



Weak Measurement Effects on Dynamics of Quantum Correlations in a Two-Atom System in Thermal Reservoirs

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Abstract

The dynamical behaviour of quantum correlations captured by different forms of Measurement-Induced Nonlocality (MIN) between two atoms coupled with thermal reservoirs is investigated and compared with the entanglement. It is shown that the MIN quantities are more robust, while noise causes sudden death in entanglement. Further, we quantified the quantum correlation with weak measurement and the effect of measurement strength is observed. The role of mean photon number and weak measurements on quantum correlation is also highlighted.

Keywords Entanglement · Quantum correlation · Dynamics · Weak measurement · Projective measurements · Nonlocality

1 Introduction

The preservation of quantum correlation in various quantum systems is one of the fundamental areas of interest in quantum communication and information theory [1–8]. In particular, the use of such a correlation in open systems is a cumbersome task due to the

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decoherence effect [9]. This unavoidable interaction between a quantum system and its environment would destroy quantum correlations between the systems and that reduce the quantum advantages of the system. Further quantum correlation dynamics provides the knowledge to design suitable mechanisms to protect quantum correlations during information processing [10]. Recently, researchers paid more attention in identifying different types of quantum correlations in various quantum systems [11–15]. In particular, generation of nonlocal correlation beyond the entanglement, because entanglement does not account for all of the non-classical properties of quantum correlations.

The present work will focus on the dynamics of quantum correlations beyond the entanglement in open quantum systems. Yi and Sun made first attempt to study the dynamics of entanglement in open systems [16]. Subsequently, Rajagopal and Rendell investigated the dynamics of entanglement by considering two coupled and initially entangled harmonic oscillators in environment [17]. Zyczkowski et al. studied dynamics of entanglement for the bipartite systems under the action of decaying channel [18]. In these papers [17, 18], it is shown that entanglement might disappear at finite times, while coherence, for the same environment would vanish asymptotically in time.

It is worth mentioning that weak measurement allows us to probe into the quantum system with minimum disturbance, which was introduced by Aharonov et al. [19]. In general, the von Neumann measurements can be realized as a sequence of weak measurements which make it as universal measurement [20]. Binding the concept of dynamics of quantum correlation and weak measurement, there were several results reported as follows. The dynamics of quantum correlation for two isolated atoms in their own thermal reservoir is studied by using quantum discord with weak measurement [21]. In addition, the quantification of quantum correlation via geometric discord using weak measurements is also demonstrated [22]. Further, the comparison between dynamics of quantum discord and entanglement of a two-qubit system in thermal reservoir is also investigated [23–25]. However, from a computational point of view, the computation of quantum discord is a tough task due to the complicated optimization procedure [26]. Consequently, a lot of alternative measures of quantum correlation have been proposed, such as geometric quantum discord, quantum deficit, measurement induced disturbance (MID), MIN [11, 12, 14]. These measures are also quantum measurement dependent. Further, it is also shown that MIN is more robust than the concurrence to decoherence [27]. Despite the correlation dynamics has been broadly studied in open quantum systems, the problem is still open for the effect of the environment on different form of MIN. Therefore MIN is thus taken as dynamical quantity with and without weak measurements, and we study its evolution under the influence of environment.

The article is organized as follows. In Section 2, we define the quantifiers of quantum correlation studied in this paper. In Section 3, we introduce the theoretical model under our investigation and the notion of intrinsic decoherence in a quantum system. The dynamics of entanglement and MINs are presented in Section 4. Finally, the conclusions are given in Section 5.

2 Quantum Correlation Measures

Entanglement: Concurrence has been recognized as a powerful measure of entanglement for arbitrary two-qubit state [28]. It can be defined for a bipartite composite state ρ shared by the subsystems a and b in the Hilbert space $\mathcal{H} = \mathcal{H}^a \otimes \mathcal{H}^b$, that is

$$C(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}, \quad (1)$$

where λ_i are the square root of eigenvalues of matrix $R = \rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$ arranged in decreasing order. The symbol $*$ denotes the usual complex conjugate in the computational basis and σ_y is Pauli spin flip operator. The concurrence varies from 0 to 1 with minimum and maximum values corresponding to the separable and maximally entangled states respectively.

Measurement induced nonlocality: Next, we employ measurement-induced nonlocality as a second measure of quantum correlations (QC). It captures the maximal nonlocal effects of bipartite state due to locally invariant projective measurements, is defined as the maximal distance between the quantum state of consideration and the corresponding state after performing a local measurement on one of the subsystems i.e., [14]

$$N_2(\rho) = \max_{\Pi^a} \|\rho - \Pi^a(\rho)\|_2^2, \tag{2}$$

where $\|\mathcal{O}\|_2^2 = \text{Tr}(\mathcal{O}\mathcal{O}^\dagger)$ is Hilbert-Schmidt norm of operator \mathcal{O} and the maximum is taken over the von Neumann projective measurements on subsystem a which does not change the state. Mathematically it is defined as $\Pi^a(\rho) = \sum_k (\Pi_k^a \otimes \mathbb{1}^b)\rho(\Pi_k^a \otimes \mathbb{1}^b)$, with $\Pi^a = \{\Pi_k^a\} = \{|k\rangle\langle k|\}$, being the projective measurements on the subsystem a , which do not change the marginal state ρ^a locally i.e., $\Pi^a(\rho^a) = \rho^a$. If ρ^a is a non-degenerate, then the maximization is not required.

In general, the arbitrary two-qubit state in the Bloch representation can be written as

$$\rho = \frac{1}{2}X_0 \otimes Y_0 + \sum_{i=1}^3 x_i(X_i \otimes Y_0) + \sum_{j=1}^3 y_j(X_0 \otimes Y_j) + \sum_{i,j=1}^3 t_{ij}X_i \otimes Y_j \tag{3}$$

where $x_i = \text{Tr}(\rho(X_i \otimes Y_0))$, $y_j = \text{Tr}(\rho(X_0 \otimes Y_j))$ are the components of Bloch vector and $t_{ij} = \text{Tr}(\rho(X_i \otimes Y_j))$ being real matrix elements of correlation matrix T . In bipartite state space, the orthonormal operators in respective state spaces are $\{X_0, X_1, X_2, X_3\} = \{\mathbb{1}, \sigma_1, \sigma_2, \sigma_3\}/\sqrt{2}$ and $\{Y_0, Y_1, Y_2, Y_3\} = \{\mathbb{1}, \sigma_1, \sigma_2, \sigma_3\}/\sqrt{2}$, where σ_i are the Pauli matrices. MIN has a closed formula as

$$N_2(\rho) = \begin{cases} \text{Tr}(TT^t) - \frac{1}{\|\mathbf{x}\|^2}\mathbf{x}^t TT^t \mathbf{x} & \text{if } \mathbf{x} \neq 0, \\ \text{Tr}(TT^t) - \lambda_{\min} & \text{if } \mathbf{x} = 0, \end{cases} \tag{4}$$

where λ_{\min} is the least eigenvalue of matrix TT^t , the superscript t stands for the transpose and the vector $\mathbf{x} = (x_1, x_2, x_3)^t$.

Trace distance-based MIN (T-MIN): It is a well-known fact that the MIN based on the Hilbert-Schmidt norm is not a credible measure in capturing nonlocal attributes of a quantum state due to the local ancilla problem [29]. In order to resolve the local ancilla problem of Hilbert-Schmidt norm of measure [31], the MIN is defined by using other distance measures. One such measure is trace distance-based MIN and is defined as [30]

$$N_1(\rho) := \max_{\Pi^a} \|\rho - \Pi^a(\rho)\|_1, \tag{5}$$

where $\|A\|_1 = \text{Tr}\sqrt{A^\dagger A}$ is the trace norm of operator A . Here again, the maximum is taken over all von Neumann projective measurements. Without loss of generality, we can rewrite Eq. 3 as

$$\rho = \frac{1}{4} \left[\mathbb{1} \otimes \mathbb{1} + \sum_{i=1}^3 x_i(\sigma_i \otimes \mathbb{1}) + \sum_{j=1}^3 y_j(\mathbb{1} \otimes \sigma_j) + \sum_{i,j=1}^3 c_{ij}\sigma_i \otimes \sigma_j \right] \tag{6}$$

where $c_{ij} = \text{Tr}(\rho(\sigma_i \otimes \sigma_j))$. For the above system, the closed formula of T-MIN is given as

$$N_1(\rho) = \begin{cases} \frac{\sqrt{\chi_+} + \sqrt{\chi_-}}{2\|\mathbf{x}\|_1} & \text{if } \mathbf{x} \neq 0, \\ \max\{|c_1|, |c_2|, |c_3|\} & \text{if } \mathbf{x} = 0, \end{cases} \tag{7}$$

where $\chi_{\pm} = \alpha \pm 2\sqrt{\beta} \|\mathbf{x}\|_1$, $\alpha = \|\mathbf{c}\|_1^2 \|\mathbf{x}\|_1^2 - \sum_i c_i^2 x_i^2$, $\beta = \sum_{(ijk)} x_i^2 c_j^2 c_k^2$, $|c_i|$ are the absolute values of c_i and the summation runs over cyclic permutation of $\{1, 2, 3\}$.

The weak measurement is an important tool to characterize the quantum correlation and is studied recently [32, 33]. Let us briefly review the quantum correlation from the perspectives of weak measurements. The weak measurement operators are given by [32]

$$P(+)=t_1\Pi_1+t_2\Pi_2, \quad P(-)=t_2\Pi_1+t_1\Pi_2 \tag{8}$$

where

$$t_1 = \sqrt{\frac{1 - \tanh x}{2}} \quad \text{and} \quad t_2 = \sqrt{\frac{1 + \tanh x}{2}} \tag{9}$$

with x denoting the strength in the weak measurement process, Π_1 and Π_2 are two orthogonal projectors with $\Pi_1 + \Pi_2 = \mathbb{1}$. We can easily show that the weak measurement operators will reduce to orthogonal projectors when $x \rightarrow \infty$ and $P^\dagger(+)P(+) + P^\dagger(-)P(-) = \mathbb{1}$. The post-measurement state $\Omega(\rho)$ after the weak measurement is

$$\Omega(\rho) = P(+)\rho P(+) + P(-)\rho P(-). \tag{10}$$

The MIN based on the weak measurement is defined as

$$N_{pW}(\rho) := \max_{\Omega} \|\rho - \Omega(\rho)\|_p^p \tag{11}$$

where $\|\dots\|_p$ denotes the Schatten p -norm.

The actual MIN related with MIN based on the weak measurement is [34]

$$N_{2W}(\rho) = (1 - t_1 t_2) N_2(\rho). \tag{12}$$

Similarly, the trace MIN based on weak measurement is

$$N_{1W}(\rho) = (1 - t_1 t_2) N_1(\rho). \tag{13}$$

Next, we show that the dynamical behaviour of correlation measure can be extracted by the MIN with weak measurements.

3 The Model and Solution

To investigate the dynamics of standard quantum correlation measure and quantum correlation with weak measurements, we consider a dissipative model, namely two two-level atoms each being trapped separately in one of the two spatially separated thermal reservoirs. Using the standard quantum reservoir theory, time evolution of the density operator for the system after tracing out the reservoir obeys the following master equation

$$\begin{aligned} \dot{\rho} = \sum_{i=1,2} \left\{ -\frac{1}{2} \gamma_i (n_i + 1) \left[\sigma_+^i \sigma_-^i \rho - 2\sigma_-^i \rho \sigma_+^i + \rho \sigma_+^i \sigma_-^i \right] \right. \\ \left. - \frac{1}{2} \gamma_i n_i \left[\sigma_-^i \sigma_+^i \rho - 2\sigma_+^i \rho \sigma_-^i + \rho \sigma_-^i \sigma_+^i \right] \right\} \tag{14} \end{aligned}$$

where γ_i is the spontaneous emission rate of atom i , n_i are mean thermal photon numbers of their local thermal reservoirs separately and σ_+^i (σ_-^i) is the usual raising (lowering) operator of atom i . Here the dynamics of a quantum system depend on the initial states of the system and environment, the types of interaction between them.

Let us consider a mixture of the Bell state and an excited state as initial state to show the evolution of entanglement and MINs, that is

$$\rho(0) = (1 - r)|11\rangle\langle 11| + r|\Phi^+\rangle\langle \Phi^+| \tag{15}$$

where $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$ is maximally entangled state and $0 \leq r \leq 1$ determines the degree of purity of initial state. When $r = 0$, the initial state is $\rho(0) = |11\rangle\langle 11|$ a separable state and does not have any quantum advantage and $r = 1$ the initial state is pure and maximally entangled state. For simplicity, we set $n_1 = n_2 = n$ and $\gamma_1 = \gamma_2 = \gamma$. Then the solution of the master equation for the given initial condition is

$$\rho(t) = \begin{pmatrix} \rho_{11}(t) & 0 & 0 & 0 \\ 0 & \rho_{22}(t) & \rho_{23}(t) & 0 \\ 0 & \rho_{32}(t) & \rho_{33}(t) & 0 \\ 0 & 0 & 0 & \rho_{44}(t) \end{pmatrix} \tag{16}$$

where

$$\begin{aligned} \rho_{11}(t) &= \frac{1}{2(2n + 1)^2} \{2(n + 1)^2 + 2(n + 1)[r(2n + 1) - 2(n + 1)]e^{-(2n+1)\gamma t} \\ &\quad - [r(n + 1)(4n + 2) - 2(n + 1)^2]e^{-2(2n+1)\gamma t}\} \\ \rho_{22}(t) &= \rho_{33}(t) = \frac{1}{2(2n + 1)^2} \{2n(n + 1) - [r(2n + 1) - 2(n + 1)]e^{-(2n+1)\gamma t} \\ &\quad + [r(n + 1)(4n + 2) - 2(n + 1)^2]e^{-2(2n+1)\gamma t}\} \\ \rho_{44}(t) &= \frac{1}{2(2n + 1)^2} \{2n^2 - 2n[r(2n + 1) - 2(n + 1)]e^{-(2n+1)\gamma t} \\ &\quad - [r(n + 1)(4n + 2) - 2(n + 1)^2]e^{-2(2n+1)\gamma t}\} \\ \rho_{23}(t) &= \rho_{32}(t) = \frac{r}{2}e^{-(2n+1)\gamma t}. \end{aligned}$$

The entanglement and MINs of the time evolved state $\rho(t)$ is computed as

$$C(\rho) = 2 \max\{0, |\rho_{23}| - \sqrt{\rho_{11}\rho_{44}}\} \tag{17}$$

$$N_2(\rho) = 2 |\rho_{23}|^2 \quad \text{and} \quad N_1(\rho) = 2 |\rho_{23}|. \tag{18}$$

Substituting the matrix elements in the above equation, we obtain the entanglement and MIN of the time evolved state.

4 Quantum Correlation Dynamics of the Two Atom Systems

To study the dynamics of quantum correlations, we plot the entanglement measured by concurrence and the quantum correlation beyond entanglement quantified by different forms of MIN as a function of scaled time γt for the fixed mean thermal photon number in Fig. 1.

First, we set $r = 1$, then the initial state is pure and maximally entangled state. For $n = 1$, we observe that at time $t = 0$, the entanglement is maximum, as time increases the entanglement decreases monotonically and vanishes at a critical point. After the critical

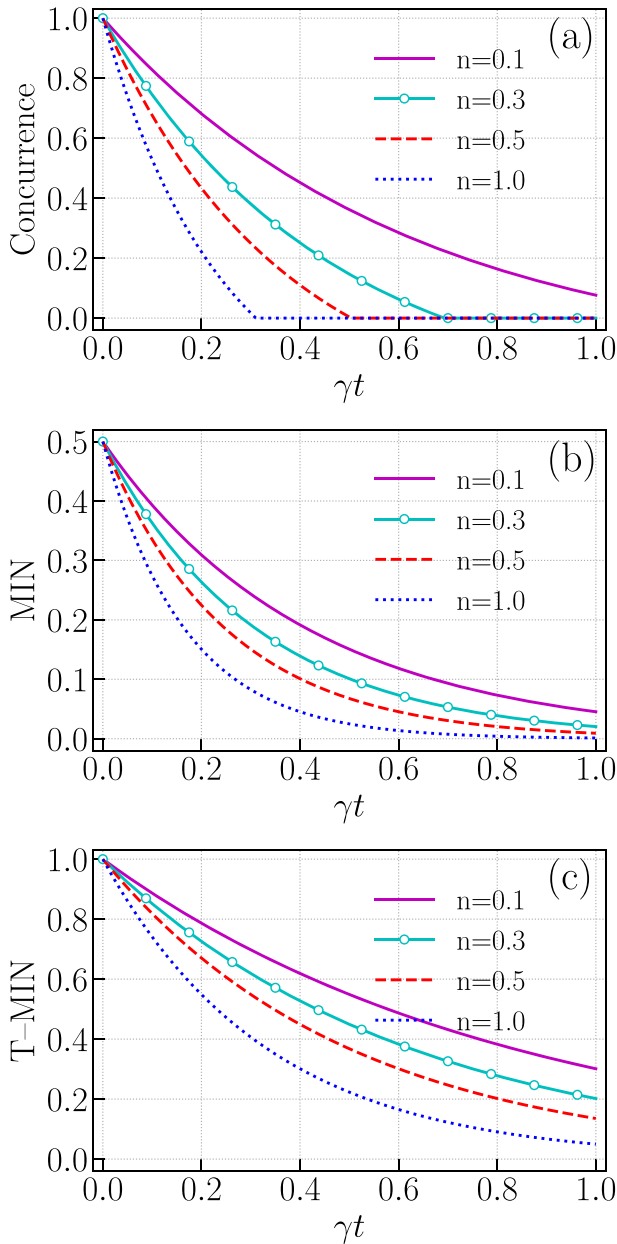


Fig. 1 (color online) (a) Entanglement, (b) Hilbert-Schmidt norm of MIN and (c) trace MIN of $\rho(t)$ for the different mean photon numbers n with $r = 1$

point, the entanglement between the qubits remains zero and this is known as sudden death of entanglement. On the other hand, both the quantities MIN and T-MIN are also maximum at $t = 0$, and further increases in time cause monotonic decrease in the quantum correlations. In contrast to the entanglement, the MIN is vanishing only at the asymptotic limit and the trace MIN persist for long time compared to entanglement and MIN. This is due to

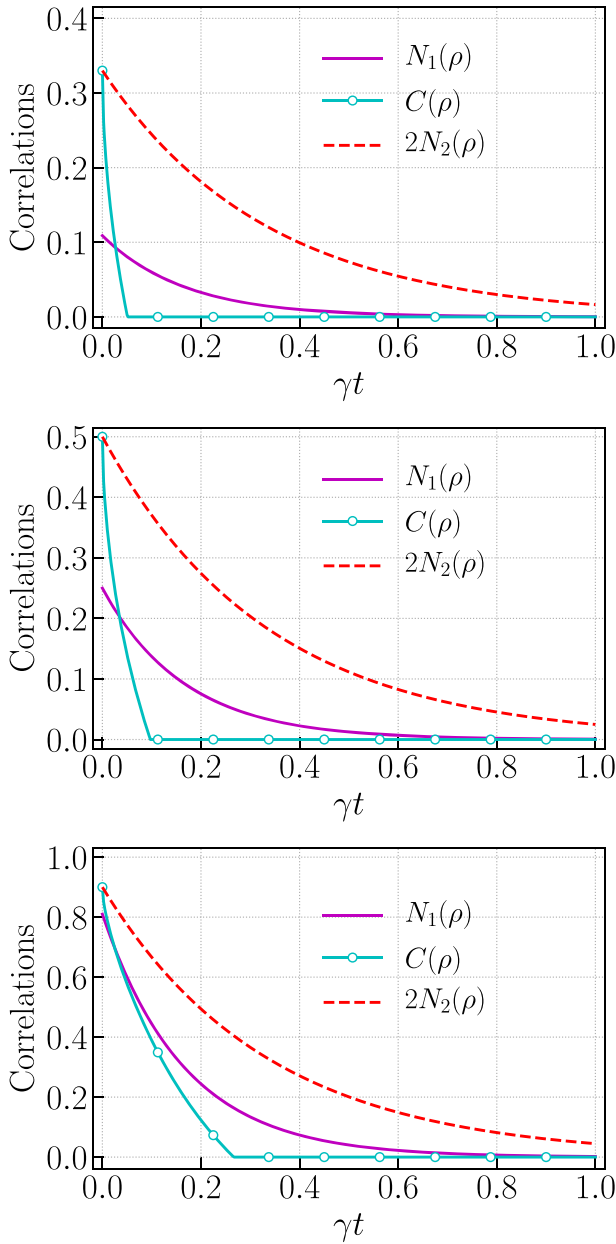


Fig. 2 (color online) Entanglement, $2N_2(\rho)$ and trace-MIN of $\rho(t)$ for (a) $r = 0.3$, (b) $r = 0.5$ and (c) $r = 0.9$. Here the mean photon number (n) is 0.1

the fact that the quantum system interaction with the dissipation environments. The above observation also highlights the robustness of MIN quantities. Further, it also emphasizes that an efficient quantum information processing based on the MIN offers more resistance to external perturbation.

To assert the prominent role of mean photon number, we plotted the entanglement and MINs for the different mean photon numbers such as $n = 1, 0.5, 0.3$ and 0.1 . From Fig. 1a, here also we observe that the entanglement suffers a sudden death at a certain time point in a thermal reservoir of nonzero mean photon number n . Further, we find that the time at which sudden death occurs increases with the decrease of mean photon number n . Here, it is obvious that the entanglement decreases with the increase of the mean photon number. On the other hand, in order to see how the dissipation affects these two quantum correlations (MIN and trace MIN) differently, we change the mean photon number in the thermal reservoirs. The decrease in mean photon number greatly enhances the strength of quantum correlation between the subsystems as shown in Fig. 1b and c.

To understand the effects of mixedness on quantum correlations, in Fig. (2) we have plotted the MIN and entanglement for the different values of r such as $r = 0.3, 0.5$, and 0.9 . For $0 < r < 1$, the state is minimally entangled state and all the correlation measures show monotonically decreasing behaviours with respect to time. It is also observed that the evolution of the entangled state under this channel exhibits entanglement sudden death that is influence of quantum noise reduces the entanglement to zero in finite time. As time increases the MIN and T-MIN decrease from a maximum value and shows that the quantum correlation vanishes asymptotically.

In order to understand the effect of weak measurements, we plot the MINs and MINs based on weak measurement for different weak measurement strength with the fixed values of mean photon numbers n and purity of the quantum state r in Fig. 3. We observe that both measures exhibit monotonically decreasing behaviors with respect to the scaled time for the fixed $n = r = 0.5$. Recently, Singh and Pati introduced super quantum discord (SQD) with weak measurement [32] and they have shown that for any state the SQD under

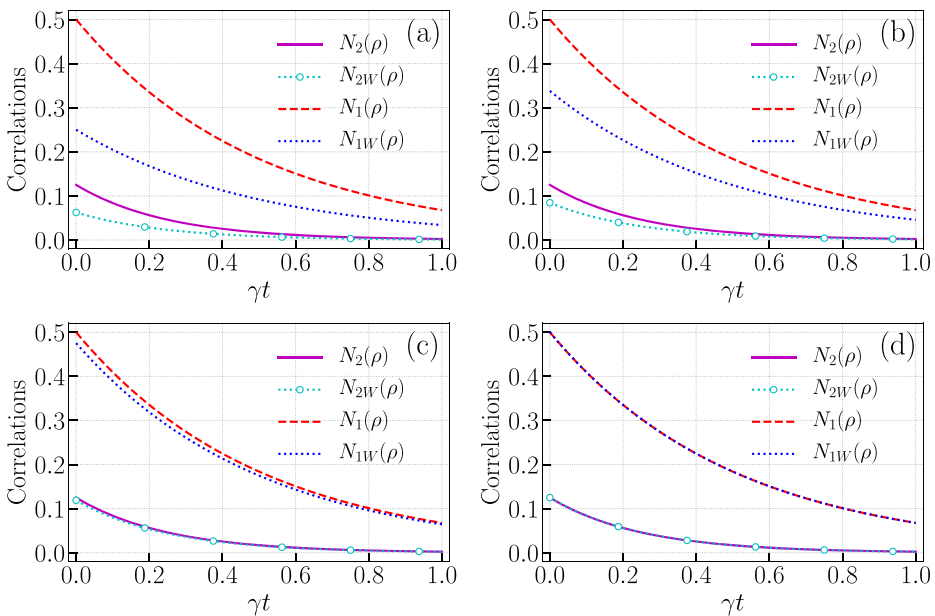


Fig. 3 (color online) MINs and Weak MINs of $\rho(t)$ for different weak measurement strengths (a) $x = 0.1$, (b) $x = 0.5$, (c) $x = 2$ and (d) $x = 50$ with the fixed parameters $n = 0.5$ and $r = 0.5$

weak measurement was always greater than quantum discord revealed by projective measurements. Here, one can see clearly for the given values of x that $N_{2W}(\rho) \leq N_2(\rho)$ and $N_{1W}(\rho) \leq N_1(\rho)$ which is in sharp contrast to the SQD [32]. On the other hand, this observation is consistent with the result presented in [34], where they proved that weak one-way deficit is smaller than one-way deficit for the given values of x . Further, the above results underscore the weak measurements are less invasive than projective measurements, so the Hilbert-Schmidt distance (trace distance) or von Neumann entropy from pre-measurement state to post-weak-measured state is less than that to post-projective-measured state.

While comparing Fig. 3, we observe that the weak MINs are monotonically increasing functions of the measurement strength x . On the other hand, one can show that SQD is a monotonically decreasing function of the measurement strength x . Hence, the above findings reveal that the SQD and weak MIN capture different nonlocal aspects of quantum system. Further, in the asymptotic limit $x \rightarrow \infty$, the MIN due to the eigenprojective measurements and weak measurement coincide with each other due to the fact that $\Pi^a(\rho) = \Omega(\rho)$.

5 Conclusions

In conclusion, we have studied the dynamics of quantum correlations of a two qubit system, each in its own thermal reservoir. Concurrence and measurement induced nonlocality are used to quantify the quantum correlations and show that the correlation measure exhibits monotonic decaying behavior with time. We observe that the entanglement suffers by sudden death while MIN quantities are more robust than entanglement. This observation implies that the presence of nonlocality (in terms of MIN) even in the absence of entanglement between the local constituents and MINs could capture more quantumness than the entanglement. In addition, we quantified the quantum correlation from the perspectives of weak measurements and also found that the weak MIN tends to increase as the measurement strength increase. It is shown that the MIN and weak MIN are identical in measuring the quantum correlation in the asymptotic limit.

Moreover, we emphasize that based on our observations, measurement-induced nonlocality offers more resistance to the effect of external perturbation and it is more robust than the entanglement.

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References

1. Einstein, A., Podolsky, B., Rosen, N.: Can quantum-mechanical description of physical reality be considered complete? *Phys. Rev.* **47**, 777 (1935)
2. Schrodinger, E.: Discussion of probability relations between separated systems. *Proc. Camb. Philos. Soc.* **31**, 555 (1935)
3. Nielsen, M., Chuang, I.: *Quantum Computation and Quantum Information*. Cambridge University Press, Cambridge (2010)
4. Knill, E., Laflamme, R.: Power of one bit of quantum information. *Phys. Rev. Lett.* **81**, 5672 (1998)
5. Datta, A., Flammia, S.T., Caves, C.M.: Entanglement and the power of one qubit. *Phys. Rev. A* **72**, 042316 (2005)

6. Datta, A., Shaji, A., Caves, C.M.: Quantum discord and the power of one qubit. *Phys. Rev. Lett.* **100**, 050502 (2008)
7. Henderson, L., Vedral, V.: Classical, quantum and total correlations. *J. Phys. A: Math. Theor.* **34**, 6899 (2001)
8. Bennett, C., DiVincenzo, D.: Quantum information and computation. *Nature* **404**, 247 (2000)
9. Breuer, H.P., Petruccione, F.: The theory of open quantum systems. Oxford University Press, Oxford (2002)
10. Zurek, W.H.: Decoherence, einselection, and the quantum origins of the classical. *Rev. Mod. Phys.* **75**, 715 (2003)
11. Ollivier, H., Zurek, W.H.: Quantum discord: a measure of the quantumness of correlations. *Phys. Rev. Lett.* **88**, 017901 (2001)
12. Luo, S.L., Fu, S.S.: Geometric measure of quantum discord. *Phys. Rev. A* **82**, 034302 (2010)
13. Luo, S.: Using measurement-induced disturbance to characterize correlations as classical or quantum. *Phys. Rev. A* **77**, 022301 (2008)
14. Luo, S., Fu, S.: Measurement-Induced Nonlocality. *Phys. Rev. Lett.* **106**, 120401 (2011)
15. Xiong, S., Zhang, W.J., Yu, C.S., Song, H.-S.: Uncertainty-induced quantum nonlocality. *Phys. Lett. A* **378**, 344 (2014)
16. Yi, X.X., Sun, C.P.: Factoring the unitary evolution operator and quantifying entanglement. *Phys. Lett. A* **262**, 287 (1999)
17. Rajagopal, A.K., Rendell, R.W.: Decoherence, correlation and entanglement in a pair of coupled dissipative oscillators. *Phys. Rev. A* **63**, 022116 (2001)
18. Zyczkowski, K., Horodecki, P., Horodecki, M., Horodecki, R.: Dynamics of quantum entanglement. *Phys. Rev. A* **65**, 012101 (2001)
19. Aharonov, Y., Albert, D.Z., Vaidman, L.: How the result of a measurement of a component of the spin of a spin-1/2 particle can turn out to be 100. *Phys. Rev. Lett.* **60**, 1351 (1988)
20. Oreshkov, O., Brun, T.A.: Weak measurements are universal. *Phys. Rev. Lett.* **95**, 110409 (2005)
21. Bai, M., Xu, H.J., Yan, X.Q.: Dynamics of the quantum discord with weak measurement for a two-atom system in thermal reservoirs. *Amer. J. Modern Phys.* **9**, 68 (2020)
22. Li, L., Wang, Q.W., Shen, S.Q., Li, M.: Geometric measure of quantum discord with weak measurements. *Quantum Inf Process* **15**, 291–300 (2016)
23. Yan, X.Q., Yue, Z.-L.: Correlated relation between quantum discord and entanglement of two-atom in thermal reservoirs. *J. Atomic Mol. Phys.*, 564805 (2014)
24. Hu, M.L.: Disentanglement, Bell-nonlocality violation and teleportation capacity of the decaying tripartite states. *Ann. Phys.* **327**, 2332 (2012)
25. Li, Z., Wang, X.-M., Zhou, W., Hu, M.L.: Geometric quantum discords of interacting qubits in thermal reservoir. *Sci. Rep.* **7**, 3342 (2017)
26. Modi, K., Brodutch, A., Cable, H., Paterek, T., Vedral, V.: The classical-quantum boundary for correlations: discord and related measures. *Rev. Mod. Phys.* **84**, 1655 (2012)
27. Ma, X.S., Cheng, M.-T., Yin, Q.: Dynamics of measurement-induced nonlocality of quantum states in a structured environment. *Int. J. Mod. Phys. B* **30**, 1650260 (2016)
28. Hill, S., Wootters, W.K.: Entanglement of a pair of quantum bits. *Phys. Rev. Lett.* **78**, 5022 (1997)
29. Chang, L., Luo, S.: Remedying the local ancilla problem with geometric discord. *Phys. Rev. A* **87**, 062303 (2013)
30. Hu, M.-L., Fan, H.: Measurement-induced nonlocality based on the trace norm. *J. Phys.* **17**, 033004 (2015)
31. Piani, M.: Problem with geometric discord. *Phys. Rev. A* **86**, 034101 (2012)
32. Singh, U., Pati, A.: Super quantum discord with weak measurements. *Ann. Phys.* **343**, 141–152 (2014)
33. Hu, M.L., Fan, H., Tian, D.P.: Role of weak measurements on states ordering and monogamy of quantum correlation. *Int. J. Theor. Phys.* **54**, 62–71 (2015)
34. Li, L., Wang, Q.-W., Shen, S.-Q., Li, M.: Geometric measure of quantum discord with weak measurements. *Quantum Inf. Process.* **15**, 291 (2016)

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