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Conormal Product for Intutionistic Anti-Fuzzy Graphs

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Abstract

This study introduces and analyzes the conormal product of intuitionistic anti-fuzzy graphs (IAFGs) and analyzes certain fundamental theorems and applications. Further, new notions on complete and regular IAFGs were introduced, and the conormal product operation was applied to these IAFGs. We showed that the conormal product of two IAFGs could be used and analyzed important results showing that the conormal product of complete, regular, and strong IAFGs is an IAFG.

Keywords: Conormal product of IFGs, Conormal product of IAFGs, Conormal product of complete IAFGs, Conormal product of regular IAFGs, Conormal product of strong IAFGs

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1. Introduction

The concept of fuzzy sets was first introduced by Zadeh [1] in 1965 and has since gained popularity. It has been used to solve many real-world decision-making problems with uncertainty. Additionally, fuzzy set theory is investigated in various domains, including mathematics, computer science, and signal processing. There have been many attempts at generalizing fuzzy sets, including interval-valued fuzzy sets (IVFSs) bipolar fuzzy sets (BPFs), intuitionistic fuzzy sets (IFSs), and picture fuzzy sets (PFSs). Graphs are a simple method of communicating data, as are connections between different substances. The substances are demonstrated by vertices and their relations by edges. Kaufmann and Zadeh [2] was the first to introduce the concept of fuzzy graphs. In 1975, Rosenfield [3] fuzzy graph concepts that broadened the scope of fuzzy sets to include graph theory. Later, Bhattachaya [4] analyzed several ideas in fuzzy graphs. Mordeson and his colleagues [5,6] analyzed certain operations on fuzzy graphs and hypergraphs. Parvathi and his colleagues [7,8] introduced intuitionistic fuzzy graphs (IFGs). Chaira and Ray [9] explained the application of intuitionistic fuzzy set theory. Nagoorgani and Radha [10] proposed certain properties and the conjunction of two fuzzy graphs. Xu and Hu [11] suggested some ideas in the projection models of IFGs. Mordeson and Nar [6] put forward basic ideas for fuzzy graphs and hypergraphs. Radha and Arumugam [12, 13] summarized the properties of strong product fuzzy graphs. The product of fuzzy graphs was established by Dogra [14], and Sahoo and Pal [15] presented product concepts based on IFGs in the same year. Rashmanlou et al. [16] explained some properties of IFGs. Products on IFGs were introduced in the same year by Sahoo et al.[18]. Sahoo and Pal [15]. The authors of [15, 17] also developed intuitionistic fuzzy competition graphs in 2016. Karunambigai et al. [18]

explained the concepts of IFGs. In the same year, Mohideen et al. [19] described the properties of regular IFGs. Peter [20] established the conormal product of two fuzzy graphs. Muthuraj and Sasireka [21, 22] provided notions on anti-fuzzy graphs. Complex IFGs were introduced by Yaqoob et al. [23]. Pal et al. [24] established important notions of fuzzy graph theory. Rashmanlou and his colleagues [16, 25-28] explained various IVFSs in 2014. In 2015, IFGs with categorical properties were developed by Rashmanlou et al. [16]. Several studies [29-33] discussed the principles of fuzzy graphs and different products in fuzzy graphs. X. Hong et al. introduced some applications of fuzzy graphs. From 2012 to 2023, the authors of [18, 34-38] explained the concepts of intuitionistic and domination fuzzy graphs. Later, they discovered the novel conormal product, utilized exclusively for IFGs. Subsequently, the conormal product was utilized for intuitionistic anti-fuzzy graphs (IAFGs).

This manuscript is organized as follows: Section 2 provides basic and important definitions. We introduce and study conormal products on IAFGs in Section 3 on fascinating IAFGs, including complete, regular, and strong IAFGs. Using a conormal product yielded interesting outcomes. The conormal product of complete IAFGs yields a complete IAFG, the conormal product of regular IAFGs yields a regular IAFG, and the conormal product of pseudo-strong IAFGs yields a strong IAFG. In Section 4, we discovered an application of the conormal product on IAFGs. Future conormal product work on IAFGs is discussed in Section 5.

2. Preliminaries

Definition 2.1 [2]. A fuzzy graph is an ordered triple $G_F(V_F, \sigma_F, \mu_F)$, where V_F is a set of vertices $\{u_{F_1}, u_{F_2}, ..., u_{F_n}\}$ and σ_F is a fuzzy subset of V_F such that $\sigma_F : V_F \to [0, 1]$ and is denoted by $\sigma_F = \{(u_{F_1}, \sigma_F(u_{F_1})), (u_{F_2}, \sigma(u_{F_2})), ..., (u_{F_n}, \sigma(u_{Fn}))\}$, and μ_F is a fuzzy relation on σ_F such that $\mu_F(u_F, v_F) \leq \sigma_F(u_F) \wedge \sigma_F(v_F)$.

Definition 2.2 [11]. A fuzzy graph G is *regular* if all its vertices have the same degree. In a fuzzy graph, if the degree of each vertex is k_F, that is, $d_F(v_F) = \sum \mu_F(u_F, v_F) = k_F$, the graph is called a k_F-regular fuzzy graph.

Definition 2.3 [6]. An IFG has the form $G_F : (V_F, E_F)$, where

(i) $V_F = \{v_{F_1}, v_{F_2}, ..., v_{F_n}\}$ such that $\mu_{F_1} : V_F \to [0, 1]$ and $V_{F_1} : v_F \to [0, 1]$ denote the degree of membership (MS) and non-membership (NMS) of the element $v_{F_i} \in V_{F_i}$, respectively, and $0 \leq \mu_{F_i}(v_{F_i}) + V_{F_i}(v_{F_i}) \leq 1$ for every $v_{F_i} \in V_{F_i}$ $(i = 1, 2, ..., n)$;

(ii) $E_F \subseteq V_F \times V_F$, where $\mu_{F_2} : V_F \times V_F \to [0, 1]$ and $V_{F_2} : V_F \times V_F \to [0, 1]$ are such that

$$\mu_{F_2}(v_{F_i}, v_{F_j}) \leq \{\mu_{F_1}(v_{F_i}) \wedge \mu_{F_1}(v_{F_j})\},$$
$$V_{F_2}(v_{F_i}, v_{F_j}) \leq \{v_{F_1}(v_{F_I}) \vee v_{F_1}(v_{F_j})\},$$

and $0 \leq \mu_{F_2}(v_{F_i}, v_{F_j}) + V_{F_2}(v_{F_i}, v_{F_j}) \leq 1$ for every $(v_{F_i}, v_{F_j}) \in E_F$, $(i, j = 1, 2, ..., n)$.

Definition 2.4 [10]. Let $G_{F_1} : (\mu_{F_1}, \mu_{F_2})$ and $G_{F_2} : (\mu'_{F_1}, \mu'_{F_2})$ be two fuzzy graphs with underlying vertex sets V_{F_1} and V_{F_2} and edge sets E_{F_1} and E_{F_2}, respectively. Then, the conormal product of IFGs G_{F_1} and G_{F_2} is a pair of functions $G_{F_1} * G_{F_2} : (\mu_{F_1} * \mu'_{F_1}, \mu_{F_2} * \mu'_{F_2})$ with the underlying vertex set $V_{F_1} \times V_{F_2} = \{(u_{F_1}, v_{F_1}); u_{F_1} \in V_{F_1}, v_{F_1} \in V_{F_2}\}$ and underlying the edge in $E_{F_2} \times E_{F_2} = \{(u_{F_1}, v_{F_1})(u_{F_2}, v_{F_2}) : u_{F_1} = u_{F_2}, v_{F_1}v_{F_2} \in E_{F_2}(or)u_{F_1}u_{F_2} \in E_{F_1}, V_{F_1} = V_{F_2}\}$. The vertex's MS and NMS values (u_F, v_F) in $G_{F_1} * G_{F_2}$ are given by

$$(\sigma_{F_1} * \sigma'_{F_1})(u_{F_1}, v_{F_1}) = \sigma_{F_1}(u_{F_1}) \wedge \sigma'_{F_1}(v_{F_1}),$$
$$(\sigma_{F_2} * \sigma'_{F_2})(u_{F_1}, v_{F_1}) = \sigma_{F_2}(u_{F_1}) \wedge \sigma'_{F_2}(v_{F_1}).$$

The extension of edge values is given by

(i) $\mu_{F_1} * \mu'_{F_1}((u_{F_1}, v_{F_1})(u_{F_2}, v_{F_2}))$

$$= \left\{\begin{array}{l} \mu_{F_1}(u_{F_1}u_{F_2}) \wedge \sigma'_{F_1}(v_{F_1}) \\ \quad \text{if } u_{F_1}u_{F_2} \in E_{F_1} \text{ and } v_{F_1} = v_{F_2}, \\ \mu'_{F_1}(v_{F_1}v_{F_2}) \wedge \sigma_{F_1}(u_{F_1}) \\ \quad \text{if } v_{F_1}v_{F_2} \in E_{F_2} \text{ and } u_{F_1} = u_{F_2}, \\ \mu_{F_1}(u_{F_1}u_{F_2}) \wedge \sigma'_{F_1}(v_{F_1}) \wedge \sigma'_{F_1}(v_{F_2}) \\ \quad \text{if } u_{F_1}u_{F_2} \in E_{F_1} \text{ and } v_{F_1}v_{F_2} \notin E_{F_2}, \\ \mu'_{F_1}(v_{F_1}v_{F_2}) \wedge \sigma_{F_1}(u_{F_1}) \wedge \sigma_{F_1}(u_{F_2}) \\ \quad \text{if } v_{F_1}v_{F_2} \in E_{F_2} \text{ and } u_{F_1}u_{F_2} \notin E_{F_1}, \\ \mu_{F_1}(u_{F_1}u_{F_2}) \wedge \mu'_{F_1}(v_{F_1}v_{F_2}) \\ \quad \text{if } u_{F_1}u_{F_2} \in E_{F_1} \text{ and } v_{F_1}v_{F_2} \in E_{F_2}. \end{array}\right\}$$

(ii) $\mu_{F_2} * \mu'_{F_2}((u_{F_1}, v_{F_1})(u_{F_2}, v_{F_2}))$

$$= \left\{\begin{array}{l} \mu_{F_2}(u_{F_1}u_{F_2}) \vee \sigma'_{F_2}(v_{F_2}) \\ \quad \text{if } u_{F_1}u_{F_2} \in E_{F_1} \text{ and } v_{F_1} = v_{F_2}, \\ \mu'_{F_2}(v_{F_1}v_{F_2}) \vee \sigma_{F_2}(u_{F_2}) \\ \quad \text{if } v_{F_1}v_{F_2} \in E_{F_2} \text{ and } u_{F_1} = u_{F_2}, \end{array}\right.$$

$$\begin{Bmatrix} \mu_{F_2}(u_{F_1}u_{F_2}) \vee \sigma'_{F_2}(v_{F_1}) \vee \sigma'_{F_2}(v_{F_2}) \\ \quad \text{if } u_{F_1}u_{F_2} \in E_{F_1} \text{ and } v_{F_1}v_{F_2} \notin E_{F_2}, \\ \mu'_{F_2}(v_{F_1}v_{F_2}) \vee \sigma_{F_2}(u_{F_1}) \vee \sigma_{F_2}(u_{F_2}) \\ \quad \text{if } v_{F_1}v_{F_2} \in E_{F_2} \text{ and } u_{F_1}u_{F_2} \notin E_{F_1}, \\ \mu_{F_2}(u_{F_1}u_{F_2}) \vee \mu'_{F_2}(v_{F_1}v_{F_2}) \\ \quad \text{if } u_{F_1}u_{F_2} \in E_{F_1} \text{ and } v_{F_1}v_{F_2} \in E_{F_2}. \end{Bmatrix}.$$

3. Conormal Product of IAFGs

The preceding paper discussed certain concepts of IAFGs. This section introduces conormal products and illustrates how to use them in IAFGs. We use the conormal product on two IFGs.

Definition 3.1. Let $G_{AF_1} : (\mu_{AF_1}, \mu_{AF_2})$ and $G_{AF_2} : (\mu'_{AF_1}, \mu'_{AF_2})$ be anti-fuzzy graphs with underlying vertex sets V_{AF_1} and V_{AF_2} and edge sets E_{AF_1} and E_{AF_2}, respectively. Then, the conormal product of IAFGs G_{AF_1} and G_{AF_2} is a pair of functions $G_{AF_1} * G_{AF_2} : (\mu_{AF_1} * \mu'_{AF_1}, \mu_{AF_2} * \mu'_{AF_2})$ with the underlying vertex set $V_{AF_1} \times V_{AF_2} = \{(u_{AF_1}, v_{AF_1});\ u_{AF_1} \in V_{AF_1},\ v_{AF_1} \in V_{AF_2}\}$ and underlying the edge in $E_{AF_1} \times E_{AF_2} = \{(u_{AF_1}, v_{AF_1})(u_{AF_2}, v_{AF_2})\text{: } u_{AF_1} = u_{AF_2}, v_{AF_1}v_{AF_2} \in E_{AF_2}(or)u_{AF_1}u_{AF_2} \in E_{AF_1}, v_{AF_1} = v_{AF_2}\}$.

The vertex's MS and NMS values (u_{AF}, v_{AF}) in $G_{AF_1} * G_{AF_2}$ are given by

(i) $(\sigma_{AF_1} * \sigma'_{AF_1})(u_{AF_1}, v_{AF_1}) = \sigma_{AF_1}(u_{AF_1}) \wedge \sigma'_{AF_1}(v_{AF_1})$,
$(\sigma_{AF_2} * \sigma'_{AF_2})(u_{AF_1}, v_{AF_1}) = \sigma_{AF_2}(u_{AF_1}) \vee \sigma'_{AF_2}(v_{AF_1})$,

(ii) $(\sigma_{AF_1} * \sigma'_{AF_1})(u_{AF_2}, v_{AF_2}) = \sigma_{AF_1}(u_{AF_2}) \wedge \sigma'_{AF_1}(v_{AF_2})$,
$(\sigma_{AF_2} * \sigma'_{AF_2})(u_{AF_2}, v_{AF_2}) = \sigma_{AF_2}(u_{AF_2}) \vee \sigma'_{AF_2}(v_{AF_2})$.

The extension of edge values is given by

(i) $\mu_{AF_1} * \mu'_{AF_1}((u_{AF_1}, v_{AF_1})(u_{AF_2}, v_{AF_2}))$

$$= \begin{Bmatrix} \mu_{AF_1}(u_{AF_1}u_{AF_2}) \vee \sigma'_{AF_1}(v_{AF_1}) \\ \quad \text{if } u_{AF_1}u_{AF_2} \in E_{AF_1} \text{ and } v_{AF_1} = v_{AF_2}, \\ \mu'_{AF_1}(v_{AF_1}v_{AF_2}) \vee \sigma_{AF_1}(u_{AF_1}) \\ \quad \text{if } v_{AF_1}v_{AF_2} \in E_{AF_2} \text{ and } u_{AF_1} = u_{AF_2}, \\ \mu_{AF_1}(u_{AF_1}u_{AF_2}) \vee \sigma'_{AF_1}(v_{AF_1}) \vee \sigma'_{AF_1}(v_{AF_2}) \\ \quad \text{if } u_{AF_1}u_{AF_2} \in E_{AF_1} \text{ and } v_{AF_1}v_{AF_2} \notin E_{AF_2}, \\ \mu'_{AF_1}(v_{AF_1}v_{AF_2}) \vee \sigma_{AF_1}(u_{AF_1}) \vee \sigma_{AF_1}(u_{AF_2}) \\ \quad \text{if } v_{AF_1}v_{AF_2} \in E_{AF_2} \text{ and } u_{AF_1}u_{AF_2} \notin E_{AF_1}, \\ \mu_{AF_1}(u_{AF_1}u_{AF_2}) \vee \mu'_{AF_1}(v_{AF_1}v_{AF_2}) \\ \quad \text{if } u_{AF_1}u_{AF_2} \in E_{AF_1} \text{ and } v_{AF_1}v_{AF_2} \in E_{AF_2}. \end{Bmatrix}$$

(ii) $\mu_{AF_2} * \mu'_{AF_2}((u_{AF_1}, v_{AF_1})(u_{AF_2}, v_{AF_2}))$

$$= \begin{Bmatrix} \mu_{AF_2}(u_{AF_1}u_{AF_2}) \wedge \sigma'_{AF_2}(v_{AF_2}) \\ \quad \text{if } u_{AF_1}u_{AF_2} \in E_{AF_1} \text{ and } v_{AF_1} = v_{AF_2}, \\ \mu'_{AF_2}(v_{AF_1}v_{AF_2}) \wedge \sigma_{AF_2}(u_{AF_2}) \\ \quad \text{if } v_{AF_1}v_{AF_2} \in E_{AF_2} \text{ and } u_{AF_1} = u_{AF_2}, \\ \mu_{AF_2}(u_{AF_1}u_{AF_2}) \wedge \sigma'_{AF_2}(v_{AF_1}) \wedge \sigma'_{AF_2}(v_{AF_2}) \\ \quad \text{if } u_{AF_1}u_{AF_2} \in E_{AF_1} \text{ and } v_{AF_1}v_{AF_2} \notin E_{AF_2}, \\ \mu'_{AF_2}(v_{AF_1}v_{AF_2}) \wedge \sigma_{AF_2}(u_{AF_1}) \wedge \sigma_{AF_2}(u_{AF_2}) \\ \quad \text{if } v_{AF_1}v_{AF_2} \in E_{AF_2} \text{ and } u_{AF_1}u_{AF_2} \notin E_{AF_1}, \\ \mu_{AF_2}(u_{AF_1}u_{AF_2}) \wedge \mu'_{AF_2}(v_{AF_1}v_{AF_2}) \\ \quad \text{if } u_{AF_1}u_{AF_2} \in E_{AF_1} \text{ and } v_{AF_1}v_{AF_2} \in E_{AF_2}. \end{Bmatrix}$$

Example 3.1. See Figure 1.

Definition 3.2. Let $G_{AF_1} : (\mu_{AF_1}, \mu_{AF_2})$ and $G_{AF_2} : (\mu'_{AF_1}, \mu'_{AF_2})$ be complete and conormal products of two IAFGs; then, the fuzzy graph is called a conormal product of complete IAFGs and is denoted by $G_{CAF_1} * G_{CAF_2}$.

Example 3.2. See Figure 2.

Example 3.3. See Figure 3.

Example 3.4. See Figure 4.

Definition 3.3. Let $G_{AF_1} : (\mu_{AF_1}, \mu_{AF_2})$ and $G_{AF_2} : (\mu'_{AF_1}, \mu'_{AF_2})$ be strong, conormal products of two IAFGs; then, the fuzzy graph is called a conormal product of strong IAFGs and

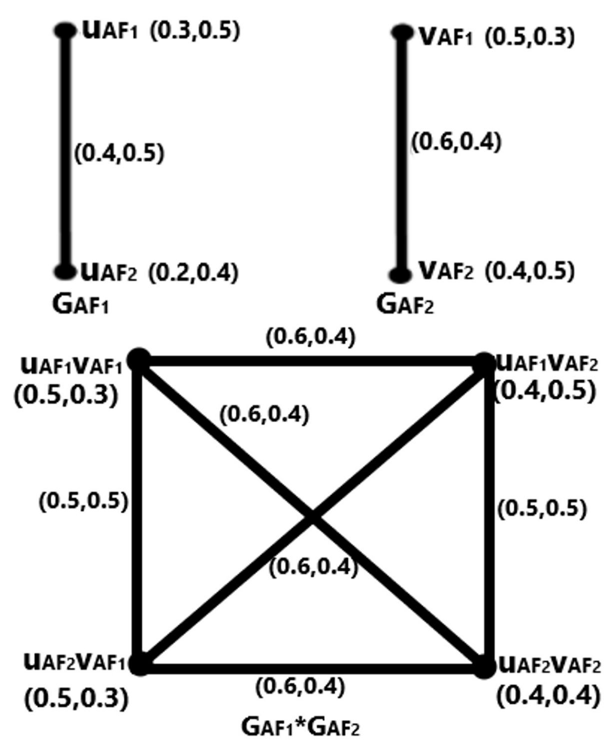

Figure 1. Conormal product of two IAFGs.

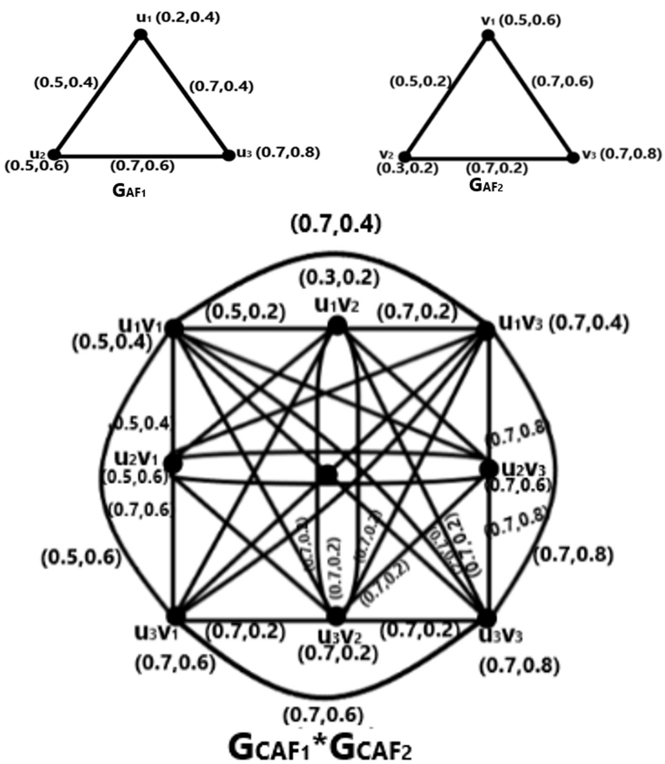

Figure 2. Conormal product of complete IAFGs.

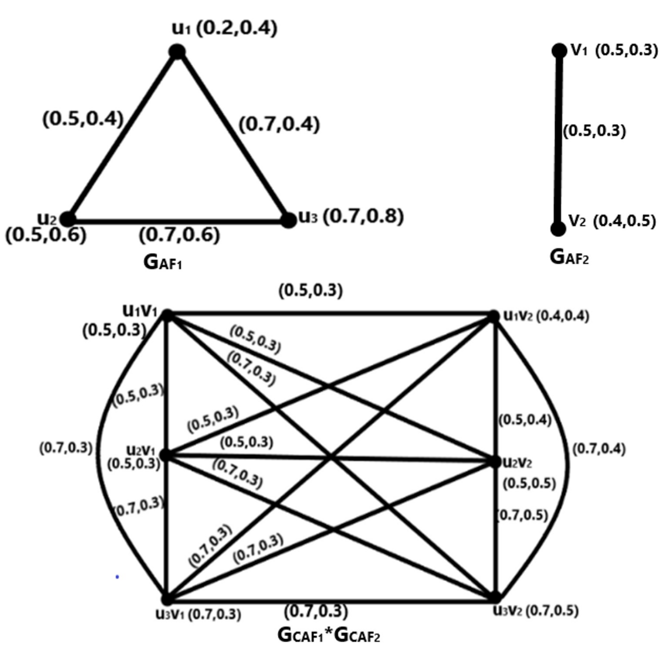

Figure 3. Conormal product of complete IAFGs.

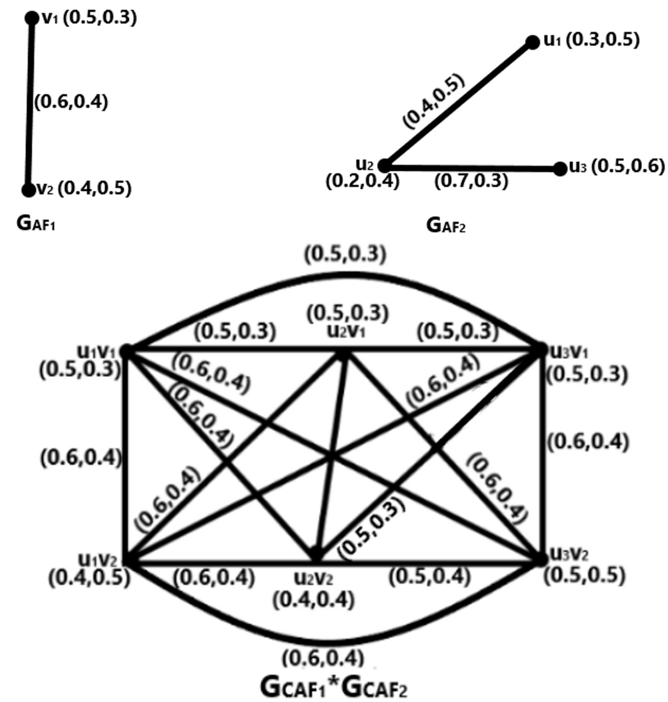

Figure 4. Conormal product of complete IAFGs.

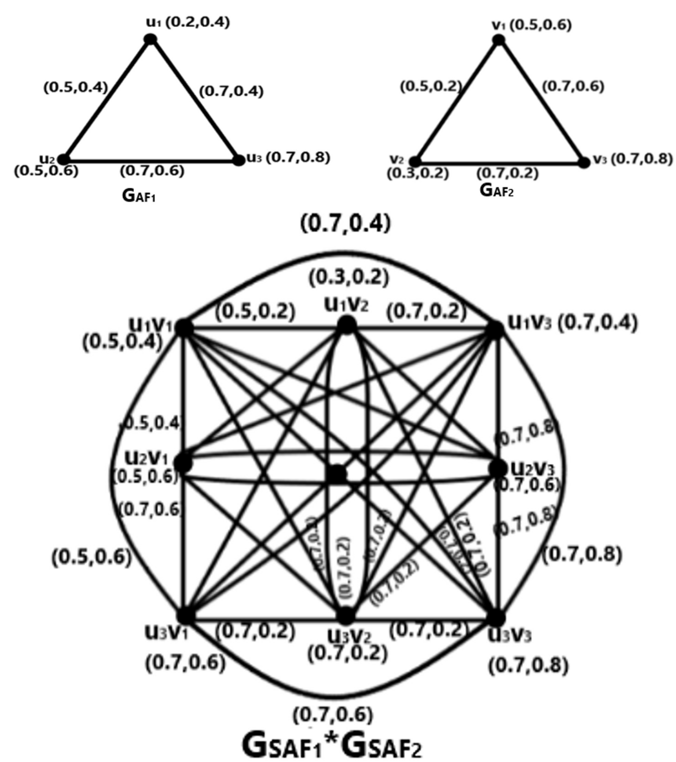

Figure 5. Conormal product of strong IAFGs.

denoted by $G_{SAF_1} * G_{SAF_2}$.

Example 3.5. See Figure 5.

Definition 3.4. Let $G_{AF_1} * G_{AF_2}$ be the conormal product of two IAFGs; then, the graph is said to be a conormal product of regular IAFGs if each vertex has the same degree.

Example 3.6 See Figure 6.

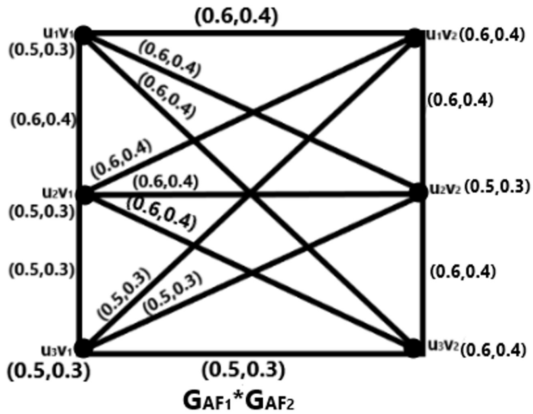

Figure 6. Conormal product of regular IAFGs.

Theorem 3.1. A conormal product of two IAFGs is an IAFGs.

Proof. Let $G_{AF_1} : (\mu_{AF_1}, \mu_{AF_2})$ and $G_{AF_2} : (\mu'_{AF_1}, \mu'_{AF_2})$ represent IAFGs with the underlying crisp graphs $G^*_{AF_1} : (V_{AF_1}, E_{AF_1})$ and $G^*_{AF_2} : (V_{AF_2}, E_{AF_2})$, respectively, and $G_{AF_1} * G_{AF_2}$ be their conormal product; we must prove that $G_{AF_1} * G_{AF_2}$ is an IAFG.

From definition, it follows that

Case (i) If $u_{AF_1}u_{AF_2} \in E_{AF_1}$ and $v_{AF_1} = v_{AF_2}$,

$$\begin{aligned}
&\mu_{AF_1} * \mu'_{AF_1}((u_{AF_1}, v_{AF_1})(u_{AF_2}, v_{AF_2}))\\
&= \mu_{AF_1}(u_{AF_1}u_{AF_2}) \vee \sigma'_{AF_1}(v_{AF_1})\\
&\leq [\sigma_{AF_1}(u_{AF_1}) \vee \sigma_{AF_1}(u_{AF_2})]\\
&\quad \vee [\sigma'_{AF_1}(v_{AF_1}) \vee \sigma'_{AF_2}(v_{AF_2})]\\
&= [\sigma_{AF_1}(u_{AF_1}) \vee \sigma'_{AF_1}(v_{AF_1})]\\
&\quad \vee [\sigma_{AF_1}(u_{AF_2}) \vee \sigma'_{AF_2}(v_{AF_2})]\\
&= (\sigma_{AF_1} * \sigma'_{AF_1})(u_{AF_1}v_{AF_1})\\
&\quad \vee (\sigma_{AF_1} * \sigma'_{AF_1})(u_{AF_2}v_{AF_2}),\\
&\mu_{AF_2} * \mu'_{AF_2}((u_{AF_1}, v_{AF_1})(u_{AF_2}, v_{AF_2}))\\
&= \mu_{AF_2}(u_{AF_1}, u_{AF_2}) \wedge \sigma'_{AF_2}(v_{AF_1})\\
&\leq [\sigma_{AF_2}(u_{AF_1}) \wedge \sigma_{AF_2}(u_{AF_2})]\\
&\quad \wedge [\sigma'_{AF_2}(v_{AF_1}) \wedge \sigma'_{AF_2}(v_{AF_2})]\\
&= [\sigma_{AF_2}(u_{AF_1}) \wedge \sigma'_{AF_2}(v_{AF_1})]\\
&\quad \wedge [\sigma_{AF_2}(u_{AF_2}) \wedge \sigma'_{AF_2}(v_{AF_2})]\\
&= (\sigma_{AF_2} * \sigma'_{AF_2})(u_{AF_1}v_{AF_1})\\
&\quad \wedge (\sigma_{AF_2} * \sigma'_{AF_2})(u_{AF_2}v_{AF_2}).
\end{aligned}$$

Case (ii) If $v_{AF_1}v_{AF_2} \in E_{AF_2}$ and $u_{AF_1} = u_{AF_2}$,

$$\begin{aligned}
&\mu_{AF_1} * \mu'_{AF_1}((u_{AF_1}, v_{AF_1})(u_{AF_2}, v_{AF_2}))\\
&= \mu'_{AF_1}(v_{AF_1}, v_{AF_2}) \vee \sigma_{AF_1}(u_{AF_1})\\
&\leq [\sigma'_{AF_1}(v_{AF_1}) \vee \sigma'_{AF_2}(v_2)]\\
&\quad \vee [\sigma_{AF_1}(u_{AF_1}) \vee \sigma_{AF_1}(u_{AF_2})]\\
&= [\sigma_{AF_1}(u_{AF_1}) \vee \sigma'_{AF_1}(v_{AF_1})]\\
&\quad \vee [\sigma_{AF_1}(u_{AF_2}) \vee \sigma'_{AF_1}(v_{AF_2})]\\
&= (\sigma_{AF_1} * \sigma'_{AF_1})(u_{AF_1}v_{AF_1})\\
&\quad \vee (\sigma_{AF_1} * \sigma'_{AF_1})(u_{AF_2}v_{AF_2}),\\
&\mu_{AF_2} * \mu'_{AF_2}((u_{AF_1}, v_{AF_1})(u_{AF_2}, v_{AF_2}))\\
&= \mu'_{AF_2}(v_{AF_1}, v_{AF_2}) \wedge \sigma'_{AF_2}(u_{AF_2})\\
&\leq [\sigma'_{AF_2}(v_{AF_1}) \wedge \sigma'_{AF_2}(v_{AF_2})]\\
&\quad \wedge [\sigma_{AF_2}(u_{AF_1}) \wedge \sigma_{AF_2}(u_{AF_2})]\\
&= [\sigma_{AF_2}(u_{AF_1}) \wedge \sigma'_{AF_2}(v_{AF_1})]\\
&\quad \wedge [\sigma_{AF_2}(u_{AF_2}) \wedge \sigma'_{AF_2}(v_{AF_2})]\\
&= (\sigma_{AF_2} * \sigma'_{AF_2})(u_{AF_1}v_{AF_1})\\
&\quad \wedge (\sigma_{AF_2} * \sigma'_{AF_2})(u_{AF_2}v_{AF_2}).
\end{aligned}$$

Case (iii) If $u_{AF_1}u_{AF_2} \in E_{AF_1}$ and $v_{AF_1}v_{AF_2} \notin E_{AF_2}$,

$$\begin{aligned}
&\mu_{AF_1} * \mu'_{AF_1}((u_{AF_1}, v_{AF_1})(u_{AF_2}, v_{AF_2}))\\
&= \mu_{AF_1}(u_{AF_1}u_{AF_2}) \vee \sigma'_{AF_1}(v_{AF_1}) \vee \sigma'_{AF_1}(v_{AF_2})\\
&\leq [\sigma_{AF_1}(u_{AF_1}) \vee \sigma_{AF_1}(u_{AF_2})]\\
&\quad \vee [\sigma'_{AF_1}(v_{AF_1}) \vee \sigma'_{AF_1}(v_{AF_2})]\\
&= [\sigma_{AF_1}(u_{AF_1}) \vee \sigma'_{AF_1}(v_{AF_1})]\\
&\quad \vee [\sigma_{AF_1}(u_{AF_2}) \vee \sigma'_{AF_1}(v_{AF_2})]\\
&= (\sigma_{AF_1} * \sigma'_{AF_1})(u_{AF_1}v_{AF_1})\\
&\quad \vee (\sigma_{AF_1} * \sigma'_{AF_1})(u_{AF_2}v_{AF_2}),\\
&\mu_{AF_2} * \mu'_{AF_2}((u_{AF_1}, v_{AF_1})(u_{AF_2}, v_{AF_2}))\\
&= \mu_{AF_2}(u_{AF_1}u_{AF_2}) \wedge \sigma'_{AF_2}(v_{AF_1}) \wedge \sigma'_{AF_2}(v_{AF_2})\\
&\leq [\sigma'_{AF_2}(v_{AF_1}) \wedge \sigma'_{AF_2}(v_{AF_2})]\\
&\quad \wedge [\sigma_{AF_2}(u_{AF_1}) \wedge \sigma_{AF_2}(u_{AF_2})]\\
&= [\sigma_{AF_2}(u_{AF_1}) \wedge \sigma'_{AF_2}(v_{AF_1})]\\
&\quad \wedge [\sigma_{AF_2}(u_{AF_2}) \wedge \sigma'_{AF_2}(v_{AF_2})]\\
&= (\sigma_{AF_2} * \sigma'_{AF_2})(u_{AF_1}v_{AF_1})\\
&\quad \wedge (\sigma_{AF_2} * \sigma'_{AF_2})(u_{AF_2}v_{AF_2}).
\end{aligned}$$

Case (iv) If $v_{AF_1}v_{AF_2} \in E_{AF_2}$ and $u_{AF_1}u_{AF_2} \notin E_{AF_1}$,

$$\begin{aligned}
&\mu_{AF_1} * \mu'_{AF_1}((u_{AF_1}, v_{AF_1})(u_{AF_2}, v_{AF_2}))\\
&= \mu_{AF_1}(v_{AF_1}v_{AF_2}) \vee \sigma'_{AF_1}(u_{AF_1}) \vee \sigma'_{A_{AF_1}}(u_{AF_2})\\
&\leq [\sigma_{AF_1}(u_{AF_1}) \vee \sigma_{AF_1}(u_{AF_2})]\\
&\quad \vee [\sigma'_{AF_1}(v_{AF_1}) \vee \sigma'_{AF_1}(v_{AF_2})]\\
&= [\sigma_{AF_1}(u_{AF_1}) \vee \sigma'_{AF_1}(v_{AF_1})]\\
&\quad \vee [\sigma_{AF_1}(u_{AF_2}) \vee \sigma'_{AF_1}(v_{AF_2})]
\end{aligned}$$

$$= (\sigma_{AF_1} * \sigma^{'}_{AF_1})(u_{AF_1} v_{AF_1})$$
$$\vee (\sigma_{AF_1} * \sigma^{'}_{AF_1})(u_{AF_2} v_{AF_2}),$$
$$\mu_{AF_2} * \mu^{'}_{AF_2}((u_{AF_1}, v_{AF_1})(u_{AF_2}, v_{AF_2}))$$
$$= \mu^{'}_{A_2}(v_{AF_1} v_{AF_2}) \wedge \sigma_{A_2}(u_{AF_1}) \wedge \sigma_{AF_2}(u_{AF_2})$$
$$\leq [\sigma^{'}_{AF_2}(v_{AF_1}) \wedge \sigma^{'}_{AF_2}(v_{AF_2})]$$
$$\wedge [\sigma_{AF_2}(u_{AF_1}) \wedge \sigma_{AF_2}(u_{AF_2})]$$
$$= [\sigma_{AF_2}(u_{AF_1}) \wedge \sigma^{'}_{AF_2}(v_{AF_1})]$$
$$\wedge [\sigma_{AF_2}(u_2) \wedge \sigma^{'}_{AF_2}(v_{AF_2})]$$
$$= (\sigma_{AF_2} * \sigma^{'}_{AF_2})(u_{AF_1} v_{AF_1})$$
$$\wedge (\sigma_{AF_2} * \sigma^{'}_{AF_2})(u_{AF_2} v_{AF_2}).$$

Case (v) If $u_{AF_1} u_{AF_2} \in E_{AF_1}$ and $v_{AF_1} v_{AF_2} \in E_{AF_2}$,

$$\mu_{AF_1} * \mu^{'}_{AF_1}((u_{AF_1}, v_{AF_1})(u_{AF_2}, v_{AF_2}))$$
$$= \mu_{AF_1}(u_{AF_1}, u_{AF_2}) \vee \sigma^{'}_{AF_1}(v_{AF_1} v_{AF_2})$$
$$\leq [\sigma_{AF_1}(u_{AF_1}) \vee \sigma_{AF_1}(u_{AF_2})]$$
$$\vee [\sigma^{'}_{AF_1}(v_{AF_1}) \vee \sigma^{'}_{AF_1}(v_{AF_2})]$$
$$= [\sigma_{AF_1}(u_{AF_1}) \vee \sigma^{'}_{AF_1}(v_{AF_1})]$$
$$\vee [\sigma_{AF_1}(u_{AF_2}) \vee \sigma^{'}_{AF_1}(v_{AF_2})]$$
$$= (\sigma_{AF_1} * \sigma^{'}_{AF_1})(u_{AF_1} v_{AF_1})$$
$$\vee (\sigma_{AF_1} * \sigma^{'}_{AF_1})(u_{AF_2} v_{AF_2}),$$
$$\mu_{AF_2} * \mu^{'}_{AF_2}((u_{AF_1}, v_{AF_1})(u_{AF_2}, v_{AF_2}))$$
$$= \mu_{AF_2}(u_{AF_1} u_{AF_2}) \vee \mu^{'}_{AF_2}(v_{AF_1} v_{AF_2})$$
$$\leq [\sigma^{'}_{AF_2}(v_{AF_1}) \wedge \sigma^{'}_{AF_2}(v_{AF_2})]$$
$$\wedge [\sigma_{AF_2}(u_{AF_1}) \wedge \sigma_{AF_2}(u_{AF_2})]$$
$$= [\sigma_{AF_2}(u_{AF_1}) \wedge \sigma^{'}_{AF_2}(v_{AF_1})]$$
$$\wedge [\sigma_{AF_2}(u_{AF_2}) \wedge \sigma^{'}_{AF_2}(v_{AF_2})]$$
$$= (\sigma_{AF_2} * \sigma^{'}_{AF_2})(u_{AF_1} v_{AF_1})$$
$$\wedge (\sigma_{AF_2} * \sigma^{'}_{AF_2})(u_{AF_2} v_{AF_2}),$$

then

$$\mu_{AF_1} * \mu^{'}_{AF_1}((u_{AF_1}, v_{AF_1})(u_{AF_2}, v_{AF_2}))$$
$$= (\sigma_{AF_1} * \sigma^{'}_{AF_1})(u_{AF_1} v_{AF_1})$$
$$\vee (\sigma_{AF_1} * \sigma^{'}_{AF_1})(u_{AF_2} v_{AF_2}),$$
$$\mu_{AF_2} * \mu^{'}_{AF_2}((u_{AF_1}, v_{AF_1})(u_{AF_2}, v_{AF_2}))$$
$$= (\sigma_{AF_2} * \sigma^{'}_{AF_2})(u_{AF_1} v_{AF_1})$$
$$\wedge (\sigma_{AF_2} * \sigma^{'}_{AF_2})(u_{AF_2} v_{AF_2}).$$

This proves that an IAFG is the conormal product of two IAFGs. □

Theorem 3.2. If $G_{AF_1} : (\mu_{AF_1}, \mu_{AF_2})$ and $G_{AF_2} : (\mu^{'}_{AF_1}, \mu^{'}_{AF_2})$ are complete IAFGs, then $G_{AF_1} * G_{AF_2}$ is also a complete IAFG.

Proof. W.K.T conormal product of two IAFGs is an IAFG.

To prove: The conormal product of both IAFGs is complete.

An IAFG is said to be complete if $\mu_{AF_1}(v_{AF_i}, v_{AF_j}) = \sigma_{AF_1}(v_{AF_i}) \vee \sigma_{AF_1}(v_{AF_j})$ and $\mu_{AF_2}(v_{AF_i}, v_{AF_j}) = \sigma_{AF_2}(v_{AF_i}) \wedge \sigma_{AF_2}(v_{AF_j})$ for all $v_{AF_i}, v_{AF_j} \in V_{AF}$.

Case (i) If $u_{AF_1} u_{AF_2} \in E_{AF_1}$ and $v_{AF_1} = v_{AF_2}$,

$$\mu_{AF_1} * \mu^{'}_{AF_1}((u_{AF_1}, v_{AF_1})(u_{AF_2}, v_{AF_2}))$$
$$= \mu_{AF_1}(u_{AF_1} u_{AF_2}) \vee \sigma^{'}_{AF_1}(v_{AF_1})$$
$$= [\sigma_{AF_1}(u_{AF_1}) \vee \sigma_{AF_1}(u_{AF_2})]$$
$$\vee [\sigma^{'}_{AF_1}(v_{AF_1}) \vee \sigma^{'}_{AF_2}(v_{AF_2})]$$
$$= [\sigma_{AF_1}(u_{AF_1}) \vee \sigma^{'}_{AF_1}(v_{AF_1})]$$
$$\vee [\sigma_{AF_1}(u_{AF_2}) \vee \sigma^{'}_{AF_2}(v_{AF_2})]$$
$$= (\sigma_{AF_1} * \sigma^{'}_{AF_1})(u_{AF_1} v_{AF_1})$$
$$\vee (\sigma_{AF_1} * \sigma^{'}_{AF_1})(u_{AF_2} v_{AF_2}),$$
$$\mu_{AF_2} * \mu^{'}_{AF_2}((u_{AF_1}, v_{AF_1})(u_{AF_2}, v_{AF_2}))$$
$$= \mu_{AF_2}(u_{AF_1}, u_{AF_2}) \wedge \sigma^{'}_{AF_2}(v_{AF_1}),$$
$$= [\sigma_{AF_2}(u_{AF_1}) \wedge \sigma_{AF_2}(u_{AF_2})]$$
$$\wedge [\sigma^{'}_{AF_2}(v_{AF_1}) \wedge \sigma^{'}_{AF_2}(v_{AF_2})]$$
$$= [\sigma_{AF_2}(u_{AF_1}) \wedge \sigma^{'}_{AF_2}(v_{AF_1})]$$
$$\wedge [\sigma_{AF_2}(u_{AF_2}) \wedge \sigma^{'}_{AF_2}(v_{AF_2})]$$
$$= (\sigma_{AF_2} * \sigma^{'}_{AF_2})(u_{AF_1} v_{AF_1})$$
$$\wedge (\sigma_{AF_2} * \sigma^{'}_{AF_2})(u_{AF_2} v_{AF_2}).$$

Case (ii) If $v_{AF_1} v_{AF_2} \in E_{AF_2}$ and $u_{AF_1} = u_{AF_2}$,

$$\mu_{AF_1} * \mu^{'}_{AF_1}((u_{AF_1}, v_{AF_1})(u_{AF_2}, v_{AF_2}))$$
$$= \mu^{'}_{AF_1}(v_{AF_1}, v_{AF_2}) \vee \sigma_{AF_1}(u_{AF_1})$$
$$= [\sigma^{'}_{AF_1}(v_{AF_1}) \vee \sigma^{'}_{AF_2}(v_2)]$$
$$\vee [\sigma_{AF_1}(u_{AF_1}) \vee \sigma_{AF_1}(u_{AF_2})]$$
$$= [\sigma_{AF_1}(u_{AF_1}) \vee \sigma^{'}_{AF_1}(v_{AF_1})]$$
$$\vee [\sigma_{AF_1}(u_{AF_2}) \vee \sigma^{'}_{AF_1}(v_{AF_2})]$$
$$= (\sigma_{AF_1} * \sigma^{'}_{AF_1})(u_{AF_1} v_{AF_1})$$
$$\vee (\sigma_{AF_1} * \sigma^{'}_{AF_1})(u_{AF_2} v_{AF_2}),$$
$$\mu_{AF_2} * \mu^{'}_{AF_2}((u_{AF_1}, v_{AF_1})(u_{AF_2}, v_{AF_2}))$$
$$= \mu^{'}_{AF_2}(v_{AF_1}, v_{AF_2}) \wedge \sigma^{'}_{AF_2}(u_{AF_2})$$
$$= [\sigma^{'}_{AF_2}(v_{AF_1}) \wedge \sigma^{'}_{AF_2}(v_{AF_2})]$$
$$\wedge [\sigma_{AF_2}(u_{AF_1}) \wedge \sigma_{AF_2}(u_{AF_2})]$$
$$= [\sigma_{AF_2}(u_{AF_1}) \wedge \sigma^{'}_{AF_2}(v_{AF_1})]$$
$$\wedge [\sigma_{AF_2}(u_{AF_2}) \wedge \sigma^{'}_{AF_2}(v_{AF_2})]$$
$$= (\sigma_{AF_2} * \sigma^{'}_{AF_2})(u_{AF_1} v_{AF_1})$$

$\wedge (\sigma_{AF_2} * \sigma'_{AF_2})(u_{AF_2} v_{AF_2})$.

Case (iii) If $u_{AF_1} u_{AF_2} \in E_{AF_1}$ and $v_{AF_1} v_{AF_2} \notin E_{AF_2}$,

$$\begin{aligned}
&\mu_{AF_1} * \mu'_{AF_1}((u_{AF_1}, v_{AF_1})(u_{AF_2}, v_{AF_2}))\\
&= \mu_{AF_1}(u_{AF_1} u_{AF_2}) \vee \sigma'_{AF_1}(v_{AF_1}) \vee \sigma'_{AF_1}(v_{AF_2})\\
&= [\sigma_{AF_1}(u_{AF_1}) \vee \sigma_{AF_1}(u_{AF_2})]\\
&\quad \vee [\sigma'_{AF_1}(v_{AF_1}) \vee \sigma'_{AF_1}(v_{AF_2})]\\
&= [\sigma_{AF_1}(u_{AF_1}) \vee \sigma'_{AF_1}(v_{AF_1})]\\
&\quad \vee [\sigma_{AF_1}(u_{AF_2}) \vee \sigma'_{AF_1}(v_{AF_2})]\\
&= (\sigma_{AF_1} * \sigma'_{AF_1})(u_{AF_1} v_{AF_1})\\
&\quad \vee (\sigma_{AF_1} * \sigma'_{AF_1})(u_{AF_2} v_{AF_2}),\\
&\mu_{AF_2} * \mu'_{AF_2}((u_{AF_1}, v_{AF_1})(u_{AF_2}, v_{AF_2}))\\
&= \mu_{AF_2}(u_{AF_1} u_{AF_2}) \wedge \sigma'_{AF_2}(v_{AF_1}) \wedge \sigma'_{AF_2}(v_{AF_2})\\
&= [\sigma'_{AF_2}(v_{AF_1}) \wedge \sigma'_{AF_2}(v_{AF_2})]\\
&\quad \wedge [\sigma_{AF_2}(u_{AF_1}) \wedge \sigma_{AF_2}(u_{AF_2})]\\
&= [\sigma_{AF_2}(u_{AF_1}) \wedge \sigma'_{AF_2}(v_{AF_1})]\\
&\quad \wedge [\sigma_{AF_2}(u_{AF_2}) \wedge \sigma'_{AF_2}(v_{AF_2})]\\
&= (\sigma_{AF_2} * \sigma'_{AF_2})(u_{AF_1} v_{AF_1})\\
&\quad \wedge (\sigma_{AF_2} * \sigma'_{AF_2})(u_{AF_2} v_{AF_2}).
\end{aligned}$$

Case (iv) If $v_{AF_1} v_{AF_2} \in E_{AF_2}$ and $u_{AF_1} u_{AF_2} \notin E_{AF_1}$,

$$\begin{aligned}
&\mu_{AF_1} * \mu'_{AF_1}((u_{AF_1}, v_{AF_1})(u_{AF_2}, v_{AF_2}))\\
&= \mu_{AF_1}(v_{AF_1} v_{AF_2}) \vee \sigma'_{AF_1}(u_{AF_1}) \vee \sigma'_{A_{AF_1}}(u_{AF_2})\\
&= [\sigma_{AF_1}(u_{AF_1}) \vee \sigma_{AF_1}(u_{AF_2})]\\
&\quad \vee [\sigma'_{AF_1}(v_{AF_1}) \vee \sigma'_{AF_1}(v_{AF_2})]\\
&= [\sigma_{AF_1}(u_{AF_1}) \vee \sigma'_{AF_1}(v_{AF_1})]\\
&\quad \vee [\sigma_{AF_1}(u_{AF_2}) \vee \sigma'_{AF_1}(v_{AF_2})]\\
&= (\sigma_{AF_1} * \sigma'_{AF_1})(u_{AF_1} v_{AF_1})\\
&\quad \vee (\sigma_{AF_1} * \sigma'_{AF_1})(u_{AF_2} v_{AF_2}),\\
&\mu_{AF_2} * \mu'_{AF_2}((u_{AF_1}, v_{AF_1})(u_{AF_2}, v_{AF_2}))\\
&= \mu'_{A_2}(v_{AF_1} v_{AF_2}) \wedge \sigma_{A_2}(u_{AF_1}) \wedge \sigma_{AF_2}(u_{AF_2})\\
&= [\sigma'_{AF_2}(v_{AF_1}) \wedge \sigma'_{AF_2}(v_{AF_2})]\\
&\quad \wedge [\sigma_{AF_2}(u_{AF_1}) \wedge \sigma_{AF_2}(u_{AF_2})]\\
&= [\sigma_{AF_2}(u_{AF_1}) \wedge \sigma'_{AF_2}(v_{AF_1})]\\
&\quad \wedge [\sigma_{AF_2}(u_2) \wedge \sigma'_{AF_2}(v_{AF_2})]\\
&= (\sigma_{AF_2} * \sigma'_{AF_2})(u_{AF_1} v_{AF_1})\\
&\quad \wedge (\sigma_{AF_2} * \sigma'_{AF_2})(u_{AF_2} v_{AF_2}).
\end{aligned}$$

Case (v) If $u_{AF_1} u_{AF_2} \in E_{AF_1}$ and $v_{AF_1} v_{AF_2} \in E_{AF_2}$,

$$\begin{aligned}
&\mu_{AF_1} * \mu'_{AF_1}((u_{AF_1}, v_{AF_1})(u_{AF_2}, v_{AF_2}))\\
&= \mu_{AF_1}(u_{AF_1}, u_{AF_2}) \vee \sigma'_{AF_1}(v_{AF_1} v_{AF_2})\\
&= [\sigma_{AF_1}(u_{AF_1}) \vee \sigma_{AF_1}(u_{AF_2})]\\
&\quad \vee [\sigma'_{AF_1}(v_{AF_1}) \vee \sigma'_{AF_1}(v_{AF_2})]\\
&= [\sigma_{AF_1}(u_{AF_1}) \vee \sigma'_{AF_1}(v_{AF_1})]\\
&\quad \vee [\sigma_{AF_1}(u_{AF_2}) \vee \sigma'_{AF_1}(v_{AF_2})]\\
&= (\sigma_{AF_1} * \sigma'_{AF_1})(u_{AF_1} v_{AF_1})\\
&\quad \vee (\sigma_{AF_1} * \sigma'_{AF_1})(u_{AF_2} v_{AF_2}),\\
&\mu_{AF_2} * \mu'_{AF_2}((u_{AF_1}, v_{AF_1})(u_{AF_2}, v_{AF_2}))\\
&= \mu_{AF_2}(u_{AF_1} u_{AF_2}) \vee \mu'_{AF_2}(v_{AF_1} v_{AF_2})\\
&= [\sigma'_{AF_2}(v_{AF_1}) \wedge \sigma'_{AF_2}(v_{AF_2})]\\
&\quad \wedge [\sigma_{AF_2}(u_{AF_1}) \wedge \sigma_{AF_2}(u_{AF_2})]\\
&= [\sigma_{AF_2}(u_{AF_1}) \wedge \sigma'_{AF_2}(v_{AF_1})]\\
&\quad \wedge [\sigma_{AF_2}(u_{AF_2}) \wedge \sigma'_{AF_2}(v_{AF_2})]\\
&= (\sigma_{AF_2} * \sigma'_{AF_2})(u_{AF_1} v_{AF_1})\\
&\quad \wedge (\sigma_{AF_2} * \sigma'_{AF_2})(u_{AF_2} v_{AF_2}).
\end{aligned}$$

Hence, $G_{AF_1} * G_{AF_2}$ is a complete IAFG. □

Theorem 3.3. If $G_{AF_1} : (\mu_{AF_1}, \mu_{AF_2})$ and $G_{AF_2} : (\mu'_{AF_1}, \mu'_{AF_2})$ are two strong IAFGs, then $G_{AF_1} * G_{AF_2}$ is also a strong IAFG.

Proof. W.K.T conormal product of two IAFGs is an IAFG.

To prove: Conormal product of both IAFGs is strong.

An IAFG is said to be strong if $\mu_{AF_1}(v_{AF_i}, v_{AF_j}) = \sigma_{AF_1}(v_{AF_i}) \vee \sigma_{AF_1}(v_{AF_j})$ and $\mu_{AF_2}(v_{AF_i}, v_{AF_j}) = \sigma_{AF_2}(v_{AF_i}) \wedge \sigma_{AF_2}(v_{AF_j})$ for all $v_{AF_i}, v_{AF_j} \in V_{AF}$.

Case (i) If $u_{AF_1} u_{AF_2} \in E_{AF_1}$ and $v_{AF_1} = v_{AF_2}$,

$$\begin{aligned}
&\mu_{AF_1} * \mu'_{AF_1}((u_{AF_1}, v_{AF_1})(u_{AF_2}, v_{AF_2}))\\
&= \mu_{AF_1}(u_{AF_1} u_{AF_2}) \vee \sigma'_{AF_1}(v_{AF_1})\\
&= [\sigma_{AF_1}(u_{AF_1}) \vee \sigma_{AF_1}(u_{AF_2})]\\
&\quad \vee [\sigma'_{AF_1}(v_{AF_1}) \vee \sigma'_{AF_2}(v_{AF_2})]\\
&= [\sigma_{AF_1}(u_{AF_1}) \vee \sigma'_{AF_1}(v_{AF_1})]\\
&\quad \vee [\sigma_{AF_1}(u_{AF_2}) \vee \sigma'_{AF_2}(v_{AF_2})]\\
&= (\sigma_{AF_1} * \sigma'_{AF_1})(u_{AF_1} v_{AF_1})\\
&\quad \vee (\sigma_{AF_1} * \sigma'_{AF_1})(u_{AF_2} v_{AF_2}),\\
&\mu_{AF_2} * \mu'_{AF_2}((u_{AF_1}, v_{AF_1})(u_{AF_2}, v_{AF_2}))\\
&= \mu_{AF_2}(u_{AF_1}, u_{AF_2}) \wedge \sigma'_{AF_2}(v_{AF_1})\\
&= [\sigma_{AF_2}(u_{AF_1}) \wedge \sigma_{AF_2}(u_{AF_2})]\\
&\quad \wedge [\sigma'_{AF_2}(v_{AF_1}) \wedge \sigma'_{AF_2}(v_{AF_2})]\\
&= [\sigma_{AF_2}(u_{AF_1}) \wedge \sigma'_{AF_2}(v_{AF_1})]\\
&\quad \wedge [\sigma_{AF_2}(u_{AF_2}) \wedge \sigma'_{AF_2}(v_{AF_2})]\\
&= (\sigma_{AF_2} * \sigma'_{AF_2})(u_{AF_1} v_{AF_1})\\
&\quad \wedge (\sigma_{AF_2} * \sigma'_{AF_2})(u_{AF_2} v_{AF_2}).
\end{aligned}$$

Case (ii) If $v_{AF_1}v_{AF_2} \in E_{AF_2}$ and $u_{AF_1} = u_{AF_2}$,

$$\begin{aligned}
&\mu_{AF_1} * \mu'_{AF_1}((u_{AF_1}, v_{AF_1})(u_{AF_2}, v_{AF_2}))\\
&= \mu'_{AF_1}(v_{AF_1}, v_{AF_2}) \vee \sigma_{AF_1}(u_{AF_1})\\
&= [\sigma'_{AF_1}(v_{AF_1}) \vee \sigma'_{AF_2}(v_2)]\\
&\quad \vee [\sigma_{AF_1}(u_{AF_1}) \vee \sigma_{AF_1}(u_{AF_2})]\\
&= [\sigma_{AF_1}(u_{AF_1}) \vee \sigma'_{AF_1}(v_{AF_1})]\\
&\quad \vee [\sigma_{AF_1}(u_{AF_2}) \vee \sigma'_{AF_1}(v_{AF_2})]\\
&= (\sigma_{AF_1} * \sigma'_{AF_1})(u_{AF_1}v_{AF_1})\\
&\quad \vee (\sigma_{AF_1} * \sigma'_{AF_1})(u_{AF_2}v_{AF_2}),\\
&\mu_{AF_2} * \mu'_{AF_2}((u_{AF_1}, v_{AF_1})(u_{AF_2}, v_{AF_2}))\\
&= \mu'_{AF_2}(v_{AF_1}, v_{AF_2}) \wedge \sigma'_{AF_2}(u_{AF_2})\\
&= [\sigma'_{AF_2}(v_{AF_1}) \wedge \sigma'_{AF_2}(v_{AF_2})]\\
&\quad \wedge [\sigma_{AF_2}(u_{AF_1}) \wedge \sigma_{AF_2}(u_{AF_2})]\\
&= [\sigma_{AF_2}(u_{AF_1}) \wedge \sigma'_{AF_2}(v_{AF_1})]\\
&\quad \wedge [\sigma_{AF_2}(u_{AF_2}) \wedge \sigma'_{AF_2}(v_{AF_2})]\\
&= (\sigma_{AF_2} * \sigma'_{AF_2})(u_{AF_1}v_{AF_1})\\
&\quad \wedge (\sigma_{AF_2} * \sigma'_{AF_2})(u_{AF_2}v_{AF_2}).
\end{aligned}$$

Case (iii) If $u_{AF_1}u_{AF_2} \in E_{AF_1}$ and $v_{AF_1}v_{AF_2} \notin E_{AF_2}$,

$$\begin{aligned}
&\mu_{AF_1} * \mu'_{AF_1}((u_{AF_1}, v_{AF_1})(u_{AF_2}, v_{AF_2}))\\
&= \mu_{AF_1}(u_{AF_1}u_{AF_2}) \vee \sigma'_{AF_1}(v_{AF_1}) \vee \sigma'_{AF_1}(v_{AF_2})\\
&= [\sigma_{AF_1}(u_{AF_1}) \vee \sigma_{AF_1}(u_{AF_2})]\\
&\quad \vee [\sigma'_{AF_1}(v_{AF_1}) \vee \sigma'_{AF_1}(v_{AF_2})]\\
&= [\sigma_{AF_1}(u_{AF_1}) \vee \sigma'_{AF_1}(v_{AF_1})]\\
&\quad \vee [\sigma_{AF_1}(u_{AF_2}) \vee \sigma'_{AF_1}(v_{AF_2})]\\
&= (\sigma_{AF_1} * \sigma'_{AF_1})(u_{AF_1}v_{AF_1})\\
&\quad \vee (\sigma_{AF_1} * \sigma'_{AF_1})(u_{AF_2}v_{AF_2}),\\
&\mu_{AF_2} * \mu'_{AF_2}((u_{AF_1}, v_{AF_1})(u_{AF_2}, v_{AF_2}))\\
&= \mu_{AF_2}(u_{AF_1}u_{AF_2}) \wedge \sigma'_{AF_2}(v_{AF_1}) \wedge \sigma'_{AF_2}(v_{AF_2})\\
&= [\sigma'_{AF_2}(v_{AF_1}) \wedge \sigma'_{AF_2}(v_{AF_2})]\\
&\quad \wedge [\sigma_{AF_2}(u_{AF_1}) \wedge \sigma_{AF_2}(u_{AF_2})]\\
&= [\sigma_{AF_2}(u_{AF_1}) \wedge \sigma'_{AF_2}(v_{AF_1})]\\
&\quad \wedge [\sigma_{AF_2}(u_{AF_2}) \wedge \sigma'_{AF_2}(v_{AF_2})]\\
&= (\sigma_{AF_2} * \sigma'_{AF_2})(u_{AF_1}v_{AF_1})\\
&\quad \wedge (\sigma_{AF_2} * \sigma'_{AF_2})(u_{AF_2}v_{AF_2}).
\end{aligned}$$

Case (iv) If $v_{AF_1}v_{AF_2} \in E_{AF_2}$ and $u_{AF_1}u_{AF_2} \notin E_{AF_1}$,

$$\begin{aligned}
&\mu_{AF_1} * \mu'_{AF_1}((u_{AF_1}, v_{AF_1})(u_{AF_2}, v_{AF_2}))\\
&= \mu_{AF_1}(v_{AF_1}v_{AF_2}) \vee \sigma'_{AF_1}(u_{AF_1}) \vee \sigma'_{A_{AF_1}}(u_{AF_2})\\
&= [\sigma_{AF_1}(u_{AF_1}) \vee \sigma_{AF_1}(u_{AF_2})]\\
&\quad \vee [\sigma'_{AF_1}(v_{AF_1}) \vee \sigma'_{AF_1}(v_{AF_2})]\\
&= [\sigma_{AF_1}(u_{AF_1}) \vee \sigma'_{AF_1}(v_{AF_1})]\\
&\quad \vee [\sigma_{AF_1}(u_{AF_2}) \vee \sigma'_{AF_1}(v_{AF_2})]\\
&= (\sigma_{AF_1} * \sigma'_{AF_1})(u_{AF_1}v_{AF_1})\\
&\quad \vee (\sigma_{AF_1} * \sigma'_{AF_1})(u_{AF_2}v_{AF_2}),\\
&\mu_{AF_2} * \mu'_{AF_2}((u_{AF_1}, v_{AF_1})(u_{AF_2}, v_{AF_2}))\\
&= \mu'_{A_2}(v_{AF_1}v_{AF_2}) \wedge \sigma_{A_2}(u_{AF_1}) \wedge \sigma_{AF_2}(u_{AF_2})\\
&= [\sigma'_{AF_2}(v_{AF_1}) \wedge \sigma'_{AF_2}(v_{AF_2})]\\
&\quad \wedge [\sigma_{AF_2}(u_{AF_1}) \wedge \sigma_{AF_2}(u_{AF_2})]\\
&= [\sigma_{AF_2}(u_{AF_1}) \wedge \sigma'_{AF_2}(v_{AF_1})]\\
&\quad \wedge [\sigma_{AF_2}(u_2) \wedge \sigma'_{AF_2}(v_{AF_2})]\\
&= (\sigma_{AF_2} * \sigma'_{AF_2})(u_{AF_1}v_{AF_1})\\
&\quad \wedge (\sigma_{AF_2} * \sigma'_{AF_2})(u_{AF_2}v_{AF_2}).
\end{aligned}$$

Case (v) If $u_{AF_1}u_{AF_2} \in E_{AF_1}$ and $v_{AF_1}v_{AF_2} \in E_{AF_2}$,

$$\begin{aligned}
&\mu_{AF_1} * \mu'_{AF_1}((u_{AF_1}, v_{AF_1})(u_{AF_2}, v_{AF_2}))\\
&= \mu_{AF_1}(u_{AF_1}, u_{AF_2}) \vee \sigma'_{AF_1}(v_{AF_1}v_{AF_2})\\
&= [\sigma_{AF_1}(u_{AF_1}) \vee \sigma_{AF_1}(u_{AF_2})]\\
&\quad \vee [\sigma'_{AF_1}(v_{AF_1}) \vee \sigma'_{AF_1}(v_{AF_2})]\\
&= [\sigma_{AF_1}(u_{AF_1}) \vee \sigma'_{AF_1}(v_{AF_1})]\\
&\quad \vee [\sigma_{AF_1}(u_{AF_2}) \vee \sigma'_{AF_1}(v_{AF_2})]\\
&= (\sigma_{AF_1} * \sigma'_{AF_1})(u_{AF_1}v_{AF_1})\\
&\quad \vee (\sigma_{AF_1} * \sigma'_{AF_1})(u_{AF_2}v_{AF_2}),\\
&\mu_{AF_2} * \mu'_{AF_2}((u_{AF_1}, v_{AF_1})(u_{AF_2}, v_{AF_2}))\\
&= \mu_{AF_2}(u_{AF_1}u_{AF_2}) \vee \mu'_{AF_2}(v_{AF_1}v_{AF_2})\\
&= [\sigma'_{AF_2}(v_{AF_1}) \wedge \sigma'_{AF_2}(v_{AF_2})]\\
&\quad \wedge [\sigma_{AF_2}(u_{AF_1}) \wedge \sigma_{AF_2}(u_{AF_2})]\\
&= [\sigma_{AF_2}(u_{AF_1}) \wedge \sigma'_{AF_2}(v_{AF_1})]\\
&\quad \wedge [\sigma_{AF_2}(u_{AF_2}) \wedge \sigma'_{AF_2}(v_{AF_2})]\\
&= (\sigma_{AF_2} * \sigma'_{AF_2})(u_{AF_1}v_{AF_1})\\
&\quad \wedge (\sigma_{AF_2} * \sigma'_{AF_2})(u_{AF_2}v_{AF_2}).
\end{aligned}$$

Hence, $G_{AF_1} * G_{AF_2}$ is also a strong IAFG. □

Theorem 3.4. If $G_{AF_1} : (\mu_{AF_1}, \mu_{AF_2})$ and $G_{AF_2} : (\mu'_{AF_1}, \mu'_{AF_2})$ are two regular IAFGs, then $G_{AF_1} * G_{AF_2}$ is also a regular IAFG.

Proof. Let $G_{AF_1} = (V_{AF}, E_{AF}, \sigma_{AF}, \mu_{AF})$ and $G_{AF_2} = (V'_{AF}, E'_{AF}, \sigma'_{AF}, \mu'_{AF})$ be the conormal product of IAFGs.

To prove: The conormal product of IAFGs is also an IAFG.

Let $G_{AF_1} = (V_{AF}, E_{AF}, \sigma_{AF}, \mu_{AF})$ and $G_{AF_2} = (V'_{AF}, E'_{AF}, \sigma'_{AF}, \mu'_{AF})$ be IAFGs and $G_{AF_1} * G_{AF_2}$ be their conor-

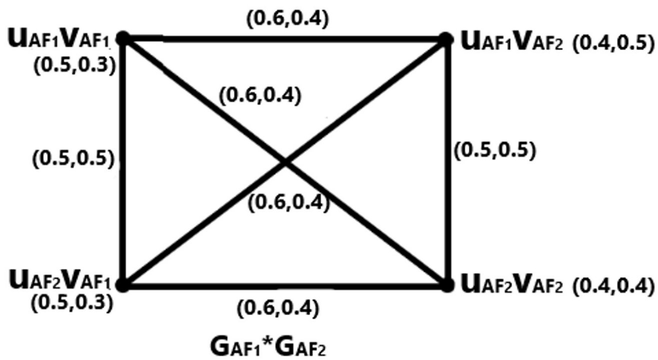

Figure 7. Conormal product of regular IAFG.

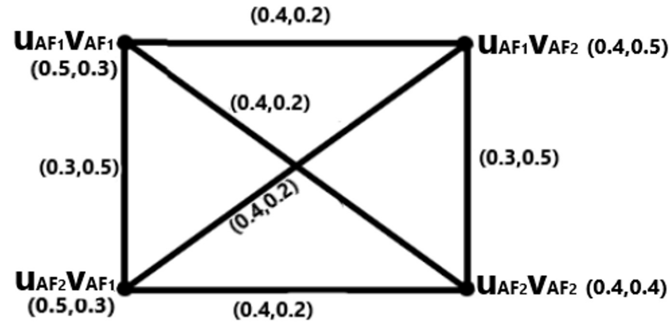

Figure 8. Regular IAFG.

mal product; then, $d_{FG_{AF_1} * G_{AF_2}}(u_{AF_i}, v_{AF_j}) = \Im$ for all i, $j = 1, 2, ..., n$. A regular fuzzy graph of degree $\Im$, or a $\Im$-regular fuzzy graph, is one in which each vertex has the same degree. An IAFG is a regular IAFG in this sense.

Example 3.7 See Figure 7.

In Fig. 7,

$$\begin{aligned} d_{AF}(u_{AF_1}, v_{AF_1}) &= 1.7, d_{AF}(u_{AF_1}, v_{AF_2}) \\ &= 1.7, d_{AF}(u_{AF_2}, v_{AF_1}) \\ &= 1.7, d_{AF}(u_{AF_2}, v_{AF_2}) \\ &= 1.7. \end{aligned}$$

From the above we get

$$\begin{aligned} d_{AF}(u_{AF_1}, v_{AF_1}) &= d_{AF}(u_{AF_1}, v_{AF_2}) \\ &= d_{AF}(u_{AF_2}, v_{AF_1}) \\ &= d_{AF}(u_{AF_2}, v_{AF_2}). \end{aligned}$$

Hence, $G_{AF_1} * G_{AF_2}$ is a regular IAFG.

Remark 3.1. Although the conormal product of IAFGs is regular, regular fuzzy graphs need not be the conormal product of IAFGs.

Example 3.8. See Figure 8.

The above example is a regular and pseudo-regular fuzzy graph. However, it is not a conormal product of IAFGs.

4. Application

In this example, we demonstrated how to use the conormal product of two IAFGs. Consider the following example, where G_{AF_1} and G_{AF_2} represent two IFAGs and two vertices representing distinct grocery goods. The MS and NMS values of the vertices denote the proportion of a product's quality and cost, respectively.

In G_{AF_1}, the MS and NMS values of the vertices reflect the percentage of a product's quality and cost, respectively. The MS and NMS values of the edges show the maximum relative product quality and minimum product cost, respectively. In addition, for G_{AF_2}, the MS and NMS values of the vertices represent the percentage of a product's quality and cost, respectively. The MS and NMS values of the edges represent the maximum relative product quality and minimum product cost, respectively. The conormal product $G_{AF_1} * G_{AF_2}$ of G_{AF_1} and G_{AF_2} shows the percentage change in product quality for the highest MS value and the percentage change in product cost for the lowest NMS value.

Consequently, the most effective combinations with the highest percentage of quality improvement and the lowest percentage of cost increase were discovered across all supermarkets.

5. Conclusion

In this manuscript, we present the conormal product of IAFGs. We define the conormal product of IAFGs, and use a few examples to demonstrate its properties. Each of the given definitions and hypothetical methodologies is appropriate for regular, complete, and strong IAFGs.

In future work, different kinds of products of different types of IFGs, like m-polar, interval-valued, vague, and hesitancy anti-fuzzy graphs, will be discussed. The above anti-fuzzy graphs are useful for practical applications. We are committed to managing other maintainable improvement objectives for a better world.

Conflict of Interest

No potential conflict of interest relevant to this article was reported.

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