ALGORITHMIC APPROACH TO FUZZY LINEAR SUM ASSIGNMENT PROBLEMS

Thesis submitted to the Bharathidasan University, Tiruchirappalli in partial fulfillment of the requirements for the Degree of

DOCTOR OF PHILOSOPHY IN MATHEMATICS

Submitted By

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CERTIFICATE

This thesis entitled "**ALGORITHMIC APPROACH TO FUZZY LINEAR SUM ASSIGNMENT PROBLEMS**" **submitted by Mr. T. SHIEK PAREETH for the fulfillment of the degree of Doctor of Philosophy in Mathematics** is a record of original work done by him under my guidance. It has not previously formed the basis for the award of any Degree, Diploma, Associateship, Fellowship or any such similar title of any University or Institution.

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DECLARATION

I do hereby declare that this work has been originally carried out by me under the guidance and supervision of **Dr. A. NAGOOR GANI**, Associate Professor, P.G. & Research Department of Mathematics, Jamal Mohamed College (Autonomous), Tiruchirappalli - 620 020 and that this work has not been submitted elsewhere for any other degree.

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AUGUST 2022

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PREFACE

Fuzzy logic is similar to how people make decisions. There are a lot of fuzzy situations that give unclear information in our universe. It deals with fuzzy and ambiguous data. This is a gross oversimplification of the difficulties in the real world. Uncertain or ambiguous items are referred regarded as fuzzy. We must characterize such activities in a fuzzy fashion since any event, process, or function that is always changing cannot always be defined as either true or false. The current study examines various algorithmic approaches to solving fuzzy linear sum assignment problems and its modifications. The study could be viewed as a modest step toward the development of the fuzzy linear sum assignment problem.

Lofti A. Zadeh's [79] 1965 research article "Fuzzy Sets" introduced fuzzy logic. He is regarded as the father of fuzzy logic. Linear sum assignment problem was originally proposed by R. Burkard, Mauro Dell'Amico, and Silvano Martello [14]. This thesis uses various fuzzy numbers, defuzzifying ω-trapezoidal fuzzy numbers, and generalised trapezoidal fuzzy numbers to identify methods and various algorithmic approaches with complimentary slackness conditions. The main concepts discussed in this thesis are some new algorithmic approaches for solving fuzzy linear sum assignment problems to obtain optimal/feasible or partial/complete matching solutions in the bipartite graph. Many authors L.Yang and B.Liu , P.S. Pundir, S.K. Porwal and Brijesh, P. Singh and U. Derigs [23] utilized methods of their own interest and made various algorithms to tackle the linear sum assignment problems and fuzzy assignment problems.

This thesis is organized into eight chapters in order to present the FLSAP, with the above methods:

Chapter I Introduction

In this chapter presents, a basic preliminaries of assignment problems, linear sum assignment problems, the basic concepts of fuzzy sets, fuzzy numbers and fuzzy linear sum assignment problems are discussed and proposed different types of fuzzy numbers, α-cut fuzzy numbers also discussed. Obtained Dual and partial feasible solutions and complete optimal solutions by using different fuzzy optimization matching techniques of fuzzy assignment problems and linear sum assignment problems. Finally, the arithmetic functions of different types of fuzzy numbers are presented. This method is given by a numerical example.

Chapter II Dual and Partial Primal Solution for Solving Fuzzy Linear Sum Feasible Assignment Problems [FLSAP]

This chapter presents, a new method by using complementary slackness conditions to calculate the fuzzy dual solution and fuzzy partial primal solution using a bipartite graph and the assignment cost were taken as $\omega -$trapezoidal fuzzy numbers. This chapter solves the primal problem first, then the dual problem, and finally the primal-dual problem. Here first construct $\omega -$ trapezoidal fuzzy Linear Sum Assignment table. Next we have to find row and column reduction, it's worth noting that the resulting dual variables of lower costs aren't negative and calculate reduced cost matrix $\bar{C}$ next to obtain partial feasible solution. This method is illustrated by a numerical example.

Chapter III Feasible Degenerate Pivoting and Optimal Non-Degenerate Pivoting for Solving Fuzzy Linear Sum Assignment Problems

This chapter proposes Feasible Degenerate Pivoting and Optimal Non-Degenerate Pivoting for Solving Fuzzy Linear Sum Assignment Problems. In this chapter the spanning trees on the associated bipartite graph G = (U, V; E). If a strongly feasible tree is producing degenerate pivoting on a backward edge and the rank of reduced fuzzy cost matrix $[\Re(\widetilde{\overline{C_{\iota J}}})]$ is negative and then the current fuzzy linear sum assignment problem is not optimal. If a strongly feasible tree is producing non-degenerate pivoting on a forward edge and the rank of the reduced cost matrix $[\Re(\widetilde{\overline{C_{\iota J}}})]$ is non-negative and $\bar{E}= \emptyset$ then the Current fuzzy linear sum assignment problem is optimal. This method is discussed by a numerical example.

Chapter IV A New Modified Optimal Perfect Matching in Partial Feasible Matching for Solving Fuzzy Linear Sum Assignment Problems [FLSAP]

This chapter proposes a new modified optimal and perfect matching from partial assignment for solving fuzzy linear sum assignment problem. The fuzzy assignment cost is we take as ω-Trapezoidal fuzzy numbers. By using ranking method, ω-Trapezoidal fuzzy numbers converted to crisp one. First calculate the rank of fuzzy dual variables and compute a partial feasible solution then calculate reduced rank of ω-trapezoidal fuzzy cost and find the new column and the new assignment introduced, Continue the process to reach optimal solution and complete bipartite matching. This method is illustrated by a numerical example.

Chapter V Spread of New Partial/ Feasible and Optimal/ Perfect Matching for Solving Interval-Valued α-Cut Fuzzy Linear Sum Bottleneck Assignment Problem.

This chapter proposes a spread of new partial/feasible and optimal/perfect matches of bipartite graphs for solving interval-valued α-cuts of generalized trapezoidal fuzzy numbers. Obtain α-cut generalized trapezoidal fuzzy numbers from generalized trapezoidal fuzzy numbers, then discuss membership functions. The α-cut generalized trapezoidal fuzzy number is transformed into an Interval-valued α-Cut of Generalized Fuzzy Numbers. The basic preliminaries and fuzzy interval operations are discussed. If the solution is maximum cost and complete match, then the solution is feasible and complete. If the solution is minimum cost and complete, then the solution is optimal and complete. If maximum cost and partial match, then the solution is feasible with partial match. If minimum cost or partial match, then the solution is optimal or feasible match. This method is presented by a numerical example.

Chapter VI A New Optimal Complete Matching of Edges with Minimum Cost by Ranking Method for Solving ω-Type -2 Fuzzy Linear Sum Assignment Problem[FLSAP]

This chapter proposes a new optimal solution and complete matching edges of bipartite graph. ω-type -2 [FLSAP] is converted to crisp one by using new ranking method for solving ω-type -2 [FLSAP]. This chapter discussed ω-trapezoidal fuzzy number, ω-type 1-Trapezoidal fuzzy number and ω-type 2-Trapezoidal fuzzy number. Create ω-type -2 [FLSAT].The rank of -type 2 Trapezoidal fuzzy number to assign each machine to a job with the lowest cost in that job for solving -type 2 [FLSAP].

Furthermore, each iteration updates a non-matched edge to a matched edge and update the corresponding dual variables. By using alternating path method to obtain a new optimal complete matching solution. This method is illustrated by a numerical example.

Chapter VII Fuzzy Multi-Objective Linear Sum Assignment Problem with Modified Partial Solution of ω - type 2 - Diamond Fuzzy Numbers[DFN] by Using Linguistic variables

This chapter proposes fuzzy multi-objective linear sum assignment problem with modified partial assignment of ω - type 2 - diamond fuzzy numbers using linguistic variables. In this chapter introduced ω - type 1 and ω - type 2 diamond fuzzy numbers. Let us consider four jobs and four machine problem and to optimize fuzzy cost , fuzzy time, fuzzy quality are each considered as a ω - type 2 - DFN. ω-type 2 DFN are converted into λ_d--cut of DFN and upper and lower ω-type 2 diamond multi-objective fuzzy numbers are converted into single objective λ_d-cut fuzzy number by using ranking method. Obtain dual variables and then calculate $[\widetilde{c_{ij}} - \widetilde{u_i} - \widetilde{v_j}]$; by using alternate path method increase the partial assignment. This method is illustrated by a numerical example, proving its efficiency.

Chapter VIII Minimum vertex cover of ω −Pentagonal Fuzzy Linear Sum Bottleneck Assignment Problem [$\omega - PFLSBAP$]

In this chapter we presented a spread of minimum solution of fuzzy optimization matching procedure in the bipartite graph. it provides minimum vertex cover with edge set E for solving $\omega - PFLSAP$. The $\omega - PFLSAP$ is minimum cost and complete matching in the bipartite graph. The Linear Sum Bottleneck Assignment Cost [LSBAC] we taken as ω −Pentagonal Fuzzy Numbers ($\omega - PFN$). If each person and each job contain exactly one matching solution with Spr (φ) = 0 or minimum Spr (φ), then the current ω-PFLSBAP is optimal. If each person and each job contain exactly one matching solution with maximum Spr (φ), then the current ω-PFLSBAP is not optimal but feasible and complete matching solution. Finally obtained the graph has minimum vertex cover of cardinality n with perfect or complete matching. This method is illustrated by a numerical example.

CONTENTS

CHAPTER I

INTRODUCTION

In this chapter presents, a basic preliminaries of assignment problems, linear sum assignment problems, fuzzy linear sum assignment problems and the basic concepts of fuzzy sets, fuzzy numbers are discussed and proposed different types of fuzzy numbers ,α-cut fuzzy numbers and also discussed . obtained Dual and partial feasible solutions and complete optimal solutions by using different fuzzy optimization matching techniques of fuzzy assignment problems and linear sum assignment problems. Finally, we present the arithmetic functions of different types of fuzzy numbers. This method is illustrated by a numerical example.

1.1 LINEAR SUM ASSIGNMENT PROBLEM

A special type of transportation problem is the assignment problem. There is just one assignment supply from all sources, and only one assignment demand from all destinations. The n × n matrix contains the assignment problem. We have 'n' jobs that are performed by 'n' workers, and our focus is to minimize costs or increase profits. Let C_{ij} be the assignment cost for the i^{th} person to the j^{th} job.

One of the most well-known problems in linear programming and combinatorial optimization is the linear sum assignment problem. We have a $(n \times n)$ cost matrix C = (c_{ij}) that we want to match each row to a different column so that the sum of the associated entries is as small as possible. To look at it another way, we want to select n elements from C so that there is exactly one element in each row and column, and the sum of the corresponding costs is as small as possible.

Bipartite graphs provide another way to describe assignments. If every edge connects a vertex of U with a vertex of V and there are no edges that have both endpoints in U and in V, a graph G = (U, V; E) with disjoint vertex sets U and V and edge set E is called bipartite. A matching M in G is a subset of the edges where every vertex of G meets at least one matching edge. Assume that the total number of vertices in U and V is n, and that |U| = |V|= n. The matching M is said to be perfect if every vertex of G coincides with an edge of the matching M in this case.

1.2 FUZZY SET THEORY

In real world, fuzzy data is frequently used in human thinking and reasoning. Humans are capable of providing satisfactory responses that are most likely correct. Because our systems are built on classical set theory, they are unable to respond to a broad range of questions. Our system should be able to handle unreliable and incomplete information, as we desire. A solution was provided through a fuzzy system.

Many real-world application problems, especially those involving elements that are only partially members of a set, cannot be explained and solved using classical set theory. Fuzzy set theory, on the other hand, allows partial memberships and so extends classical set theory in some ways. To introduce the concept of fuzzy sets, we will first go through the basics of classical mathematics' set theory. It will be seen that fuzzy set theory is both a natural extension and a formal mathematical notion of classical set theory.

The process of converting a crisp set into a fuzzy set or a fuzzy set into a fuzzier set may be used to describe it. In general, this process converts accurate, crisp input values into linguistic variables. The two key approaches of fuzzification are (i) Support Fuzzification Method and (ii) Grade Fuzzification Method.

The process transforms a fuzzy set into a crisp set or a fuzzy member into a crisp member can be used to describe it. The conversion of crisp values to fuzzy quantities is a step in the fuzzification process. Defuzzification, or rather the conversion of a "fuzzy result" to a crisp result, is required in a number of technical applications. Defuzzification is also known as "rounding it off" in mathematics. The different methods of Defuzzification are (i) Max-Membership Method, (ii) Centroid Method, (iii) Weighted Average Method, (iv) Mean-Max Membership and so on.

Lofti A. Zadeh [79] proposed membership functions for the first time in 1965. fuzzy sets After commencing, it developed in a variety of ways across a variety of fields. Fuzzy logic is used to describe fuzziness, not because it is fuzzy logic. The membership function of this fuzziness serves as its best characterization. In other terms, we may say that the membership function in fuzzy logic represents the degree of truth. Fuzzy logic can manage data obtained from computational perception and cognition that is uncertain, imprecise, vague, partially true, or lacking sharp limits. In computational challenges, fuzzy logic allows for the inclusion of fuzzy human judgments. It also gives an excellent method for resolving multiple criterion conflicts and better evaluating solutions. New computing approaches based on fuzzy logic can be applied in the creation of intelligent systems for decision making, identification, pattern recognition, optimization, and control.

1.2.1 Definition: Fuzzy Sets

A fuzzy set $\widetilde{A_1}$ in a universal set X is defined by a membership function $\mu_{\widetilde{A_1}} : X \rightarrow$[0,1].Where $\mu_{\widetilde{A_1}}\ (x)$ is the degree of membership function in the fuzzy set $\widetilde{A_1}$, and is denoted by $\widetilde{A_1}$ $= \{(x, \mu_{\widetilde{A_1}}\ (x)): x\epsilon X\}$.

1.2.2 Example:

If X = {Apple, Orange, Banana, Grapes, Cherry, Watermelon} the membership value of the set

$\widetilde{A_1}$ = (Apple, 0.4), (Orange, 0.5), (Banana, 0.75), (Grapes, 0.6), (Cherry, 0.3) , (Kuvi, 0.9)}

is a fuzzy set on X.

1.2.3 Definition:

A fuzzy set $\widetilde{A_1}$ is said to be normal if its defined in a universe of discourse holds total ordering, maximal membership value $\mu_{\widetilde{A_1}}$ is equal to one, that is $\mu_{\widetilde{A_1}}\ (x) = 1$.

1.2.4 Definition:

An element of a crisp subset of X with all of its elements with nonzero membership grades is the support of a fuzzy set $\widetilde{A_1}$, and is defined by support of $\widetilde{A_1}$ $= \{\ x\epsilon X: \mu_{\widetilde{A_1}}\ (x) > 0\}$.

1.2.5 Definition:

The crisp subset of the universe of discourse X in α -cut ($\tilde{A}\ _\alpha$) whose elements all have membership degrees greater than or equal to is α, that is $\tilde{A}\ _\alpha = \{x\ \epsilon\ X: \mu_{\widetilde{A_1}}\ (x) \geq \alpha\}$

1.2.6 Example:

If $X\ =\ \{P, Q, R, S, T\}$, membership value $\widetilde{A_1} =$

$\{(P, 0.5), (Q, 0.7), (R, 0.2), (S, 0.8), (T, 0.1)\}$ fuzzy set on X, then $\tilde{A}_{0.5}$= {P,Q,S}.

1.2.7 Definition:

A fuzzy set $\widetilde{A_1}$ is convex if $\mu_{\widetilde{A_1}}(\lambda p + (1 - \lambda) q) \geq \min \{\mu_{\widetilde{A_1}}(p), \mu_{\widetilde{A_1}}(q)\} \forall$ p,q $\epsilon\ R^n$ and $\lambda\ \epsilon$ [0,1].

1.3 FUZZY NUMBERS

In fuzzy mathematics, fuzzy numbers perform a vital role, much like ordinary numbers do in crisp mathematics. A fuzzy number is a generalization of a regular, real number in that it refers to a connected collection of potential values instead of a single value, where each potential value has a weight between 0 and 1. This factor is referred to as the membership function. A convex, normalized fuzzy set of the real line is a special case of a fuzzy number, which implies. Fuzzy numbers are a development of real numbers, much like fuzzy logic is a development of Boolean logic. Uncertainty in parameters, characteristics, geometry, initial conditions, etc., can be incorporated into calculations using fuzzy numbers.

A fuzzy number is a quantity whose value is imprecise, rather than exact as is the case with "ordinary" (single-valued) numbers. In many respects, fuzzy numbers depict the physical world more realistically than single-valued numbers.

The membership functions of these sets, which have the form $\widetilde{A_1}$: R → [0,1], have a quantitative meaning that is obvious, albeit they can occasionally be interpreted as fuzzy numbers. To interpret them in this way, they must embody our intuitive ideas of approximate numbers, such as "numbers that are close to a particular real value" or "numbers that are about a specific interval of real numbers." Determining the states of fuzzy variables requires the use of such ideas, which are crucial for many applications, such as fuzzy control, decision-making, and optimization and so on.

A fuzzy set $\widetilde{A_1}$ on R must have at least three of the following characteristics in order to be considered a fuzzy number.

- $\widetilde{A_1}$ must also be a normal fuzzy set;
- $\tilde{A}_{\alpha}$ must also be a closed interval for every $\alpha\epsilon(0,1)$;
- The support of $\widetilde{A_1}$ must always be bounded

Assuming that r fully satisfies our conception of a set of real numbers close, the fuzzy set must be normal therefore r's membership grade in any attempt to represent this conception must be 1. Each fuzzy number is a convex fuzzy set because for all α -cuts of a fuzzy number must be closed intervals for every (0,1).

One approach, in particular, to describe imprecise information, is provided by fuzzy set theory. A specific subclass of fuzzy sets on the real line is composed of fuzzy numbers. This concept's fundamental idea is motivated by the observation that people frequently use imprecise numbers to convey their basic knowledge of objects.

1.3.1 GENERALIZED FUZZY NUMBER

A fuzzy set $\widetilde{A_1}$= (p,q,r,s; ω) is said to be a generalized fuzzy number if its membership function satisfies the condition listed below. It is defined on the universal set R of real numbers.

1. The membership function $\mu_{\widetilde{A_1}}\ (x)$: R tends to [0,ω] is continuous.
2. The membership function $\mu_{\widetilde{A_1}}\ (x)$ is equal to zero for all x ∈ $(-\infty,p)$ ∪ [s,∞).
3. The membership function $\mu_{\widetilde{A_1}}\ (x) = \omega\ \forall$ x ∈ $[q,r]$, where 0 < x ≤ 1.
4. The membership function strictly increasing on [p,q] and strictly decreasing on [r,s]

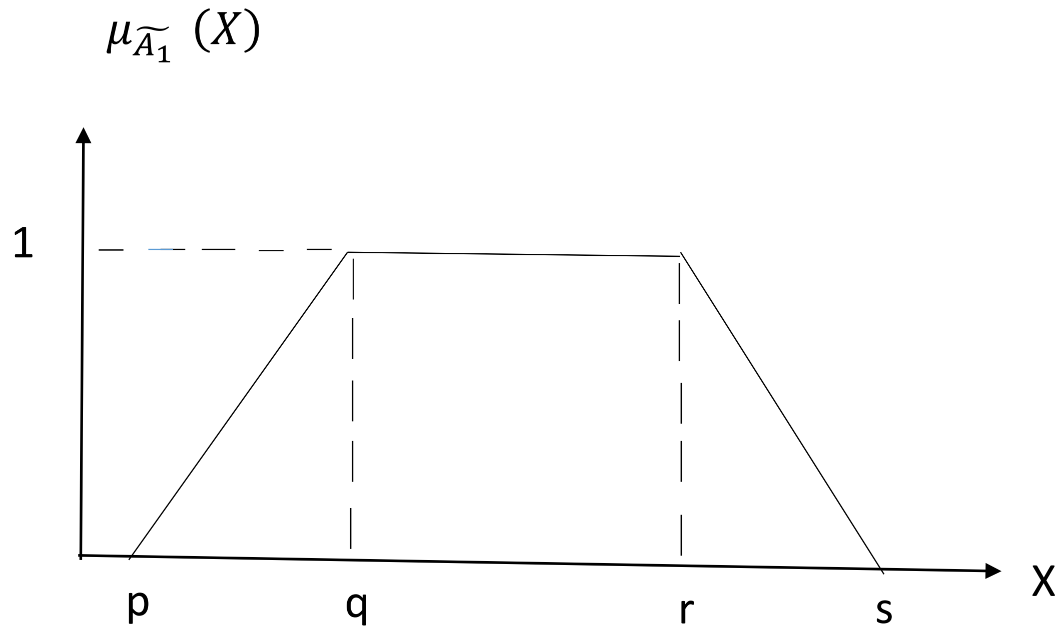
1.4 TRAPEZOIDAL FUZZY NUMBER

The membership function of a trapezoidal fuzzy number is piecewise linear and trapezoidal, which can represent the fuzziness of certain linguistic assessments. Fuzzy variables are important because they enable gradual changes in state and, as a consequence, have a built-in ability to describe and handle measurement and observational uncertainty. Fuzzy variables capture measurement uncertainties as part of experimental data, they are more attuned to reality than crisp variables.it is an interesting paradox that data based on fuzzy variables provide us, in fact, with more accurate evidence about real phenomena than data based upon crisp variables. Suppose, we take five linguistic concepts are represented by the fuzzy set: extremely low, low, medium, high, and very high. The graphs of these functions have trapezoidal forms, and they are all determined by membership functions of the form $[R_1, R_2] \rightarrow [0,1]$.

1.4.1 Definition:

The fuzzy number $\widetilde{A_1} \;=\; (p, q, r, s)$ is referred to as a *trapezoidal fuzzy number* if the membership function is as follows:

$$
\begin{aligned}
\mu_{\widetilde{A_1}}(x) &= 0 && \text{if} \quad x \leq p \\
&= \frac{x-p}{q-p} && \text{if} \quad p \leq x \leq q \\
&= 1 && \text{if} \quad q \leq x \leq r \\
&= \frac{s-x}{s-r} && \text{if} \quad r \leq x \leq s \\
&= 0 && \text{if} \quad x \geq s
\end{aligned}
$$

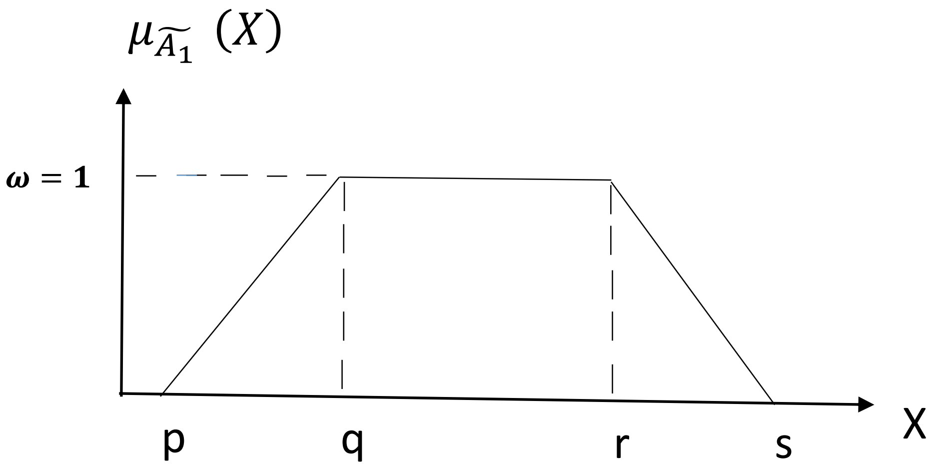


1.4.2 Definition:

The fuzzy number $\widetilde{A_1} \ = \ (p, q, r, s; \omega)$ is referred to as a *generalized trapezoidal fuzzy number* if the membership function is as follows:

$$
\begin{aligned}
\mu_{\widetilde{A_1}}\ (x) &= 0, && \text{if} \quad x < p \\
&= \omega \left(\frac{x-p}{q-p}\right), && \text{if} \quad p \le x \le q \\
&= \omega\ , && \text{if} \quad q \le x \le r \\
&= \omega \left(\frac{s-x}{s-r}\right), && \text{if} \quad r \le x \le s \\
&= 0\ , && \text{if} \quad x > s
\end{aligned}
$$

Where $\omega \in [0,1]$

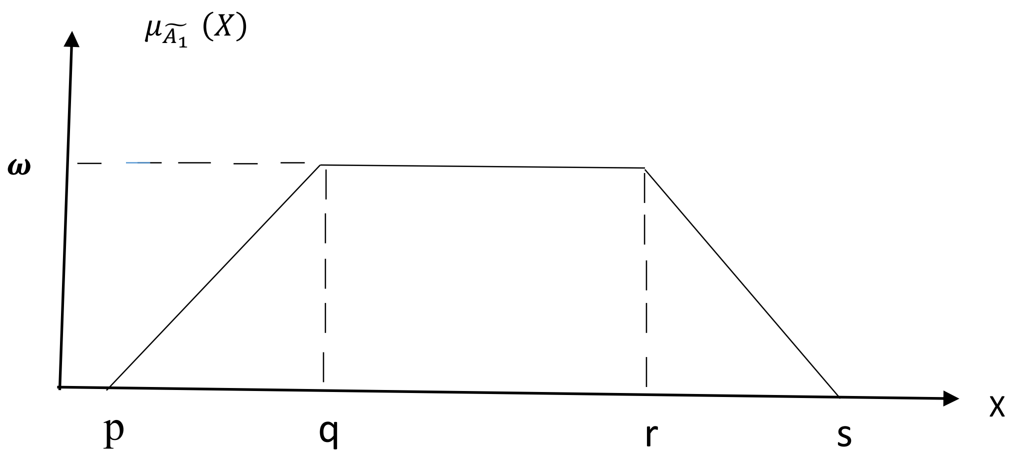


1.4.3 Definition:

The fuzzy number $\widetilde{A_1}\ =\ (p, q, r, s; \omega)$ is referred to as a ω *−trapezoidal Fuzzy Number* if the membership function is as follows:

$$\mu_{\widetilde{A_1}}\ (x) = \begin{cases} 0 & \text{if} \quad x < p \\ \omega\left(\frac{x-p}{q-p}\right) & \text{if} \quad p \leq x \leq q \\ \omega & \text{if} \quad q \leq x \leq r \\ \omega\left(\frac{s-x}{s-r}\right) & \text{if} \quad r \leq x \leq s \\ 0 & \text{if} \quad x > s \end{cases}$$

Where $\omega \in (0,1)$



1.5 DIAMOND FUZZY NUMBER

A diamond Fuzzy Number $\widetilde{A}_D$ = {$d', d^*, d''(\alpha_d, \beta_d)$} are satisfy the following conditions;

(i) $\mu_{\tilde{A}_D}$is a continuous function in interval[0,1].

(ii) $\mu_{\tilde{A}_D}$ is strictly increasing and continuous function on $[d', d^*]$

(iii) $\mu_{\tilde{A}_D}$ is strictly decreasing and continuous function on $[d^*, d'']$.

1.5.1 Definition:

The fuzzy number $\widetilde{A_D} = \{d', d^*, d''(\alpha_b, \beta_b)\}$ is referred to as a *diamond fuzzy number* if the membership function is as follows:

$$\mu_{\tilde{A}_D} = 0 \quad \text{if} \quad x \leq d'$$

$$= \frac{x-d'}{d^*-d'} \quad \text{if} \quad d' \leq x \leq d^*$$

$$= \frac{d''-x}{d''-d^*} \quad \text{if} \quad d^* \leq x \leq d''$$

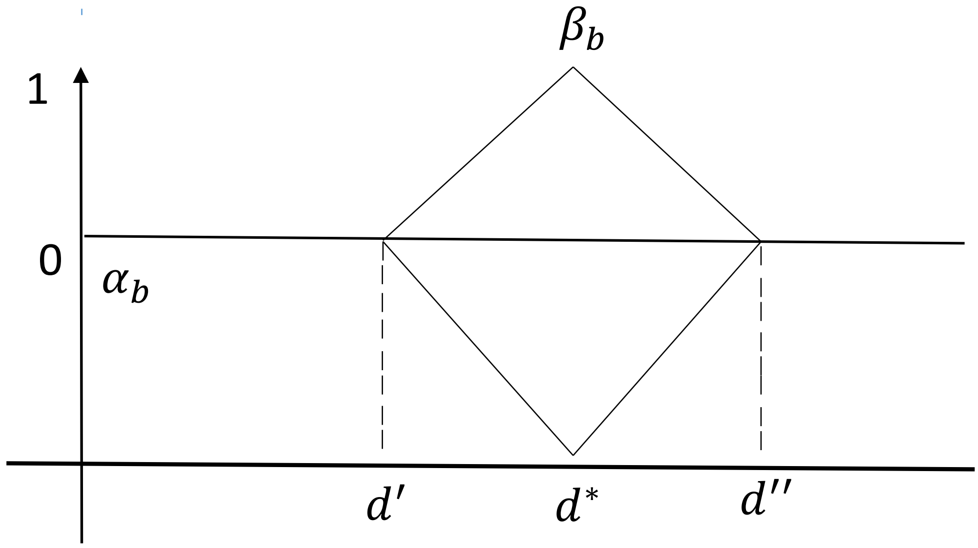
$$\alpha_b - base$$

$$= \frac{d'-x}{d'-d^*} \quad \text{if} \quad d' \leq x \leq d^*$$

$$= \frac{x-d''}{d^*-d''} \quad \text{if} \quad d^* \leq x \leq d''$$

$$= 1 \quad \text{if} \quad x = \beta_b$$

$$= 0 \quad otherwise$$



1.6 PENTAGONAL FUZZY NUMBER

A Pentagonal Fuzzy Number $\widetilde{A_1}$ = (P_1 , P_2 ,P_3 , P_4 , P_5 ;ω) are satisfy the following conditions;

(i) $\mu_{\widetilde{A_1}}(x)$ is a continuous function in the interval [0,1],

(ii) $\mu_{\widetilde{A_1}}(x)$ is strictly increasing and continuous function on [P_1 , P_2].

(iii) $\mu_{\widetilde{A_1}}(x)$ is strictly decreasing and continuous function on [, P_4 , P_5].

(iv) P_3 is the middle point and (P_1 , P_2) ; (P_4 , P_5) are the left and right side points of P_3.

(v) A Pentagonal Fuzzy Number $\tilde{P}$ = (P_1 , P_2 ,P_3 , P_4 , P_5 ;ω) where ω ∈ (0.5,1)

1.7 ARITHMETIC OPERATIONS

1.7.1 ARITHMETIC OPERATIONS ON ω-TRAPEZOIDAL FUZZY NUMBERS.

Let $\widetilde{A_1}$ = $(p_1, q_1, r_1, s_1; \omega_1)$ and $\widetilde{B_1}$ = $(p_2, q_2, r_2, s_2; \omega_2)$ be any two ω-trapezoidal fuzzy numbers, the then following operations are,

(i) $\widetilde{A_1} + \widetilde{B_1}$ = $(p_1+p_2, q_1+q_2, r_1+r_2, s_1+s_2;$ min$(\omega_1, \omega_2))$

(ii) $\widetilde{A_1} - \widetilde{B_1}$ = $(p_{1-}s_2, q_1-r_2, , r_1-q_2, s_1-p_2$; min$(\omega_1, \omega_2))$

(iii) $\gamma\widetilde{A_1} = \begin{cases} \gamma p_1, \gamma q_1, \gamma r_1, \gamma s_1;\ \omega_1), \gamma > 0 \\ \gamma s_1, \gamma r_1, \gamma q_1, \gamma p_1;\ \omega_2), \gamma < O \end{cases}$

1.7.2 ARITHMETIC OPERATIONS ON GENERALIZED TRAPEZOIDAL FUZZY NUMBERS:

Let $\widetilde{A_1}$ = $(\breve{a}^L, \breve{a}^\alpha, \breve{a}^\beta, \breve{a}^U, \omega)$ and $\widetilde{B_1}$ = $(\breve{b}^L, \breve{b}^\alpha, \breve{b}^\beta, \breve{b}^U, \omega)$ are two generalized trapezoidal fuzzy number then the following operations are,

1. $\widetilde{A_1} + \widetilde{B_1}$ = $(\breve{a}^L+\breve{b}^L, \breve{a}^\alpha + \breve{b}^\alpha, \breve{a}^\beta \widetilde{+} \breve{b}^\beta \breve{a}^U + \breve{b}^U, \omega)$ where ω = (min$(\omega_{1,}\ \omega_2)$)

2. $\widetilde{A_1} - \widetilde{B_1}$ = $(\breve{a}^L - \breve{b}^U, \breve{a}^\alpha$--$\breve{b}^\beta, \breve{a}^\beta - \breve{b}^\alpha, \breve{a}^U - \breve{b}^L, \omega)$ where ω = (min$(\omega_{1,}\ \omega_2)$).

1.7.3 ARITHMETIC OPERATIONS OF ω-TYPE-2 DIAMOND FUZZY NUMBER

Let us take two ω-type-2 diamond fuzzy number are given below $\omega_2\widetilde{F_d}^1$= [$\underline{\omega_2\widetilde{F_d}^1}$, $\overline{\omega_2\widetilde{F_d}^1}$] = $(\underline{d_1'}, \underline{d_2' d_3' d_4'}, \underline{d_5'}, \underline{d_6'}, \underline{\omega_{1F^1}}), (\overline{d_1'}, \overline{d_2'}, \overline{d_3'}, \overline{d_4'}, \overline{d_5'}, \overline{d_6'}, \overline{\omega_{1F^1}})$ and $\omega_2\widetilde{F_d}^2$= [$\underline{\omega_2\widetilde{F_d}^2}$, $\overline{\omega_2\widetilde{F_d}^2}$]=$(\underline{d_1''}, \underline{d_2''}, \underline{d_3''}, \underline{d_4''}, \underline{d_5''}, \underline{d_6''}, \underline{\omega_{1F^2}}), (\overline{d_1''}, \overline{d_2''}, \overline{d_3''}, \overline{d_4''}, \overline{d_5''}, \overline{d_6''}, \overline{\omega_{1F^2}})$.

The following arithmetic operations of $\omega_2\widetilde{F_d}^1$ and $\omega_2\widetilde{F_d}^2$.

$\omega_2\widetilde{F_d}^1 \oplus \omega_2\widetilde{F_d}^2$ =

$((\underline{d_1'}\oplus\underline{d_1''}, \underline{d_2'}\oplus\underline{d_2''}, ,\underline{d_3'}\oplus\underline{d_3''}, \underline{d_4'}\oplus\underline{d_4''}, \underline{d_5'}\oplus\underline{d_5''}, \underline{d_6'}\oplus\underline{d_6''}), \min\{\underline{\omega_{1F^1}}, \underline{\omega_{1F^2}}\}),$

$((\overline{d_1'}\oplus\overline{d_1''}, \overline{d_2'}\oplus\overline{d_2''}, ,\overline{d_3'},\oplus\overline{d_3''}, \overline{d_4'}\oplus\overline{d_4''}, \overline{d_5'}\oplus\overline{d_5''}, \overline{d_6'}\oplus\overline{d_6''}), \min\{\overline{\omega_{1F^1}}, \overline{\omega_{1F^2}}\})$

$\omega_2\widetilde{F_d}^1 \ \Theta \ \omega_2\widetilde{F_d}^2$ =

$((\underline{d_1'}\Theta\,\underline{d_6''}, \underline{d_2'}\Theta\,\underline{d_5''}, \underline{d_3'}\Theta\underline{d_4''}, \underline{d_4'}\Theta\,\underline{d_3''}, \underline{d_5'}\Theta\,\underline{d_2''}, \underline{d_6'}\Theta\,\underline{d_6''}), \min\{\underline{\omega_{1F^1}}, \underline{\omega_{1F^2}}\}),$

$((\overline{d_1'}\Theta\overline{d_6''}, \overline{d_2'}\Theta\overline{d_5''}, \overline{d_3'}\Theta\ \overline{d_4''}, \overline{d_4'}\Theta\overline{d_3''}, \overline{d_5'}\Theta\ \overline{d_2''}, \overline{d_6'}\Theta\ \overline{d_1''}), \min\{\overline{\omega_{1F^1}}, \overline{\omega_{1F^2}}\}).$

$\omega_2\widetilde{F_d}^1 \otimes \omega_2\widetilde{F_d}^2$ =

$((\underline{d_1'}\otimes\,\underline{d_1''}, \underline{d_2'}\otimes\,\underline{d_2''}, ,\underline{d_3'}\otimes\,\underline{d_3''}, \underline{d_4'}\otimes\,\underline{d_4''}, \underline{d_5'}\otimes\,\underline{d_5''}, \underline{d_6'}\otimes\,\underline{d_6''}), \min\{\underline{\omega_{1F^1}}, \underline{\omega_{1F^2}}\}),$

$(\overline{d_1'}\otimes\overline{d_1''}, \overline{d_2'}\otimes\overline{d_2''}, \overline{d_3'}\otimes\ \overline{d_3''}, \overline{d_4'}\otimes\overline{d_4''}, \overline{d_5'}\otimes\overline{d_5''}, \overline{d_6'}\otimes\overline{d_6''}), \min\{\overline{\omega_{1F^1}}, \overline{\omega_{1F^2}}\}).$

if $\alpha_k \geq 0$.

$\alpha_k(\omega_2\widetilde{F_d}^1) = ((\alpha_k\underline{d_1'}, \alpha_k\underline{d_2'}, \alpha_k\,\underline{d_3'}, \alpha_k\overline{d_4'}, \alpha_k\underline{d_5'}, \alpha_k\underline{d_6'}, \underline{\omega_{1F^1}}),$

$(\alpha_k\overline{d_1'}, \alpha_k\overline{d_2'}, \alpha_k\overline{d_3'}, \alpha_k\overline{d_4'}, \alpha_k\overline{d_5'}\alpha_k\overline{d_6'}, \overline{\omega_{1F^1}}))$

if $\alpha_k \leq 0$.

$\alpha_k(\omega_2\widetilde{F_d}^1) = ((\alpha_k\underline{d_6'}, \alpha_k\underline{d_5'}, \alpha_k\,\underline{d_3'}, \alpha_k\underline{d_2'}, \underline{\alpha_k d_1'}, \underline{\omega_{1F^1}}),$

$(\alpha_k\overline{d_6'}, \alpha_k\overline{d_5'}, \alpha_k\overline{d_4'}, \alpha_k\overline{d_3'}, \alpha_k\overline{d_2'}, \alpha_k\overline{d_1'}, \overline{\omega_{1F^1}}))$

1.7.4 ARITHMETIC OPERATIONS OF ω −PENTAGONAL FUZZY NUMBERS

Let $\widetilde{A_1}$= (p_1, p_2, p_3, p_4 ,p_5; ω_1) and $\widetilde{B_1}$= (q_1, q_2, q_3, q_4,q_5;ω_2) be a two ω-Pentagonal Fuzzy Numbers (ω-PFN) then the following arithmetic operations:

1. $\widetilde{A_1} + \widetilde{B_1}$= ($p_1$+ q_1, p_2+ q_2,p_3+q_3,p_4+ q_4 ,p_5+ q_5; min (ω_1,ω_2))

2. $\widetilde{A_1} - \widetilde{B_1}$= ($p_1$-$q_5$, p_2-q_4,p_3-q_3,p_4-q_2, p_5-q_1; (ω_1-ω_2))

3. $\widetilde{A_1}$+(-$\widetilde{B_1}$)=(p_1, p_2, p_3, p_4 ,p_5) + (-q_1, -q_2, -q_3, -q_4,-q_5) ; (ω_1-ω_2))

   = (p_1- q_1, p_2- q_2,p_3- q_3,p_4- q_4 , p_5- q_5;(ω_1-ω_2))

4. $\lambda\widetilde{A_1}$ ={λp_1,λp_2,λp_3,λp_4 , λp_5 ; $\lambda>0$ and λp_5,λp_4,λp_3,λp_2,λp_1; $\lambda<0$

5. $\widetilde{A_1}\widetilde{B_1}$ = (p_1q_1, p_2q_2,p_3q_3,p_4q_4,p_5q_5;min (ω_1,ω_2))

6. $\frac{\widetilde{A_1}}{\widetilde{\widetilde{B_1}}}=\widetilde{A_1}\overline{\widetilde{B_1}}=\left(\frac{p_1}{q_5},\frac{p_2}{q_4},\frac{p_3}{q_3},\frac{p_4}{q_2},\frac{p_5}{q_1}; \min(\frac{1}{2\omega_1},\frac{1}{2\omega_2})\right)$

7. If $\widetilde{A_1}$=(p_1, p_2, p_3, p_4 ,p_5; ω) be a ω- Pentagonal Fuzzy Numbers;

$$\widetilde{A_1}^{-1}=\frac{1}{\widetilde{\widetilde{A_1}}}=\left(\frac{1}{p_5},\frac{1}{p_4},\frac{1}{p_3},\frac{1}{p_2},\frac{1}{p_1};\frac{1}{2\omega}\right).$$

1.7.5 Example:

$\widetilde{A_1}$ = (6,11,16,21;0.7) and $\widetilde{B_1}$= (4,7,10,13; 0.5) are any any two ω-trapezoidal fuzzy numbers, the then following operations

1. $\widetilde{A_1}$+ $\widetilde{B_1}$= (6+4,11+7,10+16,13+21;min(0.5,0.7))= (10,18,26,34;0.5).
2. $\widetilde{A_1}$ - $\widetilde{B_1}$= (6 -13,11-10,16 – 7,21-4;min(0.5,0.7))= (-7,-1,11,17;0.5).

3. $2\widetilde{A_1} =$ (12,22,32,42;0.5)

4. -$2\widetilde{A_1} =$ (-12,-22,-32,-42;0.5)

1.7.6 Example:

$\omega_2\widetilde{F_d}^1$ = {(8,11,17)(5,14,21)}and $\omega_2\widetilde{F_d}^2$= {(13,16,22)(10,20,25)}are any any two ω-type-2 diamond fuzzy numbers, the then following operations.

1. $\omega_2\widetilde{F_d}^1 + \omega_2\widetilde{F_d}^2$= {(8,11,17;0.2)(5,14,21;0.4)}+ {(13,16,22;0.4)(10,20,25;0.6)}

= {(8+13,11+16,17+22; min(0.2,0.4))(5+10,14+20,21+25;min(0.4,0.6))}

= {(21,27,39;0.2)(15,34,46;0.4)}

2. $\omega_2\widetilde{F_d}^1 - \omega_2\widetilde{F_d}^2$= {(8,11,17;0.2)(5,14,21;0.4)}- {(13,16,22;0.4)(10,20,25;0.6)}

={(8-22,11-16,17-13; min(0.2,0.4)(5-25,14-20,21-10;min(0.4,0.6))}

= {(-14,-5,4;0.2)(-20,-6,11;0.4)}.

3. $\omega_2\widetilde{F_d}^1 \otimes \omega_2\widetilde{F_d}^2$= {(8,11,17;0.2)(5,14,21;0.4)} ⊗ {(13,16,22;0.4)(10,18,25;0.6)}

={(8⊗13,11⊗16,17⊗22) min (0.2,0.4)(5⊗10,14⊗18,21⊗25 min(0.4,0.6))}

= {(104,176,374;0.2) (50,252,525;0.4)}

1.7.7 Example:

Let $\widetilde{A_1}$ = (4,8,12,16,20;0.3) and $\widetilde{B_1}$= (5,10,15.20,25; 0.5) are any two ω-Pentagonal Fuzzy Numbers (ω-PFN) then the following arithmetic operations:

1. $\widetilde{A_1} + \widetilde{B_1}$= (4+5,8+10,12+15,16+20,20+25;min(0.3,0.5))= (9,18,27,36,45;0.3).

2. $\widetilde{A_1}$ - $\widetilde{B_1}$= (4-25,8-20,12-15,16-10,20-5;min(0.3,0.5))=(-21,-12,-3,6,15;0.3,0.5)).

3.$\widetilde{A_1} \otimes \widetilde{B_1}$ = (4⊗5,8⊗10,12⊗15,16⊗20,20⊗25;min(0.3,0.5))

= (20,80,180,320,500;0.3)

4. $\frac{\widetilde{A_1}}{\widetilde{\widetilde{B_1}}}=\widetilde{A_1}\widetilde{\widetilde{B_1}} = \left(\frac{4}{25},\frac{8}{20},\frac{12}{15},\frac{16}{10},\frac{20}{5};1\right)$

5. $\widetilde{A_1}^{-1}=\frac{1}{\widetilde{\widetilde{A_1}}}=\left(\frac{1}{20},\frac{1}{16},\frac{1}{12},\frac{1}{8},\frac{1}{4};\frac{1}{0.6}\right)$

1.8 FUZZY LINEAR SUM ASSIGNMENT PROBLEM

In combinatorial fuzzy optimization matching approaches, the fuzzy linear sum assignment problem is one of the most common issues. In order to reduce the total of the related entries, we must match each row to a different column in the (n$\times n$) cost matrix C = ($\tilde{C}_{ij}$) .The fuzzy costs or numbers in this case are C = ($\tilde{C}_{ij}$). The various kinds of fuzzy numbers include ω −type-1 and ω −type-2 diamond fuzzy numbers, ω −type-1 and ω −type-2 trapezoidal fuzzy numbers, generalized trapezoidal fuzzy numbers, ω −type-1 and ω −type-2 pentagonal fuzzy numbers, and so on. Assumed to be non-negative are fuzzy costs. Every fuzzy number is a convex fuzzy set since all α -cuts of a interval fuzzy number for every (0,1] However, the opposite may not always be true because some convex fuzzy sets' α -cuts may be open and half-open interval. By Fulkerson, Glicksberg, and Gross, the bottleneck assignment problem was first presented. The time required by machine j to finish task i is represented by the fuzzy cost coefficient ($\tilde{C}_{ij}$).The bottleneck fuzzy assignment problem is minimum fuzzy cost and maximum matching in the bipartite graph. The matching is partial/complete in the Bipartite graph and fuzzy optimization cost is feasible/optimal. A spanning tree produced strong feasible tree. If a strongly feasible tree is producing degenerate pivoting on a backward edge and reduced fuzzy cost matrix is negative, then current FLSAP is not optimal. If a strongly feasible tree is producing non-degenerate pivoting on a forward edge and the reduced cost matrix is non-negative. The current FLSAP is optimal.

1.8.1 Definition:

Every row and column of a bipartite graph G = (U,V;E) has a vertex of U and V, and every edge of [i,j] has a cost $\tilde{C}_{ij}$ [i,j] (i,j=1,2,...n). Finding a perfect match in G for the lowest possible cost is the problem. (Weighted bipartite matching problem) Find a set of an edges where each vertex belongs to exactly one edge and where the total cost of these edges is the minimum.

1.9 DUAL AND PARTIAL PRIMAL SOLUTION

The primal-dual approach for linear programming is known to have its roots in the Hungarian algorithm. The Hungarian algorithm uses an assignment that is only partially feasible. Row(j) is equal to I if column j is assigned to row i. Row(j) is equal to 0 if column j is not assigned to row i. Beginning with a partial primal (to which less than n rows are assigned) that satisfies the complimentary slackness conditions with respect to the feasible dual solution u, v.

Every iteration attempts to improve the cardinality of the current assignment by rearranging the edges of the partial graph of G = (U, V; E), which only comprises the edges of E with zero reduced costs. Each iteration solves a constrained primal problem independent of the costs. If successful, the attempt yields a new primal solution with one additional row assigned. Otherwise, the dual solution as it stands is updated to produce new edges with zero reduced costs.

The first polynomial-time solutions to the linear sum assignment issue were primal-dual algorithms. In 1964, Balinski and Gomory presented the first primal algorithm. A set of assigned edges and corresponding set of assigned vertices are defined on G by the present partial assignment. A basic path with alternately assigned and unassigned edges is called an alternating path. An alternate path with unassigned initial and terminal edges is known as an augmenting path. By changing the assigned and unassigned edges along P, the improved assignment is produced.

1.9.1 Definition: Fuzzy Linear Sum Assignment Problem (FLSAP)

A bipartite graph G = (U, V; E) with an edge [i, j] and a $\boldsymbol{\omega}$ -trapezoidal fuzzy cost $\widetilde{c_{ij}}$ associated with each row, column, and vertex of U, V, with (i, j=1, 2....n). Finding a perfect match in G at the lowest possible cost.

1.9.2 Definition: (Partial Feasible Assignment (PFA) / Partial Feasible Solution

Suppose that there are ‘n’ workers and ‘n’ jobs. If the column ‘j’ is assigned to row ‘i’ then row (j) = i, (less than “n” rows / ”n” columns assigned), otherwise row (j) = 0 (or) If the row ‘i’ is assigned to column ‘j’ then column (i) = j (less than “n” rows / ”n” columns assigned), otherwise column (i) = 0, that is called partial feasible assignment (or) partial feasible solution.

1.9.3 Definition (Fuzzy Partial Feasible Matching (FPFM))

A bipartite graph G = (U, V; E) with an edge [i, j] and a $\boldsymbol{\omega}$ −trapezoidal fuzzy cost $\widetilde{c_{ij}}$ associated with each row, column, and vertex of U, V, with (i, j=1, 2....n). Finding less than “n” rows /” n” columns match in G.

1.10 LITERATURE REVIEW

The main motivation for L.A. Zadeh's [79] development of fuzzy set theory was to overcome the obscurity of everyday life. After beginning, the theory expanded the scope of its applicability. The notion that a number is a significant information carrier should not come as a surprise. The concept of a fuzzy number was first proposed by Dubois and Prade in 1978, greatly extending the fuzzy set theory.

In 1970, Zadeh and Bellman proposed [10] a method for making decisions in uncertain cases. It refers to a decision-making process where the restrictions and/or goals, but not always the system under management, are uncertain. This indicates that the limitations and/or goals define classes of alternatives whose borders are not sharply defined. During the year 2004, Lin and Wen [50], presented an algorithm for labelling in the fuzzy assignment problem. In this case, the components of the cost matrix for the assignment problem are subnormal fuzzy intervals with increasing linear membership functions, while the membership function for the total cost is a fuzzy interval with decreasing linear membership functions.

In 2016 M.C Yeola., and V.A Jahav [77] Solving Multi-Objective Transportation Problem using fuzzy programming technique –Parallel method. Here, we outline a strategy for resolving the Multi-Objective Transportation Problem (MOTP). In order to solve MOTP, fuzzy programming is employed here together with a fuzzy linear membership function for varied costs. The suggested approach is comparable to the New Row Maxima Approach. It produces better outcomes when MOTP is solved in a simpler manner.

In 2007, Majumder and Bhunia [52] proposed Elitist Genetic algorithm for assignment problem with imprecise goal. It is to use an elitist genetic algorithm to solve a generalized assignment problem with approximate cost(s) and time(s) rather than an exact one. Since interval valued numbers are the best representation over others like random variable representation with a known probability distribution and fuzzy representation, the imprecision of cost(s)/time(s) has been represented here by interval valued numbers and same year Chen Liang – Hsuan and Lu Hai – Wen [15], proposed an extended assignment problem considering multiple outputs.it is presented for each potential assignment, a method is developed in this study for handling situations with multiple, for in inputs and outputs.and the same year S.J.Chen, S.M.Chen [17] Fuzzy risk analysis on the ranking of generalized trapezoidal fuzzy numbers. Here, the suggested analysis takes into consideration the centroid points and standard deviations of generalized trapezoidal fuzzy numbers while ranking them and Bao, M.C.Tsai, and Tsai [9] proposed a new approach to study the multi-objective assignment problem.

During the year 2012, Emrouznejad and Angiz, and L, W. Ho [29] proposed an alternative formulation for the fuzzy assignment problem, discussed for each potential assignment, a strategy is developed in this study to address problems with fuzzy costs or profits using the data envelopment analysis method. The objective is to maximize the profit or reduce the assignment cost while obtaining the points with the highest membership values for the fuzzy parameters.

In 1995, A. V. Goldberg, Robert Kennedy [33] proposed an efficient cost scaling algorithm for the assignment problem Andrew Programming. It has been demonstrated that the cost scaling push-relabel method is effective for resolving minimum-cost flow problems and implementations of the approach that take advantage of the unique structure of the assignment problem and apply it to the assignment problem. The outcomes demonstrate how practical application of the technology is very promising.

In 2010, Amit Kmar, Pushpinder Singh and Jagdeep Kaur [47] discussed generalized simplex algorithm to solve fuzzy linear programming problems with ranking of generalized fuzzy numbers. Here, the proposed algorithm is a direct extension of classical algorithm so it is very easy to understand and apply the proposed algorithm to find the fuzzy optimal solution of fuzzy linear programming problems occurring in the real life situations.

In the year of 2014, Supriya Kar, Kajla Basu and Sathi Mukherjee[41] presented Solution of generalized fuzzy assignment problem with restriction on costs under fuzzy environment and Kayvan Salehi [66] proposed an approach for solving multi-objective assignment problem with interval parameters. Three criteria—total cost, total profit, and total operation time of the assignment are the focus of this model. The suggested model belongs to the category of nonlinear programming.

In 2013, Thorani, and Ravi Shanker [70] proposed fuzzy assignment problem with generalized fuzzy numbers. It is means that provide new algorithms for fuzzy assignment problems with fuzzy costs based on the ranking approach in classical and linear programming. A generalized fuzzy number is used to measure the fuzzy cost. To find the minimum fuzzy cost and to account for the different forms the fuzzy assignment problem can take, we constructed the classical algorithm using the fundamental theorems of the problem. In 1987, Wang [71] proposed Fuzzy optimal assignment problem.

During the year 2018, Jishu Jana and Sankar Kumar Roy [38] Solution of Matrix Games with generalized trapezoidal Fuzzy Payoffs . It is shown that when a linear ranking function is selected, the fuzzy matrix game is transformed into a crisp one and can then be solved quickly using the suggested approach. The primary goal is to employ the ranking function to reduce the computational complexity and H. A. Khalifa, and M. Al-Shabi [45] proposed an interactive approach for solving fuzzy Multi-Objective assignment problems. The goal of this study is to investigate the fuzzy multi-objective assignment problem. The issue is taken into account by using trapezoidal fuzzy numbers. The problem under discussion is transformed into the equivalent (MOAS) with the use of α-level sets. It is proposed to use an interactive method to enhance the weights in the Weighted Tchebysheff program. The resultant solution is then matched with the stability set of the first kind without differentiability.

In 2005, Albrecher [1] proposed a note on the asymptotic behavior of bottleneck problems. In 1971, Garfinkal [32] proposed an improved algorithm for Bottleneck Assignment Problem. In 2010, Kagade and Bajaj [43] proposed a fuzzy method for solving unbalanced assignment problems with interval valued coefficients. The approach suggested in the literature is predicated on the idea that some tasks will be delegated to make-believe or fictitious machines. Jobs that are performed on dummy machines may later be ignored. In 1999, Dubois and Fortemps [26] Computing improved optimal solutions to max-min flexible constraint satisfaction problems. In 1998, Sokkalingam and Aneja [68] proposed Lexicographic bottleneck combinatorial problems. We propose an approach which solves the lex-bottleneck optimization problem by solving bottleneck and zero–one sum optimizations for at most t iterations and reducing the problem size in each iteration.

In 2015, Pathinathan and Ponnivalavan [61], proposed Diamond fuzzy number. In 2011, Pramanik and Biswas[13] Multi-objective assignment problem with fuzzy costs for the case military affairs. Each fuzzy cost is viewed in this context as a trapezoidal fuzzy number. Weights of the objectives have been taken based on their priority to create a single objective problem from a multi-objective fuzzy assignment problem.

During the year 2020, Sanjivani and Ghadle [37], discussed Optimal Solution for Fuzzy Assignment Problem and Applications. The proposed method requires fewer iterations than previous methods while still providing the best feasible answer for a balanced fuzzy assignment problem. To validate the suggested technique's procedure, which is based on the industrial, environmental, and educational domains.

In 2019, Dong and Wan [27], proposed a new method for solving fuzzy multi-objective linear programming problems. The goal of this work is to create a novel two-stage strategy for selecting engineering project portfolios in fuzzy multi-objective linear programming. All of the objective coefficients, technology coefficients, and resource coefficients in the fuzzy multi-objective linear programed are trapezoidal fuzzy numbers. The interval expectation of trapezoidal fuzzy numbers. is used to introduce an order connection for them.

In 2012, Surapati Pramanik, Pranab Biswas [69] presented Multi-Objective Assignment Problem with Generalized Trapezoidal Fuzzy Numbers. The purpose of this work is to examine a multi-objective assignment problem with inaccurate time, costs, and results in place of accurate information and P.K.De, ,Bharti Yadev [22], presented a General Approach for Solving Assignment Problems involving with Fuzzy Cost Coefficients and the same year Isabel and Uthra [64], presented an application of linguistic variables in assignment problem with fuzzy costs. The goal of this assignment problem is to reduced costs. Each fuzzy cost is viewed in this context as a triangular or trapezoidal fuzzy number. The fuzzy numbers have been ranked using Yager's ranking approach. Using linguistic variables and the Hungarian technique, the fuzzy assignment problem has been converted into a crisp one.

In 2017 Avinash,Kamble [40] proposed some notes on pentagonal fuzzy numbers. In 2015, Jatinder Pal Singh&Neha Ishesh Thakur [39] discussed a novel method to solve assignment problem in Fuzzy Environment. In the year 2014, A. Khandelwal [46] presented a modified approach for assignment method .

In 2010, Anthony Przybylski, Xavier Gandibleuxa, Matthias Ehrgott [5], proposed a two phase method for multi-objective integer programming and its application to the assignment problem with three objectives . Here, proposed an approach to compute all supported efficient solutions in the first phase. The second stage is defining and examining the search area where it is possible to find non supported non dominated locations. In the year 1998, C.H. Cheng [19], proposed a new approach for ranking fuzzy numbers by distance method.

In 2002, Haddad, Mohammadi, and Pooladkhan [36] proposed two models for the generalized assignment problem in uncertain environment. For this problem, two models are described, and novel hybrid algorithm is provided that combines the simulated annealing method with the max-min fuzzy to achieve a nearly optimal result. The effectiveness of the suggested strategy is confirmed by computational experiments.

In 2006, Mitchell [55] proposed Ranking type-2 fuzzy numbers. In 1983, McGinnis[53], Implementation and testing of a primal-dual algorithm for the assignment problem. This study shows a thorough development of a primal-dual algorithm that is computationally efficient and extensive computational comparisons to primal simplex techniques. During the year 2010, Sathi Mukherjee and Kajla Basu [56] proposed, application of fuzzy ranking method for solving assignment problems with fuzzy costs. In 2002 ,Wu,J.M Mendel [74], proposed Uncertainty bounds and their use in the design of interval type-2 fuzzy logic systems. In 2009, Wu and Mendel [73] discussed a comparative study of ranking methods, similarity measures and uncertainty measures for interval type-2 fuzzy sets.

1.11 MOTIVATION AND SCOPE OF THE THESIS

This thesis attempts a significant yield from the conventional method of handling assignment problems to fuzzy linear sum assignment problems, motivated and inspired by the works of L.W Ho, R Burkard, Dell' M Amico, and S Martello ,S Kar, K Basu, and S Mukherjee, R Garfinkel, A Emrouznejad, M. Zerafat Angiz,. In this study, different approaches to solving fuzzy linear sum assignment problems are analyzed. New approaches are then developed complementary slack conditions and a partial feasible solution/complete optimal solution. This study could be viewed as a modest addition to the improvement of the fuzzy linear sum assignments

1.12 ORGANIZATION OF THE THESIS

Chapter I Introduction

In this chapter presents, a basic preliminaries of assignment problems, linear sum assignment problems, fuzzy linear sum assignment problems and the basic concepts of fuzzy sets, fuzzy numbers are discussed and proposed different types of fuzzy numbers ,α-cut fuzzy numbers and also discussed . obtained Dual and partial feasible solutions and complete optimal solutions by using different fuzzy optimization matching techniques of fuzzy assignment problems and linear sum assignment problems. Finally, we present the arithmetic functions of different types of fuzzy numbers. This method is illustrated by a numerical example.

Chapter II Dual and Partial Primal Solution for Solving Fuzzy Linear Sum Feasible Assignment Problems [FLSAP]

This chapter presents a new method by using complementary slackness conditions to calculate the fuzzy dual solution and fuzzy partial primal solution using a bipartite graph and the assignment cost were taken as $\omega -$trapezoidal fuzzy numbers. This chapter solves the primal problem first, then the dual problem, and finally the primal-dual problem. Here first construct $\omega -$ trapezoidal fuzzy Linear Sum Assignment table. Next we have to find row and column reduction, it's worth noting that the resulting dual variables of lower costs aren't negative and calculate reduced cost matrix $\bar{C}$ next to obtain Partial Feasible solution. This method is illustrated by a numerical example.

Chapter III Feasible Degenerate Pivoting and Optimal Non-Degenerate Pivoting for Solving Fuzzy Linear Sum Assignment Problems

This chapter proposes Feasible Degenerate Pivoting and Optimal Non-Degenerate Pivoting for Solving Fuzzy Linear Sum Assignment Problems. In this chapter the spanning trees on the associated bipartite graph G = (U, V; E). If a strongly feasible tree is producing degenerate pivoting on a backward edge and the rank of reduced fuzzy cost matrix $[\Re(\widetilde{\overline{C_{ij}}})]$ is negative and then the current fuzzy linear sum assignment problem is not optimal. If a strongly feasible tree is producing non-degenerate pivoting on a forward edge and the rank of the reduced cost matrix $[\Re(\widetilde{\overline{C_{ij}}})]$ is non-negative and $\bar{E}= \emptyset$ then the Current fuzzy linear sum assignment problem is optimal. This method is illustrated by a numerical example.

Chapter IV A New Modified Optimal Perfect Matching in Partial Feasible Matching for Solving Fuzzy Linear Sum Assignment Problems [FLSAP]

This chapter proposes a new modified optimal and perfect matching from partial assignment for solving fuzzy linear sum assignment problem. The fuzzy assignment cost is we take as ω-trapezoidal fuzzy numbers. By using ranking method, ω-trapezoidal fuzzy numbers converted to crisp one. First Calculate the rank of fuzzy dual variables and Compute a partial feasible solution then Calculate reduced rank of ω--trapezoidal fuzzy cost and Compute the new column and the new assignment introduced. continue the process to reach optimal solution and complete bipartite matching. This method is illustrated by a numerical example.

Chapter V Spread of New Partial/ Feasible and Optimal/ Perfect Matching for Solving Interval-Valued α-Cut Fuzzy Linear Sum Bottleneck Assignment Problem.

This chapter proposes a spread of new partial/feasible and optimal/perfect matches of bipartite graphs for solving interval-valued α-cuts of generalized trapezoidal fuzzy numbers. Obtain α -cut generalized trapezoidal fuzzy numbers from generalized trapezoidal fuzzy numbers, then discuss membership functions. The α -cut generalized trapezoidal fuzzy number is transformed into an Interval-valued α -Cut of Generalized Fuzzy Numbers. The basic preliminaries and fuzzy interval operations are discussed. If maximum cost and complete match, then the solution is feasible and complete. If the solution is minimum cost and complete, then the solution is optimal and complete. If maximum cost and partial match, then the solution is feasible with partial match. If minimum cost or partial match, then the solution is optimal or feasible match. This method is illustrated by a numerical example.

Chapter VI A New Optimal Complete Matching of Edges with Minimum Cost by Ranking Method for Solving ω-Type -2 Fuzzy Linear Sum Assignment Problem[FLSAP]

This chapter proposes a new optimal solution and complete matching edges of bipartite graph. ω-type -2 [FLSAP] is converted to crisp one by using new ranking method for solving ω-type -2 [FLSAP]. This chapter discussed ω-trapezoidal fuzzy number, ω-type 1-trapezoidal fuzzy number and ω-type 2-trapezoidal fuzzy number.

Create ω-type -2 [FLSAT].The rank of -type 2 trapezoidal fuzzy number to assign each machine to a job with the lowest cost in that job for solving -type 2 [FLSAP].

Furthermore, each iteration updates a non-matched edge to a matched edge and update the corresponding dual variables. By using alternating path method to obtain a new optimal complete matching solution. This method is illustrated by a numerical example.

Chapter VII Fuzzy Multi-Objective Linear Sum Assignment Problem with Modified Partial Solution of $\boldsymbol{\omega}$ - type 2 - Diamond Fuzzy Numbers[DFN] by Using Linguistic variables

This chapter proposes fuzzy multi-objective linear sum assignment problem with modified partial assignment of ω - type 2 - diamond fuzzy numbers using linguistic variables. In this chapter introduced ω - type 1 and ω - type 2 diamond fuzzy numbers. Let us consider four jobs and four machine problem and to optimize fuzzy cost , fuzzy time, fuzzy quality are each considered as a ω - type 2 - DFN. ω-type 2 DFN are converted into λ_d--cut of DFN and upper and lower ω-type 2 diamond multi-objective fuzzy numbers are converted into single objective λ_d-cut fuzzy number by using ranking method. obtain dual variables and calculate $[\widetilde{c_{ij}} - \widetilde{u_i} - \widetilde{v_j}]$; by using alternate path method increase the partial assignment. This method is illustrated by a numerical example, proving its efficiency.

Chapter VIII Minimum vertex cover of $\boldsymbol{\omega}$ −Pentagonal Fuzzy Linear Sum Bottleneck Assignment Problem [$\boldsymbol{\omega - PFLSBAP}$]

In this chapter we presented a spread of minimum solution of fuzzy optimization matching procedure in the bipartite graph. it provides minimum vertex cover with edge set E for solving $\omega - PFLSAP$. The $\omega - PFLSAP$ is minimum cost and complete matching in the bipartite graph. The Linear Sum Bottleneck Assignment Cost [LSBAC] we taken as ω −Pentagonal Fuzzy Numbers ($\omega - PFN$). If each person and each job contain exactly one matching solution with Spr (φ) = 0 or minimum Spr (φ), then the current ω-PFLSBAP is optimal. If each person and each job contain exactly one matching solution with maximum Spr (φ), then the current ω-PFLSBAP is not optimal but feasible and complete matching solution. Finally obtained the graph has minimum vertex cover of cardinality n with perfect or complete matching. This method is illustrated by a numerical example.

CHAPTER II

DUAL AND PARTIAL PRIMAL SOLUTION FOR SOLVING FUZZY LINEAR SUM FEASIBLE ASSIGNMENT PROBLEMS

In this chapter presents, a new method by using complementary slackness conditions to calculate the fuzzy dual solution and fuzzy partial primal solution using a bipartite graph and the assignment cost were taken as $\omega -$*trapezoidal fuzzy numbers. This chapter solves the primal problem first, then the dual problem, and finally the primal-dual problem. Here first construct* $\omega -$ *trapezoidal fuzzy Linear Sum Assignment table. Next we have to find row and column reduction, it's worth noting that the resulting dual variables of lower costs aren't negative and calculate reduced cost matrix* $\bar{C}$ *next to obtain Partial Feasible solution. This method is illustrated by a numerical example.*

2.1 INTRODUCTION

The Fuzzy Assignment problem is a special kind of fuzzy linear programming problem. In the previously mentioned publication by Dinic and Kronrod, the first dual (non-simplex) algorithm for LSAP initially appeared. Bellman and Zadeh proposed the idea of fuzzy set theory. For the purpose of solving the linear fractional programming, Lin and Wen presented an effective algorithm based on the labelling technique**.** Different algorithms, including the mathematical programming, Hungarian algorithm, neural network, and genetic algorithm, have been designed to find optimal/feasible solutions to methods for solving.

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Many other iterations of the traditional assignment problems have been put forth during the past 52 years. Due to the significant degeneracy of the linear programme associated with LSAP, the earliest primal simplex algorithms, proposed in the middle of the year by Cunningham and Barr, Glover, and Kingman, took exponential time.

The algorithms for Fuzzy Linear Sum Assignment Problem (FLSAP) are based on various strategies: a first class of methods solves the primal problem directly, a second one solves the dual problem, and a third one uses an intermediary approach (primal-dual). In order to find a partial primal solution that meets the complimentary slackness constraints and a feasible dual solution, less than 'n' rows must be given by using $\omega -$ trapezoidal fuzzy number.

The solution to the partial feasible fuzzy assignment problem is dual and primal. in trapezoidal fuzzy numbers, we take, if 'm' people to 'n' jobs and its cost coefficient '$\widetilde{c_{ij}}$' As specified by, the storage location for the partial feasible fuzzy assignment. If column "j" is assigned to row i, then row(j) is equal to "1" and if column "j" is not assigned to row i, then row (j) is equal to "0."

2.2 Properties of $\boldsymbol{\omega -}$ Trapezoidal Fuzzy Number

1) Two $\omega -$trapezoidal fuzzy numbers are increasing order if and only if the sum of the $\omega -$trapezoidal fuzzy number is also increasing order.
2) Let $\tilde{A}_1$ =($a_1, b_1, c_{1,} d_1 : \omega_1$), and $\tilde{A}_2$ = ($a_2, b_2, c_2, d_2 : \omega_2$) are any two $\omega -$ trapezoidal fuzzy numbers is said to be equal if and only if $a_1 = a_2$, $b_1 = b_2$, $c_1 = c_2$, d_1=d_2 and ω_1=ω_2.

3) ω $-$trapezoidal fuzzy number $\tilde{A}$ = (a,b,c,d) is said to be symmetric $\omega-$ trapezoidal fuzzy number if and only if c = d i.e, $\tilde{A}$ = (a,b,c,c : ω).

2.3 MATHEMATICAL FORMULATION

Suppose that there are 'n' workers and 'n' jobs. Only one worker is permitted to perform each task at any given time. The challenge is to distribute up the tasks among the workers while attempting to keep the overall cost, which is represented by ω-trapezoidal fuzzy numbers, as low as possible.

2.3.1 Fuzzy Assignment Problem (FAP)

$$\text{Min } Z = \sum_{I=1}^{n}\sum_{j=1}^{n}\widetilde{c_{\iota J}}X_{ij} \qquad \ldots\ldots\ldots\ldots\ldots(a)$$

Subject to $\quad \sum_{j=1}^{n} x_{ij}$ =1; for i = 1,2......,n ………… (b)

$\sum_{i=1}^{n} x_{ij}$ =1; for j =1,2........,n

x_{ij} = 0 or 1

2.3.2 Fuzzy Dual Assignment Problem (FDAP)

The fuzzy dual variables $\widetilde{u_\iota}$ and $\widetilde{v_J}$ with constraints and then fuzzy dual problem is

$$\text{Max } \sum_{i=1}^{n}\widetilde{u_\iota} + \sum_{j=1}^{n}\widetilde{v_J}$$

Subject to $\quad \widetilde{u_\iota} + \widetilde{v_J} \leq \widetilde{c_{\iota J}}$ (i,j = 1,2.....,n)

2.4 Fuzzy complementary slackness conditions

A pair of solutions that are feasible for the fuzzy primal and the dual, respectively, are said to be optimal by duality theory if and only if (complementary slackness)

$$x_{ij}(\widetilde{c_{\iota J}} - \widetilde{u_\iota} - \widetilde{v_J}) = 0 \quad \text{(i,j=1,2.....n)}$$

the values

$$\widetilde{\widetilde{C_{ij}}} = \widetilde{c_{ij}} - \widetilde{u_i} - \widetilde{v_j} \quad \text{(i,j = 1,2....n)}$$

Where $\widetilde{\widetilde{C_{ij}}}$ is reduced cost. This transformation from $\widetilde{c_{ij}}$ to $\widetilde{\widetilde{c_{ij}}}$ is a special case of admissible transformation.

For any feasible solution, the transformed objective function is

$$\sum_{i=1}^{n}\sum_{j=1}^{n}(\widetilde{c_{ij}} - \widetilde{u_i} - \widetilde{v_j})\, x_{ij} = \sum_{i=1}^{n}\sum_{i=1}^{n}\widetilde{c_{ij}}\, x_{ij} - \sum_{i=1}^{n}\widetilde{u_i}_i \sum_{i=1}^{n} x_{ij} - \sum_{j=1}^{n}\widetilde{v_j}_j \sum_{i=1}^{n} x_{ij}$$

$$= \sum_{i=1}^{n}\sum_{i=1}^{n}\widetilde{c_{ij}}\, x_{ij} - \sum_{i=1}^{n}\widetilde{u_i} - \sum_{j=1}^{n}\widetilde{v_j}$$

The values of $\widetilde{u_i}$ and $\widetilde{v_j}$ determined by the first two sentences in this case fulfil the dual constraints. The x_{ij} values that were subsequently acquired ensure that the complimentary slackness conditions are satisfied While the primal constraints (a) and (b) the $\leq$ sign holds instead of = . It should be noted that an alternative technique might reduce the rows first, then the columns. getting various reduced costs and assignments. A feasible dual solution $\widetilde{u_i}$, $\widetilde{v_j}$ satisfying $\widetilde{u_i} + \widetilde{v_j} \leq \widetilde{c_{ij}}$ and partial primal solution satisfying complimentary slackness conditions with respect to $\widetilde{u_i}$, $\widetilde{v_j}$. Each iteration solves a restricted primal problem independent of the costs. Trying to increase the cardinality of the current assignment by operating on the partial graph of G that only contains the edges of E having zero reduced costs are obtained.

2.5 An Algorithm for Solving Feasible dual and Partial Primal Fuzzy Linear Sum Assignment Problem with $\omega -$ Trapezoidal Fuzzy Numbers

Step 1: Create the balanced fuzzy linear assignment table using $\omega -$ trapezoidal fuzzy numbers. If the fuzzy linear assignment table is unbalanced and introduce dummy fuzzy cost (row)/ dummy fuzzy cost (column) and convert to balanced one.

Step 2: Calculate row reduction $(\widetilde{u_i})$

$$\widetilde{u}_i = \underset{1 \le i \le n}{Min}\{\widetilde{c_{ij}}\} \quad , \qquad \text{for (j = 1,2......n).}$$

Step 3: Calculate column reduction ($\widetilde{v}_j$)

$$\widetilde{v}_j = \underset{1\le j \le n}{Min}\{\widetilde{c_{ij}} - \widetilde{u}_i\} \ , \ \text{for (i = 1,2......n).}$$

Step 4: Calculate reduced cost matrix$\overline{\widetilde{C_{ij}}}$

The reduced fuzzy cost is difference of fuzzy cost, row reduction and column reduction.

$$\overline{\widetilde{C_{ij}}} = \widetilde{c_{ij}} - \widetilde{u}_i - \widetilde{v}_j \quad \text{for} \quad \text{(i,j = 1,2......n).}$$

Step 5: [Partial assignment]

If the column ‘j’ is assigned to row ‘i’ then row (j) = i, otherwise row (j) =0 (or) If the row ‘i’ is assigned to column ‘j’ then column (i) = j, otherwise column (i) = 0,that is called partial feasible assignment (or) partial feasible solution. Suppose we select the first column having any zero and assigned to row i, then row (j) = i and cross out corresponding row and column. Similarly select the second column having any zero and assigned to row i, then row (j) = i and cross out corresponding row and column. Suppose, if the column is not assigned to row i, then row(j) = 0.

Step 6: [Partial assignment that implements the inverse of row]

Select the ‘i’ th row having any zero and assigned to column ‘j’ then column (i) = j and cross out corresponding row and column. if the row is not assigned to column j, then column(i) = 0.

Step 7: Draw a bipartite graph G = (U, V; E) with an edge [i, j] and a trapezoidal fuzzy cost $\widetilde{c_{ij}}$ associated with each row, column, and vertex of U, V, with (i, j=1, 2....n). Finding less than ‘n’ rows / ‘n’ columns match in G.

Step 8: Stop.

2.6 NUMERICAL EXAMPLE

To illustrate the proposed algorithm, let us consider an $\omega -$ trapezoidal fuzzy assignment problem with obtain 'i' th rows assigned to 'j'th column from the fuzzy cost matrix by using dual and primal fuzzy assignment problem. In 'i' th rows represented 4 persons P_1, P_2, P_3 , P_4 and in 'j' th column represented the 4 jobs J_1, J_2, J_3, J_4 . The cost matrix $[\tilde{c}_{ij}]$ is given whose elements are $\omega -$ trapezoidal fuzzy numbers. This problem is to find the feasible matching of the dual and partial primal fuzzy assignment problem.

Persons/jobs	**J_1**	**J_2**	**J_3**	**J_4**
P_1	(1,4,9,16 : 0.3)	(4,9,16,25: 0.4)	(25,37,50,65:0.8)	(4,9,16,25: 0.4)
P_2	(4,9,16,25: 0.4)	(25,37,50,65:0.8)	(16,25,37,50: 0.6)	(9,16,25,37:0.5)
P_3	(16,25,37,50:0.6)	(37,50,65,82: 0.9)	(37,50,65,82: 0.9)	(25,37,50,65:0.8)
P_4	(37,50,65,82 0.9)	(25,37,50,65:0.8)	(9,16,25,37:0.5)	(9,16,25,37:0.5)

Solution: The fuzzy assignment problem given . Construct the fuzzy assignment table for the given balanced fuzzy linear assignment problem.

The total number of jobs and the total number of persons are equal. Since the given fuzzy assignment problem is balanced. Next find row reduction $(\widetilde{u_i})$, column reduction $(\widetilde{v_j})$ and reduced cost matrix $\widetilde{c_{ij}}$. Calculate the row reduction $(\widetilde{u_i})$ the following table:

Persons/Jobs $(\widetilde{u_i})$	**J1**	**J2**	**J3**	**J4**
(1,4,9,16 : 0.3)	(1,4,9,16 : 0.3)	(4,9,16,25: 0.4)	(25,37,50,65: 0.8)	(4,9,16,25: 0.4)
(4,9,16,25: 0.4)	(4,9,16,25: 0.4)	(25,37,50,65:0.8)	(16,25,37,50: 0.6)	(9,16,25,37:0.5)
(16,25,37,50:0.6)	(16,25,37,50:0.6)	(37,50,65,82:0.9)	(37,50,65,82: 0.9)	(25,37,50,65:0.8)
(9,16,25,37:0.5)	(37,50,65,82:0.9)	(25,37,50,65:0.8)	(9,16,25,37: 0.5)	(9,16,25,37: 0.5)

Persons/jobs $\widetilde{u_i}$ / $\widetilde{v_j}$	**(0,0,0,0)**	**(3,5,7,9 ; 0.1)**	**(0,0,0,0)**	**(0,0,0,0)**
(1,4,9,16 : 0.3)	(1,4,9,16 : 0.3)	(4,9,16,25: 0.4)	(25,37,50,65:0.8)	(4,9,16,25: 0.4)
(4,9,16,25: 0.4)	(4,9,16,25: 0.4)	(25,37,50,65:0.8)	(16,25,37,50:0.6)	(9,16,25,37:0.5)
(16,25,37,50:0.6)	(16,25,37,50:0.6)	(37,50,65,82:0.9)	(37,50,65,82:0.9)	(25,37,50,65:0.8)
(9,16,25,37: 0.5)	(37,50,65,82:0.9)	(25,37,50,65:0.8)	(9,16,25,37: 0.5)	(9,16,25,37:0.5)

Given matrix $\widetilde{c_{ij}}$ and we obtain the dual variables $\widetilde{u_i}$ and $\widetilde{v_j}$ (Shown on the left and on the top of the given matrix $\widetilde{c_{ij}}$ then find the reduced cost matrix $\overline{\overline{\widetilde{C_{ij}}}}$

(0,0,0,0)	(0,0,0,0)	(24,33,41,49: 0.5)	(3,5,7,9; 0.1)
(0,0,0,0)	(18,23,27,31: 0.3)	(12,16,21,25 : 0.2)	(5,7,9,12: 0.1)
(0,0,0,0)	(18,20,21,23: 0.2)	(21,25,28,32 : 0.3)	(9,12,13,15:0.2)
(28,34,40,45: 0.2)	(13,16,18,19 : 0.2)	**(0,0,0,0)**	(0,0,0,0)

Select the ‘j’ th column having any zero and assigned to row i , then row(j) = i ,otherwise row(j) = 0. row (j) = (1,0,4,0) and inverse of row δ(i) = (1.0,0,3)

The Partial Primal Feasible Fuzzy Linear Sum Assignment Solution

{(1,4,9,16: 0.3) + (9,16,25,37:0.5)} = (10,20,34,53: 0.3)

row (j) = (1,0,4,0**)** **and**

The inverse of the row is $\boldsymbol{\delta}$(i) = (1.0,0,3).

CHAPTER III

FEASIBLE DEGENERATE PIVOTING AND OPTIMAL NON-DEGENERATE PIVOTING FOR SOLVING FUZZY LINEAR SUM ASSIGNMENT PROBLEMS

In this chapter proposes Feasible Degenerate Pivoting and Optimal Non-Degenerate Pivoting for Solving Fuzzy Linear Sum Assignment Problems. In this chapter the spanning trees on the associated bipartite graph G = (U, V; E). If a strongly feasible tree is producing degenerate pivoting on a backward edge and the rank of reduced fuzzy cost matrix $[R(\widetilde{\widetilde{C_{ij}}})]$ *is negative and then the current fuzzy linear sum assignment problem is not optimal. If a strongly feasible tree is producing non-degenerate pivoting on a forward edge and the rank of the reduced cost matrix* $[R(\widetilde{\widetilde{C_{ij}}})]$ *is non-negative and* $\bar{E} = \emptyset$ *then the Current fuzzy linear sum assignment problem is optimal. This method is illustrated by a numerical example.*

3.1 INTRODUCTION

Many applications of fuzzy assignment problems applied in real life situations, scientific, uncertainty and engineering. We compute the fuzzy optimal assignment cost for solving fuzzy linear sum assignment problem through rooted in a strongly feasible tree. In this problem $(\widetilde{c_{ij}})$ is denotes as fuzzy cost and assigned to perfect matching of the jth job to the ith person.

Here, $(\widetilde{c_{ij}})$ is ω-trapezoidal fuzzy numbers to convert crisp by using Average

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ranking technique for solving fuzzy linear sum assignment problem. In solving the fuzzy linear sum assignment problem, we compute the fuzzy optimal assignment cost using a strongly feasible tree in the bipartite graph.

In fuzzy linear sum assignment problem, there is one-to-one correspondence between primal basic solutions and spanning trees on the associated with bipartite graph G = (U, V; E). Given any feasible solution, and the associated spanning tree T consists of the 2n-1 edges corresponding to the basic columns. The strong feasible tree (T) producing degenerate pivoting on a backward edge and non-degenerate pivoting on a forward edge. If the solution is feasible a strongly feasible tree (T) is Producing degenerate pivoting on a backward edge. If the solution is optimal a strongly feasible tree (T) is Producing non- degenerate pivoting on a forward edge.

3.2 $\boldsymbol{\omega}$- Trapezoidal Fuzzy Linear Sum Assignment Problem [$\boldsymbol{\omega}$-TFLSAP]

Suppose there are ‘m’ jobs to be performed and ‘n’ persons (m = n) are available for doing the jobs. Assume that each person can do each job at a time, depending on their efficiency to do the job. Let $(\widetilde{c_{ij}})$ be the ω $-$trapezoidal fuzzy linear sum assignment cost, then the objective is to minimize the total ω- trapezoidal cost is performed i^{th} person perfect matched to the available j^{th} job or assigning all the jobs to the available persons (one job to one person).

The assignment cost as ω-trapezoidal fuzzy numbers. Here, ω-trapezoidal fuzzy assignment problem has been transformed into crisp assignment problem using Some ranking method. A strongly feasible tree T is producing fuzzy degenerate pivoting on a backward edge and T is producing the non-degenerate pivoting on a forward edge and optimal are discussed.

The ω-trapezoidal fuzzy linear sum assignment problem can be stated in the form of an mxn (m = n) Cost matrix $[\widetilde{c_{ij}}]$ of ω-trapezoidal fuzzy numbers as given in the following table:

ω- Trapezoidal Fuzzy Linear Sum Assignment Problem

Persons/jobs	1	2	3	. . . . .	N
1	$[\widetilde{c_{11}};\ \omega_{11}]$	$[\widetilde{c_{12}};\ \omega_{12}]$	$[\widetilde{c_{13}};\ \omega_{13}]$	. . . . .	$[\widetilde{c_{1n}};\ \omega_{1n}]$
2	$[\widetilde{c_{21}};\ \omega_{21}]$	$[\widetilde{c_{22}};\ \omega_{22}]$	$[\widetilde{c_{23}};\ \omega_{23}]$	. . . . .	$[\widetilde{c_{2n}};\ \omega_{2n}]$
3	$[\widetilde{c_{31}};\ \omega_{31}]$	$[\widetilde{c_{32}};\ \omega_{32}]$	$[\widetilde{c_{33}};\ \omega_{33}]$	. . . . .	$[\widetilde{c_{3n}};\ \omega_{3n}]$
. . .	. . .	. . .	. . .	. . .	. . .
M	$[\widetilde{c_{m1}};\ \omega_{m1}]$	$[\widetilde{c_{m2}};\ \omega_{m2}]$	$[\widetilde{c_{m3}};\ \omega_{m3}]$		$[\widetilde{c_{mn}};\ \omega_{mn}]$

3.3 PROPERTIES OF STRONG FEASIBLE TREE

A strongly feasible tree T is producing fuzzy degenerate pivoting on a backward edge and T is producing the non-degenerate pivoting on a forward edge and optimal. A strongly feasible tree (T) that produces degenerate pivoting on a back edge indicates that the solution is feasible. A strongly viable tree (T), which produces non-degenerate pivoting on a forward edge, is the best case situation for the solution.

3.3.1 Definition:

From the FLSAP with cost $\widetilde{c_{ij}}$ is ω-trapezoidal fuzzy number and if the reduced from cost matrix elements at least any one cost is negative then, the basis corresponding to tree is producing degenerate pivoting.

3.3.2 Definition:

From the FLSAP with cost $\widetilde{c_{ij}}$ is ω-trapezoidal fuzzy number and if the reduced from cost matrix elements for all the cost is non-negative, then the basis corresponding to tree is producing non-degenerate pivoting.

3.3.3 Definition:

Given a feasible solution x, a tree T in G = (U,V;E) rooted at r ϵ U is a strongly feasible tree if x_{ij} = 1 ∀ odd edges [i, j] ϵ T and x_{ij} = 0 ∀ even edges [i, j] ϵ T.

3.3.4 Definition

An edge [i, j] ϵ E\T with i ϵ U and j ϵ V is a forward edge if i lies on the Path that connects r to j in T.

3.3.5 Definition:

An edge [i, j] ϵ E\T with i ϵ U and j ϵ V is a backward edge if j lies on the path that connects r to i in T.

3.3.6 Definition:

An edge [i, j] ϵ E\T with i ϵ U and j ϵ V is a cross edge if it is neither forward nor backward.

3.3.7 Theorem

If reduced costs $\overline{c_{ij}}$ = c_{ij} - u_i - v_j ≥ 0 where 1≤ i,j ≤ n is corresponding LSAP it is optimal solution and T is producing a non-degenerate pivot on a forward edge.

Proof. Let us take a strongly feasible tree T in the bipartite graph and from the balanced fuzzy linear sum assignment problem and arbitrarily fixing the value of the root u_r to zero from the strong feasible tree (T) and computed dual variables from T. next execution by using duality theory, suppose $\overline{c_{ij}} = c_{ij} - u_i - v_j$ is non- negative, Since the LSAP is optimal solution and T is producing non-degenerate pivot on a forward edge. ■

3.3.8 Theorem

If $\overline{c_{ij}} = c_{ij} - u_i - v_j$ < o where 1≤ i,j ≤ n is corresponding LSAP it is not optimal solution and T is producing a degenerate on a backward edge.

Proof. Let T be a strongly feasible tree in the bipartite graph and from the balanced linear sum assignment problem and arbitrarily fixing the value of the root u_r to zero from strong feasible tree (T) and computed dual variables from T. by using duality theory $\overline{c_{ij}} = c_{ij} - u_i - v_j$, if $\overline{c_{ij}} = c_{ij} - u_i - v_j$ is negative. i.e, the LSAP is not optimal solution and T is producing degenerate pivot on a backward edge. ■

3.4 RANKING OF ω-TRAPEZOIDAL FUZZY NUMBERS

An effective algorithm developed before for comparing ω- trapezoidal fuzzy numbers is by use of ranking function [1],The given ω- trapezoidal fuzzy number to convert crisp one by using average ranking method [1] . The ranking function map $\Re$: F(R) → R, where F(R) is a set of fuzzy numbers defined on the set of real numbers, which maps each fuzzy number into the real line.

The following comparisons are exists i.e.,

(i) $\tilde{A} > \tilde{B}$ if and only if $\Re(\tilde{A}) > \Re(\tilde{B})$

(ii) $\tilde{A} < \tilde{B}$ if and only if $\Re(\tilde{A}) < \Re(\tilde{B})$

(iii) $\tilde{A} = \tilde{B}$ if and only if $\Re(\tilde{A}) = \Re(\tilde{B})$

Let $\tilde{A}$ = $(p_1, q_1, r_1, s_1; \omega_1)$ and $\tilde{B}$ = $(p_2, q_2, r_2, s_2; \omega_2)$ be any two ω-trapezoidal fuzzy numbers and ω = min(ω_1, ω_2) then

$$\Re\,(\tilde{A}) = \frac{\omega_1(p_1 + q_1 + r_1 + s_1)}{4} \quad \text{and} \quad \Re\,(\tilde{B}) = \frac{\omega_2(p_2 + q_2 + r_2 + s_2)}{4}$$

3.5 THE PROPOSED ALGORITHM.

Step 1: First test whether the given ω-trapezoidal fuzzy linear sum assignment table is balanced (or) unbalanced. If it is balanced one, the total number of persons are equal to the total number of jobs, then go to step 3. If it is unbalanced one, the total number of persons are not equal to the total number of jobs, then go to step2.

Step 2: Introduce dummy rows /dummy columns (ω –trapezoidal fuzzy Cost is zero), so unbalanced problem converts to balanced one.

Step 3: The ω –trapezoidal fuzzy number convert to crisp one by using average ranking method. Examine the rank of ω-trapezoidal fuzzy cost and defined as [$\Re$ ($\widetilde{c_{ij}}$)]

Step 4: Form a Strongly Feasible Tree(T)

Given a feasible solution x, a tree T in G = (U,V;E) rooted at r ϵ U is a strongly feasible tree if x_{ij} = 1∀ odd edges [i, j] ϵ T and x_{ij} = 0∀ even edges [i, j] ϵ T.

Step 5: Compute Rank of Fuzzy Dual Variables ($\widetilde{u_i}$ * and $\widetilde{v_j}$*);

First form a strong feasible tree(T) from the bipartite graph (G).the root $\widetilde{u_i}$*****= 0 then calculate $\widetilde{v_j}$***** = $\widetilde{c_{ij}}$ - $\widetilde{u_i}$ * ; and $\widetilde{u_{i-1}}$***** = $\widetilde{c_{ij}}$ - $\widetilde{v_j}$***** and next $\widetilde{v_{j-1}}$***** = $\widetilde{c_{ij}}$- $\widetilde{u_{i-1}}$***** and so on, similarly compute $\widetilde{u_{i+1}}$***** = $\widetilde{c_{ij}}$ - $\widetilde{v_j}$***** ; $\widetilde{v_{j-1}}$***** = $\widetilde{c_{ij}}$- $\widetilde{u_{i-1}}$***** .

Step 6: [Compute reduced rank of fuzzy cost matrix [$\Re$ **(**$\widetilde{\widetilde{c_{ij}}}$)]

The reduced rank of fuzzy cost matrix $\Re$ **(**$\widetilde{\widetilde{c_{ij}}}$)= $\Re$ [$\widetilde{c_{ij}}$ - $\widetilde{u_i}$ - $\widetilde{v_j}$]

Step 7: (Assigning the zeros)

(a) Compute the row**[**$\Re$ **(**$\widetilde{\widetilde{c_{ij}}}$)]successively until a row with exactly one unmarked zero is found. Make an assignment to this single unmarked zero by encircling it. Cross all other zeros in the column **[**$\Re$ **(**$\widetilde{\widetilde{c_{ij}}}$)]of this encircled zero.

(b) Compute the column [$\Re$ ($\widetilde{\widetilde{c_{ij}}}$)]successively until a column with exactly one unmarked zero is found. Make an assignment to this single unmarked zero by encircling it. Cross any other zeros in its row **[**$\Re$ **(**$\widetilde{\widetilde{c_{ij}}}$)]of this encircled zeros.

(c) Continue the process until in each row **[**$\Re$ **(**$\widetilde{\widetilde{c_{ij}}}$)]and each column **[**$\Re$ **(**$\widetilde{\widetilde{c_{ij}}}$)] exactly one encircled zero.

Step 8: (Apply optimal test)

(i) A strongly feasible tree is producing degenerate pivoting on a backward edge and the rank of reduced fuzzy cost matrix **[**$\Re$ **(**$\widetilde{\widetilde{c_{ij}}}$)] is negative and $\bar{E}$= {[i, j] / $\bar{E}$: $\Re[\widetilde{c_{ij}}$ - $\widetilde{u_i}$ - $\widetilde{v_j}] < 0$} then the current fuzzy linear sum assignment problem is not optimal.

(ii) A strongly feasible tree is producing non-degenerate pivoting on a forward edge and the rank the reduced cost matrix **[**$\Re$ **(**$\widetilde{\widetilde{c_{ij}}}$)]is non-negative and $\bar{\bar{\bar{E}}}$ = $\emptyset$ then the current fuzzy linear sum assignment problem is optimal.

Step 9:

The rank of reduced fuzzy cost matrix **[**$\Re$ **(**$\widetilde{\widetilde{c_{ij}}}$)] = $\Re$ [$\widetilde{c_{ij}}$ - $\widetilde{u_i}$ - $\widetilde{v_j}$] $<$ 0 and $\bar{E}$= [i, j] / $\bar{E}$: $\Re$ [$\widetilde{c_{ij}}$ - $\widetilde{u_i}$ - $\widetilde{v_j}$] $<$ 0} form a matching in a bipartite graph from **[**R**(**$\widetilde{\widetilde{c_{ij}}}$)] then select most negative edge from **[**$\Re$ **(**$\widetilde{\widetilde{c_{ij}}}$)] the most negative edge[i, j] ϵ $\bar{E}$with i ϵ u and j ϵ v and removes from the basis the unique other edge [i, l] ϵ C (T , [i,j]) incident to i with

$l \neq j$ and form a new matching in a bipartite graph and making a new strongly feasible tree. Again a strongly feasible tree T is producing degenerate pivoting on a backward edge,continue the process until T is producing the non-degenerate pivoting on a forward edge and optimal.

Step 10: Stop**.**

3.6 NUMERICAL EXAMPLE

A company has four persons P_1, P_2, P_3, P_4 and four jobs J_1, J_2, J_3, J_4 with cost matrix$[\widetilde{c_{ij}}]$ is given whose elements are ω-trapezoidal fuzzy numbers and then illustrated the proposed algorithm. Compute the optimal complete matching in the bipartite graph (G) and also discuss the strong feasible tree (T) producing degenerate pivoting on a backward edge and non-degenerate pivoting on a forward edge and optimal.

	J_1	**J_2**	**J_3**	**J_4**
P_1	(14,20,26,32;0.4)	(8,14,20,26;0.5)	(4,8,14,20;0.2)	(14,20,26,32;0.4)
P_2	(8,14,20,26;0.5)	(8,14,20,26;0.5)	(14,20,26,32;0.4)	(20,26,32,38;0.5)
P_3	(32,38,44,50;0.8)	(14,20,26,32;0.4)	(4,8,14,20;0.2)	(14,20,26,32;0.4)
P_4	(19,25,31,37;0.55)	(26,32,38,44;0.46)	(14,20,26,32;0.4)	(8,14,20,26;0.5)

Case (i): The given ω-trapezoidal fuzzy assignment problem is balanced one. Here we first obtain the matrix **[**R**(**$\widetilde{c_{ij}}$)] by using the given ranking method.

$[\Re(\widetilde{c_{ij}})]$ =

	J_1	J_2	J_3	J_4
P_1	9.20	8.50	2.30	9.20
P_2	8.50	8.50	9.20	14.5
P_3	32.8	9.20	2.30	9.20
P_4	15.4	16.1	9.20	8.50

Form a strongly feasible tree

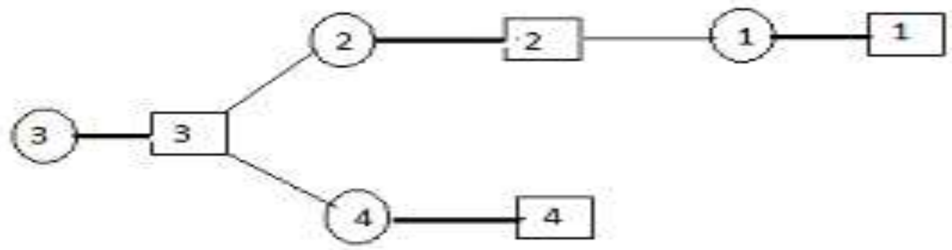

Figure:3.1

$\tilde{u}_i$* / $\tilde{v}_j^*$	**2.30**	**1.60**	**2.30**	**1.60**
6.9	9.20	8.50	2.30	9.20
6.9	8.50	8.50	9.20	14.5
0	32.8	9.20	2.30	9.20
6.9	15.4	16.1	9.20	8.50

(0)	0	***- 6.9***	0.7
- 0.7	**(0)**	0	6
30.5	7.6	**(0)**	7.6
6.2	7.6	0	**(0)**

Here $\tilde{E}$ = {[1,3], [2, 1]} ; select [i, j] = [1,3] and [i,l] = [1,2]

A strongly feasible tree (T) is producing degenerate pivoting on a backward edge

Case (ii): The above reduced rank of fuzzy cost matrix $[\Re(\widetilde{c_{ij}})]$ and strongly feasible tree (T) is producing degenerate pivoting on a backward edge. So, continue the process.

The rank of fuzzy cost matrix is given by,

$[\Re(\widetilde{c_{ij}})]$ =

	J_1	**J_2**	**J_3**	**J_4**
P_1	9.20	8.50	2.30	9.20
P_2	8.50	8.50	9.20	14.5
P_3	32.8	9.20	2.30	9.20
P_4	15.4	16.1	9.20	8.50

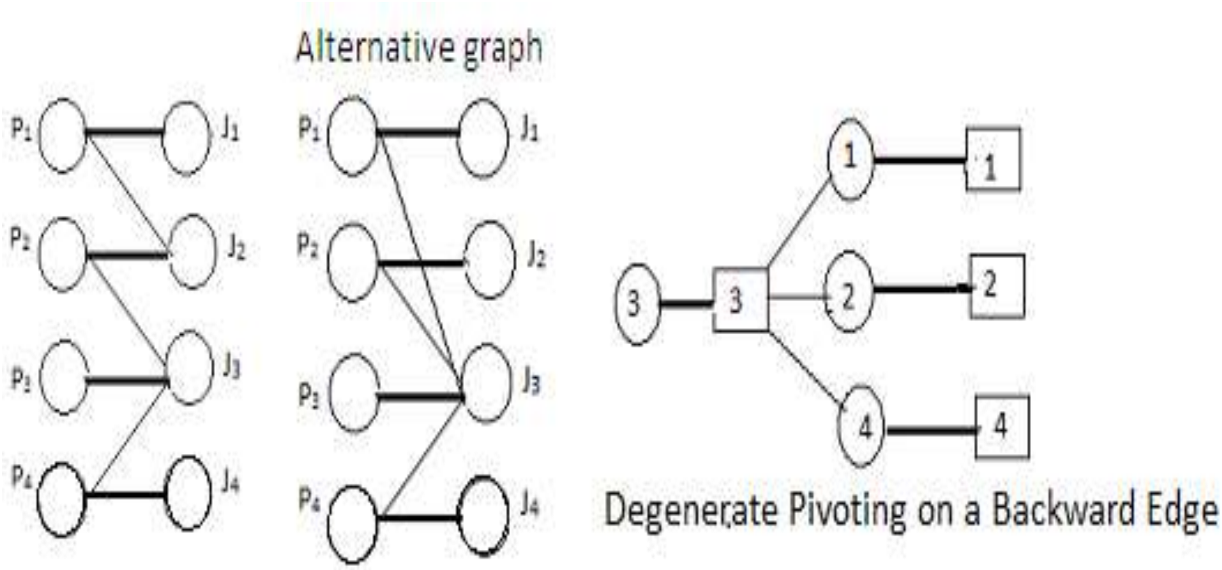

Figure :3.2

$\widetilde{u_\iota}$ * / $\tilde{v}_j{}^*$	**9.20**	**1.60**	**2.30**	**1.60**
0	9.20	8.50	2.30	9.20
6.9	8.50	8.50	9.20	14.5
0	32.8	9.20	2.30	9.20
6.9	15.4	16.1	9.20	8.50

$\Re(\widetilde{c_{\iota J}}) =$

(0)	6.9	0	7.6
- 7.6	**(0)**	0	6
23.6	7.6	**(0)**	7.6
- 0.7	7.6	0	**(0)**

Here, $\tilde{E}$ = {[4,1], [2, 1]} ; select [i, j] = [2,1] and [i,l] = [2,3]

A strongly feasible tree (T) is producing degenerate pivoting on a backward edge

Case (iii): The above reduced rank of fuzzy cost matrix $[\Re(\widetilde{c_{ij}})]$ and strongly feasible tree (T) is producing degenerate pivoting on a backward edge. So, continue the process.

The rank of fuzzy cost matrix is given by,

$[R(\widetilde{c_{ij}})]$ =

Persons /Jobs	**J_1**	**J_2**	**J_3**	**J_4**
P_1	9.20	8.50	2.30	9.20
P_2	8.50	8.50	9.20	14.5
P_3	32.8	9.20	2.30	9.20
P_4	15.4	16.1	9.20	8.50

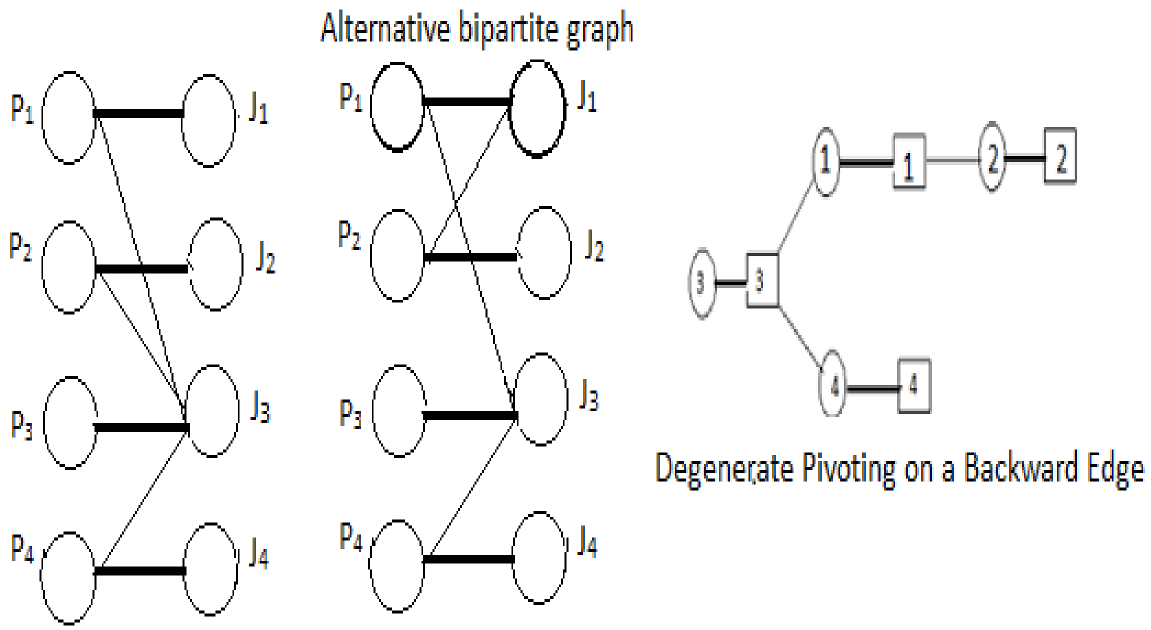

Figure:3.3

$\tilde{v}_j$ * \ $\tilde{u}_i$ *	**9.20**	**9.20**	**2.30**	**1.60**
0	9.20	8.50	2.30	9.20
- 0.7	8.50	8.50	9.20	14.5
0	32.8	9.20	2.30	9.20
6.9	15.4	16.1	9.20	8.50

$[R(\widetilde{\widetilde{c_{ij}}})]$ =

(0)	***- 0.7***	0	7.6
0	**(0)**	7.6	13.6
23.6	0	**(0)**	7.6
- 0.7	0	0	**(0)**

Here , $\tilde{E}$= {[4,1], [1,2]} ; select [i, j] = [1,2] and [i,l] = [1,1]

A strongly feasible tree (T) is producing degenerate pivoting on a backward edge

Case (iv): The above reduced rank of fuzzy cost matrix $[\Re(\widetilde{\widetilde{c_{ij}}})]$ and strongly feasible tree (T) is producing degenerate pivoting on a backward edge. So, continue the process.

The rank of fuzzy cost matrix is given by,

Persons /Jobs	J_1	J_2	J_3	J_4
P_1	9.20	8.50	2.30	9.20
P_2	8.50	8.50	9.20	14.5
P_3	32.8	9.20	2.30	9.20
P_4	15.4	16.1	9.20	8.50

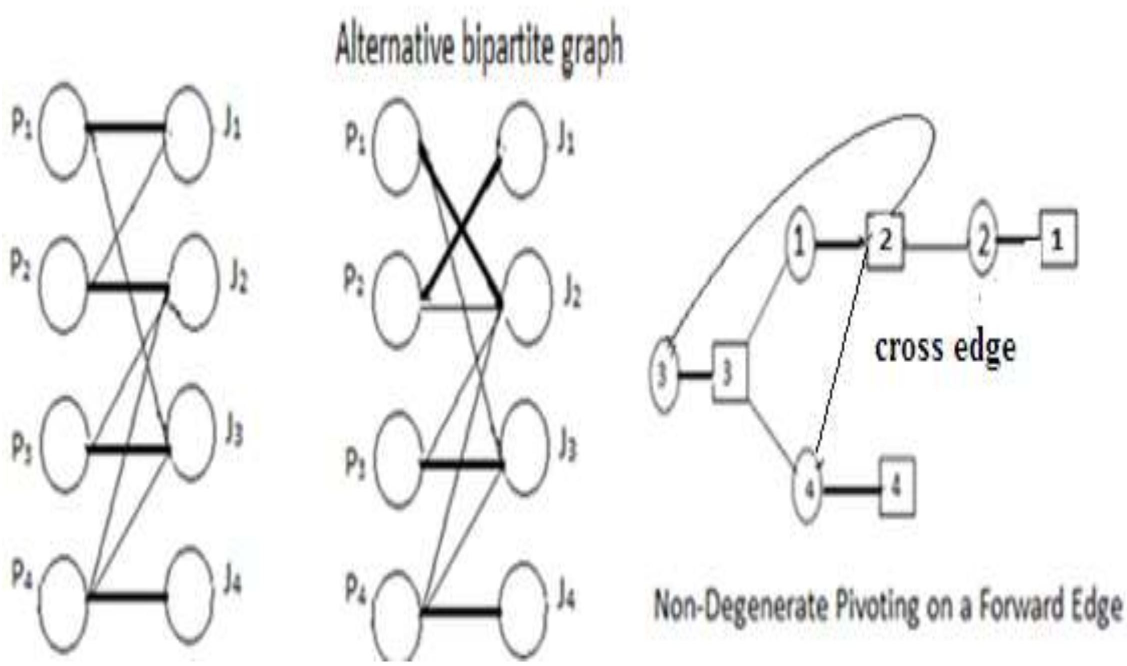

Figure: 3.4

$\tilde{v}_j$ * \ $\tilde{u}_i$ *	**8.50**	**8.50**	**2.30**	**1.60**
0	9.20	8.50	2.30	9.20
0	8.50	8.50	9.20	14.5
0	32.8	9.20	2.30	9.20
6.9	15.4	16.1	9.20	8.50

$[R(\widetilde{c_{ij}})]$ =

0.7	**(0)**	0	7.6
(0)	0	6.9	12.5
24.3	0.7	**(0)**	7.6
0	0.7	0	**(0)**

Here $\tilde{E} = \emptyset$; A strongly feasible tree (T) is Producing non- degenerate pivoting on a forward edge. Optimum reached and stop the procedure.

The optimal assignment perfect Matching schedule is $P_1 \rightarrow J_2$, $P_2 \rightarrow J_1$, $P_3 \rightarrow J_3$, $P_4 \rightarrow J_4$,

The fuzzy optimal assignment cost

$(\widetilde{C_{12}};\ \omega_{12})+(\widetilde{C_{21}};\ \omega_{21})+(\widetilde{C_{33}};\ \omega_{33})+(\widetilde{C_{44}};\ \omega_{44})$ = (8,14,20,26;0.5) + (8,14,20,26;0.5) + (4, 8, 14, 20; 0.2) + 8, 14, 20, 26; 0.5)

= *(*28,50,74, 98;0.2)

and also $[\Re(\widetilde{c_{ij}})]$ = (28,50,74,98;0.2) = *12.5.*

CHAPTER IV

A NEW MODIFIED OPTIMAL PERFECT MATCHING IN PARTIAL FEASIBLE MATCHING FOR SOLVING FUZZY LINEAR SUM ASSIGNMENT PROBLEMS

In this chapter proposed, a new modified optimal and perfect matching from partial assignment for solving fuzzy linear sum assignment problem. Here the fuzzy assignment cost is ω- trapezoidal fuzzy numbers and by using ranking method ω-trapezoidal fuzzy numbers converted to crisp one. First Calculate the rank of fuzzy dual variables and Compute a partial feasible solution then Calculate reduced rank of ω-- trapezoidal fuzzy cost and Compute the new column and the new assignment introduced. Continue the process to reach optimal solution and complete bipartite matching. A numerical example is used to illustrate this technique.

4.1 INTRODUCTION

The assignment problem is a special type of the transportation problem. The assignment problem is in the form of mxn matrix. Here, we have 'n' jobs is performed to 'm' persons and our objective is minimize the cost or maximize the profit. Let c_{ij} be the assignment cost is performed to j^{th} job to the i^{th} person.

Fuzzy assignment problem is most powerful tool in fuzzy operations research. Many applications implemented and new ideas introduced in this criteria. The Linear Sum Assignment Problem is most important and special type of linear programming and in combinatorial optimization. we are given mxn matrix and c_{ij} is denotes as cost for

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optimal assigning through the j^{th} job to the i^{th} person and total sum of the entries minimize the cost or maximize the profit. In other words, Let G be a bipartite graph. G=(U,V;E) being a vertex of U for each row, a vertex of V for each column and cost or profit associated with edge [i, j] for i = 1,2...n and j = 1,2...n then the problem is to determine a minimum cost or maximum profit is perfect matching in G. Easterfield was first initiated for Linear Sum Assignment Problem(LSAP) in 1946.Cunningham and Barr, Glover, and Kingman first initiated primal simplex algorithm in the mid-1970s.

4.2. Ranking of ω-Trapezoidal Fuzzy Numbers.

The decision maker first take ω-trapezoidal fuzzy cost and then find ranking of ω -trapezoidal fuzzy cost after the process of optimal decisions. Here, ω-trapezoidal fuzzy cost is decision variables,

The ranking of ω -trapezoidal fuzzy cost to following comparisons are exits.

- **(i)** $\tilde{A} >_R \tilde{B}$ if and only if $R(\tilde{A}) > R(\tilde{B})$.
- **(ii)** $\tilde{A} <_R \tilde{B}$ if and only if $R(\tilde{A}) < R(\tilde{B})$.
- **(iii)** $\tilde{A} =_R \tilde{B}$ if and only if $R(\tilde{A}) = R(\tilde{B})$

Let $\tilde{A} = (p_1, q_1, r_1, s_1; \omega_1)$ and $\tilde{B} = (p_2, q_2, r_2, s_2; \omega_2)$ be any two ω-trapezoidal fuzzy numbers, the following ranking function [7] is,

$$\Re(\tilde{A}) = \frac{\omega_1(2p_1+q_1+,r_1+2s_1)}{6}, \quad \Re(\tilde{B}) = \frac{\omega_2(2p_2+q_2+r_2+2s_2)}{6}.$$

4.3 A New Modified Optimal Perfect Matching in Partial Feasible Matching.

The steps for a new modified optimal matching in a partial feasible matching technique are as follows:

Step 1: First, determine if the number of people and the number of jobs are equal. If they are, then do the ω -trapezoidal fuzzy assignment cost. When it is said that the problem is balanced, go on to step 2. Otherwise, dummy rows or dummy columns are added if it is not balanced.

Step 2: ω-trapezoidal fuzzy numbers convert to crisp one by using ranking method [7]. The rank of ω-trapezoidal fuzzy cost is defined as R $[\tilde{C}_{ij}]$.

Step3: Calculate the rank of fuzzy dual variables

$$\Re[\tilde{v}_j] = \min\{\Re[\tilde{c}_{ij}];\ i = 1\ to\ n\ and\ j = 1\ to\ n.$$

$$\Re[\tilde{u}_i] = min\{\Re[\tilde{c}_{ij} - \tilde{v}_j];\ i = 1\ to\ n\ and\ j = 1\ to\ n.$$

Step4: Compute a partial feasible solution

$$\Re[\tilde{c}_{ij} - \tilde{v}_j] = \begin{cases} i & if\ \ column\ j\ is\ assigned\ to\ row\ i \\ 0 & if\ column\ j\ is\ not\ assigned\ to\ row\ i. \end{cases}$$

Step 5: Calculate reduced rank of ω-trapezoidal fuzzy cost R ($\bar{\tilde{c}}_{ij}$)

Find partial feasible matching [less than 'n' - rows / columns matching]in the bipartite graph and calculate the reduced rank of ω- trapezoidal fuzzy cost R $[\tilde{c}_{ij} - \tilde{u}_i - \tilde{v}_j]$ and performed in j^{th} job to the i^{th} person in G.

Step 6: Partial to Optimal Matching

First we take unassigned row $[i^*]$ and calculate

$g^* = \arg\min\{\Re[\tilde{c}_{ij} - \tilde{v}_j];\ j = 1,2...n\}$ and $u_{g^*} = c_{ig^*} - v_{g^*}$

$h^* = \arg\min\{\Re[\tilde{c}_{ij} - \tilde{v}_j];\ j = 1,2...n,\ g^* \neq h^*\}$ and. $u_{h^*} = c_{ih^*} - v_{h^*}$

Step 7: If $h^* > g^*$ then calculate $v_{g^{**}} = v_{g^*} - (u_{h^*} - u_{g^*})$.

Step 8: compute new column and new assignment introduced

If we consider T = $u_{h^*} - u_{g^*}$ then calculate new pivotal column $v_{g^{**}} = v_{g^*} + T$.

The new assignment is obtained $z^* = u_{g^*} + T$.

Step 9: To find entering and leaving assignment

The entering new assignment is z^* and the entering assignment column is known as key assignment column or pivot assignment column. The old assigned element of the key assignment column is leaving assignment and introduced new assignment z*. if the key assignment column already must does not assign any old assignment then we have introduced new assignment z*.

Step10: To find optimal perfect matching

- Each row has precisely one unmarked zero that is assigned, and it crosses all other zeros in the corresponding column because these won't be taken into account for any additional future allocations. Continue in this manner until each row has been encountered.
- Each column has precisely one unmarked zero that is assigned; encircle this one unmarked zero and cross any additional zeros in the column. Continue in this manner until each column has been encountered. The ideal assignment is attained at the end.
- Finally, obtain optimal and complete matching in the bipartite graph assigned from j^{th} job to the i^{th} person in G.

Step 11: STOP.

4.4 NUMERICAL EXAMPLE

The fuzzy assignment problem associated with four jobs J_1, J_2, J_3, J_4 and four persons P_1, P_2, P_3, P_4 respectively. The fuzzy assignment cost to be ω-trapezoidal fuzzy numbers and allocating each row and each column exactly one perfect person.

Solution :

First, determine if the number of people and the number of jobs are equal. If they are, then do the ω -trapezoidal fuzzy assignment cost. When it is said that the problem is balanced. Create balanced ω -trapezoidal fuzzy assignment cost.

Persons / jobs	$\mathbf{J_1}$	$\mathbf{J_2}$	$\mathbf{J_3}$	$\mathbf{J_4}$
$\mathbf{M_1}$	(12,14,16,18;0.7)	(16,18,20,22;0.9)	(18,20,22,24;0.95)	(22,24,26,28;0.99)
$\mathbf{M_2}$	(4,6,8,10;0.3)	(14,16,18,20;0.8)	(12,14,16,18;0.7)	(16,18,20,22;0.9)
$\mathbf{M_3}$	(2,4,6,8;0.2)	(10,12,14,16;0.6)	(12,14,16,18;0.7)	(20,22,24,26;0.97)
$\mathbf{M_4}$	(6,8,10,12;0.4)	(10,12,14,16;0.6)	(4,6,8,10;0.3)	(8,10,12,14;0.5)

The ω -trapezoidal fuzzy assignment cost is convert to crisp one by using ranking method. The rank of ω -trapezoidal fuzzy assignment cost is denoted as $\mathfrak{R}[\ \tilde{c}_{ij}]$,

10.5	17.1	19.95	24.75
2.1	13.6	10.5	17.1
1	7.8	10.5	22.31
3.6	7.8	2.1	5.5

First, find the minimum value of each column and next calculate the difference between the rank of ω -trapezoidal fuzzy assignment cost and minimum value of each column.

9.5	9.3	17.85	19.25
1.1	5.8	8.4	11.6
0	**0**	8.4	16.81
2.6	0	0	**0**

Calculate R [$\tilde{c}_{ij}$ -$\tilde{u}_i$- $\tilde{v}_j$]

0.2	0	8.55	9.95
0	4.7	7.3	10.5
0	**0**	8.4	16.81
2.6	0	0	**0**

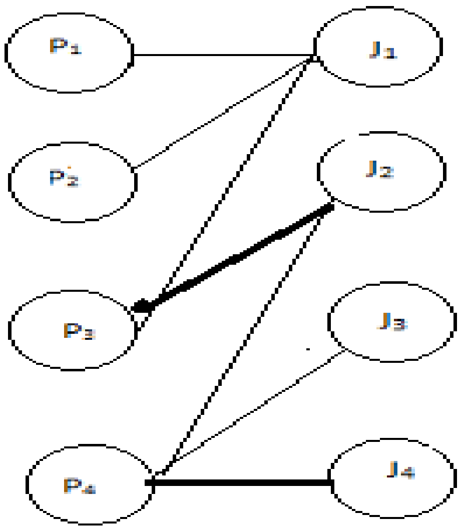

Figure: 4.1

v = (1,7.8,2.1,5.5) ; u = (9.3,1.1,0,0) ; φ = (0,0,2,4) ; ρ = (0,3,0,4).

First we take, $i^* = 1$; u_{g^*}= 9.3 ; u_{h^*}= 9.5 ; $v_{2^{**}} = v_{2^*} - (u_{h^*} - u_{g^*}) = 7.6$.

Calculate new feasible assignment

9.5	**9.5**	17.85	19.25
1.1	6	8.4	11.6
0	0.2	8.4	16.81
2.6	0.2	0	**0**

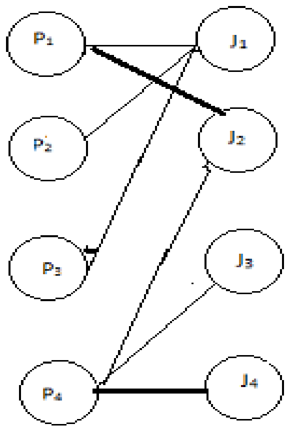

Figure:4.2

V = (1,7.6,2.1,5.5);; φ = (2,0,0,4) ; ρ = (0,1,0,4).

i* = 3 ; u_{g^*}= 0 ; u_{h^*}= 0.2 ; $v_{2^{**}} = v_{2^*} - (u_{h^*} - u_{g^*}) = 0.8$.

Calculate new feasible assignment

9.7	9.5	17.85	19.25
1.3	6	8.4	11.6
0.2	0.2	8.4	16.81
2.8	0.2	0	**0**

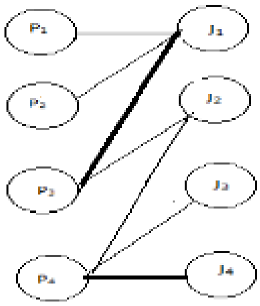

Figure:4.3

V = (0.8,7.6,2.1,5.5); φ = (0,0,1,4) ; ρ = (3,0,0,4).

i* = 2 ; u_{g^*}= 1.3 ; u_{h^*}= 6 ; $v_{2^{**}}$ = v_{2^*} - (u_{h^*} - u_{g^*}) = -3.9.

14.4	9.5	17.85	19.25
6	6	8.4	11.6
4.9	0.2	8.4	16.81
7.5	0.2	0	**0**

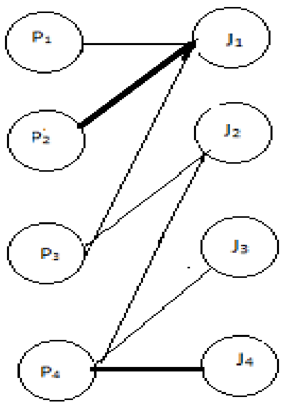

Figure:4.4

V = (-3.9,7.6,2.1,5.5); φ = (0,1,0,4) ; ρ = (2,0,0,4).

i* = 3 ; u_{g^*}= 0.2 ; u_{h^*}= 4.9 ; $v_{2^{**}}$ = v_{2^*} - (u_{h^*} - u_{g^*}) =2.9.

14.4	14.2	17.85	19.25
6	10.7	8.4	11.6
4.9	**4.9**	8.4	16.81
7.5	4.9	0	**0**

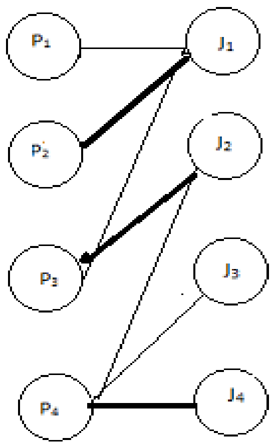

Figure:4.5

V = (-3.9,2.9,2.1,5.5); φ = (0,1,2,4) ; ρ = (2,3,0,4).

$i^* = 1$; u_{g^*}=14.2 ; u_{h^*}= 14.4 ; $v_{2^{**}} = v_{2^*} - (u_{h^*} - u_{g^*})$ =2.7 .

Calculate new feasible assignment

Partial Feasible Matching in Bipartite Graph

14.4	**14.4**	17.85	19.25
6	10.9	8.4	11.6
4.9	5.1	8.4	16.81
7.5	5.1	0	**0**

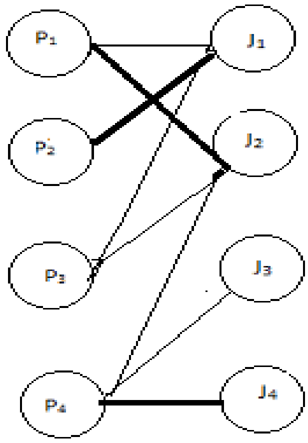

Figure:4.6

V = (-3.9,2.7,2.1,5.5); φ = (2,1,0,4) ; ρ = (2,1,0,4).

i* = 3 ; u_{g^*}= 4.9 ; u_{h^*}=5.1; $v_{2^{**}} = v_{2^*} - (u_{h^*} - u_{g^*}) = -4.1$

14.6	**14.4**	17.85	19.25
6.2	10.9	8.4	11.6
5.1	5.1	8.4	16.81
7.7	5.1	0	**0**

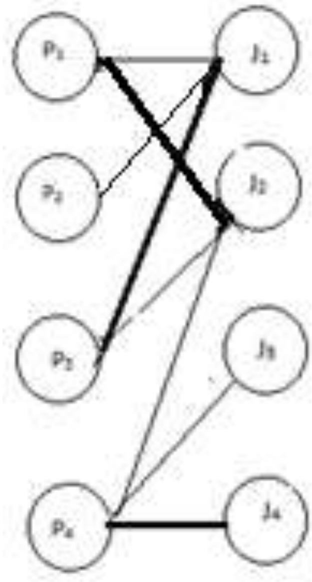

Figure: 4.7

V = (-4.1,7.6,2.1,5.5); φ = (2,0,1,4) ; ρ = (3,1,0,4).

Partial Feasible Matching in Bipartite Graph

i^* = 2 ; u_{g^*}= 6.2 ; u_{h^*}= 8.4 ; $v_{2^{**}}$ = v_{2^*} - (u_{h^*} - u_{g^*}) = -6.3

Calculate new feasible assignment

16.8	**14.4**	17.85	19.25
8.4	10.9	8.4	11.6
7.3	5.1	8.4	16.81
9.9	5.1	0	**0**

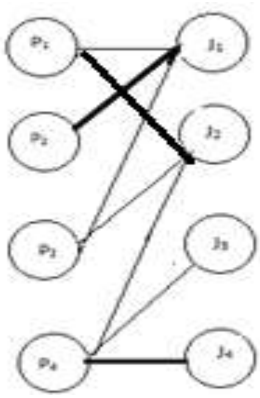

Figure: 4.8

V = (-6.3,7.6,2.1,5.5); φ = (2,1,0,4) ; ρ = (2,1,0,4).

Partial Feasible Matching in Bipartite Graph

i^*= 3 ; u_{g^*}=5.1 ; u_{h^*}= 7.3 ; $v_{2^{**}}$ = v_{2^*} - (u_{h^*} - u_{g^*}) = 5.4

16.8	16.6	17.85	19.25
8.4	13.1	8.4	11.6
7.3	**7.3**	8.4	16.81
9.9	7.3	0	**0**

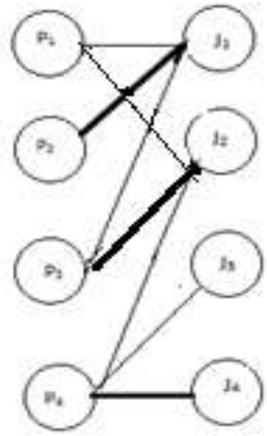

Figure: 4.9

V = (-6.3,5.4,2.1,5.5); φ = (0,1,2,4) ; ρ = (2,3,0,4).

$i^*= 1$; u_{g^*}=16.6 ; u_{h^*}= 16.8 ; $v_{2^{**}} = v_{2^*} - (u_{h^*} - u_{g^*}) = 5.2$

Calculate new feasible assignment

16.8	**16.8**	17.85	19.25
8.4	13.3	8.4	11.6
7.3	7.5	8.4	16.81
9.9	7.5	0	**0**

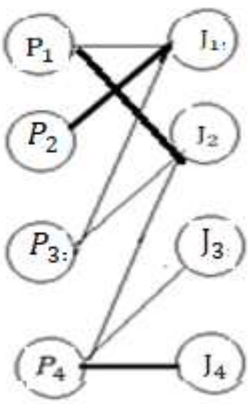

Figure: 4.10

V = (-6.3,5.2,2.1,5.5); φ = (2,1,0,4) ; ρ = (2,1,0,4).

i^* =3 ; u_{g^*}=7.3 ; u_{h^*}= 7.5 ; $v_{2^{**}} = v_{2^*}$ - (u_{h^*} - u_{g^*}) = -6.5

17	**16.8**	17.85	19.25
8.6	13.3	8.4	11.6
7.5	7.5	8.4	16.81
10.1	7.5	0	**0**

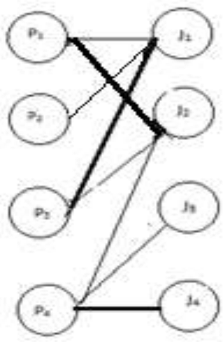

Figure: 4.11

V = (-6.5**,5.2,2.1,5.5); φ = (2,0,1,4) ; ρ = (3,1,0,4).**

i^*= 2 ; u_{g^*}=8.4 ; u_{h^*}= 8.6 ; $v_{2^{**}} = v_{2^*}$ - (u_{h^*} - u_{g^*}) = 1.

17	***16.8***	18.05	19.25
8.6	13.3	***8.6***	11.6
7.5	7.5	8.6	16.81
10.1	7.5	0.2	**0**

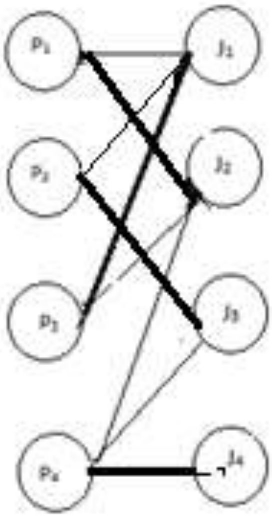

Figure: 4.12

V = (-6.5,5.2,1.9,5.5); φ = (2,3,0,4) ; ρ = (3,1,2,4).

The ω-Trapezoidal Fuzzy Optimal Assignment Table

Persons / jobs	**J_1**	**J_2**	**J_3**	**J_4**
P_1	(12,14,16,18;0.7)	***(16,18,20,22;0.9)***	(18,20,22,24;0.6)	(22,24,26,28;0.7)
P_2	(4,6,8,10;0.3)	(14,16,18,20;0.8)	***(12,14,16,18;0.7)***	(16,18,20,22;0.9)
P_3	***(2,4,6,8;0.2)***	(10,12,14,16;0.6)	(12,14,16,18;0.7)	(20,22,24,26;0.8)
P_4	(6,8,10,12;0.4)	(10,12,14,16;0.6)	(4,6,8,10;0.3)	***(8,10,12,14;0.5)***

The Optimal Perfect Matching Schedule is; **$P_1 \rightarrow J_2, P_2 \rightarrow J_3, P_3 \rightarrow J_1, P_4 \rightarrow J_4$.**

The Optimal Perfect Matching in ω-trapezoidal fuzzy assignment cost is

(2,4,6,8;0.2) + (16,18,20,22;0.9) + (12,14,16,18;0.7) + (8,10,12,14;0.5)

= (38,46,54,62 ; 0.2)

The Rank of Optimal Perfect Matching in fuzzy assignment cost is $\mathbf{R[\tilde{C}_{ij}]}$ = ***10.***

CHAPTER V

A SPREAD OUT OF NEW PARTIAL FEASIBLE AND OPTIMAL PERFECT MATCHING FOR SOLVING INTERVAL-VALUED α-Cut FUZZY LINEAR SUM BOTTLENECK ASSIGNMENT PROBLEM

In this chapter, proposed a spread of new partial/feasible and optimal/perfect matches of bipartite graph for solving interval-valued α-cuts of generalized trapezoidal fuzzy numbers. Obtain α-cut generalized trapezoidal fuzzy numbers from generalized trapezoidal fuzzy numbers, and discussed membership functions. The α-cut generalized trapezoidal fuzzy number is transformed into an Interval-valued α-Cut of Generalized Fuzzy Numbers. The basic preliminaries and fuzzy interval operations are discussed. If maximum cost and complete match, then the solution is feasible and complete solution. If the solution is minimum cost and complete match, then the solution is optimal and complete solution. If maximum cost or partial match, then the solution is feasible with partial solution. If minimum cost or partial match, then the solution is optimal or feasible solution. This method is illustrated by a numerical example.

5.1 INTRODUCTION

Let 'J' jobs and 'P' machines be given in a balanced interval valued fuzzy linear sum bottleneck assignment problem (IFLSBAP) where $\widetilde{C_{ij}}$ generalized trapezoidal fuzzy numbers. The bottleneck assignment refers to latest completion in the allocation of assignment problem. The interval-valued α-cut of generalized fuzzy linear sum bottleneck assignment problems are minimum cost maximum matching problem.

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Let G = (U,V;E) be a bipartite graph with edge set E. The edge [i,j] has a cost coefficient $\omega_{\widetilde{C_{ij}}}$ We obtain complete matching in G such that the perfect length of an edge in this matching is as small as possible. The bottleneck minimum cost maximum matching problem can be formulated as follows. Find maximum matching in G such that the maximum length of an edge in this matching is small as possible.

Fuzzy numbers have two characteristics on which fuzzy arithmetic is based. The following are the two characteristics of fuzzy numbers:

(a) Each fuzzy set, and hence every fuzzy number, may be completely and specifically represented by its α- cuts.

(b) α- cuts of each fuzzy number are closed intervals of real numbers $\forall\ \alpha \epsilon$ (0,1]. These characteristics allow us to describe arithmetic operations on fuzzy numbers in terms of arithmetic operations on their α- cuts.

The idea of using threshold methods of the interval-valued α-cut of generalized fuzzy numbers. A threshold method alternates between two phases. In the first phase a cost element (threshold value) is chosen and threshold matrix $\widetilde{C_{ij}}$ is defined by threshold matrix is equal to 1 if the cost element is greater than threshold value otherwise zero.

5.2 GENERALIZED α-CUT FUZZY NUMBERS TO FUZZY INTERVAL

$\omega_{\widetilde{C_{11}}}$ = [$\breve{a}^L$ + ($\breve{a}^\alpha$-$\breve{a}^L$) ω, -($\breve{a}^U$-$\breve{a}^\beta$) + $\breve{a}^U$] = [${a_{11}}^L{d_{11}}^U$],

$\omega_{\widetilde{C_{12}}}$ = [$\breve{a}^L$ + ($\breve{a}^\alpha$-$\breve{a}^L$) ω, -($\breve{a}^U$-$\breve{a}^\beta$) + $\breve{a}^U$] = [${a_{12}}^L{d_{12}}^U$]

$\omega_{\widetilde{C_{13}}}$ = [$\breve{a}^L$ + ($\breve{a}^\alpha$-$\breve{a}^L$) ω, -($\breve{a}^U$-$\breve{a}^\beta$) + $\breve{a}^U$] = [${a_{13}}^L{d_{13}}^U$],

$\omega_{\widetilde{C_{14}}}$ = [$\breve{a}^L$ + ($\breve{a}^\alpha$-$\breve{a}^L$) ω, -($\breve{a}^U$-$\breve{a}^\beta$) + $\breve{a}^U$] = [${a_{14}}^L{d_{14}}^U$]

$\omega_{\widetilde{C_{21}}}$ = [$\breve{a}^L$ + ($\breve{a}^\alpha$-$\breve{a}^L$) ω, -($\breve{a}^U$-$\breve{a}^\beta$) + $\breve{a}^U$] = [${a_{21}}^L {d_{21}}^U$]

$\omega_{\widetilde{C_{22}}}$ = [$\breve{a}^L$ + ($\breve{a}^\alpha$-$\breve{a}^L$) ω, -($\breve{a}^U$-$\breve{a}^\beta$) + $\breve{a}^U$] = [${a_{22}}^L {d_{22}}^U$]

$\omega_{\widetilde{C_{23}}}$ = [$\breve{a}^L$ + ($\breve{a}^\alpha$-$\breve{a}^L$) ω, -($\breve{a}^U$-$\breve{a}^\beta$) + $\breve{a}^U$] = [${a_{23}}^L {d_{23}}^U$],

$\omega_{\widetilde{C_{24}}}$ = [$\breve{a}^L$ + ($\breve{a}^\alpha$-$\breve{a}^L$) ω, -($\breve{a}^U$-$\breve{a}^\beta$) + $\breve{a}^U$] = [${a_{24}}^L {d_{24}}^U$]

$\omega_{\widetilde{C_{31}}}$ = [$\breve{a}^L$ + ($\breve{a}^\alpha$-$\breve{a}^L$) ω, -($\breve{a}^U$-$\breve{a}^\beta$) + $\breve{a}^U$] = [${a_{31}}^L {d_{31}}^U$],

$\omega_{\widetilde{C_{12}}}$ = [$\breve{a}^L$ + ($\breve{a}^\alpha$-$\breve{a}^L$) ω, -($\breve{a}^U$-$\breve{a}^\beta$) + $\breve{a}^U$] = [${a_{32}}^L {d_{32}}^U$]

$\omega_{\widetilde{C_{33}}}$ = [$\breve{a}^L$ + ($\breve{a}^\alpha$-$\breve{a}^L$) ω, -($\breve{a}^U$-$\breve{a}^\beta$) + $\breve{a}^U$] = [${a_{33}}^L {d_{33}}^U$],

$\omega_{\widetilde{C_{34}}}$ = [$\breve{a}^L$ + ($\breve{a}^\alpha$-$\breve{a}^L$) ω, -($\breve{a}^U$-$\breve{a}^\beta$) + $\breve{a}^U$] = [${a_{34}}^L {d_{34}}^U$]

$\omega_{\widetilde{C_{41}}}$ = [$\breve{a}^L$ + ($\breve{a}^\alpha$-$\breve{a}^L$) ω, -($\breve{a}^U$-$\breve{a}^\beta$) + $\breve{a}^U$] = [${a_{41}}^L {d_{41}}^U$],

$\omega_{\widetilde{C_{12}}}$ = [$\breve{a}^L$ + ($\breve{a}^\alpha$-$\breve{a}^L$) ω, -($\breve{a}^U$-$\breve{a}^\beta$) + $\breve{a}^U$] = [${a_{42}}^L {d_{42}}^U$]

$\omega_{\widetilde{C_{43}}}$ = [$\breve{a}^L$ + ($\breve{a}^\alpha$-$\breve{a}^L$) ω, -($\breve{a}^U$-$\breve{a}^\beta$) + $\breve{a}^U$] = [${a_{43}}^L {d_{43}}^U$],

$\omega_{\widetilde{C_{44}}}$ = [$\breve{a}^L$ + ($\breve{a}^\alpha$-$\breve{a}^L$) ω, -($\breve{a}^U$-$\breve{a}^\beta$) + $\breve{a}^U$] = [${a_{44}}^L {d_{44}}^U$]

5.3 MATHEMATICAL FORMULATION

Min $\max_{1 \le i,j \le n} \omega_{\widetilde{C_{ij}}} x_{ij}$

Such that $\sum_{j=1}^{n} x_{ij} = 1$ (i = 1,2…..n)

$\sum_{j=1}^{n} x_{ij} = 1$ (j =1,2…..n)

$x_{ij} \in \{0,1\}$ (i,j = 1,2…..n).

5.3.1 Definition: An interval value $\omega_{\widetilde{C_{ij}}}$ = $[a_{ij}{}^{L} d_{ij}{}^{U}]$∈ R is said to be interval value fuzzy set with membership grade $\mu_{\widetilde{C_{\omega}}}(x)$ then the following membership functions as,

$$\mu_{\widetilde{C_{\omega}}}(x) = \begin{cases} 0, & X < a_{ij}{}^{L} \\ 1, & a_{ij}{}^{L} < X < d_{ij}{}^{U} a_{ij}{}^{L} < d_{ij}{}^{U} \\ 0, & X > d_{ij}{}^{U} \end{cases}$$

5.3.2 Interval-valued α-Cut of Fuzzy Linear Sum Bottleneck Assignment Table:

$[a_{11}{}^{L} d_{11}{}^{U}]$	$[a_{12}{}^{L} d_{12}{}^{U}]$	$[a_{13}{}^{L} d_{13}{}^{U}]$	$[a_{14}{}^{L} d_{14}{}^{U}]$
$[a_{21}{}^{L} d_{21}{}^{U}]$	$[a_{22}{}^{L} d_{22}{}^{U}]$	$[a_{23}{}^{L} d_{23}{}^{U}]$	$[a_{24}{}^{L} d_{24}{}^{U}]$
$[a_{31}{}^{L} d_{31}{}^{U}]$	$[a_{32}{}^{L} d_{32}{}^{U}]$	$[a_{33}{}^{L} d_{33}{}^{U}]$	$[a_{34}{}^{L} d_{34}{}^{U}]$
$[a_{41}{}^{L} d_{41}{}^{U}]$	$[a_{42}{}^{L} d_{42}{}^{U}]$	$[a_{43}{}^{L} d_{43}{}^{U}]$	$[a_{44}{}^{L} d_{44}{}^{U}]$

5.4 α-CUT OF THRESHOLD FUZZY LINEAR SUM BOTTLENECK ASSIGNMENT PROBLEM :

In the first case α-Cut of threshold Fuzzy Linear Sum Bottleneck Assignment cost element is ($\omega_{\widetilde{C_{ij}}}{}^{*}_{M}$) and α-Cut of threshold Fuzzy Linear Sum Bottleneck Assignment are defined as,

$$\omega_{\widetilde{C_{ij}}} = \begin{cases} 1, if\ \omega_{\widetilde{C_{ij}}} > \omega_{\widetilde{C_{ij}}}{}^{*}_{M} \\ 0, otherwise \end{cases}$$

Let C = $\omega_{\widetilde{C_{ij}}}$ be n x n matrix and $\omega_{\widetilde{C_{i}}}{}^{U}_{\varphi(\mathrm{i})\mathrm{n}}$ be a fuzzy arbitrary permutation of IFLSBAP.

Spreading solution is sp (φ(i)) = max {min{$\omega_{\widetilde{C_{i}}}{}_{\varphi(\mathrm{i})\mathrm{n}}$}

5.4.1 Property:

If two elements of IFLSBAP are in increasing order, then prove that the sum of two elements of IFLSBAP is also in Increasing Order,

Proof: Let X = $[{a_{11}}^L {d_{11}}^U]$ and Y =$[{a_{22}}^L {d_{22}}^U]$ are two closed interval values in R is IFLSBAP.

Here, ${a_{ij}}^L$<${d_{ij}}^U$ and ${a_{22}}^L$<${d_{22}}^U$ are in increasing orders.

We prove that, the sum of two elements of IFLSBAP is also in increasing Order, adding X and Y. We get,

X+Y = $[{a_{11}}^L {d_{11}}^U]$ + $[{a_{22}}^L {d_{22}}^U]$ = $[{a_{11}}^L + {a_{22}}^L {d_{11}}^U + {d_{22}}^U]$.

We see that, Here, ${a_{11}}^L + {a_{22}}^L < {d_{11}}^U + {d_{22}}^U$ and ${a_{ij}}^L$<${d_{ij}}^U$ and ${a_{22}}^L$<${d_{22}}^U$.

Therefore, $[{a_{11}}^L {d_{11}}^U]$ + $[{a_{22}}^L {d_{22}}^U]$< $[{a_{11}}^L + {a_{22}}^L {d_{11}}^U + {d_{22}}^U]$. Hence, If two elements of IFLSBAP are in increasing order then the sum of two elements of IFLSBAP are also in increasing order.

5.5 THE PROPOSED ALGORITM:

Solving optimal perfect matching and feasible partial matching by using generalized α-cut trapezoidal fuzzy numbers we present in the following step by step procedure.

Step 1: Generalized α-cut trapezoidal fuzzy numbers

Let us take generalized trapezoidal fuzzy number and obtain α-cut of trapezoidal fuzzy numbers, If α = ω,then the following form.

$$\omega_{\breve{A}} = [\breve{a}^{L} + (\breve{a}^{\alpha}\text{-}\breve{a}^{L})\omega,\ \ \text{-}(\breve{a}^{U}\text{-}\breve{a}^{\beta}) + \breve{a}^{U}],$$

$$\omega_{\breve{B}} = [\breve{b}^{L} + (\breve{b}^{\alpha}\text{-}\breve{b}^{L})\omega,\ \ \text{-}(\breve{b}^{U}\text{-}\breve{b}^{\beta}) + \breve{b}^{U}].$$

Step 2: compute fuzzy interval values by using generalized α-cut trapezoidal fuzzy numbers:

$$\omega_{\widetilde{C_{ij}}} = [\breve{a}^{L} + (\breve{a}^{\alpha}\text{-}\breve{a}^{L})\ \omega,\ \ \text{-}(\breve{a}^{U}\text{-}\breve{a}^{\beta}) + \breve{a}^{U}] = [{a_{ij}}^{L}{d_{ij}}^{U}].$$

Where ${a_{ij}}^{L}<{d_{ij}}^{U}$ and ${a_{ij}}^{L}$ = lower boundary of least value

${d_{ij}}^{U}$ = upper boundary of largest value

Step 3: Forming balanced interval valued fuzzy linear sum bottleneck assignment problem (IFLSBAP): (number of Machines (M) and number of Jobs (P) are equal i.e.,$\sum_{i=1}^{n} M_i = \sum_{j=1}^{n} P_j$), if IFLSBAP $\sum_{i=1}^{n} M_i \neq \sum_{j=1}^{n} P_j$, We introduce dummy row i.e.,$\sum_{i=1}^{n} M_i + D_i$ (or), introduce dummy column $\sum_{j=1}^{n} P_j + D_j$ (where D_i = dummy row ,D_j = dummy column)

Step 4: Calculate${\omega_{\widetilde{C_{ij}}}}_{0}^{L}$, ${\omega_{\widetilde{C_{ij}}}}_{n}^{U}$.

Let $\omega_{\widetilde{C_{ij}}} = [{a_{ij}}^{L}{d_{ij}}^{U}]$ be (n x n) interval cost/time matrix;

$${\omega_{\widetilde{C_{ij}}}}_{0}^{L} = min_{ij}\{{a_{ij}}^{L}{d_{ij}}^{U}\},\ {\omega_{\widetilde{C_{ij}}}}_{n}^{U} = max_{ij}\{{a_{ij}}^{L}{d_{ij}}^{U}\}$$

Step 5: Calculate(${\omega_{\widetilde{C_{ij}}}}^{*}$) , (${\omega_{\widetilde{C_{ij}}}}_{M}^{*}$)

$${\omega_{\widetilde{C_{ij}}}}^{*} = \{\omega_{\widetilde{C_{ij}}} : {\omega_{\widetilde{C_{ij}}}}_{0}^{L} < \omega_{\widetilde{C_{ij}}} < {\omega_{\widetilde{C_{ij}}}}_{n}^{U}$$

$${\omega_{\widetilde{C_{ij}}}}_{M}^{*} = \min\{\ \omega_{\widetilde{C_{ij}}} \in {\omega_{\widetilde{C_{ij}}}}^{*} : |\{\ \omega_{\widetilde{C_{ij}}} \in {\omega_{\widetilde{C_{ij}}}}^{*} :\ \omega_{\widetilde{C_{ij}}} \leq [{a_{ij}}^{L}{d_{ij}}^{U}]\}| \geq |{\omega_{\widetilde{C_{ij}}}}^{*}| / 2.$$

Step 6: Feasibility check: Select the feasible element $\omega_{\widetilde{C_{ij}}_M}{}^{*}$, ($\omega_{\widetilde{C_{ij}}_0}{}^{L}$< $\omega_{\widetilde{C_{ij}}_1}{}^{L}$, $\omega_{\widetilde{C_{ij}}_2}{}^{L}$…..<$\omega_{\widetilde{C_{ij}}_M}{}^{*}$ = 0), select the ($\omega_{\widetilde{C_{1j}}}$) row and assigned at least only one zero , similarly column allocations are$\omega_{\widetilde{C_{2j}}}, \omega_{\widetilde{C_{3j}}}$……$\omega_{\widetilde{C_{nj}}}$, next column assigned as $\omega_{\widetilde{C_{i1}}}$, $\omega_{\widetilde{C_{i2}}}$,…. $\omega_{\widetilde{C_{in}}}$. Each row and each column at least only one zero are assigned. Every row and column has at least one matching, the feasibility is executed. The bipartite matching, if minimum cost and maximum matching is optimal and perfect, otherwise the bipartite graph is feasible and partial matching. Obtain interval valued fuzzy linear sum cost is $\sum_{i=1}^{n} \omega_{\widetilde{C_i}_{\varphi(\mathrm{i})M}}{}^{*}$ = $[a_{i\varphi(\mathrm{i})M}{}^{L} d_{i\varphi(\mathrm{i})M}{}^{U}]$.

Step 7: Backward calculation: Select the lower feasible elements of $\omega_{\widetilde{C_{ij}}_M}{}^{*}$and determine Feasible IFLSBAP

$\omega_{\widetilde{C_{ij}}_0}{}^{L}$ =0,the feasible cost/time is$\sum_{i=1}^{n} \omega_{\widetilde{C_i}_{\varphi(\mathrm{i})0}}{}^{L}$= $[a_{i\varphi(\mathrm{i})0}{}^{L} d_{i\varphi(\mathrm{i})0}{}^{U}]$

$\omega_{\widetilde{C_{ij}}_1}{}^{L}$ =0,the feasible cost/time is $\sum_{i=1}^{n} \omega_{\widetilde{C_i}_{\varphi(\mathrm{i})1}}{}^{L}$ = $[a_{i\varphi(\mathrm{i})1}{}^{L} d_{i\varphi(\mathrm{i})1}{}^{U}]$

$\omega_{\widetilde{C_{ij}}_2}{}^{L}$ =0,the feasible cost/time is $\sum_{i=1}^{n} \omega_{\widetilde{C_i}_{\varphi(\mathrm{i})2}}{}^{L}$ = $[a_{i\varphi(\mathrm{i})2}{}^{L} d_{i\varphi(\mathrm{i})2}{}^{U}]$

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$\omega_{\widetilde{C_{ij}}_{m-1}}{}^{L}$ =0 ,the feasible cost/time is $\sum_{i=1}^{n} \omega_{\widetilde{C_i}_{\varphi(\mathrm{i})\mathrm{m}-1}}{}^{L}$ = $[a_{i\varphi(\mathrm{i})m-1}{}^{L} d_{i\varphi(\mathrm{i})\mathrm{m}-1}{}^{U}]$.

Step 8: Forward calculation: Select the upper feasible elements of $\omega_{\widetilde{C_{ij}}_M}{}^{*}$and determine Feasible IFLSBAP

$\omega_{\widetilde{C_{ij}}_{m+1}}{}^{U}$ =0,the feasible cost/time is $\sum_{i=1}^{n} \omega_{\widetilde{C_i}_{\varphi(\mathrm{i})\mathrm{m}+1}}{}^{U}$ = $[a_{i\varphi(\mathrm{i})m+1}{}^{L} d_{i\varphi(\mathrm{i})\mathrm{m}\mp 1}{}^{U}]$.

$\omega_{\widetilde{C_{\iota J}}_{m+2}}{}^{U}$ =0,the feasible cost/time is $\sum_{i=1}^{n} \omega_{\widetilde{C_{\iota}}_{\varphi(\mathrm{i})\mathrm{m}+2}}{}^{U}$ = $[a_{i\varphi(\mathrm{i})m+2}{}^{L} d_{i\varphi(\mathrm{i})\mathrm{m}+2}{}^{U}]$.

$\omega_{\widetilde{C_{\iota J}}_{m+3}}{}^{U}$ =0,the feasible cost/time is $\sum_{i=1}^{n} \omega_{\widetilde{C_{\iota}}_{\varphi(\mathrm{i})\mathrm{m}+3}}{}^{U}$ = $[a_{i\varphi(\mathrm{i})m+3}{}^{L} d_{i\varphi(\mathrm{i})\mathrm{m}+3}{}^{U}$

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$\omega_{\widetilde{C_{\iota J}}_{n}}{}^{U}$ =0 ,the feasible cost/time is $\sum_{i=1}^{n} \omega_{\widetilde{C_{\iota}}_{\varphi(\mathrm{i})\mathrm{n}}}{}^{U}$ = $[a_{i\varphi(\mathrm{i})n}{}^{L} d_{i\varphi(\mathrm{i})\mathrm{n}}{}^{U}]$.

Step 9: Determine and checking Feasible/Optimal and Partial/Perfect of IFLSBAP.

$\omega_{\widetilde{C_{\iota J}}_{\delta}}{}^{L}$ = $[a_{i\varphi(\mathrm{i})\delta}{}^{L} d_{i\varphi(\mathrm{i})\delta}{}^{L}]$ = min { $\omega_{\widetilde{C_{\iota J}}_{0}}{}^{L}$, $\omega_{\widetilde{C_{\iota J}}_{1}}{}^{L}$,….$\omega_{\widetilde{C_{\iota J}}_{M}}{}^{*}$ …… $\omega_{\widetilde{C_{\iota J}}_{m+2}}{}^{U}$, $\omega_{\widetilde{C_{\iota J}}_{m+3}}{}^{U}$ .. $\omega_{\widetilde{C_{\iota J}}_{n}}{}^{U}$}

If, $\omega_{\widetilde{C_{\iota J}}_{\delta}}{}^{L}$ = $[a_{i\varphi(\mathrm{i})\delta}{}^{L} d_{i\varphi(\mathrm{i})\delta}{}^{L}] \leq [a_{i\varphi(\mathrm{i})n}{}^{L} d_{i\varphi(\mathrm{i})\mathrm{n}}{}^{U}]$ is optimal and perfect matching.

If, $\omega_{\widetilde{C_{\iota J}}_{\delta}}{}^{L}$ = $[a_{i\varphi(\mathrm{i})\delta}{}^{L} d_{i\varphi(\mathrm{i})\delta}{}^{L}{}_{i\varphi(\mathrm{i})\delta}{}^{*}] < [a_{i\varphi(\mathrm{i})n}{}^{L} d_{i\varphi(\mathrm{i})\mathrm{n}}{}^{U}]$ is feasible and partial matching.

If, $\omega_{\widetilde{C_{\iota J}}_{\delta}}{}^{U}$ = $[a_{i\varphi(\mathrm{i})\delta}{}^{U} d_{i\varphi(\mathrm{i})\delta}{}^{U}] \leq [a_{i\varphi(\mathrm{i})n}{}^{L} d_{i\varphi(\mathrm{i})\mathrm{n}}{}^{U}]$ is feasible and perfect matching.

If, $\omega_{\widetilde{C_{\iota J}}_{\delta}}{}^{U}$ = $[a_{i\varphi(\mathrm{i})\delta}{}^{U} d_{i\varphi(\mathrm{i})\delta}{}^{U}] < [a_{i\varphi(\mathrm{i})n}{}^{L} d_{i\varphi(\mathrm{i})\mathrm{n}}{}^{U}]$ is feasible and partial matching.

Step 10: Stop.

5.6 NUMERICAL EXAMPLE

Consider Generalized trapezoidal fuzzy numbers ($\widetilde{C_{ij}}$)

(9,13,17,21;0.25)	(15,20,25,30;0.20)	(4,6,8,10;0.50)	(3,5,7,9;0.50)
(5,7,9,11;0.50)	(8,10,12,14;0.50)	(4,6,8,10;0.50)	(9,13,17,21;0.25)
(2,4,6,8;0.50)	(9,13,17,21;0.25)	(13,18,23,28; 0.20)	(5,7,9,11;0.50)
(3,5,7,9;0.50)	(6,8,10,12;0.50)	(9,13,17,21;0.25)	(4,6,8,10;0.50)

Generalized α-cut trapezoidal fuzzy numbers is

$$\omega_{\widetilde{C_{ij}}} = [\breve{a}^{L} + (\breve{a}^{\alpha}\text{-}\breve{a}^{L})\ \omega,\ \text{-}(\breve{a}^{U}\text{-}\breve{a}^{\beta}) + \breve{a}^{U}] = [{a_{ij}}^{L}{d_{ij}}^{U}]$$

If α = ω, compute fuzzy interval values by using generalized α-cut trapezoidal fuzzy numbers:

$\omega_{\widetilde{C_{11}}}$ =[10 20], $\omega_{\widetilde{C_{12}}}$ =[16 29] $\omega_{\widetilde{C_{13}}}$ =[5 9], $\omega_{\widetilde{C_{14}}}$ =[4 8] ,$\omega_{\widetilde{C_{21}}}$ =[6 10], $\omega_{\widetilde{C_{22}}}$ =[9 12] ,$\omega_{\widetilde{C_{23}}}$ = [5 9]$\omega_{\widetilde{C_{24}}}$ = [10 20]$\omega_{\widetilde{C_{31}}}$ = [3 1]$\omega_{\widetilde{C_{32}}}$ = [10 20] ,$\omega_{\widetilde{C_{33}}}$ =[14 27]$\omega_{\widetilde{C_{34}}}$ = [6 1]$\omega_{\widetilde{C_{41}}}$ = [4 8] ,$\omega_{\widetilde{C_{42}}}$ =[7 11] ,$\omega_{\widetilde{C_{43}}}$ = [10 20],$\omega_{\widetilde{C_{44}}}$ =[5 9]

interval-valued fuzzy linear sum bottleneck assignment problem by using Generalized ω-cut trapezoidal fuzzy numbers

[10 20]	[16 29]	[5 9]	[4 8]
[6 10]	[9 13]	[5 9]	[10 20]
[3 7]	[10 20]	[14 27]	[6 10]
[4 8]	[7 11]	[10 20]	[5 9]

Case i:$\omega_{\widetilde{C_{ij}}}{}^{*} = \omega_{\widetilde{C_{ij}}}{}^{*} = \{\omega_{\widetilde{C_{ij}}} : \omega_{\widetilde{C_{ij}}_{0}}{}^{L} < \omega_{\widetilde{C_{ij}}} < \omega_{\widetilde{C_{ij}}_{n}}{}^{U} =$ [7 11]

[10 20]	[16 29]	0	**0**
0	[9 13]	**0**	[10 20]
0	[10 20]	[14 27]	0
0	**0**	[10 20]	0

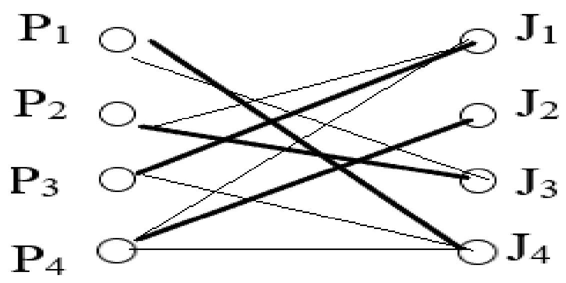

Figure: 5.1

$\sum_{i=1}^{n} \omega_{\widetilde{C_{i}}}{}^{*}_{\varphi(\mathrm{i})M} = [a_{i\varphi(\mathrm{i})M}{}^{L} d_{i\varphi(\mathrm{i})M}{}^{U}]$ = [19 35], The IFLSAP is optimal and perfect matching

Case ii:$\omega_{\widetilde{C_{ij}}}{}^{L}_{M-1} = \omega_{\widetilde{C_{ij}}}{}^{L} = \{\omega_{\widetilde{C_{ij}}} : \omega_{\widetilde{C_{ij}}_{0}}{}^{L} < \omega_{\widetilde{C_{ij}}} < \omega_{\widetilde{C_{ij}}_{n}}{}^{U} =$ [6 10]

[10 20]	[16 29]	0	**0**
0	[9 13]	**0**	[10 20]
0	[[10 20]	[14 27]	0
0	[7 11]	[10 20]	0

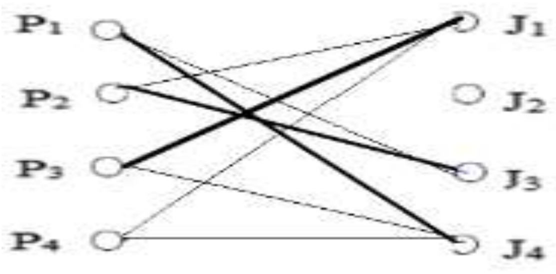

Figure: 5.2

Now $[a_{i\varphi(\text{i})M-1}{}^{L} d_{i\varphi(\text{i})M-1}{}^{U}]$ = [12 24],The IFLSAP is feasible and partial.

Case iii:$\omega_{\widetilde{C_{ij}}}{}^{L}_{M-2}$ = $\{\omega_{\widetilde{C_{ij}}} : \omega_{\widetilde{C_{ij}}}{}^{L}_{0} < \omega_{\widetilde{C_{ij}}} < \omega_{\widetilde{C_{ij}}}{}^{U}_{n}$ = [5 9]

[10 20]	[16 29]	0	**0**
[6 10]	[9 12]	**0**	[10 20]
0	[[10 20]	[14 27]	[6 10]
0	[7 11]	[10 20]	0

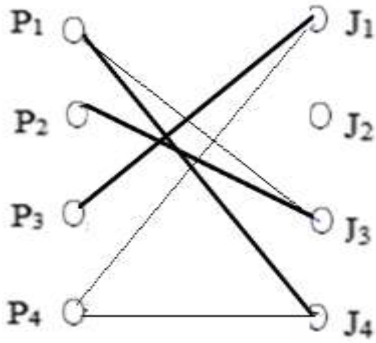

Figure: 5.3

Now $\sum_{i=1}^{n} \omega_{\widetilde{C_{i}}}{}^{*}_{\varphi(\text{i})M-2}$ = $[a_{i\varphi(\text{i})M-2}{}^{L} d_{i\varphi(\text{i})M-2}{}^{U}]$ = [12 24],The IFLSAP is feasible and

partial.

Case iv:$\omega_{\widetilde{C_{ij}}}{}^{L}_{M-3}$ = $\omega_{\widetilde{C_{ij}}}{}^{L}$ = $\{\omega_{\widetilde{C_{ij}}} : \omega_{\widetilde{C_{ij}}}{}^{L}_{0} < \omega_{\widetilde{C_{ij}}} < \omega_{\widetilde{C_{ij}}}{}^{U}_{n}$ = [4 8]

[10 20]	[16 29]	0	**0**
0	[9 12]	**0**	[10 20]
0	[10 20]	[14 27]	0
0	[7 11]	[10 20]	0

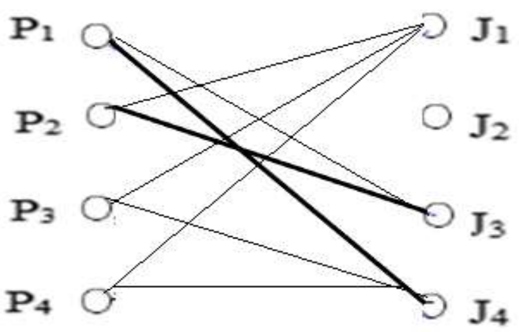

Figure: 5.4

Now $\sum_{i=1}^{n} \omega_{\tilde{C}_{i\varphi(\mathrm{i})M-3}}^{*}$ = $[a_{i\varphi(\mathrm{i})M-2}{}^{L} d_{i\varphi(\mathrm{i})M-2}{}^{U}]$ = [7 15],The IFLSAP is feasible and partial.

Case v:$\omega_{\tilde{C}_{ij}}{}^{L}_{M-4} = \omega_{\tilde{C}_{ij}}{}^{L} = \{\omega_{\tilde{C}_{ij}} : \omega_{\tilde{C}_{ij}}{}^{L}_{0} < \omega_{\tilde{C}_{ij}} < \omega_{\tilde{C}_{ij}}{}^{U}_{n}$ = [3 7]

[10 20]	[16 29]	[5 9]	[4 8]
[6 10]	[9 12]	[5 9]	[10 20]
0	[[10 20]	[14 27]	[6 10]
[4 8]	[7 11]	[10 20]	[5 9]

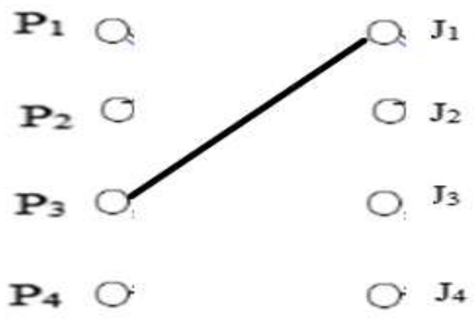

Figure: 5.5

Now $\sum_{i=1}^{n} \omega_{\breve{C}_{i}}{}^{*}_{\varphi(\mathrm{i})M-4}$ = $[a_{i\varphi(\mathrm{i})M-4}{}^{L} d_{i\varphi(\mathrm{i})M-4}{}^{U}]$ = [3 7],The IFLSAP is feasible and

partial

Case vi:$\omega_{\breve{C}_{ij}}{}^{L}_{M+1}$ = $\omega_{\breve{C}_{ij}}{}^{L}$ = $\{\omega_{\breve{C}_{ij}} : \omega_{\breve{C}_{ij}}{}^{L}_{0} < \omega_{\breve{C}_{ij}} < \omega_{\breve{C}_{ij}}{}^{U}_{n}$ = [9 13]

[10 20]	[16 29]	0	**0**
0	0	**0**	[10 20]
0	[[10 20]	[14 27]	0
0	0	[10 20]	0

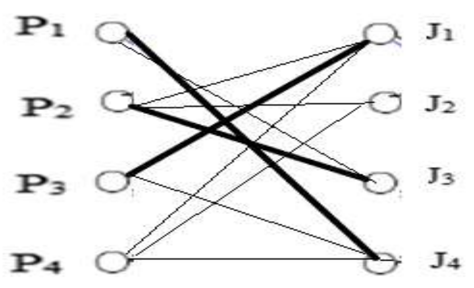

Figure: 5.6

Now $\sum_{i=1}^{n} \omega_{\breve{C}_{i}}{}^{*}_{\varphi(\mathrm{i})M+1}$ = $[a_{i\varphi(\mathrm{i})M+1}{}^{L} d_{i\varphi(\mathrm{i})M+1}{}^{U}]$ = [12 24], The IFLSAP is feasible and perfect.

Case vii: $\omega_{\tilde{C_{ij}}_{M+2}}{}^{L} = \omega_{\tilde{C_{ij}}}{}^{L} = \{\omega_{\tilde{C_{ij}}} : \omega_{\tilde{C_{ij}}_{0}}{}^{L} < \omega_{\tilde{C_{ij}}} < \omega_{\tilde{C_{ij}}_{n}}{}^{U} = [10\ 20]$

0	[16 29]	0	**0**
0	0	**0**	0
0	0	[14 27]	0
0	**0**	0	0

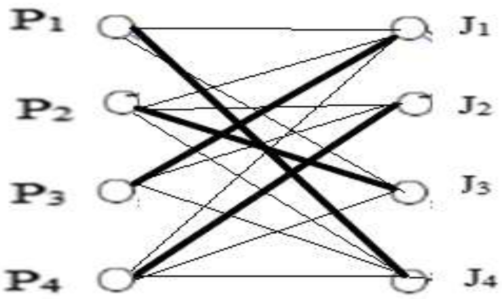

Figure: 5.7

$\sum_{i=1}^{n} \omega_{\tilde{C_i}_{\varphi(\mathrm{i})M+2}}^{*} = [a_{i\varphi(\mathrm{i})M+2}{}^{L} d_{i\varphi(\mathrm{i})M+2}{}^{U}] = [19\ 35]$,

The IFLSAP is optimal and perfect.

Case viii: $\omega_{\tilde{C_{ij}}_{M+3}}{}^{L} = \omega_{\tilde{C_{ij}}}{}^{L} = \{\omega_{\tilde{C_{ij}}} : \omega_{\tilde{C_{ij}}_{0}}{}^{L} < \omega_{\tilde{C_{ij}}} < \omega_{\tilde{C_{ij}}_{n}}{}^{U} = [14\ 27]$

0	[16 29]	0	**0**
0	0	**0**	0
0	0	0	0
0	**0**	0	0

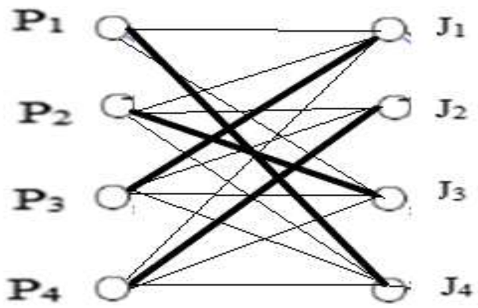

Figure: 5.8

Now $\sum_{i=1}^{n} \omega_{\breve{C}_{i\,\varphi(\mathrm{i})M+3}}^{\;*}$ = $[a_{i\varphi(\mathrm{i})M+3}{}^{L} d_{i\varphi(\mathrm{i})M+3}{}^{U}]$ = [19 35],

The IFLSAP is optimal and perfect.

Case ix:$\omega_{\widetilde{C_{ij}}_{M+4}}^{\;L}$ = $\omega_{\widetilde{C_{ij}}}^{\;L}$ = $\{\omega_{\widetilde{C_{ij}}} : \omega_{\widetilde{C_{ij}}_0}^{\;L} < \omega_{\widetilde{C_{ij}}} < \omega_{\widetilde{C_{ij}}_n}^{\;U}$ = [16 29]

0	0	0	0
0	**0**	0	0
0	0	**0**	0
0	0	0	**0**

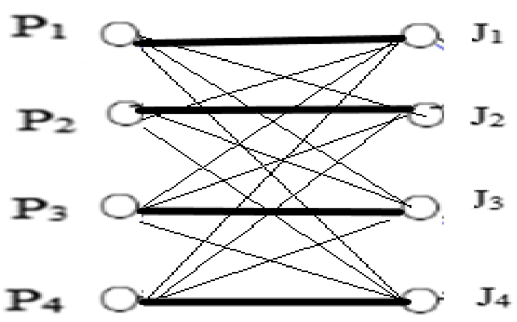

Figure: 5.9

Now $\sum_{i=1}^{n} \omega_{\breve{C}_{i\,\varphi(\mathrm{i})M+4}}^{\;*}$ = $[a_{i\varphi(\mathrm{i})M+4}{}^{L} d_{i\varphi(\mathrm{i})M+4}{}^{U}]$ = [38 69], the IFLSAP is feasible and perfect.

The optimal perfect schedule is $P_1 \rightarrow J_4, P_2 \rightarrow J_3, P_3 \rightarrow J_1, P_4 \rightarrow J_2$.

The spread of new generalized trapezoidal fuzzy optimal perfect assignment cost

$\sum \widetilde{C_{\iota\varphi(\iota)}} =$

(3,5,7,9;0.50)+ (4,6,8,10;0.50)+ (2,4,6,8;0.50)+ (6,8,10,12;0.50) = (15,23,33,39;0.5).

CHAPTER VI

A New Optimal Complete Matching of Edges with Minimum Cost by Ranking Method for Solving ω-Type -2 Fuzzy Linear Sum Assignment Problem

In this chapter, proposed a new optimal solution and complete matching edges of bipartite graph. ω-type -2 Fuzzy Linear Sum Assignment Problem[FLSAP] is converted to crisp one by using new ranking method for solving ω-type -2 [FLSAP]. This chapter discussed ω-trapezoidal fuzzy number, we introduced ω-type 1-trapezoidal fuzzy numbers and ω-type 2-trapezoidal fuzzy numbers. Create ω-type -2 Fuzzy Linear Sum Assignment Table[FLST].The rank of ω -type 2 trapezoidal fuzzy number to assign each machine to a job with the lowest cost in that job for solving -type 2 [FLSAP]. Furthermore, each iteration updates a non-matched edge to a matched edge and update the corresponding dual variables. By using alternating path method to obtain a new optimal complete matching solution. This method is illustrated by a numerical example.

6.1 INTRODUCTION:

Ranking fuzzy numbers is a vital step in the decision-making process in many applications. Linguistic decision-making and various other fuzzy application systems both depend heavily on ranking fuzzy numbers. For ranking fuzzy numbers, a variety of methods have been suggested. It has been demonstrated that in some situations, each of these strategies can lead to unexpected outcomes. Due to its simplicity and popularity, the fuzzy technique is frequently utilized to deal with decision-making

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problems. Fuzzy set theory can be used to solve difficulties in the real world. While fuzzy numbers lack this kind of inequality, real numbers can be sorted by or. Since fuzzy numbers can have a variety of outcomes, it is challenging to determine whether one is more or smaller than another. Using a ranking function to sort the fuzzy numbers is an effective method. The set of fuzzy numbers, which maps each fuzzy number to the real line in a natural order, is defined by real numbers. Fuzzy set theory has become more problematic utilizing the precise ranking of fuzzy numbers, a key step in making judgments in a fuzzy environment.

In this chapter we introduced ω-Type-1 trapezoidal fuzzy numbers and ω-Type-2 trapezoidal fuzzy numbers and we discussed ranking methods,and assign each machine to a job with minimum reduced cost in that job by using ranking method of lower and upper membership function of the ω-Type-2 trapezoidal fuzzy linear sum assignment problem ($\omega - T2TrFLSAP$) and obtain new optimal complete matching edges of G.

6.2 PRELIMINARIES OF $\boldsymbol{\omega}$-TYPE 1 AND $\boldsymbol{\omega}$-Type-2 TRAPEZOIDAL FUZZY NUMBER

We present here membership function of ω- trapezoidal fuzzy number (ω-TrFN) and minimum and maximum membership value of ω- type-1 trapezoidal fuzzy number ($\omega - T1TrFN$),lower and upper membership function of ω- type-2 trapezoidal fuzzy number ($\omega - T2TrFN$).

6.2.1 Definition:

A ω-type 1 trapezoidal fuzzy number($\omega - T1TrFN$) is denoted as $\omega_{\tilde{A}_1}$= $(a_\alpha, a_k, a_l, a_\beta, \omega_1, \omega_2)$ and ω_1 is minimum membership value of type1 trapezoidal fuzzy number and ω_2 is maximum membership value of type1 trapezoidal fuzzy number.

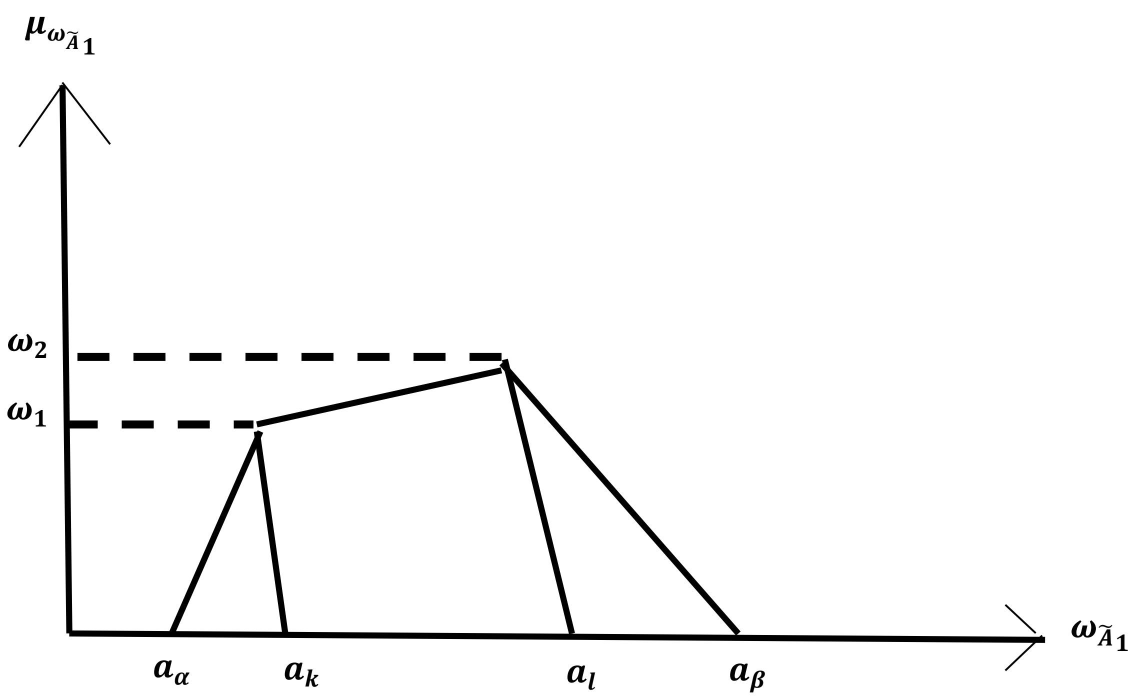
membership function of ω-type 1 trapezoidal fuzzy number :

$$\mu_{\omega_{\tilde{A}_1}} = \omega_1 \left(\frac{x-a_\alpha}{a_k-a_\alpha}\right) \qquad \text{if} \quad a_\alpha \le x \le a_k$$

$$= \omega_1 + \left(\frac{x-a_k}{a_l-a_k}\right)(\omega_2-\omega_1) \qquad \text{if} \quad a_k \le x \le a_l$$

$$= \omega_2 \left(\frac{a_\beta - x}{a_\beta - a_l}\right) \qquad \text{if} \quad a_l \le x \le a_\beta$$

Where $\omega_1 < \omega_2$



6.2.2 Definition:

A fuzzy number $\omega_{\tilde{A}_2}$ is said to be ω-type 2 trapezoidal fuzzy number ($\omega - T2TrFN$) and is defined as, ${\omega_{\tilde{A}_2}}^{LM}$ = (a_α^{LM}, a_k^{LM} , a_l^{LM}, a_β^{LM},${\omega_1}^{LM}$, ${\omega_2}^{LM}$) and , ${\omega_{\tilde{A}_2}}^{UM}$ = (a_α^{UM}, a_k^{UM} , a_l^{UM}, a_β^{UM}, ${\omega_3}^{UM}$, ${\omega_4}^{UM}$) are lower and upper membership function of ω-type 2 trapezoidal fuzzy number,(${\omega_1}^{LM}$, ${\omega_2}^{LM}$) and (${\omega_3}^{UM}$, ${\omega_4}^{UM}$) are lower and upper membership value of ω-type 2 trapezoidal fuzzy number ($\omega - T2TrFN$), if the following membership function

Membership function of upper and lower ω-type 2 trapezoidal fuzzy number :

$$\mu_{{\omega_{\tilde{A}_2}}^{LM}} = {\omega_1}^{LM}\left(\frac{x-a_\alpha^{LM}}{a_k^{LM}-a_\alpha^{LM}}\right) \qquad \text{if } a_\alpha^{LM} \le x \le a_k^{LM}$$

$$= {\omega_1}^{LM} + \left(\frac{x-a_k^{LM}}{a_l^{LM}-a_k^{LM}}\right)({\omega_2}^{LM} - {\omega_1}^{LM}) \quad \text{if } a_k^{LM} \le x \le a_l^{LM}$$

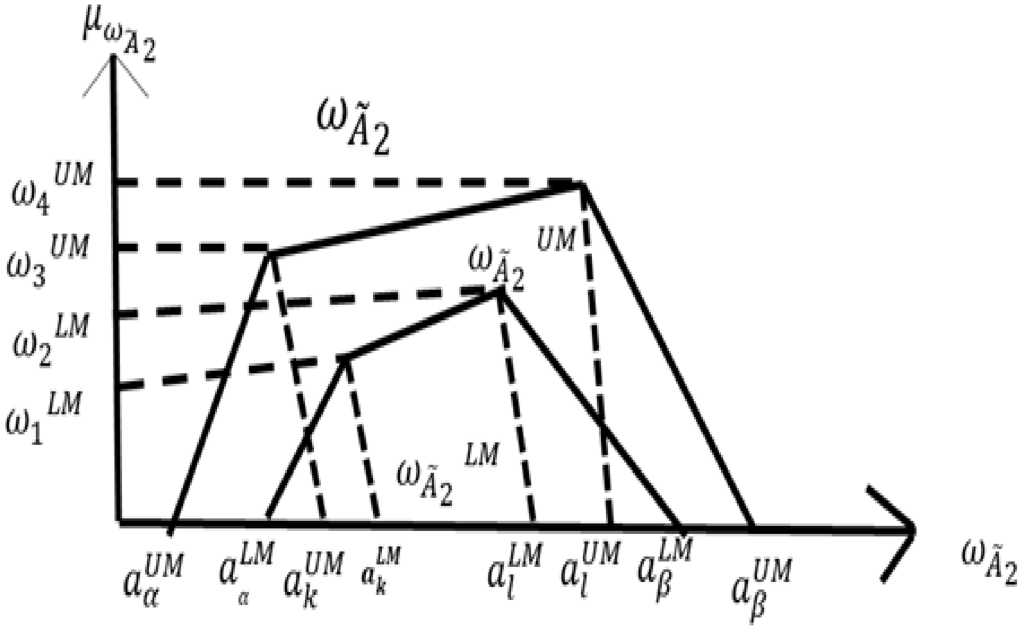
$$= {\omega_2}^{LM}\left(\frac{a_\beta^{LM}-x}{a_\beta^{LM}-a_l^{LM}}\right) \qquad \text{if } a_l^{LM} \le x \le a_\beta^{LM}$$

and

$$\mu_{{\omega_{\tilde{A}_2}}^{UM}} = {\omega_3}^{UM}\left(\frac{x-a_\alpha^{UM}}{a_k^{UM}-a_\alpha^{UM}}\right) \qquad \text{if } a_\alpha^{UM} \le x \le a_k^{UM}$$

$$= {\omega_3}^{UM} + \left(\frac{x-a_k^{UM}}{a_l^{UM}-a_k^{UM}}\right)({\omega_4}^{UM} - {\omega_3}^{UM}) \quad \text{if } a_k^{UM} \le x \le a_l^{UM}$$

$$= {\omega_4}^{UM}\left(\frac{a_\beta^{UM}-x}{a_\beta^{UM}-a_l^{UM}}\right) \qquad \text{if } a_l^{UM} \le x \le a_\beta^{UM}$$



6.3 A NEW RANKING METHOD OF ω-TYPE-2 TRAPEZOIDAL FUZZY NUMBER ($\omega - T2TrFN$)

The lower membership function of ω-type-2 trapezoidal fuzzy number is denoted as $\omega_{\tilde{A}_2}{}^{LM}$ and upper membership function of ω-type-2 trapezoidal fuzzy number is denoted as $\omega_{\tilde{A}_2}{}^{UM}$; where $\omega_{\tilde{A}_2}{}^{LM} = (a_\alpha^{LM},\ a_k^{LM},\ a_l^{LM}, a_\beta^{LM}, \omega_1{}^{LM}, \omega_2{}^{LM})$ and $\omega_{\tilde{A}_2}{}^{UM} = (a_\alpha^{UM},\ a_k^{UM},\ a_l^{UM}, a_\beta^{UM}, \omega_3{}^{UM}, \omega_4{}^{UM})$. The ranking method of lower membership function of the ω-type-2 trapezoidal fuzzy number($\omega_{\tilde{A}_2}{}^{LM}$) is defined as $R_{\omega_{\tilde{A}_2}{}^{LM}}$ and The ranking method of upper membership function of the ω-type-2 trapezoidal fuzzy number is defined as $R_{\omega_{\tilde{A}_2}{}^{UM}}$.

$$R_{\omega_{\tilde{A}_2}} = \frac{1}{2}\left[R_{\omega_{\tilde{A}_2}{}^{LM}} \min(\omega_1{}^{LM},\ \omega_2{}^{LM}) + R_{\omega_{\tilde{A}_2}{}^{UM}} \max(\omega_3{}^{UM},\ \omega_4{}^{UM})\right]$$

Where $R_{\omega_{\tilde{A}_2}{}^{LM}} \min(\omega_1{}^{LM},\ \omega_2{}^{LM}) = \left(\frac{\left(a_\alpha^{LM} + a_k^{LM} + a_l^{LM} + a_\beta^{LM}\right)\min(\omega_1{}^{LM}, \omega_2{}^{LM})}{4}\right)$;

$$R_{\omega_{\tilde{A}_2}UM} \max(\omega_3{}^{UM},\ \omega_4{}^{UM}) = \left(\frac{\left(a_\alpha^{UM} + a_k^{UM} + a_l^{UM} + a_\beta^{UM}\right) \max(\omega_3{}^{UM}, \omega_4{}^{UM})}{4}\right)$$

$R_{\omega_{\tilde{A}_2}} =$

$$\frac{1}{2}\left[\left(\frac{\left(a_\alpha^{LM} + a_k^{LM} + a_l^{LM} + a_\beta^{LM}\right) min\,(\omega_1{}^{LM},\ \omega_2{}^{LM})}{4}\right) + \left(\frac{\left(a_\alpha^{UM} + a_k^{UM} + a_l^{UM} + a_\beta^{UM}\right) max\,(\omega_3{}^{UM},\ \omega_4{}^{UM})}{4}\right)\right]$$

$R_{\omega_{\tilde{A}_2}} =$

$$\left(\frac{\left(a_\alpha^{LM} + a_k^{LM} + a_l^{LM} + a_\beta^{LM}\right) \min(\omega_1{}^{LM},\ \omega_2{}^{LM}) + \left(a_\alpha^{UM} + a_k^{UM} + a_l^{UM} + a_\beta^{UM}\right) \max(\omega_3{}^{UM},\ \omega_4{}^{UM})}{\mathbf{8}}\right)$$

6.4 A NEW OPTIMAL COMPLETE MATCHING SOLUTION WITH MINIMUM REDUCED COST BY USING RANKING METHOD

We discussed a new optimal complete matching solution for solving ω-type-2 fuzzy linear sum assignment problem by using ranking method and compute each machine to a job with minimum reduced cost in that job. We introduce the following new procedure.

6.4.1 Theorem : If $R_{\omega_{v_0}} = \emptyset$,then, Xij =1 provides an optimal complete matching of edges.

proof

First choose , $R_{\omega_{v_{12}}} \neq \emptyset$, then we proceed with the lowest possible expense for each job j ∈ $R_{\omega_{v_{12}}}$ with $R_{\omega_{\pi_j}}$= min {$R_{\omega_{c_{ij}}}$- $R_{\omega_{u_i}}$: row i is present in the tree}. pick j ∈ $R_{\omega_{v_{12}}}$ with minimum $R_{\omega_{\pi_j}}$ - $R_{\omega_{v_j}}$ and set $R_{\omega_{\pi_j}}$ = $R_{\omega_{v_j}}$ then add new edge [i',j'] to the tree, where i' is the row vertex compute $R_{\omega_{\pi_j}}$. For each i∈U ; Xi j' =1 then we proceed to $R_{\omega_{u_i}}$= $R_{\omega_{c_{ij'}}}$- $R_{\omega_{v_{j'}}}$and [i,j'] add to the tree. $R_{\omega_{v_0}}$, $R_{\omega_{v_1}}$and $R_{\omega_{v_2}}$ should all be modified and hence Continue the loop until $R_{\omega_{v_0}}$ = ∅ and then the turn paths to obtain a new optimal complete matching of edges.

6.5 THE PROPOSED METHOD

Step 1: If the total number of machines (M) is equal to the total number of jobs (J), then ω-type-2 fuzzy linear sum assignment problem is balanced. otherwise ω-type-2 fuzzy linear sum assignment problem is unbalanced. if ω-type-2 fuzzy linear sum assignment problem is unbalanced, then we introduce dummy row or dummy column.

Step 2: Balanced ω-type- 2 fuzzy linear sum assignment problem is converted to ranking of balanced ω-type-2 fuzzy linear sum assignment problem.

Step 3: Let us assume $R_{\omega_{v_0}}$, $R_{\omega_{v_1}}$and $R_{\omega_{v_2}}$ are column vertices with no machine Matched, one machine matched and more than one or two machine matched. for job j =1,2,3… n do $R_{\omega_{v_j}}$ = 0.

Step 4: construction of the tree: Choose a job r ∈ $R_{\omega_{v_2}}$as the root, and set v_r =0; for each i∈U, Xir =1 then we proceed $R_{\omega_{u_i}}$= $R_{\omega_{c_{ir}}}$ the tree with the edge [i,r] such that Xir =1.

Step 5 : Select one row assigned column $R_{\omega_{v_{12}}} \neq \emptyset$ then we proceed for each job j ∈ the tree}.

Step 6: Select j ∈ $R_{\omega_{v_{12}}}$ with minimum $R_{\omega_{\pi_j}}$ - $R_{\omega_{v_j}}$ and set $R_{\omega_{\pi_j}}$ = $R_{\omega_{v_j}}$ then add new edge [i',j'] to the tree, where i' is the row vertex obtaining $R_{\omega_{\pi_j}}$. For each i∈U ; Xi j' =1 then we proceed to $R_{\omega_{u_i}}$= $R_{\omega_{c_{ij'}}}$- $R_{\omega_{v_{j'}}}$and [i,j'] add to the tree.

Step 7: Select the unassigned job $R_{\omega_{j^*}} \in \overline{R_{\omega_{v_0}}}$ and set $R_{\omega_{u_{i^*}}} = arg\ min_{i\in U}$ {$R_{\omega_{c_{ij^*}}}$- $R_{\omega_{u_i}}$},$R_{\omega_{v_{j^*}}}$= $R_{\omega_{c_{i^*j^*}}}$-$R_{\omega_{u_{i^*}}}$. Let P be the bipartite path $R_{\omega_{u_{i^*}}}$ connecting to a column of $R_{\omega_{v_2}}$.Interchange unmatched edges to matched edges along path, $X_{i^* j^*}$=1.

Step 8: Update $R_{\omega_{v_0}}$, $R_{\omega_{v_1}}$and $R_{\omega_{v_2}}$. Continue the process until $R_{\omega_{v_0}}$ = ∅ and then alternate paths to obtain a new optimal complete matching of edges.

Step 9: Stop.

6.6 NUMERICAL EXAMPLE:

The number of machines equal to the number of jobs, so therefore the given fuzzy linear sum assignment problem is balanced otherwise unbalanced fuzzy linear sum assignment problem The balanced ω-type-2 trapezoidal fuzzy number of fuzzy linear sum assignment problem as follows. The new ranking method of ω-type-2 trapezoidal fuzzy number($\omega - T2TrFN$) is as follows:

$\omega_{\tilde{A}_2}$ = (5,11,14,20,0.2,0.4);(2,8,17,23,0.4,0.8) → $R_{\omega_{\tilde{A}_2}}$ = 6.25 ,

$\omega_{\tilde{A}_2}$ =(8,16,20,28,0.1,0.3); (4,12,24,32,0.45,0.6) → $R_{\omega_{\tilde{A}_2}}$ = 6.30.

$\omega_{\tilde{A}_2}$ = (14,26,32,44,0.25,0.3); (8,20,38,50,0.4,0.5) → $R_{\omega_{\tilde{A}_2}}$= 10.88

$\omega_{\tilde{A}_2}$ = (20,30,35,45,0.25,0.4); (15,25,40,50,0.6,0.8) → $R_{\omega_{\tilde{A}_2}}$= 17.06

$\omega_{\tilde{A}_2}$ = (20,40,50,70,0.2,0.35); (10,30,60,80,0.5,0.8) → $R_{\omega_{\tilde{A}_2}}$ = 22.50

$\omega_{\tilde{A}_2}$ = (40,60,70,90,0.3,0.5); (30,50,80,100, .65, .95) → $R_{\omega_{\tilde{A}_2}}$= 40.63

$\omega_{\tilde{A}_2}$ =(35,45,50,60,0.65,0.75) ; (30,40,55,65,0.8,0.9) → $R_{\omega_{\tilde{A}_2}}$= 36.81

$\omega_{\tilde{A}_2}$ = (60,80,90,110,0.25,0.45) ; (50,70,100,120,0.7,0.95) → $R_{\omega_{\tilde{A}_2}}$ = 51

Machine(M)/ JOB(J)	J_1	J_2	J_3	J_4
M_1	**36.81**	51	40.63	51
M_2	**6.30**	40.63	10.88	36.81
M_3	**6.25**	22.50	22.50	51
M_4	17.06	22.50	**6.30**	6.30

Figure: 6.1

Machine(M)/ JOB(J)	J_1	J_2	J_3	J_4
M_1	36.81	**51**	40.63	51
M_2	**6.30**	40.63	10.88	36.81
M_3	**6.25**	22.50	22.50	51
M_4	17.06	22.50	**6.30**	6.30

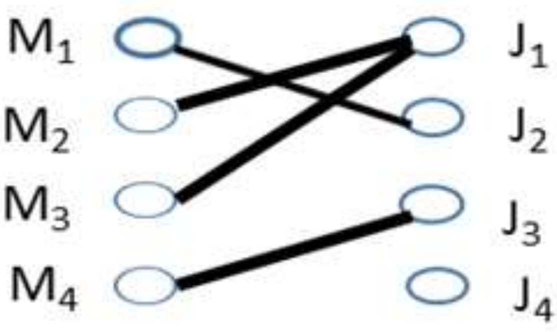


Figure:6.2

Updated dual solution and $R_{\omega_{v_{12}}}$ ={2,3} and obtain $R_{\omega_{\pi_2}}$= min {$R_{\omega_{c_{ij}}}$- $R_{\omega_{u_i}}$= 16.25 (row 3) add new edge[3,2], update $R_{\omega_{u_1}}$ = 34.75 and $R_{\omega_{\pi_3}}$= min {$R_{\omega_{c_{ij}}}$- $R_{\omega_{u_i}}$= = 4.58 (row 2), add new matching edge [2,3]; update $R_{\omega_{u_4}}$= 1.72., add new matching edge [1,2]; Update the dual solutions is $R_{\omega_{u_i}}$ = (34.75, 6.30, 6.25,1.72); $R_{\omega_{v_j}}$ = (0, 16.25, 4.58, 4.58).select unassigned column $R_{\omega_{j^*}}$= {4} $R_{\omega_{v_{j^*}}}$=($R_{\omega_{c_{i^*j^*}}}$-$R_{\omega_{u_{i^*}}}$) = 4.58 (row 4).hence, path P= {[4,3],[2,3],[2,1]}, unassigned to assigned edges are X43=0, X23 = 1, X21= 0 , X44= 1.

Figure: 6.3

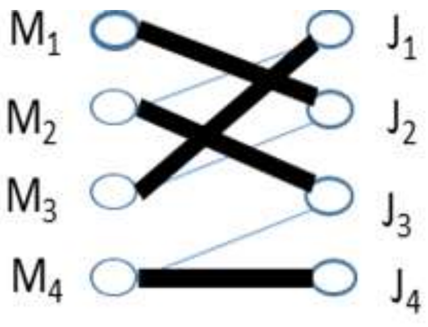


Figure: 6.4

$M_1 \rightarrow J_2$; $M_2 \rightarrow J_3$; $M_3 \rightarrow J_1$; $M_4 \rightarrow J_4$

A new Optimal complete matching solution of ranking $\omega - T2FLSAP$ is

51+10.88+6.25 + 6.30 = 74.43

A new Optimal complete solution of $\omega - T2FLSAP$ is

(87,133,156,202,0.1,0.3);(64,110,179,225,0.7,0.95)

CHAPTER – VII

SOLVING FUZZY MULTI-OBJECTIVE LINEAR SUM ASSIGNMENT PROBLEM WITH MODIFIED PARTIAL PRIMAL SOLUTION OF ω - TYPE 2 - DIAMOND FUZZY NUMBERS BY LINGUISTIC VARIABLES

In this chapter, presented fuzzy multi-objective linear sum assignment problem with modified partial assignment of ω - type 2 - Diamond Fuzzy Numbers[DFN] using linguistic variables. In this chapter introduced ω - type 1 and ω - type 2 diamond fuzzy numbers. Let us consider four jobs and four machine problem and to optimize fuzzy cost , fuzzy time, fuzzy quality are each considered as a ω - type 2 - DFN. ω-type 2 DFN are converted into λ_d--cut of DFN and upper and lower ω-type 2 diamond multi-objective fuzzy numbers are converted into single objective λ_d-cut fuzzy number by using ranking method. obtain dual variables and calculate $[\, \widetilde{c_{ij}} - \widetilde{u_i} - \widetilde{v_j}\,]$; by using alternate path method increase the partial assignment. This method is discussed by a numerical example, proving its efficiency.

7.1 INTRODUCTION:

In this chapter, we introduced ω-type 1-diamond fuzzy numbers and ω-type2-diamond fuzzy numbers are discussed . The upper and lower membership functions of diamond fuzzy numbers are described as ω-type 1 and ω-type 2-diamond fuzzy numbers. In λ_d- cut form, express the ω-type2 -diamond fuzzy numbers. Single fuzzy linear sum assignment problems are converted from fuzzy multi-objective linear sum assignment problems by using ranking method. Obtain partial feasible solution and

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complete optimal solution by using λ_d −cut of ω-type 2-diamond fuzzy numbers (ω_{t2}-DFN) . Arithmetic operations of λ_d −cut of ω-type 2-diamond fuzzy numbers (ω_{t2}-DFN) to obtain complete optimal matching.

We discussed ω-type 1 and ω-type 2-diamond fuzzy numbers. We proposed a new method for solving λ_d- cut of ω-type 2-diamond fuzzy multi-objective linear sum assignment problem and involving linguistic variables and by using alternate method and augmented method of bipartite graph to compute partial feasible solution and complete optimal solution. To modified partial primal solution and obtain complete optimal solution using the alternate path method producing augment path method of the bipartite graph.

7.2 Membership functions of ω-type1- and A ω-type2- diamond fuzzy number

In this chapter ,we introduced ω-type1- and A ω-type2- diamond fuzzy numbers and numerical examples are also discussed.

7.2.1 Definition:

A ω-type1- diamond fuzzy number is upper and lower membership function of the diamond fuzzy number is defined as $[\underline{\omega_{t1}\widetilde{F_d}}, \overline{\omega_{t1}\widetilde{F_d}}]$ where $\underline{\omega_{t1}\widetilde{F_d}} = \{\underline{d'}, d^*, \underline{d''}, (\underline{\alpha_d}, \underline{\beta_d}),$

$\overline{\omega_{t1}\widetilde{F_d}} = \{\overline{d'}, d^*, \overline{d''}, (\overline{\alpha_d}, \overline{\beta_d})\}$ and it's the following membership function is given by

<u>Membership function of A ω-type1- diamond fuzzy number</u>

$$\mu_{\omega_{t1}\widetilde{F_d}} = 0 \qquad otherwise$$

$$= \underline{\omega}\left(\frac{(x-\underline{d'})}{(d^*-\underline{d'})}\right) \qquad for\ \underline{d'} \le x \le d^*$$

$$= \underline{\omega}\left(\frac{(\underline{d''}-x)}{(\underline{d''}-d^*)}\right) \qquad for\ d^* \le x \le \underline{d''}$$

$$= \overline{\omega}\left(\frac{(x-\overline{\mathrm{d}'})}{(d^{*}-\overline{\mathrm{d}'})}\right) \qquad for\ \overline{\mathrm{d}'} \le x \le d^{*}$$

$$= \overline{\omega}\left(\frac{(\overline{d''}-x)}{(\overline{d''}-d^{*})}\right) \qquad for\ d^{*} \le x \le \overline{d''}$$

$$\alpha_d \quad - \quad base$$

$$= \underline{\omega}\left(\frac{(\underline{d'}-\mathrm{x})}{(\underline{d'}-d^{*})}\right) \qquad for\ \underline{d'} \le x \le d^{*}$$

$$= \underline{\omega}\left(\frac{(x-\underline{d''})}{(d^{*}-\underline{d''})}\right) \qquad for\ d^{*} \le x \le \underline{d''}$$

$$= \overline{\omega}\left(\frac{(\overline{\mathrm{d}'}-\mathrm{x})}{(\overline{\mathrm{d}'}-d^{*})}\right) \qquad for\ \overline{\mathrm{d}'} \le x \le d^{*}$$

$$= \overline{\omega}\left(\frac{\left(x-\overline{d''}\right)}{\left(d^{*}-\overline{d''}\right)}\right) \qquad for\ d^{*} \le x \le \overline{d''}$$

$$= \quad \omega = 1 \qquad x = \beta_d$$

$$0 \qquad otherwise$$

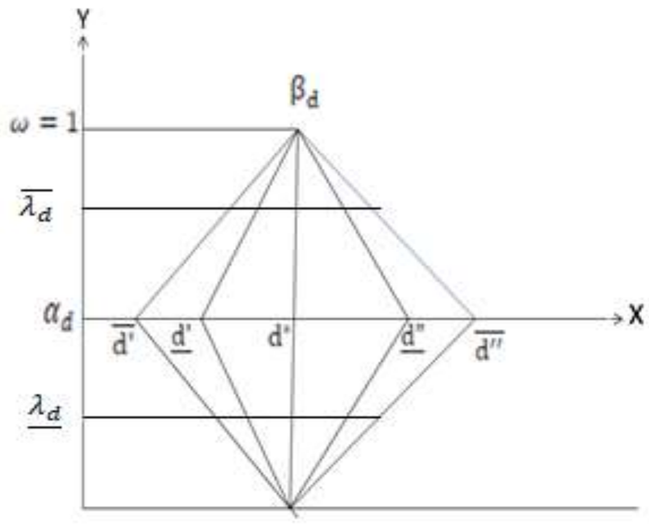

Figure: 7.1 λ_d- cut of ω-type type1- diamond fuzzy number

7.2.2 Definition:

A ω-type2 diamond fuzzy number is upper and lower membership function of the diamond fuzzy number is defined as $[\underline{\omega_{t2}\widetilde{F_d}}, \overline{\omega_{t2}\widetilde{F_d}}]$ where $\underline{\omega_{t2}\widetilde{F_d}} = \{\underline{d'}, \underline{d^*}, \underline{d''}\ (\underline{\alpha_d}, \underline{\beta_d}),$

$\overline{\omega_{t2}\widetilde{F_d}} = (\overline{d'}, \overline{d^*}, \overline{d''}\ (\overline{\alpha_d}, \overline{\beta_d}))$ and it's the following membership function.

<u>Membership function of A ω-type1- diamond fuzzy number</u>

$$\mu_{\omega_{t2}\widetilde{F_d}} = 0 \qquad otherwise$$

$$= \underline{\omega}\left(\frac{(x-\underline{d'})}{(\underline{d^*}-\underline{d'})}\right) \qquad for\ \underline{d'} \le x \le \underline{d^*}$$

$$= \underline{\omega}\left(\frac{(\underline{d''}-x)}{(\underline{d''}-\underline{d^*})}\right) \qquad for\ \underline{d^*} \le x \le \underline{d''}$$

$$= \overline{\omega}\left(\frac{(x-\overline{d'})}{(\overline{d^*}-\overline{d'})}\right) \qquad for\ \overline{d'} \le x \le \overline{d^*}$$

$$= \overline{\omega}\left(\frac{(\overline{d''}-x)}{(\overline{d''}-\overline{d^*})}\right) \qquad for\ \overline{d^*} \le x \le \overline{d''}$$

$$\alpha_d \quad - \quad base$$

$$= \underline{\omega}\left(\frac{(\underline{d'}-x)}{(\underline{d'}-\underline{d^*})}\right) \qquad for\ \underline{d'} \le x \le \underline{d^*}$$

$$= \underline{\omega}\left(\frac{(x-\underline{d''})}{(\underline{d^*}-\underline{d''})}\right) \qquad for\ \underline{d^*} \le x \le \underline{d''}$$

$$= \overline{\omega}\left(\frac{(\overline{d'}-x)}{(\overline{d'}-\overline{d^*})}\right) \qquad for\ \overline{d'} \le x \le \overline{d^*}$$

$$= \overline{\omega}\left(\frac{(x-\overline{d''})}{(\overline{d^*}-\overline{d''})}\right) \qquad for\ \overline{d^*} \le x \le \overline{d''}$$

$$0 \qquad otherwise$$

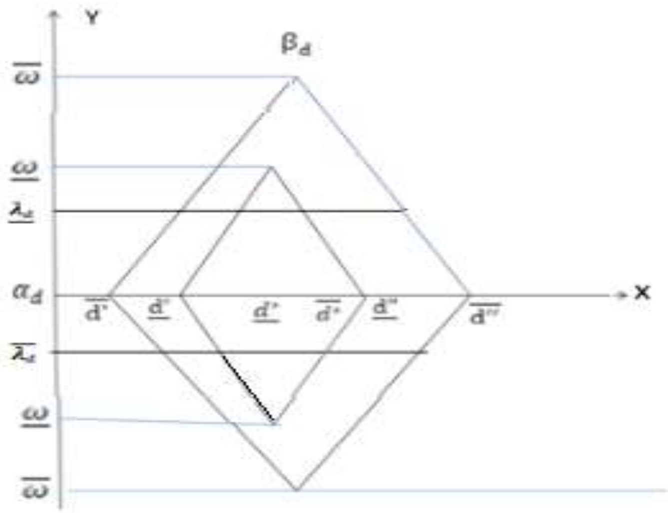

Figure: 7.2 λ_d- cut of ω-type 2- diamond fuzzy number

7.2.3 Example:

Consider two ω-type 2- diamond fuzzy number $\underline{\omega_{t2}\widetilde{F_d}}$ = {(23,30,37;0.3) , $\overline{\omega_{t2}\widetilde{F_d}}$ = (22,30,38;0.6)}then the following membership function of ω-type 2- diamond fuzzy number.

$$\mu_{\omega_{t2}\widetilde{F_d}} = 0 \qquad otherwise$$

$$= 0.3\left(\frac{(x-23)}{(30-23)}\right) \qquad for\ 23 \le x \le 30$$

$$= 0.3\left(\frac{(37-x)}{(37-30)}\right) \qquad for\ 30 \le x \le 37$$

$$= 0.6\left(\frac{(x-22)}{(30-22)}\right) \qquad for\ 22 \le x \le 30$$

$$= 0.6\left(\frac{(38-x)}{(38-30)}\right) \qquad for\ 30 \le x \le 38$$

$$\alpha_d \quad - \quad base$$

$$= 0.3\left(\frac{(23-\mathrm{x})}{(23-30)}\right) \qquad for\ 23 \le x \le 30$$

$$= 0.3\left(\frac{(x-37)}{(30-37)}\right) \qquad for\ 30 \le x \le 37$$

$= 0.6\left(\frac{(22-x)}{(22-30)}\right) \qquad for\ 22 \le x \le 30$

$= 0.6\left(\frac{(x-38)}{(30-38)}\right) \qquad for\ 30 \le x \le 38$

$0 \qquad otherwise$

7.2.4 Example:

Consider two ω-type 2- diamond fuzzy number $\underline{\omega_{t2}\widetilde{F_d}}$ = {(23,30,37;0.3) , $\overline{\omega_{t2}\widetilde{F_d}}$ = (22,31,38;0.6)}then the following membership function of ω-type 2- diamond fuzzy number.

$$\mu_{\omega_{t2}\widetilde{F_d}} = \begin{cases} 0 & otherwise \\ 0.3\left(\frac{(x-23)}{(30-23)}\right) & for\ 23 \le x \le 30 \\ 0.3\left(\frac{(37-x)}{(37-30)}\right) & for\ 30 \le x \le 37 \\ 0.6\left(\frac{(x-22)}{(31-22)}\right) & for\ 22 \le x \le 31 \\ 0.6\left(\frac{(38-x)}{(38-31)}\right) & for\ 31 \le x \le 38 \end{cases}$$

$\alpha_d - base$

$$\begin{cases} = 0.3\left(\frac{(23-x)}{(23-30)}\right) & for\ 23 \le x \le 30 \\ = 0.3\left(\frac{(x-37)}{(30-37)}\right) & for\ 30 \le x \le 37 \\ = 0.6\left(\frac{(22-x)}{(22-31)}\right) & for\ 22 \le x \le 31 \\ = 0.6\left(\frac{(x-38)}{(31-38)}\right) & for\ 31 \le x \le 38 \\ 0 & otherwise \end{cases}$$

λ_d- cut of ω-type 2- diamond fuzzy number assignment table:

Job/ Machine	J_1	J_2	J_3	J_4
M_1	$[\underline{\tilde{c}_{11}},\overline{\tilde{c}_{11}}]$ $[\underline{\tilde{t}_{11}},\overline{\tilde{t}_{11}}]$ $[\underline{\tilde{q}_{11}},\overline{\tilde{q}_{11}}]$	$[\underline{\tilde{c}_{12}},\overline{\tilde{c}_{12}}]$ $[\underline{\tilde{t}_{12}},\overline{\tilde{t}_{12}}]$ $[\underline{\tilde{q}_{12}},\overline{\tilde{q}_{12}}]$	$[\underline{\tilde{c}_{13}},\overline{\tilde{c}_{13}}]$ $[\underline{\tilde{t}_{13}},\overline{\tilde{t}_{13}}]$ $[\underline{\tilde{q}_{13}},\overline{\tilde{q}_{13}}]$	$[\underline{\tilde{c}_{14}},\overline{\tilde{c}_{14}}]$ $[\underline{\tilde{t}_{14}},\overline{\tilde{t}_{14}}]$ $[\underline{\tilde{q}_{14}},\overline{\tilde{q}_{14}}]$
M_2	$[\underline{\tilde{c}_{21}},\overline{\tilde{c}_{21}}]$ $[\underline{\tilde{t}_{21}},\overline{\tilde{t}_{21}}]$ $[\underline{\tilde{q}_{21}},\overline{\tilde{q}_{21}}]$	$[\underline{\tilde{c}_{22}},\overline{\tilde{c}_{22}}]$ $[\underline{\tilde{t}_{22}},\overline{\tilde{t}_{22}}]$ $[\underline{\tilde{q}_{22}},\overline{\tilde{q}_{22}}]$	$[\underline{\tilde{c}_{23}},\overline{\tilde{c}_{23}}]$ $[\underline{\tilde{t}_{23}},\overline{\tilde{t}_{23}}]$ $[\underline{\tilde{q}_{23}},\overline{\tilde{q}_{23}}]$	$[\underline{\tilde{c}_{24}},\overline{\tilde{c}_{24}}]$ $[\underline{\tilde{t}_{24}},\overline{\tilde{t}_{24}}]$ $[\underline{\tilde{q}_{24}},\overline{\tilde{q}_{24}}]$
M_3	$[\underline{\tilde{c}_{31}},\overline{\tilde{c}_{31}}]$ $[\underline{\tilde{t}_{31}},\overline{\tilde{t}_{31}}]$ $[\underline{\tilde{q}_{31}},\overline{\tilde{q}_{11}}]$	$[\underline{\tilde{c}_{32}},\overline{\tilde{c}_{32}}]$ $[\underline{\tilde{t}_{32}},\overline{\tilde{t}_{32}}]$ $[\underline{\tilde{q}_{32}},\overline{\tilde{q}_{32}}]$	$[\underline{\tilde{c}_{33}},\overline{\tilde{c}_{33}}]$ $[\underline{\tilde{t}_{33}},\overline{\tilde{t}_{33}}]$ $[\underline{\tilde{q}_{33}},\overline{\tilde{q}_{33}}]$	$[\underline{\tilde{c}_{34}},\overline{\tilde{c}_{34}}]$ $[\underline{\tilde{t}_{34}},\overline{\tilde{t}_{34}}]$ $[\underline{\tilde{q}_{34}},\overline{\tilde{q}_{34}}]$
M_4	$[\underline{\tilde{c}_{41}},\overline{\tilde{c}_{41}}]$ $[\underline{\tilde{t}_{41}},\overline{\tilde{t}_{41}}]$ $[\underline{\tilde{q}_{41}},\overline{\tilde{q}_{41}}]$	$[\underline{\tilde{c}_{42}},\overline{\tilde{c}_{42}}]$ $[\underline{\tilde{t}_{42}},\overline{\tilde{t}_{42}}]$ $[\underline{\tilde{q}_{42}},\overline{\tilde{q}_{42}}]$	$[\underline{\tilde{c}_{43}},\overline{\tilde{c}_{43}}]$ $[\underline{\tilde{t}_{43}},\overline{\tilde{t}_{43}}]$ $[\underline{\tilde{q}_{43}},\overline{\tilde{q}_{43}}]$	$[\underline{\tilde{c}_{44}},\overline{\tilde{c}_{44}}]$ $[\underline{\tilde{t}_{44}},\overline{\tilde{t}_{44}}]$ $[\underline{\tilde{q}_{44}},\overline{\tilde{q}_{44}}]$

7.3 RANKING FUNCTION OF ω-TYPE 2 DIAMOND FUZZY NUMBERS

Let $\omega_{t2}\widetilde{F_d}^1 = [\underline{\omega_{t2}\widetilde{F_d}^1}, \overline{\omega_{t2}\widetilde{F_d}^1}]$ and $\omega_{t2}\widetilde{F_d}^2 = [\underline{\omega_{t2}\widetilde{F_d}^2}, \overline{\omega_{t2}\widetilde{F_d}^2}]$ are two ω-type2 diamond fuzzy numbers. $\underline{\omega_{t2}\widetilde{F_d}^1}$, $\underline{\omega_{t2}\widetilde{F_d}^2}$ are lower ω-type 2 diamond fuzzy number and $\overline{\omega_{t2}\widetilde{F_d}^1}$, $\overline{\omega_{t2}\widetilde{F_d}^2}$ are upper ω-type 2 diamond fuzzy number. Then the following ranking function of ω-type2 diamond fuzzy number and defined as R($\omega_{t2}\widetilde{F_d}$)

$$R(\omega_{t2}\widetilde{F_d}) = \frac{\omega_{t2}\widetilde{F_d}^{1}+\omega_{t2}\widetilde{F_d}^{2}}{2} = \frac{(\underline{\omega_{t2}\widetilde{F_d}^{1}} + \underline{\omega_{t2}\widetilde{F_d}^{2}}) + (\overline{\omega_{t2}\widetilde{F_d}^{1}} + \overline{\omega_{t2}\widetilde{F_d}^{2}}])}{2}$$

7.4 PROPERTIES OF $\boldsymbol{\omega}$-TYPE 2 DIAMOND FUZZY NUMBERS

7.4.1 Theorem: If partial feasible matching is the minimum matching edges in any bipartite graph.

Proof: Consider the fuzzy cost matrix (nxn) is $\widetilde{c_{ij}}$ and define the fuzzy dual variables are $\tilde{u}_i$ = min $\{\widetilde{c_{ij}}\}$ and $\widetilde{v_j}$ = min $\{\widetilde{c_{ij}}$ - $\tilde{u}_i$ $\}$. Then, we have by applying complementary slackness conditions for transform cost matrix $\widetilde{c_{ij}}$ to reduced cost matrix $\overline{\widetilde{c_{ij}}}$ (ie), $\overline{\widetilde{c_{ij}}}$ = $\widetilde{c_{ij}}$ - $\tilde{u}_i$ - $\widetilde{v_j}$ = $0, \forall\ 0 \leq i, j \leq n$. Therefore, assign only one matching edge to each rows and columns but both rows and columns are less than n. Then we have a solution is partial if there are a minimum number of matching edges in any bipartite graph.

7.4.2 Theorem: If an optimal complete matching is the number of matching is equal to the order of the matrix (nxn).

Proof: From Theorem (5.1). Let us take the partial feasible matching edges in bipartite graph. The matching vertex is less then n and increase the partial solution and let 'E' be any vertex in U and choose the elementary path from 'E' whose edges are alternatively not matched and matched. In a bipartite graph, an alternating tree rooted in a vertex 'r' is a tree in which all paths emanating from 'r' alternate. adding new matching vertex is $\overline{U}$ = $\overline{U}$ ∪ {E}. Choose the minimum value of an unassigned row δ = min{$\widetilde{c_{ij}}$ - $\tilde{u}_i$ - $\widetilde{v_j}$= 0 and then updated the dual variables are $\tilde{u}_i^{*}$ = $\tilde{u}_i + \delta$: $\widetilde{v_j}^{*}$ = $\widetilde{c_{ij}}$ - $\tilde{u}_i^{*}$ then compute

$\overline{\widetilde{c_{ij}}}^{*} = \widetilde{c_{ij}} - \tilde{u}_i^{*} - \tilde{v}_j^{*} = 0$ is the new bipartite graph of the current solution. The alternate method executed again for k = δ Producing the augmented tree. Then we have a solution is complete optimal if there are a maximum number of matching is equal to the order of the matrix (nxn).

7.5 THE PROPOSED ALGORITHM

ω-type 2 diamond fuzzy numbers are considered as linguistic variables. The fuzzy cost coefficient, fuzzy time, and fuzzy quality are expressed in λ_d- cut of ω-type 2 diamond fuzzy numbers to compute the partial feasible solution and complete optimal solution.

Step 1: First let us take the cost matrix $[\widetilde{c_{ij}}]$, whose elements are linguistic variables that have been substituted by fuzzy numbers, is presented. Examine whether or not the provided ω-type 2 diamond fuzzy multi-objective linear sum assignment table is balanced.

a) If the number of machines and the number of jobs are equal, go to step 3.

b) Proceed to step 2 if the number of machines does not equal the number of jobs.

Step 2: Create ω-type 2 diamond fuzzy multi objective linear sum assignment table, add a dummy row or column. Dummy row/column cost, time, and quantity entries are always zero.

Step 3: In λ_d- cut form, express the above ω-type 2 diamond fuzzy multi-objective linear sum assignment problems. The upper and lower ω-type 2 diamond fuzzy numbers of the multi-objective linear sum assignment problem are then merged into single λ_d-cut form of ω-type 2 diamond fuzzy number of the multi-objective linear sum assignment problem.

Step 4: By applying ranking method, convert a λ_d- cut of ω-type 2-diamond fuzzy multi objective linear sum assignment problem to λ_d- cut of ω-type 2-diamond single objective fuzzy linear sum assignment problem..

Step 5: Find dual variables ($\tilde{u}_i$,$\tilde{v}_j$),

If M_i = M_1,M_2….M_n then find $\tilde{u}_i$ = min {$\widetilde{c_{ij}}$; J_i = J_1 ,J_2 …..J_n

If J_i = J_1 , J_2 …..J_n then find $\tilde{v}_j$ = min {$\widetilde{c_{ij}}$ - $\tilde{u}_i$; M_i = M_1,M_2….M_n};

Step 6: Calculate ($\overline{\overline{\widetilde{c_{ij}}}}$) and find a partial feasible solution

if j_i = J_1 , J_2 …..J_n then row (j) = 0; if M_i = M_1,M_2….M_n and j_i = J_1 , J_2 J_3…..J_n then obtain $\overline{\overline{\widetilde{c_{ij}}}}$ = $\widetilde{c_{ij}}$ - $\tilde{u}_i$ - $\tilde{v}_j$ = 0 and the solution is row (j) =i ,$\tilde{x}_{ij}$ = 1

a) If there are less than ‘n' rows of matching. go to the next step

b) An optimal solution is found if the number of matches is equal to n.

Step7: If the number of matching solution is less than (the order of the matrix) n matching solution by using the following alternative path method. The matching vertex$|\overline{\boldsymbol{U}}|$< n then increase the partial solution and let E be any vertex in U and select the elementary path from k whose edges are alternatively not assigned and assigned. If E $\notin \overline{U}$ then sink = Alternate(k); If sink > 0 then $\overline{U}$ = $\overline{U}$ ∪ {E} ; j = sink and obtain in new graph.

Step 8: update the dual variables and obtain complete optimal solution select the minimum value of an unassigned row δ = min{$\widetilde{c_{ij}}$ - $\tilde{u}_i$ - $\tilde{v}_j$= 0 and then updated dual variables are $\tilde{u}_i{}^*$ = $\tilde{u}_i$ + δ : $\tilde{v}_j{}^*$ = $\widetilde{c_{ij}}$ - $\tilde{u}_i{}^*$ then obtain $\overline{\overline{\widetilde{c_{ij}}}}^*$ = $\widetilde{c_{ij}}$ - $\tilde{u}_i{}^*$ - $\tilde{v}_j{}^*$ = 0 is the new bipartite graph of the current solution. Alternate (k) is then executed again for k = δ Producing the augmented tree. Finally, each machine (M_i) and job (j_i) has one and only matching edges, complete optimum solution is reached.

Step 9: Stop.

7.6 NUMERICAL EXAMPLE:

Let us considered the four machines given below. M_1, M_2, M_3, M_4, and four jobs J_1, J_2, J_3, J_4 respectively. To optimize the fuzzy cost, fuzzy time, and fuzzy quality are each considered as a ω-type 2 diamond fuzzy numbers. The fuzzy cost, the fuzzy time and the fuzzy quality for solving λ_d- cut of ω-type 2 diamond fuzzy numbers of multi-objective linear sum assignment problem.

Solution:

To optimize the fuzzy cost, fuzzy time, and fuzzy quality are each considered as a ω-type 2 diamond fuzzy numbers by using linguistic variables as follows:

Job/Machine	J_1	J_2	J_3	J_4
M_1	Fairly high	Very high	High	Very high
M_2	Very low	High	low	Fairly high
M_3	Extremely low	Medium	Medium	Very high
M_4	Fairly low	Medium	Very low	Very low

(23,30,37) (22,31,38) (31,38,45) (30,39,46) (36,43,50) (35,44,51)	(25,33,41) (24,34,42) (31,39,47) (30,40,48) (36,44,52) (35,45,53)	(21,27,33) (20,28,34) (18,24,30) (17,25,31) (27,33,39) (26,34,40)	(25,33,41) (24,34,42) (31,39,47) (30,40,48) (36,44,52) (35,45,53)
(1,2,4) (0,3,7) (3,4,7) (2,5,9) (4,6,8) (3,7,10)	(21,27,33) (20,28,34) (18,24,30) (17,25,31) (27,33,39) (26,34,40)	(8,13,18) (7,15,21) (14,18,24) (13,23,28) (11,13,21) (10,17,22)	(23,30,37) (22,31,38) (31,38,45) (30,39,46) (36,43,50) (35,44,51)
(1,2,4) (0,3,5) (2,3,5) (1,4,6) (4,5,7) (3,6,8)	(7,12,15) (6,13,16) (8,13,16) (7,14,17) (13,18,21) (12,19,22)	(7,12,15) (6,13,16) (8,13,16) (7,14,17) (13,18,21) (12,19,22)	(25,33,41) (24,34,42) (31,39,47) (30,40,48) (36,44,52) (35,45,53)
(3,5,9) (2,6,10) (3,4,7) (2,5,10) (4,6,8) (3,7,11)	(7,12,15) (6,13,16) (8,13,16) (7,14,17) (13,18,21) (12,19,22)	(1,2,4) (0,3,7) (3,4,7) (2,5,9) (4,6,8) (3,7,10)	(1,2,4) (0,3,7) (3,4,7) (2,5,9) (4,6,8) (3,7,10)

The fuzzy cost, fuzzy time, and fuzzy quality are each considered as a ω-type 2 diamond fuzzy numbers. ω-type 2 diamond fuzzy multi-objective linear sum assignment table is balanced. ω-type 2 diamond fuzzy numbers are converted into λ_d-cut of fuzzy numbers

	J_1	J_2	J_3	J_4
M_1	[6$\underline{\lambda}_d$ +21, 33-6$\underline{\lambda}_d$] [8$\overline{\lambda}_d$ +20, 34-6$\overline{\lambda}_d$] [6$\underline{\lambda}_d$+18,30- 6$\underline{\lambda}_d$] [8$\overline{\lambda}_d$ +17, 31-6$\overline{\lambda}_d$] [6$\underline{\lambda}_d$+27,39- 6$\underline{\lambda}_d$][8$\overline{\lambda}_d$ +26, 40-6$\overline{\lambda}_d$]	[8λ_d +25, 41-8$\underline{\lambda}_d$] [10$\overline{\lambda}_d$+24, 42-8$\overline{\lambda}_d$] [8$\underline{\lambda}_d$+31,47- 8$\underline{\lambda}_d$] [10$\overline{\lambda}_d$+30, 48- [8$\underline{\lambda}_d$+36,52- 8$\underline{\lambda}_d$][10$\overline{\lambda}_d$ +35, 53-8$\overline{\lambda}_d$]	[7$\underline{\lambda}_d$ +23, 37-7$\underline{\lambda}_d$] [9$\overline{\lambda}_d$ +22, 38-7$\overline{\lambda}_d$] [7$\underline{\lambda}_d$+31,45- 7$\underline{\lambda}_d$] [9$\overline{\lambda}_d$ +30, 46-7$\overline{\lambda}_d$] [7$\underline{\lambda}_d$+36,50- 7$\underline{\lambda}_d$][9$\overline{\lambda}_d$ +35, 51-7$\overline{\lambda}_d$]	[8λ_d +25, 41-8$\underline{\lambda}_d$] [10$\overline{\lambda}_d$ +24, 42-8$\overline{\lambda}_d$] [8$\underline{\lambda}_d$+31,47- 8$\underline{\lambda}_d$] [10$\overline{\lambda}_d$ +30, 48-8$\overline{\lambda}_d$] [8$\underline{\lambda}_d$+36,52- 8$\underline{\lambda}_d$][10$\overline{\lambda}_d$ +35, 53-8$\overline{\lambda}_d$]
M_2	[$\underline{\lambda}_d$+1, 4-2$\underline{\lambda}_d$] [3$\overline{\lambda}_d$+0, 7-4$\overline{\lambda}_d$] [$\underline{\lambda}_d$+3,7- 3$\underline{\lambda}_d$] [3$\overline{\lambda}_d$ +2, 9-4$\overline{\lambda}_d$] [2$\underline{\lambda}_d$+4,8- 2$\underline{\lambda}_d$][4$\overline{\lambda}_d$ +3, 10-3$\overline{\lambda}_d$]	[7$\underline{\lambda}_d$ +23, 37-7$\underline{\lambda}_d$] [9$\overline{\lambda}_d$ +22, 38-7$\overline{\lambda}_d$] [7$\underline{\lambda}_d$+31,45- 7$\underline{\lambda}_d$] [9$\overline{\lambda}_d$ +30, 46-7$\overline{\lambda}_d$] [7$\underline{\lambda}_d$+36,50- 7$\underline{\lambda}_d$][9$\overline{\lambda}_d$ +35, 51-7$\overline{\lambda}_d$]	[5$\underline{\lambda}_d$ +8, 18-5$\underline{\lambda}_d$] [8$\overline{\lambda}_d$ +7, 21-6$\overline{\lambda}_d$] [4$\underline{\lambda}_d$+14,24- 4$\underline{\lambda}_d$] [10$\overline{\lambda}_d$+13, 28-5$\overline{\lambda}_d$] [2$\underline{\lambda}_d$+11,21- 5$\underline{\lambda}_d$][7$\overline{\lambda}_d$ +10, 22-5$\overline{\lambda}_d$]	[6$\underline{\lambda}_d$ +21, 33-6$\underline{\lambda}_d$] [8$\overline{\lambda}_d$ +20, 34-6$\overline{\lambda}_d$] [6$\underline{\lambda}_d$+18,30- 6$\underline{\lambda}_d$] [8$\overline{\lambda}_d$ +17, 31-6$\overline{\lambda}_d$] [6$\underline{\lambda}_d$+27,39- 6$\underline{\lambda}_d$][8$\overline{\lambda}_d$ +26, 40-6$\overline{\lambda}_d$]
M_3	[$\underline{\lambda}_d$ +1, 4-2$\underline{\lambda}_d$] [3$\overline{\lambda}_d$], 5-2$\overline{\lambda}_d$]] [$\underline{\lambda}_d$+2,5- 2$\underline{\lambda}_d$] [3$\overline{\lambda}_d$] +1, 6-2$\overline{\lambda}_d$]] [$\underline{\lambda}_d$+4,7- 2$\underline{\lambda}_d$][3$\overline{\lambda}_d$] +3, 8-2$\overline{\lambda}_d$]]	[5$\underline{\lambda}_d$ +7, 15-3$\underline{\lambda}_d$] [7$\overline{\lambda}_d$ +6, 16-3$\overline{\lambda}_d$] [5$\underline{\lambda}_d$λ+8,16- 3$\underline{\lambda}_d$] [7$\overline{\lambda}_d$ +7, 17-3$\overline{\lambda}_d$] [5$\underline{\lambda}_d$+13,21- 3$\underline{\lambda}_d$][7$\overline{\lambda}_d$ +12, 22-3$\overline{\lambda}_d$]	[5$\underline{\lambda}_d$ +7, 15-3$\underline{\lambda}_d$] [7$\overline{\lambda}_d$ +6, 16-3$\overline{\lambda}_d$] [5$\underline{\lambda}_d$λ+8,16- 3$\underline{\lambda}_d$] [7$\overline{\lambda}_d$ +7, 17-3$\overline{\lambda}_d$] [5$\underline{\lambda}_d$+13,21- 3$\underline{\lambda}_d$][7$\overline{\lambda}_d$ +12, 22-3$\overline{\lambda}_d$]	[8$\underline{\lambda}_d$+ +25, 41-8$\underline{\lambda}_d$] [10$\overline{\lambda}_d$ +24, 42-8$\overline{\lambda}_d$] [8$\underline{\lambda}_d$+31,47- 8$\underline{\lambda}_d$] [10$\overline{\lambda}_d$ +30, 48-8$\overline{\lambda}_d$] [8$\underline{\lambda}_d$+36,52- 8$\underline{\lambda}_d$][10$\overline{\lambda}_d$ +35, 53-8$\overline{\lambda}_d$]
M_4	[2$\underline{\lambda}_d$ +3, 9-4$\underline{\lambda}_d$] [4$\overline{\lambda}_d$, +2, 10-4$\overline{\lambda}_d$] [$\underline{\lambda}_d$+3,7- 3$\underline{\lambda}_d$] [3$\overline{\lambda}_d$+2, 9-5$\overline{\lambda}_d$] [2$\underline{\lambda}_d$+4,8- 2$\underline{\lambda}_d$][4$\overline{\lambda}_d$+3 , 11-4$\overline{\lambda}_d$]	[5$\underline{\lambda}_d$ +7, 15-3$\underline{\lambda}_d$] [7$\overline{\lambda}_d$ +6, 16-3$\overline{\lambda}_d$] [5$\underline{\lambda}_d$λ+8,16- 3$\underline{\lambda}_d$] [7$\overline{\lambda}_d$ +7, 17-3$\overline{\lambda}_d$] [5$\underline{\lambda}_d$+13,21- 3$\underline{\lambda}_d$][7$\overline{\lambda}_d$ +12, 22-3$\overline{\lambda}_d$]	[$\underline{\lambda}_d$+1, 4-2$\underline{\lambda}_d$] [3$\overline{\lambda}_d$+0, 7-4$\overline{\lambda}_d$] [$\underline{\lambda}_d$+3,7- 3$\underline{\lambda}_d$] [3$\overline{\lambda}_d$ +2, 9-4$\overline{\lambda}_d$] [2$\underline{\lambda}_d$+4,8- 2$\underline{\lambda}_d$][4$\overline{\lambda}_d$ +3, 10-3$\overline{\lambda}_d$]	[$\underline{\lambda}_d$+1, 4-2$\underline{\lambda}_d$] [3$\overline{\lambda}_d$+0, 7-4$\overline{\lambda}_d$] [$\underline{\lambda}_d$+3,7- 3$\underline{\lambda}_d$] [3$\overline{\lambda}_d$ +2, 9-4$\overline{\lambda}_d$] [2$\underline{\lambda}_d$+4,8- 2$\underline{\lambda}_d$][4$\overline{\lambda}_d$ +3, 10-3$\overline{\lambda}_d$]

ω-type 2 diamond multi-objective fuzzy numbers are converted into single λ-cut fuzzy number

Job/ Machine	M_1	M_2	M_3	M_4
J_1	$[18\underline{\lambda_d}+66,102-18\underline{\lambda_d}]$ $[24\overline{\lambda_d}+63, 105-18\overline{\lambda_d}]$	$[4\underline{\lambda_d}+8,19-7\underline{\lambda_d}]$ $[10\overline{\lambda_d}+5, 26-11\overline{\lambda_d}]$	$[3\underline{\lambda_d}+7,16-6\underline{\lambda_d}]$ $[9\overline{\lambda_d}+4, 19-6\overline{\lambda_d}]$	$[5\underline{\lambda_d}+10,24-9\underline{\lambda_d}]$ $[11\overline{\lambda_d}+7, 30-13\overline{\lambda_d}]$
J_2	$[24\underline{\lambda_d}+92,140-24\underline{\lambda_d}]$ $[30\overline{\lambda_d}+89, 143-24\overline{\lambda_d}]$	$[21\underline{\lambda_d}+90,132-21\underline{\lambda_d}]$ $[27\overline{\lambda_d}+87, 135-21\overline{\lambda_d}]$	$[15\underline{\lambda_d}+28,52-9\underline{\lambda_d}]$ $[21\overline{\lambda_d}+25, 55-9\overline{\lambda_d}]$	$[15\underline{\lambda_d}+28,52-9\underline{\lambda_d}]$ $[21\overline{\lambda_d}+25, 55-9\overline{\lambda_d}]$
J_3	$[21\underline{\lambda_d}+90,132-21\underline{\lambda_d}]$ $[27\overline{\lambda_d}+87, 135-21\overline{\lambda_d}]$	$[11\underline{\lambda_d}++33,63-14\underline{\lambda_d}]$ $[25\overline{\lambda_d}+30, 71-16\overline{\lambda_d}]$	$[15\underline{\lambda_d}+28,52-9\underline{\lambda_d}]$ $[21\overline{\lambda_d}+25, 55-9\overline{\lambda_d}]$	$[4\underline{\lambda_d}+8,19-7\underline{\lambda_d}]$ $[10\overline{\lambda_d}+5, 26-11\overline{\lambda_d}]$
J_4	$[24\underline{\lambda_d}+92,140-24\underline{\lambda_d}]$ $[30\overline{\lambda_d}+89, 143-24\overline{\lambda_d}]$	$[18\underline{\lambda_d}+66,102-18\underline{\lambda_d}]$ $[24\overline{\lambda_d}+63, 105-18\overline{\lambda_d}]$	$[24\underline{\lambda_d}+92,140-24\underline{\lambda_d}]$ $[30\overline{\lambda_d}+89, 143-24\overline{\lambda_d}]$	$[4\underline{\lambda_d}+8,19-7\underline{\lambda_d}]$ $[10\overline{\lambda_d}+5, 26-11\overline{\lambda_d}]$

Upper and lower ω-type 2 diamond multi-objective fuzzy numbers are converted into single objective λ_d-cut fuzzy number

Job/ Machi	M_1	M_2	M_3	M_4
J_1	$[21\underline{\lambda_d}+64.5, 103-18\overline{\lambda_d}]$	$[7\underline{\lambda_d}+6.5, 22.5-9\overline{\lambda_d}]$	$[6\underline{\lambda_d}+5.5, 17.5-6\overline{\lambda_d}]$	$[8\underline{\lambda_d}+8.5, 27-11\overline{\lambda_d}]$
J_2	$[27\underline{\lambda_d}+90.5, 141.5-24\overline{\lambda_d}]$	$[24\underline{\lambda_d}+88.5, 133.5-21\overline{\lambda_d}]$	$[18\underline{\lambda_d}+26.5, 53.5-9\overline{\lambda_d}]$	$[18\underline{\lambda_d}+26.5, 53.5-9\overline{\lambda_d}]$
J_3	$[24\underline{\lambda_d}+88.5, 133.5-21\lambda]$	$[18\underline{\lambda_d}+31.5, 67-15\overline{\lambda_d}]$	$[18\underline{\lambda_d}+26.5, 53.5-9\overline{\lambda_d}]$	$[7\underline{\lambda_d}+6.5, 22.5-9\overline{\lambda_d}]$
J_4	$[27\underline{\lambda_d}+90.5, 141.5-24\overline{\lambda_d}]$	$[21\underline{\lambda_d}+64.5, 103.5-18\overline{\lambda_d}]$	$[27\underline{\lambda_d}+90.5, 141.5-24\overline{\lambda_d}]$	$[7\underline{\lambda_d}+6.5, 22.5-9\overline{\lambda_d}]$

Obtain dual variable

$\tilde{u}$ ={[21$\underline{\lambda_d}$+64.5,103-18$\overline{\lambda_d}$], [7$\underline{\lambda_d}$+6.5,25.5-9$\overline{\lambda_d}$], [6$\underline{\lambda_d}$+5.5,17.5-6$\overline{\lambda_d}$],

[7$\underline{\lambda_d}$+6.5,25.5-9$\overline{\lambda_d}$]

$\tilde{v}$ = {0, [6$\underline{\lambda_d}$+26,38.5- 6$\overline{\lambda_d}$],0 , 0}

$\underline{0}$	-[6$\underline{\lambda_d}$+26,38.5- 6$\overline{\lambda_d}$]	0	0
0	0	[3$\underline{\lambda_d}$+24,30.5-3$\overline{\lambda_d}$]	[6$\underline{\lambda_d}$+26,38.5- 6$\overline{\lambda_d}$]
0	[11$\underline{\lambda_d}$+56,72.5-6$\overline{\lambda_d}$]	[11$\underline{\lambda_d}$+25,44.5-6$\overline{\lambda_d}$]	[14$\underline{\lambda_d}$+58,81-9$\overline{\lambda_d}$]
0	[6$\underline{\lambda_d}$-5, -2.5+3$\overline{\lambda_d}$]	[12$\underline{\lambda_d}$+21,36-3$\overline{\lambda_d}$]	[21$\underline{\lambda_d}$+85,124- 18$\overline{\lambda_d}$]
[$\underline{\lambda_d}$+2.5,4.5-2$\overline{\lambda_d}$]	[5$\underline{\lambda_d}$-6,-7.5+6$\overline{\lambda_d}$]	**$\underline{0}$**	0

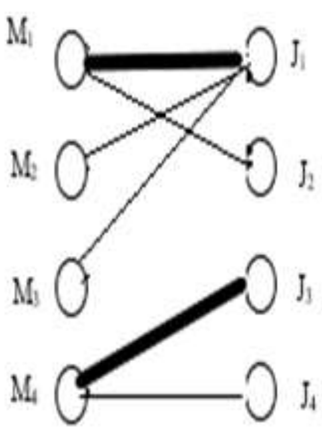

Figure :7.3

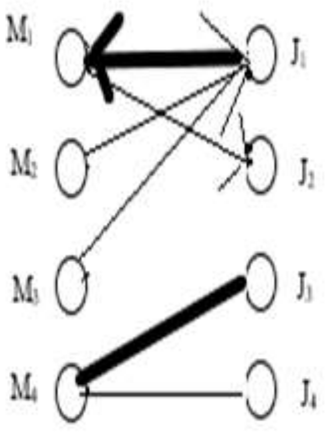

Figure :7.4

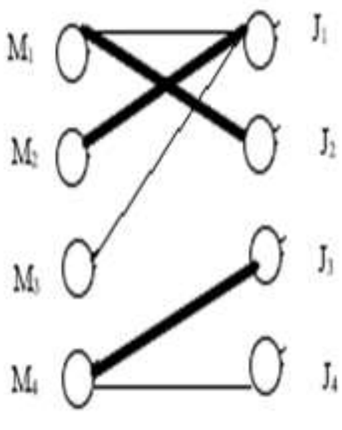

Figure :7.5

Job/ Machine	M_1	M_2	M_3	M_4
J_1	$[21\underline{\lambda_d}+64.5,103-18\overline{\lambda_d}]$	$[7\underline{\lambda_d}+6.5,22.5 -9\overline{\lambda_d}]$	$[6\underline{\lambda_d}+5.5,17.5-6\overline{\lambda_d}]$	$[8\underline{\lambda_d}+8.5,27-11\overline{\lambda_d}]$
J_2	$[27\underline{\lambda_d}+90.5,141.5- 24\overline{\lambda_d}]$	$[24\underline{\lambda_d}+88.5,133.5-21\overline{\lambda_d}]$	$[18\underline{\lambda_d}+26.5,53.5-9\overline{\lambda_d}]$	$[18\underline{\lambda_d}+26.5,53.5-9\overline{\lambda_d}]$
J_3	$[24\underline{\lambda_d}+88.5,133.5-21\lambda]$	$[18\underline{\lambda_d}+31.5,67-15\overline{\lambda_d}]$	$[18\underline{\lambda_d}+26.5,53.5-9\overline{\lambda_d}]$	$[7\underline{\lambda_d}+6.5,22.5 -9\overline{\lambda_d}]$
J_4	$[27\underline{\lambda_d}+90.5,141.5- 24\overline{\lambda_d}]$	$[21\underline{\lambda_d}+64.5,103.5-18\overline{\lambda_d}]$	$[27\underline{\lambda_d}+90.5,141.5- 24\overline{\lambda_d}]$	$[7\underline{\lambda_d}+6.5,22.5 -9\overline{\lambda_d}]$

Job/ Machine	M_1	M_2	M_3	M_4
J_1	0	0	$\underline{\mathbf{0}}$	$[12\underline{\lambda_d}+27,49-4\overline{\lambda_d}]$
J_2	$\underline{\mathbf{0}}$	$[0\underline{\lambda_d}\ -1.5,28-0\overline{\lambda_d}]$	0	$[0\underline{\lambda_d}-6,-7.5-4\overline{\lambda_d}]$
J_3	$[3\underline{\lambda_d}+24,30.5-3\overline{\lambda_d}]$	$\underline{\mathbf{0}}$	$[6\underline{\lambda_d}+26.5,38.5-0\overline{\lambda_d}]$	0
J_4	$[6\underline{\lambda_d}+26,38.5-\ 6\overline{\lambda_d}]$	$[3\underline{\lambda_d}+33,36.5+3\overline{\lambda_d}]$	$[15\underline{\lambda_d}+90.5,126.5-\ 5\overline{\lambda_d}]$	$\underline{\mathbf{0}}$

$\tilde{\boldsymbol{u}}_{\boldsymbol{i}}^{*} = \{[21\underline{\boldsymbol{\lambda}_{\boldsymbol{d}}} + 64.5, 103 - 18\overline{\boldsymbol{\lambda}_{\boldsymbol{d}}}], [18\underline{\boldsymbol{\lambda}_{\boldsymbol{d}}} + 31.5, 67 - 15\overline{\boldsymbol{\lambda}_{\boldsymbol{d}}}],$

$[12\underline{\boldsymbol{\lambda}_{\boldsymbol{d}}}, 15 - 9\overline{\boldsymbol{\lambda}_{\boldsymbol{d}}}], [7\underline{\boldsymbol{\lambda}_{\boldsymbol{d}}} + 6.5, 25.5 - 9\overline{\boldsymbol{\lambda}_{\boldsymbol{d}}}]\};$

$\tilde{\boldsymbol{v}}_{\boldsymbol{J}}^{*} = \{[-11\underline{\boldsymbol{\lambda}_{\boldsymbol{d}}}-25, -44.5+6\overline{\boldsymbol{\lambda}_{\boldsymbol{d}}}], [6\underline{\boldsymbol{\lambda}_{\boldsymbol{d}}}+26, 38.5- 6\overline{\boldsymbol{\lambda}_{\boldsymbol{d}}}], 0, 0$

Fuzzy optimal schedule $M_1 \rightarrow J_2,\ M_2 \rightarrow J_3,\ M_3 \rightarrow J_1,\ M_4 \rightarrow J_4$

$\left.\begin{array}{c}\textbf{Fuzzy}\\ \textbf{optimal}\\ \textbf{cost}\end{array}\right\}$ = {(25,33,41) + (8,13,18) + (1,2,4) + (1,2,4)}{(24,34,42) + (7,15,21) + (0,3,5) + (0,3,7)}

Fuzzy optimal cost = (35, 50, 67) (31,55,75)

Fuzzy optimal time= {(31,39,47) + (14,18,24) + (2,3,5) + (3,4,7)}{(30,40,48 + (13,23,28) + (1,4,6) + (2,5,9)}

Fuzzy optimal time = (50, 64, 83) (46,72,91)

Fuzzy optimal quality = {(36,44,52) + (4,6,8) + (4,5,7) + (4,6,8)} {(35,45,53) + (3,7,10) + (3,6,8) + (3,7,10)}

Fuzzy optimal quality = (48,61,75) (44,65,81).

CHAPTER VIII

MINIMUM VERTEX COVER OF ω-PENTAGONAL FUZZY LINEAR SUM BOTTLENECK ASSIGNMENT PROBLEM

In this chapter proposed, a spread of minimum solution of fuzzy optimization matching procedure in the bipartite graph. it provides minimum vertex cover with edge set E for solving $\omega -$ Pentagonal Fuzzy Linear Sum Bottleneck Assignment Problem [ω-PFLSBAP]. The $\omega - PFLSBAP$ is minimum cost and complete matching in the bipartite graph. The Linear Sum Bottleneck Assignment Cost [LSBAC] we taken as $\omega -$Pentagonal Fuzzy Numbers ($\omega - PFN$). If each person and each job contain exactly one matching solution with Spr (φ) = 0 or minimum Spr (φ), then the current ω-PFLSBAP is optimal. If each person and each job contain exactly one matching solution with maximum Spr (φ), then the current ω-PFLSBAP is not optimal but feasible and complete matching solution. Finally obtained the graph has minimum vertex cover of cardinality n with perfect or complete matching. This method is illustrated by a numerical example.

8.1 INTRODUCTION:

In this chapter**,** the study of fuzzy linear sum assignment problems utilizing various methodologies and various fuzzy numbers based on fuzzy optimization matching techniques is the main goal of this research study. It primarily focuses on various methods for solving fuzzy assignment problems and linear sum assignment problems, which might lead to an optimal solution or a viable solution, as well as partial or complete matching in a bipartite graph. In order to use the approaches suggested, the research aims to emphasize the concepts described. The primary contribution of this

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This content of this chapter has been communicated to Advances and Applications in Mathematical Sciences

work is the presentation of a feasible/optimal solution and partial/complete matching in a bipartite graph for solving and connecting linear sum assignment problems with fuzzy numbers. This fundamental approach should be seen as a more approachable way to assess and identify optimized matching techniques in fuzzy numbers and fuzzy logic for students who lack understanding of new ideas for achieving optimal or feasible solutions. The matching solution is the partial/complete solution of the bipartite graph and If the solution have maximum matching and minimum spread of solution. then the solution is optimal or minimum vertex cover of spread of "ω-" pentagonal fuzzy linear sum bottleneck assignment problem.

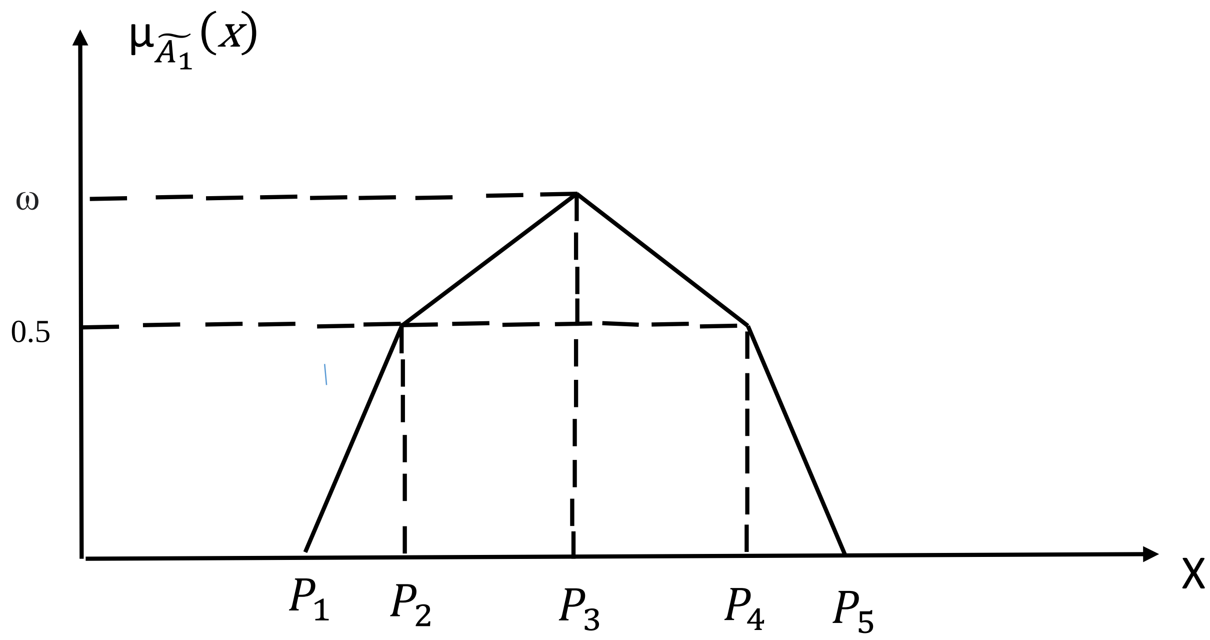
8.2 SPREADING SOLUTION OF ω-PENTAGONAL FUZZY NUMBER

8.2.1 Definition:

The fuzzy number $\widetilde{A_1}$ = (P_1 , P_2 ,P_3 , P_4 , P_5 ;ω) is referred to as a ω *-Pentagonal fuzzy number* if the membership function is as follows:

$$\mu_{\widetilde{A_1}}(x) = \begin{cases} \frac{1}{2}\left(\frac{x-P_1}{P_2-P_1}\right), & \text{if} \quad P_1 \le x \le P_2 \\ \frac{1}{2} - \left(\frac{1}{2}-\omega\right)\left(\frac{x-P_2}{P_3-P_2}\right), & \text{if} \quad P_2 \le x \le P_3 \\ \omega, & \text{if} \quad x = P_3 \\ \frac{1}{2} - \left(\frac{1}{2}-\omega\right)\left(\frac{P_4-x}{P_4-P_3}\right) & \text{if} \quad P_3 \le x \le P_4 \\ \frac{1}{2}\left(\frac{P_5-x}{P_5-P_4}\right) & \text{if} \quad P_4 \le x \le P_5 \end{cases}$$

where $\omega \in (0.5,1)$



8.2.2 Definition Let $\widetilde{C}=\widetilde{c_{ij}}$ be a given (m×n) ω-Pentagonal Fuzzy Linear Sum Bottleneck Assignment Problem (ω-PFLSBAP) and φ be an arbitrary permutation of the set then spr(φ) = $\underset{i}{\max}\{\tilde{c}_{i\varphi(i)}\}$- $\underset{i}{\min}\{\tilde{c}_{i\varphi(i)}\}$ is called spread of solution(φ).

8.2.3 Definition Let φ be a matching solution of the bipartite graph. If the solution is maximum matching and minimum spread of solution (φ) or spr (φ)=0, then the solution is optimal or minimum vertex cover of spread of ω-pentagonal fuzzy linear sum bottleneck assignment problem.

8.3 Mathematical formulation of ω −Pentagonal Fuzzy Linear Sum Bottleneck Assignment Problem (ω-PFLSBAP**)**

Let $\widetilde{C}=\widetilde{c_{ij}}$ be a given (n×n) ω-Pentagonal FuzzyLinear Sum Bottleneck Assignment Problem (ω-PFLSBAP) then the following mathematical formulation:

Objective $\underset{\varphi}{\min}$ spr(φ)

Subject to the constraints

$$\sum_{j=1}^{n} \widetilde{x_{ij}} = 1 \quad ;i=(1,2,\ldots n)$$

$$\sum_{i=1}^{n} \widetilde{x_{ij}} = 1 \quad ;j=(1,2,\ldots n)$$

$$\tilde{x}_{ij} = 0\ (or)\ 1.$$

8.4 Matching solution of the ω-Pentagonal Fuzzy Linear Sum Bottleneck Assignment Problem (ω-PFLSBAP**)**

- The matching solution $\varphi=(\varphi_1,\varphi_2,\ldots\varphi_n)$ and row(j)

  row (j) = i   if column j is matched to row i

  = 0   if column j is not matched to row i   (for j = 1,2…..n)

- The matching solution $\varphi=(\varphi_1,\varphi_2,\ldots\varphi_n)$ and implements the inverse of row

  φ = j   if row i is matched to column j

  = 0   if row i is not matched to column j   (for i = 1,2…n)

8.5 THE PROPOSED ALGORITHM

Find minimum vertex cover of optimization matching techniques in the bipartite graph for solving ω-pentagonal fuzzy linear sum bottleneck assignment problems. The Linear Sum Bottleneck Assignment Cost [LSBAC] we taken as ω-Pentagonal Fuzzy Numbers (ω-PFN).

Step 1: First check whether the given ω-Pentagonal Fuzzy Linear Sum Bottleneck Assignment Problem (ω-PFLSBAP) is balanced or not,

- If the total number of persons is equal to the total number of jobs, then ω-PFLSBAP is balanced, go to step 3.
- If the total number of persons is not equal to the total number of jobs, then ω-PFLSBAP is unbalanced, go to step 2.

Step 2: Add a dummy row/column of ω-pentagonal fuzzy linear sum bottleneck assignment cost. Entries with a cost of dummy row/dummy column are always zero.

Step 3: **Calculate the matching solution (φ) of** ω-PFLSBAP **(row minimum)**

$$\varphi = \min\left(\widetilde{C_{ij}}\right) \quad \text{for } i=1,2\ldots n$$

Step 4: **Calculate spr (φ)**

Let φ be a matching solution of ω-PFLSBAP with cost matrix $\widetilde{C_{ij}}$

$$\pi = \max_i\left(\widetilde{C}_{i\,\varphi(i)}\right)$$

$$\lambda = \min_i\left(\widetilde{C}_{i\,\varphi(i)}\right)$$

$$\text{Spr}(\varphi) = \pi + (-\lambda) \quad ; \quad T = \{(i, j) : \lambda < \widetilde{C_{ij}}$$

Step 5: **Find minimum spr (φ) or spr (φ) = 0**

In the bipartite graph (n×n) with edge set E, find the minimum vertex cover (σ). If |σ|=n then φ be a perfect matching solution of ω-PFLSBAP.

$$\pi = \max_i\left(\widetilde{C}_{i\,\varphi(i)}\right) ; \quad \lambda = \min_i\left(\widetilde{C}_{i\,\varphi(i)}\right) ; \quad \text{Spr}(\varphi) = \pi + (-\lambda)$$

Step 6: **Minimum Vertex Cover of Cardinality n with Perfect Matching**

Let us take uncovered vertex $\overline{T} = \{(i, j) \epsilon\, T, (i, j) \text{ is Uncovered}\}$. If $\overline{T}$ is not equal to ∅ then find $\pi = \min\{\widetilde{C_{ij}} : (i,j) \epsilon\, \overline{T}\}$ and $\lambda = \pi + (-\text{Spr}(\varphi))$. By completing no perfect matching through augmenting techniques, the new vertex cover is obtained from the current partial solution.

Step 7: **(Apply optimal test of** ω **-** PFLSBAP)

- If each person and each job contain exactly one matching solution with Spr (φ) = 0 or minimum Spr (φ), then the current ω-PFLSBAP is optimal.
- If each person and each job contain exactly one matching solution with maximum Spr (φ), then the current ω-PFLSBAP is not optimal but feasible and perfect/complete matching solution.

Step 8: Finally obtained the graph has minimum vertex cover of cardinality n with complete matching. Repeat the procedure (1) to (7), until an optimum ω-PFLSBA is attained.

Step 9: STOP.

8.6 NUMERICAL EXAMPLE

A company wants to assign four persons A, B, C and D to four jobs 1,2,3,4. one for each job, with no person working on more than one job. The assignment cost is considered as ω- pentagonal fuzzy number. Find minimum vertex of Cardinality n with Perfect Matching of ω-Pentagonal Fuzzy Linear Sum Bottleneck Assignment Problem.

Solution:

Consider the following ω −Pentagonal Fuzzy Linear Sum Bottleneck Assignment table

	1	2	3	4
A	(20,26,32,38,44;0.9)	(25,32,39,46,53;0.92)	(4,7,10,13,16;0.65)	(2,4,6,8,10;0.63)
B	(6,10,14,18,22;0.75)	(10,15,20,25,30; 0.7)	(4,7,10,13,16;0.65;.95)	(20,26,32,38,44;0.9)
C	(0,1,2,3,4;0.6)	(20,26,32,38,44;0.9)	(35,40,45,50,55;0.90)	(6,10,14,18,22;0.75)
D	(2,4,6,8,10;0.63)	(8,12,16,20,24;0.8)	(20,26,32,38,44;0.9)	(4,7,10,13,16;0.65)

	1	2	3	4
A	(20,26,32,38,44;0.9)	(25,32,39,46,53;0.92)	(4,7,10,13,16;0.65)	**(2,4,6,8,10;0.63)**
B	(6,10,14,18,22;0.75)	(10,15,20,25,30; 0.7)	**(4,7,10,13,16;0.65;0.95)**	(20,26,32,38,44;0.9)
C	**(0,1,2,3,4;0.6)**	(20,26,32,38,44;0.9)	(35,40,45,50,55;.90)	(6,10,14,18,22;0.75)
D	(2,4,6,8,10;0.63)	**(8,12,16,20,24;0.8)**	(20,26,32,38,44;0.9)	(4,7,10,13,16;0.65)

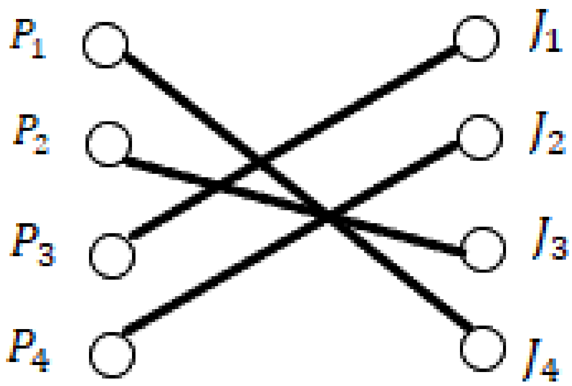


Figure: 8.1

φ=(4,3,1,2)***;*** λ=(0,1,2,3,4;0.6) ; π=(8,12,16,20,24;0.8)***;***

Spr (φ)=(8,11,14,17,20;0.2)

Table:3

(20,26,32,38,44;0.9)	(25,32,39,46,53;0.92)	**(4,7,10,13,16;0.65)**	-
(6,10,14,18,22;0.75)	(10,15,20,25,30; 0.7)	-	(20,26,32,38,44;0.9)
-	(20,26,32,38,44;0.9)	(35,40,45,50,55;.90)	**(6,10,14,18,22;0.75)**
(2,4,6,8,10;0.63)	**(8,12,16,20,24;0.8)**	(20,26,32,38,44;0.9)	(4,7,10,13,16;0.65)

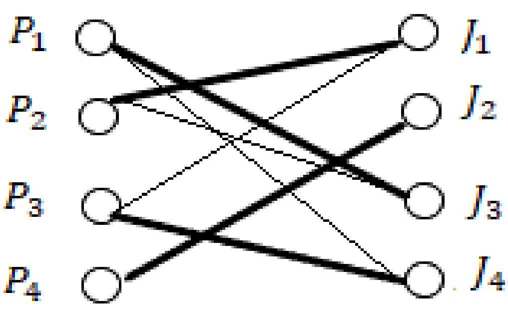


Figure: 8.2

φ=(3,1,4,2); λ=(4,7,10,13,16; 0.65) ; π=(8,12,16,20,24; 0.8);

Spr (φ)=(4,5,6,7,8;0.15)

(20,26,32,38,44;0.9)	(25,32,39,46,53;0.92)	- -	-
- -	**(10,15,20,25,30; 0.7)**	-	(20,26,32,38,44;0.9)
-	(20,26,32,38,44;0.9)	**(35,40,45,50,55;0.9)**	- -
(2,4,6,8,10;0.63)	- -	(20,26,32,38,44;0.9)	**(4,7,10,13,16;0.65)**

$\varphi = (1,2,3,4)$; $\pi = (20,26,32,38,44; 0.9)$; $\lambda = \pi + \left(-Spr\ (\varphi)\right) =$

(16,21,26,31,36;0.75)

(20,26,32,38,44;0.9)	(25,32,39,46,53;0.92)	- -	-
- -	- -- -	-	**(20,26,32,38,44;0.9)**
-	**(20,26,32,38,44;0.9)**	(35,40,45,50,55;0.9)	- -
- -- -	- -	**(20,26,32,38,44;0.9)**	- -- -

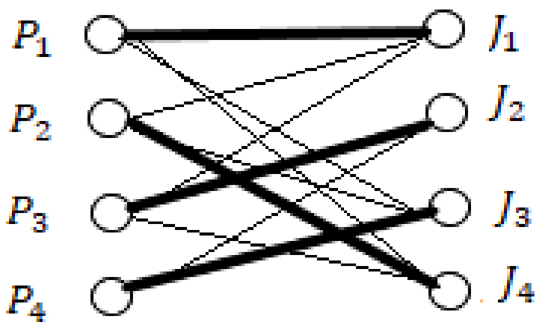


Figure: 8.3

$\varphi = (1,4,2,3)$; $\pi = (20,26,32,38,44; 0.9)$; $\lambda = (20,26,32,38,44; 0.9)$;

SPr (φ) = (0,0,0,0)

$\therefore$ **The optimal ω - PFLSBA schedule is** A → 1, B → 4, C → 2, D → 3

The optimal ω – PFLSBAP is

(20,26,32,38,44;0.9) + (20,26,32,38,44;0.9) +(20,26,32,38,44;0.9) +

(20,26,32,38,44;0.9) = (80,104,128,152,176; 0.9).

CONCLUSION

Fuzzy decision making is a group of single- or multi-criteria strategies designed to choose the optimal option when faced with ambiguous, incomplete, or inaccurate data. Establishing broad and usable fuzzy optimization methods is crucial in both theory and application since fuzzy optimization is a well-known optimization problem in artificial intelligence, manufacturing, and management. This research work's attention was primarily directed at an algorithmic approach to the fuzzy linear sum assignment problem. There are a number of papers in the literature that use fuzzy assignment costs to solve linear sum assignment problems, but no one has previously used fuzzy linear sum assignment costs. Here, the optimal results are chosen using a variety of algorithms. The techniques presented here are simple to comprehend and are applicable to all classes of linear sum assignment problems with fuzzy costs and fuzzy numbers. Numerical examples are used to explain the solution processes.

These techniques can be extended to fuzzy quadratic assignment problems, New fuzzy linear sum assignment problems, and their modifications, so they are not just limited to optimization in fuzzy linear sum assignment problems.

LIST OF PAPER PUBLICATIONS

PUBLICATIONS IN NATIONAL / INTERNATIONAL JOURNALS

1. Nagoor Gani.A., Shiek Pareeth.T, Dual and Partial Primal Solution for Solving Linear Sum Feasible Fuzzy Assignment Problem, International Journal of Fuzzy Mathematical Archive, Vol. 14, No. 2, 2017, P.171-177.

2. Nagoor Gani.A., Shiek Pareeth.T, Feasible Degenerate Pivoting and Optimal Non-Degenerate Pivoting for Solving Fuzzy Linear Sum Assignment Problems, Journal of Physical Sciences, Vol. 23, 2018, P.97-109.

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4. Nagoor Gani.A., Shiek Pareeth.T, A Spread Out of New Partial Feasible and Optimal Perfect Matching for Solving Interval-Valued α-Cut Fuzzy Linear Sum Bottleneck Assignment Problem, Advances and Applications in Mathematical Sciences, Volume 19, Issue 11,September 2020,P. 1159-1173.

5. Nagoor Gani.A., Shiek Pareeth.T, A New Optimal Complete Matching of Edges with Minimum Cost by Ranking Method for Solving ω -Type -2 Fuzzy Linear Sum Assignment Problem, Journal of Cardiovascular Disease Research,Volume 12, Issue 02, 2021,P.100-105.

6. Nagoor Gani.A., Shiek Pareeth.T, Solving Fuzzy Multi-Objective Linear Sum Assignment Problem with Modified Partial Primal Solution of **ω** - type 2 - Diamond Fuzzy Numbers by Using Linguistic Variables, Advances in Dynamical Systems and Applications. Volume 16, Number 2, (2021) ,P.1499-1514.

7. Nagoor Gani.A., Shiek Pareeth.T, Minimum Vertex Cover of ω-Pentagonal Fuzzy Linear Sum Bottleneck Assignment Problem, Advances and Applications in Mathematical Sciences

PAPER PRESENTED IN INTERNATIONAL CONFERENCES

1. Dual and Partial Primal Solution for Solving Linear Sum Feasible Fuzzy Assignment Problem in "International Conference on Mathematical Methods and Computation(ICOMAC)", on 11th December 2017 at Jamal Mohamed College, Tiruchirappalli, Tamilnadu.
2. Spread Out of New Partial Feasible and Optimal Perfect Matching for Solving Interval-Valued α-Cut Fuzzy Linear Sum Bottleneck Assignment Problem in "International Conference on "Mathematical Methods and Computation (ICOMAC)", on 20th & 21st February 2019 at Jamal Mohamed College, Tiruchirappalli, Tamilnadu.
3. A New Optimal Complete Matching of Edges with Minimum Cost by Ranking Method for Solving ω-Type -2 Fuzzy Linear Sum Assignment Problem "Heber International Conference on Applied Mathematics (HICAM)", on 26th April 2021 at Bishop Heber College, Tiruchirappalli, Tamilnadu.
4. A Spread of Minimum Vertex Cover of Optimization Matching Techniques in the Bipartite Graph for Solving ω - Pentagonal Fuzzy Linear Sum Assignment Problem International Conference on "Pure and Applied Mathematics (ICOPAM)" in Kerala Mathematical Association, on 24th & 25th March 2022 at TBML College, Porayar, Tamilnadu.

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A SPREAD OUT OF NEW PARTIAL FEASIBLE AND OPTIMAL PERFECT MATCHING FOR SOLVING INTERVAL-VALUED □-CUT FUZZY LINEAR SUM BOTTLENECK ASSIGNMENT PROBLEM

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A SPREAD OUT OF NEW PARTIAL FEASIBLE AND OPTIMAL PERFECT MATCHING FOR SOLVING INTERVAL-VALUED α-CUT FUZZY LINEAR SUM BOTTLENECK ASSIGNMENT PROBLEM

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Abstract

In this paper, we compare the spreading of new optimal perfect matching to solve interval-value α-cut fuzzy linear sum bottleneck assignment problem. If all the spreading solutions are in minimum cost/time and maximum matching, and we get the matching solution optimal and perfect. Suppose, the solution is in minimum cost/time and minimum matching, the solution is a spread out of new partial feasible matching, if the solution is in maximum cost and minimum matching ,we get a spread out of partial feasible matching.

1. Introduction

Let '*J*' jobs and '*P*' machines be given in a balanced interval valued fuzzy linear sum bottleneck assignment problem (IFLSBAP) where $\breve{C}_{ij}$ generalized trapezoidal fuzzy numbers. The bottleneck assignment refers to latest completion in the allocation of assignment problem. The interval-valued α-cut of generalized fuzzy linear sum bottleneck assignment problems are minimum cost maximum matching problem. Let $G = (U, V, E)$ be a bipartite graph with edge set *E*. The edge $[i, j]$ has a cost coefficient $\omega_{\breve{C}_{ij}}$.

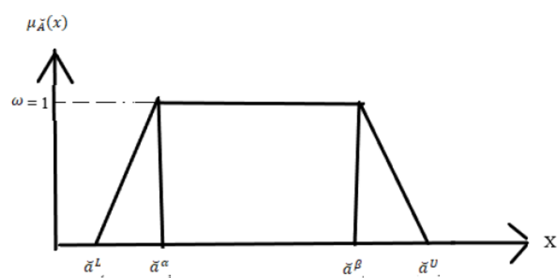
We obtain perfect matching in *G* such that the perfect length of an edge in this matching is as small as possible.

2010 Mathematics Subject Classification: Please provide.
Keywords: Trapezoidal fuzzy numbers, Generalized trapezoidal fuzzy number, Bipartite graph, α-cut of generalized trapezoidal fuzzy number, Partial matching and Perfect matching.
Received Please provide

Amit Kumar and Anil Gupta have proposed Assignment and Travelling Salesman problems with Coefficient as LR Fuzzy parameters [1]. In2019, K. Atanassov proposed extended interval-valued intuitionistic fuzzy index matrices [2]. H. Albrecher approached a note on the asymptotic behaviour of bottleneck problems [3]. Linear bottleneck assignment problems were proposed by Fulkerson, Glicksberg and Gross. In 1999, D. Dubois, and P. Fortemps have proposed Computing improved optimal solutions to max-min flexible constraint satisfaction problems [4]. In 1971, R. Garfinkal have proposed improved algorithm for bottleneck assignment problem [5]. In 2004 E. Hansen and G. W. Walster, introduced Global Optimization using interval analysis.

Definition 1.1. fuzzy number $\breve{A} = (\breve{a}^L, \breve{a}^\alpha, \breve{a}^\beta, \breve{a}^U)$ is fuzzy subset of R with membership grade $\mu_{\breve{A}}(x)$ is said to be trapezoidal fuzzy numberand then the following membership functions as,

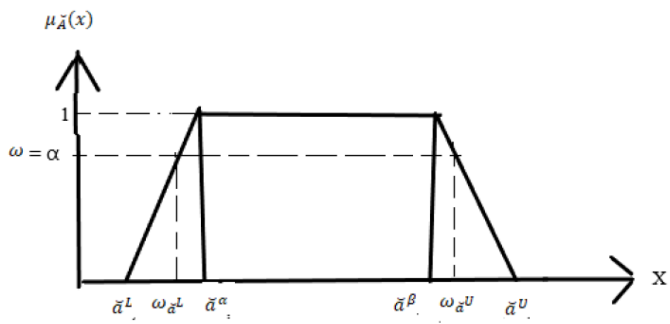
$$\mu_{\breve{A}}(x) = \begin{cases} \dfrac{X - \breve{a}^L}{\breve{a}^\alpha - \breve{a}^L}, \text{ if } \breve{a}^L \le X \le \breve{a}^\alpha \\ 1, \; if \; \breve{a}^\alpha \le X \le \breve{a}^\beta \qquad \breve{a}^L < \breve{a}^\alpha < \breve{a}^\beta < \breve{a}^U \\ \dfrac{\breve{a}^U - X}{\breve{a}^U - \breve{a}^\beta}, \; if \; \breve{a}^\beta \le X \le \breve{a}^U \end{cases}$$



Definition 1.2. A fuzzy number $\breve{A} = (\breve{a}^L, \breve{a}^\alpha, \breve{a}^\beta, \breve{a}^U, \omega)$ is said to be generalized trapezoidal fuzzy number with membership grade $\mu_{\breve{A}}(x)$ then the following membership functions as,

$$\mu_{\breve{A}}(x) = \begin{cases} \omega\left(\dfrac{X - \breve{a}^L}{\breve{a}^\alpha - \breve{a}^L}\right), \text{ if } \breve{a}^L \le X \le \breve{a}^\alpha \\ \omega = 1, \textit{ if } \breve{a}^\alpha \le X \le \breve{a}^\beta \qquad \breve{a}^L < \breve{a}^\alpha < \breve{a}^\beta < \breve{a}^U \\ \omega\left(\dfrac{\breve{a}^U - X}{\breve{a}^U - \breve{a}^\beta}\right), \textit{ if } \breve{a}^\beta \le X \le \breve{a}^U \end{cases}$$

Where $\omega \in (0, 1]$.



α-Cut of Generalized Trapezoidal Fuzzy Number

1.3. Arithmetic operations Generalized Trapezoidal Fuzzy Number:

Let $\breve{P} = (\breve{a}^L, \breve{a}^\alpha, \breve{a}^\beta, \breve{a}^U, \omega)$ and $\breve{Q} = (\breve{b}^L, \breve{b}^\alpha, \breve{b}^\beta, \breve{b}^U, \omega)$ are two generalized trapezoidal fuzzy number then the following operations are,

1. $\breve{P} + \breve{Q} = (\breve{a}^L + \breve{b}^L, \breve{a}^\alpha + \breve{b}^\alpha, \breve{a}^\beta + \breve{b}^\beta, \breve{a}^U + \breve{b}^U, \omega)$ where $\omega = (\min\ (\omega_1, \omega_2))$

2. $\breve{P} - \breve{Q} = (\breve{a}^L - \breve{b}^U, \breve{a}^\alpha - \breve{b}^\beta, \breve{a}^\beta - \breve{b}^\alpha, \breve{a}^U - \breve{b}^L, \omega)$ where $\omega = (\min\ (\omega_1, \omega_2))$.

2. α-Cut Generalized Trapezoidal Fuzzy Number

Let $\breve{P} = (\breve{a}^L, \breve{a}^\alpha, \breve{a}^\beta, \breve{a}^U, \omega)$ and $\breve{Q} = (\breve{b}^L, \breve{b}^\alpha, \breve{b}^\beta, \breve{b}^U, \omega)$ are two generalized trapezoidal fuzzy numbers and $*$ be any arithmetic operators, $\breve{P} * \breve{Q}$ define its α-cut, $\alpha_{(\breve{P} * \breve{Q})} = \alpha_{\breve{P}} * \alpha_{\breve{Q}}$. Let $\breve{P} * \breve{Q}$ can be formed as

$\breve{P} * \breve{Q} = \bigcup_{\omega \in [0, 1]} \alpha^{(\breve{P} * \breve{Q})}$. $\breve{P}$ and $\breve{Q}$ are fuzzy numbers, $\breve{P} * \breve{Q}$ is also fuzzy numbers.

Consider $\breve{P}$ and $\breve{Q}$ as two generalized fuzzy numbers and then the following membership functions are,

$$\mu_{\breve{P}}(x) = \begin{cases} \omega\left(\dfrac{X - \breve{a}^L}{\breve{a}^\alpha - \breve{a}^L}\right), & if\ \breve{a}^L \le X \le \breve{a}^\alpha \\ \omega = 1, & if\ \breve{a}^\alpha \le X \le \breve{a}^\beta \\ \omega\left(\dfrac{\breve{a}^U - X}{\breve{a}^U - \breve{a}^\beta}\right), & if\ \breve{a}^\beta \le X \le \breve{a}^U \end{cases}$$

$$\mu_{\breve{Q}}(x) = \begin{cases} \left(\dfrac{X - \breve{b}^L}{\breve{b}^\alpha - \breve{b}^L}\right), & if\ \breve{b}^L \le X \le \breve{b}^\alpha \\ \omega = 1, & if\ \breve{b}^\alpha \le X \le \breve{b}^\beta \\ \omega\left(\dfrac{\breve{a}^U - X}{\breve{a}^U - \breve{a}^\beta}\right), & if\ \breve{b}^\beta \le X \le \breve{b}^U \end{cases}.$$

Generalized α-Cut trapezoidal fuzzy numbers are defined as,

$$\alpha_{\breve{P}} = [\breve{a}^L + (\breve{a}^\alpha - \breve{a}^L)\alpha, -(\breve{a}^U - \breve{a}^\beta) + \breve{a}^U]$$

$$\alpha_{\breve{Q}} = [\breve{b}^L + (\breve{b}^\alpha - \breve{b}^L)\alpha, -(\breve{b}^U - \breve{b}^\beta) + \breve{b}^U].$$

Let us assume that if $\alpha = \omega$ Then, the following generalized α-Cut trapezoidal fuzzy numbers are formed as,

$$\omega_{\breve{P}} = [\breve{a}^L + (\breve{a}^\alpha - \breve{a}^L)\omega, -(\breve{a}^U - \breve{a}^\beta) + \breve{a}^U]$$

$$\omega_{\breve{Q}} = [\breve{b}^L + (\breve{b}^\alpha - \breve{b}^L)\omega, -(\breve{b}^U - \breve{b}^\beta) + \breve{b}^U].$$

2.1.Generalized α-Cut fuzzy numbers to Fuzzy Interval

$$\omega_{\breve{c}_{11}} = [\breve{a}^L + (\breve{a}^\alpha - \breve{a}^L)\omega - (\breve{a}^U - \breve{a}^\beta) + \breve{a}^U] = [a_{11}^L\, d_{11}^U],$$

$$\omega_{\breve{c}_{12}} = [\breve{a}^L + (\breve{a}^\alpha - \breve{a}^L)\omega - (\breve{a}^U - \breve{a}^\beta) + \breve{a}^U] = [a_{12}^L\ d_{12}^U]$$

$$\omega_{\breve{c}_{13}} = [\breve{a}^{L} + (\breve{a}^{\alpha} - \breve{a}^{L})\omega - (\breve{a}^{U} - \breve{a}^{\beta}) + \breve{a}^{U}] = [a_{13}^{L}\, d_{13}^{U}],$$

$$\omega_{\breve{c}_{14}} = [\breve{a}^{L} + (\breve{a}^{\alpha} - \breve{a}^{L})\omega - (\breve{a}^{U} - \breve{a}^{\beta}) + \breve{a}^{U}] = [a_{14}^{L}\ d_{14}^{U}]$$

$$\omega_{\breve{c}_{21}} = [\breve{a}^{L} + (\breve{a}^{\alpha} - \breve{a}^{L})\omega - (\breve{a}^{U} - \breve{a}^{\beta}) + \breve{a}^{U}] = [a_{21}^{L}\, d_{21}^{U}],$$

$$\omega_{\breve{c}_{22}} = [\breve{a}^{L} + (\breve{a}^{\alpha} - \breve{a}^{L})\omega - (\breve{a}^{U} - \breve{a}^{\beta}) + \breve{a}^{U}] = [a_{22}^{L}\ d_{22}^{U}]$$

$$\omega_{\breve{c}_{23}} = [\breve{a}^{L} + (\breve{a}^{\alpha} - \breve{a}^{L})\omega - (\breve{a}^{U} - \breve{a}^{\beta}) + \breve{a}^{U}] = [a_{23}^{L}\, d_{23}^{U}],$$

$$\omega_{\breve{c}_{24}} = [\breve{a}^{L} + (\breve{a}^{\alpha} - \breve{a}^{L})\omega - (\breve{a}^{U} - \breve{a}^{\beta}) + \breve{a}^{U}] = [a_{24}^{L}\ d_{24}^{U}]$$

$$\omega_{\breve{c}_{31}} = [\breve{a}^{L} + (\breve{a}^{\alpha} - \breve{a}^{L})\omega - (\breve{a}^{U} - \breve{a}^{\beta}) + \breve{a}^{U}] = [a_{31}^{L}\, d_{31}^{U}],$$

$$\omega_{\breve{c}_{12}} = [\breve{a}^{L} + (\breve{a}^{\alpha} - \breve{a}^{L})\omega - (\breve{a}^{U} - \breve{a}^{\beta}) + \breve{a}^{U}] = [a_{32}^{L}\ d_{32}^{U}]$$

$$\omega_{\breve{c}_{33}} = [\breve{a}^{L} + (\breve{a}^{\alpha} - \breve{a}^{L})\omega - (\breve{a}^{U} - \breve{a}^{\beta}) + \breve{a}^{U}] = [a_{33}^{L}\, d_{33}^{U}],$$

$$\omega_{\breve{c}_{34}} = [\breve{a}^{L} + (\breve{a}^{\alpha} - \breve{a}^{L})\omega - (\breve{a}^{U} - \breve{a}^{\beta}) + \breve{a}^{U}] = [a_{34}^{L}\ d_{34}^{U}]$$

$$\omega_{\breve{c}_{41}} = [\breve{a}^{L} + (\breve{a}^{\alpha} - \breve{a}^{L})\omega - (\breve{a}^{U} - \breve{a}^{\beta}) + \breve{a}^{U}] = [a_{41}^{L}\, d_{41}^{U}],$$

$$\omega_{\breve{c}_{12}} = [\breve{a}^{L} + (\breve{a}^{\alpha} - \breve{a}^{L})\omega - (\breve{a}^{U} - \breve{a}^{\beta}) + \breve{a}^{U}] = [a_{42}^{L}\ d_{42}^{U}]$$

$$\omega_{\breve{c}_{43}} = [\breve{a}^{L} + (\breve{a}^{\alpha} - \breve{a}^{L})\omega - (\breve{a}^{U} - \breve{a}^{\beta}) + \breve{a}^{U}] = [a_{43}^{L}\, d_{43}^{U}],$$

$$\omega_{\breve{c}_{44}} = [\breve{a}^{L} + (\breve{a}^{\alpha} - \breve{a}^{L})\omega - (\breve{a}^{U} - \breve{a}^{\beta}) + \breve{a}^{U}] = [a_{44}^{L}\ d_{44}^{U}].$$

2.2. Interval-valued α-Cut of Generalized Fuzzy Linear Sum Bottleneck can be modelled as

$$Min \max_{1 \leq i,\ j \geq n} \omega_{\breve{c}_{ij}} x_{ij}$$

Such that

$$\sum_{j=1}^{n} x_{ij} = 1 \ (i = 1, 2, \ldots, n)$$

$$\sum_{j=1}^{n} x_{ij} = 1 \ (j = 1, 2, \ldots, n)$$

$$x_{ij} \in \{0, 1\} \ (i, j = 1, 2, \ldots, n).$$

Definition 2.3. An interval value $\omega_{\tilde{c}_{ij}} = [a_{ij}^{L} d_{ij}^{U}] \in R$ is said to be interval value fuzzy set with membership grade $\mu_{\tilde{c}_{\omega}}(x)$ then the following membership functions as,

$$\mu_{\tilde{C}_{\omega}}(x) = \begin{cases} 0, \ X < a_{ij}^{L} \\ 1, \ a_{ij}^{L} < X < d_{ij}^{U} \\ 0, \ X > d_{ij}^{u} \end{cases}$$

Where $a_{ij}^{L} < d_{ij}^{U}$.

3. Fuzzy Interval Operations

Let $X = [a_{11}^{L} \ d_{11}^{U}]$ and $Y = [a_{22}^{L} \ d_{22}^{U}]$ are two closed interval values in R then the following operations are as,

(a) $X + Y = [a_{11}^{L} \ d_{11}^{U}] + [a_{22}^{L} \ d_{22}^{U}] = [a_{11}^{L} + a_{22}^{L} \ d_{11}^{U} + d_{22}^{U}]$

(b) $X - Y = [a_{11}^{L} \ d_{11}^{U}] - [a_{22}^{L} \ d_{22}^{U}] = [a_{11}^{L} - d_{22}^{L} \ d_{11}^{U} - a_{22}^{U}]$

(c) $X \cdot Y = (a_{11}^{L} \ a_{22}^{L}, a_{11}^{L} d_{22}^{U}, d_{11}^{U} a_{22}^{L}, d_{11}^{U} a_{22}^{L}, d_{11}^{U} d_{22}^{U}) \max \ (a_{11}^{L} a_{22}^{L}, a_{11}^{L} a_{22}^{U}, d_{11}^{U} a_{22}^{L}, d_{11}^{U} d_{22}^{U})$

(d) $X/Y = \left[\min \left(\frac{a_{11}^{L}}{d_{22}^{U}}, \frac{a_{11}^{L}}{a_{22}^{L}}, \frac{d_{11}^{U}}{d_{22}^{U}}, \frac{d_{11}^{U}}{a_{22}^{L}}\right) \max \left(\frac{a_{11}^{L}}{d_{22}^{U}}, \frac{a_{11}^{L}}{a_{22}^{L}}, \frac{d_{11}^{U}}{d_{22}^{U}}, \frac{d_{11}^{U}}{a_{22}^{L}}\right)\right].$

(e) $X \wedge Y = [a_{11}^{L} d_{11}^{U}] \wedge [a_{22}^{L} d_{22}^{U}] = [a_{11}^{L} \wedge a_{22}^{L} d_{11}^{U} \wedge d_{22}^{U}]$

(d) $X \vee Y = [a_{11}^{L} d_{11}^{U}] \vee [a_{22}^{L} d_{22}^{U}] = [a_{11}^{L} \vee a_{22}^{L} d_{11}^{U} \vee d_{22}^{U}].$

3.1. Interval-valued α-Cut of Fuzzy Linear Sum Bottleneck

Assignment Table:

$[a_{11}^L \; d_{11}^U]$	$[a_{12}^L \; d_{12}^U]$	$[a_{13}^L \; d_{13}^U]$	$[a_{14}^L \; d_{14}^U]$
$[a_{21}^L \; d_{21}^U]$	$[a_{22}^L \; d_{22}^U]$	$[a_{23}^L \; d_{23}^U]$	$[a_{24}^L \; d_{24}^U]$
$[a_{31}^L \; d_{31}^U]$	$[a_{32}^L \; d_{32}^U]$	$[a_{33}^L \; d_{33}^U]$	$[a_{34}^L \; d_{34}^U]$
$[a_{41}^L \; d_{41}^U]$	$[a_{42}^L \; d_{42}^U]$	$[a_{43}^L \; d_{43}^U]$	$[a_{44}^L \; d_{44}^U]$

3.2. α-Cut of threshold Fuzzy Linear Sum Bottleneck Assignment

In the first case α-Cut of threshold Fuzzy Linear Sum Bottleneck Assignment cost element is $(\omega_{\tilde{c}_{ij}{}_M^*})$ and α-Cut of threshold Fuzzy Linear Sum Bottleneck Assignment are defined as,

$$\omega_{\tilde{c}_{ij}} = \begin{cases} 1, \; if \;\; \omega_{\tilde{c}_{ij}} > \omega_{\tilde{c}_{ij}{}_M^*} \\ 0, \; \text{otherwise.} \end{cases}$$

Let $C = \omega_{\tilde{c}_{ij}}$ be $n \times n$ matrix and $\omega_{\tilde{c}_{ij}}{}^U_{\varphi(i)n}$ be a fuzzy arbitrary permutation of IFLSBAP.

Spreading solution is $sp(\varphi(i)) = \max \;\{\min \;\{\omega_{\tilde{c}_{\varphi(i)n}}\}$.

3.3. Property (i): If two elements of IFLSBAP are in increasing order, then prove that the sum of two elements of IFLSBAP is also in Increasing Order,

Proof. Let $X = [a_{11}^L \, d_{11}^U]$ and $Y = [a_{22}^L \, d_{22}^U]$ are two closed interval values in R is IFLSBAP.

Here, $a_{ij}^L < d_{ij}^U$ and $a_{22}^L < d_{22}^U$ are in increasing orders.

We prove that, the sum of two elements of IFLSBAP is also in increasing order, adding X and Y. We get,

$$X + Y = [a_{11}^L \, d_{11}^U] + [a_{22}^L \, d_{22}^U] = [a_{11}^L + a_{22}^L \; d_{11}^U + d_{22}^U].$$

We see that, Here, $a_{11}^L + a_{22}^L < d_{11}^U + d_{22}^U$ and $a_{ij}^L < d_{ij}^U$ and $a_{22}^L < d_{22}^U$.

Therefore, $[a_{11}^{L}\, d_{11}^{U}] + [a_{22}^{L}\, d_{22}^{U}] < [a_{11}^{L} + a_{22}^{L}\, d_{11}^{U} + d_{22}^{U}]$. Hence, If two elements of IFLSBAP are in increasing order then the sum of two elements of IFLSBAP are also in increasing order.

4. Algorithm

Solving optimal perfect matching and feasible partial matching by using generalized α-cut trapezoidal fuzzy numbers we present in the following step by step procedure.

Step 1. Generalized α-cut trapezoidal fuzzy numbers

Let us take generalized trapezoidal fuzzy number and obtain α-cut of trapezoidal fuzzy numbers, If $\alpha = \omega$, then the following form.

$$\omega_{\breve{A}} = [\breve{a}^{L} + (\breve{a}^{\alpha} - \breve{a}^{L})\omega - (\breve{a}^{U} - \breve{a}^{\beta}) + \breve{a}^{U}],$$

$$\omega_{\breve{B}} = [\breve{b}^{L} + (\breve{b}^{\alpha} - \breve{b}^{L})\omega - (\breve{b}^{U} - \breve{b}^{\beta}) + \breve{b}^{U}].$$

Step 2. compute fuzzy interval values by using generalized α-cut trapezoidal fuzzy numbers:

$$\omega_{\breve{C}_{ij}} = [\breve{a}^{L} + (\breve{a}^{\alpha} - \breve{a}^{L})\omega - (\breve{a}^{U} - \breve{a}^{\beta}) + \breve{a}^{U}] = [a_{ij}^{L}\, d_{ij}^{U}].$$

Where $a_{ij}^{L} < d_{ij}^{U}$ and $a_{ij}^{L} =$ lower boundary of least value

$d_{ij}^{U} =$ upper boundary of largest value

Step 3. Forming balanced interval valued fuzzy linear sum bottleneck assignment problem (IFLSBAP):

Let us consider balanced interval valued fuzzy linear sum bottleneck assignment problem (IFLSBAP) (number of Machines (M) and number of Jobs (P) are equal i.e., $\sum_{i=1}^{n} M_i = \sum_{j=1}^{n} P_j$), if IFLSBAP $\sum_{i=1}^{n} M_i \neq \sum_{j=1}^{n} P_j$, We Introduce dummy row i.e., $\sum_{i=1}^{n} M_i + D_i$ (or), introduce dummy column $\sum_{j=1}^{n} P_j + D_j$ (where $D_i =$ dummy row, $D_j =$

dummy column)

Step 4: Calculate $\omega_{\tilde{c}_{ij}}{}_0^L$, $\omega_{\tilde{c}_{ij}}{}_n^U$.

Let $\omega_{\tilde{c}_{ij}} = [a_{ij}^L d_{ij}^U]$ be $(n \times n)$ interval cost/time matrix;

$$\omega_{\tilde{c}_{ij}}{}_0^L = \min_{ij} \{a_{ij}^L d_{ij}^U\},\ \omega_{\tilde{c}_{ij}}{}_n^U = \max_{ij} \{a_{ij}^L d_{ij}^U\}.$$

Step 5: Calculate $(\omega_{\tilde{c}_{ij}}{}^*)$, $(\omega_{\tilde{c}_{ij}}{}_M^*)$

$$\omega_{\tilde{c}_{ij}}{}^* = \{\omega_{\tilde{c}_{ij}} : \omega_{\tilde{c}_{ij}}{}_0^L < \omega_{\tilde{c}_{ij}} < \omega_{\tilde{c}_{ij}}{}_n^U$$

$$\omega_{\tilde{c}_{ij}}{}_M^* = \min \{\omega_{\tilde{c}_{ij}} \in \omega_{\tilde{c}_{ij}}{}^* : |\{\omega_{\tilde{c}_{ij}} \in \omega_{\tilde{c}_{ij}}{}^* : \omega_{\tilde{c}_{ij}} \le [a_{ij}^L d_{ij}^U]\} \ge \left| \omega_{\tilde{c}_{ij}}{}^* \right| / 2.$$

Step 6: Feasibility check: Select the feasible element $\omega_{\tilde{c}_{ij}}{}_M^*$, $(\omega_{\tilde{c}_{ij}}{}_0^L < \omega_{\tilde{c}_{ij}}{}_1^L, \omega_{\tilde{c}_{ij}}{}_2^L \ldots \omega_{\tilde{c}_{ij}}{}_M^* = 0)$, select the $(\omega_{\tilde{c}_{ij}})$ row and assigned at least only one zero, similarly column allocations are $\omega_{\tilde{c}_{2j}}, \omega_{\tilde{c}_{3j}} \ldots, \omega_{\tilde{c}_{nj}}$, next column assigned as $\omega_{\tilde{c}_{i1}}, \omega_{\tilde{c}_{i2}} \ldots, \omega_{\tilde{c}_{in}}$. Each row and each column at least only one zero are assigned. Every row and column has at least one matching, the feasibility is executed. The bipartite matching, if minimum cost and maximum matching is optimal and perfect, otherwise the bipartite graph is feasible and partial matching. Obtain interval valued fuzzy linear sum cost is $\sum_{i=1}^n \omega_{\tilde{c}}{}^*_{i_{\varphi(i)M}} = [a_{i\varphi(i)M^L} d_{i\varphi(i)M^U}]$.

Step 7: Backward calculation: Select the lower feasible elements of $\omega_{\tilde{c}_{ij}}{}_M^*$ and determine Feasible IFLSBAP

$\omega_{\tilde{c}_{ij}}{}_0^L = 0$, the feasible cost/time is $\sum_{i=1}^n \omega_{\tilde{c}}{}^L_{i_{\varphi(i)0}} = [a_{i\varphi(i)0^L} d_{i\varphi(i)0^U}]$.

$\omega_{\tilde{c}_{ij}}{}_1^L = 0$, the feasible cost/time is $\sum_{i=1}^n \omega_{\tilde{c}}{}^L_{i_{\varphi(i)1}} = [a_{i\varphi(i)1^L} d_{i\varphi(i)1^U}]$.

$\omega_{\tilde{c}_{ij}}{}_2^L = 0$, the feasible cost/time is $\sum_{i=1}^n \omega_{\tilde{c}}{}^L_{i_{\varphi(i)2}} = [a_{i\varphi(i)2^L} d_{i\varphi(i)2^U}]$.

$\omega_{\tilde{c}_{ij}}{}_{M-1}^{L} = 0$, the feasible cost/time is $\sum_{i=1}^{n} \omega_{\tilde{c}}{}_{i_{\varphi(i)}m-1}^{L} = [a_{i\varphi(i)m-1^L} d_{i\varphi(i)m-1^U}]$.

Step 8. Forward calculation: Select the upper feasible elements of $\omega_{\tilde{c}_{ij}}{}_{M}^{*}$ and determine Feasible IFLSBAP

$\omega_{\tilde{c}_{ij}}{}_{m+1}^{U} = 0$, the feasible cost/time is $\sum_{i=1}^{n} \omega_{\tilde{c}}{}_{i_{\varphi(i)}m+1}^{U}$ $= [a_{i\varphi(i)m+1^L} d_{i\varphi(i)m\mp1^U}]$.

$\omega_{\tilde{c}_{ij}}{}_{m+2}^{U} = 0$, the feasible cost/time is $\sum_{i=1}^{n} \omega_{\tilde{c}}{}_{i_{\varphi(i)}m+2}^{U} = [a_{i\varphi(i)m+2^L} d_{i\varphi(i)m+2^U}]$.

$\omega_{\tilde{c}_{ij}}{}_{m+3}^{U} = 0$, the feasible cost/time is $\sum_{i=1}^{n} \omega_{\tilde{c}}{}_{i_{\varphi(i)}m+3}^{U} = [a_{i\varphi(i)m+3^L} d_{i\varphi(i)m+3^U}]$.

$\omega_{\tilde{c}_{ij}}{}_{n}^{U} = 0$, the feasible cost/time is $\sum_{i=1}^{n} \omega_{\tilde{c}}{}_{i_{\varphi(i)n}}^{U} = [a_{i\varphi(i)n^L} d_{i\varphi(i)n^U}]$.

Step 9. Determine and checking Feasible/Optimal and Partial/Perfect of IFLSBAP.

$$\omega_{\tilde{c}_{ij}}{}_{\delta}^{L} = [a_{i\varphi(i)\delta^L} d_{i\varphi(i)\delta^L}] = \min\ \{\omega_{\tilde{c}_{ij}}{}_{0}^{L}, \omega_{\tilde{c}_{ij}}{}_{1}^{L}, \ldots, \omega_{\tilde{c}_{ij}}{}_{M}^{*} \ldots \omega_{\tilde{c}_{ij}}{}_{m+2}^{U}, \omega_{\tilde{c}_{ij}}{}_{m+3}^{U} \ldots \omega_{\tilde{c}_{ij}}{}_{n}^{U}\}..$$

If, $\omega_{\tilde{c}_{ij}}{}_{\delta}^{L} = [a_{i\varphi(i)\delta^L} d_{i\varphi(i)\delta^L}] \leq [a_{i\varphi(i)n^L} d_{i\varphi(i)n^U}]$ is optimal and perfect matching.

If, $\omega_{\tilde{c}_{ij}}{}_{\delta}^{L} = [a_{i\varphi(i)\delta^L} d_{i\varphi(i)\delta^*}] < [a_{i\varphi(i)n^L} d_{i\varphi(i)n^U}]$ is feasible and partial matching.

If, $\omega_{\tilde{c}_{ij}}{}_{\delta}^{U} = [a_{i\varphi(i)\delta^U} d_{i\varphi(i)\delta^U}] \leq [a_{i\varphi(i)n^L} d_{i\varphi(i)n^U}]$ is feasible and perfect matching.

If, $\omega_{\breve{c}_{ij}}{}_{\delta}^{U} = [a_{i\varphi(i)\delta^U} d_{i\varphi(i)\delta^U}] < [a_{i\varphi(i)n^L} d_{i\varphi(i)n^U}]$ is feasible and partial matching.

Step 10: Stop.

Example: Consider generalized trapezoidal fuzzy numbers $(\breve{C}_{ij})$

(9,13,17,21;0.25)	(15,20,25,30;0.20)	(4, 6, 8, 10; 0.50)	(3, 5, 7, 9; 0.50)
(5,7,9,11;0.50)	(8, 10, 12, 14; 0.50)	(4, 6, 8, 10; 0.50)	(9,13,17,21;0.25)
(2,4,6,8;0.50)	(9, 13, 17, 21; 0.25)	(13,18,23,28;0.20)	(5, 7, 9, 11; 0.50)
(3,5,7,9;0.50)	(6, 8, 10, 12; 0.50)	(9, 13, 17, 21; 0.25)	(4, 6, 8, 10; 0.50)

Generalized α-cut trapezoidal fuzzy numbers is

$$\omega_{\breve{c}_{ij}} = [\breve{a}^{L} + (\breve{a}^{\alpha} - \breve{a}^{L})\omega - (\breve{a}^{U} - \breve{a}^{\beta}) + \breve{a}^{U}] = [a_{ij}^{L} d_{ij}^{U}].$$

If $\alpha = \omega$, compute fuzzy interval values by using generalized α-cut trapezoidal fuzzy numbers:

$\omega_{\breve{c}_{11}} = [10, 20], \omega_{\breve{c}_{12}} = [16, 29], \omega_{\breve{c}_{13}} = [5, 9], \omega_{\breve{c}_{14}} = [4, 8], \omega_{\breve{c}_{21}} = [6, 10], \omega_{\breve{c}_{22}} = [9, 12],$

$\omega_{\breve{c}_{23}} = [5, 9], \omega_{\breve{c}\,24} = [10, 20], \omega_{\breve{c}_{31}} = [3, 1], \omega_{\breve{c}_{32}} = [10, 20], \omega_{\breve{c}_{33}} = [14, 27], \omega_{\breve{c}_{34}} = [6, 1],$

$\omega_{\breve{c}_{41}} = [4, 8], \omega_{\breve{c}_{42}} = [7, 11], \omega_{\breve{c}_{43}} = [10, 20], \omega_{\breve{c}_{44}} = [5, 9].$

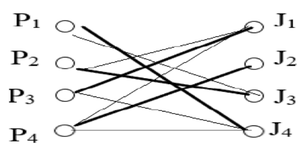
Interval-valued fuzzy linear sum bottleneck assignment problem by using Generalized α-cut trapezoidal fuzzy numbers

Table 2

[10 20]	[16 29]	[5 9]	[4 8]
[6 10]	[9 13]	[5 9]	[10 20]
[3 7]	[10 20]	[14 27]	[6 10]
[4 8]	[7 11]	[10 20]	[5 9]

Case i. $\omega_{\breve{c}_{ij}^{*}} = \omega_{\breve{c}_{ij}^{*}} = \{\omega_{\breve{c}_{ij}} : \omega_{\breve{c}_{ij}}{}_{0}^{L} < \omega_{\breve{c}_{ij}} < \omega_{\breve{c}_{ij}}{}_{n}^{U} = [7, 11]$

Table 3

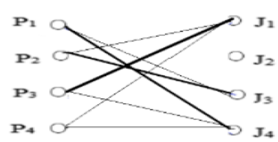


[10 20]	[16 29]	0	0
0	[9 13]	0	[10 20]
0	[10 20]	[14 27]	0
0	0	[10 20]	0

$\sum_{i=1}^{n} \omega_{\tilde{c}^*_{i\varphi(i)M}} = [a_{i\varphi(i)M}{}^{L}\ d_{i\varphi(i)M}{}^{U}] = [19, 35]$. The IFLSAP is optimal and perfect matching

Case ii: $\omega_{\tilde{c}_{ij}}{}^{*}_{M-1} = \omega_{\tilde{c}_{ij}^{L}} = \{\omega_{\tilde{c}_{ij}} : \omega_{\tilde{c}_{ij}}{}^{L}_{0} < \omega_{\tilde{c}_{ij}} < \omega_{\tilde{c}_{ij}}{}^{U}_{n} = [6, 10]$.

Table 4

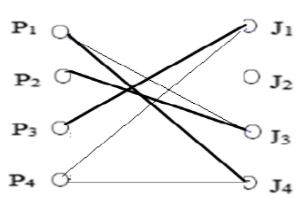


[10 20]	[16 29]	0	**0**
0	[9 13]	**0**	[10 20]
0	[[10 20]	[14 27]	0
0	[7 11]	[10 20]	0

Now $[a_{i\varphi(i)M-1}{}^{L}\ d_{i\varphi(i)M-1}{}^{U}] = [12, 24]$. The IFLSAP is feasible and partial.

Case iii: $\omega_{\tilde{c}_{ij}}{}^{*}_{M-2} = \omega_{\tilde{c}_{ij}^{L}} = \{\omega_{\tilde{c}_{ij}} : \omega_{\tilde{c}_{ij}}{}^{L}_{0} < \omega_{\tilde{c}_{ij}} < \omega_{\tilde{c}_{ij}}{}^{U}_{n} = [5, 9]$.

Table 5

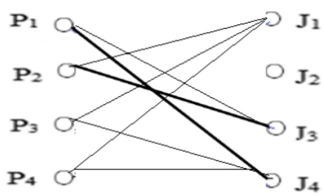


[10 20]	[16 29]	0	**0**
[6 10]	[9 12]	**0**	[10 20]
0	[[10 20]	[14 27]	[6 10]
0	[7 11]	[10 20]	0

Now $\sum_{i=1}^{n} \omega_{\tilde{c}^*_{i\varphi(i)M-2}} = [a_{i\varphi(i)M-2}{}^{L}\ d_{i\varphi(i)M-2}{}^{U}] = [12, 24]$. The IFLSAP is feasible and partial.

Case iv: $\omega_{\tilde{c}_{ij}}{}^{L}_{M-3} = \omega_{\tilde{c}^{L}_{ij}} = \{\omega_{\tilde{c}_{ij}} : \omega_{\tilde{c}_{ij}}{}^{L}_{0} < \omega_{\tilde{c}_{ij}} < \omega_{\tilde{c}_{ij}}{}^{U}_{n} = [4, 8].$

Table 6.

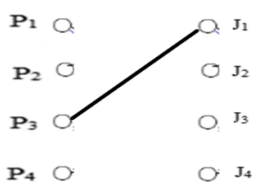


[10 20]	[16 29]	0	**0**
0	[9 12]	**0**	[10 20]
0	[[10 20]	[14 27]	0
0	[7 11]	[10 20]	0

Now $\sum_{i=1}^{n} \omega_{\tilde{c}^{*}_{i\varphi(i)M-3}} = [a_{i\varphi(i)M-2^{L}}\, d_{i\varphi(i)M-2^{U}}] = [7, 15].$ The IFLSAP is feasible and partial.

Case v: $\omega_{\tilde{c}_{ij}}{}^{L}_{M-4} = \omega_{\tilde{c}^{L}_{ij}} = \{\omega_{\tilde{c}_{ij}} : \omega_{\tilde{c}_{ij}}{}^{L}_{0} < \omega_{\tilde{c}_{ij}} < \omega_{\tilde{c}_{ij}}{}^{U}_{n} = [3, 7].$

Table 7

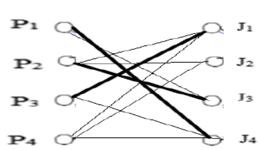


[10 20]	[16 29]	[5 9]	[4 8]
[6 10]	[9 12]	[5 9]	[10 20]
0	[[10 20]	[14 27]	[6 10]
[4 8]	[7 11]	[10 20]	[5 9]

Now $\sum_{i=1}^{n} \omega_{\tilde{c}^{*}_{i\varphi(i)M-4}} = [a_{i\varphi(i)M-4^{L}}\, d_{i\varphi(i)M-4^{U}}] = [3, 7].$ The IFLSAP is feasible and partial.

Case vi: $\omega_{\tilde{c}_{ij}}{}^{L}_{M+1} = \omega_{\tilde{c}^{L}_{ij}} = \{\omega_{\tilde{c}_{ij}} : \omega_{\tilde{c}_{ij}}{}^{L}_{0} < \omega_{\tilde{c}_{ij}} < \omega_{\tilde{c}_{ij}}{}^{U}_{n} = [9, 13].$

Table 8

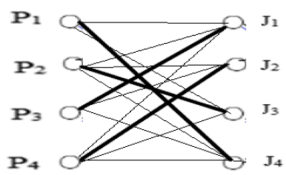


[10 20]	[16 29]	0	**0**
0	0	**0**	[10 20]
0	[[10 20]	[14 27]	0
0	0	[10 20]	0

Now $\sum_{i=1}^{n} \omega_{\tilde{c}^{*}_{i\varphi(i)M+1}} = [a_{i\varphi(i)M+1^{L}}\, d_{i\varphi(i)M+1^{U}}] = [12, 24].$ The IFLSAP is feasible and perfect.

Case vii: $\omega_{\tilde{c}_{ij}}{}^{L}_{M+2} = \omega_{\tilde{c}^{L}_{ij}} = \{\omega_{\tilde{c}_{ij}} : \omega_{\tilde{c}_{ij}}{}^{L}_{0} < \omega_{\tilde{c}_{ij}} < \omega_{\tilde{c}_{ij}}{}^{U}_{n} = [10, 20]$.

Table 9

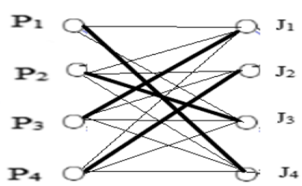


0	[16 29]	0	**0**
0	0	**0**	0
0	0	[14 27]	0
0	**0**	0	0

$\sum_{i=1}^{n} \omega_{\tilde{c}^{*}_{i\varphi(i)M+2}} = [a_{i\varphi(i)M+2^{L}}\, d_{i\varphi(i)M+2^{U}}] = [19, 35]$. The IFLSAP is optimal and perfect.

Case viii: $\omega_{\tilde{c}_{ij}}{}^{L}_{M+3} = \omega_{\tilde{c}^{L}_{ij}} = \{\omega_{\tilde{c}_{ij}} : \omega_{\tilde{c}_{ij}}{}^{L}_{0} < \omega_{\tilde{c}_{ij}} < \omega_{\tilde{c}_{ij}}{}^{U}_{n} = [14, 27]$.

Table 10

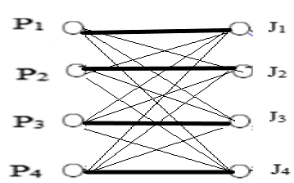


0	[16 29]	0	**0**
0	0	**0**	0
0	0	0	0
0	**0**	0	0

Now $\sum_{i=1}^{n} \omega_{\tilde{c}^{*}_{i\varphi(i)M+3}} = [a_{i\varphi(i)M+3^{L}}\, d_{i\varphi(i)M+3^{U}}] = [19, 35]$. The IFLSAP is optimal and perfect.

Case ix: $\omega_{\tilde{c}_{ij}}{}^{L}_{M+4} = \omega_{\tilde{c}^{L}_{ij}} = \{\omega_{\tilde{c}_{ij}} : \omega_{\tilde{c}_{ij}}{}^{L}_{0} < \omega_{\tilde{c}_{ij}} < \omega_{\tilde{c}_{ij}}{}^{U}_{n} = [16, 29]$.

Table 11



0	0	0	0
0	**0**	0	0
0	0	**0**	0
0	0	0	**0**

Now $\sum_{i=1}^{n} \omega_{\breve{c}^*_{i\varphi(i)M+4}} = [a_{i\varphi(i)M+4^L}\ d_{i\varphi(i)M+4^U}] = [38,\ 69]$. the IFLSAP is feasible and perfect.

The optimal perfect schedule is $P_1 \to J_4,\ P_2 \to J_3,\ P_3 \to J_1,\ P_4 \to J_2$.

The spread of new generalized trapezoidal fuzzy optimal perfect assignment cost is $\sum \breve{C}_{i\varphi(i)} = (3, 5, 7, 9, 0.50) + (4, 6, 8, 10; 0.50) + (2, 4, 6, 8; 0.50) + (6, 8, 10, 12; 0.50) = (15, 23, 33, 39; 0.50)$.

6. Conclusion

We discussed above concepts of optimal perfect matching and partial feasible matching for solving interval valued fuzzy linear sum bottleneck assignment problem by using α-cut of generalized trapezoidal fuzzy number. The machine completed maximum jobs with minimum cost or time then the solution is optimal and perfect, otherwise partial and feasible.

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Solving Fuzzy Multi-objective Linear Sum Assignment Problem with Modified Partial Primal Solution of ω - type 2 - Diamond Fuzzy Numbers by Using Linguistic Variables

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Abstract

In this paper, we introduce ω-type 1 and ω-type 2-diamond fuzzy numbers and solving a fuzzy multi-objective linear sum assignment problem (FMOLSAP) with linguistic variables whose λ_d −cut are expressed as ω-type 2-diamond fuzzy numbers (ω_{t2}-DFN). To solve the FMOLSAP, we can apply arithmetic operations of λ_d −cut of ω-type 2-diamond fuzzy numbers (ω_{t2}-DFN) to compute complete optimal matching.

Keywords: ω -type 1 and ω -type 2-diamond fuzzy numbers, Fuzzy assignment problem,

AMS Mathematics Subject Classification (2010): 90B80, 90C29, 03E72

1. Introduction

In 2010, Kagade and Bajaj[6] discussed for solving fuzzy multi-objective assignment problem with interval cost. Isabel et.al [5] an application of linguistic variables in fuzzy assignment problem, In 2014, Gupta and Mehlawat[3] For treating the fuzzy multi-objective assignment problems, a novel possibility programming technique was provided, Kayvan Salehi[7] proposed an approach for solving MOAP with interval parameters. In 2015, Pathinathan and Ponnivalavan [11] discussed diamond fuzzy number. In 2017, Nagoor Gani and Shiek Pareeth [9] are discussed dual variables and partial solution for solving FLSAP. In recently, many of the researchers work in this area of fuzzy multi-objective assignment problems like [1-4,8,10,12, 13-15].

We introduced ω-type 1-diamond fuzzy numbers and ω-type2-diamond fuzzy numbers are discussed in this paper. The upper and lower membership functions of diamond fuzzy numbers are described as ω-type 1 and ω-type 2-diamond fuzzy numbers. In λ_d- cut form, express the ω-type2 -diamond fuzzy numbers. Single fuzzy linear sum assignment problems are converted from fuzzy multi-objective linear sum assignment problems by using ranking method. obtain partial feasible solution and complete optimal solution by using λ_d −cut of ω-type 2-diamond fuzzy numbers (ω_{t2}-DFN) . Arithmetic operations of λ_d −cut of ω-type 2-diamond fuzzy numbers (ω_{t2}-DFN) to obtain complete optimal matching

2. Preliminaries

2.1. Definition: [11]A fuzzy set F is defined as $\widetilde{F_d} = \{d', d^*, d''\ (\alpha_d, \beta_d)\}$is called diamond fuzzy number and it's the following membership function is given by

$$\mu_{\widetilde{F_d}} = \begin{cases} 0 & for\ x \leq d' \\ \frac{(x-d')}{(d^*-d')} & for\ d' \leq x \leq d^* \\ \frac{(d''-x)}{(d''-d^*)} & for\ d^* \leq x \leq d'' \\ & \alpha_d \\ \frac{(d'-x)}{(d'-d^*)} & for\ d' \leq x \leq d^* \\ \frac{(x-d'')}{(d^*-d'')} & for\ d^* \leq x \leq d'' \\ 1 & x = \beta_d \\ 0 & otherwise \end{cases}$$

2.2.Definition: A ω-type1- diamond fuzzy number is upper and lower membership function of the diamond fuzzy number is defined as $[\underline{\omega_{t1}\widetilde{F_d}}, \overline{\omega_{t1}\widetilde{F_d}}]$ where $\underline{\omega_{t1}\widetilde{F_d}}$ $=\{\underline{d'}, d^*, \underline{d''}, (\underline{\alpha_d}, \underline{\beta_d}), \overline{\omega_{t1}\widetilde{F_d}} = \{\overline{d'}, d^*, \overline{d''}, (\overline{\alpha_d}, \overline{\beta_d})\}$ and it's the following membership function is given by

$$
\mu_{\omega_{t1}\widetilde{F_d}} = \begin{cases}
0 & otherwise \\
\underline{\omega}\left(\dfrac{(x-\underline{d'})}{(d^{*}-\underline{d'})}\right) & for\ \underline{d'} \le x \ \le d^{*} \\
\underline{\omega}\left(\dfrac{(\underline{d''}-x)}{(\underline{d''}-d^{*})}\right) & for\ d^{*} \le x \le \underline{d''} \\
\overline{\omega}\left(\dfrac{(x-\overline{\mathrm{d'}})}{(d^{*}-\overline{\mathrm{d'}})}\right) & for\ \overline{\mathrm{d'}} \le x \ \le d^{*} \\
\overline{\omega}\left(\dfrac{(\overline{d''}-x)}{(\overline{d''}-d^{*})}\right) & for\ d^{*} \le x \le \overline{d''} \\
\alpha_d \ - \ base & \\
\underline{\omega}\left(\dfrac{(\underline{d'}-\mathrm{x})}{(\underline{d'}-d^{*})}\right) & for\ \underline{d'} \le x \ \le d^{*} \\
\underline{\omega}\left(\dfrac{(x-\underline{d''})}{(d^{*}-\underline{d''})}\right) & for\ d^{*} \le x \le \underline{d''} \\
\overline{\omega}\left(\dfrac{(\overline{\mathrm{d'}}-\mathrm{x})}{(\overline{\mathrm{d'}}-d^{*})}\right) & for\ \overline{\mathrm{d'}} \le x \ \le d^{*} \\
\overline{\omega}\left(\dfrac{\left(x-\overline{d''}\right)}{\left(d^{*}-\overline{d''}\right)}\right) & for\ d^{*} \le x \le \overline{d''} \\
\omega = 1 & x = \beta_d \\
0 & otherwise
\end{cases}
$$

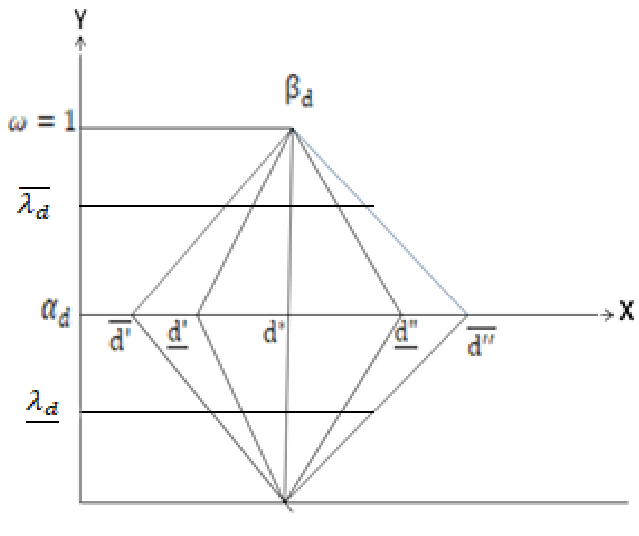

Figure 1: ω-type 1- diamond fuzzy number

2.3. Definition: A ω-type2 diamond fuzzy number is upper and lower membership function of the diamond fuzzy number is defined as $[\underline{\omega_{t2}\widetilde{F_d}}, \overline{\omega_{t2}\widetilde{F_d}}]$ where $\underline{\omega_{t2}\widetilde{F_d}}$

$=\{\underline{d'}, \underline{d^*}, \underline{d''}\ (\underline{\alpha_d}, \underline{\beta_d}),\ \overline{\omega_{t2}\widetilde{F_d}} = (\overline{d'}, \overline{d^*}, \overline{d''}\ (\overline{\alpha_d}, \overline{\beta_d}))$ and it's the following membership function is given by

$$\mu_{\omega_{t1}\widetilde{F_d}} = \begin{cases} 0 & otherwise \\ \underline{\omega}\left(\frac{(x-\underline{d'})}{(d^*-\underline{d'})}\right) & for\ \underline{d'} \le x \le \underline{d^*} \\ \underline{\omega}\left(\frac{(\underline{d''}-x)}{(\underline{d''}-d^*)}\right) & for\ \underline{d^*} \le x \le \underline{d''} \\ \overline{\omega}\left(\frac{(x-\overline{d'})}{(d^*-\overline{d'})}\right) & for\ \overline{d'} \le x \le d^* \\ \overline{\omega}\left(\frac{(\overline{d''}-x)}{(\overline{d''}-d^*)}\right) & for\ d^* \le x \le \overline{d''} \\ \alpha_d - base & \\ \underline{\omega}\left(\frac{(\underline{d'}-x)}{(\underline{d'}-d^*)}\right) & for\ \underline{d'} \le x \le d^* \\ \underline{\omega}\left(\frac{(x-\underline{d''})}{(d^*-\underline{d''})}\right) & for\ d^* \le x \le \underline{d''} \\ \overline{\omega}\left(\frac{(\overline{d'}-x)}{(\overline{d'}-d^*)}\right) & for\ \overline{d'} \le x \le d^* \\ \overline{\omega}\left(\frac{(x-\overline{d''})}{(d^*-\overline{d''})}\right) & for\ d^* \le x \le \overline{d''} \\ \omega = 1 & x = \beta_d \\ 0 & otherwise \end{cases}$$

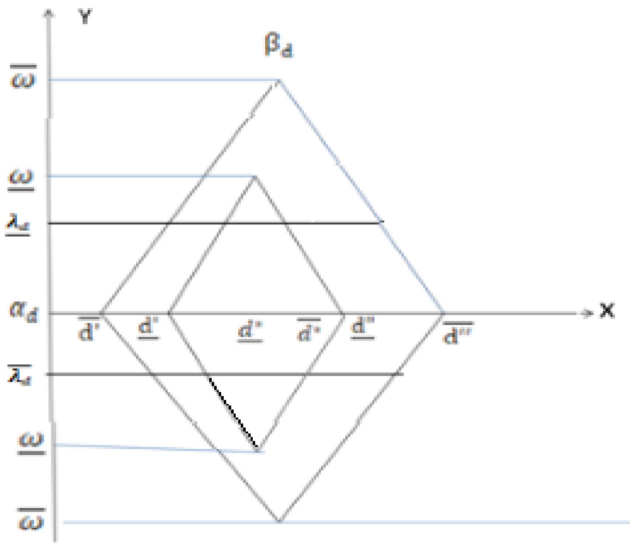

Figure 2: λ_d- cut of ω-type 2- diamond fuzzy number

3. Arithmetic Operations of ω-type2- Diamond Fuzzy Number ($\omega_{t2}\widetilde{F_d}$) :

Let us take two ω-type2 diamond fuzzy number are given below

$\omega_{t2}\widetilde{F_d}^1 = [\underline{\omega_{t2}\widetilde{F_d}^1}, \overline{\omega_{t2}\widetilde{F_d}^1}] = (\underline{d_1'}, \underline{d_2'}\underline{d_3'}\underline{d_4'}, \underline{d_5'}, \underline{d_6'}, \underline{\omega_{1F^1}}), (\overline{d_1'}, \overline{d_2'}, \overline{d_3'}, \overline{d_4'}, \overline{d_5'}, \overline{d_6'}, \overline{\omega_{1F^1}})$ and

$\omega_{t2}\widetilde{F_d}^2 = [\underline{\omega_{t2}\widetilde{F_d}^2}, \overline{\omega_{t2}\widetilde{F_d}^2}] = (\underline{d_1''}, \underline{d_2''}, \underline{d_3''}, \underline{d_4''}, \underline{d_5''}, \underline{d_6''}, \underline{\omega_{1F^2}}), (\overline{d_1''}, \overline{d_2''}, \overline{d_3''}, \overline{d_4''}, \overline{d_5''}, \overline{d_6''}, \overline{\omega_{1F^2}})$.

The following arithmetic operations of $\omega_{t2}\widetilde{F_d}^1$ and $\omega_{t2}\widetilde{F_d}^2$.

Addition

$\omega_{t2}\widetilde{F_d}^1 \oplus \omega_{t2}\widetilde{F_d}^2 =$

$((\underline{d_1'} \oplus \underline{d_1''}, \underline{d_2'} \oplus \underline{d_2''}, \underline{d_3'} \oplus \underline{d_3''}, \underline{d_4'} \oplus \underline{d_4''}, \underline{d_5'} \oplus \underline{d_5''}, \underline{d_6'} \oplus \underline{d_6''}), \min\{\underline{\omega_{1F^1}}, \underline{\omega_{1F^2}}\}),$

$((\overline{d_1'} \oplus \overline{d_1''}, \overline{d_2'} \oplus \overline{d_2''}, \overline{d_3'} \oplus \overline{d_3''}, \overline{d_4'} \oplus \overline{d_4''}, \overline{d_5'} \oplus \overline{d_5''}, \overline{d_6'} \oplus \overline{d_6''}), \min\{\overline{\omega_{1F^1}}, \overline{\omega_{1F^2}}\})$

Subtraction

$\omega_{t2}\widetilde{F_d}^1 \ominus \omega_{t2}\widetilde{F_d}^2 =$

$((\underline{d_1'} \ominus \underline{d_6''}, \underline{d_2'} \ominus \underline{d_5''}, \underline{d_3'} \ominus \underline{d_4''}, \underline{d_4'} \ominus \underline{d_3''}, \underline{d_5'} \ominus \underline{d_2''}, \underline{d_6'} \ominus \underline{d_6''}), \min\{\underline{\omega_{1F^1}}, \underline{\omega_{1F^2}}\}),$

$((\overline{d_1'} \ominus \overline{d_6''}, \overline{d_2'} \ominus \overline{d_5''}, \overline{d_3'} \ominus \overline{d_4''}, \overline{d_4'} \ominus \overline{d_3''}, \overline{d_5'} \ominus \overline{d_2''}, \overline{d_6'} \ominus \overline{d_1''}), \min\{\overline{\omega_{1F^1}}, \overline{\omega_{1F^2}}\}).$

Multiplication

$\omega_{t2}\widetilde{F_d}^1 \otimes \omega_{t2}\widetilde{F_d}^2 =$

$((\underline{d_1'} \otimes \underline{d_1''}, \underline{d_2'} \otimes \underline{d_2''}, \underline{d_3'} \otimes \underline{d_3''}, \underline{d_4'} \otimes \underline{d_4''}, \underline{d_5'} \otimes \underline{d_5''}, \underline{d_6'} \otimes \underline{d_6''}), \min\{\underline{\omega_{1F^1}}, \underline{\omega_{1F^2}}\}),$

$((\overline{d_1'} \otimes \overline{d_1''}, \overline{d_2'} \otimes \overline{d_2''}, \overline{d_3'} \otimes \overline{d_3''}, \overline{d_4'} \otimes \overline{d_4''}, \overline{d_5'} \otimes \overline{d_5''}, \overline{d_6'} \otimes \overline{d_6''}), \min\{\overline{\omega_{1F^1}}, \overline{\omega_{1F^2}}\}).$

Scalar multiplication

$\alpha_k(\omega_{t2}\widetilde{F_d}^1) =$

$((\alpha_k\underline{d_1'}, \alpha_k\underline{d_2'}, \alpha_k\underline{d_3'}, \alpha_k\overline{d_4'}, \alpha_k\underline{d_5'}, \alpha_k\underline{d_6'}, \underline{\omega_{1F^1}}),), (\alpha_k\overline{d_1'}, \alpha_k\overline{d_2'}, \alpha_k\overline{d_3'}, \alpha_k\overline{d_4'},$

$\alpha_k\overline{d_5'}, \alpha_k\overline{d_6'}, \overline{\omega_{1F^1}}))$ if $\alpha_k \geq 0$.

$\alpha_k(\omega_{t2}\widetilde{F_d}^1) =$

$((\alpha_k\underline{d_6'}, \alpha_k\underline{d_5'}, \alpha_k\underline{d_3'}, \alpha_k\underline{d_2'}, \underline{\alpha_k d_1'}, \underline{\omega_{1F^1}}), (\alpha_k\overline{d_6'}, \alpha_k\overline{d_5'}, \alpha_k\overline{d_4'}, \alpha_k\overline{d_3'}, \alpha_k\overline{d_2'}, \alpha_k\overline{d_1'}, \overline{\omega_{1F^1}}))$

if $k \leq 0$.

4. Mathematical Form of Multi-objective Fuzzy Linear Sum Assignment Problem is defined as:

Minimize $\tilde{z}^{k} = \sum_{i=1}^{n} \tilde{c}_{ij}^{k} \sum_{j=1}^{n} \tilde{x}_{ij}$

Subject to $\sum_{i=1}^{n} \tilde{x}_{i} = 1$, for j = 1,2,3,4…n

$\sum_{i=1}^{n} \tilde{x}_{j} = 1$, for i = 1,2,3,4…n

and $$\tilde{x}_{ij} = \begin{cases} 1, & if \;\; job\ 'j'\ is\ matched\ to\ machine\ 'm' \\ 0, & otherwise \end{cases}$$

where $\tilde{z}^{k} = \{\tilde{z}^{1}, \tilde{z}^{2}, \tilde{z}^{3} \ldots\ldots \tilde{z}^{k}\}$ is vector of multi objectives

Table 1.

Job/ Machine	J_1	J_2	J_3	J_4
M_1	$[\underline{\tilde{c}_{11}}, \overline{\tilde{c}_{11}}]$ $[\underline{\tilde{t}_{11}}, \overline{\tilde{t}_{11}}]$ $[\underline{\tilde{q}_{11}}, \overline{\tilde{q}_{11}}]$	$[\underline{\tilde{c}_{12}}, \overline{\tilde{c}_{12}}]$ $[\underline{\tilde{t}_{12}}, \overline{\tilde{t}_{12}}]$ $[\underline{\tilde{q}_{12}}, \overline{\tilde{q}_{12}}]$	$[\underline{\tilde{c}_{13}}, \overline{\tilde{c}_{13}}]$ $[\underline{\tilde{t}_{13}}, \overline{\tilde{t}_{13}}]$ $[\underline{\tilde{q}_{13}}, \overline{\tilde{q}_{13}}]$	$[\underline{\tilde{c}_{14}}, \overline{\tilde{c}_{14}}]$ $[\underline{\tilde{t}_{14}}, \overline{\tilde{t}_{14}}]$ $[\underline{\tilde{q}_{14}}, \overline{\tilde{q}_{14}}]$
M_2	$[\underline{\tilde{c}_{21}}, \overline{\tilde{c}_{21}}]$ $[\underline{\tilde{t}_{21}}, \overline{\tilde{t}_{21}}]$ $[\underline{\tilde{q}_{21}}, \overline{\tilde{q}_{21}}]$	$[\underline{\tilde{c}_{22}}, \overline{\tilde{c}_{22}}]$ $[\underline{\tilde{t}_{22}}, \overline{\tilde{t}_{22}}]$ $[\underline{\tilde{q}_{22}}, \overline{\tilde{q}_{22}}]$	$[\underline{\tilde{c}_{23}}, \overline{\tilde{c}_{23}}]$ $[\underline{\tilde{t}_{23}}, \overline{\tilde{t}_{23}}]$ $[\underline{\tilde{q}_{23}}, \overline{\tilde{q}_{23}}]$	$[\underline{\tilde{c}_{24}}, \overline{\tilde{c}_{24}}]$ $[\underline{\tilde{t}_{24}}, \overline{\tilde{t}_{24}}]$ $[\underline{\tilde{q}_{24}}, \overline{\tilde{q}_{24}}]$
M_3	$[\underline{\tilde{c}_{31}}, \overline{\tilde{c}_{31}}]$ $[\underline{\tilde{t}_{31}}, \overline{\tilde{t}_{31}}]$ $[\underline{\tilde{q}_{31}}, \overline{\tilde{q}_{11}}]$	$[\underline{\tilde{c}_{32}}, \overline{\tilde{c}_{32}}]$ $[\underline{\tilde{t}_{32}}, \overline{\tilde{t}_{32}}]$ $[\underline{\tilde{q}_{32}}, \overline{\tilde{q}_{32}}]$	$[\underline{\tilde{c}_{33}}, \overline{\tilde{c}_{33}}]$ $[\underline{\tilde{t}_{33}}, \overline{\tilde{t}_{33}}]$ $[\underline{\tilde{q}_{33}}, \overline{\tilde{q}_{33}}]$	$[\underline{\tilde{c}_{34}}, \overline{\tilde{c}_{34}}]$ $[\underline{\tilde{t}_{34}}, \overline{\tilde{t}_{34}}]$ $[\underline{\tilde{q}_{34}}, \overline{\tilde{q}_{34}}]$
M_4	$[\underline{\tilde{c}_{41}}, \overline{\tilde{c}_{41}}]$ $[\underline{\tilde{t}_{41}}, \overline{\tilde{t}_{41}}]$ $[\underline{\tilde{q}_{41}}, \overline{\tilde{q}_{41}}]$	$[\underline{\tilde{c}_{42}}, \overline{\tilde{c}_{42}}]$ $[\underline{\tilde{t}_{42}}, \overline{\tilde{t}_{42}}]$ $[\underline{\tilde{q}_{42}}, \overline{\tilde{q}_{42}}]$	$[\underline{\tilde{c}_{43}}, \overline{\tilde{c}_{43}}]$ $[\underline{\tilde{t}_{43}}, \overline{\tilde{t}_{43}}]$ $[\underline{\tilde{q}_{43}}, \overline{\tilde{q}_{43}}]$	$[\underline{\tilde{c}_{44}}, \overline{\tilde{c}_{44}}]$ $[\underline{\tilde{t}_{44}}, \overline{\tilde{t}_{44}}]$ $[\underline{\tilde{q}_{44}}, \overline{\tilde{q}_{44}}]$

5. Ranking function of ω-type 2 diamond fuzzy numbers

Let $\omega_{t2}\widetilde{F_d}^1$ = $[\underline{\omega_{t2}\widetilde{F_d}^1}, \overline{\omega_{t2}\widetilde{F_d}^1}]$ and $\omega_{t2}\widetilde{F_d}^2$ = $[\underline{\omega_{t2}\widetilde{F_d}^2}, \overline{\omega_{t2}\widetilde{F_d}^2}]$ are two ω-type2 diamond fuzzy numbers. $\underline{\omega_{t2}\widetilde{F_d}^1}$, $\underline{\omega_{t2}\widetilde{F_d}^2}$ are lower ω-type 2 diamond fuzzy number and $\overline{\omega_{t2}\widetilde{F_d}^1}$, $\overline{\omega_{t2}\widetilde{F_d}^2}$ are upper ω-type 2 diamond fuzzy number. Then the following ranking function of ω-type2 diamond fuzzy number and defined as $R(\omega_{t2}\widetilde{F_d})$

$$R(\omega_{t2}\widetilde{F_d}) = \frac{\omega_{t2}\widetilde{F_d}^1+\omega_{t2}\widetilde{F_d}^2}{2} = \frac{(\underline{\omega_{t2}\widetilde{F_d}^1} + \underline{\omega_{t2}\widetilde{F_d}^2}) + (\overline{\omega_{t2}\widetilde{F_d}^1} + \overline{\omega_{t2}\widetilde{F_d}^2}])}{2}$$

6. Algorithm and Properties:

6.1: A New Algorithm for fuzzy multi-objective linear sum assignment:

ω-type 2 diamond fuzzy numbers are considered as linguistic variables. The fuzzy cost coefficient, fuzzy time, and fuzzy quality are expressed in λ_d- cut of ω-type 2 diamond fuzzy numbers to compute the partial feasible solution and complete optimal solution.

Step 1: First let us take the cost matrix $[\widetilde{c_{ij}}]$, whose elements are linguistic variables that have been substituted by fuzzy numbers, is presented. Examine whether or not the provided ω-type 2 diamond fuzzy multi-objective linear sum assignment table is balanced.

a) If the number of machines and the number of jobs are equal, go to step 3.

b) Proceed to step 2 if the number of machines does not equal the number of jobs.

Step 2: In the ω-type 2 diamond fuzzy multi objective linear sum assignment table, add a dummy row or column. Dummy row/column cost, time, and quantity entries are always zero.

Step 3: In λ_d- cut form, express the above ω-type 2 diamond fuzzy multi-objective linear sum assignment problems. The upper and lower ω-type 2 diamond fuzzy numbers of the multi-objective linear sum assignment problem are then merged into single λ_d- cut form of ω-type 2 diamond fuzzy number of the multi-objective linear sum assignment problem.

Step 4: By applying ranking method, convert a λ_d- cut of ω-type 2-diamond fuzzy multi objective linear sum assignment problem to λ_d- cut of ω-type 2-diamond fuzzy linear sum assignment problem..

Step 5: Find dual variables $(\tilde{u}_i, \tilde{v}_j)$,

If $M_i = M_1, M_2 \ldots M_n$ then find $\tilde{u}_i = \min\{\widetilde{c_{ij}}$; $j_i = J_1, J_2 \ldots J_n$

If $j_i = J_1, J_2 \ldots J_n$ then find $\tilde{v}_j = \min\{\widetilde{c_{ij}} - \tilde{u}_i$; $M_i = M_1, M_2 \ldots M_n\}$;

Step 6: Calculate ($\overline{\widetilde{c_{ij}}}$) and find a partial feasible solution

if j_i = J_1 , J_2 …..J_n then row (j) = 0;

if $M_i = M_1, M_2 \dots M_n$ and j_i = J_1 , J_2 …..J_n then obtain $\overline{\widetilde{c_{ij}}}$ = $\widetilde{c_{ij}}$ - $\tilde{u}_i$ - $\tilde{v}_j$ = 0 and the solution is

row (j) =i , $\tilde{x}_{ij}$ = 1 and

a) If there are less than ‘n' rows of matching. go to the next step

b) An optimal solution is found if the number of matches is equal to n..

Step7: If the number of matching solution is less than (the order of the matrix) n matching solution by using the following alternative path method.

The matching vertex$|\overline{\boldsymbol{U}}|$< n then increase the partial solution and let E be any vertex in U and select the elementary path from k whose edges are alternatively not assigned and assigned.

If E $\notin \overline{U}$ then sink = Alternate(k);

If sink > 0 then $\overline{U}$ = $\overline{U} \cup$ {E} ; j = sink and obtain in new graph.

Step 8: update the dual variables and obtain complete optimal solution

Select the minimum value of an unassigned row δ = min{$\widetilde{c_{ij}}$ - $\tilde{u}_i$ - $\tilde{v}_j$= 0 and then updated dual variables are $\tilde{u}_i^*$ = $\tilde{u}_i + \delta$: $\tilde{v}_j^*$ = $\widetilde{c_{ij}}$ - $\tilde{u}_i^*$ then obtain $\overline{\widetilde{c_{ij}}}^*$ = $\widetilde{c_{ij}}$ - $\tilde{u}_i^*$ - $\tilde{v}_j^*$ = 0 is the new bipartite graph of the current solution. Alternate (k) is then executed again for k = δ Producing the augmented tree. Finally, each machine (M_i) and job (j_i) has one and only matching edges, complete optimum solution is reached.

Step 9: Stop.

6.2. Properties on fuzzy matching:

6.2.1. Theorem: If partial feasible matching is the minimum matching edges in any bipartite graph.

Proof: Consider the fuzzy cost matrix (nxn) is $\widetilde{c_{ij}}$ and define the fuzzy dual variables are $\tilde{u}_i$ = min {$\widetilde{c_{ij}}$} and $\tilde{v}_j$ = min {$\widetilde{c_{ij}}$ - $\tilde{u}_i$ }. Then, we have by applying complementary slackness conditions for transform cost matrix $\widetilde{c_{ij}}$ to reduced cost matrix $\overline{\widetilde{c_{ij}}}$ (ie), $\overline{\widetilde{c_{ij}}}$ = $\widetilde{c_{ij}}$ - $\tilde{u}_i$ - $\tilde{v}_j$ = $0, \forall\, 0 \le i, j \le n$. Therefore, assign only one matching edge to each rows and columns but both rows and columns are less than n. Then we have a solution is partial if there are a minimum number of matching edges in any bipartite graph.

6.2.2. Theorem: If an optimal complete matching is the number of matching is equal to the order of the matrix (nxn).

Proof: From Theorem (5.1). Let us take the partial feasible matching edges in bipartite graph. The matching vertex is less then n and increase the partial solution and let 'E' be any vertex in U and choose the elementary path from 'E' whose edges are alternatively not matched and matched. In a bipartite graph, an alternating tree rooted in a vertex 'r' is a tree in which all paths emanating from 'r' alternate. adding new matching vertex is $\overline{U}\ \ = \overline{U} \cup \{E\}$. Choose the minimum value of an unassigned row δ = min{$\widetilde{c_{ij}}$ - $\tilde{u}_i$ - $\tilde{v}_j$= 0 and then updated the dual variables are $\tilde{u}_i{}^* = \tilde{u}_i + \delta$: $\tilde{v}_j{}^* = \widetilde{c_{ij}} - \tilde{u}_i{}^*$ then compute $\overline{\widetilde{c_{ij}}}^* = \widetilde{c_{ij}} - \tilde{u}_i{}^* - \tilde{v}_j{}^* = 0$ is the new bipartite graph of the current solution. The alternate method executed again for k = δ Producing the augmented tree. Then we have a solution is complete optimal if there are a maximum number of matching is equal to the order of the matrix (nxn).

7. Example:

Let us considered the four machines given below. M_1, M_2, M_3, M_4, and four jobs J_1, J_2, J_3, J_4 respectively. To optimize the fuzzy cost, fuzzy time, and fuzzy quality are each considered as a ω-type 2 diamond fuzzy numbers. The fuzzy cost, the fuzzy time and the fuzzy quality for solving λ_d- cut of ω-type 2 diamond fuzzy numbers of multi-objective linear sum assignment problem.

Table 2: ω-type 2 diamond fuzzy numbers are representing to the linguistic variables.

Job/Machine	J_1	J_2	J_3	J_4
M_1	Fairly high	Very high	High	Very high
M_2	Very low	High	low	Fairly high
M_3	Extremely low	Medium	Medium	Very high
M_4	Fairly low	Medium	Very low	Very low

Table 3.

Job/Machine	J_1	J_2	J_3	J_4
M_1	(23,30,37) (22,31,38) (31,38,45) (30,39,46) (36,43,50) (35,44,51)	(25,33,41) (24,34,42) (31,39,47) (30,40,48) (36,44,52) (35,45,53)	(21,27,33) (20,28,34) (18,24,30) (17,25,31) (27,33,39) (26,34,40)	(25,33,41) (24,34,42) (31,39,47) (30,40,48) (36,44,52) (35,45,53)
M_2	(1,2,4) (0,3,7) (3,4,7) (2,5,9) (4,6,8) (3,7,10)	(21,27,33) (20,28,34) (18,24,30) (17,25,31) (27,33,39) (26,34,40)	(8,13,18) (7,15,21) (14,18,24) (13,23,28) (11,13,21) (10,17,22)	(23,30,37) (22,31,38) (31,38,45) (30,39,46) (36,43,50) (35,44,51)
M_3	(1,2,4) (0,3,5) (2,3,5) (1,4,6) (4,5,7) (3,6,8)	(7,12,15) (6,13,16) (8,13,16) (7,14,17) (13,18,21) (12,19,22)	(7,12,15) (6,13,16) (8,13,16) (7,14,17) (13,18,21) (12,19,22)	(25,33,41) (24,34,42) (31,39,47) (30,40,48) (36,44,52) (35,45,53)
M_4	(3,5,9) (2,6,10) (3,4,7) (2,5,10) (4,6,8) (3,7,11)	(7,12,15) (6,13,16) (8,13,16) (7,14,17) (13,18,21) (12,19,22)	(1,2,4) (0,3,7) (3,4,7) (2,5,9) (4,6,8) (3,7,10)	(1,2,4) (0,3,7) (3,4,7) (2,5,9) (4,6,8) (3,7,10)

Table 4: ω-type 2 diamond fuzzy numbers are converted into λ_d-cut of fuzzy numbers

Job/ Machine	J_1	J_2	J_3	J_4
M_1	[6$\underline{\lambda_d}$ +21, 33-6$\underline{\lambda_d}$] [8$\overline{\lambda_d}$ +20, 34-6$\overline{\lambda_d}$] [6$\underline{\lambda_d}$+18,30- 6$\underline{\lambda_d}$] [8$\overline{\lambda_d}$ +17, 31-6$\overline{\lambda_d}$] [6$\underline{\lambda_d}$+27,39-6$\underline{\lambda_d}$][8$\overline{\lambda_d}$ +26, 40-6$\overline{\lambda_d}$]	[8λ_d +25, 41-8$\underline{\lambda_d}$] [10$\overline{\lambda_d}$+24, 42-8$\overline{\lambda_d}$] [8$\underline{\lambda_d}$+31,47- 8$\underline{\lambda_d}$] [10$\overline{\lambda_d}$+30, 48- [8$\underline{\lambda_d}$+36,52-8$\underline{\lambda_d}$][10$\overline{\lambda_d}$ +35, 53-8$\overline{\lambda_d}$]	[7$\underline{\lambda_d}$ +23, 37-7$\underline{\lambda_d}$] [9$\overline{\lambda_d}$ +22, 38-7$\overline{\lambda_d}$] [7$\underline{\lambda_d}$+31,45- 7$\underline{\lambda_d}$] [9$\overline{\lambda_d}$ +30, 46-7$\overline{\lambda_d}$] [7$\underline{\lambda_d}$+36,50-7$\underline{\lambda_d}$][9$\overline{\lambda_d}$ +35, 51-7$\overline{\lambda_d}$]	[8λ_d +25, 41-8$\underline{\lambda_d}$] [10$\overline{\lambda_d}$ +24, 42-8$\overline{\lambda_d}$] [8$\underline{\lambda_d}$+31,47- 8$\underline{\lambda_d}$] [10$\overline{\lambda_d}$ +30, 48-8$\overline{\lambda_d}$] [8$\underline{\lambda_d}$+36,52- 8$\underline{\lambda_d}$][10$\overline{\lambda_d}$ +35, 53-8$\overline{\lambda_d}$]
M_2	[$\underline{\lambda_d}$+1, 4-2$\underline{\lambda_d}$] [3$\overline{\lambda_d}$+0, 7-4$\overline{\lambda_d}$] [$\underline{\lambda_d}$+3,7- 3$\underline{\lambda_d}$] [3$\overline{\lambda_d}$ +2, 9-4$\overline{\lambda_d}$] [2$\underline{\lambda_d}$+4,8-2$\underline{\lambda_d}$][4$\overline{\lambda_d}$ +3, 10-3$\overline{\lambda_d}$]	[7$\underline{\lambda_d}$ +23, 37-7$\underline{\lambda_d}$] [9$\overline{\lambda_d}$ +22, 38-7$\overline{\lambda_d}$] [7$\underline{\lambda_d}$+31,45- 7$\underline{\lambda_d}$] [9$\overline{\lambda_d}$ +30, 46-7$\overline{\lambda_d}$] [7$\underline{\lambda_d}$+36,50-7$\underline{\lambda_d}$][9$\overline{\lambda_d}$ +35, 51-7$\overline{\lambda_d}$]	[5$\underline{\lambda_d}$ +8, 18-5$\underline{\lambda_d}$] [8$\overline{\lambda_d}$ +7, 21-6$\overline{\lambda_d}$] [4$\underline{\lambda_d}$+14,24- 4$\underline{\lambda_d}$] [10$\overline{\lambda_d}$+13, 28-5$\overline{\lambda_d}$] [2$\underline{\lambda_d}$+11,21-5$\underline{\lambda_d}$][7$\overline{\lambda_d}$ +10, 22-5$\overline{\lambda_d}$]	[6$\underline{\lambda_d}$ +21, 33-6$\underline{\lambda_d}$] [8$\overline{\lambda_d}$ +20, 34-6$\overline{\lambda_d}$] [6$\underline{\lambda_d}$+18,30- 6$\underline{\lambda_d}$] [8$\overline{\lambda_d}$ +17, 31-6$\overline{\lambda_d}$] [6$\underline{\lambda_d}$+27,39- 6$\underline{\lambda_d}$][8$\overline{\lambda_d}$ +26, 40-6$\overline{\lambda_d}$]
M_3	[$\underline{\lambda_d}$ +1, 4-2$\underline{\lambda_d}$] [3$\overline{\lambda_d}$], 5-2$\overline{\lambda_d}$]] [$\underline{\lambda_d}$+2,5- 2$\underline{\lambda_d}$] [3$\overline{\lambda_d}$] +1, 6-2$\overline{\lambda_d}$]] [$\underline{\lambda_d}$+4,7-2$\underline{\lambda_d}$][3$\overline{\lambda_d}$] +3, 8-2$\overline{\lambda_d}$]]	[5$\underline{\lambda_d}$ +7, 15-3$\underline{\lambda_d}$] [7$\overline{\lambda_d}$ +6, 16-3$\overline{\lambda_d}$] [5$\underline{\lambda_d}\lambda$+8,16- 3$\underline{\lambda_d}$] [7$\overline{\lambda_d}$ +7, 17-3$\overline{\lambda_d}$] [5$\underline{\lambda_d}$+13,21-3$\underline{\lambda_d}$][7$\overline{\lambda_d}$ +12, 22-3$\overline{\lambda_d}$]	[5$\underline{\lambda_d}$ +7, 15-3$\underline{\lambda_d}$] [7$\overline{\lambda_d}$ +6, 16-3$\overline{\lambda_d}$] [5$\underline{\lambda_d}\lambda$+8,16- 3$\underline{\lambda_d}$] [7$\overline{\lambda_d}$ +7, 17-3$\overline{\lambda_d}$] [5$\underline{\lambda_d}$+13,21-3$\underline{\lambda_d}$][7$\overline{\lambda_d}$ +12, 22-3$\overline{\lambda_d}$]	[8$\underline{\lambda_d}$+ +25, 41-8$\underline{\lambda_d}$] [10$\overline{\lambda_d}$ +24, 42-8$\overline{\lambda_d}$] [8$\underline{\lambda_d}$+31,47- 8$\underline{\lambda_d}$] [10$\overline{\lambda_d}$ +30, 48-8$\overline{\lambda_d}$] [8$\underline{\lambda_d}$+36,52- 8$\underline{\lambda_d}$][10$\overline{\lambda_d}$ +35, 53-8$\overline{\lambda_d}$]
M_4	[2$\underline{\lambda_d}$ +3, 9-4$\underline{\lambda_d}$] [4$\overline{\lambda_d}$, +2, 10-4$\overline{\lambda_d}$] [$\underline{\lambda_d}$+3,7- 3$\underline{\lambda_d}$] [3$\overline{\lambda_d}$+2, 9-5$\overline{\lambda_d}$] [2$\underline{\lambda_d}$+4,8-2$\underline{\lambda_d}$][4$\overline{\lambda_d}$+3 , 11-4$\overline{\lambda_d}$]	[5$\underline{\lambda_d}$ +7, 15-3$\underline{\lambda_d}$] [7$\overline{\lambda_d}$ +6, 16-3$\overline{\lambda_d}$] [5$\underline{\lambda_d}\lambda$+8,16- 3$\underline{\lambda_d}$] [7$\overline{\lambda_d}$ +7, 17-3$\overline{\lambda_d}$] [5$\underline{\lambda_d}$+13,21-3$\underline{\lambda_d}$][7$\overline{\lambda_d}$ +12, 22-3$\overline{\lambda_d}$]	[$\underline{\lambda_d}$+1, 4-2$\underline{\lambda_d}$] [3$\overline{\lambda_d}$+0, 7-4$\overline{\lambda_d}$] [$\underline{\lambda_d}$+3,7- 3$\underline{\lambda_d}$] [3$\overline{\lambda_d}$ +2, 9-4$\overline{\lambda_d}$] [2$\underline{\lambda_d}$+4,8- 2$\underline{\lambda_d}$][4$\overline{\lambda_d}$ +3, 10-3$\overline{\lambda_d}$]	[$\underline{\lambda_d}$+1, 4-2$\underline{\lambda_d}$] [3$\overline{\lambda_d}$+0, 7-4$\overline{\lambda_d}$] [$\underline{\lambda_d}$+3,7- 3$\underline{\lambda_d}$] [3$\overline{\lambda_d}$ +2, 9-4$\overline{\lambda_d}$] [2$\underline{\lambda_d}$+4,8- 2$\underline{\lambda_d}$][4$\overline{\lambda_d}$ +3, 10-3$\overline{\lambda_d}$]

Table 5: ω-type 2 diamond multi-objective fuzzy numbers are converted into single λ-cut fuzzy number

Job/ Machine	J_1	J_2	J_3	J_4
M_1	[$18\underline{\lambda_d}$+66,102- $18\underline{\lambda_d}$] [$24\overline{\lambda_d}$ +63, 105-$18\overline{\lambda_d}$]	[$24\underline{\lambda_d}$+92,140- $24\underline{\lambda_d}$] [$30\overline{\lambda_d}$ +89, 143- $24\overline{\lambda_d}$]	[$21\underline{\lambda_d}$+90,132- $21\underline{\lambda_d}$] [$27\overline{\lambda_d}$ +87, 135- $21\overline{\lambda_d}$]	[$24\underline{\lambda_d}$+92,140- $24\underline{\lambda_d}$] [$30\overline{\lambda_d}$ +89, 143- $24\overline{\lambda_d}$]
M_2	[$4\underline{\lambda_d}$+8,19-$7\underline{\lambda_d}$] [$10\overline{\lambda_d}$ +5, 26-$11\overline{\lambda_d}$]	[$21\underline{\lambda_d}$+90,132- $21\underline{\lambda_d}$] [$27\overline{\lambda_d}$ +87, 135- $21\overline{\lambda_d}$]	[$11\underline{\lambda_d}$++33,63-$14\underline{\lambda_d}$] [$25\overline{\lambda_d}$ +30, 71-$16\overline{\lambda_d}$]	[$18\underline{\lambda_d}$ +66,102- $18\underline{\lambda_d}$] [$24\overline{\lambda_d}$ +63, 105- $18\overline{\lambda_d}$]
M_3	[$3\underline{\lambda_d}$+7,16-$6\underline{\lambda_d}$] [$9\overline{\lambda_d}$ +4, 19-$6\overline{\lambda_d}$]	[$15\underline{\lambda_d}$+28,52- $9\underline{\lambda_d}$] [$21\overline{\lambda_d}$ +25, 55-$9\overline{\lambda_d}$]	[$15\underline{\lambda_d}$+28,52- $9\underline{\lambda_d}$] [$21\overline{\lambda_d}$ +25, 55-$9\overline{\lambda_d}$]	[$24\underline{\lambda_d}$+92,140- $24\underline{\lambda_d}$] [$30\overline{\lambda_d}$ +89, 143- $24\overline{\lambda_d}$]
M_4	[$5\underline{\lambda_d}$+10,24-$9\underline{\lambda_d}$] [$11\overline{\lambda_d}$ +7, 30-$13\overline{\lambda_d}$]	[$15\underline{\lambda_d}$+28,52- $9\underline{\lambda_d}$] [$21\overline{\lambda_d}$ +25, 55-$9\overline{\lambda_d}$]	[$4\underline{\lambda_d}$ +8,19-$7\underline{\lambda_d}$] [$10\overline{\lambda_d}$ +5, 26-$11\overline{\lambda_d}$]	[$4\underline{\lambda_d}$ +8,19-$7\underline{\lambda_d}$] [$10\overline{\lambda_d}$ +5, 26-$11\overline{\lambda_d}$]

Table 6: Upper and lower ω-type 2 diamond multi-objective fuzzy numbers are converted into single objective λ_d-cut fuzzy number

Job/ Machine	J_1	J_2	J_3	J_4
M_1	[$21\underline{\lambda_d}$+64.5,103- $18\overline{\lambda_d}$]	[$27\underline{\lambda_d}$+90.5,141.5- $24\overline{\lambda_d}$]	[$24\underline{\lambda_d}$+88.5,133.5- 21λ]	[$27\underline{\lambda_d}$+90.5,141.5- $24\overline{\lambda_d}$]
M_2	[$7\underline{\lambda_d}$+6.5,22.5 -$9\overline{\lambda_d}$]	[$24\underline{\lambda_d}$+88.5,133.5- $21\overline{\lambda_d}$]	[$18\underline{\lambda_d}$+31.5,67- $15\overline{\lambda_d}$]	[$21\underline{\lambda_d}$+64.5,103.5- $18\overline{\lambda_d}$]
M_3	[$6\underline{\lambda_d}$+5.5,17.5-$6\overline{\lambda_d}$]	[$18\underline{\lambda_d}$+26.5,53.5- $9\overline{\lambda_d}$]	[$18\underline{\lambda_d}$+26.5,53.5- $9\overline{\lambda_d}$]	[$27\underline{\lambda_d}$+90.5,141.5- $24\overline{\lambda_d}$]
M_4	[$8\underline{\lambda_d}$+8.5,27-$11\overline{\lambda_d}$]	[$18\underline{\lambda_d}$+26.5,53.5- $9\overline{\lambda_d}$]	[$7\underline{\lambda_d}$+6.5,22.5 -$9\overline{\lambda_d}$]	[$7\underline{\lambda_d}$+6.5,22.5 -$9\overline{\lambda_d}$]

Obtain dual variables

$\tilde{u}$ ={[$21\underline{\lambda_d}$+64.5,103-$18\overline{\lambda_d}$], [$7\underline{\lambda_d}$+6.5,25.5-$9\overline{\lambda_d}$], [$6\underline{\lambda_d}$+5.5,17.5-$6\overline{\lambda_d}$], [$7\underline{\lambda_d}$+6.5,25.5-$9\overline{\lambda_d}$]

$\tilde{v}$ = {0, [$6\underline{\lambda_d}$+26,38.5- $6\overline{\lambda_d}$],0 , 0}

Table 7.

$\overline{\widetilde{c_{ij}}}$ =

$\underline{\mathbf{0}}$	-[6$\underline{\lambda_d}$+26,38.5- 6$\overline{\lambda_d}$]	0	0
0	0	[3$\underline{\lambda_d}$+24,30.5-3$\overline{\lambda_d}$]	[6$\underline{\lambda_d}$+26,38.5- 6$\overline{\lambda_d}$]
0	[11$\underline{\lambda_d}$+56,72.5-6$\overline{\lambda_d}$]	[11$\underline{\lambda_d}$+25,44.5-6$\overline{\lambda_d}$]	[14$\underline{\lambda_d}$+58,81-9$\overline{\lambda_d}$]
0	[6$\underline{\lambda_d}$-5, -2.5+3$\overline{\lambda_d}$]	[12$\underline{\lambda_d}$+21,36-3$\overline{\lambda_d}$]	[21$\underline{\lambda_d}$+85,124- 18$\overline{\lambda_d}$]
[$\underline{\lambda_d}$+2.5,4.5-2$\overline{\lambda_d}$]	[5$\underline{\lambda_d}$-6,-7.5+6$\overline{\lambda_d}$]	$\underline{\mathbf{0}}$	0

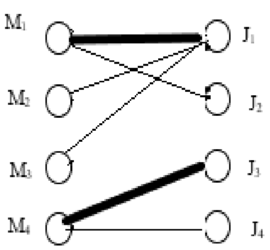

Figure :1

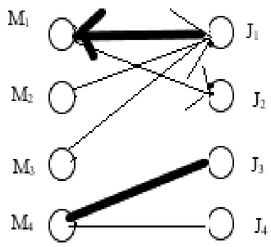

Figure : 2

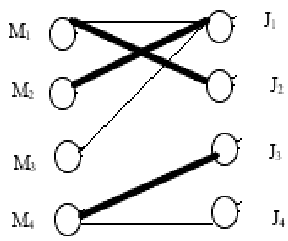

Figure : 3

Table 8.

Job/ Machine	J_1	J_2	J_3	J_4
M_1	[21$\underline{\lambda_d}$+64.5,103-18$\overline{\lambda_d}$]	[27$\underline{\lambda_d}$+90.5,141.5-24$\overline{\lambda_d}$]	[24$\underline{\lambda_d}$+88.5,133.5-21λ]	[27$\underline{\lambda_d}$+90.5,141.5-24$\overline{\lambda_d}$]
M_2	[7$\underline{\lambda_d}$+6.5,22.5 -9$\overline{\lambda_d}$]	[24$\underline{\lambda_d}$+88.5,133.5-21$\overline{\lambda_d}$]	[18$\underline{\lambda_d}$+31.5,67-15$\overline{\lambda_d}$]	[21$\underline{\lambda_d}$+64.5,103.5-18$\overline{\lambda_d}$]
M_3	[6$\underline{\lambda_d}$+5.5,17.5-6$\overline{\lambda_d}$]	[18$\underline{\lambda_d}$+26.5,53.5-9$\overline{\lambda_d}$]	[18$\underline{\lambda_d}$+26.5,53.5-9$\overline{\lambda_d}$]	[27$\underline{\lambda_d}$+90.5,141.5-24$\overline{\lambda_d}$]
M_4	[8$\underline{\lambda_d}$+8.5,27-11$\overline{\lambda_d}$]	[18$\underline{\lambda_d}$+26.5,53.5-9$\overline{\lambda_d}$]	[7$\underline{\lambda_d}$+6.5,22.5 -9$\overline{\lambda_d}$]	[7$\underline{\lambda_d}$+6.5,22.5 -9$\overline{\lambda_d}$]

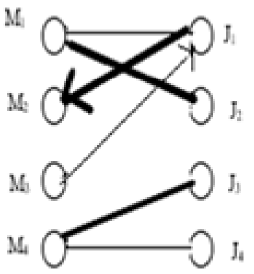

Figure :4

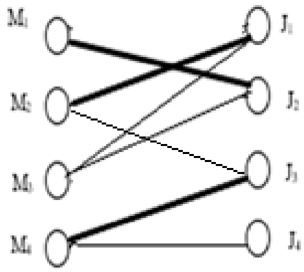

Figure : 5

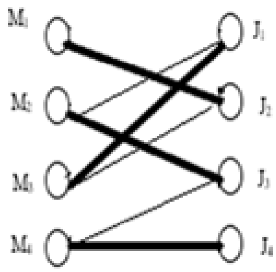

Figure : 6

Updated dual variables

$\tilde{u}_i^* = \{[21\underline{\lambda_d}+64.5, 103-18\overline{\lambda_d}], [18\underline{\lambda_d}+31.5, 67-15\overline{\lambda_d}], [12\underline{\lambda_d}, 15-9\overline{\lambda_d}], [7\underline{\lambda_d}+6.5, 25.5-9\overline{\lambda_d}]\};$

$\tilde{v}_j^* = \{[-11\underline{\lambda_d}-25, -44.5+6\overline{\lambda_d}], [6\underline{\lambda_d}+26, 38.5-6\overline{\lambda_d}], 0, 0\}$

Table 9.

$\overline{\tilde{c}_{ij}}^* =$

Job/ Machine	J_1	J_2	J_3	J_4
M_1	0	**0**	$[3\underline{\lambda_d}+24, 30.5-3\overline{\lambda_d}]$	$[6\underline{\lambda_d}+26, 38.5-6\overline{\lambda_d}]$
M_2	0	$[0\underline{\lambda_d}-1.5, 28-0\overline{\lambda_d}]$	**0**	$[3\underline{\lambda_d}+33, 36.5+3\overline{\lambda_d}]$
M_3	**0**	0	$[6\underline{\lambda_d}+26.5, 38.5-0\overline{\lambda_d}]$	$[15\underline{\lambda_d}+90.5, 126.5-15\overline{\lambda_d}]$
M_4	$[12\underline{\lambda_d}+27, 49-4\overline{\lambda_d}]$	$[0\underline{\lambda_d}-6, -7.5-4\overline{\lambda_d}]$	0	**0**

Fuzzy optimal schedule $\mathbf{M_1 \rightarrow J_2}$, $\mathbf{M_2 \rightarrow J_3}$, $\mathbf{M_3 \rightarrow J_1}$, $\mathbf{M_4 \rightarrow J_4}$

Fuzzy optimal cost = **{(25,33,41) + (8,13,18) + (1,2,4) + (1,2,4)}{(24,34,42) + (7,15,21) + (0,3,5) + (0,3,7)}**

Fuzzy optimal cost = (35, 50, 67) (31,55,75)

Fuzzy optimal time= **{(31,39,47) + (14,18,24) + (2,3,5) + (3,4,7)}{(30,40,48 + (13,23,28) + (1,4,6) + (2,5,9)}**

Fuzzy optimal time = (50, 64, 83) (46,72,91)

Fuzzy optimal quality = **{(36,44,52) + (4,6,8) + (4,5,7) + (4,6,8)} {(35,45,53) + (3,7,10) + (3,6,8) + (3,7,10)}**

Fuzzy optimal quality = (48,61,75) (44,65,81)

CONCLUSION

We discussed ω-type 1 and ω-type 2-diamond fuzzy numbers. We proposed a new method for solving λ_d- cut of ω-type 2-diamond fuzzy multi-objective linear sum assignment problem and involving linguistic variables and by using alternate method and augmented method of bipartite graph to compute partial feasible solution and complete optimal solution. To modified partial primal solution and obtain complete optimal solution using the alternate path method producing augment path method of the bipartite graph.

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