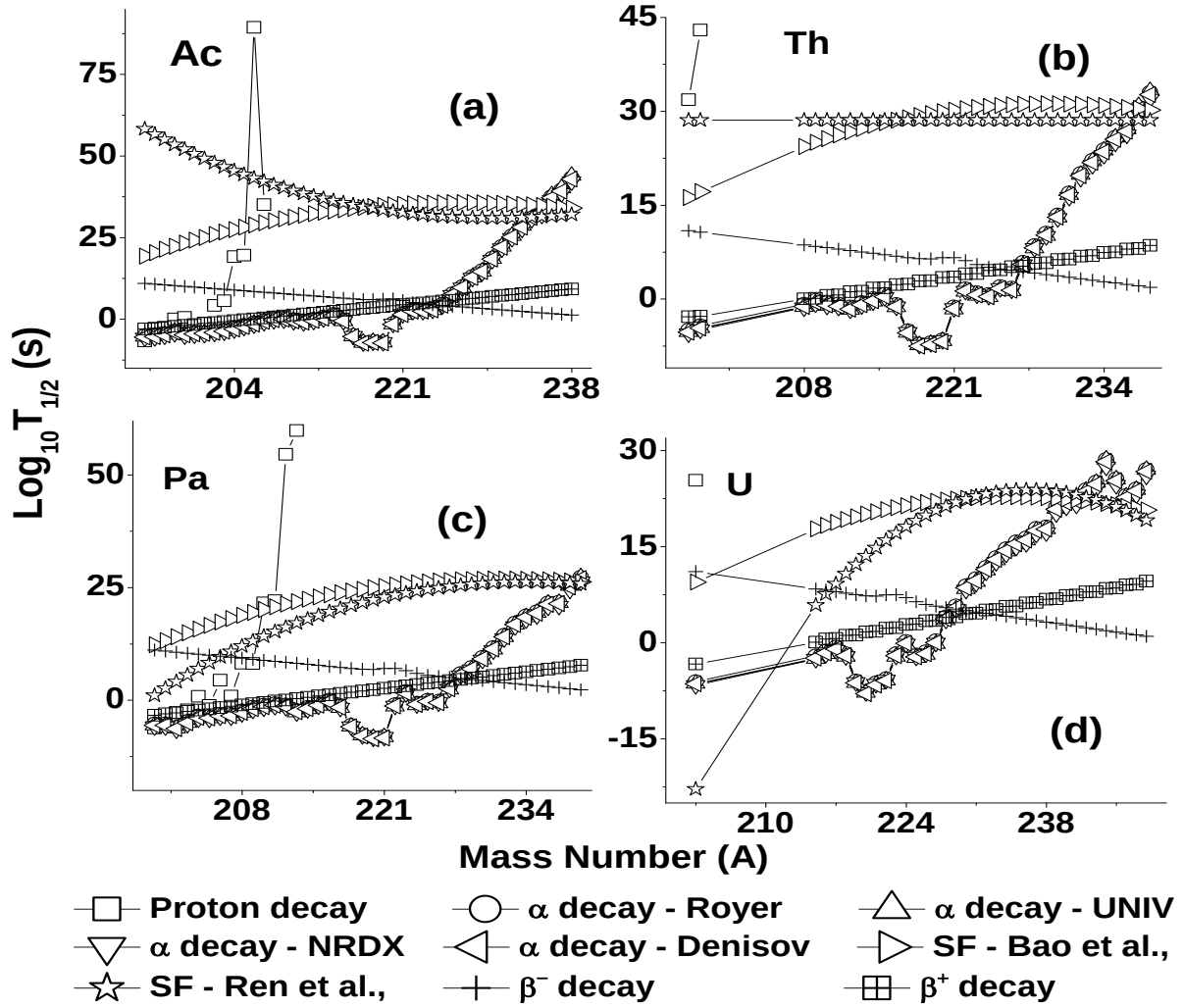


Fig. 4.16 Variation of $\log(T_{1/2})$ of proton activity, alpha decay, spontaneous fission and beta decay as a function of mass number of the parent nuclei (A).

Proton decay from rich proton emitters in the actinide region is studied using the Coulomb and proximity potential model(CPPM). The penetration probability and proton decay half-lives in the actinide region are calculated.logarithmic half-lives of alpha decay, spontaneous fission half-lives, β^- decay and β^+ decay half-lives are also studied. The comparison of $\log(T_{1/2})$ of proton

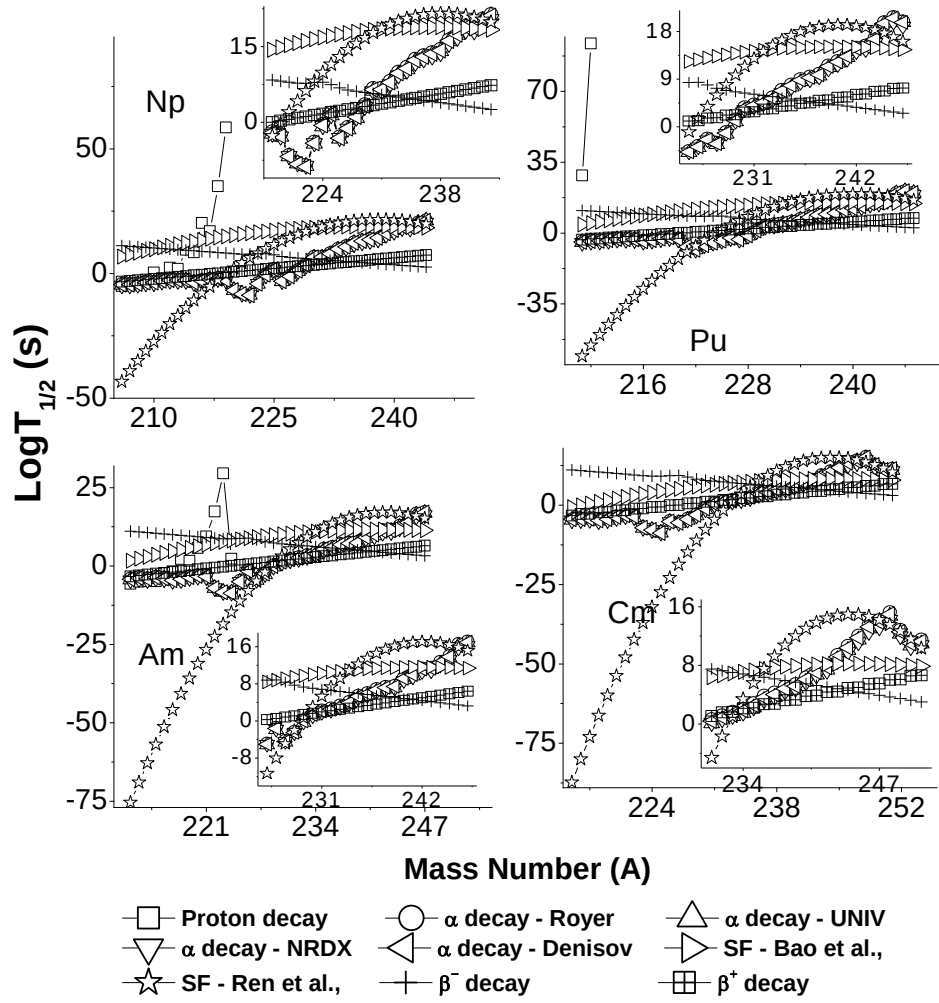


Fig. 4.17 Variation of $\log(T_{1/2})$ of proton activity, alpha decay, spontaneous fission and beta decay as a function of mass number of the parent nuclei (A).

decay with that of other decay modes such as alpha decay, spontaneous fission and beta decay as a function of mass number of parent nuclei is as presented in the figure 4.16. From the figure 4.16(a) it is observed that proton decay for Actinium (A_c) is energetically possible for the mass number of $195 < A < 207$. Figure 4.16(a) gives the comparison of proton decay with that of other decay modes such as alpha decay, spontaneous fission and beta decay. Similarly from the figure 4.16(b) for Th, proton decay is energetically possible in the mass number region $195 < A < 207$, from the figure. 4.16(c) for Pa, proton decay is energetically possible in the mass number region $200 < A < 209$ and $212 < A < 213$, and in figure 4.16(d) for U proton decay is energetically

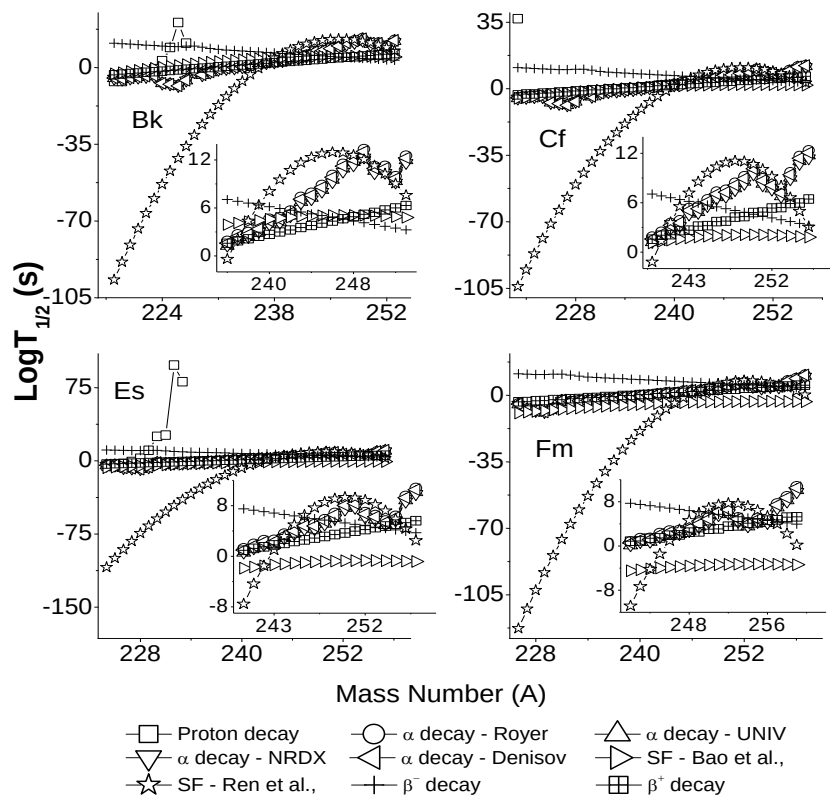


Fig. 4.18 Variation of $\log(T_{1/2})$ of proton activity, alpha decay, spontaneous fission and beta decay as a function of mass number of the parent nuclei (A).

Table 4.8 List of studied actinide nuclei for proton decay

Z	A-Range	A-Q+VE	A-Q-VE
89	195-293	195-207	208-293
90	198-296	198-199	200-296
91	200-300	200-209, 212-213	210-211, 214-300
92	203-303	203	204-303
93	206-306	206-217	218-306
94	209-309	209	210-309
95	212-313	212-224	225-313
96	215-316	215	216-316
97	218-319	218-227	228-319
98	221-322	221	222-322
99	224-326	224-231	232-326
100	226-329	-	226-329
101	229-332	229-239	240-332
102	232-335	232-233	234-335
103	235-339	235-243	244-339

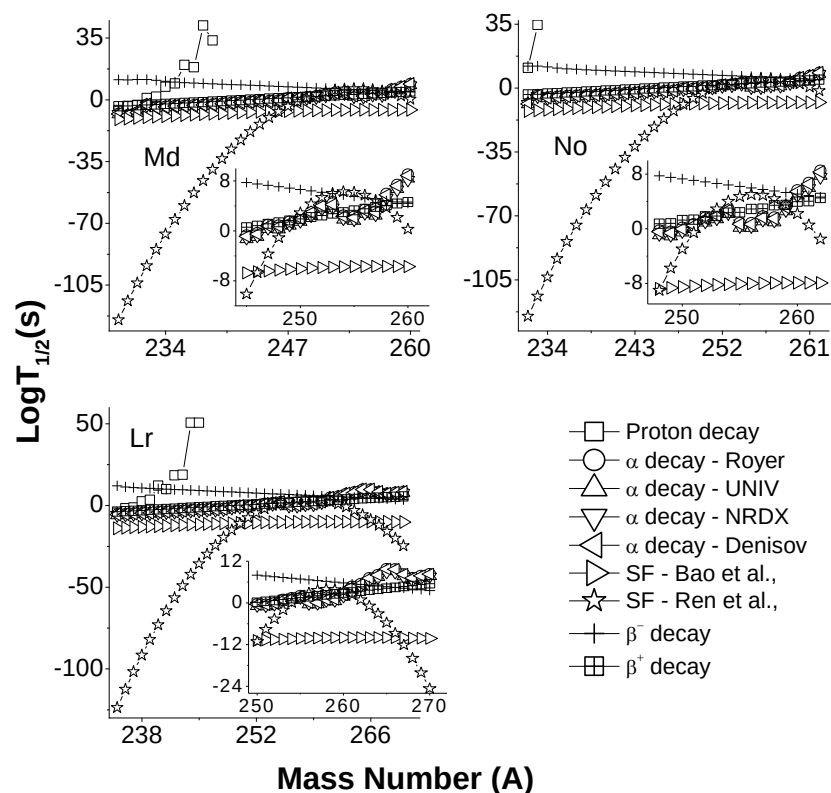


Fig. 4.19 Variation of $\log(T_{1/2})$ of proton activity, alpha decay, spontaneous fission and beta decay as a function of mass number of the parent nuclei (A).

possible in the mass number region 203. Similarly the figure 4.17-4.19 gives the comparison of proton decay with that of alpha decay, spontaneous fission and beta decay in the actinide region $Z=93-103$. The energetically favor proton emission is tabulated in table 4.8.

For better understanding of predictable decay modes, a graph is plotted with the logarithmic half-lives of different decay modes such as proton decay, spontaneous fission, alpha decay and beta decay half-lives and it is presented in figure 4.20. From the figure 4.20(a) it is observed that the ^{194}Ac is a proton emitter, alpha decay mode is observed in the mass number of range $^{195-209}\text{Ac}$ and $^{211-224}\text{Ac}$, β^+ decay and β^- decay is energetically possible in the nuclei ^{210}Ac and $^{225-239}\text{Ac}$ respectively. Similarly the decay modes for actinide nuclei with $Z=90-103$ (Th-Lr) are shown in the figure 4.20. The predicted energy released during proton decay, penetration probability and half-lives for $Z=89-103$ and the results are tabulated in table 4.9. In order to predict the dominant

Table 4.9 Energy released, penetration factor and logarithmic half-lives for proton decay in actinide nuclei

Nuclei	Q	P	T	Nuclei	Q	P	T	Nuclei	Q	P	T
¹⁹⁵ Ac	2.161	2.142×10 ⁻¹⁵	1.892×10 ⁻⁷	²¹⁰ Np	1.301	5.444×10 ⁻²⁴	7.631×10 ⁺¹	²²⁷ Bk	0.771	4.652×10 ⁻³⁶	9.165×10 ⁺¹³
¹⁹⁶ Ac	1.591	1.396×10 ⁻¹⁹	2.907×10 ⁻³	²¹¹ Np	1.561	5.185×10 ⁻²¹	8.025×10 ⁻²	²²¹ Cf	0.281	6.088×10 ⁻⁶⁹	6.941×10 ⁺⁴⁶
¹⁹⁷ Ac	1.591	1.391×10 ⁻¹⁹	2.923×10 ⁻³	²¹² Np	1.151	3.491×10 ⁻²⁶	1.193×10 ⁺⁴	²²⁴ Es	2.181	1.426×10 ⁻¹⁷	2.976×10 ⁻⁵
¹⁹⁸ Ac	1.321	1.775×10 ⁻²²	2.294×10 ⁰	²¹³ Np	1.181	1.025×10 ⁻²⁵	4.072×10 ⁺³	²²⁵ Es	2.181	1.419×10 ⁻¹⁷	2.995×10 ⁻⁵
¹⁹⁹ Ac	1.331	2.277×10 ⁻²²	1.791×10 ⁰	²¹⁴ Np	0.771	2.498×10 ⁻³⁴	1.673×10 ⁺¹²	²²⁶ Es	1.771	1.186×10 ⁻²⁰	3.589×10 ⁻²
²⁰⁰ Ac	1.441	4.324×10 ⁻²¹	9.452×10 ⁻²	²¹⁵ Np	0.811	3.254×10 ⁻³³	1.286×10 ⁺¹¹	²²⁷ Es	1.541	6.434×10 ⁻²³	6.626×10 ⁰
²⁰¹ Ac	1.371	6.726×10 ⁻²²	6.087×10 ⁻¹	²¹⁶ Np	0.471	4.977×10 ⁻⁴⁷	8.426×10 ⁺²⁴	²²⁸ Es	1.251	1.236×10 ⁻²⁶	3.454×10 ⁺⁴
²⁰² Ac	0.971	6.033×10 ⁻²⁸	6.797×10 ⁺⁵	²¹⁷ Np	0.541	4.641×10 ⁻⁴³	9.05×10 ⁺²⁰	²²⁹ Es	0.821	2.005×10 ⁻³⁵	2.132×10 ⁺¹³
²⁰³ Ac	1.018	4.688×10 ⁻²⁷	8.762×10 ⁺⁴	²⁰⁹ Pu	0.351	2.896×10 ⁻⁵⁷	1.432×10 ⁺³⁵	²³⁰ Es	0.441	1.03×10 ⁻⁵²	4.157×10 ⁺³⁰
²⁰⁴ Ac	0.595	1.424×10 ⁻³⁸	2.889×10 ⁺¹⁶	²¹² Am	2.051	1.838×10 ⁻¹⁷	2.267×10 ⁻⁵	²³¹ Es	0.421	2.493×10 ⁻⁵⁴	1.719×10 ⁺³²
²⁰⁵ Ac	0.707	1.575×10 ⁻³⁴	2.616×10 ⁺¹²	²¹³ Am	1.951	3.465×10 ⁻¹⁸	1.204×10 ⁻⁴	²²⁹ Md	2.251	1.482×10 ⁻¹⁷	2.883×10 ⁻⁵
²⁰⁶ Ac	0.383	5.019×10 ⁻⁵¹	8.225×10 ⁺²⁸	²¹⁴ Am	1.911	1.705×10 ⁻¹⁸	2.451×10 ⁻⁴	²³⁰ Md	1.921	6.453×10 ⁻²⁰	6.636×10 ⁻³
²⁰⁷ Ac	0.277	4.149×10 ⁻⁶²	9.965×10 ⁺³⁹	²¹⁵ Am	1.931	2.449×10 ⁻¹⁸	1.709×10 ⁻⁴	²³¹ Md	1.871	2.514×10 ⁻²⁰	1.705×10 ⁻²
¹⁹⁸ Th	0.291	2.163×10 ⁻⁵⁶	1.883×10 ⁺³⁴	²¹⁶ Am	1.601	3.564×10 ⁻²¹	1.176×10 ⁻¹	²³² Md	1.441	1.172×10 ⁻²⁴	3.662×10 ⁺²
¹⁹⁹ Th	0.201	1.218×10 ⁻⁷⁶	3.35×10 ⁺⁵⁴	²¹⁷ Am	1.531	6.843×10 ⁻²²	6.138×10 ⁻¹	²³³ Md	1.381	1.935×10 ⁻²⁵	2.222×10 ⁺³
²⁰⁰ Pa	2.111	3.807×10 ⁻¹⁶	1.073×10 ⁻⁶	²¹⁸ Am	1.191	3.191×10 ⁻²⁶	1.318×10 ⁺⁴	²³⁴ Md	1.001	8.656×10 ⁻³²	4.975×10 ⁺⁹
²⁰¹ Pa	2.091	2.964×10 ⁻¹⁶	1.381×10 ⁻⁶	²¹⁹ Am	1.231	1.29×10 ⁻²⁵	3.264×10 ⁺³	²³⁶ Md	0.911	7.3×10 ⁻³⁴	5.909×10 ⁺¹¹
²⁰² Pa	1.751	1.028×10 ⁻¹⁸	3.988×10 ⁻⁴	²²⁰ Am	0.971	3.403×10 ⁻³⁰	1.24×10 ⁺⁸	²³⁷ Md	0.561	2.862×10 ⁻⁴⁶	1.509×10 ⁺²⁴
²⁰³ Pa	1.491	3.735×10 ⁻²¹	1.099×10 ⁻¹	²²¹ Am	0.801	2.486×10 ⁻³⁴	1.7×10 ⁺¹²	²³⁸ Md	0.591	9.869×10 ⁻⁴⁵	4.383×10 ⁺²²
²⁰⁴ Pa	1.221	1.868×10 ⁻²⁴	2.202×10 ⁺²	²²² Am	0.551	1.326×10 ⁻⁴³	3.189×10 ⁺²¹	²³⁹ Md	0.251	2.607×10 ⁻⁷⁶	1.661×10 ⁺⁵⁴
²⁰⁵ Pa	1.391	2.895×10 ⁻²²	1.423×10 ⁰	²²³ Am	0.341	5.349×10 ⁻⁵⁹	7.923×10 ⁺³⁶	²³² No	0.831	2.217×10 ⁻³⁶	1.936×10 ⁺¹⁴
²⁰⁶ Pa	0.981	1.738×10 ⁻²⁸	2.375×10 ⁺⁶	²²⁴ Am	1.181	2.105×10 ⁻²⁶	2.016×10 ⁺⁴	²³³ No	0.331	6.422×10 ⁻⁶⁵	6.697×10 ⁺⁴²
²⁰⁷ Pa	1.221	1.842×10 ⁻²⁴	2.244×10 ⁺²	²¹⁵ Cm	0.221	8.665×10 ⁻⁷⁸	4.832×10 ⁺⁵⁵	²³⁵ Lr	2.161	1.317×10 ⁻¹⁸	3.273×10 ⁻⁴
²⁰⁸ Pa	0.801	1.222×10 ⁻³²	3.388×10 ⁺¹⁰	²¹⁸ Bk	2.241	1.015×10 ⁻¹⁶	4.142×10 ⁻⁶	²³⁶ Lr	1.781	1.25×10 ⁻²¹	3.455×10 ⁻¹
²⁰⁹ Pa	0.801	1.215×10 ⁻³²	3.412×10 ⁺¹⁰	²¹⁹ Bk	2.231	8.983×10 ⁻¹⁷	4.69×10 ⁺⁶	²³⁷ Lr	1.731	4.252×10 ⁻²²	1.017×10 ⁰
²¹² Pa	0.42	3.479×10 ⁻⁴⁹	1.197×10 ⁺²⁷	²²⁰ Bk	1.841	1.507×10 ¹⁹	2.799×10 ⁻³	²³⁸ Lr	1.361	2.602×10 ⁻²⁶	1.66×10 ⁺⁴
²¹³ Pa	0.283	4.874×10 ⁻¹⁴	8.563×10 ⁻⁹	²²¹ Bk	1.871	2.73×10 ⁻¹⁹	1.547×10 ⁻³	²³⁹ Lr	1.281	1.766×10 ⁻²⁷	2.455×10 ⁺⁵
²⁰³ U	0.381	3.469×10 ⁻⁵³	1.184×10 ⁺³¹	²²² Bk	1.611	1.288×10 ⁻²¹	3.283×10 ⁻¹	²⁴⁰ Lr	0.811	2.142×10 ⁻³⁷	2.027×10 ⁺¹⁵
²⁰⁶ Np	1.911	5.525×10 ⁻¹⁸	7.471×10 ⁻⁵	²²³ Bk	1.461	2.993×10 ⁻²³	1.416×10 ⁺¹	²⁴¹ Lr	0.901	6.58×10 ⁻³⁵	6.61×10 ⁺¹²
²⁰⁷ Np	1.881	3.226×10 ⁻¹⁸	1.281×10 ⁻⁴	²²⁴ Bk	1.171	3.263×10 ⁻²⁷	1.3×10 ⁺⁵	²⁴² Lr	0.611	8.46×10 ⁻⁴⁵	5.149×10 ⁺²²
²⁰⁸ Np	1.751	3.04×10 ⁻¹⁹	1.362×10 ⁻³	²²⁵ Bk	0.841	4.615×10 ⁻³⁴	9.21×10 ⁺¹¹	²⁴³ Lr	0.601	2.884×10 ⁻⁴⁵	1.512×10 ⁺²³
²⁰⁹ Np	1.691	8.868×10 ⁻²⁰	4.677×10 ⁻³	²²⁶ Bk	0.501	1.852×10 ⁻⁴⁷	2.298×10 ⁺²⁵				

decay mode in the atomic number range Z=89-103, branching ratios are calculated. The branching ratio of proton decay to alpha decay is defined as,

$$BR = \frac{\lambda_p}{\lambda_{\alpha/SF/\beta^+/\beta^-}} \quad (4.10)$$

Where λ_p is the decay constant corresponding to proton emission and $\lambda_{\alpha/SF/\beta^+/\beta^-}$ is the decay constant corresponding to alpha decay, spontaneous fission, β^+ decay and β^- decay respectively. The variation of branching ratios of proton decay with respect to the alpha decay (NRDX), SF (Bao et al.), beta (minus) decay and beta (plus) decay as a function mass number of the nuclei is as shown in figure 4.21. From the figure 4.21(a) it is observed that the branching ratio of $\lambda_p/\lambda_{\alpha/\beta^+}$ values are higher in the mass number range ^{194–197}Ac and gradually decreases with increase in mass number range above ^{197–207}Ac. The branching ratio of $\lambda_p/\lambda_{sf/\beta^-}$ val-

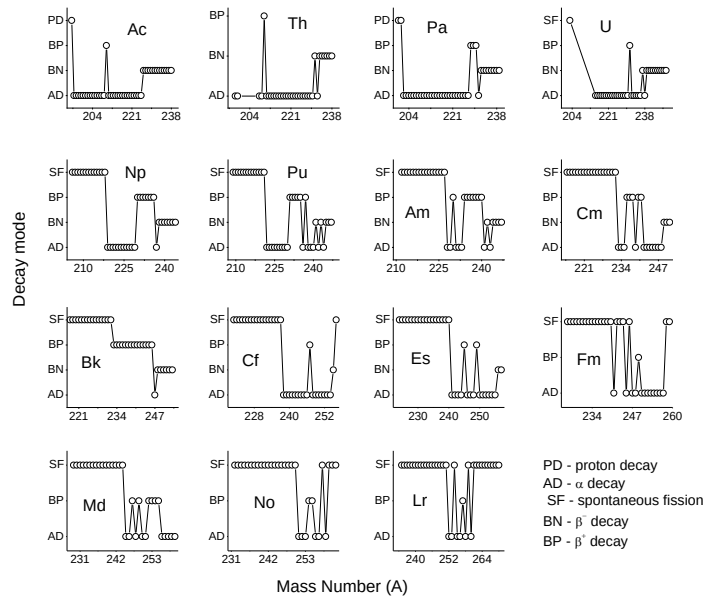


Fig. 4.20 Variation of decay modes such as proton activity, alpha decay, spontaneous fission, beta plus decay and beta minus decay as a function of mass number of the parent nuclei (A).

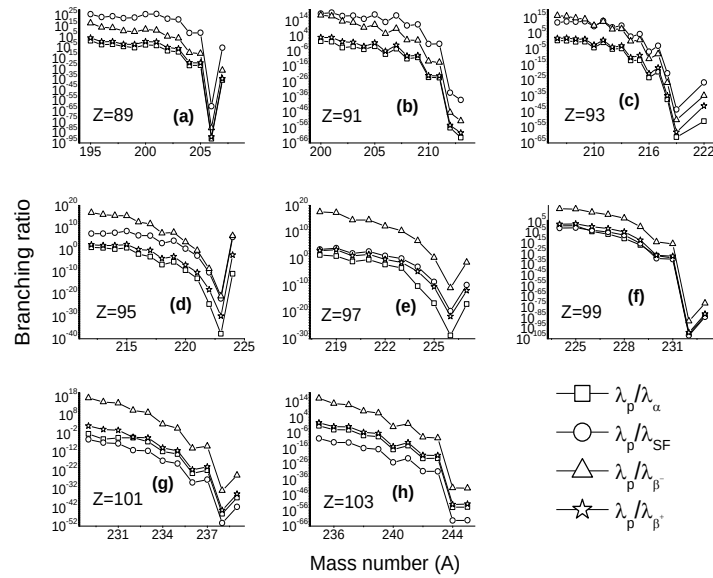


Fig. 4.21 A variation of branching ratios of proton decay to the alpha decay (NRDX), SF (Bao et al., [295]), beta (minus) decay and beta (plus) decay as a function mass number of the nuclei.

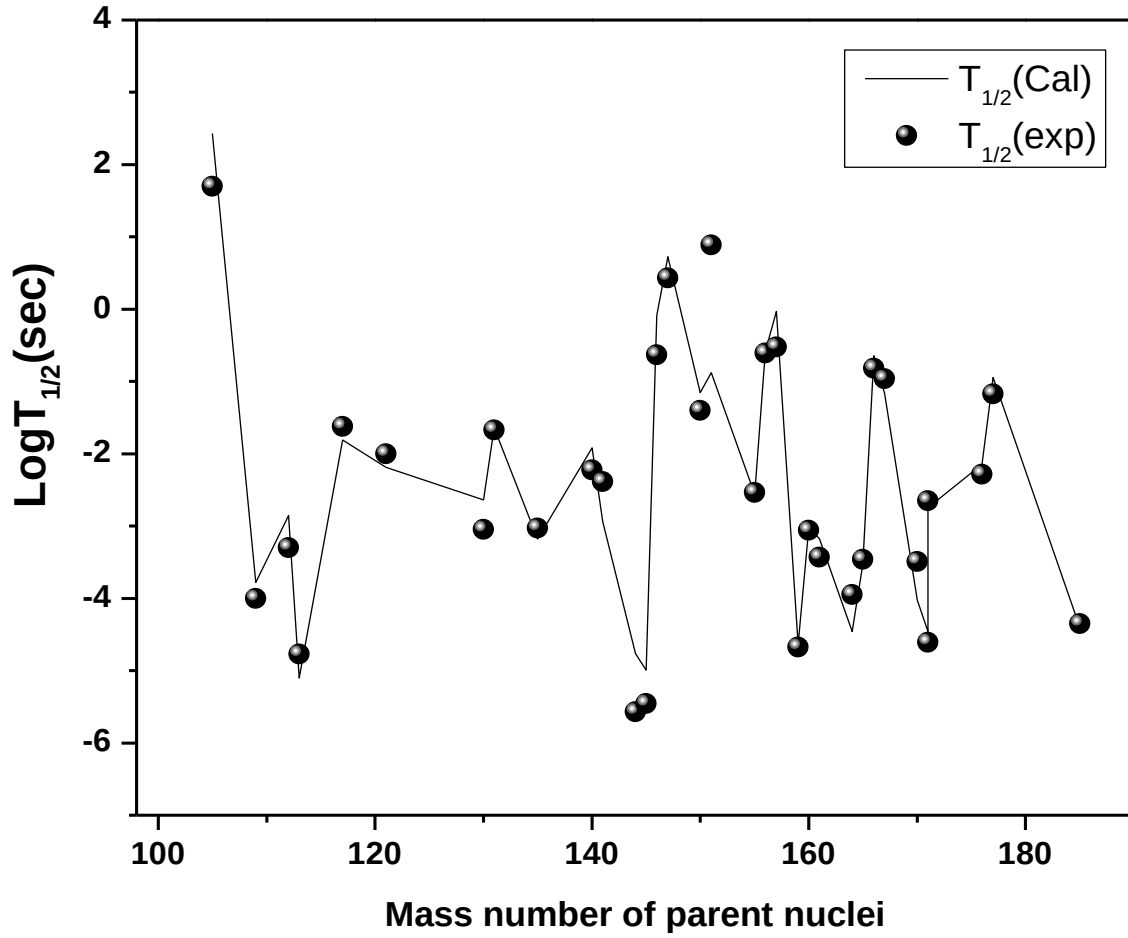


Fig. 4.22 The variation of calculated logarithmic half-lives of proton decay of different mass number of proton emitters with the available experimental values [6, 34, 35, 115, 117, 198, 216, 234, 246–248, 296–303] in the actinide range.

ues are higher in the mass number range $^{194-203}\text{Ac}$ and decreases with increase in mass number range $^{204-207}\text{Ac}$. Similarly from the figure 4.21(b) to 4.21(h) it is observed that the values of branching ratios gradually decreases with increase in mass number. The figure 4.22 denotes the variation of calculated logarithmic half-lives with the available experimental values [6, 34, 35, 115, 117, 198, 216, 234, 246–248, 296–303]. The continuous line represents the calculated logarithmic half-lives and dots represents the experimental logarithmic half-lives values of proton emitters. Table 4.10 also lists the experimental half-life values, energy released during the proton decay and calculated half-lives of proton emitters. From the figure 4.22 and table 4.10 it is

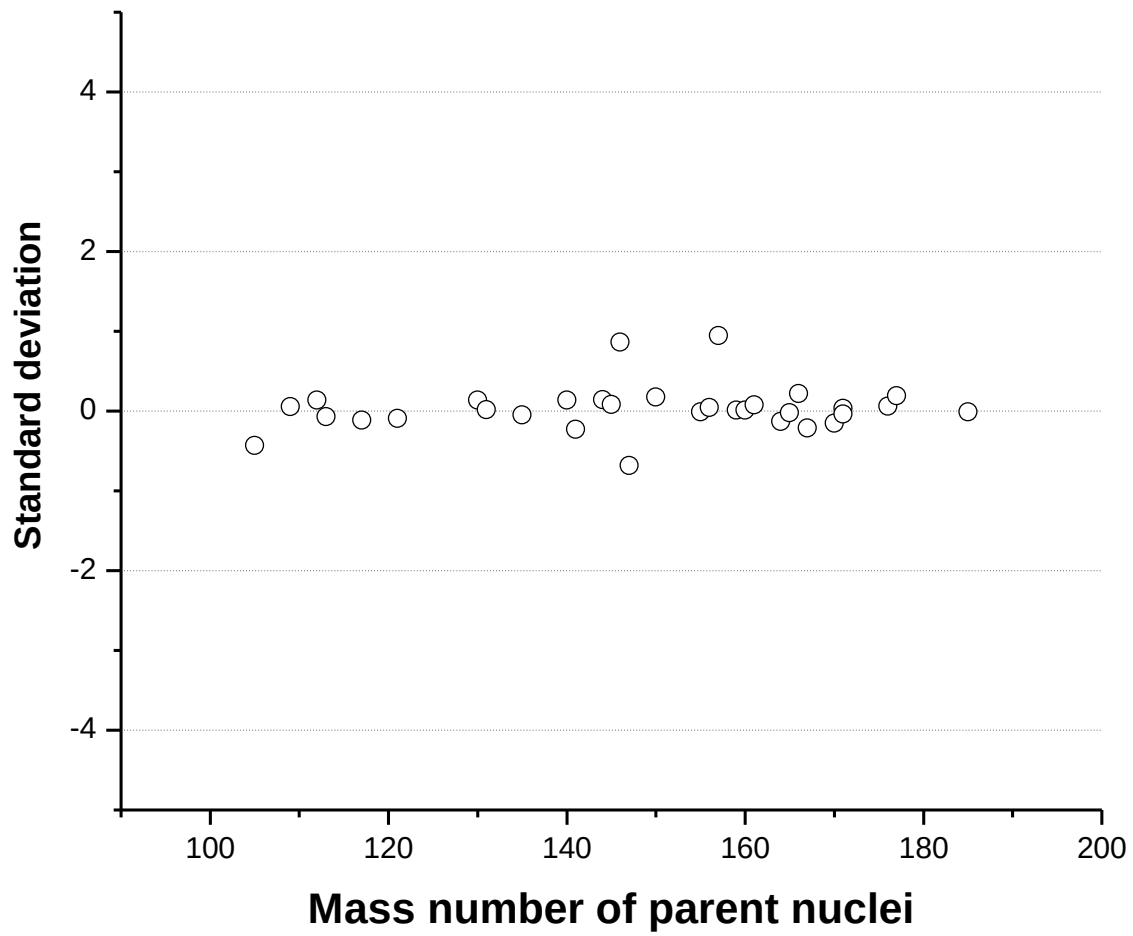


Fig. 4.23 A standard deviation of proton decay from the experimental values with that of mass number of parent nuclei.

Table 4.10 The comparison of calculated half-lives with the experimental values [6, 34, 35, 115, 117, 198, 216, 234, 246–248, 296–303].

Isotopes	β_2	β_4	ℓ	$\log 1/2(\text{exp.})$	Ref.	Isotopes	β_2	β_4	ℓ	$\log 1/2(\text{exp.})$	Ref.
¹⁰⁵ Sb	0.081	0.051	2	1.7	[296]	¹⁵⁵ Ta	0.008	0	5	-2.538	[6, 34, 35, 198, 234]
¹⁰⁹ I	0.16	0.06	2	-4	[117, 246, 297]	¹⁵⁶ Ta	-0.053	0.001	2	-0.609	[6, 34, 35, 198, 234]
¹¹² Cs	0.208	0.067	2	-3.3	[247]	¹⁵⁷ Ta	0.045	0.001	0	-0.523	[6, 34, 35, 198, 234]
¹¹³ Cs	0.207	0.052	2	-4.77	[247, 297]	¹⁵⁹ Re	0.053	-0.007	5	-4.678	[6, 34, 35, 198, 234]
¹¹⁷ La	0.29	0.1	2	-1.623	[6, 34, 35, 198, 234]	¹⁶⁰ Re	0.08	0.002	2	-3.06	[300]
¹²¹ Pr	0.318	0.075	2	-2	[6, 34, 35, 198, 234]	¹⁶¹ Re	0.08	-0.006	3	-3.43	[301]
¹³⁰ Eu	0.331	0	2	-3.046	[6, 34, 35, 198, 234]	¹⁶⁴ Ir	0.089	-0.006	5	-3.947	[6, 34, 35, 198, 234]
¹³¹ Eu	0.331	0	2	-1.67	[6, 34, 35, 198, 234]	¹⁶⁵ Ir	0.099	-0.012	5	-3.46	[302]
¹³⁵ Tb	0.325	-0.046	3	-3.027	[6, 34, 35, 198, 234]	¹⁶⁶ Ir	0.107	-0.004	2	-0.82	[302]
¹⁴⁰ Ho	0.297	-0.07	3	-2.222	[6, 34, 35, 198, 234]	¹⁶⁷ Ir	0.116	-0.011	0	-0.96	[302]
¹⁴¹ Ho	0.286	-0.063	3	-2.387	[6, 34, 35, 198, 234]	¹⁷⁰ Au	-0.096	-0.012	2	-3.493	[6, 34, 35, 198, 234]
¹⁴⁴ Tm	0.258	-0.077	5	-5.569	[6, 34, 35, 198, 234]	¹⁷¹ Au	-0.105	-0.011	0	-4.611	[6, 34, 35, 198, 234]
¹⁴⁵ Tm	0.249	-0.078	5	-5.456	[6, 34, 35, 198, 234]	¹⁷¹ Au	-0.105	-0.011	4	-2.65	[302]
¹⁴⁶ Tm	-0.199	-0.038	5	-0.63	[248]	¹⁷⁶ Tl	-0.053	-0.007	0	-2.284	[6, 34, 35, 198, 234]
¹⁴⁷ Tm	-0.19	-0.04	5	0.43	[216, 246, 298]	¹⁷⁷ Tl	-0.053	-0.007	0	-1.174	[6, 34, 35, 198, 234]
¹⁵⁰ Lu	-0.164	-0.05	5	-1.4	[246, 299]	¹⁸⁵ Bi	-0.052	0.016	0	-4.35	[303]
¹⁵¹ Lu	-0.156	-0.045	5	0.89	[115, 246]						

Table 4.11 Standard deviation of proton decay half-lives in the nuclei region Z=51-83.

n	Parent	σ
21	o-o	1.09065
12	o-e	0.879551

observed that the calculated half lives of proton emitters are in good agreement with the experimental values [6, 34, 35, 115, 117, 198, 216, 234, 246–248, 296–303]. The standard deviation of calculated proton decay half lives with the experimental values is studied. The standard deviation of present work with the experimental values is presented in figure 4.23. The standard root mean square deviation of calculated logarithmic half-lives are evaluated using the following equation:

$$\sigma = \left\{ \sum_{i=1}^n [\log_{10} (T_{cal}/T_{exp})]^2 / (n - 1) \right\}^{1/2} \quad (4.11)$$

The overall standard deviation of proton decay half-lives in the odd-odd nuclei are observed to be 1.09065 and for odd-even were observed to be 0.879551 and are tabulated in table 4.11. From the predictions of proton decay in the atomic nuclei range Z=51-83, it is observed that our calculations are in good agreement with the experimental values. Hence forth, we have predicted half-lives of proton decay in the actinide region.

4.2.4 Systematics of actinides

The proton decay rates are sensitive to amount of energy released (Q_p) and the orbital angular momentum of the emitted proton. The total interacting potential which is a sum of Coulomb, proximity and angular potential is studied as explained in the theory. During the proton emission, the ground state to ground state transitions has zero angular momentum $\ell = 0$. Thus the effects of angular potential in case of proton emission is neglected and the deformed nuclei is also considered in the present work. The penetration probability is evaluated using WKB approximation and studied logarithmic half-lives of proton decay in the actinide region. The amount of energy

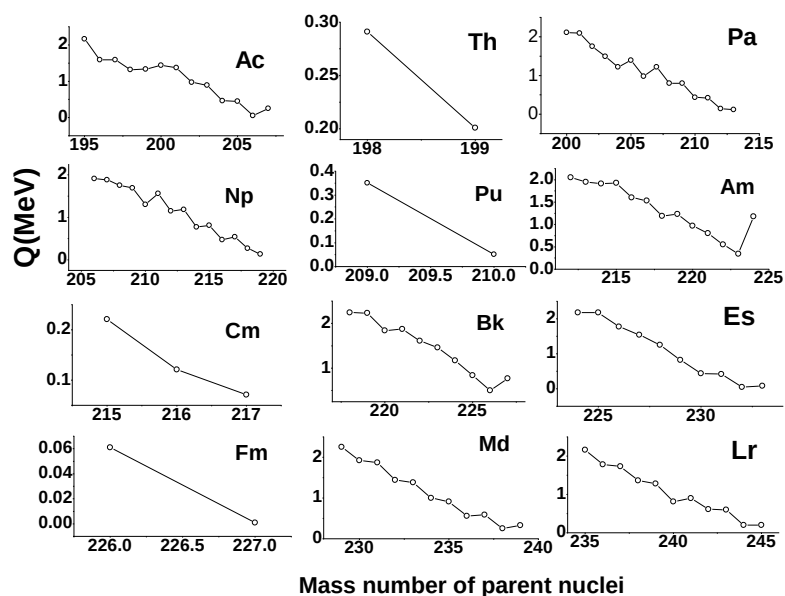


Fig. 4.24 The variation of amount of energy released during proton decay with the mass number of parent nuclei in the actinide region.

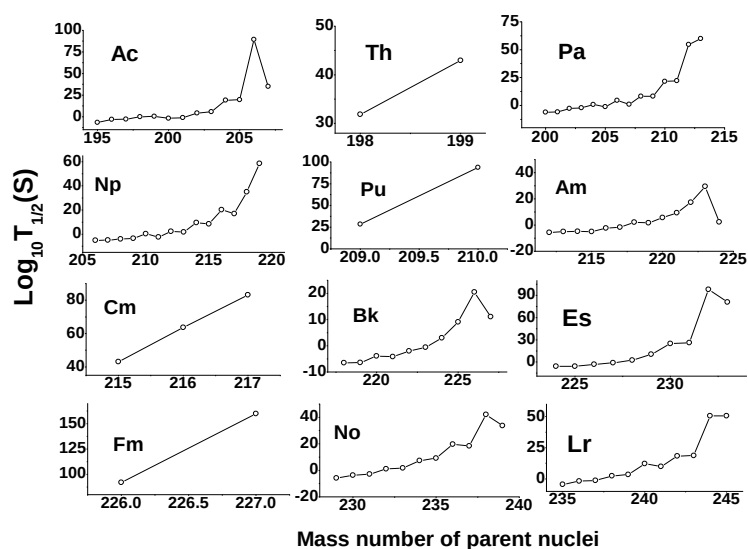


Fig. 4.25 The variation of logarithmic half-lives of proton decay with the mass number of parent nuclei in the actinide region.

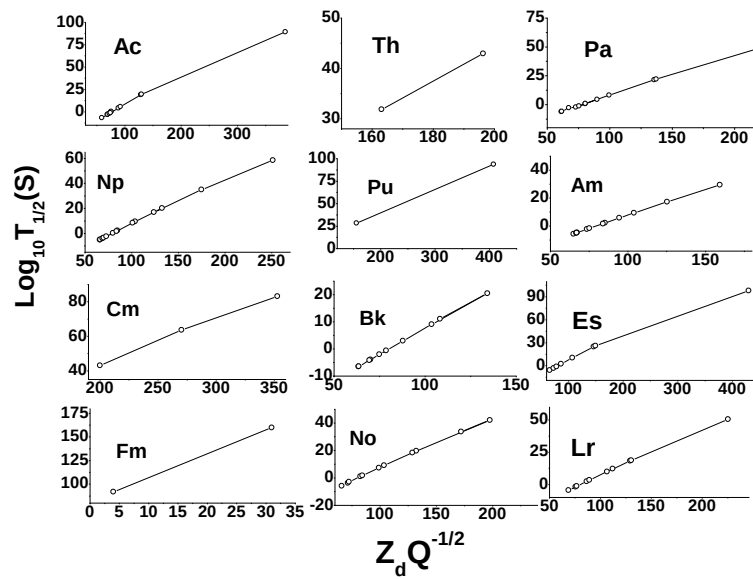


Fig. 4.26 Variation of logarithmic half-lives of proton decay with the product of $Z_d Q^{-1/2}$ in the actinide region.

Table 4.12 Proton decay half-lives, penetration factor and amount of energy released during proton decay in actinides.

Nuclei	Q (MeV)	Penetration factor	$\log T_{1/2}(s)$
^{195}Ac	2.161	2.14×10^{-15}	1.89×10^{-7}
^{200}Pa	2.111	3.8×10^{-16}	1.07×10^{-6}
^{206}Np	1.911	5.52×10^{-18}	1.47×10^{-5}
^{212}Am	2.051	1.83×10^{-17}	2.26×10^{-5}
^{218}Bk	2.241	1.01×10^{-16}	4.14×10^{-6}
^{224}Es	2.181	1.42×10^{-17}	2.97×10^{-5}
^{229}Md	2.251	1.48×10^{-17}	2.88×10^{-5}
^{235}Lr	2.161	1.31×10^{-18}	3.27×10^{-4}

released during proton decay as function of mass number of parent nuclei in the actinide region as shown in figure 4.24. From the figure it is observed that the amount of energy released during proton decay gradually decreases with the increase in mass number of parent nuclei.

The studied logarithmic half-lives of proton decay in the actinide region is plotted as function of the mass number of parent nuclei is presented in figure 4.25. The figure indicates that the logarithmic half-lives increases with increase in mass number of parent nuclei. The half-lives values are of the order of 10^{-6} to 10^{-4} S for the actinides ^{195}Ac , ^{200}Pa , ^{206}Np , ^{212}Am , ^{218}Bk ,

Table 4.13 A comparison of logarithmic half-lives of proton decay with Royer, Univ, NRDX, Denisov and Bao.

Parent Nuclei	log T _{1/2} (s)						Decay Mode	Parent Nuclei	log T _{1/2} (s)						Decay Mode
	Proton	Sf	α-decay						Proton	Sf	α-decay				
		Royer	UNIV	NRDX	Denisov	Royer				UNIV	NRDX	Denisov			
¹⁹⁵ Ac	-6.723	19.285	1.9019	1.389	1.0527	1.4794	p	²¹⁹ Am	3.5138	5.7441	8.3273	7.6976	7.1848	7.9916	p
¹⁹⁶ Ac	-2.536	20.325	4.1825	3.6576	3.281	3.7588	p	²²⁰ Am	8.0934	6.2466	9.6523	9.0362	8.4594	9.3163	p
¹⁹⁷ Ac	-2.53	21.33	4.161	3.637	3.283	3.739	p	²²¹ Am	12.231	6.7266	10.5613	9.9572	9.3406	10.2256	sf
¹⁹⁸ Ac	0.3608	22.3023	5.3396	4.8169	4.4464	4.9171	p	²²² Am	21.504	7.184	11.9864	11.4037	10.7102	11.6503	sf
¹⁹⁹ Ac	0.2533	23.2399	5.2725	4.751	4.4043	4.8513	p	²²³ Am	36.899	7.6186	13.2599	12.6999	11.9365	12.9237	sf
²⁰⁰ Ac	-1.0245	24.1421	4.7543	4.234	3.9256	4.3348	p	²²⁴ Am	4.3046	8.0304	8.4788	7.8558	7.4335	8.1488	p
²⁰¹ Ac	-0.2156	25.0094	5.0479	4.5289	4.2323	4.6293	p	²¹⁵ Cm	55.684	-0.757	14.7612	14.1951	13.0602	14.4243	sf
²⁰² Ac	5.8323	25.8415	6.9328	6.4209	6.0791	6.5134	p	²¹⁸ Bk	-5.3828	-3.0072	4.7539	4.0643	3.6171	4.442	p
²⁰³ Ac	4.9426	26.6385	6.6779	6.1659	5.855	6.26	p	²¹⁹ Bk	-5.3288	-2.467	4.774	4.0855	3.6571	4.4633	p
²⁰⁴ Ac	16.461	27.4004	8.8704	8.7363	7.9999	8.4513	α	²²⁰ Bk	-2.5529	-1.9435	6.41	5.7228	5.2133	6.0987	p
²⁰⁵ Ac	12.42	28.1272	8.2391	7.7403	7.4114	7.8219	α	²²¹ Bk	-2.8104	-1.4371	6.2571	5.5707	5.0908	5.9472	p
²⁰⁶ Ac	28.92	28.8189	10.038	9.5593	9.1753	9.6197	α	²²² Bk	-0.4836	-0.9482	7.4155	6.7345	6.1991	7.1056	sf
²⁰⁷ Ac	39.985	29.4757	10.65	10.18	9.79	10.2318	α	²²³ Bk	1.1511	-0.4774	8.1082	7.4325	6.8702	7.7987	sf
¹⁹⁸ Th	34.275	16.1819	11.31	10.813	10.1	10.8917	α	²²⁴ Bk	5.1142	-0.0248	9.5409	8.8787	8.2362	9.231	sf
¹⁹⁹ Th	54.525	17.1443	11.845	11.357	10.641	11.4272	α	²²⁵ Bk	11.964	0.4093	11.307	10.6676	9.9153	10.9964	sf
²⁰⁰ Pa	-5.9691	12.2615	2.9029	2.341	1.9651	2.5071	p	²²⁶ Bk	25.362	0.8245	13.297	12.6906	11.805	12.9856	sf
²⁰¹ Pa	-5.8597	13.1791	2.96	2.3991	2.0424	2.5654	p	²²⁷ Bk	13.962	1.2206	11.666	11.0339	10.2944	11.3574	sf
²⁰² Pa	-3.3992	14.068	4.3279	3.7624	3.3775	3.933	p	²²¹ Cf	46.841	-5.1307	15.364	14.767	13.5179	15.0549	sf
²⁰³ Pa	-0.9587	14.9279	5.4434	4.8786	4.4705	5.0485	p	²²⁴ Es	-4.5262	-7.1242	5.7952	5.0641	4.5505	5.5104	sf
²⁰⁴ Pa	2.3428	15.7583	6.679	6.119	5.679	6.284	p	²²⁵ Es	-4.5235	-6.6849	5.7749	5.0449	4.5523	5.4914	sf
²⁰⁵ Pa	0.1534	16.5588	5.8572	5.2957	4.9125	5.4643	p	²²⁶ Es	-1.445	-6.2589	7.5555	6.8314	6.2318	7.2712	sf
²⁰⁶ Pa	6.3757	17.3293	7.8264	7.2758	6.8254	7.4327	p	²²⁷ Es	0.8213	-5.8467	8.6204	7.9043	7.2448	8.3362	sf
²⁰⁷ Pa	2.3511	18.0696	6.6162	6.0597	5.6857	6.2249	p	²²⁸ Es	4.5384	-5.4488	10.0548	9.3534	8.6021	9.7703	sf
²⁰⁸ Pa	10.53	18.7794	8.726	8.1856	7.734	8.3337	α	²²⁹ Es	13.329	-5.0656	12.393	11.7244	10.8019	12.1071	sf
²⁰⁹ Pa	10.533	19.4586	8.7054	8.1659	7.7364	8.3143	α	²³⁰ Es	30.619	-4.6974	14.6914	14.0653	12.9649	14.4042	sf
²¹² Pa	27.078	21.3128	10.794	10.281	9.8076	10.4027	α	²³¹ Es	32.236	-4.3445	14.8004	14.1776	13.0875	14.5144	sf
²¹³ Pa	-8.0673	21.8698	11.6	11.1	10.61	11.213	α	²²⁹ Md	-4.54	-11.0676	6.3166	5.5442	4.9712	6.0582	sf
²⁰³ U	31.073	9.436	11.764	11.23	10.42	11.3731	α	²³⁰ Md	-2.1781	-10.7089	7.7399	6.9732	6.3081	7.4812	sf
²⁰⁶ Np	-4.1266	6.7169	4.5136	3.9048	3.4685	4.1449	p	²³¹ Md	-1.7682	-10.3599	7.9478	7.1833	6.5207	7.69	sf
²⁰⁷ Np	-3.8922	7.4948	4.6184	4.0107	3.5904	4.2508	p	²³² Md	2.5638	-10.0213	9.9984	9.2518	8.4382	9.7396	sf
²⁰⁸ Np	-2.8658	8.2485	5.1529	4.546	4.1214	4.786	p	²³³ Md	3.3468	-9.6935	10.2843	9.5419	8.7231	10.0264	sf
²⁰⁹ Np	-2.33	8.9775	5.3939	4.7883	4.373	5.0279	p	²³⁴ Md	9.9699	-9.3769	12.309	11.5948	10.6168	12.0501	sf
²¹⁰ Np	1.8826	9.6813	7.1678	6.5688	6.0847	6.8012	p	²³⁵ Md	11.772	-9.0719	12.8028	12.0975	11.0941	12.5445	sf
²¹¹ Np	-1.0955	10.3596	5.9324	5.3299	4.9296	5.5683	p	²³⁶ Md	24.179	-8.779	14.9003	14.2344	13.0556	14.6409	sf
²¹² Np	4.0769	11.0122	7.8628	7.2705	6.7906	7.4977	p	²³⁷ Md	22.642	-8.4983	14.6912	14.0222	12.8826	14.4333	sf
²¹³ Np	3.6098	11.6388	7.6927	7.1005	6.6504	7.3291	p	²³⁸ Md	54.221	-8.2302	16.915	16.295	14.9613	16.6559	sf
²¹⁴ Np	12.224	12.2391	9.8169	9.2454	8.6965	9.4522	α	²³⁹ Md	42.18	-7.975	16.3479	18.4435	21.3829	16.0906	sf
²¹⁵ Np	11.11	12.8132	9.5771	9.004	8.4897	9.2138	α	²³² No	14.287	-12.61	13.8567	13.1417	11.8994	13.6041	sf
²¹⁶ Np	24.926	13.3607	11.5	10.95	10.34	11.1356	α	²³³ No	42.826	-12.2943	17.0247	16.3769	14.8403	16.77	sf
²¹⁷ Np	20.957	13.8818	11.065	10.516	9.951	10.702	α	²³⁵ Lr	-3.485	-14.0132	7.5036	6.6935	6.0196	7.2718	sf
²⁰⁹ Pu	35.156	4.1089	12.929	12.374	11.439	12.5655	α	²³⁶ Lr	-0.4616	-13.7382	9.222	8.4237	7.6179	8.9895	sf
²¹² Am	-4.6445	1.6163	4.7376	4.0882	3.6407	4.3973	p	²³⁷ Lr	0.0075	-13.4702	9.442	8.6467	7.8401	9.2105	sf
²¹³ Am	-3.9192	2.2689	5.1358	4.487	4.0386	4.7962	p	²³⁸ Lr	4.2212	-13.2097	11.2859	10.512	9.5541	11.0536	sf
²¹⁴ Am	-3.6105	2.9012	5.2852	4.6376	4.2014	4.9467	p	²³⁹ Lr	5.3901	-12.9572	11.6911	10.9237	9.9465	11.4596	sf
²¹⁵ Am	-3.7671	3.5128	5.1793	4.5329	4.1226	4.8422	p	²⁴⁰ Lr	15.307	-12.7131	14.3468	13.6242	12.4066	14.1136	sf
²¹⁶ Am	-0.9294	4.1033	6.616	5.9729	5.5024	6.2785	p	²⁴¹ Lr	12.82	-12.4779	13.7897	13.0578	11.9148	13.5583	sf
²¹⁷ Am	-0.212	4.6721	6.919	6.2783	5.8103	6.5824	p	²⁴² Lr	22.712	-12.2519	15.5493	14.8527	13.5519	15.3172	sf
²¹⁸ Am	4.12	5.2192	8.5489	7.9199	7.3731	8.2118	p	²⁴³ Lr	23.179	-12.0354	15.5979	14.8991	13.6128	15.3628	sf

Table 4.14 The comparison of calculated half-lives with the experimental values [117, 246, 247],[6, 34, 35, 115, 198, 216, 234, 248, 296–303].

Parent	daughter	l	logt-pw	log T _{1/2} -exp	ref	parent	Daughter	l	logt-pw	log T _{1/2} -exp	ref
¹⁰⁵ Sb	¹⁰¹ In	2	2.43	1.70	[296]	¹⁵⁵ Ta	¹⁵¹ Lu	5	-2.56	-2.54	[6, 34, 35, 198, 234]
¹⁰⁹ I	¹⁰⁵ Sb	2	-3.78	-4.00	[117, 246, 297]	¹⁵⁶ Ta	¹⁵² Lu	2	-0.58	-0.61	[6, 34, 35, 198, 234]
¹¹² Cs	¹⁰⁸ I	2	-2.86	-3.30	[247]	¹⁵⁷ Ta	¹⁵³ Lu	0	-0.03	-0.52	[6, 34, 35, 198, 234]
¹¹³ Cs	¹⁰⁹ I	2	-5.10	-4.77	[247, 297]	¹⁵⁹ Re	¹⁵⁵ Ta	5	-4.64	-4.68	[6, 34, 35, 198, 234]
¹¹⁷ La	¹¹³ Cs	2	-1.81	-1.62	[6, 34, 35, 198, 234]	¹⁶⁰ Re	¹⁵⁶ Ta	2	-3.03	-3.06	[300]
¹²¹ Pr	¹¹⁷ La	2	-2.19	-2.00	[6, 34, 35, 198, 234]	¹⁶¹ Re	¹⁵⁷ Ta	3	-3.17	-3.43	[301]
¹³⁰ Eu	¹²⁶ Pm	2	-2.64	-3.05	[6, 34, 35, 198, 234]	¹⁶⁴ Ir	¹⁶⁰ Re	5	-4.46	-3.95	[6, 34, 35, 198, 234]
¹³¹ Eu	¹²⁷ Pm	2	-1.64	-1.67	[6, 34, 35, 198, 234]	¹⁶⁵ Ir	¹⁶¹ Re	5	-3.53	-3.46	[302]
¹³⁵ Tb	¹³¹ Eu	3	-3.18	-3.03	[6, 34, 35, 198, 234]	¹⁶⁶ Ir	¹⁶² Re	2	-0.64	-0.82	[302]
¹⁴⁰ Ho	¹³⁶ Tb	3	-1.92	-2.22	[6, 34, 35, 198, 234]	¹⁶⁷ Ir	¹⁶³ Re	0	-1.16	-0.96	[302]
¹⁴¹ Ho	¹³⁷ Tb	3	-2.94	-2.39	[6, 34, 35, 198, 234]	¹⁷⁰ Au	¹⁶⁶ Ir	2	-4.03	-3.49	[6, 34, 35, 198, 234]
¹⁴⁴ Tm	¹⁴⁰ Ho	5	-4.77	-5.57	[6, 34, 35, 198, 234]	¹⁷¹ Au	¹⁶⁷ Ir	0	-4.46	-4.61	[6, 34, 35, 198, 234]
¹⁴⁵ Tm	¹⁴¹ Ho	5	-5.00	-5.46	[6, 34, 35, 198, 234]	¹⁷¹ Au	¹⁶⁷ Ir	4	-2.75	-2.65	[302]
¹⁴⁶ Tm	¹⁴² Ho	5	-0.09	-0.63	[248]	¹⁷⁶ Tl	¹⁷² Au	0	-2.15	-2.28	[6, 34, 35, 198, 234]
¹⁴⁷ Tm	¹⁴³ Ho	5	0.72	0.43	[246, 296, 297]	¹⁷⁷ Tl	¹⁷³ Au	0	-0.95	-1.17	[6, 34, 35, 198, 234]
¹⁵⁰ Lu	¹⁴⁶ Tm	5	-1.15	-1.40	[246, 299]	¹⁸⁵ Bi	¹⁸¹ Tl	0	-4.40	-4.35	[303]
¹⁵¹ Lu	¹⁴⁷ Tm	5	-0.88	0.89	[246, 299]						

²²⁴Es, ²²⁹Md and ²³⁵Lr and the corresponding values of Q(MeV), penetration factor and half-lives are tabulated in table 4.12. Hence proton decay is favourably observed in the actinides such as ¹⁹⁵Ac, ²⁰⁰Pa, ²⁰⁶Np, ²¹²Am, ²¹⁸Bk, ²²⁴Es, ²²⁹Md and ²³⁵Lr. The logarithmic half-lives of proton decay are plotted against the product of $Z_d Q^{-1/2}$ in the actinide region and is as shown in figure 4.26. From the figure it is observed that there is a linear variation in half-lives with the product of $Z_d Q^{-1/2}$. The logarithmic half-lives of proton decay with that of alpha decay are also compared. (Royer81[304], Univ82[305], NRD83[43], Denisov84[40]) and spontaneous fission (Bao85[295]) and are tabulated in table 4.13. From the table it is clear that the predicted isotopes such as ^{195–203}Ac, ^{200–207}Pa, ^{212–220,224}Am, and ^{218–221}Bk are having less half-lives compared to alpha decay and spontaneous fission decay mode. The dominant decay mode is identified and specified in the actinide region Z=89-103 in the corresponding table. Due to non-availability of experimental values in the actinide region, the predictive power is tested by comparing the available experimental values with the present work and it is tabulated in table 4.14. From the table it is observed that studied values obtained from the present work agrees well with the available experimental values.

4.2.5 Proton radioactivity of superheavy nuclei ($104 \leq Z \leq 126$)

The energy released during the proton decay (Q) is calculated using the difference of mass excess values available in the literature. The experimental mass excess values are used [306]. For those nuclei, where experimental mass excess was unavailable, recent theoretical values are used

Table 4.15 List of studied superheavy nuclei for proton decay

Z	Mass number (A)	Mass Number Q_p positive	Z	Mass number (A)	Mass Number Q_p positive	Z	Mass number (A)	Mass Number Q_p positive
104	240-339	240	112	262-339	262-265	120	287-339	287-292
105	241-339	241-251	113	266-339	266-276	121	290-339	290-303
106	244-339	240-243	114	269-339	269-271	122	294-339	294-299
107	247-339	247-257	115	272-339	272-280,291	123	297-339	297-309
108	250-339	250-253	116	275-339	275-279	124	300-339	3,00,301
109	253-339	253-263	117	278-339	278-287,291	125	303-339	303-315
110	256-339	256-261	118	281-339	281-285	126	306-339	308-329
111	259-339	259-267	119	284-339	284-296			

[36, 307]. List of studied superheavy nuclei for proton decay is given in the table 4.15. In this table, the nuclei for which proton decay is possible are highlighted. The nuclei highlighted in this table are important proton emitters in the superheavy nuclei region. The energy released during the proton decay (Q_P), penetration factor (P), normalisation factor (F) and logarithmic half-lives for proton decay in superheavy nuclei is also given in the table 4.16 and 4.17.

To study the competition between different decay modes, the alpha decay half-lives and spontaneous fission half-lives are also calculated. Alpha decay half-lives are evaluated using the semi empirical relations such as Royer[308], UNIV[309], NRDX[310] and Denisov[311]. Spontaneous fission half-lives are evaluated using semi empirical formula [312]. Figure 4.27 shows the competition between different decay modes such as proton decay, spontaneous fission and alpha decay for superheavy elements. From the detail study of comparison among the different decay modes, it is observed that proton decay half-lives in the superheavy region is greater than that of alpha decay. For most of the superheavy nuclei proton decay half lives are greater than that of spontaneous fission.

Table 4.16 Energy released, penetration factor, normalisation factor and logarithmic half-lives for proton decay in superheavy nuclei

Nuclei	Q (MeV)	Penetration factor	Normalization factor	$\log T_{1/2}$ (s)	Nuclei	Q (MeV)	Penetration factor	Normalization factor	$\log T_{1/2}$ (s)
²⁴⁰ Rf	0.011	6.29×10^{-48}	7.56×10^{-02}	25.96	²⁵⁷ Ds	0.561	2.03×10^{-43}	7.48×10^{-02}	21.46
²⁴¹ Db	2.131	5.45×10^{-21}	7.82×10^{-02}	-0.99	²⁵⁸ Ds	0.321	3.6×10^{-46}	7.43×10^{-02}	24.21
²⁴² Db	1.711	1.48×10^{-24}	7.75×10^{-02}	2.58	²⁵⁹ Ds	0.241	4.98×10^{-47}	7.4×10^{-02}	25.08
²⁴³ Db	1.691	1.04×10^{-24}	7.72×10^{-02}	2.74	²⁶⁰ Ds	0.001	1.85×10^{-49}	7.36×10^{-02}	27.51
²⁴⁴ Db	1.341	4.5×10^{-29}	7.65×10^{-02}	7.1	²⁶² Ds	0.011	2.29×10^{-49}	7.34×10^{-02}	27.42
²⁴⁵ Db	1.331	3.15×10^{-29}	7.63×10^{-02}	7.26	²⁵⁹ Rg	2.301	3.1×10^{-21}	7.66×10^{-02}	-0.73
²⁴⁶ Db	0.961	4.92×10^{-36}	7.56×10^{-02}	14.07	²⁶⁰ Rg	2.021	2.31×10^{-23}	7.61×10^{-02}	1.4
²⁴⁷ Db	0.941	1.95×10^{-36}	7.54×10^{-02}	14.47	²⁶¹ Rg	1.961	5.64×10^{-24}	7.58×10^{-02}	2.01
²⁴⁸ Db	0.531	1.84×10^{-42}	7.47×10^{-02}	20.5	²⁶² Rg	1.601	1.86×10^{-27}	7.52×10^{-02}	5.5
²⁴⁹ Db	0.501	7.78×10^{-43}	7.45×10^{-02}	20.88	²⁶³ Rg	1.591	1.51×10^{-27}	7.5×10^{-02}	5.59
²⁵⁰ Db	0.121	4.38×10^{-47}	7.39×10^{-02}	25.13	²⁶⁴ Rg	1.251	1.82×10^{-32}	7.44×10^{-02}	10.51
²⁵¹ Db	0.091	2.12×10^{-47}	7.37×10^{-02}	25.45	²⁶⁵ Rg	1.231	9.59×10^{-33}	7.42×10^{-02}	10.79
²⁴⁴ Sg	0.241	5.08×10^{-46}	7.58×10^{-02}	24.06	²⁶⁶ Rg	0.851	6.92×10^{-40}	7.36×10^{-02}	17.93
²⁴⁵ Sg	0.421	5.13×10^{-44}	7.57×10^{-02}	22.05	²⁶⁷ Rg	0.731	1.41×10^{-41}	7.33×10^{-02}	19.63
²⁴⁶ Sg	0.071	8.37×10^{-48}	7.51×10^{-02}	25.84	²⁶² Cn	0.691	2.44×10^{-42}	7.46×10^{-02}	20.38
²⁴⁷ Sg	0.021	2.6×10^{-48}	7.48×10^{-02}	26.35	²⁶³ Cn	0.711	4.38×10^{-42}	7.44×10^{-02}	20.13
²⁴⁷ Bh	1.881	1.34×10^{-23}	7.73×10^{-02}	1.63	²⁶⁴ Cn	0.391	6.57×10^{-46}	7.39×10^{-02}	23.96
²⁴⁸ Bh	1.761	1.13×10^{-24}	7.69×10^{-02}	2.7	²⁶⁵ Cn	0.321	1.12×10^{-46}	7.37×10^{-02}	24.72
²⁴⁹ Bh	1.701	4.02×10^{-25}	7.66×10^{-02}	3.15	²⁶⁶ Nh	2.251	2.71×10^{-22}	7.58×10^{-02}	0.33
²⁵⁰ Bh	1.341	9.7×10^{-30}	7.6×10^{-02}	7.77	²⁶⁷ Nh	2.151	5.04×10^{-23}	7.55×10^{-02}	1.06
²⁵¹ Bh	1.161	1.17×10^{-32}	7.55×10^{-02}	10.69	²⁶⁸ Nh	1.751	1.42×10^{-26}	7.49×10^{-02}	4.61
²⁵² Bh	0.771	7.75×10^{-40}	7.49×10^{-02}	17.88	²⁶⁹ Nh	1.681	2.79×10^{-27}	7.46×10^{-02}	5.32
²⁵³ Bh	0.861	1.74×10^{-38}	7.48×10^{-02}	16.53	²⁷⁰ Nh	1.321	4.65×10^{-32}	7.41×10^{-02}	10.1
²⁵⁴ Bh	0.511	3×10^{-43}	7.42×10^{-02}	21.29	²⁷¹ Nh	1.251	4.34×10^{-33}	7.38×10^{-02}	11.14
²⁵⁵ Bh	0.391	1.17×10^{-44}	7.39×10^{-02}	22.71	²⁷² Nh	0.921	1.94×10^{-39}	7.33×10^{-02}	17.49
²⁵⁶ Bh	0.101	8.89×10^{-48}	7.34×10^{-02}	25.83	²⁷³ Nh	0.881	4.69×10^{-40}	7.31×10^{-02}	18.11
²⁵⁷ Bh	0.091	6.94×10^{-48}	7.32×10^{-02}	25.94	²⁷⁴ Nh	0.521	9.75×10^{-45}	7.26×10^{-02}	22.79
²⁵⁰ Hs	0.521	2.32×10^{-43}	7.55×10^{-02}	21.4	²⁷⁵ Nh	0.551	2.16×10^{-44}	7.24×10^{-02}	22.45
²⁵¹ Hs	0.521	2.29×10^{-43}	7.53×10^{-02}	21.4	²⁷⁶ Nh	0.101	3.15×10^{-49}	7.18×10^{-02}	27.29
²⁵² Hs	0.261	2.57×10^{-46}	7.48×10^{-02}	24.36	²⁶⁹ Fl	0.651	2.13×10^{-43}	7.39×10^{-02}	21.45
²⁵³ Hs	0.241	1.56×10^{-46}	7.45×10^{-02}	24.58	²⁷⁰ Fl	0.271	1.08×10^{-47}	7.34×10^{-02}	25.74
²⁵³ Mt	2.211	1.92×10^{-21}	7.7×10^{-02}	-0.53	²⁷¹ Fl	0.261	8.42×10^{-48}	7.32×10^{-02}	25.85
²⁵⁴ Mt	1.851	1.83×10^{-24}	7.64×10^{-02}	2.5	²⁷² Mc	1.981	5.97×10^{-25}	7.51×10^{-02}	2.99
²⁵⁵ Mt	1.931	1.15×10^{-23}	7.63×10^{-02}	1.7	²⁷³ Mc	1.941	2.72×10^{-25}	7.48×10^{-02}	3.33
²⁵⁶ Mt	1.611	7.6×10^{-27}	7.57×10^{-02}	4.88	²⁷⁴ Mc	1.651	4.7×10^{-28}	7.43×10^{-02}	6.1
²⁵⁷ Mt	1.431	5.41×10^{-29}	7.53×10^{-02}	7.03	²⁷⁵ Mc	1.611	1.02×10^{-28}	7.41×10^{-02}	6.76
²⁵⁸ Mt	1.091	9.34×10^{-35}	7.47×10^{-02}	12.8	²⁷⁶ Mc	1.221	1.95×10^{-34}	7.36×10^{-02}	12.49
²⁵⁹ Mt	1.051	1.63×10^{-35}	7.45×10^{-02}	13.56	²⁷⁷ Mc	1.191	6.44×10^{-35}	7.33×10^{-02}	12.97
²⁶⁰ Mt	0.701	2.12×10^{-41}	7.39×10^{-02}	19.45	²⁷⁸ Mc	0.861	6.09×10^{-41}	7.29×10^{-02}	18.99
²⁶¹ Mt	0.661	6.23×10^{-42}	7.37×10^{-02}	19.98	²⁷⁹ Mc	0.181	6.86×10^{-49}	7.21×10^{-02}	26.95
²⁶² Mt	0.481	3.77×10^{-44}	7.33×10^{-02}	22.2	²⁸⁰ Mc	0.061	4.45×10^{-50}	7.18×10^{-02}	28.14
²⁶³ Mt	0.371	2.06×10^{-45}	7.3×10^{-02}	23.46	²⁷⁵ Lv	0.691	1.91×10^{-43}	7.36×10^{-02}	21.49
²⁵⁶ Ds	0.581	3.6×10^{-43}	7.5×10^{-02}	21.21	²⁷⁶ Lv	0.371	3.96×10^{-47}	7.31×10^{-02}	25.18

Table 4.17 Energy released, penetration factor, normalisation factor and logarithmic half-lives for proton decay in superheavy nuclei

Nuclei	Q (MeV)	Penetration factor	Normalization factor	$\log T_{1/2}$ (s)	Nuclei	Q (MeV)	Penetration factor	Normalization factor	$\log T_{1/2}$ (s)
^{277}Lv	0.351	2.39×10^{-47}	7.29×10^{-02}	25.4	$^{298}121$	0.831	4.28×10^{-43}	7.16×10^{-02}	21.16
^{278}Lv	0.011	9.3×10^{-51}	7.24×10^{-02}	28.81	$^{299}121$	0.741	3.02×10^{-44}	7.13×10^{-02}	22.31
^{279}Lv	0.001	7.37×10^{-51}	7.22×10^{-02}	28.92	$^{300}121$	0.401	4.15×10^{-48}	7.09×10^{-02}	26.17
^{278}Ts	1.961	1.23×10^{-25}	7.46×10^{-02}	3.68	$^{301}121$	0.331	7.66×10^{-49}	7.07×10^{-02}	26.91
^{279}Ts	1.591	1.52×10^{-29}	7.41×10^{-02}	7.59	$^{302}121$	0.091	3.25×10^{-51}	7.04×10^{-02}	29.28
^{280}Ts	1.681	2.62×10^{-28}	7.4×10^{-02}	6.35	$^{303}121$	0.091	3.19×10^{-51}	7.02×10^{-02}	29.29
^{281}Ts	1.391	3.58×10^{-32}	7.35×10^{-02}	10.22	$^{294}122$	0.481	1.94×10^{-47}	7.21×10^{-02}	25.5
^{282}Ts	1.011	3.03×10^{-39}	7.3×10^{-02}	17.3	$^{295}122$	0.531	6.69×10^{-47}	7.2×10^{-02}	24.96
^{283}Ts	1.021	4.4×10^{-39}	7.28×10^{-02}	17.14	$^{296}122$	0.311	3.14×10^{-49}	7.17×10^{-02}	27.29
^{284}Ts	0.661	4×10^{-44}	7.23×10^{-02}	22.18	$^{297}122$	0.511	3.91×10^{-47}	7.17×10^{-02}	25.19
^{285}Ts	0.641	2.26×10^{-44}	7.21×10^{-02}	22.43	$^{298}122$	0.041	7.28×10^{-52}	7.11×10^{-02}	29.93
^{286}Ts	0.331	7.63×10^{-48}	7.17×10^{-02}	25.9	$^{299}122$	0.101	2.62×10^{-51}	7.11×10^{-02}	29.37
^{287}Ts	0.261	1.42×10^{-48}	7.15×10^{-02}	26.64	$^{297}123$	2.031	1.38×10^{-26}	7.34×10^{-02}	4.64
^{281}Og	0.691	5.47×10^{-44}	7.32×10^{-02}	22.04	$^{298}123$	1.881	4.21×10^{-28}	7.31×10^{-02}	6.15
^{282}Og	0.301	2.36×10^{-48}	7.26×10^{-02}	26.41	$^{299}123$	1.791	6.53×10^{-29}	7.29×10^{-02}	6.96
^{283}Og	0.391	2.04×10^{-47}	7.25×10^{-02}	25.47	$^{300}123$	1.431	1.1×10^{-33}	7.24×10^{-02}	11.74
^{284}Og	0.011	3.23×10^{-51}	7.2×10^{-02}	29.28	$^{301}124$	1.311	1.21×10^{-35}	7.21×10^{-02}	13.7
^{285}Og	0.041	6.11×10^{-51}	7.19×10^{-02}	29	$^{302}125$	0.871	4.07×10^{-43}	7.16×10^{-02}	21.18
$^{284}119$	2.001	7.96×10^{-26}	7.43×10^{-02}	3.87	$^{303}123$	0.371	6.91×10^{-49}	7.1×10^{-02}	26.95
$^{285}119$	2.091	4.82×10^{-25}	7.42×10^{-02}	3.09	$^{304}123$	1.021	5.25×10^{-41}	7.15×10^{-02}	19.07
$^{286}119$	1.681	4.64×10^{-29}	7.36×10^{-02}	7.11	$^{305}123$	0.991	1.82×10^{-41}	7.13×10^{-02}	19.53
$^{287}119$	1.421	2.43×10^{-32}	7.32×10^{-02}	10.39	$^{306}123$	0.691	2.15×10^{-45}	7.09×10^{-02}	23.46
$^{288}119$	1.171	8.18×10^{-37}	7.28×10^{-02}	14.87	$^{307}123$	0.711	3.66×10^{-45}	7.08×10^{-02}	23.23
$^{289}119$	1.311	4.14×10^{-34}	7.28×10^{-02}	12.16	$^{308}123$	0.381	8.03×10^{-49}	7.04×10^{-02}	26.89
$^{290}119$	0.941	5.62×10^{-41}	7.23×10^{-02}	19.03	$^{309}123$	0.321	1.92×10^{-49}	7.02×10^{-02}	27.51
$^{291}119$	0.661	1.15×10^{-44}	7.18×10^{-02}	22.73	$^{300}124$	0.801	2.94×10^{-44}	7.21×10^{-02}	22.32
$^{292}119$	0.531	3.52×10^{-46}	7.16×10^{-02}	24.24	$^{301}124$	0.621	2.1×10^{-46}	7.18×10^{-02}	24.46
$^{293}119$	0.361	4.99×10^{-48}	7.13×10^{-02}	26.09	$^{303}125$	1.391	7.81×10^{-35}	7.25×10^{-02}	12.89
$^{295}119$	0.081	7.72×10^{-51}	7.08×10^{-02}	28.9	$^{304}125$	1.151	2.04×10^{-39}	7.22×10^{-02}	17.47
$^{296}119$	0.011	1.65×10^{-51}	7.06×10^{-02}	29.57	$^{305}125$	1.051	4.02×10^{-41}	7.19×10^{-02}	19.18
$^{287}120$	0.641	4.03×10^{-45}	7.28×10^{-02}	23.17	$^{306}125$	4.651	2.12×10^{-14}	7.53×10^{-02}	-7.56
$^{288}120$	0.491	7.88×10^{-47}	7.25×10^{-02}	24.88	$^{307}125$	1.951	5.77×10^{-28}	7.25×10^{-02}	6.02
$^{289}120$	0.541	2.78×10^{-46}	7.23×10^{-02}	24.34	$^{308}125$	1.731	2.47×10^{-30}	7.21×10^{-02}	8.39
$^{291}120$	0.181	4.68×10^{-50}	7.17×10^{-02}	28.12	$^{309}125$	1.721	1.87×10^{-30}	7.2×10^{-02}	8.51
$^{292}120$	0.011	1.09×10^{-51}	7.14×10^{-02}	29.75	$^{310}125$	1.271	5.13×10^{-37}	7.14×10^{-02}	15.08
$^{290}121$	2.021	3.61×10^{-26}	7.39×10^{-02}	4.22	$^{311}125$	1.361	1.77×10^{-35}	7.14×10^{-02}	13.54
$^{291}121$	1.981	1.81×10^{-26}	7.37×10^{-02}	4.52	$^{312}125$	0.721	1.44×10^{-45}	7.07×10^{-02}	23.63
$^{292}121$	1.581	9.49×10^{-31}	7.32×10^{-02}	8.8	$^{313}125$	0.571	2.76×10^{-47}	7.04×10^{-02}	25.35
$^{293}121$	1.561	3.91×10^{-31}	7.3×10^{-02}	9.19	$^{314}125$	0.211	5.37×10^{-51}	7×10^{-02}	29.07
$^{294}121$	1.411	2.48×10^{-33}	7.27×10^{-02}	11.39	$^{315}125$	0.151	1.4×10^{-51}	6.98×10^{-02}	29.65
$^{295}121$	1.571	7.27×10^{-31}	7.27×10^{-02}	8.92	$^{308}126$	0.721	8.91×10^{-46}	7.15×10^{-02}	23.84
$^{296}121$	1.271	1.25×10^{-35}	7.23×10^{-02}	13.69	$^{309}126$	0.661	1.77×10^{-46}	7.13×10^{-02}	24.54
$^{297}121$	1.031	3.22×10^{-40}	7.19×10^{-02}	18.28	$^{310}126$	0.071	1.7×10^{-52}	7.06×10^{-02}	30.56

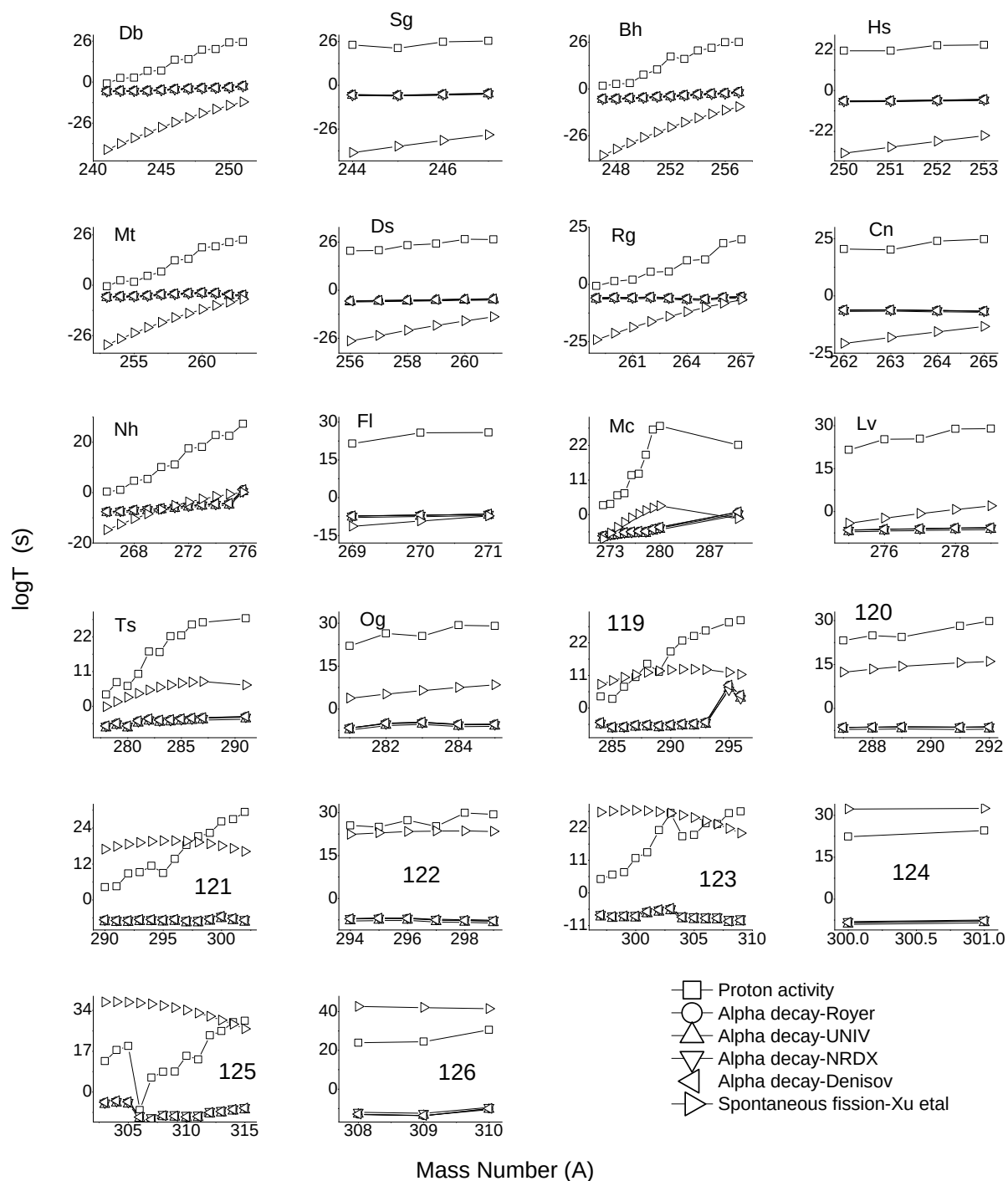


Fig. 4.27 Competition between different decay modes such as proton decay, spontaneous fission and alpha decay for superheavy elements.

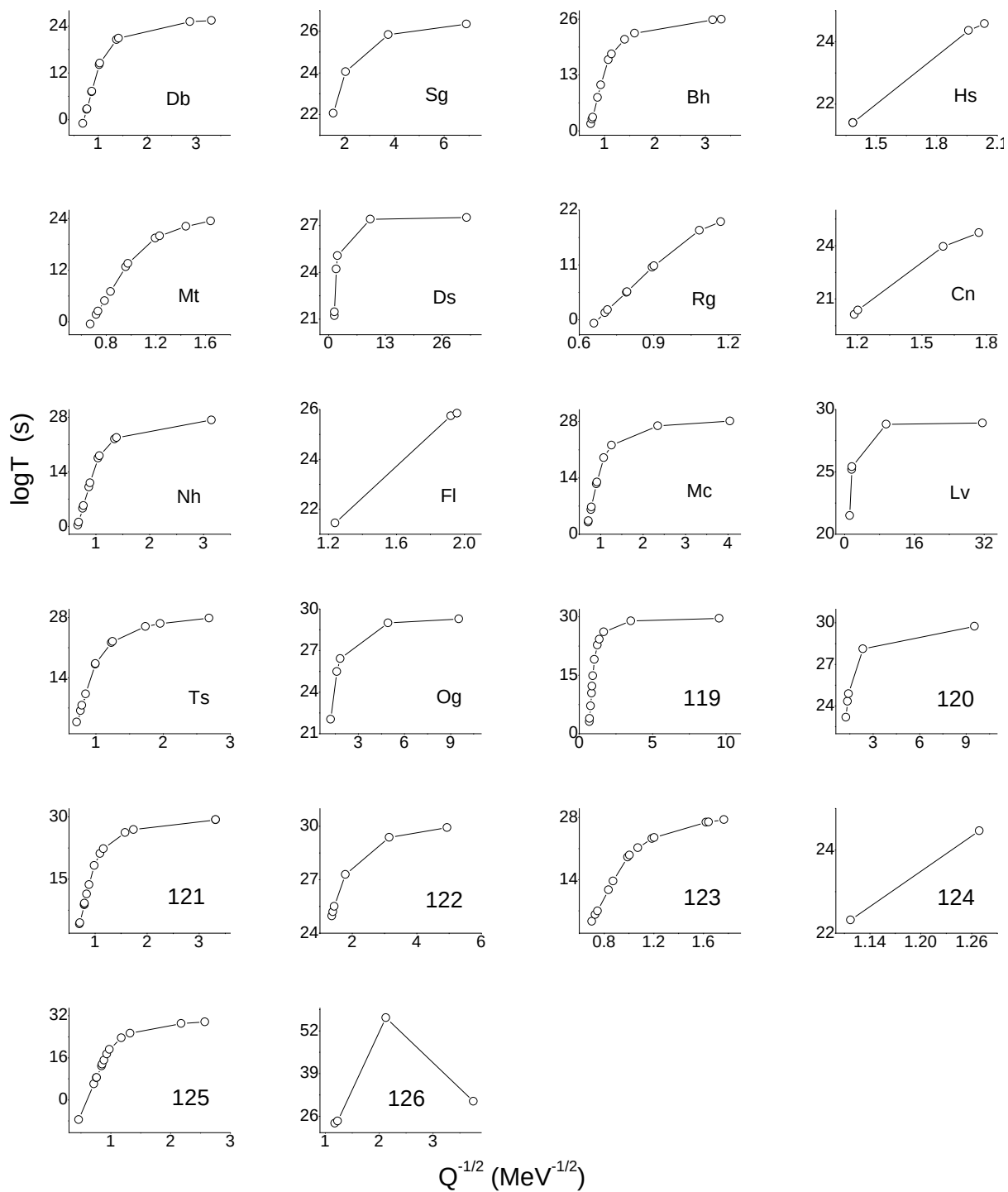


Fig. 4.28 Variation of logarithmic proton decay half-lives versus $1/\sqrt{Q}$

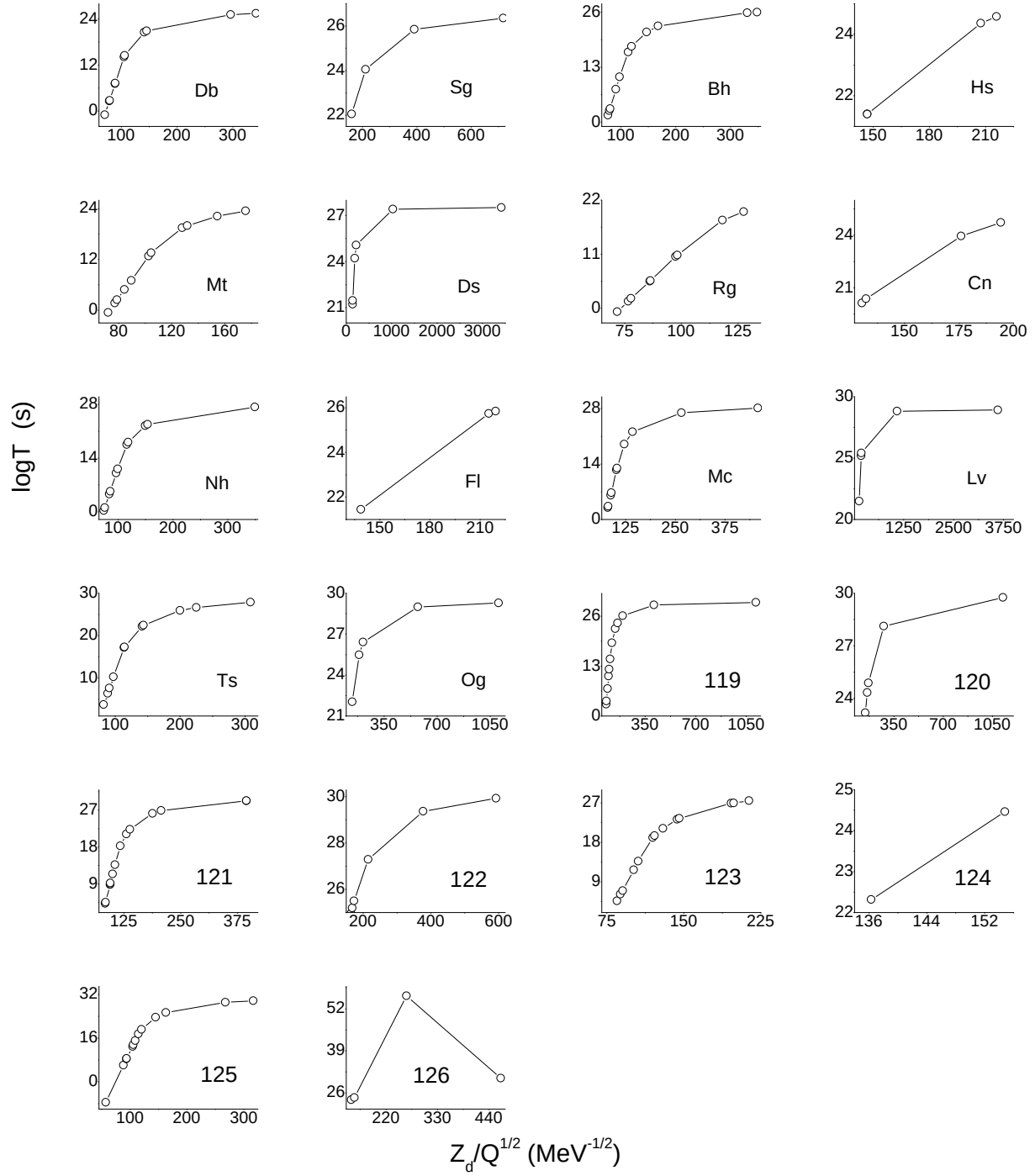


Fig. 4.29 Variation of logarithmic proton decay half-lives versus $Z_d/\sqrt{Q}$

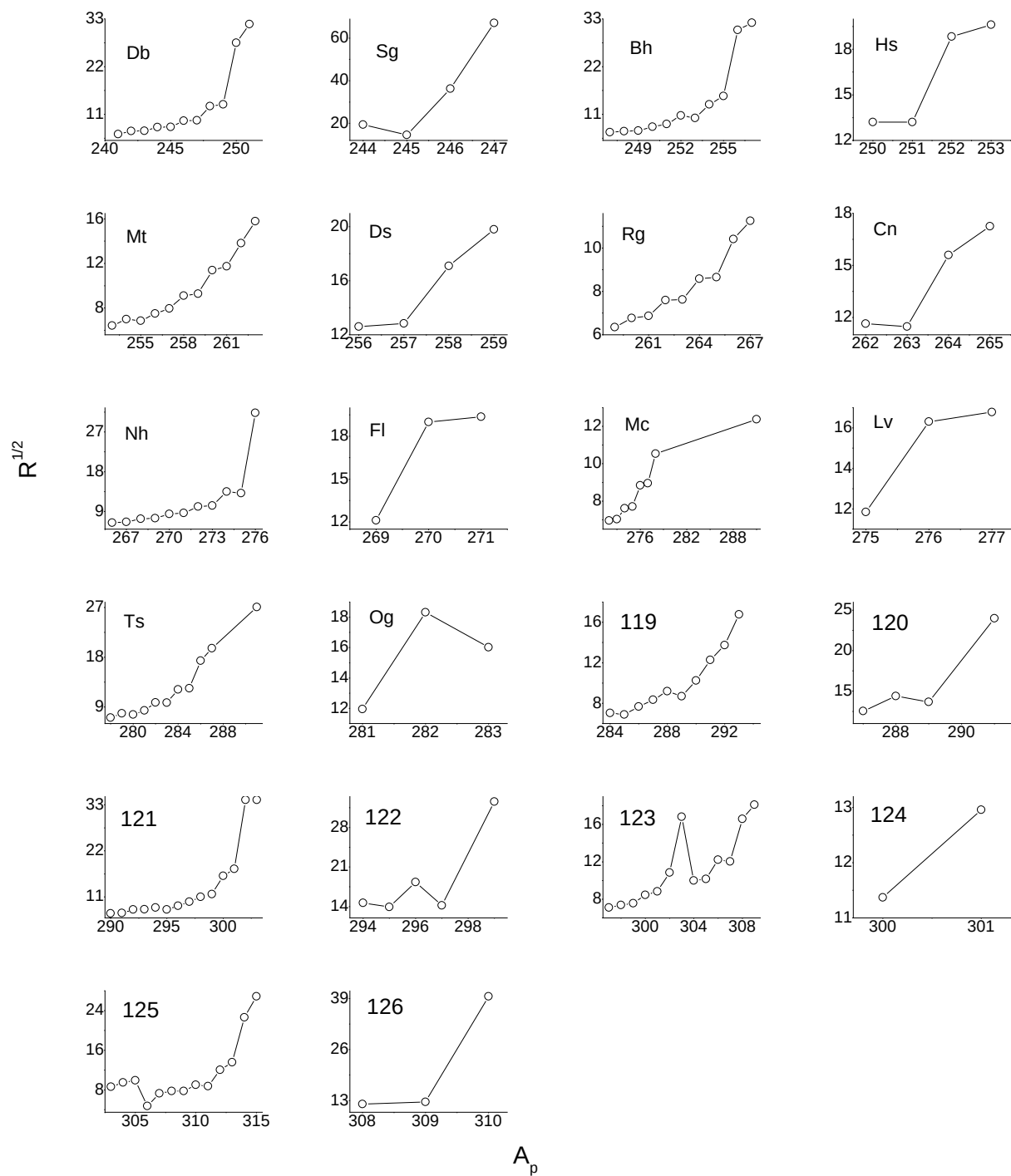


Fig. 4.30 Variation of $\sqrt{R}$ against mass number of the parent nuclei A

To check the Geiger-Nuttall law for proton decay in superheavy nuclei, the logarithmic proton decay half-lives versus $1/\sqrt{Q}$ are plotted. From this variation, it is found that proton decay half-lives do not vary linearly with $1/\sqrt{Q}$. Figure 4.29 shows the variation of logarithmic proton decay half-lives with $Z_d/\sqrt{Q}$ and it is found that proton decay half-lives do not vary linearly with $Z_d/\sqrt{Q}$ also. This fact clearly indicates that proton decay does not follow Geiger-Nuttall law.

The nuclear charge radii is possible to derive from the proton decay half-lives. The nuclear charge radii are evaluated using the semi-empirical relation explained in the literature [313]. Figure 4.30 shows the variation of $\sqrt{R}$ against mass number of the parent nuclei. From this variation it is observed that nuclear charge radii of superheavy nuclei does not vary systematically with mass number of parent nuclei.

The Proton decay in almost all superheavy nuclei with atomic number $Z=104-126$ is studied and it is listed in table 4.15. Proton decay half-lives are also longer than that of other decay modes. The competition of proton decay with different decay modes such as alpha decay and spontaneous fission reveals that proton decay is not dominant decay mode in the superheavy nuclei region. This means superheavy nuclei including Dubnium is stable against the proton decay.

4.2.5.1 systematics of proton radioactivity of Dubnium

The amount of energy released during one proton radioactivity is studied using the mass excess values available in the literature [164, 280]. The penetration factor (P), normalization factor (F) and logarithmic half-lives for proton decay in the heavy nuclei of $^{241-251}\text{Db}$ is studied. The spontaneous fission half-lives and alpha decay half-lives of the heavy nuclei of $^{241-251}\text{Db}$ is also studied. The comparison of the proton decay with the spontaneous fission and alpha decay half-lives are as shown in figure 4.31. From the figure it is observed that the spontaneous fission half-lives are

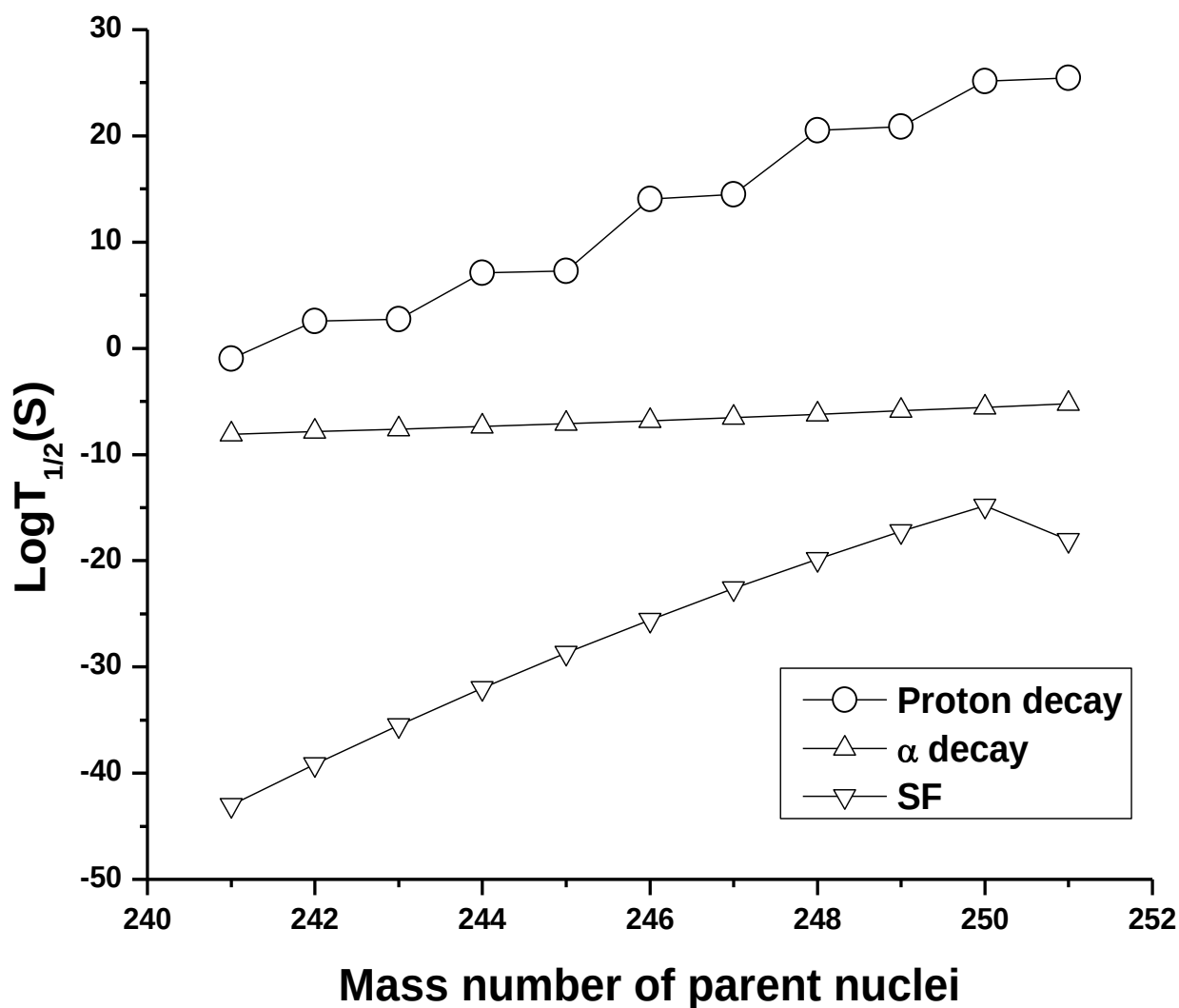


Fig. 4.31 The variation of logarithmic half-lives of the proton decay, spontaneous fission and alpha decay with the mass number of parent nuclei

smaller compared to proton and alpha decay.

The figure 4.32(a) explains the variation of amount of energy released with the mass number of parent nuclei and it decreases with increase in the mass number of parent nuclei, 4.32(b), 4.32(c) represents the penetration probability and normalization factor with the mass number of parent nuclei and both decreases with the increase in mass number of parent nuclei and 4.32(d) depicts the variation of logarithmic half lives with the product of atomic number and energy released during proton decay.

The proton decay in the heavy nuclei $^{241-251}\text{Db}$ is studied. From the figure 4.31 and 4.32 it

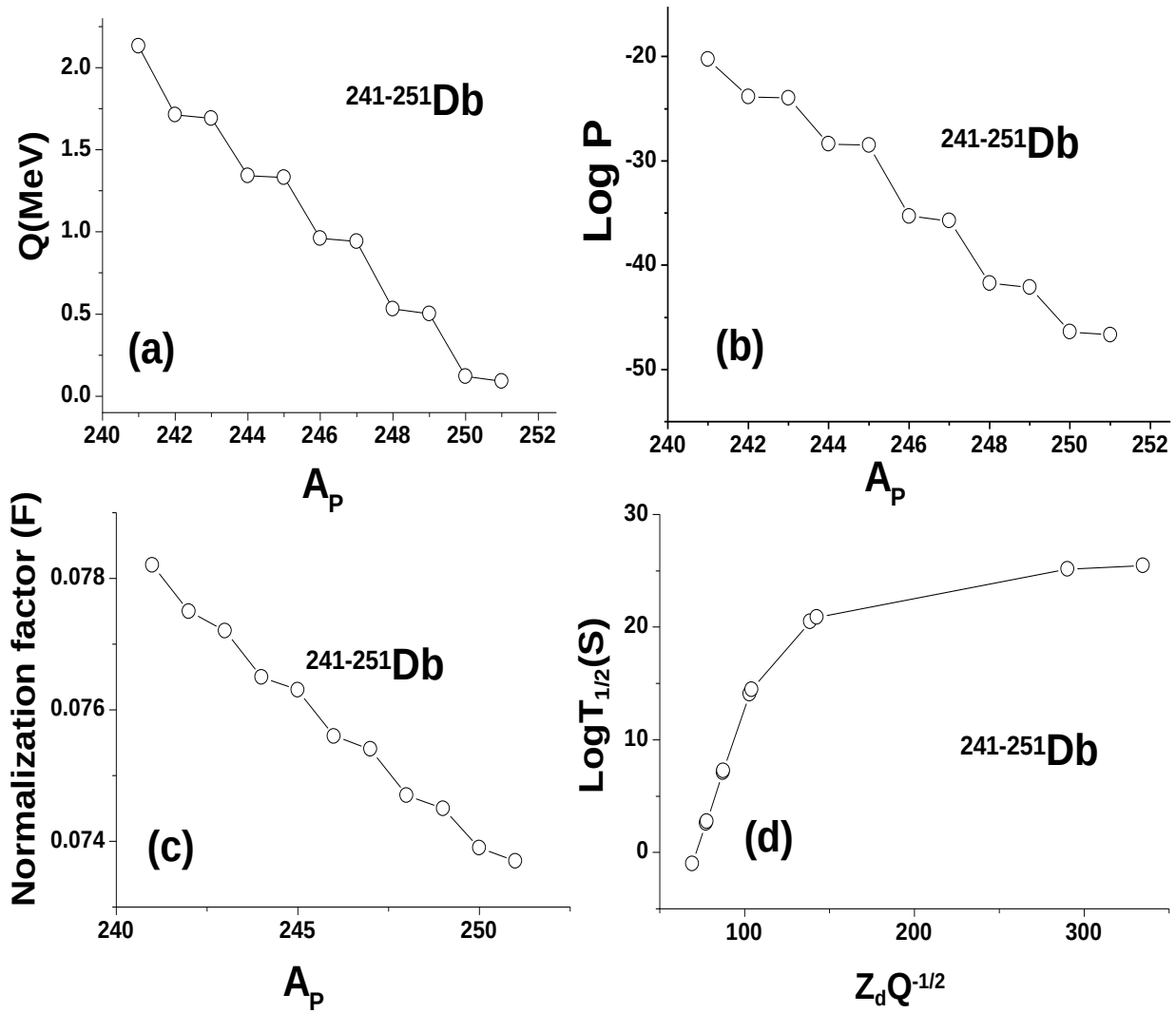


Fig. 4.32 The variation of amount of energy released, penetration probability and normalization factor with the mass number of parent nuclei and the variation of logarithmic half-lives with the product of atomic number and energy released during proton decay.

is observed that the proton decay half-lives are also longer than that of spontaneous fission and alpha decay. The competition of proton decay with different decay modes such as alpha decay and spontaneous fission reveals that proton decay is not dominant decay mode in the heavy nuclei $^{241-251}\text{Db}$.

CHAPTER 5

Comparison of present work with Microscopic models

5.1 Theory of Macroscopic models

To validate the present work, the values evaluated by the macroscopic models are compared with that of microscopic models. Brief explanation of CPPM, MGLDM and ELDM were explained in chapter 3. Eventually the other models such as GLDM, UDLP, GLM and UFM are explained below.

5.1.1 Generalized Liquid Drop Model (GLDM)

Royer et al., [170, 171] presented the Generalized Liquid Drop Model, which incorporates nuclear proximity energy and quasi molecular structures. The macroscopic energy terms including the surface, volume, Coulomb and proximity energies during an evaluation of proton decay half-lives [2] is expressed as;

$$E = E_v + E_s + E_c + E_{Prox} + E_\ell \quad (5.1)$$

The volume (E_v), surface (E_s) and coulomb energies (E_c) are given by;

$$E_v = -15.494(1 - 1.8I^2)A \text{ MeV} \quad (5.2)$$

$$E_s = 17.9439(1 - 2.6I^2)A^{2/3} \frac{S}{4\pi R_0^2} \text{ MeV} \quad (5.3)$$

$$E_c = 0.6e^2(Z^2/R_0) \times 0.5 \quad (5.4)$$

$$\int (V(\theta)/V_0)(R(\theta)/R_0)^3 \sin(\theta) d\theta \text{ MeV}$$

V_0 is the surface potential of the sphere and $V(\theta)$ is the electrostatic potential at the surface. For post-session region, the total potential and half-lives are evaluated as explained in the literature [2].

5.1.2 universal decay law for proton emission (UDLP)

The universal decay law for proton emission [116] is evaluated as follows;

$$\log T_{1/2} = a\chi' + b\rho' + d\ell(\ell + 1)/\rho' + c \quad (5.5)$$

The values of a , b , c and d are taken from the table 1 of reference [116]. In equation 5.5 the term $\rho = \sqrt{AZ_p Z_d (A_d^{1/3} + A_p^{1/3})}$, $\chi' = A^{1/2} Z_p Z_d Q_p^{-1/2}$ and $A = A_d A_p / (A_d + A_p)$. In case of one proton radioactivity $Z_p = A_p = 1$.

5.1.3 Gamow-like model for Proton decay (GLM)

As similar to an alpha and cluster radioactivity, proton decay is also understood by quantum tunneling through one-dimensional barrier [10]. The potential energy for Gamow like model is

evaluated as follows;

$$V(r) = \begin{cases} -V_0 & 0 \leq r \leq R_{in} \\ V_C(r) + V_\ell(R) & r > R_{in} \end{cases} \quad (5.6)$$

The Coulomb potential is given by;

$$V_C(r) = \frac{Z_p Z_d e^2}{r} \quad (5.7)$$

and the centrifugal potential is given by

$$V_\ell(r) = \frac{\hbar^2 \ell(\ell + 1)}{2\mu r^2} \quad (5.8)$$

where ℓ is the angular momentum and μ is the reduced mass. The penetration probability evaluated using boundary conditions and estimation of proton decay half-lives using Nilsson potential is carried out using the set of equations given in reference [10].

5.1.4 Unified Fission model for Proton decay (UFM)

The unified fission model uses mainly two conditions with $r \geq R_1 + R_2$ and $r < R_1 + R_2$. The term $V(r)$ is a polynomial function with R_0 as a radius of parent nuclei, R_1 and R_2 are the radii of daughter and emitted proton nucleus respectively. The term R_i is given by;

$$R_i = (1.28A_i^{1/3} - 0.76 + 0.8A_i^{-1/3})fm, \quad i = 0, 1, 2 \quad (5.9)$$

The total potential is evaluated as follows;

$$V(r) = \begin{cases} a_0 + a_1 r + a_2 r^2 & \text{for } R_0 \leq r \leq R_1 + R_2 \\ V_P(r) + V_\ell(r) + \frac{Z_1 Z_2 e^2}{r} & \text{for } r \geq R_1 + R_2 \end{cases} \quad (5.10)$$

here Z_1 and Z_2 are the atomic number of daughter and emitted particle respectively. In the above equation, the coefficient a_0 , a_1 and a_2 are evaluated using boundary conditions;

a) At $r = R_0 = R_{in}$, $V(r)=Q$

b) At $r = R_1 + R_2$, $V(r) = V(R_1 + R_2)$

c) At $r = R_1 + R_2$, $\frac{dV(r)}{dr} = \frac{dV(R_1+R_2)}{dr}$.

The penetration probability is evaluated using WKB integral and boundary conditions are evaluated as explained in detail in literature [102]. The half-lives are evaluated by $T_P = \ln 2 / \nu_0 P$, here ν_0 is the assault frequency [102].

5.2 Results

The main objective of the present work is to carry out comparative study of different macroscopic models such as CPPM [314], ELDM [14, 175], GLDM [170, 171], UDLP [116], GLM [10] and UFM [102] used to evaluate proton decay half-lives with that of microscopic models such as DDM3Y [94, 95], JLM [51], M3Y+EX [96], R3Y+EX [97], SLy4[98], SRG [100], Skc and SkD [93]. Also to determine which macroscopic model and microscopic model can give minimum value of statistical treatments in elucidation of experimental data.

The Q-values play a major role in the evaluation of half-lives. A small change in the value of 0.1MeV changes the half-life value of the magnitude of one to two order. Since, the sensitivity of half-lives depends on Q-values, the selection of exact Q-value is more important. Hence, the predictive power of the different models such as Lublin–Strasbourg drop model (LSD) [315], Fi-

Table 5.1 Tabulation of rms and average value ($\bar{\delta}$) for mass excess produced by different theoretical models.

Model	rms	$\bar{\delta}$
LSD	0.608	-0.027
FRDM	0.654	-0.059
FRDM12	0.579	-0.01
TF	0.649	0.027
HFB21	0.572	0.03
GHFB	0.789	-0.103
DZ	0.394	-0.032
KTUY	0.701	-0.058
INM	0.362	-0.011
WS3+	0.248	-0.008
WS4+	0.17	0

nite Range Droplet Model (FRDM) [290], Finite Range Liquid Droplet Model (FRLDM) [316], Thomas–Fermi model (TF) [317], HFB21 [318], D1MGogny forces (GHFB) [319], Duflo and Zuker (DZ) [320], Koura et al., (KTUY) [321], Nayak and Satpathy (INM) [322], WS3+ [323] and WS4+ [323] were studied in detail by previous researchers [324]. They also investigated the ability of the above nuclear mass models to predict experimental mass excess values. The accuracy of the different mass models were evaluated using root mean square and average values of the discrepancies is as follows;

$$rms^2 = \frac{1}{N_{nucl}} \sum_{i=1}^{N_{nucl}} (m_{th} - m_{exp})^2 \quad (5.11)$$

and

$$\bar{\delta} = \frac{1}{N_{nucl}} \sum_{i=1}^{N_{nucl}} (m_{th} - m_{exp}) \quad (5.12)$$

The values of rms, $\bar{\delta}$ and number of nuclei considered along with the mass model were extracted from the table A of reference [324]. The consolidated values from the table A of previous reference is tabulated in table 5.1. The two most recent Chinese versions i.e WS3+ and WS4+ achieve

the highest accuracy. However, in the present calculations we have considered experimental mass excess values available in literature and wherever experimental data is not available we have considered theoretical mass excess values from WS4+.

Table 5.2 The comparison logarithm half-lives of proton radioactivity produced by different macroscopic models CPPM, ELDM, UDLP, GLM and UFM with that of experiments [325] along with decay energies and angular momentum.

Parent nuclei	ℓ	Q_{exp}	$logT_{1/2}$						
			exp	CPPM	ELDM	GLDM	UDLP	GLM	UFM
¹⁰⁹ I	2	0.83	-3.99	-4.77	-5.64	-4.58	-3.76	-4.86	-5.44
¹¹² Cs	2	0.82	-3.31	-2.56	-2.23	-2.41	-2.88	-2.31	-2.91
¹¹³ Cs	2	0.98	-4.78	-5.10	-3.52	-4.22	-5.04	-9.02	-9.61
¹¹⁷ La	2	0.82	-1.61	-4.64	-3.25	-4.64	-2.06	0.01	-0.60
¹²¹ Pr	2	0.91	-3.05	-4.56	-4.86	-4.21	-2.50	-1.88	-2.18
¹³⁰ Eu	2	1.03	-1.58	-3.68	-3.96	-5.59	-2.81	-3.25	-3.87
¹³¹ Eu	2	0.95	-2.99	-3.16	-3.86	-5.26	-1.75	0.05	-0.57
¹³⁵ Tb	3	1.19	-3.03	-2.86	-3.85	-3.75	-3.24	-7.60	-8.21
¹⁴⁰ Ho	3	1.11	-2.22	-2.64	-3.56	-4.25	-1.64	-2.79	-3.43
¹⁴¹ Ho	3	1.19	-2.39	-2.94	-2.12	-1.98	-2.64	-5.89	-6.52
¹⁴⁴ Tm	5	1.73	-5.57	-4.28	-2.58	-3.68	-5.22	-8.32	-8.93
¹⁴⁵ Tm	5	1.75	-1.92	-2.58	-2.68	-2.16	-1.39	-8.90	-9.50
¹⁴⁶ Tm	5	1.21	-1.18	-1.28	-2.54	-2.62	-1.20	-4.80	-5.43
¹⁴⁷ Tm	5	1.07	0.59	-0.88	-0.78	-1.02	0.36	0.49	-0.15
¹⁵⁰ Lu	5	1.28	-1.18	-1.81	-3.44	-2.99	-1.32	-5.81	-6.45
¹⁵¹ Lu	5	1.25	-0.89	-1.92	-2.58	-3.58	-1.05	-4.90	-5.54
¹⁵⁵ Ta	5	1.79	-4.92	-4.03	-3.51	-4.87	-4.64	-7.27	-7.89
¹⁵⁶ Ta	2	1.03	-0.62	-1.64	-1.62	-1.95	-0.84	6.07	5.41
¹⁵⁷ Ta	0	0.95	-0.52	-0.09	-1.25	-0.98	-0.43	2.14	5.48
¹⁵⁹ Re	5	1.84	-4.68	-4.40	-4.54	-3.39	-4.42	-6.82	-1.44
¹⁶⁰ Re	2	1.28	-3.05	-4.46	-4.76	-5.28	-3.19	-2.40	-3.05
¹⁶¹ Re	0	1.21	-3.43	-3.03	-2.70	-3.11	-3.21	0.11	-0.54
¹⁶⁴ Ir	5	1.84	0.59	0.72	0.35	0.57	-3.99	-5.98	-6.60
¹⁶⁵ Ir	5	1.73	-3.47	-3.17	-2.62	-3.32	-3.28	-3.64	-4.27
¹⁶⁶ Ir	2	1.17	-0.82	-0.64	-0.76	-1.10	-1.23	-3.42	-2.76
¹⁶⁷ Ir	0	1.08	-0.96	-2.15	-1.38	-1.87	-0.93	6.97	6.31
¹⁷⁰ Au	5	1.49	-4.01	-0.58	-2.28	-0.64	-3.92	-6.18	-6.83
¹⁷¹ Au	0	1.47	-4.77	2.43	-3.56	1.83	-4.42	-5.38	-6.03
¹⁷⁶ Tl	0	1.28	-2.28	-1.16	-0.35	-1.07	-1.88	2.69	2.02
¹⁷⁷ Tl	0	1.18	-1.17	-0.95	-0.39	-1.05	-0.73	6.61	5.94
¹⁸⁵ Bi	0	1.62	-4.23	-0.03	0.46	-0.17	-4.51	-7.10	-7.76

The proton radioactivity half-lives of parent nuclei in the atomic number range $53 \leq Z \leq$

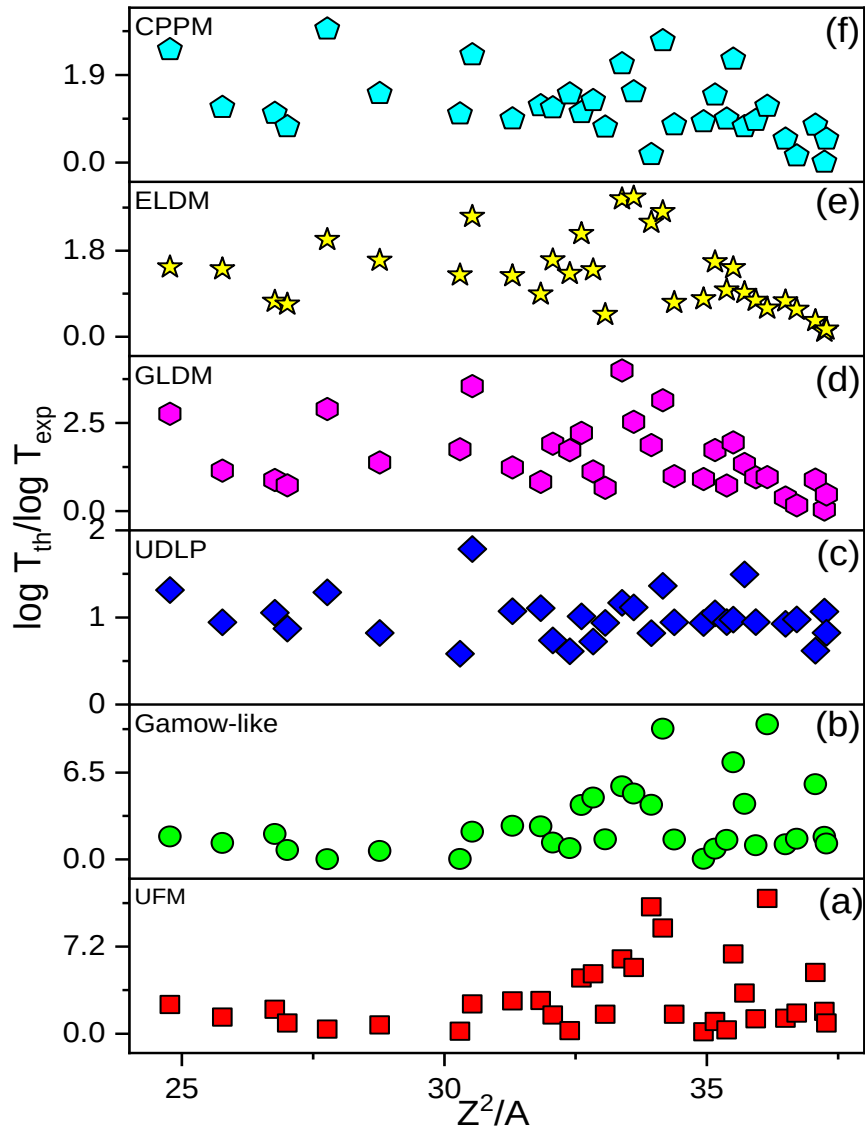


Fig. 5.1 Deviations between macroscopic models such as UFM, GLM, UDLP, GLDM, ELDM and CPPM with that of experimental logarithmic half-lives of proton radioactivity from ^{109}I to ^{209}Bi as a function of Z^2/A .

83 are studied using macroscopic models such as CPPM, ELDM, UDLP, GLM and UFM as explained in the section 5.1. The experimental half-lives of proton radioactivity is extracted from latest nuclear data [325]. For an instance, in case of ^{109}I nuclei different theoretical macroscopic models have been employed in order to evaluate its proton decay half-lives. The angular momentum is evaluated using spin-parity selection rules with; The selection rule for proton decay [109]

is as follows;

$$J_p = J_d + J_{p^\ell} \quad (5.13)$$

and

$$\pi_p = \pi_d \pi_{p^\ell} (-1)^\ell \quad (5.14)$$

where J_p , π_p , J_d , π_d , J_{p^ℓ} and π_{p^ℓ} are spin and parity values of the parent, daughter and outgoing proton particle respectively. Among 32 proton emitters available in literature, the nuclei ^{105}Sb is emitted because it is no longer a proton emitter [326, 327]. Hence, around 31 proton emitters are evaluated using macroscopic theoretical models such as CPPM, ELDM, UDLP, GLM and UFM. The proton decay half-lives obtained using these macroscopic models are tabulated in table 5.2. The closure look of the table gives an insight in to the logarithmic half-lives produced by each model. For an instance the nuclei ^{109}I whose experimental $\log T_{1/2}$ value is -3.987 [94] when $\ell = 2$. From the comparison between experimental and macroscopic models it is clear that UDLP produces close value when compared to other models studied if we adopt experimental Q-value of 0.829MeV. Similarly, in all other cases we have also observed closer reproduction of experimental proton decay half-lives. These striking results are due to the coefficients (a-d) which were used to fit available experimental data. Since, the UDLP is good enough to reproduce the experimental value with root mean square deviation (σ) of 0.93, however beyond atomic number range $53 \leq Z \leq 83$ one should require a model which can effectively reproduce the experimental data as well as prediction of new proton emitter. In this aspect, the ELDM model closely reproduces the experimental data within an error of $\sigma=1.61$ and also effectively predicts logarithmic half-lives in the unexplored isotopes.

In order to give intuitive comparisons of the experimental proton decay half-lives with the cal-

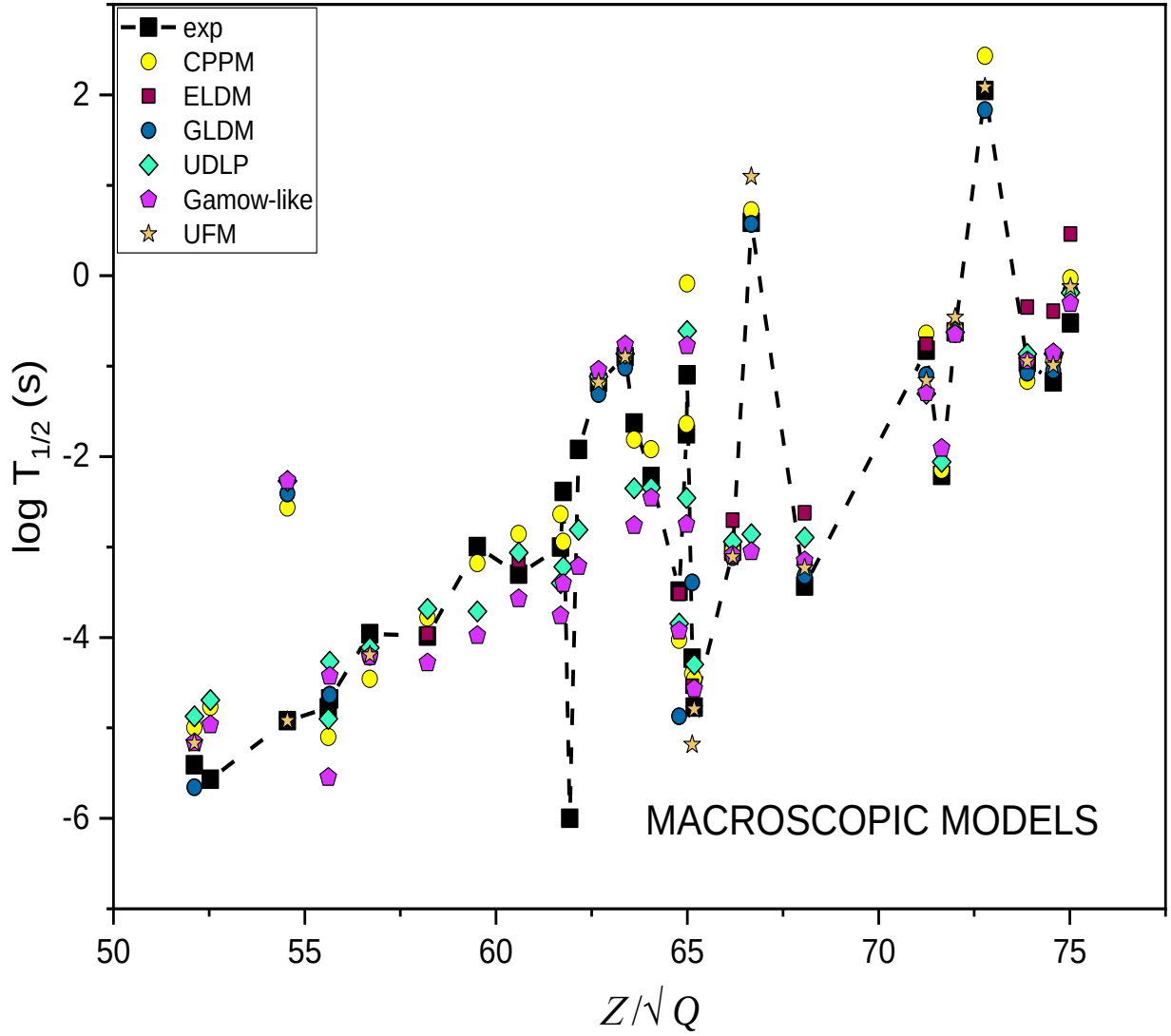


Fig. 5.2 Comparison of proton decay half-lives produced by macroscopic models such as CPPM, ELDM, GLDM, UDLP, GLM and UFM with that of experiments for various $Z/\sqrt{Q}$ values.

culated values, the deviations calculated using various macroscopic models such as GLDM, CPPM, ELDM, UDLP, GLM and UFM with that of experimental logarithmic half-lives of proton radioactivity from ^{109}I to ^{209}Bi as a function of Z^2/A are presented and is shown in figure 5.1(a-f). From these figures, it is clear that the proton radioactivity half-lives calculated using GLM and UFM are significantly different with the large value of up to 8. For the same experimental data, the models such as CPPM, ELDM and GLDM produces the deviation up to 2. However, the UDLP produces the deviation nearly equal to 1 in majority of cases. The logarithmic half-lives and the experi-

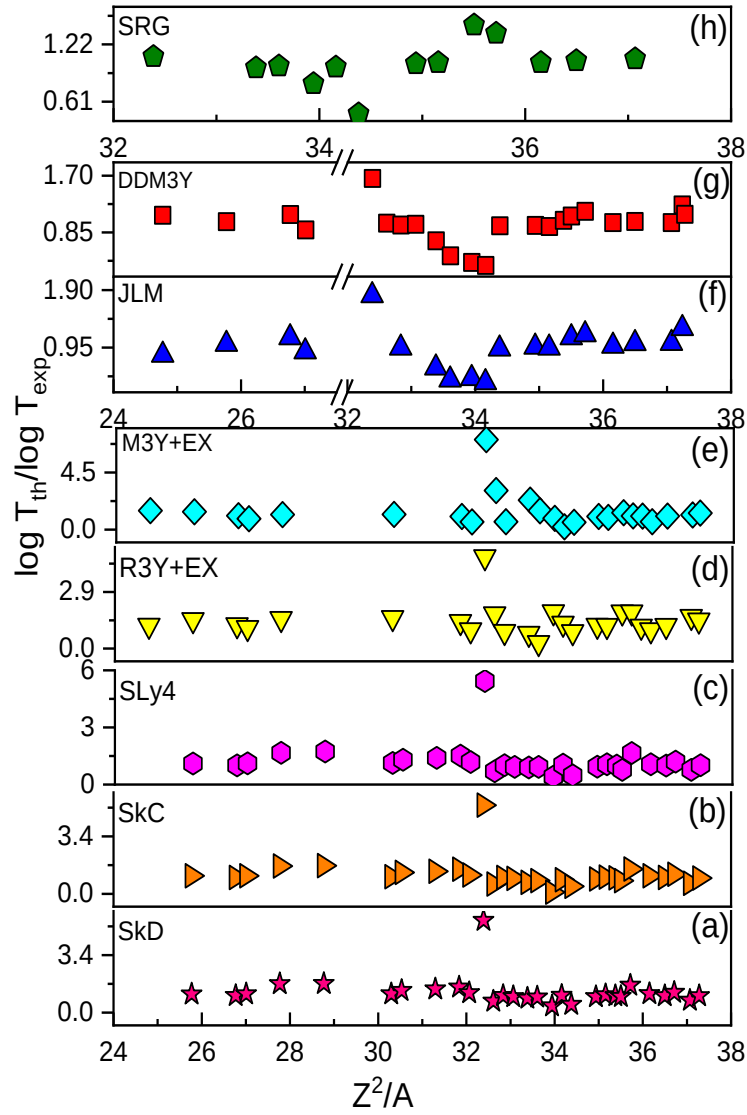


Fig. 5.3 Deviation between microscopic models such as DDM3Y [94, 95], JLM [51], M3Y+EX [96], R3Y+EX [97], SLy4[98], SRG[100], Skc and SkD [93] with that of experimental logarithmic half-lives of proton radioactivity from The nuclei ^{109}I to ^{209}Bi for various Z^2/A values.

mental values are plotted as function of $Z/\sqrt{Q}$ using different macroscopic models is as shown in figure 5.2. The closer look of the figure reveals that both ELDM and UDLP models achieve high accuracy of half-lives when compared to other macroscopic models such as CPPM, GLDM, GLM and UFM.

Furthermore, an information is gathered regarding proton decay half-lives produced using microscopic models such as DDM3Y [94, 95], JLM [51], M3Y+EX [96], R3Y+EX [97], SLy4[98], SRG [100], Skc and SkD [93]. A plot of $\log T_{th}/\log T_{exp}$ using microscopic models as a function

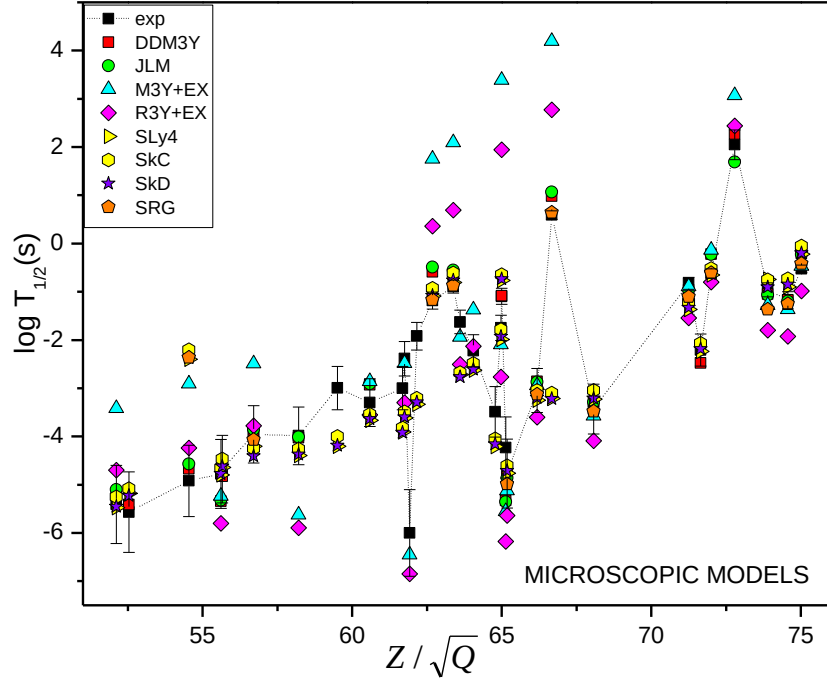


Fig. 5.4 Comparison of proton decay half-lives produced by microscopic models such as DDM3Y, JLM, M3Y+EX, R3Y+EX, SLy4, SRG, SkC and SkD with that of experiments for various $Z/\sqrt{Q}$ values.

of Z^2/A is presented in figure 5.3(a-h). One can see that the results obtained using microscopic models are in good agreement with the experimental values in majority of the cases. In some cases, even the microscopic models deviate from the experimental data. For an example in case of ^{147}Tm the final outcome is off by a factor of 3 and above in all investigated microscopic models. Similarly, in case of ^{156}Ta nuclei the results of JLM and DDM3Y are lowered by an order of one. The SRG shows deviation up to 1 in case of ^{167}Ir . Further, in all other cases, the experimental values are well reproduced in by a microscopic models. Eventually, the plot of $\log T_{1/2}$ as function of $Z/\sqrt{Q}$ using the data extracted from different microscopic models are shown in figure 5.4. One can observe from the figure that there are large deviations in case of M3Y+EX and R3Y+Ex when compared to other microscopic models. This is not to say that these two models provide the worst results. But comparatively, M3Y+EX and R3Y+Ex produces large deviation than that of other microscopic models investigated.

Table 5.3 The root mean square error (RMSE) and root mean square deviations (σ) of the calculations from macroscopic models such as CPPM, ELDM, GLDM, UDLP, GLM and UFM with respect to the experimental data and microscopic models such as DDM3Y, JLM, M3Y+EX, R3Y+EX, SLy4, SRG, Skc and SkD with that of experiments.

Model	RMSE	σ
DDM3Y	0.35	0.5
JLM	0.41	0.59
M3Y+EX	1.19	1.06
R3Y+EX	1.19	0.98
SLy4	1.01	0.77
SkC	0.99	0.77
SkD	1.01	0.77
SRG	0.69	0.55
CPPM	2.21	1.24
ELDM	1.93	1.26
GLDM	2.71	1.43
UDLP	0.91	0.68
GAMOW	3.47	1.68
UFM	3.7	1.76

The root mean square error (RMSE) and root mean square deviation (σ) between the macroscopic, microscopic and experimental are given by;

$$RMSE = \sqrt{\frac{1}{N} \sum \log \left(\frac{T_{th}}{T_{exp}} \right)^2} \quad (5.15)$$

and

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^N (\log(T^{exp}) - \log(T^{th}))^2} \quad (5.16)$$

where N is the number of nuclei considered to evaluate proton decay half-lives using microscopic and macroscopic models. The quantified deviations are tabulated in table 5.3. From the table it is clearly seen that RMSE and σ are smaller for the UDLP model. Since, the UDLP model is depended on coefficients which were fitted by taking an experimental proton decay half-lives. Applicability of UDLP beyond $53 \leq Z \leq 83$ is not known clearly. Hence the other possibility

is the ELDM with 1.93 and 1.26 of RMSE and σ respectively. Hence, during the prediction of proton decay half-lives ELDM macroscopic model is more effective in the unexplored region. Similarly, DDM3Y and JLM microscopic models effectively produces proton decay half-lives with less deviation with respect to experiments than that of other microscopic models investigated.

CHAPTER 6

Summary

6.1 Semi-empirical formula for one and two proton radioactivity

A thorough study on one proton and two proton radioactivity is carried in the atomic number range $3 \leq Z \leq 126$ using well accepted Coulomb and proximity potential model (CPPM), effective liquid drop model(ELDM), generalized liquid drop model(GLDM) and modified generalised liquid drop model(MGLDM) models. semi empirical formula for one and two proton decay half-lives is constructed by including angular momentum term . The values produced by the present formula is also compared with experiments. Around 241 and 174 energetically feasible one and two proton emitters were identified in the atomic number range $3 \leq Z \leq 126$.

6.2 Verification of Geiger-Nuttall law for Proton Radioactivity.

In order to construct a empirical relation, we have considered logarithmic half-lives of proton decay available in the reference [328] and [277] as a function of $Z_D/\sqrt{Q}$ and it is as follows;

$$\log T_{1/2} = f \left(\frac{Z_D^n}{\sqrt{Q}} \right) \quad (6.1)$$

From the study of logarithmic half-lives of proton decay as a function of $Z_D/\sqrt{Q}$, we have observed the systematic variation as shown in figure 6.1. The search was made to parameterize the

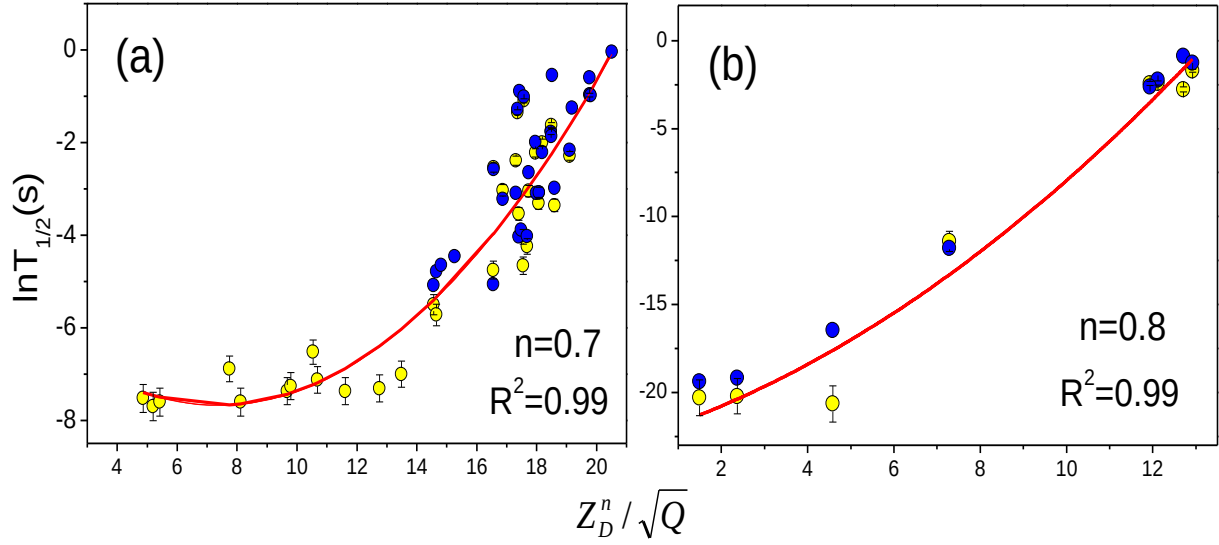


Fig. 6.1 Variation of logarithmic half lives as function of $Z_D^n / Q^{1/2}$. (a) One proton decay - Yellow circles represents the data corresponds to experiments [271], the blue colour circles represents the values produced by previous formula [277] and continues red line represents values obtained from the present formula and (b) Two proton decay- Yellow circles represents the two proton decay half-lives corresponds to experiments [271], blue colour circles represents the values produced by previous formula [329] and continues red line represents values obtained from the present formula.

logarithmic half-lives of one proton decay and 2-proton decay. We have tried different functions such as $a(Z_D / \sqrt{Q}) + b$, $a \ln(Z_D / \sqrt{Q}) + b$, $\frac{1}{a(Z_D / \sqrt{Q}) + b}$, $a \exp^{(Z_D / \sqrt{Q})} + b$, $a \exp \frac{b}{(Z_D / \sqrt{Q})}$ and polynomial functions to fit the available experimental values of one and two proton decay. Among different equations mentioned above, we have considered a second order polynomial equation whose residual sum of squares(RSS) are nearly equal to 1. The plot of logarithmic half-lives with the $Z_D / \sqrt{Q}$ is shown in the figure 6.1. Finally the constructed semi-empirical formulae for the one and two proton decay based on Geiger Nuttall law is as follows;

$$\log T_{1/2}^P = a \times \left(\frac{Z_D^n}{\sqrt{Q}} \right)^2 + b \times \left(\frac{Z_D^n}{\sqrt{Q}} \right) + c \quad (6.2)$$

Where a , b and c are the fitting parameter and the coefficients corresponding to these values were tabulated in table 6.1.

Table 6.1 The index “n” and fitting coefficients a , b and c of one and two proton decay.

Decay type	range	n	a	b	c
1P	$5 \leq Z \leq 83$	0.7	0.04405	-0.65101	-5.26603
2P	$4 \leq Z \leq 36$	0.8	0.06954	0.76778	-22.58463

6.3 Studies on Proton radioactivity using Macroscopic models

A thorough study on one proton and two proton radioactivity is carried in the atomic number range $3 \leq Z \leq 126$ using well accepted Coulomb and proximity potential model (CPPM), effective liquid drop model(ELDM), generalized liquid drop model(GLDM) and modified generalised liquid drop model(MGLDM) models. The proton decay half-lives produced by these models are compared with the experiments. Predictive power of these models are assessed by evaluation of the mean squared error. Present work is validated by comparing with experiments. Among the studied three models, CPPM model produces proton decay half-lives close to experiments than the other two models.

6.4 Proton radioactivity in Lanthanides ($57 \leq Z \leq 71$)

The proton decay half-lives in the lanthanide region have been systematically studied using different proximity functions such as Prox. 13, Prox. 77, MP 77, Ng 80 and Bass 80. Though the experimental values are found to be in good agreement with the proximity function of Ng 80, empirical formula to calculate the half-lives of such proton-emitting lanthanides is developed. The half-lives values produced by the present formula is compared with that of NG 80. The present formula produces the half lives with simple inputs of Z_d and Q values and hence it may called as pocket formula.

The competition between the evaluated proton decay half-lives with other competent decay modes such as alpha, β^+ , β^- , spontaneous fission and proton decay are also studied.Eventually,

24 new proton decay emitters in the Lanthanide region are identified. Newly identified proton emitters find application in radiation therapy.

The proton radioactivity in Dysprosium isotopes have been examined using effective liquid drop model. Proton radioactivity is energetically feasible in the isotopes Dysprosium from ^{133}Dy to ^{135}Dy . The different decay modes such as β^- -decay, β^+ -decay, and α decay are evaluated in the isotopes of $^{133-173}\text{Dy}$. The half-lives against proton decay have been compared to competing decay modes such as β^- -decay, β^+ -decay, and α decay. In the isotopes of $^{133-149}\text{Dy}$ and $^{151-154}\text{Dy}$ β^+ -decay is dominant. In ^{150}Dy , α -decay is dominant, whereas β^- -decay is dominating in $^{155-173}\text{Dy}$. The decay chains of $^{133-135}\text{Dy}$ were investigated. The detail investigations of $^{133-135}\text{Dy}$ shows that the probability of observing proton radioactivity when $^{133-135}\text{Tb}$ is converted to $^{132-134}\text{Gd}$ with the half-life of order of nanoseconds. Further, these identified proton emitters may find useful in the field of diagnosis and radiotherapy.

6.5 Proton radioactivity of Heavy nuclei ($72 \leq Z \leq 88$)

The proton radioactivity of heavy nuclei of atomic number range $72 < Z < 88$ is studied. The energy released during the proton decay (Q_P), and half-lives of proton decay are calculated. The competition between different decay modes is studied, by comparing the proton decay half lives with that of other decay modes such as alpha decay, beta decay, and spontaneous fission. To check the Geiger-Nuttall law for proton decay, the logarithmic proton decay half-lives are plotted against $1/\sqrt{Q}$. Also the possible proton emitters are also highlighted with the corresponding energies and half-lives in the atomic number range $72 < Z < 88$.

Using different models such as Coulomb and proximity potential model, effective liquid drop model and modified generalised liquid drop model, 1P-radioactivity of Tantalum is studied. The decay constant(λ) and half-lives($T_{1/2}$) of $^{151-157}\text{Ta}$ were predicted.

The calculated half-lives from the present work are compared with the available experiments. The decay energy is feasible for the nuclei $^{151-157}\text{Ta}$. The identified 1P-radioactivity of $^{151-157}\text{Ta}$ along with half-lives and decay energies plays an important role in the future experiments and may find useful applications in radiotherapy and diagnosis.

The different decay modes such as proton decay, beta-decay and an alpha decay have been evaluated using CPPM and semi-empirical relations in the isotopes of Bismuth. The values obtained from the present work were comparable with the experiments. Around 9 α emitters, one proton emitter, 18 β^+ emitters and 33 β^- emitters were identified. Among the β^- emitters, around 25 new emitters from ^{220}Bi to ^{244}Bi were newly identified. These identified new β^- emitters are useful in the field of radiotherapy.

6.6 Proton radioactivity of Actinide nuclei ($89 \leq Z \leq 103$)

The one-proton emission in the actinide region from $Z=89-103$ is theoretically studied. The half-lives of one-proton emitters in the actinide region are presented using the coulomb and proximity potential method. One-proton decay half-lives of present study have shown good agreement with the available experimental values. The studied proton decay half-lives are compared with that of other decay modes such as alpha decay, spontaneous fission and beta decay.

The proton emitters in the actinide region have been identified. The prediction of the new proton emitters in the actinide region are having measurable half-lives can be retrieved in future with developing experimental techniques.

The systematics of one proton decay in the actinide region through the study of energy released, penetration probability and logarithmic half-lives in the actinide region is studied. The studied half-lives of present work is compared with the different decay modes such as alpha decay and spontaneous fission. The possible proton emitters with the corresponding energies and half-

lives in the actinide region are identified. The possible proton emitters in the actinide region are $^{195-203}\text{Ac}$, $^{200-207}\text{Pa}$, $^{212-220,224}\text{Am}$, and $^{218-221}\text{Bk}$. The proton emitters in the unexplored isotopes of actinide region have been identified which is not specified in the nuclear chart.

6.7 Proton radioactivity of Superheavy nuclei ($104 \leq Z \leq 126$)

The proton decay in almost all superheavy nuclei with atomic number $Z=104-126$ is studied, out of which proton decay is possible in few superheavy nuclei. We have calculated the energy released during the proton decay (Q_P), penetration factor (P), normalisation factor (F) and half-lives of proton decay. Proton decay half-lives are also longer than that of other decay modes such as alpha decay and spontaneous fission. The competition of proton decay with different decay modes reveals that proton decay is not dominant decay mode in the superheavy nuclei region. This means superheavy nuclei is stable against the proton decay.

The proton radioactivity of Dubnium is studied. The amount of energy released during the proton decay, penetration probability, normalization factor and logarithmic half-lives in the superheavy nuclei of $^{241-251}\text{Db}$ is calculated. The Proton decay half lives of Dubnium are compared with the spontaneous fission and alpha decay. From the results it can be concluded that the superheavy nuclei of $^{241-251}\text{Db}$ are having half-lives greater than the spontaneous fission and alpha decay. Hence, the superheavy nuclei $^{241-251}\text{Db}$ is stable against the proton decay.

6.8 Scope of research work

The prediction of the new proton emitters in the actinide region having measurable half-lives can be retrieved in future with developing experimental techniques. Proton therapy is a type of radiation used to treat cancer. Proton therapy sends positively charged atomic particles called protons [330]. Proton therapy is used to treat breast cancer [331, 332], brain cancer [333], head and neck cancer [334] and hepatocellular carcinoma of a liver tissue [335, 336]. Radio nuclides with different energy ranges up to 5 MeV are used in the radiotherapy [336, 337]. The identified new proton emitters having decay energy between 0.378 - 4.321 KeV which clearly suggests that the proton emitters identified in the lanthanide region might find application in radiation therapy. There is a need to make progress in preclinical proton radiation biology to give accessible data to medical physicists and practicing radiation oncologists. Proton radioactivity studies are becoming important tool for nuclear structure.

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LIST OF PUBLICATIONS

Journal publications

- [1] M.G.Srinivas, H.C.Manjunatha, K.N.Sridhar, N.Sowmya and Alfred Cecilraj. Proton decay of actinide nuclei. *Nuclear physics A* 995(2020)121689. ISSN : 0375-9474.
- [2] M.G.Srinivas, H.C.Manjunatha, N.Sowmya, Damodara Gupta and Alfred Cecil Raj. Systematics of proton decay of actinides. *Indian Journal of Pure and Applied Physics* Vol.58, April 2020,pp.255-262. ISSN : 0975-1041.
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- [6] M.G. Srinivas, N. Sowmya, H.C. Manjunatha, P.S. Damodara Gupta and R. Munirathnam and N.Manjunatha Proton Radioactivity of Dysprosium, DOI-10.1142 / S0218301322500434.

Int. mod. phys. E 2022.

- [7] M.G.Srinivas, H.C.Manjunatha, K.N.Sridhar, and Alfred Cecil Raj. A Systematic study of proton decay in super heavy elements. (accepted in Ukranian Journal of Physics)
- [8] M.G.Srinivas, H.C. Manjunatha, N.Sowmya, N.Manjunatha, and S.Alfred Cecil Raj. Semi-empirical formulae for one and two proton radioactivity. (Communicated to Indian Journal of Physics.)
- [9] M.G. Srinivas , R. Munirathnam, N. Sowmya, H.C. Manjunatha, and S.Alfred Cecil Raj. Predictive power of macroscopic and microscopic models for proton decay. (Communicated to European Physical Journal plus).

Full papers in conference proceedings

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Proton decay of actinide nuclei

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Abstract

The half-lives of one-proton emitters in the actinide region are presented using the coulomb and proximity potential method. The studied proton decay half-lives are compared with that of other decay modes such as alpha decay, spontaneous fission and beta decay. We have identified proton emitters in the actinide region and also it is the competing decay mode for all the observed proton emitters. We have included the effects of deformations in the present study. One-proton decay half-lives of present study have shown good agreement with the available experimental values.

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1. Introduction

The phenomenon of proton emission from nuclear ground states limits the possibilities of creation of more exotic proton rich nuclei which are generally produced by fusion-evaporation nuclear reactions. In the energy domain of radioactivity, proton can be considered as a point charge having highest probability of being present in the parent nucleus. In the early 60s, Goldanskii [1] experimentally proposed one and two proton radioactivity for both odd and even atomic

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number. Mukha et al. [2,3] observed one and two-proton radioactivity of more than 21 high-spin isomers in ^{94}Ag . Routray et al. [4] estimated the half-lives of proton radioactivity with Yukawa effective interaction. Deng et al. [5] used different proximity potentials to study the proton activity, α decay and heavy particle radioactivity. In 2015 previous researchers [6] measured the proton activity in ^{67}Kr at the RIKEN Nishina Center. Using silicon detectors Giovinazzo et al. [7] experimentally studied two-proton activity in ^{45}Fe . Previous workers [8,9] experimentally studied one and two proton radioactivity of ^{45}Fe and ^{54}Zn . Theoretical models of proton radioactivity from spherical nuclei can predict the systematics of proton decay and spectroscopic factors [10]. Santhosh et al. [11] described proton activity of nuclei with $Z > 50$ using CPPMDN model. The researchers identified proton active nucleus beyond proton drip line using the new detector system designed at SHIP [12]. Auranen et al. [13] experimentally observed the proton decay of ^{108}I . Alavi et al. [14] evaluated the proton decay half-lives using WKB method. Earlier workers [15–17] theoretically studied proton decay emission in alkaline metals and actinides. Pfutzner et al. [18] studied the proton emission phenomenon of odd Z nuclei yielding detailed structural information. The experiments are intended at finding exotic decay, are normally based on establishment of the radioactive atoms and subsequent decay of the radioactive atoms. The two-proton radioactivity was experimentally observed in ^{45}Fe [19,20], and later in ^{19}Mg [21,22], ^{48}Ni [23], and ^{54}Zn [24]. In the year 1970, the proton radioactivity was experimentally confirmed by Jackson [25] by detecting emission of proton from the ^{53}Co to the ground state of ^{52}Fe .

Subsequently many theoretical models were put forward to study half-lives of spherical and deformed nuclei [26–33]. Earlier workers [34–38] also attended to study half-lives of superheavy nuclei using different proximity potentials. Examination for proton emitting nuclei

will results in determining nuclear stability in proton rich nuclei. Although the proton emission is difficult process, the simplified method of the one-proton penetrating the coulomb barrier will explain the process with great extent. Compared to all other decay modes, proton emissions are categorised with lowest coulomb potential and less reduced mass. From the literature, it is clearly observed that a systematic study of one-proton decay half-lives are required in the actinide region. The main objective of our work is to study proton decay half-lives in the actinide region. The coulomb and proximity potential model, which have been applied for alpha and cluster decay over many years were used to study proton radioactivity. It is predicted from the study that the proton unstable nuclei near or outside the proton drip line will be discovered in the future experiments. We have predicted the half-lives of proton emitters in the actinide region, which have not been detected experimentally. The present work is organised in the following order. Section 2 explains the theoretical framework, detailed results and discussion in section 3 and conclusion in section 4.

2. Theoretical framework

2.1. Proton decay

The proton emission half-life in the actinide region is given by

$$T_{1/2} = \frac{\ln(2)}{\nu P} \quad (1)$$

where ν is the assault frequency of proton against the potential energy barrier and P is penetration probability. The assault frequency approximation is given by the harmonic oscillator frequency using Nilson's potential and it is given by [39],

$$\nu = \frac{41}{hA^{1/3}} \text{MeV} \quad (2)$$

where h is the planks constant and A is Proton mass number. Similar to spontaneous fission, ternary fission, alpha and cluster decay, and proton emission can be expressed in terms of quantum tunnelling through one dimensional barrier. Penetration probability P is calculated using WKB approximation [40].

$$P = \exp \left[-\frac{2}{h} \int_{R_{in}}^{R_{out}} \sqrt{2\mu(V-Q)} dr \right] \quad (3)$$

Where μ is reduced mass of emitted proton, $\mu = (m_P m_{A-1} / m_P + m_{A-1}) = 938.3(A-1)/A \text{ MeV}/c^2$ and Q is the energy released. R_{in} and R_{out} are the classical inner and outer turning points. The inner turning point is equal to the radius of the spherical square well in which the proton is trapped before emission and it is given by

$$R_{in} = r_0(A_1^{1/3} + A_2^{1/3}) \quad (4)$$

Where $A_1 = 1$ and $A_2 = A - 1$ for proton emission. R_{out} is determined by the condition $V = Q$. The proton–nucleus potential should consist of a Coulomb potential V_C and Proximity potential V_P .

$$V = V_C + V_P. \quad (5)$$

Where V_C Coulomb interaction potential, V_P is the proximity potential. The coulomb interaction potential is given by,

$$V_C = \frac{Z_1 Z_2 e^2}{r} \left[1 + \frac{3R^2}{5r^2} \beta_2 Y_{20}(\theta) + \frac{3R^4}{9r^4} \beta_4 Y_{40} \right] \quad (6)$$

Where Z_1 and Z_2 are the atomic numbers of proton and daughter nuclei respectively, and ' r ' is the distance between fragment centres. R , β , $Y_{20}(\theta)$ is the radius of the nuclei, quadrupole deformation parameter and spherical harmonic function respectively. Proximity potential consists of two parts, one depends on shape and geometry of the nuclei and the other is the universal function which depends on distance between two nuclei. Proximity potential given by [41,42],

$$V_P = 4\pi\gamma b \left[\frac{C_1 C_2}{C_1 + C_2} \right] \phi \quad (7)$$

where $b = 0.99 \text{ fm}$ is the width of the nuclear surface, ϕ is the universal function, C_1 and C_2 are the Susmann central radii and γ is the nuclear surface tension coefficient and it is given by

$$\gamma = \gamma_0 \left[1 - K_S \left(\frac{N-Z}{A} \right)^2 \right] \text{MeV/fm}^2 \quad (8)$$

N , A and Z are neutron, mass and proton number of the parent nuclei respectively. Where $\gamma_0 = 1.460734 \text{ MeV/fm}^2$ and $K_S = 4.0$ [43]. Susmann central radii C_1 and C_2 are obtained by using equation,

$$C_i = R_i - \left(\frac{b^2}{R_i} \right) \quad \text{for } i = 1, 2 \quad (9)$$

Where sharp radii R_i can be obtained by

$$R_i = 1.28A_i^{1/3} - 0.76 + 0.8A_i^{-1/3} \quad \text{for } i = 1, 2 \quad (10)$$

ϕ represents the universal proximity potential given as [16]

$$\Phi(z) = \begin{cases} -4.41 \exp(\frac{-z}{0.7176}) & \text{for } \varepsilon \geq 1.9475 \\ -1.7817 + 0.9270z + 0.0169z^2 - 0.05148z^3 & \text{for } 0 \leq \varepsilon \leq 1.9475 \end{cases} \quad (11)$$

2.2. Alpha decay

In the present work, we have evaluated alpha decay half-lives using the semi empirical models such as Royer formula [44], UNIV [45], NRDX [46] and Denisov Khudenko [47].

2.2.1. Royer formula (Royer)

Royer [44] proposed the semi-empirical formula for alpha decay half-lives using potential energy within a liquid drop model including proximity effects between the alpha particle and the daughter nucleus. The alpha decay half-lives depends on the decay energy, atomic number (Z_P), and mass number of parent nuclei (A_P) respectively.

$$\log T_{1/2}^R = aZQ^{-1/2} + bA^{1/6}Z^{1/2} + c \quad (12)$$

where a , b and c are adjustable parameters and are depend on parity of the parent nucleus combination (Z_P , N_P). The fitting parameters are $a_{e-e} = 1.5864$, $a_{e-o} = 1.5848$, $a_{o-e} = 1.592$, $a_{o-o} = 1.6971$, $b_{e-e} = -1.1629$, $b_{e-o} = -1.0859$, $b_{o-e} = -1.1423$, $b_{o-o} = -1.113$, $c_{e-e} = -25.31$, $c_{e-o} = -26.65$, $c_{o-o} = -29.48$, $c_{o-e} = -25.68$.

2.2.2. UNIV formula

Poenaru et al. [45] derived universal (UNIV) curve for α decay and cluster radio activities by plotting the sum of the decimal logarithm of the half-life and cluster preformation probability versus the decimal logarithm of the penetrability of external barrier. The alpha decay half-lives is given as,

$$\log T_{1/2}^{UNIV} = -\log P_S - 22.169 + 0.598(A_e - 1) \quad (13)$$

A_e is the mass number of emitted particle and

$$\text{where } -\log P_S = c_{AZ}(\arccos \sqrt{r} - \sqrt{r(1-r)}) \quad (14)$$

$$\text{with } c_{AZ} = 0.22873(\mu_A Z_d Z_e R_b)^{1/2} \quad (15)$$

where Z_d , $Z_e R_b$ are the atomic number of daughter, emitted cluster and classical turning point respectively and $r = R_t/R_b$, $R_t = 1.2249(A_d^{1/3} + A_e^{1/3})$, $R_b = 1.43998Z_d Z_e/Q$ and $\mu_A = A_d A_e/A$, A_d , A_e are the mass number of daughter and emitted particle respectively, $R_t = R_d$ is the first turning point and Q is the amount of energy released during the decay process.

2.2.3. NRDX formula

Ni et al. [46] proposed semi-empirical formula for alpha decay and cluster decay half-lives derived from the WKB barrier penetration probability and it is given by

$$\log T_{1/2}^{NRDX} = a\sqrt{\mu}Z_\alpha Z_d Q^{-1/2} + b\sqrt{\mu}(Z_\alpha Z_d)^{1/2} + c \quad (16)$$

where a , b , and c are fitting coefficients and the corresponding values are 6.8, 6.9 and -22.4 respectively.

2.2.4. Denisov Khudenko formula

Denisov and Khudenko [47] constructed the empirical formula for alpha decay half-lives by including terms related to the orbital moment, parity of alpha transition and electron screening effect. The semi empirical formula for the evaluation of alpha decay in even–even, even–odd, odd–even and odd–odd are as follows;

$$\log T_{1/2}^{e-e} = -26.1779 - 1.1521 \frac{A^{1/6} Z^{1/2}}{\mu} + \frac{1.6068 Z}{\sqrt{Q}} \quad (17)$$

$$\log T_{1/2}^{e-o} = -30.3391 - 1.0785 \frac{A^{1/6} Z^{1/2}}{\mu} + \frac{1.6068 Z}{\sqrt{Q}} + \frac{0.2688 \sqrt{\ell(\ell+1)}}{Q A^{-1/6}} - 0.6784((-1)^\ell - 1) \quad (18)$$

$$\log T_{1/2}^{o-e} = -30.2138 - 1.0841 \frac{A^{1/6} Z^{1/2}}{\mu} + \frac{1.6949 Z}{\sqrt{Q}} + \frac{0.1302 \sqrt{\ell(\ell+1)}}{Q A^{-1/6}} - 0.5972((-1)^\ell - 1) \quad (19)$$

$$\log T_{1/2}^{o-o} = -30.3526 - 1.0149 \frac{A^{1/6} Z^{1/2}}{\mu} + \frac{1.6609 Z}{\sqrt{Q}} + \frac{0.2762 \sqrt{\ell(\ell+1)}}{Q A^{-1/6}} - 0.2209((-1)^\ell - 1) \quad (20)$$

where A and Z are the mass number and atomic number of parent nuclei. Q and ℓ are the amount of energy released during the reaction and orbital angular momentum of the emitted alpha particle respectively. The reduced mass is given by $\mu = (A/A - 4)^{1/6}$.

2.3. Spontaneous fission

In the present work, we have evaluated spontaneous fission half-lives using the semi empirical models such as Bao et al. [49] and Ren et al. [51].

2.3.1. Bao et al. [49] formula

In the year 1955 Swiatecki [48] proposed the significance of shell structure in the spontaneous fission process. He explained the irregularities observed if the fission half-lives with respect to Z^2/A . Bao et al. [49] used the modified Swiatecki's formula by considering the shell effects and observed systematic variation for spontaneous fission half-lives. The modified formula for spontaneous fission half-lives by considering shell correction and isospin effect is as follows,

$$\log_{10}[T_{1/2}(\text{yr})] = c_1 + c_2 \left(\frac{Z^2}{(1 - kI^2)A} \right) + c_3 \left(\frac{Z^2}{(1 - kI^2)A} \right)^2 + c_4 E_{sh} + h_i. \quad (21)$$

Where $Z^2/(1 - kI^2)A$ is the fissionability parameter which includes isospin effect and the constants are $c_1 = 1174.353441$, $c_2 = -47.666855$, $c_3 = 0.471307$, $c_4 = 3.378848$. The constant $k = 2.6$ [50]. The “ h_i ” is the blocking factor of unpaired nucleon. The values of $h_{e-o} = 2.609374$ for the odd- N nuclei and $h_{o-e} = 2.619768$ for the odd- Z nuclei.

2.3.2. Ren et al. [51] formula

The generalised spontaneous fission including pairing, shell model calculations and valence nucleons, Ren et al. [51] constructed a semi-empirical formula for spontaneous fission half-lives is as follows;

$$\log_{10}[T_{1/2}(\text{yr})] = 21.08 + c_1 \frac{Z - 90 - v}{A} + c_2 \frac{(Z - 90 - v)^2}{A} + c_3 \frac{(Z - 90 - v)^3}{A} + c_4 \frac{(Z - 90 - v)}{A} (N - Z - 52)^2 \quad (22)$$

where $c_1 = -548.825021$, $c_2 = -5.359139$, $c_3 = 0.767379$, $c_4 = -4.28222$ and $v = 0$ for even-even nuclei and $v = 2$ for odd-A nuclei.

2.4. β^- decay formula

β^- decay process occurs in proton rich nuclei. Zhang et al. [52] constructed a semi-empirical formula for β^- decay half-lives and it is expressed as,

$$\log_{10} T_{1/2} = (c_1 Z + c_2)N + c_3 Z + c_4 + \text{shell}(Z, N) \quad (23)$$

where shell correction term is expressed as;

$$\text{shell}(Z, N) = c_5 \left(e^{-(N-29)^2/15} + e^{-(N-50)^2/37} + e^{-(N-85)^2/9} + e^{-(N-131)^2/3} \right) + c_6 e^{-[(Z-51.5)^2 + (N-80.5)^2]/1.9} \quad (24)$$

Z and N are the proton and neutron number of the parent nuclei respectively. $T_{1/2}$ is the half-life of β^- decay. The parameters are $c_1 = 3.37 \times 10^{-4}$, $c_2 = -0.2558$, $c_3 = 0.4028$, $c_4 = -1.01$, $c_5 = 0.9039$, and $c_6 = 7.7139$.

2.5. β^+ decay formula

Zhang et al. [53] proposed semi empirical formula for β^+ decay and it is expressed as;

$$\log_{10} T_{1/2} = (c_1 Z + c_2)N + c_3 Z + c_4 \quad (25)$$

Z and N are the proton and neutron number respectively. The parameters c_1 , c_2 , c_3 and c_4 are different for different order. The first and second forbidden transition for β^+ decay and the different parameters are in detail explained in [53]. The effect of even-odd effects are also considered in the above equation.

3. Results and discussions

Using coulomb and proximity potential model, we have studied proton decay from the proton rich emitters in the actinide region. The proton emission is energetically possible only when the amount energy released is positive and it is given by;

$$Q = \delta M_p - (\delta M_d + \delta M_z) + k(Z_p^\varepsilon - Z_d^\varepsilon) \quad (26)$$

where δM_p , δM_d , and δM_z are the mass excess values of parent, daughter and emitted proton respectively. The term $kZ_{p(d)}^\varepsilon$ is the total binding energy of electrons in the parent or daughter nuclei, where $k = 8.7$ eV and $\varepsilon = 2.517$ for the nuclei $Z > 60$. The value of $k = 13.6$ eV and $\varepsilon = 2.408$ for the nuclei $Z \leq 60$ [54]. The experimental mass excess values are extracted from [55]. Wherever experimental mass excess values are not available, we have used the theoretical mass excess values available in the literature [56–59]. We have calculated penetration probability and proton decay half-lives in the actinide region. We have also studied logarithmic half-lives of

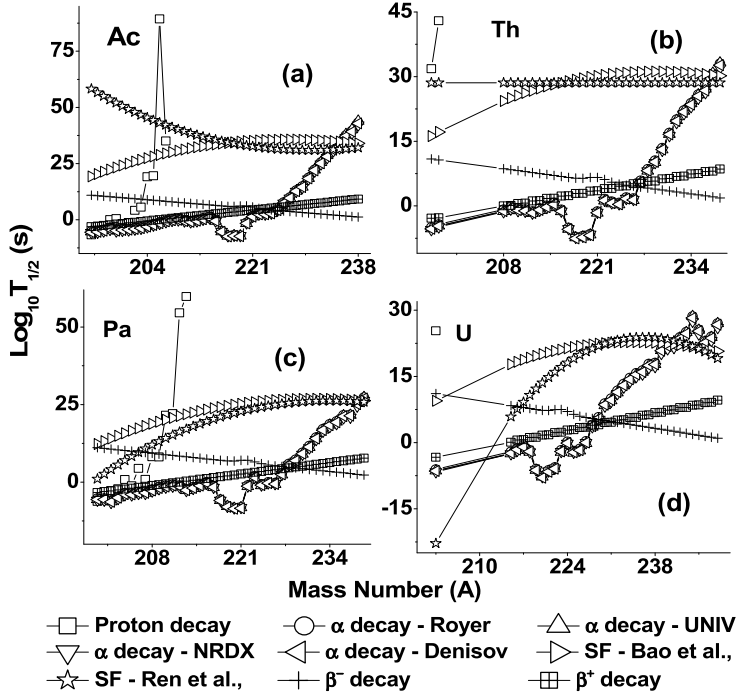


Fig. 1. Variation of $\log(T_{1/2})$ of proton activity, alpha decay, spontaneous fission and beta decay as a function of mass number of the parent nuclei (A).

alpha decay, spontaneous fission half-lives, β^- decay and β^+ decay half-lives as explained in section 2.

The comparison of $\log(T_{1/2})$ of proton emission with that of alpha decay, spontaneous fission and beta decay as a function of mass number of parent nuclei is as presented in figures. From the Fig. 1(a) it is observed that proton decay for Actinium (Ac) is energetically possible for the mass number of $195 < A < 207$. The Fig. 1(a) gives the comparison of proton decay with that of alpha decay, spontaneous fission and beta decay. Similarly from the Fig. 1(b) for Th, proton decay is energetically possible in the mass number region $195 < A < 207$. From the Fig. 1(c) for Pa, it is observed that the proton decay is energetically possible in the mass number region $200 < A < 209$ and $212 < A < 213$. From the Fig. 1(d) for U, proton decay is energetically possible for ^{203}U . Similarly the Figure 2–4 gives the comparison of proton decay with that of alpha decay, spontaneous fission and beta decay in the actinide region $Z = 93 - 103$. The energetically favour proton emission is tabulated in Table 1.

For better understanding of predictable decay modes, a graph is plotted with the logarithmic half-lives of different decay modes such as proton decay, spontaneous fission, alpha decay and beta decay half-lives and it is presented in Fig. 5. From the Fig. 5(a) it is observed that the ^{194}Ac is a proton emitter, alpha decay mode is observed in the mass number of range $^{195-209}\text{Ac}$ and $^{211-224}\text{Ac}$, β^+ decay and β^- decay is energetically possible in the nuclei ^{210}Ac and $^{225-239}\text{Ac}$ respectively. Similarly the decay modes for actinide nuclei with $Z = 90 - 103$ (Th-Lr) are shown in the Fig. 5. We have predicted the energy released during proton decay, penetration probability and half-lives for $Z = 89 - 103$ and the results are tabulated in Table 2. In order to predict the

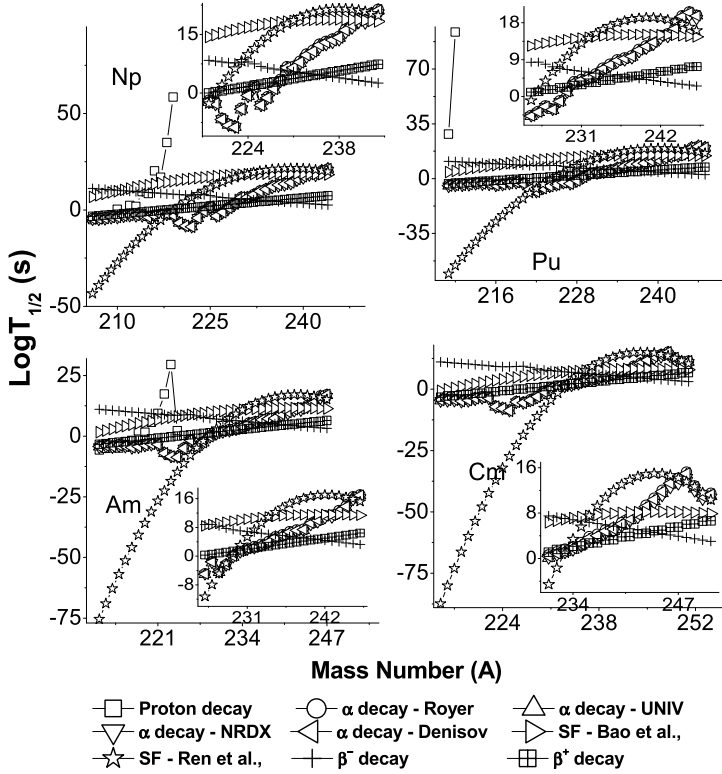


Fig. 2. Variation of $\log(T_{1/2})$ of proton activity, alpha decay, spontaneous fission and beta decay as a function of mass number of the parent nuclei (A).

dominant decay mode in the atomic number range $Z = 89 - 103$, we have calculated branching ratios. The branching ratio of proton decay to alpha decay is defined as,

$$BR = \frac{\lambda_P}{\lambda_{\alpha/SF/\beta^+/\beta^-}}. \quad (27)$$

Where λ_P is the decay constant corresponding to proton emission and $\lambda_{\alpha/SF/\beta^+/\beta^-}$ is the decay constant corresponds to alpha decay, spontaneous fission, β^+ decay and β^- decay respectively. The variation of branching ratios of proton decay with respect to the alpha decay, SF, β^- and β^+ decay as a function mass number of the nuclei is as shown in Fig. 6. From the Fig. 6(a) it observed that the branching ratio of $\lambda_P/\lambda_{\alpha/\beta^+}$ values are higher in the mass number range $^{194-197}\text{Ac}$ and gradually decreases with increase in mass number above $^{197-207}\text{Ac}$. The branching ratio of $\lambda_P/\lambda_{SF/\beta^-}$ values are higher in the mass number range $^{194-203}\text{Ac}$ and decreases with increase in mass number range $^{204-207}\text{Ac}$. Similarly from the Fig. 6(b) to 6(h) it is observed that the values of branching ratios gradually decreases with increase in mass number. The Fig. 7 denotes the variation of calculated logarithmic half-lives with the available experimental values [60–78]. The continuous line represents the calculated half-lives of proton emitters and dotted line represents the experimental logarithmic half-lives values. Table 3 also lists the experimental half-life values, proton energy released during the proton decay and calculated half-lives. From the Fig. 7 and Table 3 it is observed that our results are good agreement with the experimental

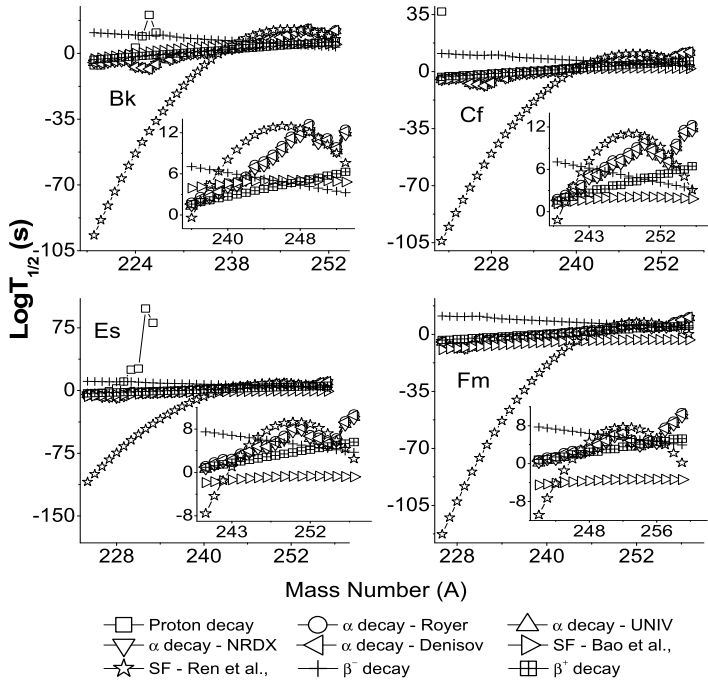


Fig. 3. Variation of $\log(T_{1/2})$ of proton activity, alpha decay, spontaneous fission and beta decay as a function of mass number of the parent nuclei (A).

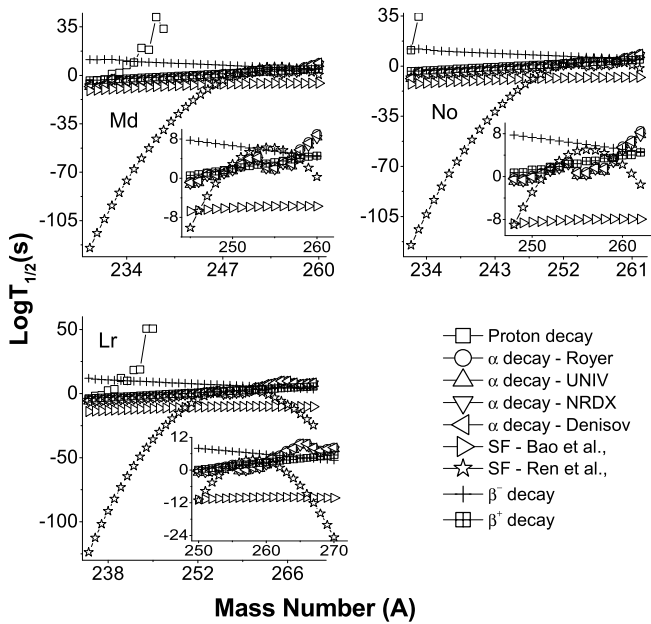


Fig. 4. Variation of $\log(T_{1/2})$ of proton activity, alpha decay, spontaneous fission and beta decay as a function of mass number of the parent nuclei (A).

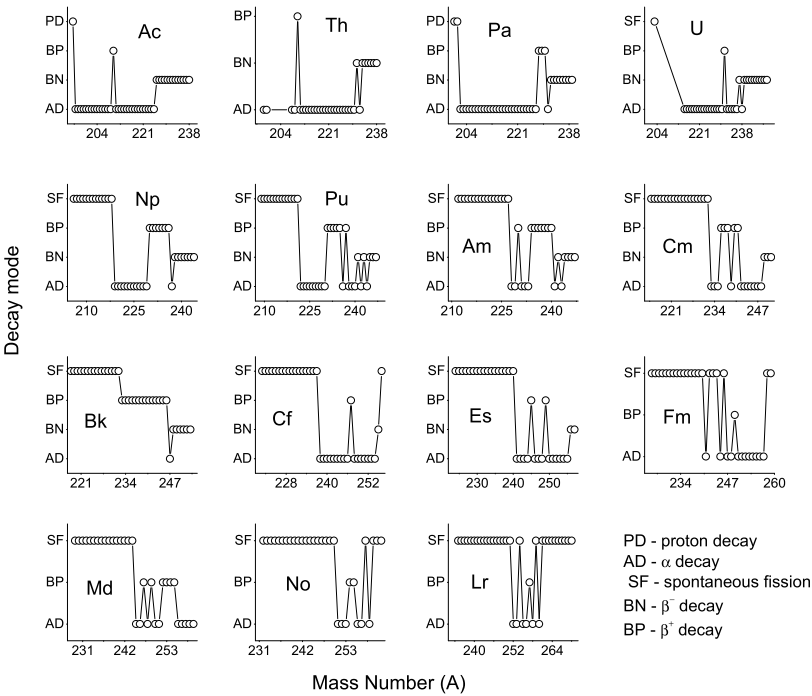


Fig. 5. Variation of decay modes such as proton activity, alpha decay, spontaneous fission, beta plus decay and beta minus decay as a function of mass number of the parent nuclei (A).

Table 1
List of studied actinide nuclei for proton decay.

Z	Mass number range studied	Mass number range where $Q + V_e$	Mass number range where $Q - V_e$
89	195–293	195–207	208–293
90	198–296	198–199	200–296
91	200–300	200–209, 212–213	210–211, 214–300
92	203–303	203	204–303
93	206–306	206–217	218–306
94	209–309	209	210–309
95	212–313	212–224	225–313
96	215–316	215	216–316
97	218–319	218–227	228–319
98	221–322	221	222–322
99	224–326	224–231	232–326
100	226–329	–	226–329
101	229–332	229–239	240–332
102	232–335	232–233	234–335
103	235–339	235–243	244–339

values [60–78]. We also studied standard deviation of calculated proton decay with the experimental values. The standard deviation of present work with the experimental is presented in

Table 2

Energy released, penetration factor and logarithmic half-lives for proton decay in actinide nuclei.

Nuclei	Q	P	T	Nuclei	Q	P	T
¹⁹⁵ Ac	2.161	2.142×10^{-15}	1.892×10^{-7}	²²⁴ Am	1.181	2.105×10^{-26}	2.016×10^4
¹⁹⁶ Ac	1.591	1.396×10^{-19}	2.907×10^{-3}	²¹⁵ Cm	0.221	8.665×10^{-78}	4.832×10^{55}
¹⁹⁷ Ac	1.591	1.391×10^{-19}	2.923×10^{-3}	²¹⁸ Bk	2.241	1.015×10^{-16}	4.142×10^{-6}
¹⁹⁸ Ac	1.321	1.775×10^{-22}	2.294	²¹⁹ Bk	2.231	8.983×10^{-17}	4.690×10^6
¹⁹⁹ Ac	1.331	2.277×10^{-22}	1.791	²²⁰ Bk	1.841	1.507×10^{19}	2.799×10^{-3}
²⁰⁰ Ac	1.441	4.324×10^{-21}	9.452×10^{-2}	²²¹ Bk	1.871	2.730×10^{-19}	1.547×10^{-3}
²⁰¹ Ac	1.371	6.726×10^{-22}	6.087×10^{-1}	²²² Bk	1.611	1.288×10^{-21}	3.283×10^{-1}
²⁰² Ac	0.971	6.033×10^{-28}	6.797×10^5	²²³ Bk	1.461	2.993×10^{-23}	1.416×10^1
²⁰³ Ac	1.018	4.688×10^{-27}	8.762×10^4	²²⁴ Bk	1.171	3.263×10^{-27}	1.300×10^5
²⁰⁴ Ac	0.595	1.424×10^{-38}	2.889×10^{16}	²²⁵ Bk	0.841	4.615×10^{-34}	9.210×10^{11}
²⁰⁵ Ac	0.707	1.575×10^{-34}	2.616×10^{12}	²²⁶ Bk	0.501	1.852×10^{-47}	2.298×10^{25}
²⁰⁶ Ac	0.383	5.019×10^{-51}	8.225×10^{28}	²²⁷ Bk	0.771	4.652×10^{-36}	9.165×10^{13}
²⁰⁷ Ac	0.277	4.149×10^{-62}	9.965×10^{39}	²²¹ Cf	0.281	6.088×10^{-69}	6.941×10^{46}
¹⁹⁸ Th	0.291	2.163×10^{-56}	1.883×10^{34}	²²⁴ Es	2.181	1.426×10^{-17}	2.976×10^{-5}
¹⁹⁹ Th	0.201	1.218×10^{-76}	3.350×10^{54}	²²⁵ Es	2.181	1.419×10^{-17}	2.995×10^{-5}
²⁰⁰ Pa	2.111	3.807×10^{-16}	1.073×10^{-6}	²²⁶ Es	1.771	1.186×10^{-20}	3.589×10^{-2}
²⁰¹ Pa	2.091	2.964×10^{-16}	1.381×10^{-6}	²²⁷ Es	1.541	6.434×10^{-23}	6.626
²⁰² Pa	1.751	1.028×10^{-18}	3.988×10^{-4}	²²⁸ Es	1.251	1.236×10^{-26}	3.454×10^4
²⁰³ Pa	1.491	3.735×10^{-21}	1.099×10^{-1}	²²⁹ Es	0.821	2.005×10^{-35}	2.132×10^{13}
²⁰⁴ Pa	1.221	1.868×10^{-24}	2.202×10^2	²³⁰ Es	0.441	1.030×10^{-52}	4.157×10^{30}
²⁰⁵ Pa	1.391	2.895×10^{-22}	1.423	²³¹ Es	0.421	2.493×10^{-54}	1.719×10^{32}
²⁰⁶ Pa	0.981	1.738×10^{-28}	2.375×10^6	²²⁹ Md	2.251	1.482×10^{-17}	2.883×10^{-5}
²⁰⁷ Pa	1.221	1.842×10^{-24}	2.244×10^2	²³⁰ Md	1.921	6.453×10^{-20}	6.636×10^{-3}
²⁰⁸ Pa	0.801	1.222×10^{-32}	3.388×10^{10}	²³¹ Md	1.871	2.514×10^{-20}	1.705×10^{-2}
²⁰⁹ Pa	0.801	1.215×10^{-32}	3.412×10^{10}	²³² Md	1.441	1.172×10^{-24}	3.662×10^2
²¹² Pa	0.42	3.479×10^{-49}	1.197×10^{27}	²³³ Md	1.381	1.935×10^{-25}	2.222×10^3
²¹³ Pa	0.283	4.874×10^{-14}	8.563×10^{-9}	²³⁴ Md	1.001	8.656×10^{-32}	4.975×10^9
²⁰³ U	0.381	3.469×10^{-53}	1.184×10^{31}	²³⁶ Md	0.911	7.300×10^{-34}	5.909×10^{11}
²⁰⁶ Np	1.911	5.525×10^{-18}	7.471×10^{-5}	²³⁷ Md	0.561	2.862×10^{-46}	1.509×10^{24}
²⁰⁷ Np	1.881	3.226×10^{-18}	1.281×10^{-4}	²³⁸ Md	0.591	9.869×10^{-45}	4.383×10^{22}
²⁰⁸ Np	1.751	3.040×10^{-19}	1.362×10^{-3}	²³⁹ Md	0.251	2.607×10^{-76}	1.661×10^{54}
²⁰⁹ Np	1.691	8.868×10^{-20}	4.677×10^{-3}	²³² No	0.831	2.217×10^{-36}	1.936×10^{14}
²¹⁰ Np	1.301	5.444×10^{-24}	7.631×10^1	²³³ No	0.331	6.422×10^{-65}	6.697×10^{42}
²¹¹ Np	1.561	5.185×10^{-21}	8.025×10^{-2}	²³⁵ Lr	2.161	1.317×10^{-18}	3.273×10^{-4}
²¹² Np	1.151	3.491×10^{-26}	1.193×10^4	²³⁶ Lr	1.781	1.250×10^{-21}	3.455×10^{-1}
²¹³ Np	1.181	1.025×10^{-25}	4.072×10^3	²³⁷ Lr	1.731	4.252×10^{-22}	1.017
²¹⁴ Np	0.771	2.498×10^{-34}	1.673×10^{12}	²³⁸ Lr	1.361	2.602×10^{-26}	1.66×10^4
²¹⁵ Np	0.811	3.254×10^{-33}	1.286×10^{11}	²³⁹ Lr	1.281	1.766×10^{-27}	2.455×10^5
²¹⁶ Np	0.471	4.977×10^{-47}	8.426×10^{24}	²⁴⁰ Lr	0.811	2.142×10^{-37}	2.027×10^{15}
²¹⁷ Np	0.541	4.641×10^{-43}	9.050×10^{20}	²⁴¹ Lr	0.901	6.580×10^{-35}	6.610×10^{12}
²⁰⁹ Pu	0.351	2.896×10^{-57}	1.432×10^{35}	²⁴² Lr	0.611	8.460×10^{-45}	5.149×10^{22}
²¹² Am	2.051	1.838×10^{-17}	2.267×10^{-5}	²⁴³ Lr	0.601	2.884×10^{-45}	1.51235×10^{23}
²¹³ Am	1.951	3.465×10^{-18}	1.204×10^{-4}	²³² No	0.831	2.217×10^{-36}	1.936×10^{14}
²¹⁴ Am	1.911	1.705×10^{-18}	2.451×10^{-4}	²³³ No	0.331	6.422×10^{-65}	6.697×10^{42}
²¹⁵ Am	1.931	2.449×10^{-18}	1.709×10^{-4}	²³⁵ Lr	2.161	1.317×10^{-18}	3.273×10^{-4}
²¹⁶ Am	1.601	3.564×10^{-21}	1.176×10^{-1}	²³⁶ Lr	1.781	1.250×10^{-21}	3.455×10^{-1}
²¹⁷ Am	1.531	6.843×10^{-22}	6.138×10^{-1}	²³⁷ Lr	1.731	4.252×10^{-22}	1.017
²¹⁸ Am	1.191	3.191×10^{-26}	1.318×10^4	²³⁸ Lr	1.361	2.602×10^{-26}	1.66×10^4

(continued on next page)

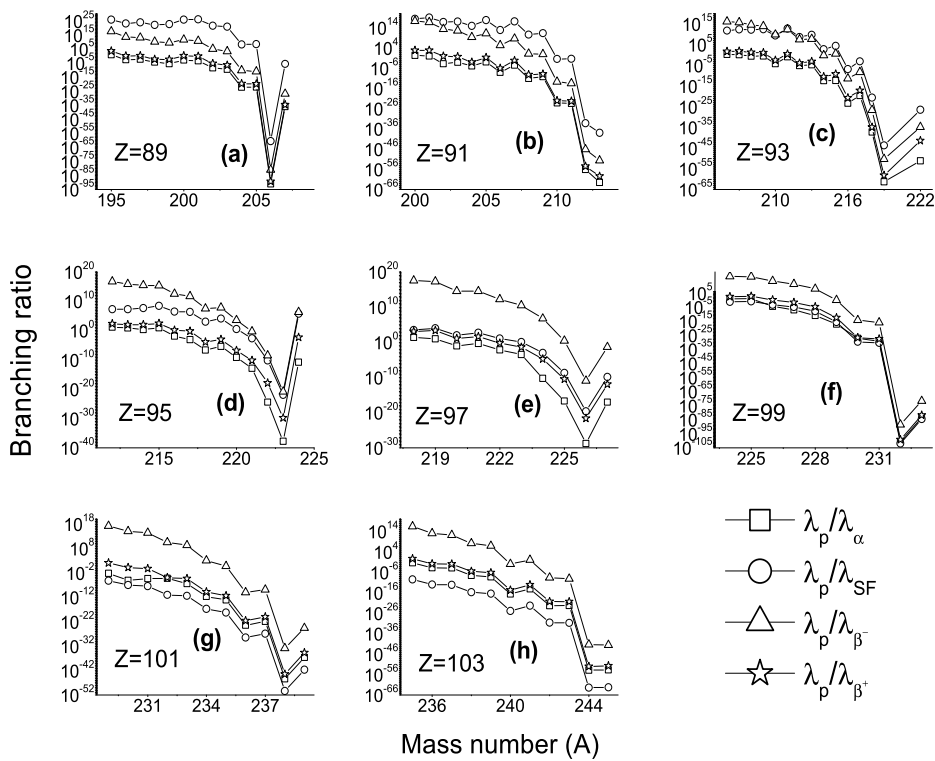


Fig. 6. A variation of branching ratios of proton decay to the alpha decay (NRDX), SF (Bao et al. [49]), beta (minus) decay and beta (plus) decay as a function mass number of the nuclei.

Table 2 (continued)

Nuclei	Q	P	T	Nuclei	Q	P	T
^{219}Am	1.231	1.290×10^{-25}	3.264×10^3	^{239}Lr	1.281	1.766×10^{-27}	2.455×10^5
^{220}Am	0.971	3.403×10^{-30}	1.240×10^8	^{240}Lr	0.811	2.142×10^{-37}	2.027×10^{15}
^{221}Am	0.801	2.486×10^{-34}	$1.700 \times 10^{+12}$	^{241}Lr	0.901	6.580×10^{-35}	6.610×10^{12}
^{222}Am	0.551	1.326×10^{-43}	3.189×10^{21}	^{242}Lr	0.611	8.460×10^{-45}	5.149×10^{22}
^{223}Am	0.341	5.349×10^{-59}	7.923×10^{36}	^{243}Lr	0.601	2.884×10^{-45}	1.51235×10^{23}

Fig. 8. The standard root mean square deviation of calculated logarithmic half-lives are calculated using following equation;

$$\sigma = \left\{ \sum_{i=1}^n [\log_{10}(T_{cal}/T_{exp})]^2 / (n-1) \right\}^{1/2}. \quad (28)$$

The overall standard deviation of proton decay half-lives in the odd–odd nuclei are observed to be 1.09065 and for odd–even were observed to be 0.879551 and are tabulated in Table 4. From the predictions of proton decay in the atomic nuclei range $Z = 51 - 83$, it is observed that our calculations are good agreement with the experimental values. Hence forth, we have predicted half-lives of proton decay in the actinide region.

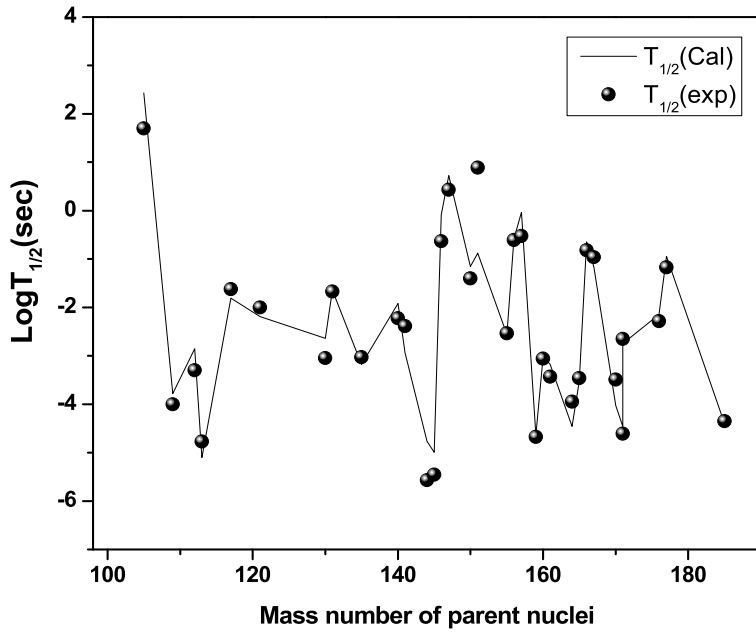


Fig. 7. The variation of calculated logarithmic half-lives of proton decay of different mass number of proton emitters with the available experimental values.

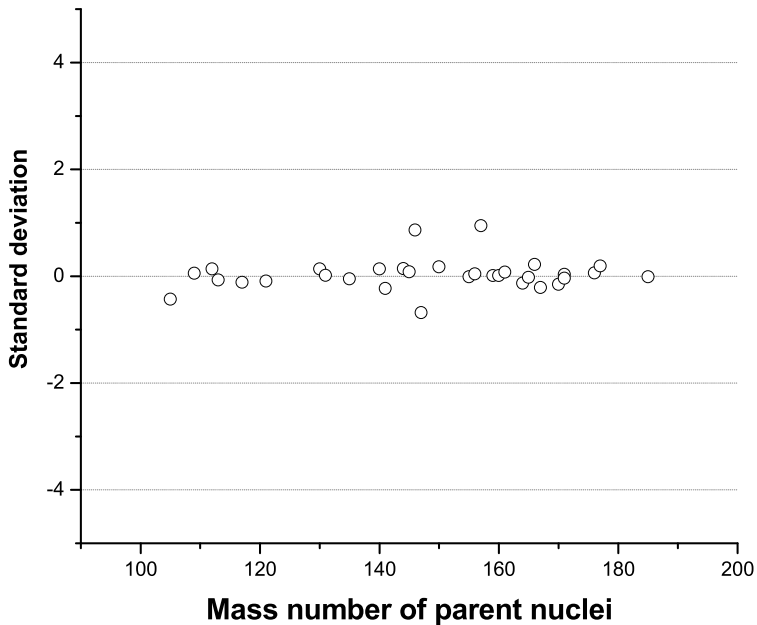


Fig. 8. A standard deviation of proton decay from the experimental values with that of mass number of parent nuclei.

Table 3
The comparison of calculated half-lives are compared with the experimental values [60,62,62–78].

Isotopes	Q_P	β_2	β_4	l	$\log T$	$\log T_{1/2} \text{ exp}$	Ref.
^{105}Sb	0.48	0.081	0.051	2	2.430	1.7	[60]
^{109}I	0.82	0.16	0.06	2	-3.779	-4	[61–63]
^{112}Cs	0.81	0.208	0.067	2	-2.856	-3.3	[64]
^{113}Cs	0.98	0.207	0.052	2	-5.100	-4.77	[61,64]
^{117}La	0.814	0.29	0.1	2	-1.812	-1.623	[74–78]
^{121}Pr	0.9	0.318	0.075	2	-2.188	-2	[74–78]
^{130}Eu	1.039	0.331	0	2	-2.637	-3.046	[74–78]
^{131}Eu	0.959	0.331	0	2	-1.639	-1.67	[74–78]
^{135}Tb	1.2	0.325	-0.046	3	-3.178	-3.027	[74–78]
^{140}Ho	1.106	0.297	-0.07	3	-1.920	-2.222	[74–78]
^{141}Ho	1.19	0.286	-0.063	3	-2.941	-2.387	[74–78]
^{144}Tm	1.725	0.258	-0.077	5	-4.767	-5.569	[74–78]
^{145}Tm	1.753	0.249	-0.078	5	-4.996	-5.456	[74–78]
^{146}Tm	1.13	-0.199	-0.038	5	-0.086	-0.63	[65]
^{147}Tm	1.06	-0.19	-0.04	5	0.723	0.43	[63,66,67]
^{150}Lu	1.27	-0.164	-0.05	5	-1.153	-1.4	[63,68]
^{151}Lu	1.24	-0.156	-0.045	5	-0.876	0.89	[63,69]
^{155}Ta	1.468	0.008	0	5	-2.563	-2.538	[74–78]
^{156}Ta	1.03	-0.053	0.001	2	-0.584	-0.609	[74–78]
^{157}Ta	0.947	0.045	0.001	0	-0.030	-0.523	[74–78]
^{159}Re	1.816	0.053	-0.007	5	-4.636	-4.678	[74–78]
^{160}Re	1.28	0.08	0.002	2	-3.026	-3.06	[70]
^{161}Re	1.36	0.08	-0.006	3	-3.171	-3.43	[71]
^{164}Ir	1.844	0.089	-0.006	5	-4.459	-3.947	[74–78]
^{165}Ir	1.7	0.099	-0.012	5	-3.529	-3.46	[72]
^{166}Ir	1.12	0.107	-0.004	2	-0.642	-0.82	[72]
^{167}Ir	1.11	0.116	-0.011	0	-1.163	-0.96	[72]
^{170}Au	1.488	-0.096	-0.012	2	-4.027	-3.493	[74–78]
^{171}Au	1.464	-0.105	-0.011	0	-4.458	-4.611	[74–78]
^{171}Au	1.51	-0.105	-0.011	4	-2.745	-2.65	[72]
^{176}Tl	1.282	-0.053	-0.007	0	-2.148	-2.284	[74–78]
^{177}Tl	1.18	-0.053	-0.007	0	-0.947	-1.174	[74–78]
^{185}Bi	1.56	-0.052	0.016	0	-4.402	-4.35	[73]

Table 4
Standard deviation of proton decay half-lives in the nuclei region $Z = 51 - 83$.

n	Parent	σ
21	$o-o$	1.09065
12	$o-e$	0.879551

4. Conclusions

In summary, we have theoretically studied the one-proton emission in the actinide region from $Z = 89 - 103$. We have also compared one-proton decay half-lives with alpha decay, spontaneous fission and beta decay. In the present work the calculated one-proton decay half-live values compare fairly well with the available experimental values. The standard deviation for

odd–odd nuclei is found to 1.09065 and for odd–even nuclei the deviation is 0.879551. We have also identified proton emitters in the actinide region and also it is the competing decay mode for all observed proton emitters. The prediction of the new proton emitters in the actinide region are having measurable half-lives can be retrieved in future with developing experimental techniques.

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Systematics of proton decay of actinides

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The phenomenon of proton emission from nuclear ground states limits the possibilities of the creation of more exotic proton rich nuclei that are usually produced by fusion-evaporation nuclear reactions. In the energy domain of radioactivity, proton can be considered as a point charge having highest probability of being present in the parent nucleus. Conclaves *et al.*¹ studied the two-proton radioactivity of nuclei of mass number $A < 70$ using the effective liquid drop model. Delion *et al.*² reviewed the theories of proton emission to analyse the properties of nuclear matter. Maglione *et al.*³ analysed the proton emission from the some deformed nuclei. We have studied proton decay in almost all actinide nuclei. We have calculated the energy released during the proton decay (Q_p), penetration factor (P), and half-lives of proton decay. Proton decay half-lives are also longer than that of other decay modes such as alpha decay and spontaneous fission. To check the Geiger-Nuttall law for proton decay in actinide nuclei, we have plotted the logarithmic proton decay half-lives versus $1/\sqrt{Q}$. The competition of proton decay with different decay modes such as alpha decay and spontaneous fission are also studied. We have also highlighted possible proton emitters with the corresponding energies and half-lives in the actinide region.

Keywords: Proton decay, Half-lives, Probability, Geiger-Nuttall law

1 Introduction

The nuclei beyond the proton drip line with the $Q_p > 0$ are the one with proton unstable and also exhibit exotic decay modes. The understanding of the proton decay is important to study the nuclear structure. The exotic nuclei exists away from the stability. The binding energy of protons above the drip line gradually decreases and hence one-proton and two proton decay is predicted. Brown⁴ studied two proton decay in $Z=22-28$ in the ground state. Goldanskii⁵ for the first time studied the one proton and two proton decay for odd and even atomic number. Janecke⁶ studied the emission of protons from the light nuclei $^{12,13}\text{O}$, ^{21}Mg and $^{24, 32}\text{Si}$. The spherical proton and deformed proton emitters were investigated in lanthanides and transaction metals. Previous workers⁷⁻¹⁸ experimentally observed one and two proton decay in proton rich nuclei. They are several theoretical models¹⁹⁻²¹, studied one proton and two proton activity in light nuclei. Using different proximity potentials previous workers²²⁻²⁴ studied proton activity in the light nuclei. The emission of heavy particles such as one proton, one neutron, two

protons, 2 neutrons and alpha particle emission takes place when the nuclei are proton rich, neutron rich and very heavy nuclei. Successively many theoretical models²⁵⁻³² were presented to study the half-lives of spherical and deformed nuclei. Dobaczewski and Nazarewicz³³ studied two-proton stability in doubly magic nuclei ^{100}Sn using self-consistent Skyrme-Hartree-Fock-Bogoliubov theory. Olsen *et al.*³⁴⁻³⁵ investigated two-proton decay in even-even nuclei and also studied competition between proton decay and alpha decay. Poenaru *et al.*³⁶ measured half-lives and branching ratios for ^{12}C , ^{16}O and ^{28}Si and proton and neutron rich nuclei with $Z=56-64$.

The observations of the proton decay is quite recent, they are several approaches to study this proton decay process, such as distorted-wave Born approximation³⁷, the study of effective interaction by the density dependent M3Y (DDM3Y)^{38,39}. The construction of proton nucleus potential by Jeukenne, Lejeune and Mahaux (JLM) applied to finite nuclei in the Local Density Approximation⁴⁰, the unified fission model², the coupled-channels approach⁴¹ and also generalized liquid drop models⁴²⁻⁴⁴. Earlier workers⁴⁵⁻⁵³ studied half-lives of spontaneous fission, ternary fission, cluster decay and alpha decay in the

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superheavy region using different proximity potentials. Faestermann⁵⁴ experimentally observed proton decay half-lives and proton energies in ¹¹³Cs and ¹⁰⁹I. Sellin⁵⁵ experimentally measured proton decay half-lives in ¹⁵⁰Lu, ¹⁵¹Lu and ¹⁴⁷Tm. Page *et al.*⁵⁶ reported proton emitter ¹¹²Cs with the half-life of $500 \pm 100 \mu\text{s}$. Livingston⁵⁷ experimentally observed proton emission from the ¹⁴⁶Tm. The two proton radioactivity⁵⁸⁻⁶³ was experimentally observed ⁴⁵Fe, ¹⁹Mg, ⁴⁸Ni and ⁵⁴Zn. In the year 1970, Jackson⁶⁴ confirmed the proton radioactivity from the proton emitter ⁵³Co.

The proton radioactivity is applied for nuclear astrophysics. In the nuclear astrophysics, the process of two-proton radiation capture process is considered, which is important for extremely high densities and temperatures. The example of such an astrophysical environment is the sources of gamma bursts related with the explosive burning of deposited hydrogen on the surface of neutron stars. Previous workers⁶⁵⁻⁶⁷ explained the astrophysical applications of the two-proton radioactivity.

From the available literature, the study on one proton emission in the actinide region is required. The study on the proton decay not only provides information about the drip line, but also provides spectroscopic information on the unpaired proton not substantial in its orbit. Hence, in the present work we want to emphasize on the possible proton emitters in the actinide region and also prediction of half-lives in the same region. The main objective is to systematically study the one proton decay half-lives of spherical and deformed nuclei in the actinide region.

2 Theoretical Framework

2.1 Proton emission half-lives

The reaction of nuclear one proton decay can be written as:

$${}^A_Z(X)_N \rightarrow {}^{A-1}_{Z-1}(Y)_N + {}^1_1(H) + Q_p \quad \dots (1)$$

where Q_p is the amount of energy released during proton decay. To study the proton decay, we have used preformed cluster model^{68,69}. The decay constant and half-lives is defined as

$$\lambda = \nu P P_0 \quad \dots (2)$$

$$T_{1/2} = \frac{\ln 2}{\lambda} \quad \dots (3)$$

where ν , P and P_0 are the assault frequency⁷⁹ with which proton hits the barrier, probability of

penetration barrier and preformation probability respectively. In the present work we have selected $P_0=1$ for the emitted proton. The penetration probability is solved numerically using WKB approximation⁷¹.

$$P = \exp \left[-\frac{2}{\hbar} \int_{R_{in}}^{R_{out}} \sqrt{2\mu(V - Q_p)} dr \right] \quad \dots (4)$$

where μ is reduced mass of proton decay, Q_p is the energy released during proton decay. R_{in} and R_{out} are the inner and outer turning points. The inner turning point is given by:

$$R_{in} = r_0 (A_1^{1/3} + A_2^{1/3}) \quad \dots (5)$$

where $A_1=1$ and $A_2=A-1$ for proton emission. R_{out} is determined by the condition $V=Q$. The r_0 is the effective nuclear constant. The total interacting potential is defined by:

$$V = V_c + V_p + V_l \quad \dots (6)$$

where V_c Coulomb interaction potential⁷², V_p is the proximity potential and V_l is the angular potential. Proximity potential⁷³⁻⁷⁴ given by:

$$V_p = 4\pi\gamma b \left[\frac{C_1 C_2}{C_1 + C_2} \right] \phi \quad \dots (7)$$

where $b \approx 1$ fm is the width of the nuclear surface, ϕ is the universal function³⁴, C_1 and C_2 are the Susmann central radii and γ is the nuclear surface tension coefficient it is given by:

$$\gamma = \gamma_0 \left[1 - K_s \left(\frac{N-Z}{A} \right)^2 \right] \text{MeV/fm}^2 \quad \dots (8)$$

neutron mass (N), atomic mass (A) and proton number (Z) of the parent nuclei. Where $\gamma_0 = 1.460734$ MeV/fm² and $K_s=4.0$ ⁷⁵. C_1 and C_2 are the Susmann central radii, R_i is the sharp radii⁷⁴ of the daughter nuclei.

3 Results and Discussion

The proton decay rates are sensitive to amount of energy released (Q_p) and the orbital angular momentum of the emitted proton. The proton emission is energetically possible when Q_p is positive and it is given by:

$$Q_p = \Delta M(A, Z) - \sum_i^n \Delta M(A_i, Z_i) \quad \dots (9)$$

where $\Delta M(A, Z)$ and $\Delta M(A_i, Z_i)$ are mass excess of the parent and emitted daughter and proton nuclei, respectively. We have selected experimental and theoretical values in case of non-availability of experimental values available in the literature⁷⁶⁻⁸⁰. We have studied total interacting potential which is a sum of coulomb, proximity and angular potential as explained in the theory. During the proton emission, the ground state to ground state transactions has zero angular momentum $\ell = 0$. Thus we neglect the effects of angular potential in case of proton emission and we have also considered deformed nuclei in the present work. We have evaluated penetration probability using WKB approximation and studied logarithmic half-lives of proton decay in the actinide region. The amount of energy released during proton decay as function of mass number of parent nuclei in the actinide region as shown in Fig. 1. From the figure it is observed that the amount of energy released during proton decay gradually decreases with the increase in mass number of parent nuclei.

The studied logarithmic half-lives of proton decay in the actinide region is plotted as function of the mass number of parent nuclei is presented in Fig. 2. The figure indicates that the logarithmic half-lives increases with increase in mass number of parent nuclei. The half-lives values are of the order of 10^{-6} to 10^{-4} s for the actinides ^{195}Ac , ^{200}Pa , ^{206}Np , ^{212}Am , ^{218}Bk , ^{224}Es , ^{229}Md and ^{235}Lr and the corresponding values of $Q(\text{MeV})$, penetration factor and half-lives are tabulated in Table 1. Hence proton decay is

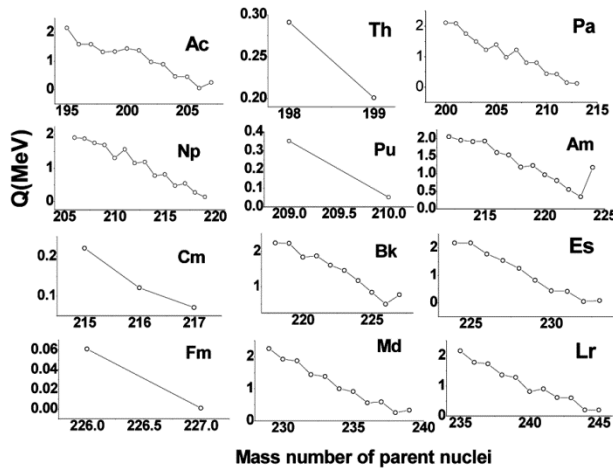


Fig. 1 — The variation of amount of energy released during proton decay with the mass number of parent nuclei in the actinide region.

favourably observed in the actinides such as ^{195}Ac , ^{200}Pa , ^{206}Np , ^{212}Am , ^{218}Bk , ^{224}Es , ^{229}Md and ^{235}Lr . Then we have also plotted logarithmic half-lives of proton decay with the product of $Z_d Q^{-1/2}$ in the actinide region and is as shown in Fig. 3. From the

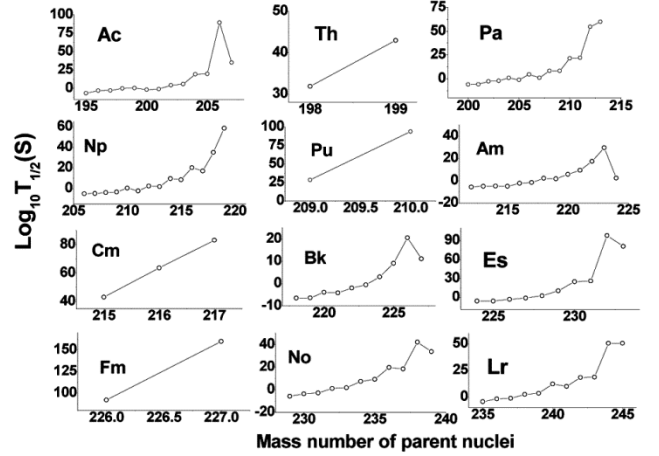


Fig. 2 — The variation of logarithmic half-lives of proton decay with the mass number of parent nuclei in the actinide region.

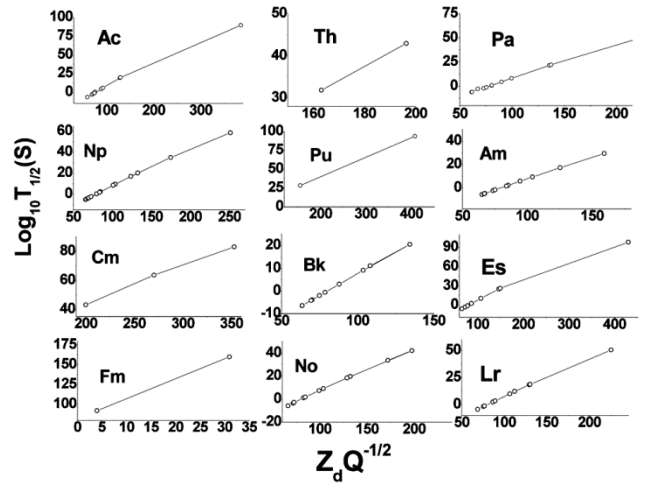


Fig. 3 — Variation of logarithmic half-lives of proton decay with the product of $Z_d Q^{-1/2}$ in the actinide region.

Table 1 — Proton decay half-lives, penetration factor and amount of energy released during proton decay in actinides.

Nuclei	Q(MeV)	Penetration factor	LogT _{1/2}
^{195}Ac	2.161	2.142×10^{-15}	1.892×10^{-7}
^{200}Pa	2.111	3.807×10^{-16}	1.073×10^{-6}
^{206}Np	1.911	5.525×10^{-18}	7.471×10^{-5}
^{212}Am	2.051	1.838×10^{-17}	2.267×10^{-5}
^{218}Bk	2.241	1.015×10^{-16}	4.142×10^{-6}
^{224}Es	2.181	1.426×10^{-17}	2.976×10^{-5}
^{229}Md	2.251	1.482×10^{-17}	2.883×10^{-5}
^{235}Lr	2.161	1.317×10^{-18}	3.273×10^{-4}

Table 2 — A comparison of logarithmic half-lives of proton decay with Royer, Univ, NRDX, Denisov and Bao.

Z_p	A_p	Proton	Log $T_{1/2}(S)$					Decay mode
			Royer(α)	UNIV(α)	NRDX(α)	Denisov(α)	Bao(SF)	
89	195	-6.7230	1.9019	1.3890	1.0527	1.4794	19.2850	Proton decay
89	196	-2.5360	4.1825	3.6576	3.2810	3.7588	20.3250	Proton decay
89	197	-2.5300	4.1610	3.6370	3.2830	3.7390	21.3300	Proton decay
89	198	0.3608	5.3396	4.8169	4.4464	4.9171	22.3023	Proton decay
89	199	0.2533	5.2725	4.7510	4.4043	4.8513	23.2399	Proton decay
89	200	-1.0245	4.7543	4.2340	3.9256	4.3348	24.1421	Proton decay
89	201	-0.2156	5.0479	4.5289	4.2323	4.6293	25.0094	Proton decay
89	202	5.8323	6.9328	6.4209	6.0791	6.5134	25.8415	Proton decay
89	203	4.9426	6.6779	6.1659	5.8550	6.2600	26.6385	Proton decay
89	204	16.461	8.8704	8.3763	7.9999	8.4513	27.4004	α decay
89	205	12.42	8.2391	7.7403	7.4114	7.8219	28.1272	α decay
89	206	28.92	10.038	9.5593	9.1753	9.6197	28.8189	α decay
89	207	39.985	10.65	10.180	9.7900	10.2318	29.4757	α decay
90	198	34.275	11.31	10.813	10.10	10.8917	16.1819	α decay
90	199	54.525	11.845	11.357	10.641	11.4272	17.1443	α decay
91	200	-5.9691	2.9029	2.3410	1.9651	2.5071	12.2615	Proton decay
91	201	-5.8597	2.9600	2.3991	2.0424	2.5654	13.1791	Proton decay
91	202	-3.3992	4.3279	3.7624	3.3775	3.9330	14.0680	Proton decay
91	203	-0.9587	5.4434	4.8786	4.4705	5.0485	14.9279	Proton decay
91	204	2.3428	6.6790	6.1190	5.6790	6.2840	15.7583	Proton decay
91	205	0.1534	5.8572	5.2957	4.9125	5.4643	16.5588	Proton decay
91	206	6.3757	7.8264	7.2758	6.8254	7.4327	17.3293	Proton decay
91	207	2.3511	6.6162	6.0597	5.6857	6.2249	18.0696	Proton decay
91	208	10.530	8.7260	8.1856	7.7340	8.3337	18.7794	α decay
91	209	10.533	8.7054	8.1659	7.7364	8.3143	19.4586	α decay
91	212	27.078	10.794	10.281	9.8076	10.4027	21.3128	α decay
91	213	-8.0673	11.60	11.10	10.61	11.2130	21.8698	α decay
92	203	31.073	11.764	11.23	10.42	11.3731	9.4360	α decay
93	206	-4.1266	4.5136	3.9048	3.4685	4.1449	6.7169	Proton decay
93	207	-3.8922	4.6184	4.0107	3.5904	4.2508	7.4948	Proton decay
93	208	-2.8658	5.1529	4.5460	4.1214	4.7860	8.2485	Proton decay
93	209	-2.3300	5.3939	4.7883	4.3730	5.0279	8.9775	Proton decay
93	210	1.8826	7.1678	6.5688	6.0847	6.8012	9.6813	Proton decay
93	211	-1.0955	5.9324	5.3299	4.9296	5.5683	10.3596	Proton decay
93	212	4.0769	7.8628	7.2705	6.7906	7.4977	11.0122	Proton decay
93	213	3.6098	7.6927	7.1005	6.6504	7.3291	11.6388	Proton decay
93	214	12.224	9.8169	9.2454	8.6965	9.4522	12.2391	α decay
93	215	11.11	9.5771	9.0040	8.4897	9.2138	12.8132	α decay
93	216	24.926	11.5	10.95	10.34	11.1356	13.3607	α decay
93	217	20.957	11.065	10.516	9.9510	10.7020	13.8818	α decay
94	209	35.156	12.929	12.374	11.439	12.5655	4.1089	α decay
95	212	-4.6445	4.7376	4.0882	3.6407	4.3973	1.6163	Proton decay
95	213	-3.9192	5.1358	4.4870	4.0386	4.7962	2.2689	Proton decay
95	214	-3.6105	5.2852	4.6376	4.2014	4.9467	2.9012	Proton decay
95	215	-3.7671	5.1793	4.5329	4.1226	4.8422	3.5128	Proton decay
95	216	-0.9294	6.6160	5.9729	5.5024	6.2785	4.1033	Proton decay
95	217	-0.2120	6.9190	6.2783	5.8103	6.5824	4.6721	Proton decay
95	218	4.1200	8.5489	7.9199	7.3731	8.2118	5.2192	Proton decay
95	219	3.5138	8.3273	7.6976	7.1848	7.9916	5.7441	Proton decay
95	220	8.0934	9.6523	9.0362	8.4594	9.3163	6.2466	Proton decay
95	221	12.231	10.5613	9.9572	9.3406	10.2256	6.7266	SF
95	222	21.504	11.9864	11.4037	10.7102	11.6503	7.1840	SF
95	223	36.899	13.2599	12.6999	11.9365	12.9237	7.6186	SF

(Contd.)

Table 2 — A comparison of logarithmic half-lives of proton decay with Royer, Univ, NRDX, Denisov and Bao. (*Contd.*)

Z_p	A_p	Proton	Log $T_{1/2}(S)$					Decay mode
			Royer(α)	UNIV(α)	NRDX(α)	Denisov(α)	Bao(SF)	
95	224	4.3046	8.4788	7.8558	7.4335	8.1488	8.0304	Proton decay
96	215	55.684	14.7612	14.1951	13.0602	14.4243	-0.7570	SF
97	218	-5.3828	4.7539	4.0643	3.6171	4.4420	-3.0072	Proton decay
97	219	-5.3288	4.7740	4.0855	3.6571	4.4633	-2.4670	Proton decay
97	220	-2.5529	6.4100	5.7228	5.2133	6.0987	-1.9435	Proton decay
97	221	-2.8104	6.2571	5.5707	5.0908	5.9472	-1.4371	Proton decay
97	222	-0.4836	7.4155	6.7345	6.1991	7.1056	-0.9482	SF
97	223	1.1511	8.1082	7.4325	6.8702	7.7987	-0.4774	SF
97	224	5.1142	9.5409	8.8787	8.2362	9.2310	-0.0248	SF
97	225	11.964	11.3070	10.6676	9.9153	10.9964	0.4093	SF
97	226	25.362	13.2970	12.6906	11.8050	12.9856	0.8245	SF
97	227	13.962	11.6660	11.0339	10.2944	11.3574	1.2206	SF
98	221	46.841	15.3640	14.7670	13.5179	15.0549	-5.1307	SF
99	224	-4.5262	5.7952	5.0641	4.5505	5.5104	-7.1242	SF
99	225	-4.5235	5.7749	5.0449	4.5523	5.4914	-6.6849	SF
99	226	-1.4450	7.5555	6.8314	6.2318	7.2712	-6.2589	SF
99	227	0.8213	8.6204	7.9043	7.2448	8.3362	-5.8467	SF
99	228	4.5384	10.0548	9.3534	8.6021	9.7703	-5.4488	SF
99	229	13.329	12.3930	11.7244	10.8019	12.1071	-5.0656	SF
99	230	30.619	14.6914	14.0653	12.9649	14.4042	-4.6974	SF
99	231	32.236	14.8004	14.1776	13.0875	14.5144	-4.3445	SF
101	229	-4.5400	6.3166	5.5442	4.9712	6.0582	-11.0676	SF
101	230	-2.1781	7.7399	6.9732	6.3081	7.4812	-10.7089	SF
101	231	-1.7682	7.9478	7.1833	6.5207	7.6900	-10.3599	SF
101	232	2.5638	9.9984	9.2518	8.4382	9.7396	-10.0213	SF
101	233	3.3468	10.2843	9.5419	8.7231	10.0264	-9.6935	SF
101	234	9.6969	12.3090	11.5948	10.6168	12.0501	-9.3769	SF
101	235	11.772	12.8028	12.0975	11.0941	12.5445	-9.0719	SF
101	236	24.179	14.9003	14.2344	13.0556	14.6409	-8.7790	SF
101	237	22.642	14.6912	14.0222	12.8826	14.4333	-8.4983	SF
101	238	54.221	16.9150	16.2950	14.9613	16.6559	-8.2302	SF
101	239	42.18	16.3479	18.4435	21.3829	16.0906	-7.9750	SF
102	232	14.287	13.8567	13.1417	11.8994	13.6041	-12.6100	SF
102	233	42.826	17.0247	16.3769	14.8403	16.7700	-12.2943	SF
103	235	-3.4850	7.5036	6.6935	6.0196	7.2718	-14.0132	SF
103	236	-0.4616	9.2220	8.4237	7.6179	8.9895	-13.7382	SF
103	237	0.0075	9.4420	8.6467	7.8401	9.2105	-13.4702	SF
103	238	4.2212	11.2859	10.5120	9.5541	11.0536	-13.2097	SF
103	239	5.3901	11.6911	10.9237	9.9465	11.4596	-12.9572	SF
103	240	15.307	14.3468	13.6242	12.4066	14.1136	-12.7131	SF
103	241	12.82	13.7897	13.0578	11.9148	13.5583	-12.4779	SF
103	242	22.712	15.5493	14.8527	13.5519	15.3172	-12.2519	SF
103	243	23.179	15.5979	14.8991	13.6128	15.3628	-12.0354	SF

figure we have observed that there is a linear variation half-lives with the product of $Z_d Q^{-1/2}$. We have also compared logarithmic half-lives of proton decay with that of alpha decay (Royer⁸¹, Univ⁸², NRDX⁸³, Denisov⁸⁴) and spontaneous fission (Bao⁸⁵) and are tabulated in Table 2. From the table it is clear that the predicted isotopes such as ¹⁹⁵⁻²⁰³Ac, ²⁰⁰⁻²⁰⁷Pa, ^{212-220,224}Am, and ²¹⁸⁻²²¹Bk are having less half-lives compared to alpha decay and spontaneous fission

decay mode. We have also identified and specified the dominant decay mode in the actinide region $Z=89-103$ in the corresponding table. Due to non-availability of experimental values in the actinide region, the predictive power is tested by comparing the available experimental values with the present work and it is tabulated in Table 3. From the table it is observed that studied values obtained from the present work agrees well with the available experimental values.

Table 3 — The comparison of calculated half-lives with the experimental values^{54-56,87-102}.

Isotopes	Z _d	A _d	l	logT (present work)	logT _{1/2} (experimental)	Ref.
¹⁰⁵ Sb	49	101	2	2.430	1.7	87
¹⁰⁹ I	51	105	2	-3.779	-4	88,54-55
¹¹² Cs	53	108	2	-2.856	-3.3	56
¹¹³ Cs	53	109	2	-5.100	-4.77	88, 56
¹¹⁷ La	55	113	2	-1.812	-1.623	98-102
¹²¹ Pr	57	117	2	-2.188	-2	98-102
¹³⁰ Eu	61	126	2	-2.637	-3.046	98-102
¹³¹ Eu	61	127	2	-1.639	-1.67	98-102
¹³⁵ Tb	63	131	3	-3.178	-3.027	98-102
¹⁴⁰ Ho	65	136	3	-1.920	-2.222	98-102
¹⁴¹ Ho	65	137	3	-2.941	-2.387	98-102
¹⁴⁴ Tm	67	140	5	-4.767	-5.569	98-102
¹⁴⁵ Tm	67	141	5	-4.996	-5.456	98-102
¹⁴⁶ Tm	67	142	5	-0.086	-0.63	89
¹⁴⁷ Tm	67	143	5	0.723	0.43	55,87-88
¹⁵⁰ Lu	69	146	5	-1.153	-1.4	55,92
¹⁵¹ Lu	69	147	5	-0.876	0.89	55,92
¹⁵⁵ Ta	71	151	5	-2.563	-2.538	98-102
¹⁵⁶ Ta	71	152	2	-0.584	-0.609	98-102
¹⁵⁷ Ta	71	153	0	-0.030	-0.523	98-102
¹⁵⁹ Re	73	155	5	-4.636	-4.678	98-102
¹⁶⁰ Re	73	156	2	-3.026	-3.06	94
¹⁶¹ Re	73	157	3	-3.171	-3.43	95
¹⁶⁴ Ir	75	160	5	-4.459	-3.947	98-102
¹⁶⁵ Ir	75	161	5	-3.529	-3.46	96
¹⁶⁶ Ir	75	162	2	-0.642	-0.82	96
¹⁶⁷ Ir	75	163	0	-1.163	-0.96	96
¹⁷⁰ Au	77	166	2	-4.027	-3.493	98-102
¹⁷¹ Au	77	167	0	-4.458	-4.611	98-102
¹⁷¹ Au	77	167	4	-2.745	-2.65	96
¹⁷⁶ Tl	79	172	0	-2.148	-2.284	98-102
¹⁷⁷ Tl	79	173	0	-0.947	-1.174	98-102
¹⁸⁵ Bi	81	181	0	-4.402	-4.35	97

4 Conclusions

We have studied one proton decay in the actinide region through the study of energy released, penetration probability and logarithmic half-lives in the actinide region. The studied half-lives of present work is compared with the different decay modes such as alpha decay and spontaneous fission. We have identified the possible proton emitters with the corresponding energies and half-lives in the actinide region. The possible proton emitters in the actinide region are ¹⁹⁵⁻²⁰³Ac, ²⁰⁰⁻²⁰⁷Pa, ^{212-220,224}Am, and ²¹⁸⁻²²¹Bk. We have identified the proton emitters in the unexplored isotopes of actinide region which is not specified in the nuclear chart⁸⁶.

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PHYSICS OF ELEMENTARY PARTICLES
AND ATOMIC NUCLEI. THEORY

Proton Radioactivity of Heavy Nuclei of Atomic Number Range $72 < Z < 88$

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Abstract—We have studied proton radioactivity of heavy nuclei of atomic number range $72 < Z < 88$. We have calculated the energy released during the proton decay (Q_p) and half-lives of proton decay. To study the competition between different decay modes, we have compared the proton decay half-lives with that of the decay modes such as alpha decay, beta decay, cluster decay and spontaneous fission. To check the Geiger–Nuttall law for proton decay, we have plotted the logarithmic proton decay half-lives versus $1/\sqrt{Q}$. We have also highlighted possible proton emitters with the corresponding energies and half-lives in the atomic number range $72 < Z < 88$.

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1. INTRODUCTION

In the energy domain of radioactivity, proton can be considered as a point charge having highest probability of being present in the parent nucleus. Conclaves et al. [1] studied the two-proton radioactivity of nuclei of mass number $A < 70$ using the effective liquid drop model. Delion et al. [2] reviewed the theories of proton emission to analyse the properties of nuclear matter. Maglione et al. [3] analysed the proton emission half-lives from the deformed nuclei $^{109}_{53}\text{I}$, $^{131}_{63}\text{Eu}$ and $^{141}_{67}\text{Ho}$. Delsanto et al. [4] investigated the β -delayed proton emission of ^{69}Kr and ^{68}Se and extracted their proton separation energies, half-lives and excitation energies. Alavi et al. [5] calculated the proton radioactivity half-lives of 45 proton emitters by WKB Method and observed the decrease in values of calculated half-lives using the orientation angle dependent formalism. Raciti et al. [6] measured the emission of two protons from the decay of ^{18}Ne . Baye et al. [7] evaluated the decay probability per second for ^{11}Be , ^{19}C and ^{31}Ne one-neutron halo nuclei. Feix et al. [8] computed the decay widths of nuclear proton emission for $Z = 51$ to 71 nuclei using Droplet Model. Anguiano et al. [9] investigated the photo-emission of two protons from the ^{12}C , ^{16}O and ^{40}Ca nuclei for the study of short range correlations. Coniglione et al. [10] explored high energy proton emission in heavy ion reactions close to the Fermi energy by investigating the production

mechanism of energetic protons in an experiment performed with the MEDEA detector. Giusti et al. [11] developed the theoretical frame work of emission of two protons in electron induced reactions. Ludewigt et al. [12] studied the proton emission in α -induced reactions at 43 MeV nucleon. Guzman et al. [13] analysed the proton emission from proton-rich nuclei and calculated the half-lives using the effective liquid drop model. Delion et al. [14] proposed semi empirical formula for logarithmic half-lives of proton decay. Dong et al. [15] theoretically calculated the half-lives of proton emitters using generalized liquid drop model (GLDM) and WKB approximation. Maglione et al. [16] studied the proton emission from ^{125}Pm and the behaviour of the half-lives were discussed as a function of deformation, spin of the decaying state, and energy of the emitted protons. Arumugam et al. [17] investigated the proton emission, gamma deformation, and the spin of the isomeric state of ^{141}Ho and revealed that proton deformations and other structural properties of exotic nuclei beyond the proton drip-line. Duarte et al. [18] explored the half-lives for proton emission, alpha decay, cluster radioactivity, and cold fission processes theoretically. Ferreira et al. [19] planned to study the proton radioactivity from spherical nuclei theoretically based on relativistic density functional derived from meson exchange and point coupling models. Ginter et al. [20] studied the proton emission from ^{150}Lu and new proton emitting state was

observed. Delion et al. [21] investigated proton decay from tri-axially deformed nuclei ^{161}Re and ^{185}Bi and studied the dependence of angular distribution of decaying particle on triaxial deformation parameters. Earlier workers [22–24] studied one and two proton decay half-lives of ^{131}Eu , ^{45}Fe and also studied proton emission from the deformed nuclei. In the literature, different theoretical approaches are available [25–41] to study different decay modes including proton decay. Aim of the present work is to study the proton radioactivity of heavy nuclei in the atomic number range $72 < Z < 88$.

2. THEORY

The proton emission half-life in the region $72 < Z < 88$ is given by

$$T_{1/2} = \frac{\ln(2)}{\nu_0 P}, \quad (1)$$

where ν_0 is the assault frequency, which is related to oscillator frequency related to ω ;

$$\nu_0 = \frac{\omega}{2\pi} = \frac{(2n_r + \ell + \frac{3}{2})}{2\pi\mu R_n^2} = \frac{(G + \frac{3}{2})}{1.2\mu R_0^2}, \quad (2)$$

where $R_n^2 = \frac{3}{5} R_0^2$ and $R_0 = 1.14$ fm is used here.

$G = 2n_r + \ell$ is the principal quantum number. For the proton radioactivity, we have selected $G = 4$ or 5 corresponding to the $4\hbar\omega$ or $5\hbar\omega$ oscillator shell depending on the individual nucleus. The penetration probability P is evaluated using WKB approximation [42].

$$P = \exp\left[-\frac{2}{\hbar} \int_{R_{\text{in}}}^{R_{\text{out}}} \sqrt{2\mu(V - Q)} dr\right], \quad (3)$$

where μ is reduced mass of emitted proton, $\mu = (m_p m_{A-1} / (m_p + m_{A-1})) = 938.3(A - 1)/A$ MeV/ c^2 and Q is the amount of energy released. R_{in} and R_{out} are the classical inner and outer turning points. The inner turning point is given by;

$$R_{\text{in}} = r_0(A_1^{1/3} + A_2^{1/3}), \quad (4)$$

where $A_1 = 1$ and $A_2 = A - 1$ for proton emission. R_{out} is determined by the condition $V = Q$. The total potential is given as;

$$V = V_C + V_P + V_\ell, \quad (5)$$

where V_C Coulomb interaction potential, V_P is the proximity potential and V_ℓ is the centrifugal potential. The V_C is given by,

$$V_C = \frac{Z_1 Z_2 e^2}{r} \left[1 + \frac{3R^2}{5r^2} \beta_2 Y_{20}(\theta) + \frac{3R^4}{9r^4} \beta_4 Y_{40} \right], \quad (6)$$

where Z_1 and Z_2 are the atomic numbers of proton and daughter nuclei respectively, and “ r ” is the distance between fragment centres. R , β , $Y_{20}(\theta)$ is the radius of the nuclei, quadrupole deformation parameter and spherical harmonic function respectively. Proximity potential given by [43, 44],

$$V_P(Z) = 4\pi\gamma\bar{R}\Phi(s). \quad (7)$$

Φ is the universal proximity potential which is independent of the shapes of nuclei or the geometry of the nuclear system, but depends on the minimum separation distance $s = z/b$. z is the distance between the near surfaces of the fragments and $b = 0.99$ is the nuclear surface thickness. In Eq. (7), $\bar{R}$ is the mean curvature radius and it is expressed as;

$$\bar{R} = \frac{C_1 C_2}{C_1 + C_2}. \quad (8)$$

The C_i is the Sussmann central radii of the fragments related to sharp radii R_i is expressed as;

$$C_i = R_i - \left(\frac{b^2}{R_i}\right). \quad (9)$$

For sharp radii R_i ,

$$R_i = 1.28A_i^{1/3} - 0.76 + 0.8A_i^{-1/3}. \quad (10)$$

In Eq. (7), γ is given as;

$$\gamma = \gamma_0 \left[1 - K_s \left(\frac{N - Z}{A} \right)^2 \right] \text{ MeV/fm}^2 \quad (11)$$

$\gamma_0 = 1.460734$ MeV/fm² and $K_s = 4.0$ [26].

Universal proximity potential is given by [26]

$$\Phi(\epsilon) = \begin{cases} -4.41 \exp\left(\frac{-\epsilon}{0.7176}\right) & \text{for } 0 \geq \epsilon \geq 1.9475 \\ -1.7817 + 0.9270\epsilon + 0.0169\epsilon^2 - 0.05148\epsilon^3 & \text{for } 0 \leq \epsilon \leq 1.9475 \end{cases}. \quad (12)$$

The centrifugal potential V_ℓ is expressed as

$$V_\ell(r) = \frac{\hbar^2 \ell(\ell + 1)}{2\mu r^2}, \quad (13)$$

where μ , ℓ and r are the reduced mass, the angular momentum and the distance between the mass centre's respectively.

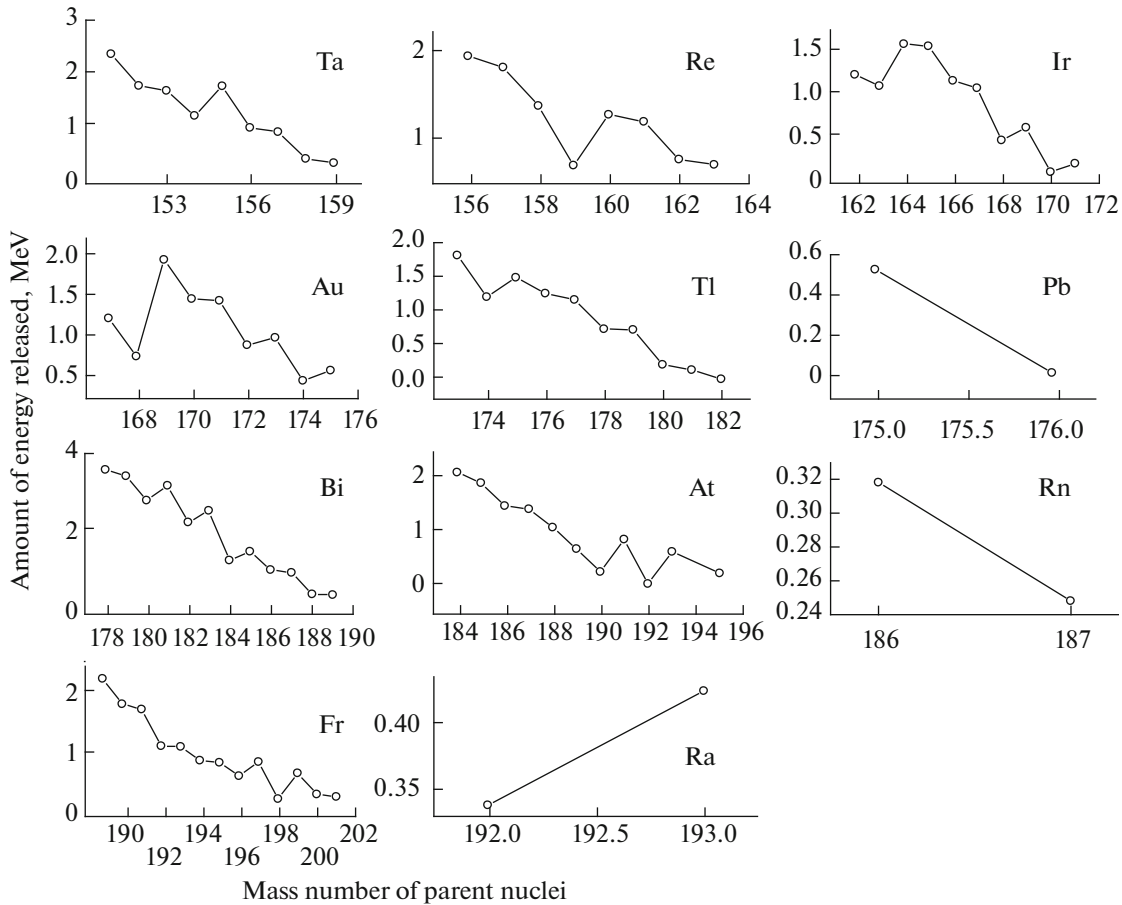


Fig. 1. A variation of amount of energy released during the proton emission in the nuclei region $72 < Z < 88$ with the mass number of parent nuclei.

3. RESULTS AND DISCUSSIONS

The amount of energy released during proton decay are studied using mass excess values available in [45–49]. We have studied driving potential, penetration factor and half-lives of proton emission in the nuclei region $72 < Z < 88$ as explained in the theory section. The variation of amount of energy released during proton decay with the mass number of parent nuclei is as shown in Fig. 1. From the figure we have observed that as the mass number of the parent nuclei increases the energy released decreases. The variation of logarithmic half-lives for the proton emission with the mass number of parent nuclei is as shown in Fig. 2. From the figure it depicts that the logarithmic half-lives for the proton emission increases with the mass number of parent nuclei. To check the Geiger–Nuttall law for proton decay, we have plotted the logarithmic proton decay half-lives versus $1/\sqrt{Q}$ and it is as shown in Fig. 3. From the figure we have observed linear variation of logarithmic half-lives with the $Z_d Q^{-1/2}$. We have also studied the competition

between different decay modes such as alpha decay, β^+ -decay, β^- -decay and proton decay. The half-lives corresponding to β^+ -decay and β^- -decay are evaluated using the semi empirical formula available in the literature [53, 54]. Alpha decay half-lives are evaluated using the procedure explained in the previous work [24]. The plot of different decay modes are as shown in Fig. 4 and also highlighted possible proton emitters with the corresponding energies and half-lives in the atomic number range $72 < Z < 88$. To validate the present work, The proton emission half-lives produced by the present work is compared with that of experiments and available semi empirical formulae such as Hatsukawa et al. [55] and Gamow [42] it is tabulated in Table 1. We have also compared the proton radioactivity logarithmic half-lives of present work with that of available experimental values and it is depicted in Fig. 5. From Fig. 5 and Table 1, it is clearly observed that the present work is in close agreement with the experimental values.

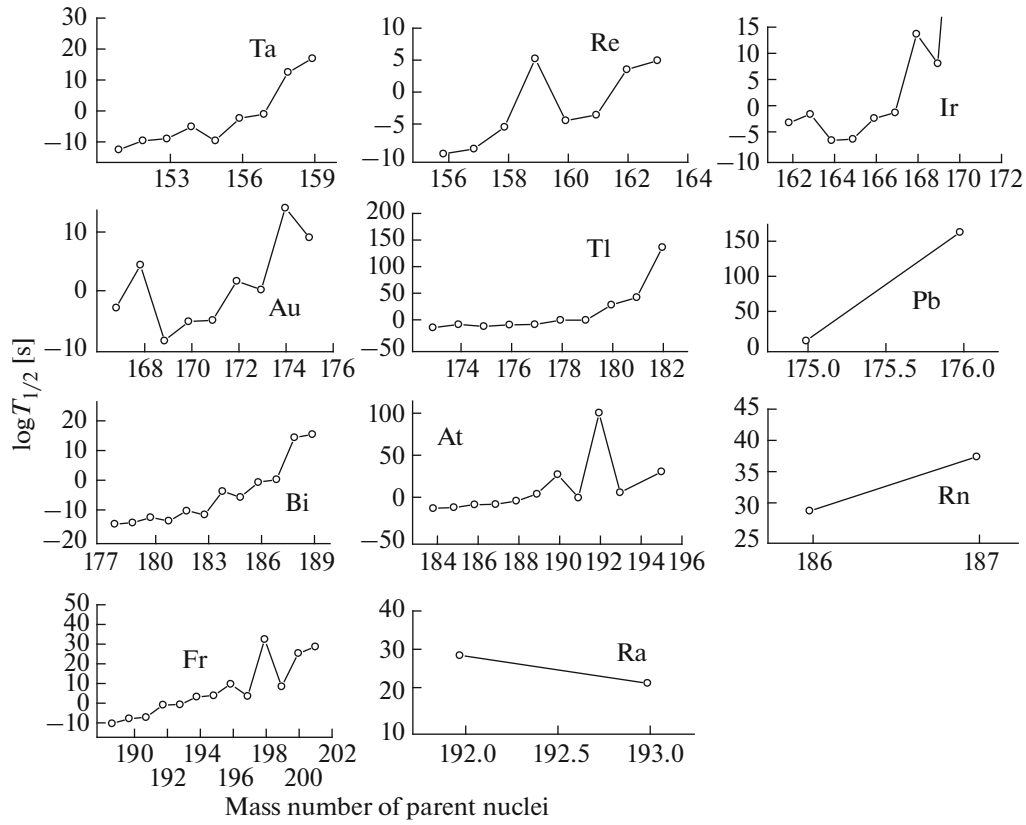


Fig. 2. A variation of logarithmic half-lives for the proton emission in the nuclei region $72 < Z < 88$ with the mass number of cluster.

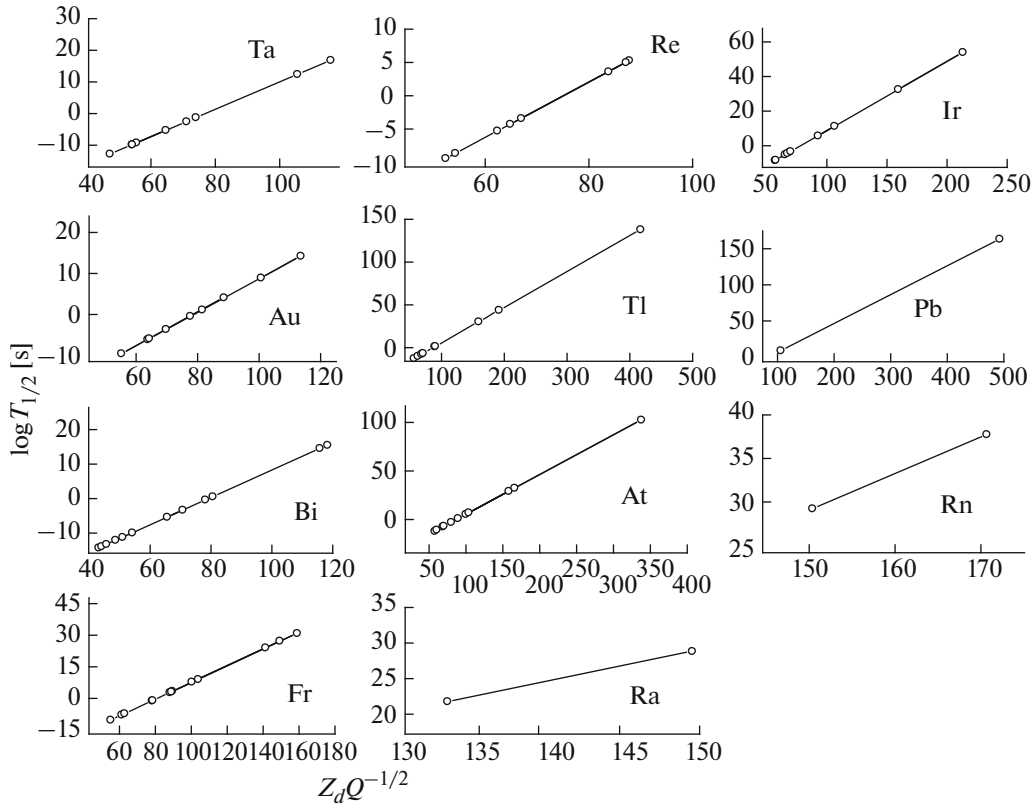
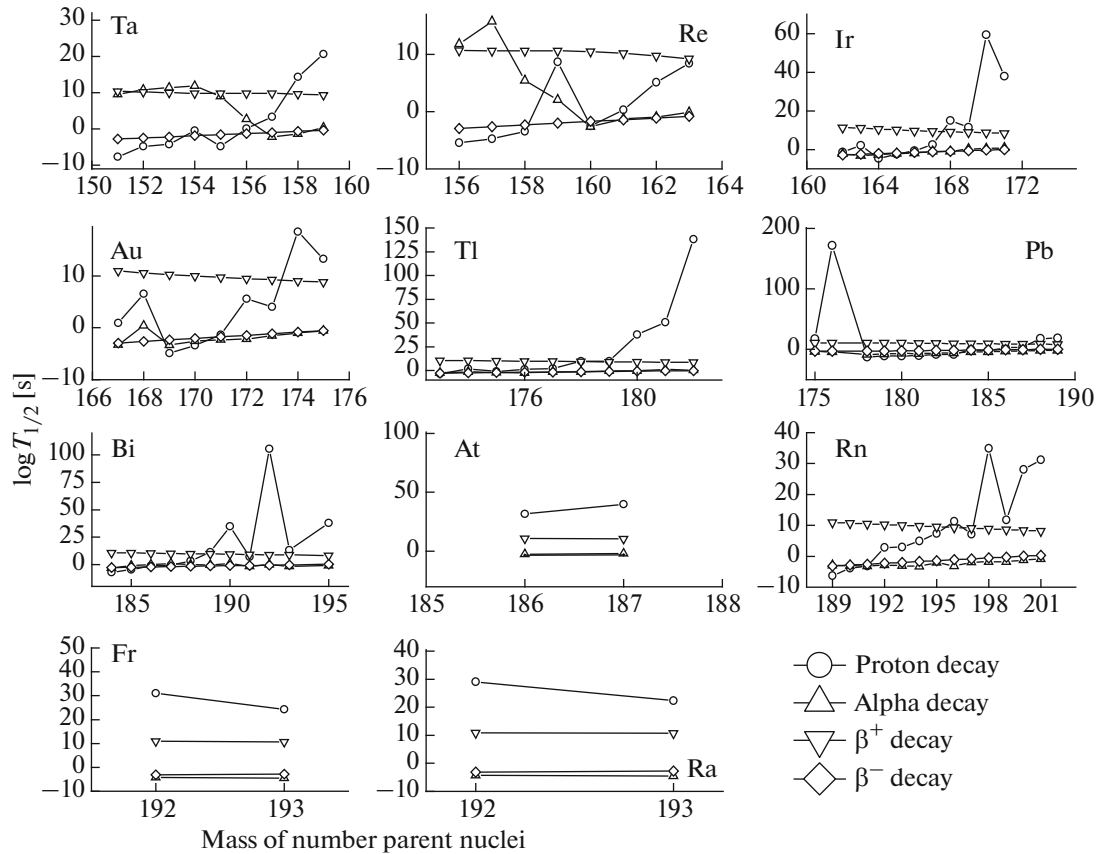


Fig. 3. A variation of logarithmic half-lives for the proton emission in the nuclei region $72 < Z < 88$ with the product of atomic number of daughter and amount of energy released during proton emission ($Z_d Q^{-1/2}$).

Table 1. Comparison of present work with experiments [2, 50–52] and other models

Parent nuclei	Q [Expt.], MeV	$\log T_{1/2}$ Expt. [2, 50–52]	Q (present work), MeV	ℓ	$\log T_{1/2}$				
					present work	CPPMDN [51]	CPPM [51]	Hatsukawa et al. [55]	Gamow [42]
^{155}Ta	1.453	−2.538	1.789	5	−2.836	−3.302	−3.218	−5.998	−5.490
^{156}Ta	1.020	−0.609	1.027	2	−0.618	−0.854	−0.620	−1.221	−0.716
^{160}Re	1.267	−3.060	1.292	2	−2.292	−2.339	−1.986	−3.732	−3.231
^{161}Re	1.197	−3.357	1.211	0	−3.614	−1.981	−1.603	−2.955	−2.454
^{164}Ir	1.540	−3.947	1.584	5	−3.599	−3.601	−3.182	−5.904	−5.406
^{166}Ir	1.152	−0.818	1.166	2	−0.874	−0.542	−0.097	−1.887	−1.392
^{167}Ir	1.070	−0.959	1.084	0	−1.016	0.298	0.752	−0.673	−0.179
^{170}Au	1.472	−3.493	1.489	2	−3.503	−3.054	−2.959	−4.643	−4.152
^{171}Au	1.447	−4.611	1.467	0	−4.593	−3.331	−3.102	−4.393	−3.903
^{176}Tl	1.265	−2.284	1.265	0	−2.362	−0.739	−0.501	−1.883	−1.399
^{177}Tl	1.159	−1.174	1.177	0	−1.184	0.633	0.878	−0.545	−0.063
^{185}Bi	1.526	−4.237	1.559	4	−4.119	−5.268	−3.507	−4.769	−4.293


Fig. 4. Variation of $\log(T_{1/2})$ of proton activity, alpha decay, β^+ and β^- decay as a function of mass number of the parent nuclei (A).

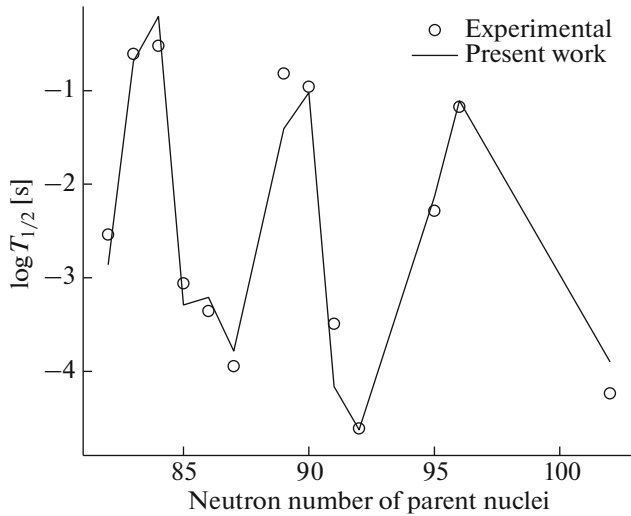


Fig. 5. A comparison of logarithmic half-lives of proton radioactivity of present work with that of available experimental values.

4. CONCLUSIONS

We have studied the amount of energy released during the proton decay (Q_p) and half-lives of proton decay. We have studied the competition between different decay modes by comparing the proton decay half-lives with that of the other decay modes such as alpha decay, β^+ and β^- decay. We have also checked the Geiger–Nuttall law for proton decay by plotting the logarithmic proton decay half-lives versus $1/\sqrt{Q}$. We have highlighted possible proton emitters with the corresponding energies and half-lives in the atomic number range $72 < Z < 88$.

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Exploring new proton emitting isotopes of Lanthanides

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Abstract: Present work explores the 24 new proton decay emitters in the Lanthanide region by studying the competition between the different possible decay modes such as proton decay, alpha decay, beta decay and spontaneous fission. The proton emission half-lives of different lanthanide isotopes have been studied using different proximity functions such as Prox. 13, Prox. 77, MP 77, Ng 80 and Bass 80. Though the experimental values are found to be in good agreement with the proximity function of Ng 80, we have developed an empirical formula to calculate the half-lives of such proton-emitting lanthanides. The half-life values produced by the present formula are compared with that of NG 80. The present formula produces the half lives with simple inputs of Z_d and Q values, and hence, we may call this as pocket formula. Newly identified proton emitters are presented in a nuclide chart. The identified proton emitters may find applications in radiation therapy.

1. Introduction

Light and medium nuclei mostly show proton decay, whereas lanthanides show the proton and β decay. Furthermore, heavy nuclei ($Z = 72-88$) show β^+ and β^- decay, actinides ($Z = 89-103$) and superheavy nuclei or transactinides decay through α particles. It is also predicted that nuclei with $Z \geq 126$ may undergo cluster decay/exotic decay [1]. The competition between decay modes depends sensitively on the Coulomb and centrifugal barriers. Proton radioactivity studies provide a unique insight into the structure of nuclei beyond the drip line limit [2]. Proton emission from long-lived excited states has been investigated since the 1970s in ^{53m}Co [3]. Subsequent discoveries of proton decay from ^{151}Lu [4], ^{113}Cs , ^{109}I [5] and eventually other exotic heavy isotopes like ^{117}La [6] and ^{135}Tb [7] have motivated additional measurements.

The Lanthanide series includes 14 elements having atomic numbers from 58 to 71. The lanthanides and their analogs finds importance in radiotherapy due to their physical properties physical half-life, type(s) of decay emission(s), energy of the emission, cost and availability, and specific activity [8]. Few studies have been devoted toward the different decay modes of actinides, lanthanides and transactinides [9–14]. Sridhar et al. [15] studied the cluster radioactivity in actinide nuclei. Quadrelli et al. [16] analyzed the quadratic decay observed for Ln(III) ionic radii and calculated bond distances and lanthanide atomic orbital expectation values. Nitscke et al. [17] identified a total of 24 new β -delayed proton precursors and several new decay branches in the region of $56 < Z < 72$ and $N < 82$ using OASIS online mass separator facility. Davids et al. [18] identified proton decay from ^{141}Ho and ^{131}Eu . Sowmya et al. [19] studied the competition between different decay modes such as binary, ternary, cluster radioactivity and alpha decay of Darmstadtium. Although different decay modes are explained for few series of actinide and the heavy elements, the lanthanide series yet to be explored.

The presence of high coulomb barrier for heavy nuclei ($Z > 52$), reduces the proton barrier penetration probability to the extent that proton decay taking place from

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the ground states of nuclei have measurable long half lives [13]. There are various methods to investigate the proton radioactivity such as the density-dependent M3Y effective interaction [20, 21], the Jeukenne-Lejeune-Mahaux (JLM) interaction [20], the unified fission model [22, 23], the generalized liquid drop model [24], the cluster model [25], the deformed density model [26], the coulomb and proximity potential model [27], the covariant density functional theory [28]. These nuclear proximity potentials provide the phenomenological potentials for nuclear reaction and structure including nuclear decay [29]. Santhosh et al. [27] explained the half life predictions for the proton emitters with $Z > 50$ in the ground state and isomeric state using coulomb and proximity potential model for the deformed nuclei Dong et al. [30]. The α -decay was studied using double-folding potentials from chiral effective field theory for the nuclei ^{104}Te .

The coulomb and nuclear proximity potential provides another simple and practical formalism to estimate the strength of the nuclear interactions during collision of heavy-ions. When two surfaces are approaching each other, approximately at a distance of 2–3 fm, an additional force due to the proximity of surfaces will appear which is called proximity potential [31]. There are adjustable parameters in various parts of the proximity formalism such as the radius parameter R , the surface energy coefficient and the universal function which lead to introduce different versions of the proximity potentials. Santhosh et al. [32] studied the coulomb and proximity potential as interacting barrier for post-scission region and calculated half-life time for different modes of exotic decay treating parent and fragments as spheres and these values are compared with experimental data. Dutta et al. [33] performed a detailed comparative study of fusion barriers for asymmetric colliding nuclei using the different versions of phenomenological proximity potential as well as other parameterizations within the proximity concept.

From the detailed analysis of the literature, it is found that, there is no systematic study of proton decay in the lanthanide region. The aim of the study is to predict the unexplored proton emitters in the lanthanide region. Experimentally, only 11 proton emitters were identified in the lanthanide region. From the present study, it is of first kind where we systematically explored the unexplored 24 proton decay emitters in the lanthanide region. This article is organized into four sections. The second section explains the theory used for the study. Results and discussion are presented in the third section, whereas fourth section concentrates on the summary of the present work.

2. Theory

The macroscopic modified generalized liquid drop energy between the two nuclei is given by [1]

$$E = E_V + E_S + E_C + E_{\text{prox.}} + E_\ell \quad (1)$$

$E_V, E_S, E_C, E_{\text{prox.}}$ and E_ℓ be the volume, surface, Coulomb, proximity and centrifugal energies, respectively. The centrifugal energies depend on the angular momentum. The selection rule for proton decay is as follows;

$$J_i = J_f + J_{p_i} \quad (2)$$

where $p_i = 1, 2$ for one and two proton decay, respectively. The conservation of parity is expressed as;

$$\pi_i = \pi_f \pi_{p_i} (-1)^\ell \quad (3)$$

where i and f are the initial and final states in the proton decay. ℓ is the angular momentum at the proton transition and p_i is nonzero and zero for both one and two proton, respectively. The selection rules for the minimum angular momentum are expressed as;

$$\ell_{\min} = \begin{cases} \Delta_j & \text{for even } \Delta_j \text{ and } \pi_p = \pi_d \\ \Delta_j + 1 & \text{for even } \Delta_j \text{ and } \pi_p \neq \pi_d \\ \Delta_j & \text{for odd } \Delta_j \text{ and } \pi_p \neq \pi_d \\ \Delta_j + 1 & \text{for odd } \Delta_j \text{ and } \pi_p = \pi_d \end{cases} \quad (4)$$

where $\Delta_j = |J_p - J_d - J_{p_i}|$.

The term $E_{\text{prox.}}$ is used to calculate nuclear part of the potential. It is very difficult to calculate the nuclear potential part. Many models such as double-folding, proximity potential, liquid drop model have been used. Among these, the proximity model has been easily and successfully used to calculate the nuclear interaction between two nuclei. It is mainly composed of two parts. One depends on the shape and geometry of two nuclei, and the other is the universal function $\Phi(s_0)$ only related to the short separation distance between two nuclei. The proximity energy is defined as.

$$E_{\text{prox.}}(Z) = 4\pi\gamma\bar{R}\Phi\left(\frac{z}{b}\right) \quad (5)$$

Here Φ is universal proximity potential function, and z is distance between the near surfaces of the fragments, respectively. $b \approx 0.99$ is the nuclear surface thickness. In the above Eq. (5) $\bar{R}$ is the mean curvature radius and γ is the surface energy co-efficient [1]. To Evaluate $E_{\text{prox.}}$ we have used following five different proximity functions.

2.1. Prox 13(Prox. 13)

The idea of the universal function is the fundamental advantage of the proximity potential model. Because of the

unique nature of the nucleus where the density distribution is different for different nucleus. Double-folding model (DFM) with the density-dependent nucleon-nucleon interaction gives the average results of the effective nucleon-nucleon interaction at all nuclear densities. The microscopic double-folding potential is proved more potential in studying the nuclear structural information, such as nuclear deformation parameters, charge root mean square radius and alpha-preformation factors [34–41]. Zhang et al. [42] used the density-dependent nucleon-nucleon interaction to calculate the nuclear potential and then deduced the universal function. This is termed as “Prox.2013” and the proposed universal function can be expressed as;

$$\Phi(\epsilon) = \frac{p_1}{1 + \exp\left(\frac{s_0 + p_2}{p_3}\right)} \quad (6)$$

With $s_0 = \frac{R - R_1 - R_2}{b}$

Here p_1 , p_2 and p_3 are -7.65 , 1.02 and 0.89 , respectively. Previous researchers [43] also used this proximity potential to study the proton radio activities of some nuclei and compared with that experiments.

2.2. Prox 1977 (Prox. 77)

Blocki et al. [44] suggested a generalized proximity theorem that leads to the formula for the interaction potential between the two nuclei and a function of simple geometrical factor and universal separation function. This theorem is important for discussing the interaction between types of surfaces for which the curvatures at the point of least separation are no longer small compared to the diffuseness of the surface region. Proximity function based on this proximity theorem is called prox 77 and it is expressed as;

$$\phi(s) = \begin{cases} -0.5(S - 2.54)^2 - \\ 0.0852(S - 2.54)^3 & S < 1.2511 \\ -3.437 \exp(-S/0.75) & S > 1.2511 \end{cases}$$

With $S = (r - C_1 - C_2)/b$ and $b \approx 1$. This proximity function was successfully applied in studying potential energy surfaces (PES) in the ground-state decay [45], quasielastic scattering [46] and barrier distribution [31].

2.3. Modified Prox 1977 (MP77)

The original form of the proximity potential 1977 overestimates the experimental data by 4% for fusion barrier heights [47]. Several improvements/modifications were made over the original proximity potential 1977 to remove the discrepancy between theory and data. It included either the better form of the surface energy coefficient [48] or universal function and/or nuclear radius [47]. Later, Ishwar

Dutt [49] modified the original proximity potential 77 which includes the reactions with combine mass between $A = 19$ and $A = 294$ units, totally 390 reactions were experimentally studied by considering symmetric as well as asymmetric colliding partners. The modified form of Prox 1977 [44] is expressed as [49];

$$\phi(S) = \begin{cases} -1.7817 + 0.9827S + 0.143S^2 \\ -0.09S^3 & S < 0 \\ -1.7817 + 0.01696S^2 \\ -0.05148S^3 & 0 < S < 1.9475 \\ -4.41 \exp(-S/0.7176) & S > 1.9475 \end{cases} \quad (7)$$

with $S = (r - C_1 - C_2)/b$ and b is the surface width, $b = (\pi/\sqrt{3})a$ with $a = 55$ fm and it is nearly equal to unity. Furthermore, This modified proximity potential has been successfully used for the prediction of different decay modes of superheavy elements α decays of the yet-to-be-discovered superheavy element $Z = 119$ in the mass range of $A = 274$ – 313 [50–53].

2.4. Prox Ngo 1980 (Ng80)

In 1980 Ngo [54] proposed a proximity function based on the calculated interaction potential between two nuclei using the energy density formalism and Fermi distributions for the nuclear densities. The proposed proximity function is expressed as;

$$\phi(S) = \begin{cases} -33 + 5.4(S - S_0)^2 & \text{for } S < S_0 \\ -33 \exp\left[-\frac{1}{5}\right](S - S_0)^2 & \text{for } S \geq S_0 \end{cases} \quad (8)$$

where $S_0 = -1.6$ fm. Furthermore, this proximity function was used by the previous researchers [55] along with the dynamical Cluster decay Model to produce the alpha decay half-lives. This proximity function was also employed in the study of nucleus-nucleus interactions such as fusion [56].

2.5. Bass Model 1980 (Bass 80)

Bass in 1977 [57] derived a universal nucleus-nucleus potential from a classical analysis of experimental fusion cross sections. The deduced potential is consistent with the liquid-drop model at small separation, and with quantum analyses of elastic scattering at large separation. Bass potentials do not have a repulsive core at shorter distance. A newer version of the Bass potential referred as “Bass 80” [48] shows slight improvement over Bass 77 and Bass 80 reproduce the experimental data within 1.5%. The proposed proximity function is given by

$$\Phi(S) = [0.033 \exp(S/3.5) + B \exp(S/0.65)]^{-1} \quad (9)$$

This proximity potential was also employed in the elastic scattering cross section [58, 59]. This proximity function was also successfully used in the study of decay process such as alpha decay and cluster decay [29, 60–62].

3. Method of calculation of half-life

According to WKB approximation (Wentzel–Kramers–Brillouin) of the penetration probability P through the potential barrier was studied for the cluster and alpha decay by the following equation;

$$P = \exp \left\{ -\frac{2}{\hbar} \int_{R_a}^{R_b} \sqrt{2\mu(E - Q)dr} \right\} \quad (10)$$

where the total energy is evaluated using Eq. (1) by using different proximity functions, where μ is the reduced mass of proton decay system, R_a and R_b are the inner and outer turning points and these turning points were evaluated using following conditions;

$$V_T(R_a) = Q = V_T(R_b) \quad (11)$$

The alpha decay half-life is studied using following equation;

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{\nu P S_p} \quad (12)$$

where λ is decay constant and ν is the assault frequency. S_p is the spectroscopic factor and it is model dependent and very sensitive to decay energy. The accurate consideration decay energies in the calculation results half-lives close to experiment and spectroscopic factors close to one. It is also evident from the literature [63, 64] that the spectroscopic factors are assumed as one in proton decay half-life calculation while using the WKB approximation. In the present work, we have used WKB approximation and accurate recent mass excess values in the calculation of decay energies. Thus the spectroscopic factors are assumed to be one. E_v is the empirical vibration energy and it is evaluated using the following equations;

$$\nu = \frac{\omega}{2\pi} = \frac{2E_v}{\hbar} \quad (13)$$

$$E_v = Q \left\{ 0.056 + 0.039 \exp \left[\frac{4 - A_2}{2.5} \right] \right\} \text{ for } A_2 \geq 4 \quad (14)$$

4. Present formula

The variation of experimental $\log(T_{1/2})$ of proton decay in the lanthanide region as a function of $Z_d/\sqrt{Q}$ is shown in Fig. 1. We have fitted empirical relation for experimental

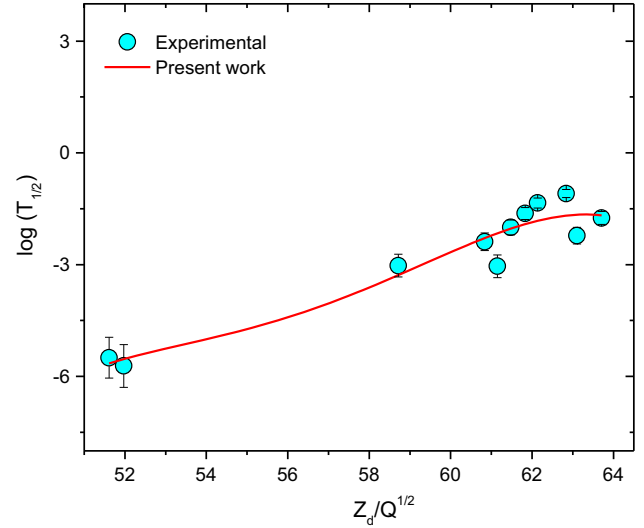


Fig. 1 Variation of experimental and present formula produced proton decay half lives as a function of $Z_d/\sqrt{Q}$

$\log(T_{1/2})$ such a way that it should have maximum R^2 and minimum residual sum of squares. Hence, proposed empirical formula for $\log(T_{1/2})$ of proton decay is given below; the half-lives for proton decay in the lanthanide region as a function of fissility parameter $Z_d/\sqrt{Q}$ is given by:

$$\log(T_{1/2}) = \sum_{i=0}^{i=4} A_i \left(\frac{Z_d}{\sqrt{Q}} \right)^i \quad (15)$$

where Z_d is the atomic number of the daughter nuclei and Q is the decay energy. The fitting parameters A_0, A_1, A_2, A_3 and A_4 are having the values $-1.61, -20.82 \times 10^{-2}, 71 \times 10^{-4}, -8.18 \times 10^{-5}$ and $-3.11 \times 10^{-7} \text{ MeV}^{1/2}$ s, respectively.

5. Results and discussion

The phenomenon of proton decay is treated as the transmission of the proton across a potential barrier developed due to combined effect of coulombic and nuclear potential [65]. Experimentally there are 11 proton emitters were identified in the lanthanide region. We have studied the proton decay for lanthanide nuclei in which its decay energy (Q_p) is positive. In the present work, it is of first kind where we systematically explored the unexplored 24 proton decay emitters in the lanthanide region. These proton emitters having half-lives in terms of $1 \text{ s} - 1 \mu\text{s}$. Generally, the half-lives of proton emitters nuclei have been determined by quantum-mechanical tunneling calculation through a potential barrier [66].

The universal function proposed by five different versions of coulomb and nuclear proximity potentials such as Prox. 13, Prox. 77, MP 77, Ng 80 and Bass 80 are used to calculate the half lives of proton emitters in the lanthanide region for different isotopes of lanthanides. Table 1 gives the range of studied lanthanide isotopes having positive proton decay energy.

In order to study whether the shape of the potential leads to different half-lives, the different proximity potentials are plotted as a function of R as shown in Fig. 2. X-axis corresponds to the distance between interacting nuclei, and Y-axis corresponds to interacting potential. The area under the potential curve is directly proportional to the penetration probability. If the area under the potential curve is more, the probability of penetration is more which clearly indicates the short half-life of the decay particle and vice versa. In the present study, from Fig. 2, it is observed that, the area under the curve is found to be maximum for Bass 80 and then follows the order Prox. 13, Mod. Prox. 77, Prox. 77 and Ng 80.

The calculated proton decay half lives are compared with the experiments. The calculated Mean square error of different proximity functions with respect to experiments is shown in Table 3. The sum of the squared residuals between the $\log(T_{1/2})$ of experimental and different proximity potentials ($SSR = \sum_{i=1}^n e_i^2$), where e_i is the i th residual or difference and n is the number of data points. Mean square error with respect to experiments for different proximity functions and proposed present formula ($\sigma_\epsilon^2 = \frac{SSR}{n-2}$) are shown in Table 3. From Table 3, it is

Table 1 The range of lanthanide isotopes having positive proton decay energy

Z	Range of mass number studied
57	$110 \leq A \leq 119$
58	$113 \leq A \leq 115$
59	$115 \leq A \leq 123$
60	$118 \leq A \leq 119$
61	$120 \leq A \leq 128$
62	$123 \leq A \leq 125$
63	$125 \leq A \leq 135$
64	$128 \leq A \leq 130$
65	$130 \leq A \leq 139$
66	$133 \leq A \leq 135$
67	$136 \leq A \leq 143$
68	$138 \leq A \leq 139$
69	$141 \leq A \leq 149$
70	$143 \leq A \leq 147$
71	$146 \leq A \leq 155$

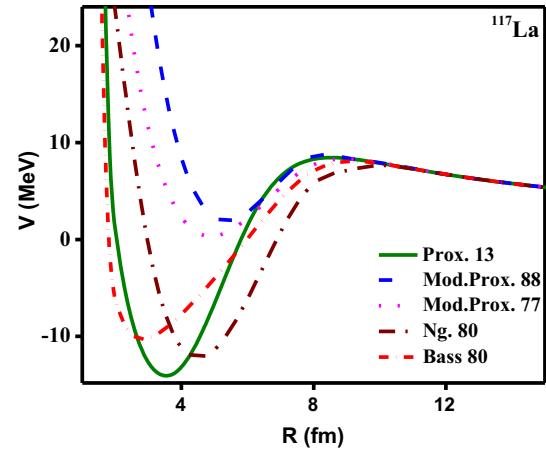


Fig. 2 Variation of potential energy as a function of R for different proximity potentials

observed that the mean square error was found to be less for Ng 80 compared to other proximity potentials. The experimental values are found to be agree well with Ng 80 among the studied proximity functions. Thus, Ng 80 proximity potential was used to study the competition between different decay modes in the lanthanide region.

We have constructed new simple empirical relation to calculate the half life of proton emitters in the lanthanide region for different isotopes of lanthanides other than the above-mentioned models. The constructed empirical formula is given in Eq. 15. The half-lives values produced with proximity function NG80 is close to the experiment. From the comparison of mean square error it shows that MP88 is better than the present empirical formula. Mean square error difference between Mp88 and Present Formula is 0.08, and it is almost negligible means both methods used to calculate half lives will produce the almost same deviation. But, to calculate half lives using the MP88 proximity function involves many physical quantities. Whereas, the present formula produces the half lives with simple inputs of Z_d and Q values and this we may call pocket formula. So that the present formula is more advantageous than the MP88. The evaluated proton decay half-lives using present formula and different proximity functions along with the experiments are presented in Table 2. From this table, it is found that the present formula produces proton decay half lives close to the experiments. Proton decay energies are also presented in this Table 2.

Dominant decay mode can be identified by studying the competition between the different possible decay modes such as alpha, β^+ , β^- , Spontaneous fission (SF) and proton decay. We have also calculated the half lives of possible decay modes using the well established formulae available in the literature [alpha [67], β^+ [68], β^- [68] and SF [69]].

Table 2 Comparison of evaluated proton decay half-lives using different proximity functions with that of the experiments

Proton emitter	Q_p MeV	$\log T_{1/2}(s)$						
		Expt	Ng. 80	Mp. 88	Mp. 77	Bass. 80	Prox. 13	Present formula
$^{111}\text{La} \rightarrow ^{110}\text{Ba}$	4.321	—	−3.80	−6.96	−6.82	−17.94	−17.76	−3.51
$^{112}\text{La} \rightarrow ^{111}\text{Ba}$	3.791	—	−3.62	−6.62	−6.46	−17.18	−17.03	−3.45
$^{113}\text{La} \rightarrow ^{112}\text{Ba}$	3.071	—	−3.38	−5.85	−5.67	−15.86	−15.51	−3.35
$^{114}\text{La} \rightarrow ^{113}\text{Ba}$	1.891	—	−3.13	−4.25	−4.00	−12.21	−12.07	−2.98
$^{115}\text{La} \rightarrow ^{114}\text{Ba}$	2.517	—	−3.01	−5.19	−4.99	−14.51	−14.13	−3.21
$^{116}\text{La} \rightarrow ^{115}\text{Ba}$	1.206	—	−2.77	−2.18	−1.89	−7.89	−7.76	−2.51
$^{117}\text{La} \rightarrow ^{116}\text{Ba}$	0.803	−1.63	−2.58	−2.11	−2.43	−2.99	−2.92	−2.12
$^{118}\text{La} \rightarrow ^{117}\text{Ba}$	0.378	—	−2.36	−2.33	−3.68	−2.00	−6.22	−2.13
$^{113}\text{Ce} \rightarrow ^{112}\text{La}$	1.971	—	−3.45	−4.15	−3.88	−12.32	−11.94	−2.98
$^{114}\text{Ce} \rightarrow ^{113}\text{La}$	1.491	—	−3.26	−2.91	−2.61	−9.76	−9.37	−2.71
$^{115}\text{Ce} \rightarrow ^{114}\text{La}$	0.891	—	−3.06	−0.15	−0.79	−3.94	−3.55	−2.17
$^{115}\text{Pr} \rightarrow ^{114}\text{Ce}$	3.861	—	−3.44	−6.58	−6.41	−17.03	−16.91	−3.43
$^{117}\text{Pr} \rightarrow ^{116}\text{Ce}$	2.811	—	−3.00	−5.43	−5.24	−14.95	−14.61	−3.25
$^{119}\text{Pr} \rightarrow ^{118}\text{Ce}$	1.411	—	−2.59	−2.74	−2.43	−8.98	−8.88	−2.61
$^{121}\text{Pr} \rightarrow ^{120}\text{Ce}$	0.837	−2	−2.24	0.37	0.02	−2.80	−2.43	−2.11
$^{122}\text{Pr} \rightarrow ^{121}\text{Ce}$	0.526	—	−2.05	3.44	3.79	1.11	4.00	−2.01
$^{123}\text{Pr} \rightarrow ^{123}\text{Ce}$	0.209	—	−0.56	7.73	7.58	3.93	9.76	−0.46
$^{118}\text{Nd} \rightarrow ^{117}\text{Pr}$	1.131	—	−2.89	−1.44	−1.11	−6.21	−6.15	−2.33
$^{119}\text{Nd} \rightarrow ^{118}\text{Pr}$	0.741	—	−2.71	1.36	0.50	−0.93	−0.31	−2.01
$^{121}\text{Pm} \rightarrow ^{120}\text{Nd}$	3.301	—	−2.70	−6.01	−5.82	−15.77	−15.69	−3.31
$^{123}\text{Pm} \rightarrow ^{122}\text{Nd}$	1.981	—	−2.28	−4.09	−3.81	−11.78	−11.44	−2.89
$^{125}\text{Pm} \rightarrow ^{124}\text{Nd}$	0.438	—	−1.85	5.48	6.15	2.47	6.45	−2.16
$^{127}\text{Pm} \rightarrow ^{126}\text{Nd}$	0.545	—	−1.56	3.58	3.94	1.44	4.63	−2.01
$^{124}\text{Sm} \rightarrow ^{123}\text{Pm}$	0.481	—	−2.16	5.08	5.12	2.35	6.02	−2.11
$^{129}\text{Eu} \rightarrow ^{128}\text{Sm}$	1.459	—	−1.61	−2.47	−2.14	−8.31	−7.98	−2.49
$^{130}\text{Eu} \rightarrow ^{129}\text{Sm}$	1.028	−3.05	−2.43	−0.31	−1.69	−4.14	−4.15	−2.15
$^{131}\text{Eu} \rightarrow ^{130}\text{Sm}$	0.939	−1.75	−1.28	0.04	0.40	−2.94	−2.64	−2.10
$^{133}\text{Eu} \rightarrow ^{132}\text{Sm}$	0.675	—	−0.96	2.53	3.81	0.63	1.84	−1.98
$^{135}\text{Tb} \rightarrow ^{134}\text{Gd}$	0.524	−3.03	−2.91	−4.75	5.18	2.66	6.43	−2.11
$^{140}\text{Ho} \rightarrow ^{139}\text{Dy}$	1.094	−2.23	−2.50	−0.36	0.06	−3.69	−3.44	−2.11
$^{141}\text{Ho} \rightarrow ^{140}\text{Dy}$	1.176	−2.39	−2.35	−0.61	−2.01	−4.65	−4.72	−2.16
$^{144}\text{Tm} \rightarrow ^{143}\text{Er}$	1.712	−5.73	−4.12	−6.34	−6.83	−4.11	−8.66	−5.53
$^{145}\text{Tm} \rightarrow ^{144}\text{Er}$	1.736	−5.49	−5.07	−2.59	−1.24	−8.79	−8.84	−5.63
$^{150}\text{Lu} \rightarrow ^{149}\text{Yb}$	1.27	−1.35	−1.10	−0.58	−2.28	−4.49	−4.61	−2.13
$^{151}\text{Lu} \rightarrow ^{150}\text{Yb}$	1.241	−1.09	−1.78	−0.64	−0.22	−4.19	−3.99	−2.11

Table 3 Mean square error with respect to experiments for different proximity functions and proposed present formula

Proximity function	Ng. 80	MP. 88	MP. 77	Bass. 80	Prox. 13	Present formula
σ	1.23	1.52	1.64	1.82	2.12	1.60

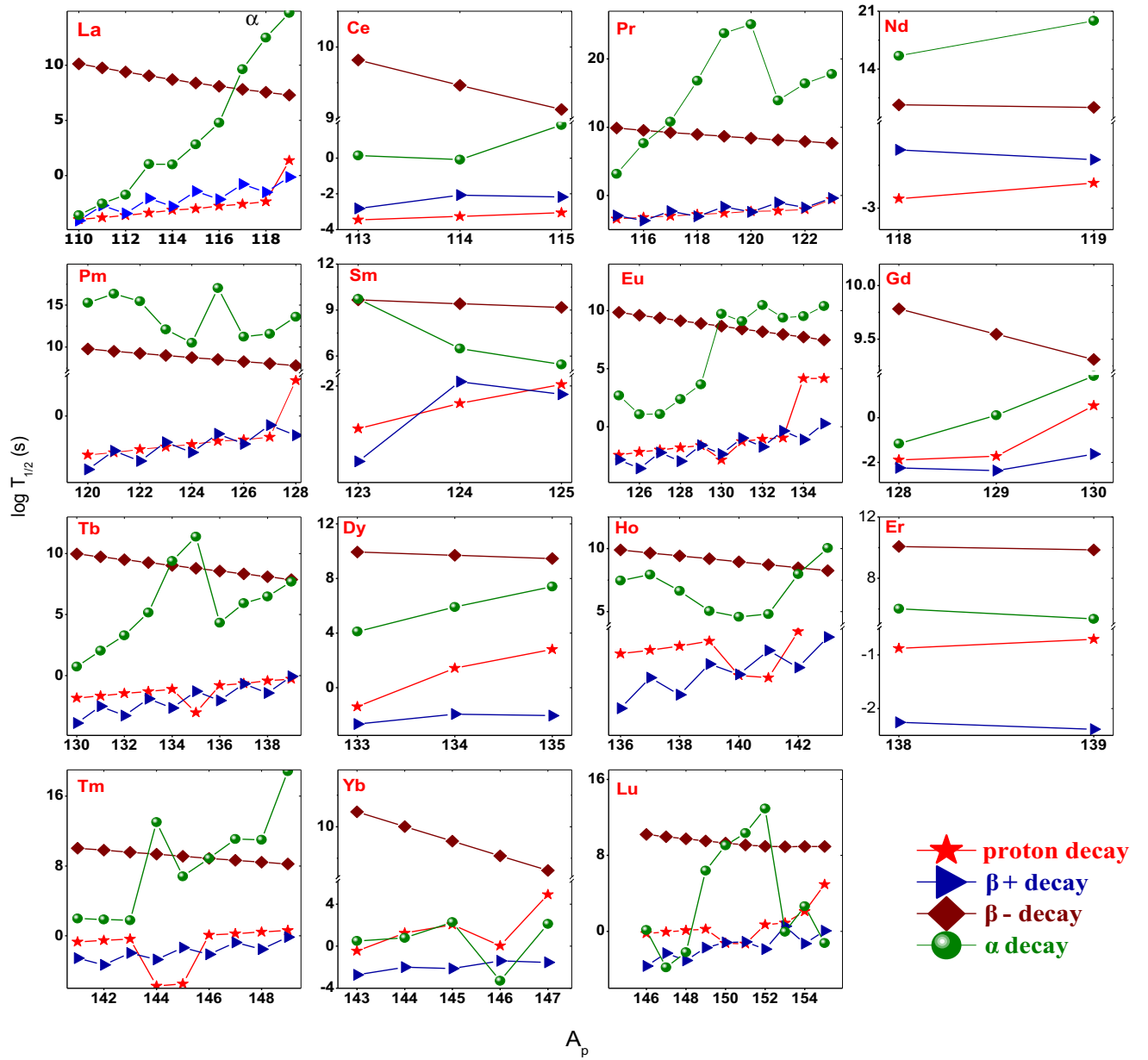


Fig. 3 Competition between different decay modes for lanthanide nuclei

The competition between different decay modes in the studied lanthanide region is shown in Fig. 3. The decay mode which is having shorter half life among the possible decay modes will be identified as the dominant decay mode. The observation of Fig. 3 clearly indicates that some isotopes of lanthanides with atomic number ranging between 57 and 63 (La, Ce, Pr, Nd, Pm, Sm and Eu) are newly identified as proton emitters in the lanthanide region, whereas the Gadolinium, Dysprosium and Erbium show β^+ decay as a dominant decay mode. In Terbium, Holmium, Thulium, even though maximum isotopes are β^+ decay emitters, few of them are proton decay emitters. In Ytterbium and Lutetium, few isotopes are β^+ decay

emitters and few of them are α decay emitters. The newly identified 24 proton emitters in the lanthanide region are ^{111}La , ^{112}La , ^{113}La , ^{114}La , ^{115}La , ^{116}La , ^{118}La , ^{113}Ce , ^{114}Ce , ^{115}Ce , ^{115}Pr , ^{117}Pr , ^{119}Pr , ^{122}Pr , ^{123}Pr , ^{118}Nd , ^{119}Nd , ^{121}Pm , ^{123}Pm , ^{125}Pm , ^{127}Pm , ^{124}Sm , ^{129}Eu , ^{133}Eu . The different α , β^+ , existing proton emitters, nuclei with electron capture decay mode and the formula predicted new proton emitters are shown in the Nucleide chart (Fig. 4). The predicted new 24 proton emitters are highlighted in pink color, whereas α , β^+ , electron capture and existing proton emitters are highlighted in yellow, green, aqua blue and brick red, respectively.

6. Conclusions

We have systematically studied proton decay half-lives in the lanthanide region using different proximity functions such as Prox. 13, Prox. 77, MP 77, Ng 80 and Bass 80. The competition between the evaluated proton decay half-lives with other competent decay modes such as alpha, β^+ , β^- , spontaneous fission and proton decay are also studied. Eventually, 24 new proton decay emitters in the Lanthanide region are identified. Furthermore, present work also proposed empirical formula to calculate the half lives of proton emitters in the lanthanide region. Newly identified proton emitters may be useful in radiation therapy.

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PROTON RADIOACTIVITY OF TANTALUM

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ABSTRACT

Using different models such as Coulomb and proximity potential model, effective liquid drop model and modified generalised liquid drop model, we have studied all possible one proton radioactivity tantalum. The calculated half-lives from the present work are compared with the available experiments. One proton decay energy is studied using recent mass excess values [Chinese Physics C Vol. 45, No. 3 (2021) 030003]. The angular momentum dependence of potential have been considered. The penetration probability (P) is studied using WKB integral. The decay constant (λ) and half-lives ($T_{1/2}$) of $^{151-157}\text{Ta}$ were predicted. The identified one proton radioactivity of $^{151-157}\text{Ta}$ along with half-lives and decay energies plays an important role in the future experiments. Present work may find useful applications in radiotherapy and diagnosis.

Keywords: Proton decay, Half-lives, Penetration probability, Decay constant.

1. INTRODUCTION

Proton decay is one of the key predictions of the various grand unified theories (GUTS) proposed in the 1970s, another major one being the existence of magnetic monopoles. Both concepts have been the focus of major experimental physics efforts since the early 1980s. The proton decay hypothesis was first formulated by Andrei Sakharov in 1967 [1]. During the year 1981 at GSI Darmstadt one proton (1P) ground decay was observed [2]. Half-lives of proton emission of nuclei such as ^{151}Lu , ^{53}Co and so on have been studied [3, 4]. A many theoretical models [5-9] have been made used to study 1P-decay. M.Pfutzner et al., [10] observed the decays of fine ^{45}Fe atoms at the fragment separator of GSI. Bajc et al., [11] systematically studied proton decay in the minimal super symmetric SU(5) grand unified theory. Goldman and Ross [12] predicted theoretical upper limit for proton decay. Two proton decay of ^{67}Kr is experimentally observed [13]. The life time of proton has been identified by earlier researchers [14]. Santosh & Indu sukumaran [15] theoretically predicted half-lives of proton emitters with the atomic number of $Z > 50$. The proton radioactivity has been studied using various proximity potentials [16]. Experimental evidence shows proton drip line of ^{45}Fe [17]. After bombardment of ^{92}Mo target nuclei with ^{50}C , Woods et al. [18] observed

proton decay [18]. Developmental theories of proton decay has been predicted by Maglione et al., [19]. Detail analysis of proton decay has been by Rykaczewska et al., [20]. Ferreira et al., [21] based on relativistic density functional theory, the proton radioactivity from spherical nuclei were studied.

Delion et al., [22] examined the characteristics of nuclear matter by reviewing proton emission hypotheses. Recent literature [23-25] also predicts proton emitters in the atomic number range $72 < Z < 88$ and actinides. Many theoretical studies shows the prediction of possible decay mode in the superheavy region [26-38]. Hence, in the present work we made an attempt to study one proton radioactivity of Tantalum using different models such as Coulomb and proximity potential model (CPPM), effective liquid drop model (ELDM) and modified generalised liquid drop model (GLDM).

2. THEORETICAL FRAMEWORK

2.1. Proton emission half-lives

2.1.1. 1P-decay using Coulomb and proximity potential model (CPPM)

The one proton decay is expressed as;

$${}^A_Z(X)_N \rightarrow {}^{A-1}_{Z-1}(Y)_N + {}^1_1(H) + Q_p \quad (1)$$

where Q_p is the amount of energy released during 1P decay. The decay constant and half-lives are defined as

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{\nu P P_0} \quad (2)$$

where ν is the assault frequency [39], P is the probability of penetration barrier and P_0 is the preformation probability. In the present work we have selected $P_0=1$ for one proton decay. The penetration probability using WKB approximation [40] is given by;

$$P = \exp \left[-\frac{2}{\hbar} \int_{R_{in}}^{R_{out}} \sqrt{2\mu(V - Q_p)} dr \right] \quad (3)$$

where μ is reduced mass, R_{in} and R_{out} are the inner and outer turning points. The inner turning point R_{in} is expressed as;

$$R_{in} = r_0 (A_1^{1/3} + A_2^{1/3}) \quad (4)$$

where $A_1=1$ and $A_2=A-1$ for proton emission. R_{out} is determined by the condition $V=Q$. The r_0 is the effective nuclear constant. The total potential is evaluated as explained in [25].

2.1.2. 1P-decay using Effective liquid drop model (ELDM)

$$V_c = \frac{8\pi}{9} a^5 \varepsilon(\theta_{2p}, \theta_D) \rho_c \quad (5)$$

where ρ_c is the initial charge density, $\varepsilon(\theta_{2p}, \theta_D)$ is a function of the angular variables, and a is the radius of the sharp neck. The surface potential energy is expressed as;

$$V_s = \sigma_{eff} (S_{2p} + S_D) \quad (6)$$

The term effective surface tension σ_{eff} is expressed as;

$$\frac{3}{5} e^2 \left[\frac{Z_p^2}{R_p} - \frac{Z_{1p}^2}{R_{2p}} - \frac{Z_D^2}{R_D} \right] + 4\pi\sigma_{eff} (R_p^2 - R_{1p}^2 - R_D^2) = Q \quad (7)$$

Where Z_p is the atomic number of parent nuclei, Z_{1p} is the atomic number of emitted proton and Z_D is the atomic number of daughter nuclei and other notations are as usual explained in reference [33]. The effect of the centrifugal potential energy is defined as;

$$V_\ell = \frac{\ell(\ell+1)\hbar^2}{2\mu\zeta^2} \quad (8)$$

Here μ represents the reduced mass of the system. Therefore, the effective total potential energy is constructed as;

$$V = V_c + V_s + V_\ell \quad (9)$$

The penetrability factor G is evaluated as explained in reference [33].

2.1.3. 1P-decay using Modified generalised liquid drop model (MGLDM)

The total energy of the system is given by;

$$E = E_v + E_s + E_c + E_{prox} + E_l \quad (10)$$

The total potential is evaluated as explained in reference [33]

The barrier penetration probability is expressed as;

$$P = \exp \left[-\frac{2}{\hbar} \int_{R_{in}}^{R_{out}} \sqrt{2B(r)(E(r) - E(sphere))} \right] \quad (11)$$

Where $R_{in} = R_d + R_\alpha$ and $B(r) = \mu$ is the reduced mass and $R_{out} = e^2 Z_d Z_\alpha / Q_\alpha$. The decay half-life is defined as;

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{\nu_0 P} \quad (12)$$

here ν_0 is the assault frequency and whose value is 10^{20} s^{-1} and P is the barrier penetration probability.

3. RESULTS AND DISCUSSIONS

Using three models such as CPPM, ELDM and MGLDM, we have studied proton decay from the proton rich emitter Tantalum. The 1P-decay is energetically possible only when Q-value of the reaction is positive. The decay energy is evaluated using the following equation;

$$Q = \delta M_p - (\delta M_d + \delta M_z) + k(Z_p^\varepsilon - Z_d^\varepsilon) \quad (13)$$

where δM_p is the mass excess of the parent nuclei, δM_d is the mass excess of the daughter nuclei and δM_z is the mass excess of the emitted proton. The term $kZ_{p(d)}^\varepsilon$ is the total binding energy of electrons in the parent or daughter nuclei. The value of $k=13.6 \text{ eV}$ and $\varepsilon=2.408$ for the nuclei $Z \leq 60$ and $k=8.7 \text{ eV}$ and $\varepsilon=2.517$ for the nuclei $Z > 60$ [25]. The recent mass excess values are taken from the reference [42]. Fig. 1 shows a plot of Q-values during 1P-decay with the mass number of parent nuclei. The minimum Q-value is observed in case of ^{157}Ta with 0.941 MeV and maximum is observed for ^{151}Ta with 2.361 MeV when compared to their neighboring one.

Then, we have calculated total potential using three models in nuclei $^{151-157}\text{Ta}$, the studied potential as function of separation distance is shown in Fig. 2. From the Fig., the minimum potential is observed when the separation energy is 6.5 fm. Then the potential gradually increases and area below the curve gives information on penetration probability.

Later, the evaluated penetration probability and 1P-decay half-lives in $^{151-157}\text{Ta}$ using three models and were tabulated in table 1. The evaluated $\log T_{1/2}$ value varies between -11.21s to -0.35s in case of CPPM. However, in case of ELDM it varies between -10.55s to -0.58s and in case of MGLDM the $\log T_{1/2}$ varies between -10.18s to -0.51s for the nuclei $^{151-157}\text{Ta}$. The values obtained

using present work is compared with the available experimental value [43]. The studied $\log T_{1/2}$ corresponding to $^{151-157}\text{Ta}$ shows close agreement with the available experimental values. However, the value obtained using MGLDM produces experimental half-lives more accurately.

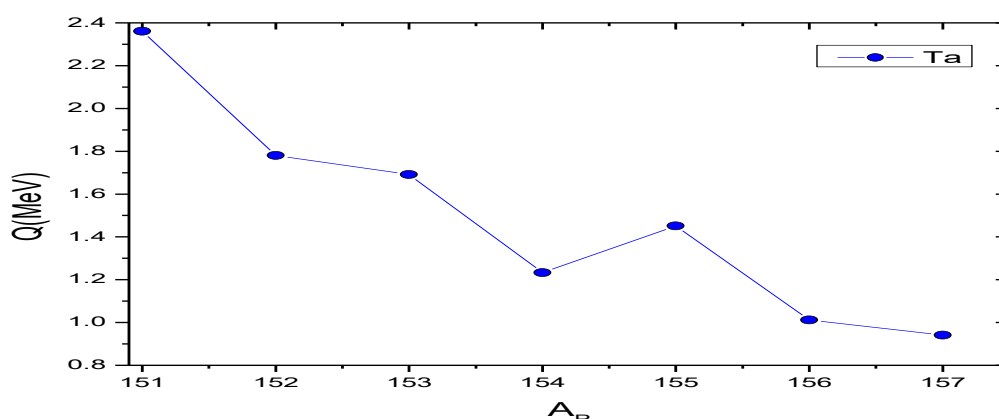


Fig. 1: A plot of Q-values during 1P- decay with the mass number of parent nuclei for the $^{151-157}\text{Ta}$ nuclei

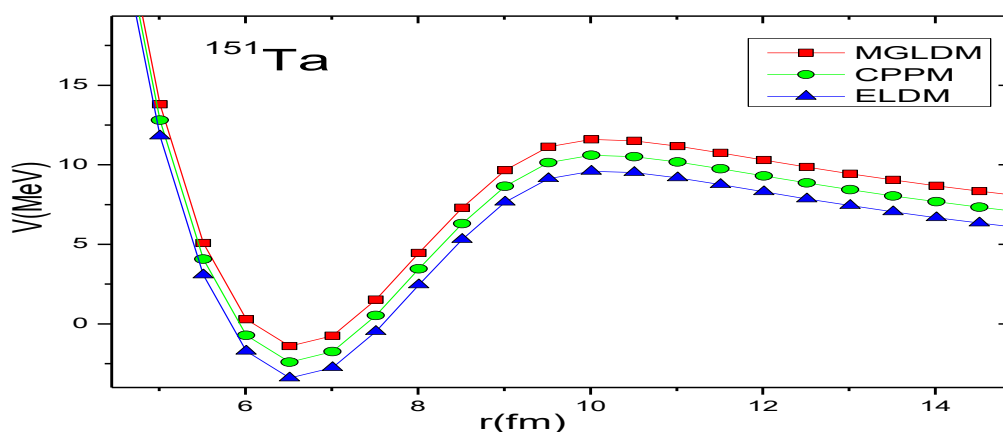


Fig. 2: Variation of total potential using three models such as CPPM, ELDM and MGLDM as function of separation distance in ^{151}Ta nuclei

Table 1: Tabulation of $\log T_{1/2}$ using three different models such as CPPM, ELDM and MGLDM for predicted proton emitters from $^{151-157}\text{Ta}$ is compared to available experiments.

Parent nuclei	Daughter nuclei	Q(MeV)	ℓ	$\log T_{1/2}$			
				Expt [43]	CPPM	ELDM	MGLDM
^{151}Ta	^{150}Hf	2.361	5	-	-11.21	-10.55	-10.18
^{152}Ta	^{151}Hf	1.781	5	-	-8.67	-7.46	-7.9
^{153}Ta	^{152}Hf	1.691	5	-	-5.6	-5.84	-7.43
^{154}Ta	^{153}Hf	1.233	5	-	-5.28	-4.03	-4.1
^{155}Ta	^{154}Hf	1.451	5	-2.49	-2.68	-2.12	-2.51
^{156}Ta	^{155}Hf	1.012	2	-0.83	-0.55	-0.5	-0.85
^{157}Ta	^{156}Hf	0.941	0	-0.53	-0.35	-0.58	-0.51

4. CONCLUSIONS

Using three different models 1P-radioactivity tantalum is studied. The calculated half-lives from the present work are compared with the available experiments. The decay energy is feasible for the nuclei $^{151-157}\text{Ta}$. The angular momentum corresponding to these isotopes varies between 0 to $5\hbar$. The evaluated logarithmic half-life value varies between -11.21s to -0.35s in case of CPPM, in case of ELDM it varies between -10.55s to -0.58s and in MGLDM the logarithmic half-lives varies between -10.18s to -0.51s for the nuclei $^{151-157}\text{Ta}$. The identified 1P-radioactivity of $^{151-157}\text{Ta}$ along with half-lives and decay energies plays an important role in the future experiments. The identified proton emitters with typical half-lives and decay energies may find useful applications in radiotherapy and diagnosis.

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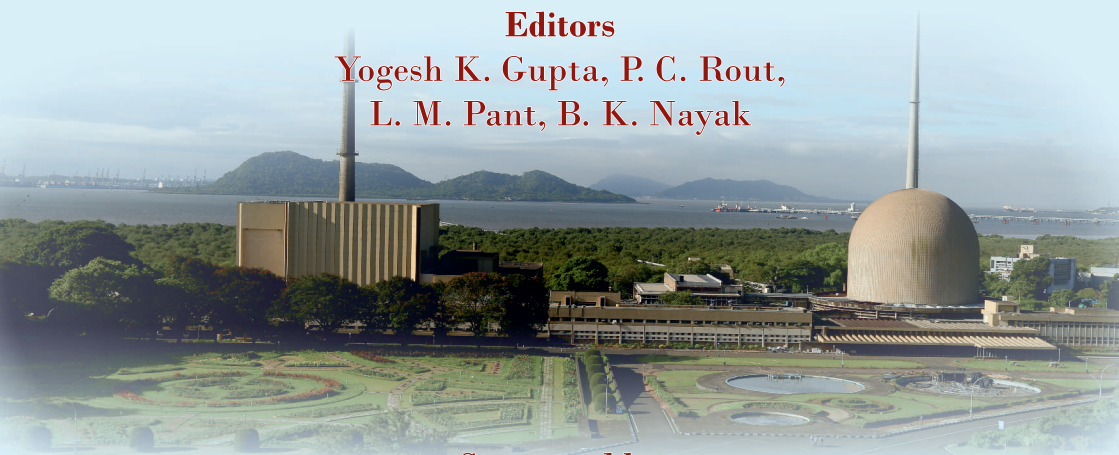
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Proton radioactivity of $^{241-251}\text{Db}$

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Introduction

In the line of stability, the excess protons still adequately bound to the nucleus with the nuclear forces, hence direct emission of proton is not possible. However, while beyond the line of stability the protons are no longer bound by the nuclear forces. In order to study the proton emission beyond the stability line Conclaves et al., [1] studied two-proton radioactivity in the mass number $A < 70$ using liquid drop model. Earlier [2-3] studied proton emission from the deformed nuclei. One proton, two proton, β decay [4-8] were studied using droplet model and WKB approximation. Giusti et al., [9] established theoretical frame work for the emission of two protons in electron induced reactions. Using generalized liquid drop model and WKB approximation, Dong et al., [10] theoretically studied proton decay half-lives of spherical proton emitters.

Previous workers [11-12] theoretically studied half-lives of proton radioactivity. Earlier workers [13-17] were studied ternary fission, binary fission, cluster radioactivity and alpha decay in the superheavy region using different proximity functions. From the available literature, it is essential to study the proton radioactivity in the Dubnium. Hence, in the present work we made a first attempt to study proton radioactivity in the isotopes of Dubnium.

Theory:

The half-lives of proton is studied using the following expression,

$$T_{1/2} = \frac{h \ln(2)}{2\pi \Gamma} \quad (1)$$

here Γ is the decay width and it is calculated using the relation

$$\Gamma = \frac{S \bar{F} h^2 \bar{P}}{16\pi^2 m} \quad (2)$$

here S, F and P are spectroscopic, normalisation and penetration factor respectively and in detail explained in previous work [18]. The average normalization factor is expressed as [18]

$$\bar{F} = \frac{2}{\pi} \int_0^{\pi/2} F(\theta) d\theta \quad (3)$$

where $F(\theta)$ is angle dependent normalization factor.

$$\bar{P} = \int_0^{\pi/2} P(\theta) \sin(\theta) d\theta \quad (4)$$

The total potential is the sum of nuclear, coulomb and centrifugal terms [16].

Results and discussions:

The amount of energy released during one-proton radioactivity is studied using the mass excess values available in the literature [16,19]. We have also studied penetration factor (P), normalization factor (F) and logarithmic half-lives for proton decay in the heavy nuclei of $^{241-251}\text{Db}$. We have also studied the spontaneous fission half-lives and alpha decay half-lives of the heavy nuclei of $^{241-251}\text{Db}$. The comparison of the proton decay with the spontaneous fission and alpha decay half-lives are as shown in figure 1. From the figure we have observed that the spontaneous fission half-lives are smaller compared to proton and alpha decay.

The figure 2(a) explains the variation of amount of energy released with the mass number of parent nuclei and it decreases with increase in the mass number of parent nuclei, 2(b), 2(c) represents the penetration probability and normalization factor with the mass number of parent nuclei and both decreases with the increase in mass number of parent nuclei and 2(d) depicts the variation of logarithmic half-

lives with the product of atomic number and energy released during proton decay.

We have studied proton decay in the heavy nuclei $^{241-251}\text{Db}$. From the figure 1 and 2 it is observed that the proton decay half-lives are also longer than that of spontaneous fission and alpha decay. The competition of proton decay with different decay modes such as alpha decay and spontaneous fission reveals that proton decay is not dominant decay mode in the heavy nuclei $^{241-251}\text{Db}$.

Fig. 1: The variation of logarithmic half-lives of the proton decay, spontaneous fission and alpha decay with the mass number of parent nuclei

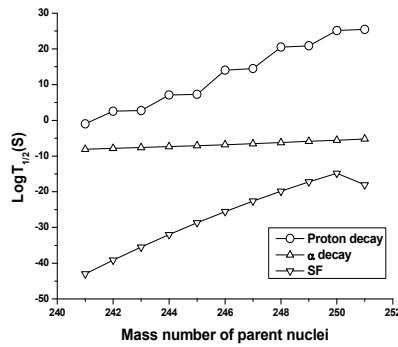
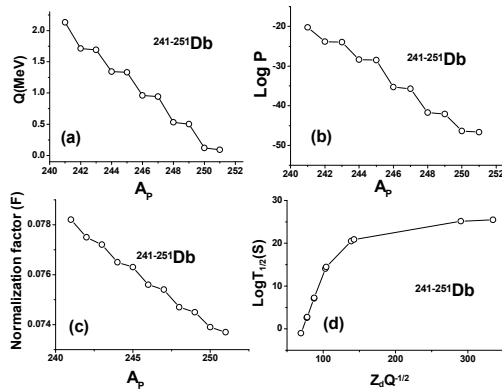


Fig. 2: The variation of amount of energy released, penetration probability and normalization factor with the mass number of parent nuclei and the variation of logarithmic half-lives with the product of atomic number and energy released during proton decay.



Conclusions:

To summarize the present work, we have studied amount of energy released during the proton decay, penetration probability, normalization factor and logarithmic half-lives in the heavy nuclei of $^{241-251}\text{Db}$. We have also compared present work with the spontaneous fission and alpha decay. From the results we can conclude that the heavy nuclei of $^{241-251}\text{Db}$ are having half-lives greater than the spontaneous fission and alpha decay. Hence, the heavy nuclei $^{241-251}\text{Db}$ is stable against the proton decay.

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Competition between different decay modes in Bismuth

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Introduction

During the last two decades, significant progress has been made in the experimental investigation of processes leading to super heavy nuclei, their decay properties and structure. The most stable super heavies are anticipated to be positioned along the β -stability line, which is unreachable by fusion reactions with stable beams. The literature studies shows the competition between different decay modes [1-2]. The proton decay half-lives of Lanthanides and actinides were studied [3-6]. Qian et al., [7] systematically studied α -decay half-lives of heavy and super heavy elements. Tan et al., [8] investigated the β^+ decays of some medium-mass nuclei.

Many theoretical models have been proposed to explore the half-lives of spherical and deformed nuclei. Earlier workers [9] have studied different decay modes of super heavy nuclei. Hence, in the present work we have examined possible decay modes such as proton decay using Coulomb and Proximity potential Model (CPPM), $\beta^\pm$ -decay and an alpha decay are evaluated using semi-empirical relations in the isotopes of Bismuth.

Theoretical Frame work

The proton decay half-lives are evaluated using Coulomb and proximity potential model by including deformation effects and angular momentum. The assault frequency term in half-lives are evaluated using harmonic oscillator frequency is given by [3],

$$\nu = \frac{41}{hA^{1/3}} \text{ MeV} \quad (1)$$

The proton-nucleus total potential will consist of Coulomb V_C and Proximity potential V_P is expressed as

$$V = V_C + V_P \quad (2)$$

The Coulomb interaction (V_C) potential is given by,

$$V_C = \frac{Z_1 Z_2 e^2}{r} \left[1 + \frac{3R^2}{5r^2} \beta_2 Y_{20}(\theta) + \frac{3R^4}{9r^4} \beta_4 Y_{40} \right] \quad (3)$$

here Z_i is the atomic numbers of proton or daughter nuclei. The term 'r' is the separation distance. R is the radius of the nuclei, β is quadrupole deformation parameter and $Y_{20}(\theta)$ is the spherical harmonic function. Proximity potential is evaluated as follows;

$$V_P = 4\pi\gamma b \left[\frac{C_1 C_2}{C_1 + C_2} \right] \phi \quad (4)$$

The penetration probability and half-lives are evaluated as explained in detail in literature [3]. The alpha-decay and beta decay half-lives are also evaluated using semi-empirical relations [3].

Results and Discussions:

The proton decay half-lives are studied in the isotopes of heavy nuclei Bismuth (Bi) using CPPM with harmonic oscillator frequency. However, an alpha-decay and $\beta^\pm$ -decay half-lives are evaluated using semi-empirical relations. If the Q-value of the reaction in proton decay is positive, then the proton radioactivity is energetically feasible [6]. The mass excess values in order to evaluate Q-value of the reaction is taken by recent mass excess data available in literature [10].

The proton decay, an alpha-decay and beta-decay half-lives obtained from the present work are compared with available experiments. The figure 1 shows comparison of proton, an alpha and beta-decay half-lives using CPPM and semi-empirical relations with that of available experiments.

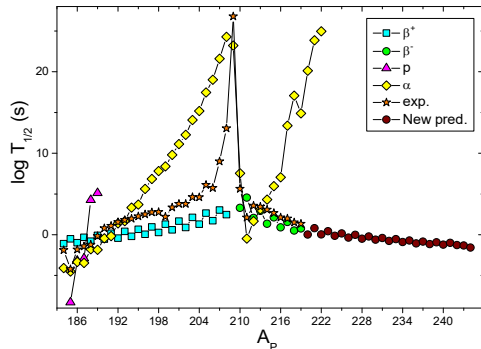


Fig 1: A comparison of proton-decay, an alpha-decay and beta-decay half-lives using CPPM and semi-empirical relations with that of available experiments.

From this comparison it is observed that the nuclei $^{184,186-189}\text{Bi}$ and $^{191,209,211-212}\text{Bi}$ which possess an alpha decay half-lives are in good agreement with the available experimental alpha decay half-lives. Similarly, the nuclei $^{190,192-208}\text{Bi}$, $^{210,213-244}\text{Bi}$ and ^{185}Bi are having β^+ , β^- and proton decay half-lives respectively are in close agreement with the available experimental values.

Table-1: Prediction of logarithmic half-lives of β^- - decay in the isotopes of heavy nuclei $^{220-244}\text{Bi}$.

A_p	$T_{1/2}$	A_p	$T_{1/2}$	A_p	$T_{1/2}$
^{220}Bi	0.01	^{229}Bi	-0.22	^{237}Bi	-0.85
^{221}Bi	0.79	^{230}Bi	-0.62	^{238}Bi	-1.16
^{222}Bi	0.04	^{231}Bi	-0.4	^{239}Bi	-0.94
^{223}Bi	0.42	^{232}Bi	-0.77	^{240}Bi	-1.22
^{224}Bi	-0.14	^{233}Bi	-0.55	^{241}Bi	-1.01
^{225}Bi	0.15	^{234}Bi	-0.9	^{242}Bi	-1.28
^{226}Bi	-0.35	^{235}Bi	-0.71	^{243}Bi	-1.33
^{227}Bi	-0.01	^{236}Bi	-1.08	^{244}Bi	-1.59
^{228}Bi	-0.49				

From this comparison it is clear that the values obtained using different decay modes are comparable with the experiments, hence we have extended our studies to isotopes of Bismuth from ^{220}Bi to ^{244}Bi . Then we have studied all possible decay modes such as proton, beta and an alpha decay half-lives. Among all the studied half-lives

the β^- -decay in the isotopes of heavy nuclei $^{220-244}\text{Bi}$ shows shorter half-lives when compared to other decay modes. Hence, the possible decay mode in heavy nuclei $^{220-244}\text{Bi}$ is β^- -decay only. The table-1 shows the predicted β^- -decay half-lives in the heavy nuclei $^{220-244}\text{Bi}$. These predicted half-lives are in seconds to ms.

Conclusions:

The different decay modes such as proton decay, beta-decay and an alpha decay have been evaluated using CPPM and semi-empirical relations in the isotopes of Bismuth. The values obtained from the present work were comparable with the experiments. Around 9 α emitters, one proton emitter, 18 β^+ emitters and 33 β^- emitters were identified. Among the β^- emitters, around 25 new emitters from ^{220}Bi to ^{244}Bi were newly identified. These identified new β^- emitters are useful in the field of radiotherapy.

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