

Article

The First Rational Type Revised Fuzzy-Contractions in Revised Fuzzy Metric Spaces with an Applications

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Abstract: This paper aims to introduce the concept of rational type revised fuzzy-contraction mappings in revised fuzzy metric spaces. Fixed point results are proven under the rational type revised fuzzy-contraction conditions in revised fuzzy metric spaces with illustrative examples provided to support the results. A significant role will be played by this new concept in the theory of revised fuzzy fixed point results, and it can be generalized for different contractive type mappings in the context of revised fuzzy metric spaces. Additionally, an application of a nonlinear integral type equation is presented to obtain the existing result in a unique solution to support the work.

Keywords: t-conorm; Revised fuzzy metric space; rational type contraction; fixed point

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1. Introduction

In the year 2018, Alexander Sostak [1–3] introduced the idea of revised fuzzy metrics, which allow for the progressive evaluation of an element's inclusion in a collection. Revised fuzzy contraction mappings were described by Muraliraj and Thangathamizh [4–7], and the existence of fixed points was established for it. Cone Revised fuzzy metric space and revised fuzzy modular metric space are also specified. Numerous general topology ideas and findings were subsequently applied to the revised fuzzy topological space.

It is well-known that GV-fuzzy metrics are non-decreasing in the third variable. From here, or independently, by analyzing the definition of an RGV-fuzzy metric, we conclude that RGV-fuzzy metrics are non-increasing in the third variable. This allows us to give the following visual interpretation of an RGV-fuzzy metric. Assume that we are looking from a distance ($t \in (0, +\infty)$) at a plane filled up with pixels. We estimate the distance between pixels x and y by means of an RGV-fuzzy metric $\mathcal{U}_t(p, q, t)$. Being close to the plane, we see quite clearly how far the two pixels x and y are. However, going further from the plane, our ability to distinguish the real distance between different pixels becomes weaker, and, at some moment, two different pixels can merge into one in our eye-pupil.

RGV-fuzzy metrics are equivalent to GV-fuzzy metrics; the theories based on these concepts are equivalent. The difference is in the definitions, the proofs, and the interpretations of results. In particular, in the case of revised fuzzy metrics, we have the natural interpretation of the standard situation: the longer the segments of two infinite words taken

into consideration, the more precise the obtained information about the closeness of the two words.

The concept of an intuitionistic fuzzy metric on a set X used two functions $M, N : X^2 \times (0, +\infty) \rightarrow (0, 1]$ satisfying inequality $M(x, y, t) + N(x, y, t) \leq 1$ for all $x, y \in X, t > 0$. The first one of these functions, $M(x, y, t)$, describes the degree of nearness, while $N(x, y, t)$ describes the degree of non-nearness of points x, y on the level t . So, actually, M in definition is an ordinary GV-fuzzy metric, and therefore, it is based on the use of a t-norm $*$. On the other hand, function N , which in some sense complements function M , is based on a t-conorm $\Diamond$ (that is, probably, unrelated to the t-norm $*$). In contrast to the case of an intuitionistic fuzzy metric, we, when defining an RGV-fuzzy metric, started with a “classic” GV-metric and just reformulated the axioms from [3] by using involution. So, in our approach, a t-conorm $\oplus$ in the definition of a fuzzy metric is used to evaluate the degree of nearness of two points, and hence, it is opposite to the role of a t-conorm in the definition of an intuitionistic fuzzy metric.

The following articles [2–5,8–21] contain some triangular characteristic and integral type application findings in the theory of fixed point.

The aim of this research is to introduce the concept of rational type revised fuzzy-contraction mappings in G-complete RFM-spaces. This new theory is crucial in the study of revised fuzzy fixed point results and can be generalized for various contractive type mappings in the context of revised fuzzy metric spaces. Additionally, an integral type application is presented in the space, and a result is proved for a unique solution to support the work. The application section of the paper is of utmost importance as this concept can be utilized to present different types of nonlinear integral equations for the existence of unique solutions for their results.

2. Preliminaries

Definition 1 [1]. A binary operation of the form $\oplus : [0, 1]^2 \rightarrow [0, 1]$ is said to be a t-conorm if it satisfies the following conditions:

- (a) $\oplus$ is associative and commutative, continuous.
- (b) $p \oplus 0 = p$, for all $p \in [0, 1]$,
- (c) $p \oplus q \leq u \oplus v$. Whenever, $p \leq u$ and $q \leq v$. For all $p, q, u, v \in [0, 1]$.

Example 1 [1].

- i. Lukasiewicz t-conorm: $p \oplus q = \max\{p, q\}$,
- ii. Product t-conorm: $p \oplus q = p + q - pq$,
- iii. Minimum t-conorm:

$$p \oplus q = \min(p + q, 1) \quad (1)$$

Definition 2 [1]. Let $\mathfrak{M}$ be a set and $\oplus : [0, 1]^2 \rightarrow [0, 1]$ is a continuous t-conorm. A Revised fuzzy metric or an (shortly, RFM), on the set $\mathfrak{M}$ is a pair $(\mathfrak{U}_h, \oplus)$ or simply $\mathfrak{U}_h$, where the mapping $\mathfrak{U}_h : \mathfrak{M} \times \mathfrak{M} \rightarrow [0, 1]$ satisfying the following conditions, for all $p, q, u \in \mathfrak{M}$ and $t, s > 0$,

- (RF 1) $\mathfrak{U}_h(p, q, t) < 1$, for all $t > 0$
- (RF 2) $\mathfrak{U}_h(p, q, t) = 0 \iff p = q > 0$
- (RF 3) $\mathfrak{U}_h(p, q, t) = \mathfrak{U}_h(q, p, t)$
- (RF 4) $\mathfrak{U}_h(p, q, t + s) \leq \mathfrak{U}_h(p, u, t) \oplus \mathfrak{U}_h(u, q, s)$
- (RF 5) $\mathfrak{U}_h(p, q, -) : (0, \infty) \rightarrow [0, 1]$ is right continuous. Then, $(\mathfrak{U}_h, \oplus)$ is said to be a

Revised fuzzy metric on $\mathfrak{M}$.

Definition 3 [6]. Let the triple $(\mathfrak{M}, \mathfrak{U}_h, \oplus)$ be a RFM-space and $\mathcal{G} : \mathfrak{M} \rightarrow \mathfrak{M}$. Then, $\mathcal{G}$ known as a revised fuzzy contractive, if there is $0 < \mathcal{M} < 1$ so that for all $t > 0, j \geq 1$.

$$\mathfrak{U}_h(p_j, p_{j+1}, t) \leq \mathcal{M}(\mathfrak{U}_h(p_{j-1}, p_j, t)) \quad (2)$$

Definition 4 [8]. Let the triple $(\mathfrak{M}, \mathfrak{U}_h, \oplus)$ be an RFM-space and the $\mathfrak{U}_h$ is triangular if

$$\mathfrak{U}_h(p, q, t) \leq \mathfrak{U}_h(p, u, t) + \mathfrak{U}_h(u, q, t) \quad (3)$$

for all $p, q, u \in \mathfrak{M}$ and $t > 0, j \geq 1$.

Definition 5 [6]. Let the triple $(\mathfrak{M}, \mathfrak{U}_h, \oplus)$ be an RFM-space and $\mathcal{G} : \mathfrak{M} \rightarrow \mathfrak{M}$. Then, $\mathcal{G}$ known as a revised fuzzy contraction, if $0 < m < 1$ so that for all $t > 0$, for all $p, q \in \mathfrak{M}$.

$$\mathfrak{U}_h(\mathcal{G}p, \mathcal{G}q, t) \leq m(\mathfrak{U}_h(p, q, t)) \quad (4)$$

Lemma 1 [6]. Let the triple $(\mathfrak{M}, \mathfrak{U}_h, \oplus)$ be an RFM-space and let a sequence $\{p_j\}$ in $\mathfrak{M}$ converge to a point $\omega_1 \in \mathfrak{M}$ iff $\mathfrak{U}_h(p_j, \omega_1, t) \rightarrow 0$, as $j \rightarrow \infty$, for $t > 0$.

Definition 6. Consider a nonempty $\mathfrak{M}$ and a mapping $\mathfrak{U}_h : [\mathfrak{M}]^2 \times [0, \infty) \rightarrow [0, 1]$. Define a set

$$\mathbb{X}(\mathfrak{U}_h, \mathfrak{M}, p) = \left\{ \{p_n\} \subset \mathfrak{M} : \lim_{n \rightarrow \infty} \mathfrak{U}_h(p_n, p, t) = 0, \text{ for all } t > 0 \right\}$$

for every $p \in \mathfrak{M}$ then $\mathfrak{U}_h$ is said to be generalized revised fuzzy metric (shortly, $\mathbb{G}$ -RFM) for all $p, q, u \in \mathfrak{M}$ and $t, s > 0$, it satisfies the following conditions:

- ($\mathbb{G}\mathcal{RF}$ 1) $\mathfrak{U}_h(p, q, t) < 1$
- ($\mathbb{G}\mathcal{RF}$ 2) $\mathfrak{U}_h(p, q, t) = 0 \implies p = q$
- ($\mathbb{G}\mathcal{RF}$ 3) $\mathfrak{U}_h(p, q, t) = \mathfrak{U}_h(q, p, t)$
- ($\mathbb{G}\mathcal{RF}$ 4) there exist $\mathfrak{a} \geq 1$ such that if $\{p_n\} \in \mathbb{X}(\mathfrak{U}_h, \mathfrak{M}, p)$ then

$$\mathfrak{U}_h(p, q, t) \leq \liminf_{n \rightarrow \infty} \mathfrak{U}_h\left(p_n, q, \frac{t}{\mathfrak{a}}\right)$$

($\mathbb{G}\mathcal{RF}$ 5) $\mathfrak{U}_h(p, q, -) : (0, \infty) \rightarrow [0, 1]$ is continuous and $\lim_{n \rightarrow \infty} \mathfrak{U}_h(p_n, q, t) = 0$. Then, $(\mathfrak{M}, \mathfrak{U}_h, \oplus)$ is said to be Generalized revised fuzzy metric space (shortly $\mathbb{G}$ -RFMS).

Example 2. Consider a generalized metric space $(\mathfrak{U}_h, d)$. Define a mapping $\mathfrak{U}_h : [\mathfrak{M}]^2 \times (0, \infty) \rightarrow [0, 1]$ by $\mathfrak{U}_h(p, q, t) = e^{-\frac{d(p, q)}{t}(e^{\frac{d(p, q)}{t}} - 1)}$ and $\mathbb{X}(\mathfrak{U}_h, \mathfrak{M}, p) = \left\{ \{p_n\} \subset \mathfrak{M} : \lim_{n \rightarrow \infty} \mathfrak{U}_h(p_n, p, t) = 0 \right\}$ for every $p \in \mathfrak{M}$ and $t > 0$. Then, $(\mathfrak{M}, \mathfrak{U}_h, \oplus)$ is Generalized fuzzy metric space ($\mathbb{G}$ -RFMS), where the t -conorm " $\oplus$ " is taken as product norm. i.e., $p \oplus q = p + q - pq$.

Proposition 1. Every revised fuzzy metric space $(\mathfrak{M}, \mathfrak{U}_h, \oplus)$ is a generalized revised fuzzy metric space ($\mathbb{G}$ -RFMS).

Definition 7. Let $(\mathfrak{M}, \mathfrak{U}_h, \oplus)$ be a generalized revised fuzzy metric space ($\mathbb{G}$ -RFMS). A sequence $\{p_n\}$ in $\mathfrak{M}$ is said to be $\mathbb{G}$ -convergent sequence if $p \in \mathfrak{M}$, $\{p_n\} \in \mathbb{X}(\mathfrak{U}_h, \mathfrak{M}, p)$.

Definition 8. Let $(\mathfrak{M}, \mathfrak{U}_h, \oplus)$ be a generalized revised fuzzy metric space ($\mathbb{G}$ -RFMS). A sequence $\{p_n\}$ in $\mathfrak{M}$ is said to be $\mathbb{G}$ -Cauchy sequence if $\lim_{n \rightarrow \infty} \mathfrak{U}_h(p_n, p_{n+m}, t) = 0$ for all $t \geq 0$.

Definition 9. A generalized revised fuzzy metric space in which every $\mathbb{G}$ -Cauchy sequence is $\mathbb{G}$ -convergent is called a $\mathbb{G}$ -complete generalized revised fuzzy metric space (shortly, $\mathbb{G}$ -complete RFM-space).

3. Main Results

In this section, we define rational type revised fuzzy-contraction maps and prove some unique fixed-point theorems under the rational type revised fuzzy-contraction mappings in $\mathbb{G}$ -complete RFM-spaces.

Definition 10. Let the triple $(\mathfrak{M}, \mathfrak{U}_h, \oplus)$ be an RFM-space; a mapping $\mathcal{G} : \mathfrak{M} \rightarrow \mathfrak{M}$ is said to be a rational type revised fuzzy-contraction if for all $m, n \in [0, 1)$, such that

$$\mathfrak{U}_h(\mathcal{G}p, \mathcal{G}q, t) \leq m(\mathfrak{U}_h(p, q, t)) + n \left(\frac{\mathfrak{U}_h(p, \mathcal{G}p, t) \oplus \mathfrak{U}_h(q, \mathcal{G}p, 2t)}{\mathfrak{U}_h(p, q, t)} \right) \quad (5)$$

for all $t > 0, p, q \in \mathfrak{M}$.

Theorem 1. Let the triple $(\mathfrak{M}, \mathfrak{U}_h, \oplus)$ be a $\mathbb{G}$ -complete RFM-space in which $\mathfrak{U}_h$ is triangular and a mapping $\mathcal{G} : \mathfrak{M} \rightarrow \mathfrak{M}$ is said to be a rational type revised fuzzy-contraction satisfying (5) with $m + n < 1$. Then, $\mathcal{G}$ has a fixed point in $\mathfrak{M}$.

Proof. Let $p_j \in \mathfrak{M}$ and $p_{j+1} = \mathcal{G}p_j, j \geq 0$. Then, by (5), for $t > 0, j \geq 0$,

$$\begin{aligned} \mathfrak{U}_h(p_j, p_{j+1}, t) &= (\mathfrak{U}_h(\mathcal{G}p_{j-1}, \mathcal{G}p_j, t)) \\ &\leq m(\mathfrak{U}_h(p_{j-1}, p_j, t)) + n \left(\frac{\mathfrak{U}_h(p_{j-1}, \mathcal{G}p_{j-1}, t) \oplus \mathfrak{U}_h(p_j, \mathcal{G}p_{j-1}, 2t)}{\mathfrak{U}_h(p_{j-1}, p_j, t)} \right) \\ &= m(\mathfrak{U}_h(p_{j-1}, p_j, t)) + n \left(\frac{\mathfrak{U}_h(p_{j-1}, p_j, t) \oplus \mathfrak{U}_h(p_j, p_j, 2t)}{\mathfrak{U}_h(p_{j-1}, p_j, t)} \right) \end{aligned} \quad (6)$$

and after being simplified,

$$\mathfrak{U}_h(p_j, p_{j+1}, t) \leq m(\mathfrak{U}_h(p_{j-1}, p_j, t)), \text{ for } t > 0. \quad (7)$$

$$\text{i.e., } \mathfrak{U}_h(p_{j-1}, p_j, t) \leq m(\mathfrak{U}_h(p_{j-2}, p_{j-1}, t)), \text{ for } t > 0. \quad (8)$$

Now, by inference, for $t > 0$, we have that from (7) and (8).

$$\mathfrak{U}_h(p_j, p_{j+1}, t) \leq \left\{ m(\mathfrak{U}_h(p_{j-1}, p_j, t)) \leq m^2(\mathfrak{U}_h(p_{j-2}, p_{j-1}, t)) \right\} \rightarrow 0, \text{ as } j \rightarrow \infty. \quad (9)$$

Consequently, Revised fuzzy contractive sequence in $(\mathfrak{M}, \mathfrak{U}_h, \oplus)$ is represented by $\{p_j\}$, then,

$$\lim_{j \rightarrow \infty} \mathfrak{U}_h(p_j, p_{j-1}, t) = 0, \text{ for } t > 0. \quad (10)$$

We now demonstrate that $\{p_j\}$ is a $\mathbb{G}$ -Cauchy sequence, assuming that $j \in \mathbb{N}$, and that there exists a fixed $q \in \mathbb{N}$, such that

$$\mathfrak{U}_h(p_j, p_{j+q}, t) \leq \mathfrak{U}_h\left(p_j, p_{j+1}, \frac{t}{q}\right) \oplus \mathfrak{U}_h\left(p_{j+1}, p_{j+2}, \frac{t}{q}\right) \oplus \dots \oplus \mathfrak{U}_h\left(p_{j+q-1}, p_{j+q}, \frac{t}{q}\right) \rightarrow 0 \oplus 0 \oplus \dots \oplus 0 = 0, \text{ as } j \rightarrow \infty. \quad (11)$$

Thus, it is established that the sequence $\{p_j\}$ is a $\mathbb{G}$ -Cauchy. Given that $(\mathfrak{M}, \mathfrak{U}_h, \oplus)$ is $\mathbb{G}$ -complete, for all $\omega_1 \in \mathfrak{M}$ such that $p_j \rightarrow \omega_1$, as $j \rightarrow \infty$,

$$\lim_{j \rightarrow \infty} \mathfrak{U}_h(p_j, \omega_1, t) = 0, \text{ for } t > 0. \quad (12)$$

Since $\mathfrak{U}_h$ is triangular, we can derive $t > 0$ from (5), (10), and (12),

$$\begin{aligned}
& \mathfrak{U}_h(\omega_1, \mathcal{G}\omega_1, t) \leq \mathfrak{U}_h(\omega_1, p_{j+1}, t) + \mathfrak{U}_h(\mathcal{G}p_j, \mathcal{G}\omega_1, t) \\
& \leq \mathfrak{U}_h(\omega_1, p_{j+1}, t) + m(\mathfrak{U}_h(p_j, \omega_1, t)) + n\left(\frac{\mathfrak{U}_h(p_j, \mathcal{G}p_j, t) \oplus \mathfrak{U}_h(\omega_1, \mathcal{G}p_j, 2t)}{\mathfrak{U}_h(p_j, \omega_1, t)}\right) \\
& = \left\{ \begin{aligned} & \mathfrak{U}_h(\omega_1, p_{j+1}, t) + m(\mathfrak{U}_h(p_j, \omega_1, t)) \\ & + n\left(\frac{\mathfrak{U}_h(p_j, p_{j+1}, t) \oplus \mathfrak{U}_h(\omega_1, p_{j+1}, 2t)}{\mathfrak{U}_h(p_j, \omega_1, t)}\right) \end{aligned} \right\} \longrightarrow 0, \text{ as } j \rightarrow \infty.
\end{aligned} \tag{13}$$

$$\begin{aligned}
& \mathfrak{U}_h(\omega_1, s_1, t) = \mathfrak{U}_h(\mathcal{G}\omega_1, \mathcal{G}s_1, t) \\
& \leq m(\mathfrak{U}_h(\omega_1, s_1, t)) + n\left(\frac{\mathfrak{U}_h(\omega_1, \mathcal{G}\omega_1, t) \oplus \mathfrak{U}_h(s_1, \mathcal{G}\omega_1, 2t)}{\mathfrak{U}_h(\omega_1, s_1, t)}\right) \\
& \leq m(\mathfrak{U}_h(\omega_1, s_1, t)) + n\left(\frac{\mathfrak{U}_h(\omega_1, \omega_1, t) \oplus \mathfrak{U}_h(s_1, \omega_1, t)}{\mathfrak{U}_h(\omega_1, s_1, t)}\right) \\
& = m(\mathfrak{U}_h(\omega_1, s_1, t)) = m(\mathfrak{U}_h(\mathcal{G}\omega_1, \mathcal{G}s_1, t)) \\
& \leq m^2(\mathfrak{U}_h(\omega_1, s_1, t)) \leq \dots \leq m^j(\mathfrak{U}_h(\omega_1, s_1, t)) \longrightarrow 0, \text{ as } j \rightarrow \infty.
\end{aligned} \tag{14}$$

Thus, it is established that $\mathfrak{U}_h(\omega_1, s_1, t) = 0 \implies \omega_1 = s_1$. $\square$

Corollary 1. (Revised fuzzy Banach contraction principle).

Let $(\mathfrak{M}, \mathfrak{U}_h, \oplus)$ be a $\mathbb{G}$ -complete RFM-space in which $\mathfrak{U}_h$ is triangular and a mapping $\mathcal{G} : \mathfrak{M} \longrightarrow \mathfrak{M}$ is a revised fuzzy-contraction satisfying (4) with $m \in (0, 1)$. Then, $\mathcal{G}$ has a unique fixed point in $\mathfrak{M}$.

Example 3. Let $\mathfrak{M} = [0, \infty)$, $\oplus$ be a continuous t -conorm, and $\mathfrak{U}_h : \mathfrak{M}^2 \times (0, \infty) \longrightarrow [0, 1]$ be defined as

$$\mathfrak{U}_h(p, q, t) = \frac{|(4p - 4q)/5|}{t + |(4p - 4q)/5|}, \text{ for all } p, q \in \mathfrak{M}, t > 0. \tag{15}$$

The one can easily verify that $\mathfrak{U}_h$ is triangular and $(\mathfrak{M}, \mathfrak{U}_h, \oplus)$ is a $\mathbb{G}$ -complete RFM space. Now we define a mapping $\mathcal{G} : \mathfrak{M} \longrightarrow \mathfrak{M}$ as

$$\mathcal{G}(p) = \begin{cases} \frac{3p}{4}, & \text{if } p \in [0, 1], \\ \frac{2p}{4} + 8, & \text{if } p \in (0, \infty). \end{cases} \tag{16}$$

Then, we have

$$\mathfrak{U}_h(\mathcal{G}p, \mathcal{G}q, t) = \frac{3}{4}(\mathfrak{U}_h(p, q, t)), \text{ for all } p, q \in \mathfrak{M}, t > 0. \tag{17}$$

Hence, a mapping $\mathcal{G}$ is a revised fuzzy contraction. Now, from Example 1 (iii), for $t > 0$,

$$\begin{aligned}
& \frac{\mathfrak{U}_h(p, \mathcal{G}p, t) \oplus \mathfrak{U}_h(q, \mathcal{G}p, 2t)}{\mathfrak{U}_h(p, q, t)} \leq \frac{\mathfrak{U}_h(p, \mathcal{G}p, t) \oplus \mathfrak{U}_h(q, p, t) \oplus \mathfrak{U}_h(p, \mathcal{G}p, t)}{\mathfrak{U}_h(p, q, t)} \\
& = \mathfrak{U}_h(p, \mathcal{G}p, t) \oplus \mathfrak{U}_h(p, \mathcal{G}p, t) = \mathfrak{U}_h(p, \mathcal{G}p, t)^2 = \frac{2p}{5t^2} \left(\frac{p}{5} + t \right)
\end{aligned} \tag{18}$$

Hence, all the conditions of Theorem 1 are satisfied with $m = \frac{3}{4}$, $n = \frac{2}{9}$.

A mapping $\mathcal{G}$ has a fixed point. i.e., $\mathcal{G}(24) = 24 \in [0, \infty)$.

Now, we prove a generalized rational type revised fuzzy contraction theorem.

Theorem 2. Let $(\mathfrak{M}, \mathfrak{U}_h, \oplus)$ is a $\mathbb{G}$ -complete RFM-space. Which $\mathfrak{U}_h$ is triangular and a mapping $\mathcal{G} : \mathfrak{M} \rightarrow \mathfrak{M}$ satisfies

$$\mathfrak{U}_h(\mathcal{G}p, \mathcal{G}q, t) \leq \left\{ \begin{array}{l} m(\mathfrak{U}_h(p, q, t)) + n\left(\frac{\mathfrak{U}_h(p, \mathcal{G}p, t) \oplus \mathfrak{U}_h(p, \mathcal{G}q, 2t)}{\mathfrak{U}_h(p, q, t) \oplus \mathfrak{U}_h(q, \mathcal{G}q, t)}\right) \\ + \mathfrak{k}\left(\frac{\mathfrak{U}_h(p, \mathcal{G}q, 2t)}{\mathfrak{U}_h(p, \mathcal{G}p, t)} + \frac{\mathfrak{U}_h(p, \mathcal{G}q, 2t)}{\mathfrak{U}_h(q, \mathcal{G}q, t)}\right) + \ell(\mathfrak{U}_h(p, \mathcal{G}p, t) + \mathfrak{U}_h(q, \mathcal{G}q, t)) \end{array} \right\} \quad (19)$$

for all $p, q \in \mathfrak{M}, t > 0$ and $m, n, \mathfrak{k}, \ell \geq 0$ with $m + n + 2\mathfrak{k} + 2\ell < 1$. Then, $\mathcal{G}$ has a unique fixed point.

Proof. Let $p_j \in \mathfrak{M}$ and $p_{j+1} = \mathcal{G}p_j, j \geq 0$. Then, by (19), for $t > 0, j \geq 0$,

$$\begin{aligned} \mathfrak{U}_h(p_j, p_{j+1}, t) &= (\mathfrak{U}_h(\mathcal{G}p_{j-1}, \mathcal{G}p_j, t)) \\ &\leq \left\{ \begin{array}{l} m(\mathfrak{U}_h(p_{j-1}, p_j, t)) + n\left(\frac{\mathfrak{U}_h(p_{j-1}, \mathcal{G}p_{j-1}, t) \oplus \mathfrak{U}_h(p_{j-1}, \mathcal{G}p_j, 2t)}{\mathfrak{U}_h(p_{j-1}, p_j, t) \oplus \mathfrak{U}_h(p_j, \mathcal{G}p_j, t)}\right) \\ + \mathfrak{k}\left(\frac{\mathfrak{U}_h(p_{j-1}, \mathcal{G}p_j, 2t)}{\mathfrak{U}_h(p_{j-1}, \mathcal{G}p_{j-1}, t)} + \frac{\mathfrak{U}_h(p_{j-1}, \mathcal{G}p_j, 2t)}{\mathfrak{U}_h(p_j, \mathcal{G}p_j, t)}\right) + \ell(\mathfrak{U}_h(p_{j-1}, \mathcal{G}p_{j-1}, t) + \mathfrak{U}_h(p_j, \mathcal{G}p_j, t)) \end{array} \right\} \\ &= \left\{ \begin{array}{l} m(\mathfrak{U}_h(p_{j-1}, p_j, t)) + n\left(\frac{\mathfrak{U}_h(p_{j-1}, p_{j+1}, t) \oplus \mathfrak{U}_h(p_{j-1}, p_{j+1}, 2t)}{\mathfrak{U}_h(p_{j-1}, p_j, t) \oplus \mathfrak{U}_h(p_j, p_{j+1}, t)}\right) \\ + \mathfrak{k}\left(\frac{\mathfrak{U}_h(p_{j-1}, p_{j+1}, 2t)}{\mathfrak{U}_h(p_{j-1}, p_j, t)} + \frac{\mathfrak{U}_h(p_{j-1}, p_{j+1}, 2t)}{\mathfrak{U}_h(p_j, p_{j+1}, t)}\right) + \ell(\mathfrak{U}_h(p_{j-1}, p_j, t) + \mathfrak{U}_h(p_j, p_{j+1}, t)) \end{array} \right\} \end{aligned} \quad (20)$$

By the Example 1 (iii), $\mathfrak{U}_h(p_{j-1}, p_{j+1}, 2t) \leq \mathfrak{U}_h(p_{j-1}, p_j, t) \oplus \mathfrak{U}_h(p_j, p_{j+1}, t)$, and after simplification, we have

$$\mathfrak{U}_h(p_j, p_{j+1}, t) \leq \varphi(\mathfrak{U}_h(p_{j-1}, p_j, t)), \text{ where } \varphi = \frac{m + n + \mathfrak{k} + \ell}{1 - \mathfrak{k} - \ell} < 1. \quad (21)$$

Similarly, for $t > 0$, we have

$$\mathfrak{U}_h(p_{j-1}, p_j, t) \leq \varphi(\mathfrak{U}_h(p_{j-2}, p_{j-1}, t)), \text{ where } \varphi = \frac{m + n + \mathfrak{k} + \ell}{1 - \mathfrak{k} - \ell} < 1. \quad (22)$$

Now, from (21) and (22) by induction, for $t > 0$, we have that

$$\mathfrak{U}_h(p_j, p_{j+1}, t) \leq \left\{ \begin{array}{l} \varphi(\mathfrak{U}_h(p_{j-1}, p_j, t)) \leq \varphi^2(\mathfrak{U}_h(p_{j-2}, p_{j-1}, t)) \\ \leq \dots \leq \varphi^j(\mathfrak{U}_h(p_0, p_1, t)) \end{array} \right\} \rightarrow 0, \quad (23)$$

As $j \rightarrow \infty$. Then, $\{p_j\}$ is revised fuzzy contractive sequence in $(\mathfrak{M}, \mathfrak{U}_h, \oplus)$; therefore,

$$\lim_{j \rightarrow \infty} \mathfrak{U}_h(p_j, p_{j+1}, t) = 0, \text{ for } t > 0. \quad (24)$$

Now, to prove that $\{p_j\}$ is a $\mathbb{G}$ -Cauchy sequence, let $j \in \mathbb{N}$, and there is a fixed $q \in \mathbb{N}$, such that

$$\begin{aligned} \mathfrak{U}_h(p_j, p_{j+q}, t) &\leq \mathfrak{U}_h\left(p_j, p_{j+1}, \frac{t}{q}\right) \oplus \mathfrak{U}_h\left(p_{j+1}, p_{j+2}, \frac{t}{q}\right) \oplus \dots \oplus \mathfrak{U}_h\left(p_{j+q-1}, p_{j+q}, \frac{t}{q}\right) \\ &\rightarrow 0 \oplus 0 \oplus \dots \oplus 0 = 0, \text{ as } j \rightarrow \infty. \end{aligned} \quad (25)$$

Hence, it is shows that $\{p_j\}$ is a $\mathbb{G}$ -Cauchy sequence. Since $(\mathfrak{M}, \mathfrak{U}_h, \oplus)$ is $\mathbb{G}$ -complete, for all $\omega_1 \in \mathfrak{M}$, such that $p_j \rightarrow \omega_1$, as $j \rightarrow \infty$,

$$\text{i.e., } \lim_{j \rightarrow \infty} \mathfrak{U}_h(p_j, \omega_1, t) = 0, \text{ for } t > 0. \quad (26)$$

Since $\mathfrak{U}_h$ is triangular,

$$\mathfrak{U}_h(\omega_1, \mathcal{G}\omega_1, t) \leq \mathfrak{U}_h(\omega_1, \mathfrak{p}_{j+1}, t) + \mathfrak{U}_h(\mathfrak{p}_{j+1}, \mathcal{G}\omega_1, t), \text{ for } t > 0. \quad (27)$$

Now, from (19), (24) and (26), for $t > 0$, we have

$$\begin{aligned} \mathfrak{U}_h(\mathfrak{p}_{j+1}, \mathcal{G}\omega_1, t) &= \mathfrak{U}_h(\mathcal{G}\mathfrak{p}_j, \mathcal{G}\omega_1, t) \\ &\leq \left\{ \begin{aligned} &m(\mathfrak{U}_h(\mathfrak{p}_j, \omega_1, t)) + n \left(\frac{\mathfrak{U}_h(\mathfrak{p}_j, \mathcal{G}\mathfrak{p}_j, t) \oplus \mathfrak{U}_h(\mathfrak{p}_j, \mathcal{G}\omega_1, 2t)}{\mathfrak{U}_h(\mathfrak{p}_j, \omega_1, t) \oplus \mathfrak{U}_h(\omega_1, \mathcal{G}\omega_1, t)} \right) \\ &+ \mathfrak{k} \left(\frac{\mathfrak{U}_h(\mathfrak{p}_j, \mathcal{G}\omega_1, 2t)}{\mathfrak{U}_h(\mathfrak{p}_j, \mathcal{G}\mathfrak{p}_j, t)} + \frac{\mathfrak{U}_h(\mathfrak{p}_j, \mathcal{G}\omega_1, 2t)}{\mathfrak{U}_h(\omega_1, \mathcal{G}\omega_1, t)} \right) + \ell(\mathfrak{U}_h(\mathfrak{p}_j, \mathcal{G}\mathfrak{p}_j, t) + \mathfrak{U}_h(\omega_1, \mathcal{G}\omega_1, t)) \end{aligned} \right\} \\ &\leq \left\{ \begin{aligned} &m(\mathfrak{U}_h(\mathfrak{p}_j, \omega_1, t)) + n \left(\frac{\mathfrak{U}_h(\mathfrak{p}_j, \mathfrak{p}_{j+1}, t) \oplus \mathfrak{U}_h(\mathfrak{p}_j, \mathcal{G}\omega_1, 2t)}{\mathfrak{U}_h(\mathfrak{p}_j, \omega_1, t) \oplus \mathfrak{U}_h(\omega_1, \mathcal{G}\omega_1, t)} \right) \\ &+ \mathfrak{k} \left(\frac{\mathfrak{U}_h(\mathfrak{p}_j, \mathcal{G}\omega_1, 2t)}{\mathfrak{U}_h(\mathfrak{p}_j, \mathfrak{p}_{j+1}, t)} + \frac{\mathfrak{U}_h(\mathfrak{p}_j, \mathcal{G}\omega_1, 2t)}{\mathfrak{U}_h(\omega_1, \mathcal{G}\omega_1, t)} \right) + \ell(\mathfrak{U}_h(\mathfrak{p}_j, \mathfrak{p}_{j+1}, t) + \mathfrak{U}_h(\omega_1, \mathcal{G}\omega_1, t)) \end{aligned} \right\} \end{aligned} \quad (28)$$

By the Example 1 (iii), $\mathfrak{U}_h(\mathfrak{p}_j, \mathcal{G}\omega_1, 2t) \leq \mathfrak{U}_h(\mathfrak{p}_j, \omega_1, t) \oplus \mathfrak{U}_h(\omega_1, \mathcal{G}\omega_1, t)$, and after simplification, we have

$$\begin{aligned} \mathfrak{U}_h(\mathfrak{p}_{j+1}, \mathcal{G}\omega_1, t) &\leq \left\{ \begin{aligned} &m(\mathfrak{U}_h(\mathfrak{p}_j, \omega_1, t)) \\ &+ n \left(\frac{\mathfrak{U}_h(\mathfrak{p}_j, \mathfrak{p}_{j+1}, t) \oplus \mathfrak{U}_h(\mathfrak{p}_j, \omega_1, t) \oplus \mathfrak{U}_h(\omega_1, \mathcal{G}\omega_1, t)}{\mathfrak{U}_h(\mathfrak{p}_j, \omega_1, t) \oplus \mathfrak{U}_h(\omega_1, \mathcal{G}\omega_1, t)} \right) \\ &+ \mathfrak{k} \left(\frac{\mathfrak{U}_h(\mathfrak{p}_j, \omega_1, t) \oplus \mathfrak{U}_h(\omega_1, \mathcal{G}\omega_1, t)}{\mathfrak{U}_h(\mathfrak{p}_j, \mathfrak{p}_{j+1}, t)} + \frac{\mathfrak{U}_h(\mathfrak{p}_j, \omega_1, t) \oplus \mathfrak{U}_h(\omega_1, \mathcal{G}\omega_1, 2t)}{\mathfrak{U}_h(\omega_1, \mathcal{G}\omega_1, t)} \right) \\ &+ \ell(\mathfrak{U}_h(\mathfrak{p}_j, \mathfrak{p}_{j+1}, t) + \mathfrak{U}_h(\omega_1, \mathcal{G}\omega_1, t)) \end{aligned} \right\} \\ &\rightarrow (\mathfrak{k} + \ell)(\mathfrak{U}_h(\omega_1, \mathcal{G}\omega_1, t)), \quad j \rightarrow \infty. \end{aligned} \quad (29)$$

Then,

$$\liminf_{n \rightarrow \infty} (\mathfrak{U}_h(\mathfrak{p}_{j+1}, \mathcal{G}\omega_1, t)) \leq (\mathfrak{k} + \ell)(\mathfrak{U}_h(\omega_1, \mathcal{G}\omega_1, t)), \text{ for } t > 0. \quad (30)$$

Now, from (26), (27), and (30), as $j \rightarrow \infty$, we get that

$$\mathfrak{U}_h(\mathfrak{p}_{j+1}, \mathcal{G}\omega_1, t) \leq (\mathfrak{k} + \ell)(\mathfrak{U}_h(\omega_1, \mathcal{G}\omega_1, t)), \text{ for } t > 0. \quad (31)$$

and $(\mathfrak{k} + \ell) < 1$, where $m + n + 2\mathfrak{k} + 2\ell < 1$, and hence $\mathfrak{U}_h(\omega_1, \mathcal{G}\omega_1, t) = 0$, i.e., $\mathcal{G}\omega_1 = \omega_1$, for $t > 0$.

Uniqueness. Let $\mathfrak{s}_1 \in \mathfrak{M}$, such that $\mathcal{G}\mathfrak{s}_1 = \mathfrak{s}_1$ and $\mathcal{G}\omega_1 = \omega_1$. Then, from (19) and Example 1 (iii), for $t > 0$, we have

$$\begin{aligned}
& \mathfrak{U}_h(\omega_1, s_1, t) = \mathfrak{U}_h(\mathcal{G}\omega_1, \mathcal{G}s_1, t) \\
& \leq \left\{ m(\mathfrak{U}_h(\omega_1, s_1, t)) + n \left(\frac{\mathfrak{U}_h(\omega_1, \mathcal{G}\omega_1, t) \oplus \mathfrak{U}_h(\omega_1, \mathcal{G}s_1, 2t)}{\mathfrak{U}_h(\omega_1, s_1, t) \oplus \mathfrak{U}_h(s_1, \mathcal{G}s_1, t)} \right) \right. \\
& \quad \left. + \mathcal{K} \left(\frac{\mathfrak{U}_h(\omega_1, \mathcal{G}s_1, 2t)}{\mathfrak{U}_h(\omega_1, \mathcal{G}\omega_1, t)} + \frac{\mathfrak{U}_h(\omega_1, \mathcal{G}s_1, 2t)}{\mathfrak{U}_h(s_1, \mathcal{G}s_1, t)} \right) + \ell(\mathfrak{U}_h(\omega_1, \mathcal{G}\omega_1, t) + \mathfrak{U}_h(s_1, \mathcal{G}s_1, t)) \right\} \\
& = \left\{ m(\mathfrak{U}_h(\omega_1, s_1, t)) + n \left(\frac{\mathfrak{U}_h(\omega_1, s_1, 2t)}{\mathfrak{U}_h(\omega_1, s_1, t)} \right) + \mathcal{K}(\mathfrak{U}_h(\omega_1, s_1, 2t) + \mathfrak{U}_h(s_1, s_1, 2t)) \right\} \\
& = \left\{ m(\mathfrak{U}_h(\omega_1, s_1, t)) + n \left(\frac{\mathfrak{U}_h(\omega_1, s_1, t) \oplus \mathfrak{U}_h(s_1, s_1, t)}{\mathfrak{U}_h(\omega_1, s_1, t)} \right) \right. \\
& \quad \left. + \mathcal{K}(\mathfrak{U}_h(\omega_1, s_1, t) \oplus \mathfrak{U}_h(s_1, s_1, t) + \mathfrak{U}_h(\omega_1, s_1, t) \oplus \mathfrak{U}_h(s_1, s_1, t)) \right\} \\
& = (m + 2\mathcal{K})(\mathfrak{U}_h(\omega_1, s_1, t)) = (m + 2\mathcal{K})(\mathfrak{U}_h(\omega_1, s_1, t)) \\
& \leq (m + 2\mathcal{K})^2(\mathfrak{U}_h(\mathcal{G}\omega_1, \mathcal{G}s_1, t)) \leq \dots \leq (m + 2\mathcal{K})^j(\mathfrak{U}_h(\mathcal{G}\omega_1, \mathcal{G}s_1, t)) \\
& \rightarrow 0, \text{ as } j \rightarrow \infty, \text{ where } (m + 2\mathcal{K}) < 1.
\end{aligned} \tag{32}$$

Hence, $\mathfrak{U}_h(\omega_1, s_1, t) = 0$, and this implies that $\omega_1 = s_1$, for $t > 0$. $\square$

Corollary 2. Let $(\mathfrak{M}, \mathfrak{U}_h, \oplus)$ is a $\mathbb{G}$ -complete RFM-space in which $\mathfrak{U}_h$ is triangular and a mapping $\mathcal{G} : \mathfrak{M} \rightarrow \mathfrak{M}$ satisfies

$$\mathfrak{U}_h(\mathcal{G}p, \mathcal{G}q, t) \leq \left\{ m(\mathfrak{U}_h(p, q, t)) + n \left(\frac{\mathfrak{U}_h(p, \mathcal{G}p, t) \oplus \mathfrak{U}_h(p, \mathcal{G}q, 2t)}{\mathfrak{U}_h(p, q, t) \oplus \mathfrak{U}_h(q, \mathcal{G}q, t)} \right) \right. \\
\left. + \ell(\mathfrak{U}_h(p, \mathcal{G}p, t) + \mathfrak{U}_h(q, \mathcal{G}q, t)) \right\}, \tag{33}$$

for all $p, q \in \mathfrak{M}, t > 0$ and $m, n, \ell \geq 0$ with $m + n + 2\ell < 1$. Then, $\mathcal{G}$ has a unique fixed point.

Corollary 3. Let $(\mathfrak{M}, \mathfrak{U}_h, \oplus)$ is a $\mathbb{G}$ -complete RFM-space in which $\mathfrak{U}_h$ is triangular and a mapping $\mathcal{G} : \mathfrak{M} \rightarrow \mathfrak{M}$ satisfies

$$\mathfrak{U}_h(\mathcal{G}p, \mathcal{G}q, t) \leq \left\{ m(\mathfrak{U}_h(p, q, t)) + \mathcal{K} \left(\frac{\mathfrak{U}_h(p, \mathcal{G}q, 2t)}{\mathfrak{U}_h(p, \mathcal{G}p, t)} + \frac{\mathfrak{U}_h(p, \mathcal{G}q, 2t)}{\mathfrak{U}_h(q, \mathcal{G}q, t)} \right) \right. \\
\left. + \ell(\mathfrak{U}_h(p, \mathcal{G}p, t) + \mathfrak{U}_h(q, \mathcal{G}q, t)) \right\}, \tag{34}$$

for all $p, q \in \mathfrak{M}, t > 0$ and $m, \mathcal{K}, \ell \geq 0$ with $m + 2\mathcal{K} + 2\ell < 1$. Then, $\mathcal{G}$ has a unique fixed point.

Corollary 4. Let $(\mathfrak{M}, \mathfrak{U}_h, \oplus)$ is a $\mathbb{G}$ -complete RFM-space in which $\mathfrak{U}_h$ is triangular and a mapping $\mathcal{G} : \mathfrak{M} \rightarrow \mathfrak{M}$ satisfies

$$\mathfrak{U}_h(\mathcal{G}p, \mathcal{G}q, t) \leq \{m(\mathfrak{U}_h(p, q, t)) + \ell(\mathfrak{U}_h(p, \mathcal{G}p, t) + \mathfrak{U}_h(q, \mathcal{G}q, t))\}, \tag{35}$$

for all $p, q \in \mathfrak{M}, t > 0$ and $m, \ell \geq 0$ with $m + 2\ell < 1$. Then, $\mathcal{G}$ has a unique fixed point.

Example 4. By the Example 3, Define $\mathfrak{U}_h$ as

$$\mathfrak{U}_h(p, q, t) = \frac{|(p - q)/2|}{t + (p - q)/2}, \quad p, q \in \mathfrak{M}, t > 0 \tag{36}$$

$$\mathcal{G}(p) = \begin{cases} \frac{3p}{7}, & \text{if } p \in [0, 1], \\ \frac{3p}{4} + 1, & \text{if } p \in (0, \infty). \end{cases} \tag{37}$$

Then, we have

$$\mathfrak{U}_h(\mathcal{G}p, \mathcal{G}q, t) = \frac{3}{7}(\mathfrak{U}_h(p, q, t)), \text{ for all } p, q \in \mathfrak{M}, t > 0. \quad (38)$$

A mapping $\mathcal{G}$ is a revised fuzzy contraction. Now, by the Example 1 (iii), for $t > 0$, $\mathfrak{U}_h(p, \mathcal{G}q, 2t) \leq \mathfrak{U}_h(p, q, t) \oplus \mathfrak{U}_h(q, \mathcal{G}q, t)$ and after simplification, we get the following result

$$\begin{aligned} \left(\frac{\mathfrak{U}_h(p, \mathcal{G}p, t) \oplus \mathfrak{U}_h(p, \mathcal{G}q, 2t)}{\mathfrak{U}_h(p, q, t) \oplus \mathfrak{U}_h(q, \mathcal{G}q, t)} \right) &\leq \mathfrak{U}_h(p, \mathcal{G}p, t) = \frac{2p}{7t} \\ \left(\frac{\mathfrak{U}_h(p, \mathcal{G}q, 2t)}{\mathfrak{U}_h(p, \mathcal{G}p, t)} + \frac{\mathfrak{U}_h(p, \mathcal{G}q, 2t)}{\mathfrak{U}_h(q, \mathcal{G}q, t)} \right) &\leq \frac{10}{7}(\mathfrak{U}_h(p, q, t)) = \frac{5|p - q|}{7t} \\ \mathfrak{U}_h(p, \mathcal{G}p, t) \oplus \mathfrak{U}_h(q, \mathcal{G}q, t) &= \frac{2|p + q|}{5t} \end{aligned} \quad (39)$$

It shows, all the conditions of Theorem 2 are satisfied with $m = \frac{3}{7}$, $n = \frac{1}{9}$, and $\ell = \frac{1}{12}$, and $\mathcal{G}$ has a fixed point, i.e., $\mathcal{G}(4) = 4 \in [0, \infty)$.

4. Application

In this section, we present an integral type application to support our work. Let $\mathfrak{M} = \mathcal{C}([0, \gamma], \mathfrak{R})$ be the space of all $\mathfrak{R}$ -valued continuous functions on the interval $[0, \gamma]$, where $0 < \gamma \in \mathfrak{R}$. The nonlinear integral equation is

$$p(\tau) = \int_0^\tau T(\tau, u, p(u))du, \text{ for all } p \in \mathfrak{M} \quad (40)$$

where $\tau, u \in [0, \gamma]$ and $T : [0, \gamma] \times [0, \gamma] \times \mathfrak{R} \rightarrow \mathfrak{R}$. The induced metric $\alpha : \mathfrak{M}^2 \rightarrow \mathfrak{R}$ can be defined as

$$\alpha(p, q) = \inf_{\tau \in [0, \gamma]} |p(\tau) - q(\tau)| = \|p - q\|, \text{ where } \forall p, q \in \mathcal{C}[0, \gamma] = \mathfrak{M}. \quad (41)$$

The operation $\oplus$ is defined by $p \oplus q = p + q - pq$, $p, q \in [0, \gamma]$. A standard revised fuzzy metric $\mathfrak{U}_h : \mathfrak{M}^2 \times (0, \infty) \rightarrow [0, 1]$ can be defined as

$$\mathfrak{U}_h(p, q, t) = \frac{\alpha(p, q)}{t + \alpha(p, q)}, \text{ for } t > 0, \text{ for all } p, q \in \mathfrak{M}. \quad (42)$$

Hence, one can easily verify that $\mathfrak{U}_h$ is triangular and $(\mathfrak{M}, \mathfrak{U}_h, \oplus)$ is a $\mathbb{G}$ -complete RFM-space.

Theorem 3. Let the integral equation be defined in (40), and such that $\varphi \in (0, 1)$ satisfies

$$\alpha(\mathcal{G}p, \mathcal{G}q) \leq \varphi(M(\mathcal{G}, p, q)), \text{ for all } p, q \in \mathfrak{M}, \quad (43)$$

where

$$M(\mathcal{G}, p, q) = \min\{\|p - q\|, 2\|p - \mathcal{G}p\|\}, \text{ for all } p, q \in \mathfrak{M}, \quad (44)$$

So, M is the only place where the integral problem in (40) can be solved.

Proof. Define the integral operator $\mathcal{G} : \mathfrak{M} \rightarrow \mathfrak{M}$ by

$$\mathcal{G}p(\tau) = \int_0^\tau T(\tau, u, p(u))du, \text{ for all } p \in \mathfrak{M}, \quad (45)$$

$\mathcal{G}$ is clearly specified, and (40) has a singular answer only if $\mathcal{G}$ has a singular fixed point in $\mathfrak{M}$. We must now demonstrate that the integral operator $\mathcal{G}$ is covered by Theorem 1. Then, for all $p, q \in \mathfrak{M}$, we have the subsequent two cases:

(a) If $M(\mathcal{G}, p, q) = \|p - q\|$ then, from (42) and (43), we have (44),

$$\mathfrak{U}_h(\mathcal{G}p, \mathcal{G}q, t) = \frac{a(\mathcal{G}p, \mathcal{G}q)}{t} \leq \varphi\left(\frac{M(\mathcal{G}, p, q)}{t}\right) = \varphi\left(\frac{\|p - q\|}{t}\right) = \varphi(\mathfrak{U}_h(p, q, t)), \quad (46)$$

$$\implies \mathfrak{U}_h(\mathcal{G}p, \mathcal{G}q, t) \leq \varphi(\mathfrak{U}_h(p, q, t)), \quad t > 0, \quad (47)$$

for all $p, q \in \mathfrak{M}$, such that $\mathcal{G}p \neq \mathcal{G}q$, inequality (47) is true. With $\varphi = m$ and $n = 0$, the integral operator $\mathcal{G}$ thus meets all the requirements of Theorem 1 (5). The answer to (40) exists in $\mathfrak{M}$, making it the only fixed point for the integral operator $\mathcal{G}$.

(b) If $M(\mathcal{G}, p, q) = \|p - \mathcal{G}p\|$ then, from (42) and (43), we have (44),

$$\mathfrak{U}_h(\mathcal{G}p, \mathcal{G}q, t) = \frac{a(\mathcal{G}p, \mathcal{G}q)}{t} \leq \varphi\left(\frac{M(\mathcal{G}, p, q)}{t}\right) = \varphi\left(\frac{\|p - \mathcal{G}p\|}{t}\right) \leq 2\varphi\left(\frac{\|p - \mathcal{G}p\|}{t}\right) \quad (48)$$

$$\implies \mathfrak{U}_h(\mathcal{G}p, \mathcal{G}q, t) \leq 2\varphi\left(\frac{\|p - \mathcal{G}p\|}{t}\right), \quad t > 0, \quad (49)$$

Here, we condense the expression $\left(\frac{\mathfrak{U}_h(p, \mathcal{G}p, t) \oplus \mathfrak{U}_h(q, \mathcal{G}p, 2t)}{\mathfrak{U}_h(p, q, t)}\right)$, and by applying Example 1 (iii) and (42) we obtain for $t > 0$,

$$\begin{aligned} \frac{\mathfrak{U}_h(p, \mathcal{G}p, t) \oplus \mathfrak{U}_h(q, \mathcal{G}p, 2t)}{\mathfrak{U}_h(p, q, t)} &\leq \left(\frac{\mathfrak{U}_h(p, \mathcal{G}p, t) \oplus \mathfrak{U}_h(q, \mathcal{G}p, 2t)}{\mathfrak{U}_h(p, q, t)}\right) = 2\frac{a(p, \mathcal{G}p)}{t} + \left(\frac{a(p, \mathcal{G}p)}{t}\right)^2 \\ &= 2\left(\frac{\|p - \mathcal{G}p\|}{t}\right) + \left(\frac{\|p - \mathcal{G}p\|}{t}\right)^2 \end{aligned} \quad (50)$$

$$\implies \frac{\mathfrak{U}_h(p, \mathcal{G}p, t) \oplus \mathfrak{U}_h(q, \mathcal{G}p, 2t)}{\mathfrak{U}_h(p, q, t)} \leq 2\left(\frac{\|p - \mathcal{G}p\|}{t}\right) + \left(\frac{\|p - \mathcal{G}p\|}{t}\right)^2, \quad \text{for } t > 0. \quad (51)$$

Now that we have (49) and (51)

$$\mathfrak{U}_h(\mathcal{G}p, \mathcal{G}q, t) \leq \frac{\mathfrak{U}_h(p, \mathcal{G}p, t) \oplus \mathfrak{U}_h(q, \mathcal{G}p, 2t)}{\mathfrak{U}_h(p, q, t)}, \quad \text{for } t > 0. \quad (52)$$

Now, $q \in \mathfrak{M}$, such that $\mathcal{G}p \neq \mathcal{G}q$. Inequality (52) holds if $\mathcal{G}p = \mathcal{G}q$. Thus, the integral operator $\mathcal{G}$ satisfies all the conditions of Theorem 1 with $\varphi = n$ and $m = 0$ in (5). The integral operator $\mathcal{G}$ has a unique fixed point; i.e., Equation (40) has a solution in $\mathfrak{M}$. $\square$

5. Conclusions

The concept of rational type revised fuzzy-contraction maps in RFM-spaces is presented in this paper, and some rational type fixed point theorems are proved in $\mathbb{G}$ -complete RFM-spaces under the rational type revised fuzzy-contraction conditions, utilizing the “triangular property of revised fuzzy metric.” In the final section, an integral type application for rational type revised fuzzy-contraction maps is presented, and a result of a unique solution for an integral operator in RFM-space is proved. In this direction, more rational type revised fuzzy-contraction results in $\mathbb{G}$ -complete-spaces with various types of applications can be demonstrated.

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