

A STUDY ON M-N FUZZY, M-N ANTI FUZZY SOFT SUBGROUPS AND NORMAL SOFT SUBGROUPS OF A GROUP

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By

S. RUMENAKA

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Under the Supervision of

**Dr. M. KALIRAJA, M. Sc., M. Phil., B. Ed., PGDCA., Ph. D.
ASSISTANT PROFESSOR**



**PG & RESEARCH DEPARTMENT OF MATHEMATICS
H. H. THE RAJAH'S COLLEGE (AUTONOMOUS)**

(Accredited B⁺ Grade by NAAC)

Affiliated to Bharathidasan University

Pudukkottai – 622 001.

Tamil Nadu, India

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Dr. M. KALIRAJA, M.Sc., M.Phil, B.Ed., PGDCA., Ph.D.
Research Advisor,
PG and Research Department of Mathematics,
H. H. The Rajah's College (Autonomous),
Pudukkottai- 622 001,
Tamilnadu – India.



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This is to certify that the thesis entitled “**A STUDY ON M-N FUZZY, M-N ANTI FUZZY SOFT SUBGROUPS AND NORMAL SOFT SUBGROUPS OF A GROUP**” in partial fulfilment of the requirement of the award of the degree of **DOCTOR OF PHILOSOPHY IN MATHEMATICS** is a bonafide record of original research work done by the candidate **S. RUMENAKA** (Ref: No. 7327/Ph.D. K3/Mathematics/Part Time/April 2017/ Dated: 07.04.2017) during the period of her study from 2017 to 2022 under the Part – Time program of Bharthidasan University, Trichirappalli at H. H. The Rajah's College, Pudukkottai, Tamilnadu, India under my guidance and supervision. I further certify that this thesis is original in character and it has not previously formed the basis for the award of any Degree, Diploma, Fellowship, Associateship or any other similar title. This thesis represents independent original contributions of the candidate.

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TABLE OF CONTENTS

S. No	TITLE	PAGE NO
	ABSTRACT	
1.	INTRODUCTION	1 - 11
	1.1 <i>Review of Literature</i>	1
	1.2 <i>Preliminaries and notations</i>	3
	1.3 <i>Summary of Results</i>	10
2.	M – N FUZZY SOFT SUBGROUPS OF A GROUP	12 - 30
	2.1 <i>M – N Fuzzy Soft Subgroups</i>	12
	2.2 <i>M – N Fuzzy Normal Soft Subgroups</i>	18
	2.3 <i>M-N Homomorphism of fuzzy soft subgroups</i>	25
3.	CONJUGATE OF M-N FUZZY SOFT SUBGROUPS OF A GROUP	31 - 39
	3.1 <i>Conjugate of M-N fuzzy soft subgroups</i>	31
	3.2 <i>M-N Fuzzy Soft Middle Coset</i>	34
4.	M – N ANTI FUZZY SOFT SUBGROUPS OF A GROUP	40 - 59
	4.1 <i>M – N Anti Fuzzy Soft Subgroups</i>	40
	4.2 <i>M – N Anti Fuzzy Normal Soft Subgroups</i>	47
	4.3 <i>M-N Anti Homomorphism of fuzzy soft subgroups</i>	54
5.	CONJUGATE OF M-N ANTI FUZZY SOFT SUBGROUPS OF A GROUP	60 - 66
	5.1 <i>Conjugate of M-N anti-fuzzy soft subgroups</i>	60
	5.2 <i>M-N Anti Fuzzy Soft Middle Coset</i>	63
6.	CONCLUSION	67
	LIST OF PUBLICATIONS	68- 69
	REFERENCES	70 - 77

A STUDY ON M-N FUZZY, M-N ANTI FUZZY SOFT SUBGROUPS AND NORMAL SOFT SUBGROUPS OF A GROUP

ABSTRACT:

Fuzzy group is an extension of theories of fuzzy sets and the studies of fuzzy group theory were initiated by Rosenfeld in the year 1971. He originated this fuzzy group theory by introducing the concepts of fuzzy subgroupoid and fuzzy subgroup which laid the foundation for further studies of various types of fuzzy algebraic substructures. In this thesis, we have introduced the concept of M-N fuzzy and anti-fuzzy soft subgroups. The concept of M-N fuzzy normal soft subgroups and anti-fuzzy normal soft subgroups are also discussed.

This thesis consists of six chapters. Initially, we have précised the basic definitions of fuzzy sets, fuzzy subgroups and normal fuzzy subgroups are needed in the succeeding chapters. Followed by, we defined the concept of M-N fuzzy soft subgroups based on the concept of fuzzy group. We have also defined the M-N fuzzy soft set and the M-N level subsets of a fuzzy soft subgroup. We defined the M-N normal fuzzy soft set, discussed the intersection of two and three variables of M-N level subsets of normal fuzzy soft subgroups with some of its elementary properties. Further, we defined the M-N homomorphism of fuzzy soft subgroups and defined the M-N Homomorphism of level subsets of fuzzy soft subgroups.

Subsequently, we discussed the Conjugate of M-N fuzzy soft subgroups based on the concept of fuzzy soft group. The conjugate of M-N fuzzy soft subgroups along with some of its elementary properties are also discussed. In addition, we defined and discussed the concept of M- N fuzzy soft middle coset with some related results.

Furthermore, we defined the M-N anti-fuzzy soft set and described the M-N level subsets of Anti fuzzy soft subgroups. We have also discussed the concept of M-N Anti fuzzy soft subgroups with some of its elementary properties. In addition, we defined the M-N Anti fuzzy normal soft set and the intersection of two and three variables of M-N level subsets of

anti-fuzzy normal soft subgroup. Furthermore, we defined the M-N anti-homomorphism of fuzzy soft subgroup and the M-N Homomorphism of anti-level subsets of fuzzy soft subgroups. We discussed the Conjugate of M-N anti-fuzzy soft subgroups based on the concept of fuzzy soft group.

Moreover, we defined the conjugate of M-N anti-fuzzy soft subgroups and some of the elementary properties are also discussed. Finally, we defined the concept of M- N anti-fuzzy soft middle coset and discussed about some of the related results.

CHAPTER – 1

INTRODUCTION

1. INTRODUCTIONS

1.1 Literature Reviews

Fuzzy set is a class with unsharp boundaries. Initially, Zadeh has proposed the notion of a fuzzy set theory in the year 1965 [84, 85] for the purpose of developing expert systems and soft computing by means of demonstrating mathematically as any imprecise (or) vague system of information in the real world. The concept of fuzzy subset theory was expounded in the first publication of Zadeh [84] and then Goguen [18] has shown the intention to generalize the classical set. A classical set A is defined as a collection of objects x which belongs to a universal set X . Each member of X can either belong to or not belong to A . In the first case, the statement “ x belongs to A ” is true, and in the latter case this statement is false. Such a classical set can be defined in different ways. Either we can list the objects that belong to the set or describe the set analytically or define the member elements by using the *characteristic function* χ_A defined from X to $\{0, 1\}$ in which 1 indicates the membership and 0 non-membership. In the general context, a *membership function* allows various degrees of membership for the elements of a given set. The range of the membership function is usually taken as $[0; 1]$. It has been noticed that the membership function is not limited to the values between 0 and 1. The set defined by a membership function is called a *fuzzy set*. In short, a fuzzy set is a generalization of a classical set and the membership function that of the characteristic function. In fact, we do not distinguish between these two notions.

In 1999, concept of soft sets has introduced Molodtsov [1] and the fundamental result of the new theory has also been established. A collection of approximate descriptions of an object is a soft set. Each approximate description has two parts: a predicate and an approximate value set. In classical mathematics, we construct a mathematical model of an object and define the notion of the exact solution of this model. Later other authors like

Maji et al. [27, 28, 29 and 81] have further studied the theory of soft sets and used this theory to solve some decision making problems. Authors have also introduced the notion of fuzzy soft set, a more generalised concept, which is a combination of fuzzy set and soft set and they have studied its properties. The new definition of soft sets and soft groups depending on inclusion relation and intersection of sets have been introduced by Aktas and Cagman [4] in the year 2007. Several investigators have explored the different types of fuzzy soft sets in [3, 5, 7 and 10].

There were numerous attempts have been taken to fuzzify various mathematical structures. Rosenfeld [61] has initiated the fuzzification of algebraic structures. In 1971, Rosenfeld has introduced the concepts of fuzzy subgroupoids and fuzzy subgroups; and some of its basic properties are also obtained and the concept of M-group M-subgroup has introduced by Jacobson [21]. The Fuzzy groups and level subgroups have been studied by Das [12]. Many other researchers have investigated fuzzy subgroups in [1, 6, 14, 20, 24, 37, 64, 66, 83 and 86].

Wu [78] was revealed the fuzzy normal subgroup during 1981. In addition, the Smarandache Fuzzy Algebra has been introduced by Vasantha Kandasamy [23] in 2003. Sarala and Suganya [62, 63] have unravelled the normal fuzzy soft subgroup during the year 2014. Further, R. Patel et. al., [58] have developed the three variables on normal fuzzy soft subgroup in 2015. Numerous investigators have also studied the fuzzy normal subgroups in [25, 26, 30, 38, 43, 57 and 56]. The normal fuzzy subgroups and conjugate fuzzy subgroups have been discussed by Zhang and Zou [87] in 1993. As well, Shukla Shobha has studied the conjugate fuzzy subgroup in [70]. In [22], Kandasamy, W., and Meiyappan, D were discussed the Fuzzy symmetric subgroups and conjugate fuzzy subgroups of a group in 1998.

In [8] Biswas introduced the concept of fuzzy subgroups and anti- fuzzy subgroup of groups. Further, Shen researched anti- fuzzy subgroups of a group in [69] and Dong [15] studied the product of anti- fuzzy subgroups. Feng and Yao [17] studied the concept of (λ, μ) anti- fuzzy subgroups. In [56, 57], Pandiamml et al, (2010) have defined a new algebraic structure of anti L- fuzzy normal M- subgroups and ant L- fuzzy M-Cosets of M- group. Massa'deh, M. O. (2012) discussed On M-Fuzzy Cosets, M-Conjugate of M-Upper Fuzzy Subgroups over M-Groups in [34]. Wang, S. H [76] further obtained some basic properties of anti-fuzzy subgroups and anti-fuzzy normal subgroups of group. In [54, 55], Onasanya, have studied some reviews about fuzzy subgroups and anti-fuzzy subgroups in 2016. Various other investigators have studied the anti-fuzzy and anti-fuzzy normal subgroups in [38, 44, 49, 68, 72 and 73].

In [48], The M-Homomorphism and M-Anti Homomorphism of an M-Fuzzy Subgroup and its Level M-Subgroups were introduced by R. Muthuraj et al. in 2010. In [35], Mourad Oqla Massa'deh has discussed the M-N-homomorphism and M-N-anti homomorphism over M-N- fuzzy subgroups in 2012.

1.2 Preliminaries and Notations:

This chapter contains basic definitions, properties and theorems which are required to develop the thesis.

Definition 1.2.1 [85]:

Let X be a non – empty set. A **fuzzy subset** A of X is a function $A: x \rightarrow [0, 1]$.

Example 1.2.2 [85]:

Let $X = \{1, 2, 3, 4, 5\}$ be a set. Then $A = \{(1, 0.2), (2, 0.4), (3, 0.6), (4, 0.7), (5, 0.8)\}$ is a fuzzy subset of X .

Definition 1.2.3[62]:

Let G be a group. A fuzzy subset A of G is called a **fuzzy subgroup** if for $x, y \in G$

1. $A(xy) \geq \min \{A(x), A(y)\}$
2. $A(x^{-1}) = A(x)$

Theorem 1.2.4:

Let G be an $M - N$ group, A and B both be $M - N$ fuzzy subgroup of G . Then $A \cap B$ is an $M - N$ fuzzy subgroup of G .

Example 1.2.5:

$A: G \rightarrow [0, 1]$ where $G = \{1, -1, i, -i\}$ defined as $A(1) = 0.8$, $A(-1) = 0.6$, $A(i) = 0.4$, $A(-i) = 0.4$

Definition 1.2.6[9]:

Let G be a group. A fuzzy subset A of G is called an **anti-fuzzy subgroup** if for $x, y \in G$

- (1) $A(xy) \leq \max \{A(x), A(y)\}$
- (2) $A(x^{-1}) = A(x)$

Theorem 1.2.7:

Let G be an $M - N$ group, A and B both be $M - N$ anti fuzzy subgroup of G . Then $A \cup B$ is an $M - N$ anti fuzzy subgroup of G .

Example 1.2.8:

$A: G \rightarrow [0, 1]$ where $G = \{1, -1, i, -i\}$ defined as $A(1) = 0.2$, $A(-1) = 0.4$, $A(i) = 0.6$, $A(-i) = 0.6$

Definition 1.2.9[12]:

Let A be a fuzzy subgroup of a group G . For $t \in [0, 1]$ the level subset of A is the set $A_t = \{x \in G / A(x) \geq t\}$. This is called a **level subset** of A .

Definition 1.2.10[15]:

Let A be an anti-fuzzy subgroup of a group G . For $t \in [0, 1]$ the anti-level subset of A is the set $A_t = \{x \in G / A(x) \leq t, A\}$. This is called an **anti-level subset** of A

Definition 1.2.11:

A subgroup H of G is called a **normal subgroup** of G if $aH = Ha$ for all $a \in G$

Definition 1.2.12 [43]:

Let G be a group. A fuzzy subgroup A of G is called **fuzzy normal** if $A(x) = A(y^{-1}xy)$, for all $x, y \in G$.

Example 1.3.13:

$A : G \rightarrow [0, 1]$ where $x = \{1, 2\}$ G defined as $A(1) = 0.8, A(2) = 0.6$

Definition 1.2.14:

Let G be a group. A fuzzy subgroup A of G is said be an **anti-fuzzy normal subgroup**, if for all $x, y \in G$ and $A(xyx^{-1}) = A(y)$ (or) $A(xy) \leq A(yx)$.

Definition 1.3.15[42]:

Let U be an initial universe set and E be the set of parameter. Let $P(U)$ denote the power set of U . A pair (F, E) is called a **soft set** over U , where F is mapping given by $F: E \rightarrow P(U)$

Definition 1.2.16 [1]:

Let (F, A) be a soft set over G . Then (F, A) is called a **soft group** over G if $F(a)$ is a group G for all $a \in A$.

Definition 1.2.17[28]:

Let U be an initial universe set and E be the set of parameters. Let $A \subseteq E$. A pair (F, A) is called **fuzzy soft** over U , where F is a mapping given by $F: A \rightarrow I^U$ where I^U denote the collection of all fuzzy subset of U .

Definition 1.2.18 [42]:

Let (F, A) and (G, B) be two fuzzy soft set over U . Then (F, A) is called a **fuzzy soft subset** of (G, B) denoted by $(F, A) \subseteq (G, B)$ if

$$(1) A \subseteq B$$

$$(2) F(a) \text{ is a fuzzy subset of } G(a), \text{ for each } a \in A.$$

Definition 1.2.19[35]:

Let M, N be left and right operator sets of group G respectively, if $(m x) n = m (x n)$ for all $x \in G, m \in M, n \in N$. Then G is said be an **$M - N$ group**.

Definition 1.2.20[35]:

If A is an $M - N$ fuzzy subgroup of an $M - N$ group G . Then the following statement holds for all $x, y \in G, m \in M$, and $n \in N$

$$(1) A(m(x y)n) \geq \min\{A(x), A(y)\}$$

$$(2) A(m x^{-1} n) \geq A(x)$$

Definition 1.2.21[9]:

If A is an $M - N$ anti fuzzy subgroup of an $M - N$ group G . Then the following statement holds for all $x, y \in G, m \in M$, and $n \in N$

$$(1) A(m(x y)n) \leq \max\{A(x), A(y)\}$$

$$(2) A(m x^{-1} n) \leq A(x)$$

Definition 1.3.22[34]:

Let G be an $M - N$ group and A be a fuzzy subgroup of G if

$$(1) A(m x) \geq A(x)$$

$$(2) A(x n) \geq A(x) \text{ hold for any } x \in G, m \in M \text{ and } n \in N, \text{ then } A \text{ is said be an}$$

$M - N$ fuzzy subgroup of G .

Definition 1.2.23[35]:

Let G be an $M - N$ group and A be an anti-fuzzy soft subgroup of G if

$$(1) A(m x) \leq A(x)$$

$$(2) A(x n) \leq A(x) \text{ hold for any } x \in G, m \in M \text{ and } n \in N, \text{ then } A \text{ is said be an}$$

$M - N$ anti fuzzy soft subgroup of G .

Proposition 1.2.24[35]:

Let G be an $M - N$ group, A and B both be $M - N$ fuzzy subgroup of a group G . Then $A \cap B$ is an $M - N$ fuzzy subgroup of a group G .

Proposition 1.2.25[35]:

Let G be an $M - N$ group, A and B both be $M - N$ anti fuzzy subgroup of a group G . Then $A \cup B$ is an $M - N$ anti fuzzy subgroup of a group G .

Proposition 1.2.26[35]:

Let G be an $M - N$ group, A be a fuzzy set of G , then A is $M - N$ fuzzy subgroup of a group G iff for any $t \in [0,1]$, A_t is an $M - N$ fuzzy subgroup of a group G when $A_t \neq \emptyset$.

Proposition 1.2.27[35]:

Let G be an $M - N$ group, A be a anti fuzzy set of G , then A is $M - N$ anti fuzzy subgroup of a group G iff for any $t \in [0,1]$, A_t is an $M - N$ anti fuzzy subgroup of a group G when $A_t \neq \emptyset$.

Proposition 1.2.28:

Let G and G^1 both be $M - N$ groups and f an $M - N$ homomorphism from G onto G^1 . If A^1 an $M - N$ fuzzy subgroup of a group G^1 , then $f^{-1}(A^1)$ is an $M - N$ fuzzy subgroups of a group G .

Proposition 1.2.29[35]:

Let G and G^1 both be M- N groups, f an M- N homomorphism from G onto G^1 , and A an M – N fuzzy subgroup of a group G , then $f(A)$ is an M – N fuzzy subgroups of a group G^1 .

Proposition 1.2.30[35]:

Let f be a homomorphism from the M – N group G onto the M –N group G^1 . Then the preimage which can be written as $f^{-1}(A^1)$ of A^1 under f where A^1 is an M – N fuzzy normal subgroup of G^1 is an M – N fuzzy normal subgroup of G .

Proposition 1.2.31[35]:

Let f be an M – N homomorphism from the M –N group G to the M – N group G^1 . Then the image which can be written as $f(A)$ of under f is an M – N fuzzy normal subgroup in case of A being an M – N fuzzy normal subgroup of G .

Let G be a M – N group and B be an M – N fuzzy normal subgroup of G . Wu [79] had proved that G/B is a M – N group.

Definition 1.2.32[63]:

Let G be an M –N group . A is said be an **M- N normal fuzzy subgroup** of G if A is not only an M- N fuzzy subgroup of G , but also normal fuzzy subgroup of G .

Definition 1.2.33:

Let G be an M –N group . A is said be an M- N **anti normal fuzzy subgroup** of G if A is not only an M- N fuzzy subgroup of G , but also anti fuzzy normal subgroup of G .

Proposition 1.2.34:

Let A be an M – N fuzzy subgroup of a group G , while the identity operator is included is $M \cap N$. Then A is an M – N fuzzy normal subgroup of G iff

$$(1) A(m(xy x^{-1})) = A(my), x, y \in G, m \in M.$$

$$(2) A((xy x^{-1})n) = A(yn), x, y \in G, n \in N$$

Proposition 1.2.35:

Let A be an $M - N$ anti fuzzy subgroup of a group G , while the identity operator is included is $M \cup N$. Then A is an $M - N$ anti fuzzy normal subgroup of G iff

$$(1) A(m(xy x^{-1})) = A(my), x, y \in G, m \in M.$$

$$(2) A((xy x^{-1})n) = A(yn), x, y \in G, n \in N$$

Proposition 1.2.36:

Let A be an $M - N$ fuzzy subgroup of $M - N$ group G , while the identity operator is included is $M \cap N$. Then A is an $M - N$ fuzzy normal subgroup of G iff $A(m(xy)) = A(m(yx))$ and $A((xy)n) = A(yx)n$ for all $x, y \in G, m \in M$ and $n \in N$.

Proposition 1.2.37:

Let A be an $M - N$ fuzzy subgroup of $M - N$ anti group G , while the identity operator is included is $M \cup N$. Then A is an $M - N$ anti fuzzy normal subgroup of G iff $A(m(xy)) = A(m(yx))$ and $A((xy)n) = A(yx)n$ for all $x, y \in G, m \in M$ and $n \in N$.

Definition 1.2.38[34]:

Let μ and λ be two fuzzy subgroup of G , then μ and λ are said to be **conjugate of fuzzy subgroup** of G if for some $g \in G, \mu(x) = \lambda(g^{-1}xg)$ for every $x \in G$.

In this thesis, we used the following notations and symbols:

M	—	Left Operator
N	—	Right Operator
$\max$	—	Maximum
$\min$	—	Minimum
(f, A)	—	Fuzzy Soft Subgroup
f_a, g_a, h_a	—	Fuzzy Soft Set
λ_a, μ_a	—	Fuzzy Soft Set
Sup	—	Supremum
Inf	—	Infimum

1.3 Summary of Results:

A short account of the results obtained in this thesis is given below:

The thesis is divided into five chapters and each chapter is further subdivided into a number sections.

In the initial chapter, we have given some basic definitions, results and examples are required for the development of the thesis.

In the second chapter, we defined the concept of M-N fuzzy soft subgroups based on the concept of fuzzy group. We have also defined the M-N fuzzy soft set and the M-N level subsets of a fuzzy soft subgroup. This work has been published in the “*Journal of Applied Science and Computations*”. In the second section of this chapter, we defined the M-N normal fuzzy soft set, discussed the intersection of two and three variables of M-N level subsets of normal fuzzy soft subgroups and some of its elementary properties that has been published in “*International Journal of Fuzzy Mathematical Archive*”. In the final section, we defined the M-N homomorphism of fuzzy soft subgroup and defined the M-N Homomorphism of level subsets of fuzzy soft subgroups. This part of content has been communicated in the Journal “*Advances and Applications in Mathematical Sciences*”.

In the Third chapter, we discussed the Conjugate of M-N fuzzy soft subgroups based on the concept of fuzzy soft group. The conjugate of M-N fuzzy soft subgroups and some elementary properties are also discussed. In the second section of this chapter, we defined and discussed the concept of M- N fuzzy soft middle coset with some related results. This content has been published in the “*Advances in Mathematics: Scientific Journal*”.

In the fourth chapter, we defined the M-N anti fuzzy soft set and described the M-N level subsets of Anti fuzzy soft subgroups. We have also discussed the concept of M-N Anti fuzzy soft subgroups with some of its elementary properties. This part of work has been

published in “*Journal of Computational Information Systems*”. In addition, we defined the M-N Anti fuzzy normal soft set and the intersection of two and three variables of M-N level subsets of anti-fuzzy normal soft subgroup. This part is published in “*International Journal of Mathematics and its Applications*”. Furthermore, we defined the M-N anti-homomorphism of fuzzy soft subgroup and the M-N Homomorphism of anti-level subsets of fuzzy soft subgroups. This part of content has been communicated in the Journal “*South East Asian Journal of Mathematics and Mathematical Sciences*”.

In the fifth chapter, we discussed the Conjugate of M-N anti-fuzzy soft subgroups based on the concept of fuzzy soft group. Further, we defined the conjugate of M-N anti-fuzzy soft subgroups and some of the elementary properties are also discussed. In the second section of this chapter, we defined the concept of M- N anti-fuzzy soft middle coset and discussed about some of the related results. This both content of the material has been published in the “*International Journal of Control and Automation*”.

CHAPTER – 2

M – N FUZZY SOFT SUBGROUPS OF A GROUP

2. M – N FUZZY SOFT SUBGROUPS OF A GROUP

In this chapter, we have intersected into three sections to explore the concept of M-N fuzzy soft subgroups of a group. The intersected sections are as follows,

2.1 M – N Fuzzy Soft Subgroups

In this Section, we defined the concept of M-N fuzzy subgroup based on the concept of fuzzy group. Here, M and N are left and right operator of a non – empty set. Soft set theory, proposed by Molodtsov [42], has been regarded as an effective mathematical tool to deal with uncertainties and completely new approach, by establishing the relation of M-N fuzzy soft subgroup of a group. We have defined the M-N fuzzy soft set and the M-N level subsets of a fuzzy soft subgroup. The concept of M-N fuzzy soft groups of some of its elementary properties is also discussed.

Definition 2.1.1

Let G be a group and (f, A) be an M – N fuzzy soft set over G. Then (f, A) is said to be a **M – N fuzzy soft group** over G iff for each $a \in A$ and $x, y \in G$,

$$(1) f_a \{m(x y) n\} \geq \min \{f_a(x), f_a(y)\}$$

$$(2) f_a \{(m x^{-1}) n\} \geq f_a(x) \text{ hold for each } a \in A, m \in M, n \in N, f_a \text{ is a M – N fuzzy soft subgroup of a group G.}$$

Theorem 2.1.2

Let G be a group and (f, A) be an M – N fuzzy soft set over G. Then (f, A) is said be an M – N fuzzy soft subgroup on a group G iff for each $a \in A$ and $x, y \in G$

Proof:

Assume that (f, A) is a M – N fuzzy soft subgroup of a group G

To prove that $f_a (m (xy^{-1}) n) \geq \min \{f_a(x), f_a(y)\}$

Let $x, y \in G$ and $a \in A$

$$\begin{aligned} \text{We have } f_a(m(xy^{-1})n) &\geq \min \{f_a(mx), f_a(y^{-1}n)\} \\ &\geq \min \{f_a(x), f_a(y^{-1})\} \quad (\text{by definition 2.1.1}) \\ &\geq \min \{f_a(x), f_a(y)\} \quad \text{by } A(x) = A(x^{-1}) \end{aligned}$$

$$\text{Therefore } f_a(m(xy^{-1})n) \geq \min \{f_a(x), f_a(y)\}$$

Conversely,

$$\text{Assume that, } f_a(m(xy^{-1})n) \geq \min \{f_a(x), f_a(y)\}$$

To prove that (f_a, A) is an M – N fuzzy subgroup of a group G

$$\begin{aligned} \text{Now } f_a(men) &= f_a(mxx^{-1}n) \\ &\geq \min \{f_a(mx), f_a(x^{-1}n)\} \\ &\geq \min \{f_a(x), f_a(x^{-1})\} \\ &\geq \min \{f_a(x), f_a(x)\} \\ &= f_a(x) \end{aligned}$$

Therefore $f_a(men) = f_a(x)$ where e is the identity element of G.

$$\begin{aligned} \text{Next } f_a(mx^{-1}n) &= f_a(mex^{-1}n) \\ &\geq \min \{f_a(me), f_a(x^{-1}n)\} \\ &\geq \min \{f_a(e), f_a(x^{-1})\} \\ &\geq \min \{f_a(e), f_a(x)\} \end{aligned}$$

$$f_a(mx^{-1}n) = f_a(x)$$

On the other hand, for each $a \in A$ and $x, y \in G$

$$\begin{aligned} f_a(mxyn) &= f_a(mx(y^{-1})^{-1}n) \\ &\geq \min \{f_a(mx), f_a((y^{-1})^{-1}n)\} \\ &\geq \min \{f_a(mx), f_a(y^{-1}n)\} \\ &\geq \min \{f_a(mx), f_a(yn)\} \end{aligned}$$

$$f_a(mxyn) \geq \min \{f_a(x), f_a(y)\}$$

Hence the proof.

Theorem 2.1.3

If (f, A) is an $M - N$ fuzzy soft set and e is the unit element of G . Then for each $a \in A$ and for each $x \in X$,

$$(1). f_a(x^{-1}) \geq f_a(x)$$

$$(2). f_a(e) = f_a(x)$$

Theorem 2.1.4

Let f_a and g_a be two $M - N$ fuzzy soft group of G , then $f_a \cap g_a$ is an $M - N$ fuzzy soft group of G .

Proof:

To prove that (1) $(f_a \cap g_a)(mxyn) \geq \min \{ (f_a \cap g_a)(x), (f_a \cap g_a)(y) \}$

$$(2) (f_a \cap g_a)(mx^{-1}n) = (f_a \cap g_a)(x)$$

$$(1) \Rightarrow (f_a \cap g_a)(mxyn) = \min \{ f_a(mxyn), g_a(mxyn) \}$$

$$\geq \min \{ \min \{ f_a(mx), f_a(yn) \}, \min \{ g_a(mx), g_a(yn) \} \}$$

$$\geq \min \{ \min \{ f_a(x), f_a(y) \}, \min \{ g_a(x), g_a(y) \} \}$$

$$\geq \min \{ \min \{ f_a(x), g_a(x) \}, \min \{ f_a(y), g_a(y) \} \}$$

$$(f_a \cap g_a)(mxyn) \geq \min \{ (f_a \cap g_a)(x), (f_a \cap g_a)(y) \}$$

$$(2) \Rightarrow (f_a \cap g_a)(mx^{-1}n) = (f_a \cap g_a)(mx^{-1}xx^{-1}n)$$

$$\geq \min \{ (f_a \cap g_a)(mx^{-1}x), (f_a \cap g_a)(x^{-1}n) \}$$

$$\geq \min \{ \min \{ (f_a \cap g_a)(mx^{-1}), (f_a \cap g_a)(x) \}, (f_a \cap g_a)(x^{-1}n) \}$$

$$\geq \min \{ \min \{ (f_a \cap g_a)(x^{-1}), (f_a \cap g_a)(x) \}, (f_a \cap g_a)(x^{-1}) \}$$

$$\geq \min \{ \min \{ (f_a \cap g_a)(x), (f_a \cap g_a)(x) \}, (f_a \cap g_a)(x) \}$$

$$\geq \min \{ (f_a \cap g_a)(x), (f_a \cap g_a)(x) \}$$

$$= (f_a \cap g_a)(x)$$

$$(f_a \cap g_a)(x^{-1}) = (f_a \cap g_a)(x)$$

Hence the proof.

Proposition 2.1.5

If f_a and g_a are two $M - N$ fuzzy soft group of G , then the following statements holds for all $x, y \in G, m \in M, n \in N$ and $a \in A$

$$(1) (f_a \wedge g_a)(mxyn) \geq \min \{ (f_a \wedge g_a)(x), (f_a \wedge g_a)(y) \}$$

$$(2) (f_a \wedge g_a)(mx^{-1}n) = (f_a \wedge g_a)(x)$$

Proposition 2.1.6

If f_a is an $M - N$ fuzzy soft subgroup of a group over G , then $f_a(m(xy)n)^2 = f_a(x^2y^2)$ is an $M - N$ fuzzy soft subgroup of a group G .

Proof:

Let $x, y \in G, m \in M, n \in N$ and $a \in A$

$$\begin{aligned} (1) \quad & \Rightarrow f_a(m(xy)n)^2 = f_a((m(xy)n), (m(xy)n)) \\ & \geq \min \{ f_a(m(xy)n), f_a(m(xy)n) \} \\ & \geq \min \{ \min \{ f_a(mx), f_a(yn) \}, \min \{ f_a(mx), f_a(yn) \} \} \\ & \geq \min \{ \min \{ f_a(x), f_a(y) \}, \min \{ f_a(x), f_a(y) \} \} \\ & \geq \min \{ \min \{ f_a(x), f_a(x) \}, \min \{ f_a(y), f_a(y) \} \} \\ & \geq \min \{ f_a(x \cdot x), f_a(y \cdot y) \} \\ & \geq \min \{ f_a(x^2), f_a(y^2) \} \\ & = f_a(x^2y^2) \end{aligned}$$

$$f_a(m(xy)n)^2 = f_a(x^2y^2)$$

$$\begin{aligned} (2) \quad & \Rightarrow f_a(m(xy)^{-1}n) = f_a(my^{-1}x^{-1}n) \\ & \geq \min \{ f_a(my^{-1}), f_a(x^{-1}n) \} \\ & \geq \min \{ f_a(y^{-1}), f_a(x^{-1}) \} \\ & \geq f_a(yx) \\ & \geq f_a(xy) \\ & f_a(m(xy)^{-1}n) = f_a(xy). \end{aligned}$$

Proposition 2.1.7

If $f_{ai}, i \in f_a$ is an $M - N$ fuzzy soft group of G , then $U f_{ai}$ is an $M - N$ fuzzy soft group G whose element $U f_{ai} = \{x, V f_{ai}(x) / x \in G\}$, where $i \in f_a$.

Proof:

Let $x, y \in G, m \in M, n \in N, a \in A$ and $i \in f_a$.

$$(1) \quad U f_{ai}(m(xy)n) \geq \min \{U f_{ai}(x), U f_{ai}(y)\}$$

$$\begin{aligned} U f_{ai}(m(xy)n) &= V f_{ai}(m(xy)n) \\ &\geq V \min \{f_{ai}(mx), f_{ai}(yn)\} \\ &\geq V \min \{f_{ai}(x), f_{ai}(y)\}, A(mx) \geq A(x) \\ &\geq \min \{V f_{ai}(x), V f_{ai}(y)\} \\ &\geq \min \{U f_{ai}(x), U f_{ai}(y)\} \end{aligned}$$

$$U f_{ai}(m(xy)n) \geq \min \{U f_{ai}(x), U f_{ai}(y)\}$$

$$(2) \quad U f_{ai}(m(x^{-1})n) \geq U f_{ai}(x)$$

$$\begin{aligned} U f_{ai}(m(x^{-1})n) &= U f_{ai}(m(x^{-1}xx^{-1})n) \\ &\geq V f_{ai}(m(x^{-1}xx^{-1})n) \\ &\geq V \min \{f_{ai}(mx^{-1}x), f_{ai}(x^{-1}n)\} \\ &\geq V \min \{\min \{f_{ai}(mx^{-1}), f_{ai}(x)\}, f_{ai}(x^{-1}n)\} \\ &\geq V \min \{\min \{f_{ai}(x^{-1}), f_{ai}(x)\}, f_{ai}(x^{-1})\}, \text{since } A(mx) \geq A(x) \\ &\geq V \min \{\min \{f_{ai}(x), f_{ai}(x)\}, f_{ai}(x)\}, \text{since } A(x) = A(x^{-1}) \\ &\geq V \min \{f_{ai}(x), f_{ai}(x)\} \\ &\geq V f_{ai}(x) \\ &\geq (U f_{ai})(x) \end{aligned}$$

$$\bigcup f_{ai}(m(x^{-1})n) \geq \bigcup f_{ai}(x)$$

Hence $\bigcup f_{ai}(x)$ is an M – N fuzzy soft group of G.

Proposition 2.1.8

Let G_1 and G_2 both be M – N group and ϕ be a soft homomorphism from G_1 and G_2 . If f_a is an M – N fuzzy soft group of G_2 , then the pre – image $\phi^{-1}(f_a)$ is an M – N fuzzy soft group of G_1

Proposition 2.1.9

Let $\phi : G_1 \rightarrow G_2$ be an epimorphism and f_a be an M – N fuzzy soft set in G_2 . If $\phi^{-1}(f_a)$ is an M – N fuzzy soft group of G_1 , f_a is an M – N fuzzy soft group of G_2 .

Proposition 2.1.10

Let f_a be an M – N fuzzy soft groups over G, and ϕ is endomorphism of G, then $f_a[\phi]$ is an M- N fuzzy soft group of G.

Proof:

Let $x, y \in G, m \in M, n \in N$ and $a \in A$

$$(1) \Rightarrow f_a[\phi](m(xy)n) = f_a[\phi](m(xy)n)$$

$$\geq \min \{f_a[\phi](mx), f_a[\phi](yn)\}$$

$$\geq \min \{f_a(\phi(x)), f_a(\phi(y))\}, \text{ since } A(mx) \geq A(x)$$

$$\geq \min \{f_a(\phi x), f_a(\phi y)\}$$

$$\geq \min \{f_a[\phi](x), f_a[\phi](y)\}$$

$$f_a[\phi](m(xy)n) \geq \min \{f_a[\phi](x), f_a[\phi](y)\}$$

$$(2) \Rightarrow f_a[\phi](m(x^{-1})n) = f_a[\phi](m(x^{-1}xx^{-1})n)$$

$$\geq \min \{f_a[\phi](mx^{-1}x), f_a[\phi](x^{-1}n)\}$$

$$\geq \min \{\min \{f_a[\phi](mx^{-1}), f_a[\phi](x)\}, f_a[\phi](x^{-1}n)\}$$

$$\begin{aligned}
&\geq \min \{ \min \{ f_a [\emptyset](x^{-1}), f_a [\emptyset](x) \}, f_a [\emptyset](x^{-1}) \} \\
&\geq \min \{ \{ f_a [\emptyset](x^{-1}), f_a [\emptyset](x^{-1}) \} \\
&\geq \min \{ f_a [\emptyset](x), f_a [\emptyset](x) \}, \text{ since } A(x^{-1}) = A(x) \\
&\geq f_a [\emptyset](x)
\end{aligned}$$

$$f_a [\emptyset] (m(x^{-1})n) \geq f_a [\emptyset](x)$$

Hence $f_a [\emptyset]$ is an M – N fuzzy soft group of G.

2.2 M – N Fuzzy Normal Soft Subgroups

In this Section, we have discussed the concept of M-N fuzzy normal soft subgroups based on the concept of fuzzy normal soft group [58 and 62]. We also discuss some results based on the fuzzy normal soft subgroups. We defined the M-N fuzzy normal soft set and also defined the intersection of two and three variables of M-N level subsets of a normal fuzzy soft subgroup with some of its elementary properties.

Definition 2.2.1

Let G be an M – N group and (F, A) be a fuzzy soft subgroup of G if

$$(1) F \{m(x y) n\} \geq \min \{F(x), F(y)\}$$

$$(2) F \{(m x^{-1}) n\} \geq F(x) \text{ hold for any } x, y \in G, m \in M, n \in N, \text{ then } (F, A) \text{ is}$$

said be an M – N fuzzy soft subgroup of G. Here $F: A \rightarrow P(G)$

Definition 2.2.2

Let G be an M – N group and (F, A) be a fuzzy soft subgroup of G if

$$(1) F(m x) \geq F(x)$$

$$(2) F(x n) \geq F(x) \text{ hold for any } x \in G, m \in M, \text{ and } n \in N, \text{ then } (F, A) \text{ is said be}$$

an M – N fuzzy soft subgroup of G.

Definition 2.2.3

Let G be an $M - N$ group. (F, A) is said be an $M - N$ fuzzy normal soft subgroup of G if (F, A) is not only an $M - N$ soft fuzzy subgroup of G , but also fuzzy normal soft subgroup of G .

Theorem 2.2.4

If α, β and γ are three $M - N$ fuzzy soft subgroup of G , then $\alpha \cap \beta \cap \gamma$ is a $M - N$ fuzzy soft subgroup of G .

Proof:

Let α, β and γ be three $M - N$ fuzzy soft subgroup of G .

$$(1) (\alpha \cap \beta \cap \gamma)(m(xy^{-1})n) \geq \min \{ (\alpha \cap \beta \cap \gamma)(m x), (\alpha \cap \beta \cap \gamma)(y^{-1}n) \}$$

$$(\alpha \cap \beta \cap \gamma)(m(xy^{-1})n) \geq \min \{ (\alpha \cap \beta)(m(xy^{-1})n), \gamma(m(xy^{-1})n) \}$$

$$\geq \min \{ \min \{ (\alpha \cap \beta)(m x), (\alpha \cap \beta)(y^{-1}n) \}, \min \{ \gamma(m x), \gamma(y^{-1}n) \} \}$$

$$\geq \min \{ \min \{ (\alpha \cap \beta)(m x), \gamma(m x) \}, \min \{ (\alpha \cap \beta)(y^{-1}n), \gamma(y^{-1}n) \} \}$$

$$\geq \min \{ (\alpha \cap \beta \cap \gamma)(m x), (\alpha \cap \beta \cap \gamma)(y^{-1}n) \}$$

$$(2) (\alpha \cap \beta \cap \gamma)(m x n) = (\alpha \cap \beta \cap \gamma)(m x^{-1}n)$$

$$(\alpha \cap \beta \cap \gamma)(m x n) = \{ (\alpha \cap \beta)(m x n), \gamma(m x n) \}$$

$$= \{ [\alpha(m x n), \beta(m x n)], \gamma(m x n) \}$$

$$= \{ [\alpha(m x^{-1}n), \beta(m x^{-1}n)], \gamma(m x n) \}$$

$$= \{ (\alpha \cap \beta)(m x^{-1}n), \gamma(m x^{-1}n) \}$$

$$= (\alpha \cap \beta \cap \gamma)(m x^{-1}n)$$

Hence $\alpha \cap \beta \cap \gamma$ is an $M - N$ fuzzy soft subgroup of G .

Theorem 2.2.5

The intersection of any three $M - N$ fuzzy normal soft subgroups of G is also an $M - N$ fuzzy normal soft subgroup G .

Proof:

Let α, β and γ are three $M - N$ fuzzy normal soft subgroup of G .

By the previous theorem we know that, $\alpha \cap \beta \cap \gamma$ is an $M - N$ fuzzy soft subgroup of G .

Let $x, y \in G$, $m \in M$, and $n \in N$

To prove that $(\alpha \cap \beta \cap \gamma)(m y x y^{-1} n) = (\alpha \cap \beta \cap \gamma)(m x n)$

$$\begin{aligned} \text{Now } (\alpha \cap \beta \cap \gamma)(m y x y^{-1} n) &= \min \{ (\alpha \cap \beta)(m(y x y^{-1})n), \gamma(m(y x y^{-1})n) \} \\ &= \min \{ [\alpha(m(y x y^{-1})n), \beta(m(y x y^{-1})n), \gamma(m(y x y^{-1})n)] \} \\ &= \min \{ [\alpha(m x n), \beta(m x n), \gamma(m x n)] \} \\ &= \min \{ (\alpha \cap \beta)(m x n), \gamma(m x n) \} \\ &= (\alpha \cap \beta \cap \gamma)(m x n) \end{aligned}$$

Hence $(\alpha \cap \beta \cap \gamma)(m y x y^{-1} n) = (\alpha \cap \beta \cap \gamma)(m x n)$.

Hence $\alpha \cap \beta \cap \gamma$ is an $M - N$ fuzzy normal soft subgroup G .

Note 2.2.6

If $(\alpha \cap \beta)_i, i \in \Delta$ are $M - N$ fuzzy normal soft subgroup of G , then $\bigcap_{i \in \Delta} (\alpha \cap \beta)_i$ is a $M - N$ fuzzy normal soft subgroup of G .

Definition 2.2.7

Let G be a group, α is a $M - N$ fuzzy soft subgroup of G is said be a $M - N$ fuzzy normal soft subgroup if

$$\alpha(m(x y x^{-1})n) = \alpha(m y n) \text{ (or)}$$

$$\alpha(m(x y)n) \geq \alpha(m(y x)n) \text{ for all } x, y \in G, m \in M, \text{ and } n \in N$$

Theorem 2.2.8

Let α is an M – N fuzzy normal soft subgroup of G, then for any $y \in G$ we have

$$\alpha(m(y^{-1}xy) n) = \alpha(m(yxy^{-1}) n)$$

Proof:

Let α be an M- N fuzzy normal soft subgroup G, then for any $y \in G$.

$$\begin{aligned} \text{Now } \alpha(m(y^{-1}xy) n) &= \alpha(m(xy^{-1}y) n) \\ &= \alpha(m(x) n) \\ &= \alpha(m(yy^{-1}x) n) \\ &= \alpha(m(yxy^{-1}) n) \end{aligned}$$

Therefore $\alpha(m(y^{-1}xy) n) = \alpha(m(yxy^{-1}) n)$

Hence the theorem

Theorem 2.2.9

If α is an M- N fuzzy normal soft subgroup G, then $g\alpha g^{-1}$ is also M- N fuzzy normal soft subgroup G, for all $g \in G$.

Proof:

Let α be an M- N fuzzy normal soft subgroup G, then $g\alpha g^{-1}$ is an M- N fuzzy subgroup G, for all $g \in G$

$$\begin{aligned} \text{Now } g\alpha g^{-1}(m(yxy^{-1}) n) &= \alpha(g^{-1}m(yxy^{-1}) n) g \\ &= \alpha(m(yxy^{-1}) n) \\ &= \alpha(m(x) n) \\ &= \alpha(g(m(xn)) g^{-1}) \\ &= g\alpha g^{-1}(m(x) n) \end{aligned}$$

Therefore $g \alpha g^{-1} (m (y x y^{-1}) n) = g \alpha g^{-1} (m x n)$

Hence the theorem.

Theorem 2.2.10

Let $\alpha \cap \beta$ is a M – N fuzzy normal soft subgroup of G, then $(\alpha \cap \beta) (m (y^{-1} x y) n) = (\alpha \cap \beta) (m x n)$.

Proof:

Let $\alpha \cap \beta$ is an M – N fuzzy normal soft subgroup of G.

Let x, y G

$$\begin{aligned} \text{Now } (\alpha \cap \beta) (m (y^{-1} x y) n) &= (\alpha \cap \beta) (m (xy^{-1}y) n) \\ &= (\alpha \cap \beta) (m (x) n) \\ &= (\alpha \cap \beta) (m (yy^{-1} x) n) \\ &= (\alpha \cap \beta) (m (y xy^{-1}) n) \end{aligned}$$

$$(\alpha \cap \beta) (m (y^{-1} x y) n) = (\alpha \cap \beta) (m (y xy^{-1}) n)$$

Hence the proof

Theorem 2.2.11

If $\alpha \cap \beta$ is a M- N fuzzy normal soft subgroup G, then $g(\alpha \cap \beta) g^{-1}$ is also a M- N fuzzy normal soft subgroup G, for all $g \in G$.

Proof:

If $\alpha \cap \beta$ is a M- N fuzzy normal soft subgroup G, then $g(\alpha \cap \beta) g^{-1}$ is also a M- N fuzzy normal soft subgroup G, for all $g \in G$.

$$\text{To prove that } g(\alpha \cap \beta) g^{-1} (m (y x y^{-1}) n) = g(\alpha \cap \beta) g^{-1} (m x n)$$

$$\begin{aligned} \text{Now } g(\alpha \cap \beta) g^{-1} (m (y x y^{-1}) n) &= (\alpha \cap \beta)(g^{-1} m (y x y^{-1}) n) g \\ &= (\alpha \cap \beta)(m (y x y^{-1}) n) \end{aligned}$$

$$= (\alpha \cap \beta)(m \times n)$$

$$= (\alpha \cap \beta)(g(m \times n)g^{-1})$$

$$= g(\alpha \cap \beta)g^{-1}(m \times n)$$

$$\text{Therefore } g(\alpha \cap \beta)g^{-1}(m(y \times y^{-1})n) = g(\alpha \cap \beta)g^{-1}(m \times n)$$

Hence the theorem

Definition 2.2.12

Let $\alpha \cap \beta$ be an M – N fuzzy soft subgroup of a group G. For any $t \in [0, 1]$, we define the M – N level subset of $\alpha \cap \beta$ is the set

$$(\alpha \cap \beta)_t = \{x \in G / (\alpha \cap \beta)(m \times x) \geq t, (\alpha \cap \beta)(x \times n) \geq t \text{ for all } m \in M, n \in N\}$$

Theorem 2.2.13

Let G be a group and $\alpha \cap \beta$ be a fuzzy subset of G. Then $\alpha \cap \beta$ is a M – N fuzzy normal soft subgroup of G iff the level subset $(\alpha \cap \beta)_t, t \in [0, 1]$ are M- N subgroup of G.

Proof:

Let $\alpha \cap \beta$ be an M – N fuzzy normal soft subgroup of G and the level subset

$$(\alpha \cap \beta)_t = \{x \in G / (\alpha \cap \beta)(m \times x) \geq t, (\alpha \cap \beta)(x \times n) \geq t, t \in [0, 1] m \in M, n \in N\}$$

$$\text{Let } x, y \in (\alpha \cap \beta)_t \text{ then } (\alpha \cap \beta)(m \times x) \geq t \text{ and } (\alpha \cap \beta)(x \times n) \geq t$$

$$\text{Now } (\alpha \cap \beta)(m \times y^{-1}n) \geq \min \{(\alpha \cap \beta)(m \times x), (\alpha \cap \beta)(y^{-1}n)\}$$

$$= \min \{(\alpha \cap \beta)(m \times x), (\alpha \cap \beta)(y \times n)\}$$

$$\geq \min \{t, t\}$$

$$(\alpha \cap \beta)(m \times y^{-1}n) \geq t$$

$$m \times y^{-1}n \in (\alpha \cap \beta)_t$$

Therefore $(\alpha \cap \beta)_t$ is a M – N subgroup of G.

Conversely,

Let us assume that $(\alpha \cap \beta)_t$ is an M – N subgroup G.

Let $x, y \in (\alpha \cap \beta)_t$ then $(\alpha \cap \beta)(mx) \geq t$ and $(\alpha \cap \beta)(xn) \geq t$

Also $(\alpha \cap \beta)(m(xy^{-1})n) \geq t$

Since $m(xy^{-1})n \in (\alpha \cap \beta)_t = \min\{t, t\}$

$$= \min\{(\alpha \cap \beta)(mx), (\alpha \cap \beta)(xn)\}$$

Therefore $(\alpha \cap \beta)(m(xy^{-1})n) \geq \min\{(\alpha \cap \beta)(mx), (\alpha \cap \beta)(xn)\}$

Hence $\alpha \cap \beta$ is an M – N fuzzy normal soft subgroup of G.

Definition 2.2.14

Let G be a group and $\alpha \cap \beta$ be an M – N fuzzy normal soft subgroup of G.

Let $N(\alpha \cap \beta) = \{y \in G / (\alpha \cap \beta)(m(yxy^{-1})n) = (\alpha \cap \beta)(mxn) \text{ for all } x \in G, m \in M, n \in N\}$, then $N(\alpha \cap \beta)$ is called the M – N fuzzy soft Normalizer of $\alpha \cap \beta$.

Theorem 2.2.15

Let G be a group and $\alpha \cap \beta$ be a fuzzy subset of G. Then $\alpha \cap \beta$ is a M – N fuzzy normal soft subgroup of G iff the level subset $(\alpha \cap \beta)_t, t \in [0, 1]$ are M- N normal subgroup of G.

Proof:

Let $\alpha \cap \beta$ be a M – N fuzzy normal soft subgroup of G and level subset $(\alpha \cap \beta)_t, t \in [0, 1]$

Let $x \in G$ and $y \in (\alpha \cap \beta)_t$, then $(\alpha \cap \beta)(my n) \geq t$, for all $m \in M, n \in N$

Now $(\alpha \cap \beta)(m(xyx^{-1})n) = (\alpha \cap \beta)(my n) \geq t$

Since $\alpha \cap \beta$ is an M – N fuzzy normal soft subgroup of G.

That is $(\alpha \cap \beta)(m(xy x^{-1})n) \geq t$

Therefore $m(xy x^{-1})n \in (\alpha \cap \beta)_t$

Hence $(\alpha \cap \beta)_t$ is an M – N normal subgroup of G.

2.3 M-N Homomorphism of fuzzy soft subgroups

In this section, we shall define the M-N homomorphism of fuzzy soft subgroup and define the M-N level subsets of a fuzzy soft subgroup. We have also discussed the concept of M-N homomorphism of fuzzy soft groups and some of its elementary properties.

Definition 2.3.1

Let G and G^1 be any two $M - N$ groups. If (f, A) is an fuzzy soft subgroup of an $M - N$ group G , then the function $f_a : G \rightarrow G^1$ is said be an $M - N$ homomorphism of fuzzy soft subgroup if

- (1) $f_a(xy) = f_a(x) f_a(y)$ for all $x, y \in G, a \in A$
- (2) $f_a(mx) = m f_a(x)$, for all $x \in G, a \in A$ and $m \in M$
- (3) $f_a(y_n) = f_a(y)n$, for all $y \in G, a \in A$ and $n \in N$

Note 2.3.2

If λ is a constant and $\ker f_a$ is an $M - N$ fuzzy soft subgroup, then

- (1) $f_a(\lambda) f_a(mx) = \lambda(mx) = \lambda(x)$, for all $x \in G, a \in A$ and $m \in M$.
- (2) $f_a(\lambda) f_a(xn) = \lambda(xn) = \lambda(x)$, for all $x \in G, a \in A$ and $n \in N$.

Theorem 2.3.3

Let f_a be an $M - N$ homomorphism of fuzzy soft subgroup from an $M - N$ group G onto an $M - N$ group G^1 . If λ is an $M - N$ fuzzy subgroup of G and λ is an f_a -soft invariant, then $f_a(\lambda)$ is an $M - N$ fuzzy soft subgroup of G^1 .

Proof:

We know that λ is a constant and $\ker f_a$ is an $M - N$ fuzzy soft subgroup.

$$\begin{aligned} \text{Now } f_a(\lambda) (f_a(x) f_a(y)) &= f_a(\lambda) (f_a(xy)) , \text{ for all } x, y \in G, a \in A \\ &= \lambda(xy) , \text{ since by note} \end{aligned}$$

$$\geq \min \{ \lambda(x), \lambda(y) \}$$

$$\geq \min \{ f_a(\lambda)(f_a(x), f_a(\lambda) f_a(y)) \}$$

Therefore $f_a(\lambda)(f_a(x) f_a(y)) \geq \min \{ f_a(\lambda)(f_a(x), f_a(\lambda) f_a(y)) \}$

Clearly $f_a(\lambda)$ is an fuzzy soft subgroup of G^1 .

To prove that $f_a(\lambda)$ is an M – N fuzzy soft subgroup of G^1

Let $f_a(\lambda) \in G^1$, then

$$\begin{aligned} (1) \quad f_a(\lambda)(m f_a(x)) &= f_a(\lambda)(f_a(mx)) \\ &= \lambda(mx) \\ &\geq \lambda(x), \text{ by the definition } A(mx) \geq A(x) \\ &= f_a(\lambda) f_a(x) \end{aligned}$$

$$\text{Therefore } f_a(\lambda)(m f_a(x)) = f_a(\lambda) f_a(x)$$

$$\begin{aligned} (2) \quad f_a(\lambda)(f_a(x)n) &= f_a(\lambda)(f_a(xn)) \\ &= \lambda(xn) \\ &\geq \lambda(x), \text{ by the definition } A(xn) \geq A(x) \\ &= f_a(\lambda) f_a(x) \end{aligned}$$

$$\text{Therefore } f_a(\lambda)(f_a(x)n) = f_a(\lambda) f_a(x)$$

Hence $f_a(\lambda)$ is an M – N fuzzy soft subgroup of G^1 .

Theorem 2.3.4

The M – N homomorphic pre- image of an M – N fuzzy soft subgroup of an M – N group G^1 is an M – N fuzzy soft subgroup of an M – N group G .

Proof:

Let $f_a: G \rightarrow G^1$ is said be an M – N homomorphism of fuzzy soft subgroup

Let μ be an fuzzy set on the $M - N$ fuzzy subgroup of G^1 .

Now $\lambda(xy) = \mu(f_a(xy))$ for all $a \in A$ and $x, y \in G$

$$= \mu(f_a(x) f_a(y)), \text{ since } f_a \text{ is an homomorphism}$$

$$\geq \min \{ \mu f_a(x), \mu f_a(y) \}, \text{ since } \mu \text{ is an fuzzy subgroup of } G^1$$

$$= \min \{ \lambda(x), \lambda(y) \}$$

That is $\lambda(xy) \geq \min \{ \lambda(x), \lambda(y) \}$

Let $x \in G$,

$$\lambda(X^{-1}) = \mu(f_a(x^{-1}))$$

$$= \mu(f_a(x)^{-1}), \text{ since } f_a \text{ is an homomorphism of fuzzy soft subgroup}$$

$$= \mu(f_a(x)), \text{ since } \mu \text{ is an } M - N \text{ fuzzy subgroup of } G^1$$

$$= \lambda(x)$$

$$\lambda(X^{-1}) \geq \lambda(x)$$

Clearly $\lambda(mx) = \mu(f_a(mx))$ for some $m \in M$ and $x \in G$

$$= \mu(mf_a(x)), \text{ since } f_a \text{ is an } M - N \text{ homomorphism of an fuzzy soft group}$$

$$\geq \mu f_a(x), \text{ since } \mu \text{ } M - N \text{ fuzzy subgroup of } G$$

$$= \lambda(x)$$

That is $\lambda(mx) \geq \lambda(x)$

Next

$$\lambda(xn) = \mu(f_a(xn)), \text{ for some } n \in N \text{ and } x \in G$$

$$= \mu(nf_a(x)), \text{ since } f_a \text{ is an } M - N \text{ homomorphism of an fuzzy soft group}$$

$$\geq \mu f_a(x), \text{ since } \mu \text{ } M - N \text{ fuzzy subgroup of } G$$

$$= \lambda(x)$$

That is, $\lambda(xn) \geq \lambda(x)$

Hence, λ is an $M - N$ fuzzy subgroup of G .

Theorem 2.3.4

If $f_a: G \rightarrow G^1$ is an M – N homomorphism of an fuzzy soft subgroup of a group G, then,

$$(1) f_a(e) = e^1, \text{ where } e^1 \text{ is the unity element of } G^1$$

$$(2) f_a(x^{-1}) = f_a(x)^{-1} \text{ for all } x \in G.$$

Proof:

Given that $f_a: G \rightarrow G^1$ is an M – N homomorphism of an fuzzy soft subgroup of a group G,

$$(1) \Rightarrow \text{Suppose } f_a(mx) e^1 = f_a(mx) = f_a(x), \text{ for some } m \in M, a \in A \text{ and } x \in G$$

$$= f_a(xe), \text{ since } e \text{ is an identity element in } G$$

$$= f_a(x) f_a(e), \text{ since } f_a \text{ is an homomorphism of an fuzzy soft subgroup}$$

$$f_a(x) e^1 = f_a(x) f_a(e), \text{ by left cancellation law}$$

$$\text{Therefore } f_a(e) = e^1$$

Similarly, we can prove that $f_a(xn) e^1 = f_a(xn) = f_a(x)$, for some $n \in N, a \in A$ and $x \in G$

$$\text{That implies } f_a(e) = e^1$$

$$(2) \Rightarrow \text{We know that } e^1 = f_a(me), \text{ since } A(mx) \geq A(x)$$

$$= f_a(e)$$

$$= f_a(xx^{-1})$$

$$= f_a(x) f_a(x^{-1}) \text{ since } f_a \text{ is an homomorphism of an fuzzy soft subgroup}$$

$$e^1 (f_a(x))^{-1} = f_a(x^{-1})$$

$$(f_a(x))^{-1} = f_a(x^{-1})$$

Similarly, we can prove that $e^1 = f_a(en)$, since $A(xn) \geq A(x)$

$$\text{That implies } (f_a(x))^{-1} = f_a(x^{-1})$$

Hence the proof

Definition 2.3.5

Let μ be an M – N fuzzy subgroup of an M – N group G. Then the M – N subgroup μ_t for $t \in [0, 1]$ and $t \geq \mu(e)$, are called level M – N subgroup of μ .

Theorem 2.3.6

The M – N homomorphic image of a level M – N subgroup of an M – N fuzzy subgroup μ of an M – N group G is a level M – N subgroup of an M – N fuzzy soft subgroup $f_a(\mu)$ of an M – N soft subgroup G^1 , where μ is f_a – soft invariant.

Proof:

Let G and G^1 be any two M – N group.

Let $f_a: G \rightarrow G^1$ be an M – N homomorphism of an fuzzy soft subgroup of a group G.

Let μ be an M – N fuzzy subgroup of G.

Clearly $f_a(\mu)$ is an M – N fuzzy soft subgroup of G^1 .

Let μ_t be a level M – N subgroup of an M – N fuzzy subgroup μ of G.

Since f_a is an M – N homomorphism fuzzy soft subgroup,

$f_a(\mu)$ is an M – N soft subgroup $f_a(\mu)$ of G^1 and $f_a(\mu_t) = (f_a(\mu))_t$

Hence $(f_a(\mu))_t$ is a level M – N soft subgroup $f_a(\mu)$ of G^1 .

Theorem 2.3.7

The M – N homomorphism pre – image of a level M – N soft subgroup of an M – N fuzzy subgroup μ of an M – N group G^1 is a level M – N subgroup of an M- N fuzzy soft subgroup $f_a^{-1}(\mu)$ of an M – N group G.

Proof:

Let $a \in A$, $m \in M$ and $n \in N$

Let G and G^1 be any two M – N group.

Let $f_a: G \rightarrow G^1$ be an M – N homomorphism of an fuzzy soft subgroup of a group G.

Let μ be an M – N fuzzy subgroup of G^1 .

Clearly $f_a^{-1}(m\mu) = f_a^{-1}(\mu)$ and $f_a^{-1}(\mu n) = f_a^{-1}(\mu)$ is an M – N fuzzy soft subgroup of G.

Let μ_t be a level M – N subgroup of an M – N fuzzy subgroup μ of G^1 .

Since f_a is an M – N homomorphism fuzzy soft subgroup,

$f_a^{-1}(\mu_t)$ is an M – N soft subgroup of $f_a^{-1}(\mu)$ of G and $f_a^{-1}(\mu_t) = (f_a^{-1}(\mu))_t$, is an M – N soft subgroup of an M – N fuzzy soft subgroup $f_a^{-1}(\mu)$ of G.

That is $(f_a^{-1}(\mu))_t$ is a level M – N subgroup of an M – N fuzzy soft subgroup $f_a^{-1}(\mu)$ of G .

Hence the proof

CHAPTER – 3

CONJUGATE OF M-N FUZZY SOFT SUBGROUPS OF A GROUP

3. CONJUGATE OF M-N FUZZY SOFT SUBGROUPS OF A GROUP

In this chapter, we have intersected into two different sections to expose the Conjugate of M-N fuzzy soft subgroups of a group. The intersected sections are as follows,

3.1 Conjugate of M-N fuzzy soft subgroups

In this section, we have defined the conjugate of M-N fuzzy soft subgroups. We have discussed the Conjugate of M-N fuzzy soft subgroups based on the concept of fuzzy soft group [5 and 34], with some of its elementary properties.

Definition 3.1.1

Let μ_a and λ_a be two M - N fuzzy soft subgroup of G, then μ_a and λ_a are said to be **conjugate of M - N fuzzy soft subgroup** of G if for some $g \in G$ and $a \in A$.

$$(1) \mu_a(mx) = \lambda_a(g^{-1}xg) \text{ for every } x \in G, m \in M \text{ and } a \in A$$

$$(2) \mu_a(xn) = \lambda_a(g^{-1}xg) \text{ for every } x \in G, n \in N \text{ and } a \in A$$

Definition 3.1.2

Let G be an M – N group and (μ, A) be a conjugate of fuzzy soft subgroup of G if

$$(1) \mu_a(mx) \geq \mu_a(x) = \lambda_a(g^{-1}xg)$$

$$(2) \mu_a(y n) \geq \mu_a(y) = \lambda_a(g^{-1}y g) \text{ hold for any } x, y \in G, m \in M, \text{ and } n \in N,$$

then (μ, A) is said to be a **conjugate of M – N fuzzy soft subgroup** of G.

Theorem 3.1.3

Let μ_a and λ_a be any M - N fuzzy soft subgroup of the group G, then μ_a and λ_a are conjugate of M - N fuzzy soft subgroup of G iff $\mu_a = \lambda_a$

Proof:

Given that μ_a and λ_a are conjugate of M - N fuzzy soft subgroup of the group G

To prove that $\mu_a = \lambda_a$

Since μ_a and λ_a are conjugate of M - N fuzzy soft subgroup of the group G

By the definition there exists $g \in G$, such that

$$\mu_a(mx) = \lambda_a(g^{-1}xg) \text{ for every } x \in G, m \in M \text{ and } a \in A.$$

$$\mu_a(xn) = \lambda_a(g^{-1}xg) \text{ for every } x \in G, n \in N \text{ and } a \in A.$$

Let $mx = gmx$ for all $g, x \in G, m \in M$, then

$$\mu_a(gmx) = \lambda_a(g^{-1}gxg)$$

$$\mu_a(gmx) = \lambda_a(xg)$$

$$\mu_a(gx) = \lambda_a(xg), \text{ since } \mu(mx) \geq \mu(x)$$

And let $xn = gxn$ for all $g, x \in G, n \in N$, then

$$\mu_a(gxn) = \lambda_a(g^{-1}gxg)$$

$$\mu_a(gxn) = \lambda_a(xg)$$

$$\mu_a(gx) = \lambda_a(xg), \text{ since } \mu(xn) \geq \mu(x)$$

For some $g = e \in G, m \in M$

We have $\mu_a(mex) = \lambda_a(mxe)$

$$\mu_a(mx) = \lambda_a(mx), \text{ since } \mu(xm) \geq \mu(x)$$

$$\mu_a(x) = \lambda_a(x)$$

$$\mu_a = \lambda_a$$

Similarly we can prove that $\mu_a(exn) = \lambda_a(xen)$

$$\text{Hence } \mu_a = \lambda_a$$

Conversely,

To prove that, μ_a and λ_a are conjugate of M - N fuzzy soft subgroup of G

$$\text{Let } \mu_a = \lambda_a$$

$$\mu_a(mx) = \lambda_a(mx)$$

$$\mu_a(mx) = \lambda_a(x) = \lambda(x), \text{ since } \mu(xm) \geq \mu(x)$$

By the definition, $\mu_a(mx) = \lambda_a(e^{-1}xe)$

Similarly we can prove that $\mu_a(xn) = \lambda_a(e^{-1}xe)$

Hence μ_a and λ_a are conjugate of M - N fuzzy soft subgroup of G

Theorem 3.1.4

Let λ_a be an M - N fuzzy soft subgroup of a group G, and μ_a be a fuzzy soft subset of G. If μ_a and λ_a are conjugate of M - N fuzzy soft subgroup of the group G, then μ_a is an M - N fuzzy soft subgroup of a group G.

Proof:

Let e be an identity element of the group G.

If μ_a and λ_a are conjugate of M - N fuzzy soft subgroup of the group G

By the definition, since there exists an element $g \in G$, such that

$$\lambda_a(x) \leq \lambda_a(mx) = \mu_a(g^{-1}xg) \text{ for all } x \in G \text{ and } a \in A.$$

$$\Rightarrow \lambda_a(x) = \mu_a(g^{-1}xg) \text{ and}$$

$$\lambda_a(x) \leq \lambda_a(xn) = \mu_a(g^{-1}xng) \text{ for all } x \in G, m \in M, \text{ and } n \in N,$$

$$\Rightarrow \lambda_a(x) = \mu_a(g^{-1}xng)$$

$$\text{Also } \lambda_a(x) \leq \mu_a(mx) = \mu_a(exe)$$

$$\mu_a(x) = \mu_a(g^{-1}gxg^{-1}g)$$

$$= \lambda_a(gxg^{-1})$$

$$\text{Therefore } \mu_a(x) = \lambda_a(gxg^{-1})$$

$$\text{Similarly we can prove that } \mu_a(x) \leq \mu_a(xn) = \mu_a(gxg^{-1})$$

To prove that, μ_a is an M - N fuzzy soft subgroup of a group G.

Since λ_a be an M - N fuzzy soft subgroup of a group G,

$$\text{Now } \mu_a(xy) \leq \mu_a(mxy) = \mu_a(emxeyne)$$

$$\mu_a(mxy) = \mu_a(g^{-1}gm x g^{-1}g y n g^{-1})$$

$$= \lambda_a(gm x g^{-1}g y n g^{-1})$$

$$\geq \min \{ \lambda_a (g m x g^{-1}), \lambda_a (g y n g^{-1}) \}, \text{ since } \mu (xm) \geq \mu (x)$$

$$\mu_a (mxyn) \geq \min \{ \lambda_a (gx g^{-1}), \lambda_a (gy g^{-1}) \}$$

$$\mu_a (xy) \geq \min \{ \mu_a (x), \mu_a (y) \}$$

$$\text{Also } \mu_a (mx^{-1}n) = \lambda_a (gm xy^{-1} g^{-1} g n y g^{-1})$$

$$\geq \min \{ \lambda_a (g m x g^{-1} g y^{-1} g^{-1}), \lambda_a (g y n g^{-1}) \},$$

$$\geq \min \{ \min \{ \lambda_a (g m x g^{-1}) \lambda_a (g y^{-1} g^{-1}) \}, \lambda_a (g y n g^{-1}) \}, \text{ since } \mu (xm) \geq \mu (x)$$

$$\geq \min \{ \min \{ \lambda_a (gx g^{-1}) \lambda_a (g y^{-1} g^{-1}) \}, \lambda_a (gy g^{-1}) \},$$

$$\geq \min \{ \{ \lambda_a (gx g^{-1}), \lambda_a (gy g^{-1}) \}, \mu (y) = \mu (y^{-1})$$

$$\geq \min \{ \lambda_a (gx g^{-1}), \lambda_a (gy^{-1} g^{-1}) \}$$

$$= \lambda_a (gx g^{-1})$$

$$\mu_a (mx^{-1}n) = \lambda_a (gx g^{-1})$$

Hence μ_a is an M - N fuzzy soft subgroup of a group G.

3.2 M-N Fuzzy Soft Middle Coset.

In this section, we define the concept of M- N fuzzy soft middle coset subgroups of a group and discuss some related results.

Definition 3.2.1

Let λ_a be an M- N fuzzy soft subgroup of a group G. then for any $s, t \in G$ the **M – N fuzzy soft middle coset** $s \lambda_a t$ of the G is defined by $(s \lambda_a t) (mxn) = \lambda_a (s^{-1} x t^{-1})$ for all $x \in G$.

Theorem 3.2.2

If λ_a is an M – N fuzzy soft subgroup of a group G, then for any $a \in G$, the M – N fuzzy soft middle coset $s \lambda_a s^{-1}$ of the group G is also a M – N fuzzy soft subgroup of the group G.

Proof:

Let λ_a be an M – N fuzzy soft subgroup of a group G and $a \in G$

Let $x, y \in G$, $m \in M$ and $n \in N$, then

$$\begin{aligned} (s \lambda_a s^{-1})(m xy^{-1}n) &= \lambda_a (ms^{-1} xy^{-1} sn) \\ &= \lambda_a (ms^{-1} xss^{-1} y^{-1} sn) \\ &= \lambda_a (m (s^{-1} xs) (s^{-1} y^{-1} s)n) \\ &\geq \min \{ \lambda_a (m(s^{-1}xs)), \lambda_a (s^{-1}y^{-1} s)n \}, \text{ since } f_a(mx) \geq f_a(x) \text{ and } f_a(yn) \geq f_a(y) \\ &\geq \min \{ \lambda_a ((s^{-1}xs), \lambda_a (s^{-1}y^{-1} s) \}, \end{aligned}$$

Since λ_a is an M – N fuzzy soft subgroup of a group G

Therefore $(s \lambda_a s^{-1})(m xy^{-1}n) \geq \min \{ \lambda_a ((s^{-1}xs), \lambda_a (s^{-1}y^{-1} s)) \}$

Hence $s \lambda_a s^{-1}$ is an M – N fuzzy soft subgroup of the group G

Theorem 3.2.3

Let λ_a be any M – N fuzzy soft subgroup of a group G and $s \lambda_a s^{-1}$ be an M – N fuzzy soft middle coset of G, then $o(s \lambda_a s^{-1}) = o(\lambda_a)$ for any $s \in G$.

Proof:

Let λ_a be an M – N fuzzy soft subgroup of a group G and $s \in G$

By the above theorem (That is Theorem 3.2.2)

$s \lambda_a s^{-1}$ is an M – N fuzzy soft subgroup of the group G

Thus $(s \lambda_a s^{-1})(m xn) = \lambda_a (ms^{-1} x sn)$, for all $x \in G$, $m \in M$ and $n \in N$

Therefore λ_a and $s \lambda_a s^{-1}$ are conjugate of M - N fuzzy soft subgroup of G

We know that the theorem if λ_a and μ_a are conjugate of M –N fuzzy soft subgroup of the group G, then $o(\lambda_a) = o(\mu_a)$

Hence $o(s \lambda_a s^{-1}) = o(\lambda_a)$ for any $s \in G$.

Definition 3.2.4

Let λ and μ be an M – N fuzzy soft subgroup of the group G, and f be a positive fuzzy set, then for a $\in G$ we define the **M – N positive double fuzzy soft coset**

$$(\lambda a \mu)^f \text{ by } (\lambda a \mu)^f = \min \{ (a \lambda)^f, (a \mu)^f \}$$

Theorem 3.2.5

The $M - N$ positive double fuzzy soft coset $(\lambda_a \text{ s } \mu_a)^f$ is $M - N$ fuzzy soft subgroup of the group G , when λ_a, μ_a are $M - N$ fuzzy soft subgroup of G

Proof:

Let $x, y \in G, m \in M$ and $n \in N$

$$\begin{aligned} \text{Now } (\lambda_a \text{ s } \mu_a)^f (mxy^{-1}n) &= \min \{ (s \lambda_a)^f (mxy^{-1}n), (s \mu_a)^f (mxy^{-1}n) \} \\ &= \min \{ f(s) \lambda_a (mxy^{-1}n), f(s) \mu_a (mxy^{-1}n) \} \\ &\geq f(s) \min \{ \min \{ \lambda_a (mx), \lambda_a (y^{-1}n) \}, \min \{ \mu_a (mx), \mu_a (y^{-1}n) \} \} \end{aligned}$$

Since $\lambda_a (mx) \geq \lambda_a (x)$, $\lambda_a (x^{-1}) = \lambda_a (x)$ and $\mu_a (xn) \geq \mu_a (x)$, $\mu_a (x^{-1}) = \mu_a (x)$

$$\begin{aligned} &\geq f(s) \min \{ \min \{ \lambda_a (x), \lambda_a (y) \}, \min \{ \mu_a (x), \mu_a (y) \} \} \\ &\geq f(s) \min \{ \min \{ \lambda_a (x), \mu_a (x) \}, \min \{ \lambda_a (y), \mu_a (y) \} \} \\ &= \min \{ f(s) \min \{ \lambda_a (x), \mu_a (x) \}, f(s) \min \{ \lambda_a (y), \mu_a (y) \} \} \\ &= \min \{ (\lambda_a \text{ s } \mu_a)^f (x), (\lambda_a \text{ s } \mu_a)^f (y) \} \end{aligned}$$

Therefore $(\lambda_a \text{ s } \mu_a)^f (mxy^{-1}n) \geq \min \{ (\lambda_a \text{ s } \mu_a)^f (x), (\lambda_a \text{ s } \mu_a)^f (y) \}$

Hence $(\lambda_a \text{ s } \mu_a)^f$ is $M - N$ fuzzy soft subgroup of the group G .

Theorem 3.2.6

If $x^{-1} f_a x$ is an soft middle coset of a group G , for some $x \in G$ and $a \in A$, then for each x form the normalize $N(f_a)$ of $M - N$ fuzzy soft subgroup f_a of G iff f_a is $M - N$ fuzzy normal soft subgroup.

Proof:

By the definition of normalize of f by $N(f) = \{x \in G: f(xy x^{-1}) = f(x)\}$

By the definition of $M - N$ fuzzy normal soft subgroup of f_a by

$$N(f_a) = \{x \in G \text{ a } \in A: f_a(mxyx^{-1}) = f_a(x) \text{ and } f_a(xyx^{-1}n) = f_a(x) \}$$

Then $f_a(mxyx^{-1}) = f_a(xyx^{-1}) = f_a(y)$ and $f_a(xyx^{-1}n) = f_a(xyx^{-1}) = f_a(y)$

Since, $f_a(mx) \geq f_a(x)$ and $f_a(xn) \geq f_a(x)$

iff f_a is an M – N fuzzy normal soft subgroups.

So that $f_a(mxyx^{-1}x) = f_a(mxyx^{-1}x)$

$$= f_a(myx)$$

$$= f_a(yx)$$

$$\Leftrightarrow f_a(xy) = f_a(yx)$$

Similarly, we can prove that $f_a(xyx^{-1}xn) = f_a(yx)$

Conversely,

Suppose f_a is an M – N fuzzy normal soft subgroup and $x^{-1} f_a x$ is an M – N fuzzy soft middle coset in G, then for all $x, y \in G$ and $a \in A$

$$(x^{-1} f_a x)(my) = f_a(xmyx^{-1})$$

$$= f_a(xx^{-1} my)$$

$$(x^{-1} f_a x)(my) = f_a(y)$$

$$(x^{-1} f_a x) = f_a(y)$$

Next,

$$(x^{-1} f_a x)(yn) = f_a(xynx^{-1})$$

$$= f_a(xx^{-1} yn)$$

$$(x^{-1} f_a x)(yn) = f_a(y)$$

$$(x^{-1} f_a x) = f_a(y)$$

$$\text{Hence } \langle x \rangle = f_a(y)$$

Proposition 3.2.7

Let f_a be an M – N fuzzy normal soft subgroup of a group G by x and y, then every fuzzy middle soft coset $xf_a y$ coincides with some left and right cosets $s(f_a)$ and $f_a(s)$ respectively, where s^{-1} is the product of $y^{-1} x^{-1}$

Proof:

We know that by the association in G , we have f_a is called a fuzzy normal soft subgroup of G if $f_a(xy) = f_a(yx)$ for all $x, y \in G, a \in A$

Now $(xf_a y)(mg) = f_a(x^{-1}mg).y^{-1}$

$$= f_a(y^{-1}(x^{-1}mg))$$

$$= f_a(s^{-1}mg)$$

$$f_a(y^{-1}x^{-1}mg) = f_a(s^{-1}mg)$$

$$= f_a(mgs^{-1}), \text{ since by associative law}$$

Thus $(xf_a y)(mg) = s f_a(mg)$.

Similarly we can prove that $(xf_a y)(gn) = s f_a(gn)$.

Hence $(xf_a y) = s f_a = f_a s$

Theorem 3.2.8

Let G be an $M - N$ group of order 2 and f_a is an $M - N$ fuzzy normal soft subgroup of G , then for some $x, y \in G$ the middle coset x of y coincides with fuzzy subgroup f_a .

Proof:

In the middle coset x of y , take $x = y$

By associative law in G , we have

$$(xf_a x)(mg) = f_a(x^{-1}mg).x^{-1}$$

$$= f_a(x^{-1}x^{-1}mg)$$

$$= f_a(x^{-2}mg)$$

$$= f_a(m(x^{-2}g).)$$

Since $x^{-2}G$ and G is of order 2

$$f_a(m(x^{-2}g).) = f_a(m(x^{-1})^2g.)$$

$$= f_a(meg)$$

$$= f_a(mg)$$

$$(xf_a x)(mg) = f_a(mg)$$

Similarly we can prove that $(xf_a x)(gn) = f_a(gn)$

$$\text{Hence } (xf_a x) = f_a$$

CHAPTER - 4

M - N ANTI FUZZY SOFT SUBGROUPS OF A GROUP

4. M – N ANTI FUZZY SOFT SUBGROUPS OF A GROUP

In this chapter, we have intersected into three sections to describe the concept of M-N Anti fuzzy soft subgroups of a group. The intersected sections are as follows,

4.1 M – N Anti Fuzzy Soft Subgroups

In this Section, we defined the concept of M-N anti fuzzy soft subgroups based on the concept of fuzzy group. We have also defined the M-N anti fuzzy soft set and the M-N level subsets of the anti fuzzy soft subgroup. Further the concepts of M-N anti fuzzy soft groups of some of its elementary properties are also discussed.

Definition 4.1.1

Let G be a group and (f, A) be an $M - N$ anti fuzzy soft set over G . Then (f, A) is said to be a $M - N$ anti fuzzy soft group over G iff for each $a \in A$ and $x, y \in G$,

$$(1) f_a \{m(xy)n\} \leq \max \{f_a(x), f_a(y)\}$$

(2) $f_a \{(mx^{-1})n\} \leq f_a(x)$ hold for each $a \in A, m \in M, n \in N$, f_a is a $M - N$ anti fuzzy soft subgroup of a group G .

Theorem 4.1.2

Let G be a group and (f, A) be an $M - N$ anti fuzzy soft set over G . Then (f, A) is said to be an $M - N$ anti fuzzy soft subgroup on a group G iff for each $a \in A$ and $x, y \in G$

Proof:

Assume that (f, A) is a $M - N$ anti fuzzy soft subgroup of a group G

To prove that $f_a \{(m(xy^{-1})n)\} \leq \max \{f_a(x), f_a(y)\}$

Let $x, y \in G$ and $a \in A$

We have $f_a \{(m(xy^{-1})n)\} \leq \max \{f_a(mx), f_a(y^{-1}n)\}$

$$\leq \max \{f_a(x), f_a(y^{-1})\} \quad (\text{by definition 1.2.23})$$

$$\leq \max \{f_a(x), f_a(y)\} \quad \text{by } A(x) = A(x^{-1})$$

$$\text{Therefore } f_a(m(xy^{-1})n) \leq \max \{f_a(x), f_a(y)\}$$

Conversely,

$$\text{Assume that } f_a(m(xy^{-1})n) \leq \max \{f_a(x), f_a(y)\}$$

To prove that (f_a, A) is an M – N anti fuzzy subgroup of a group G

$$\begin{aligned} \text{Now } f_a(men) &= f_a(mxx^{-1}n) \\ &\leq \max \{f_a(mx), f_a(x^{-1}n)\} \\ &\leq \max \{f_a(x), f_a(x^{-1})\} \\ &\leq \max \{f_a(x), f_a(x)\} \\ &= f_a(x) \end{aligned}$$

Therefore, $f_a(men) = f_a(x)$ where e is the identity element of G.

$$\begin{aligned} \text{Next, } f_a(mx^{-1}n) &= f_a(mex^{-1}n) \\ &\leq \max \{f_a(me), f_a(x^{-1}n)\} \\ &\leq \max \{f_a(e), f_a(x^{-1})\} \\ &\leq \max \{f_a(e), f_a(x)\} \end{aligned}$$

$$f_a(mx^{-1}n) = f_a(x)$$

On the other hand, for each $a \in A$ and $x, y \in G$

$$\begin{aligned} f_a(mxyn) &= f_a(mxy^{-1})^{-1}n) \\ &\leq \max \{f_a(mx), f_a((y^{-1})^{-1}n)\} \\ &\leq \max \{f_a(mx), f_a(y^{-1}n)\} \\ &\leq \max \{f_a(mx), f_a(yn)\} \end{aligned}$$

$$f_a(mxyn) \leq \max \{f_a(x), f_a(y)\}$$

Hence the proof.

Theorem 4.1.3

If (f, A) is an M – N anti fuzzy soft set and e is the unit element of G. Then for each

$a \in A$ and for each $x \in X$,

$$(1) \quad f_a(x^{-1}) \leq f_a(x)$$

$$(2) \quad f_a(e) = f_a(x)$$

Theorem 4.1.4

Let f_a and g_a be two $M - N$ anti fuzzy soft group of G , then $f_a \cap g_a$ is an $M - N$ anti fuzzy soft group of G .

Proof

To prove that (1) $(f_a \cap g_a)(mxyn) \leq \max \{ (f_a \cap g_a)(x), (f_a \cap g_a)(y) \}$

$$(2) \quad (f_a \cap g_a)(mx^{-1}n) = (f_a \cap g_a)(x)$$

$$(1) \Rightarrow (f_a \cap g_a)(mxyn) = \max \{ f_a(mxyn), g_a(mxyn) \}$$

$$\leq \max \{ \max \{ f_a(mx), f_a(yn) \}, \max \{ g_a(mx), g_a(yn) \} \}$$

$$\leq \max \{ \max \{ f_a(x), f_a(y) \}, \max \{ g_a(x), g_a(y) \} \}$$

$$\leq \max \{ \max \{ f_a(x), g_a(x) \}, \max \{ f_a(y), g_a(y) \} \}$$

$$(f_a \cap g_a)(mxyn) \leq \max \{ (f_a \cap g_a)(x), (f_a \cap g_a)(y) \}$$

$$(2) \Rightarrow (f_a \cap g_a)(mx^{-1}n) = (f_a \cap g_a)(mx^{-1}xx^{-1}n)$$

$$\leq \max \{ (f_a \cap g_a)(mx^{-1}x), (f_a \cap g_a)(x^{-1}n) \}$$

$$\leq \max \{ \max \{ (f_a \cap g_a)(mx^{-1}), (f_a \cap g_a)(x) \}, (f_a \cap g_a)(x^{-1}n) \}$$

$$\leq \max \{ \max \{ (f_a \cap g_a)(x^{-1}), (f_a \cap g_a)(x) \}, (f_a \cap g_a)(x^{-1}) \}$$

$$\leq \max \{ \max \{ (f_a \cap g_a)(x), (f_a \cap g_a)(x) \}, (f_a \cap g_a)(x) \}$$

$$\leq \max \{ (f_a \cap g_a)(x), (f_a \cap g_a)(x) \}$$

$$= (f_a \cap g_a)(x)$$

$$f_a \cap g_a)(x^{-1}) = (f_a \cap g_a)(x)$$

Hence the proof.

Proposition 4.1.5

If f_a and g_a are two $M - N$ anti fuzzy soft group of G , then the following statements holds for all $x, y \in G, m \in M, n \in N$

$$(1) (f_a \wedge g_a)(mxyn) \leq \max \{ (f_a \wedge g_a)(x), (f_a \wedge g_a)(y) \}$$

$$(2) (f_a \wedge g_a)(mx^{-1}n) = (f_a \wedge g_a)(x)$$

Proof:

Straight forward.

Proposition 4.1.6

If f_a is an $M - N$ anti fuzzy soft subgroup of a group over G , then $f_a(m(xy)n)^2 = f_a(x^2y^2)$ is an $M - N$ anti fuzzy soft subgroup of a group G .

Proof:

Let $x, y \in G, m \in M, n \in N$

$$\begin{aligned} (1) f_a(m(xy)n)^2 &= f_a((m(xy)n), (m(xy)n)) \\ &\leq \max \{ f_a(m(xy)n), f_a(m(xy)n) \} \\ &\leq \max \{ \max \{ f_a(mx), f_a(yn) \}, \max \{ f_a(mx), f_a(yn) \} \} \\ &\leq \max \{ \max \{ f_a(x), f_a(y) \}, \max \{ f_a(x), f_a(y) \} \} \\ &\leq \max \{ \max \{ f_a(x), f_a(x) \}, \max \{ f_a(y), f_a(y) \} \} \\ &\leq \max \{ f_a(x.x), f_a(y.y) \} \\ &\leq \max \{ f_a(x^2), f_a(y^2) \} \\ &= f_a(x^2y^2) \end{aligned}$$

$$f_a(m(xy)n)^2 = f_a(x^2y^2)$$

$$\begin{aligned} (2) f_a(m(xy)^{-1}n) &= f_a(my^{-1}x^{-1}n) \\ &\leq \max \{ f_a(my^{-1}), f_a(x^{-1}n) \} \\ &\leq \max \{ f_a(y^{-1}), f_a(x^{-1}) \} \end{aligned}$$

$$\leq f_a(yx)$$

$$\leq f_a(xy)$$

$$f_a(m(xy)^{-1}n) = f_a(xy)$$

Proposition 4.1.7

If $\{f_{ai}\}_i \in f_a$ is an M – N anti fuzzy soft group of G, then $\cup f_{ai}$ is an M – N anti fuzzy soft group G whose element $\cup f_{ai} = \{x, \vee f_{ai}(x) / x \in G\}$, where $i \in f_a$.

Proof:

Let $x, y \in G, m \in M, n \in N$ and $i \in f_a$.

$$(1) \cup f_{ai}(m(xy)n) \leq \max \{ \cup f_{ai}(x), \cup f_{ai}(y) \}$$

$$\cup f_{ai}(m(xy)n) = \vee f_{ai}(m(xy)n)$$

$$\leq \vee \max \{ f_{ai}(mx), f_{ai}(yn) \}$$

$$\leq \vee \max \{ f_{ai}(x), f_{ai}(y) \}, \text{ since } A(mx) \leq A(x)$$

$$\leq \max \{ \vee f_{ai}(x), \vee f_{ai}(y) \}$$

$$\leq \max \{ \cup f_{ai}(x), \cup f_{ai}(y) \}$$

$$\cup f_{ai}(m(xy)n) \leq \max \{ \cup f_{ai}(x), \cup f_{ai}(y) \}$$

$$(2) \cup f_{ai}(m(x^{-1})n) \leq \cup f_{ai}(x)$$

$$\cup f_{ai}(m(x^{-1})n) = \cup f_{ai}(m(x^{-1}xx^{-1})n)$$

$$\leq \vee f_{ai}(m(x^{-1}xx^{-1})n)$$

$$\leq \vee \max \{ f_{ai}(mx^{-1}x), f_{ai}(x^{-1}n) \}$$

$$\leq \vee \max \{ \max \{ f_{ai}(mx^{-1}), f_{ai}(x) \}, f_{ai}(x^{-1}n) \}$$

$$\leq \vee \max \{ \max \{ f_{ai}(x^{-1}), f_{ai}(x) \}, f_{ai}(x^{-1}) \}, \text{ since } A(mx) \leq A(x)$$

$$\leq \vee \max \{ \max \{ f_{ai}(x), f_{ai}(x) \}, f_{ai}(x) \} \quad A(x) = A(x^{-1})$$

$$\leq \vee \max \{ f_{ai}(x), f_{ai}(x) \}$$

$$\leq \bigvee f_{ai}(x)$$

$$\leq (\bigcup f_{ai})(x)$$

$$\bigcup f_{ai}(m(x^{-1})n) \leq \bigcup f_{ai}(x)$$

Hence $\bigcup f_{ai}(x)$ is an M – anti fuzzy soft group of G.

Definition 4.1.8

Let $f_a \cap g_a$ be an M – N anti fuzzy soft subgroup of a group G. For any $t \in [0, 1]$, we define the M – N anti level subset of $f_a \cap g_a$ is the set

$$(f_a \cap g_a)_t = \{x \in G / (f_a \cap g_a)(mx) \leq t, (f_a \cap g_a)(xn) \leq t \text{ for all } m \in M, n \in N\}$$

Theorem 4.1.9

Let G be a group and $f_a \cap g_a$ be an M- N anti fuzzy soft subgroup of G. Then the anti-level subset $(f_a \cap g_a)_t, t \in [0, 1]$ is an M- N anti fuzzy soft subgroup of G.

Proof:

Let $f_a \cap g_a$ be an M – N anti fuzzy soft subgroup of G and the anti-level subset

$$(f_a \cap g_a)_t = \{x \in G / (f_a \cap g_a)(mx) \leq t, (f_a \cap g_a)(xn) \leq t, t \in [0, 1] \text{ } m \in M, n \in N\}$$

Let $x, y \in (f_a \cap g_a)_t$, then $(f_a \cap g_a)(mx) \leq t$ and $(f_a \cap g_a)(xn) \leq t$

$$\text{Now } (f_a \cap g_a)(mxy^{-1}n) \leq \max \{(f_a \cap g_a)(mx), (f_a \cap g_a)(y^{-1}n)\}$$

$$= \max \{(f_a \cap g_a)(mx), (f_a \cap g_a)(yn)\}$$

$$\leq \max \{t, t\}$$

$$(f_a \cap g_a)(mxy^{-1}n) \leq t$$

$$mxy^{-1}n \in (f_a \cap g_a)_t$$

Therefore $(f_a \cap g_a)_t$ is an M – N anti fuzzy soft subgroup of G.

Proposition 4.1.10

Let G_1 and G_2 both are M – N anti group and ϕ be a soft homomorphism from G_1 and G_2 . If f_a is an M – N anti fuzzy soft group of G_2 , then the pre – image $\phi^{-1}(f_a)$ is an M – N anti fuzzy soft group of G_1

Proposition 4.1.11

Let $\phi : G_1 \rightarrow G_2$ be an epimorphism and f_a be an M – N anti fuzzy soft set in G_2 . If $\phi^{-1}(f_a)$ is an M – N anti fuzzy soft group of G_1 , f_a is an M – N anti fuzzy soft group of G_2 .

Proposition 4.1.12

Let f_a be an M – N anti fuzzy soft groups over G , and ϕ is endomorphism of G , then $f_a[\phi]$ is an M- N anti fuzzy soft group of G .

Proof:

Let $x, y \in G$, $m \in M$, and $n \in N$

$$(1) \quad f_a[[\phi](m(xy)n)] = f_a[\phi](m(xy)n)$$

$$\leq \max \{f_a[\phi](mx), f_a[\phi](yn)\}$$

$$\leq \max \{f_a(\phi(x)), f_a(\phi(y))\}, \text{ since } A(mx) \leq A(x)$$

$$\leq \max \{f_a(\phi x), f_a(\phi y)\}$$

$$\leq \max \{f_a[\phi](x), f_a[\phi](y)\}$$

$$f_a[[\phi](m(xy)n)] \leq \max \{f_a[\phi](x), f_a[\phi](y)\}$$

$$(2) \quad f_a[[\phi](m(x^{-1})n)] = f_a[\phi](m(x^{-1}xx^{-1})n)$$

$$\leq \max \{f_a[\phi](mx^{-1}x), f_a[\phi](x^{-1}n)\}$$

$$\leq \max \{\max \{f_a[\phi](mx^{-1}), f_a[\phi](x)\}, f_a[\phi](x^{-1}n)\}$$

$$\leq \max \{\max \{f_a[\phi](x^{-1}), f_a[\phi](x)\}, f_a[\phi](x^{-1})\}$$

$$\leq \max \{f_a[\phi](x^{-1}), f_a[\phi](x^{-1})\}$$

$$\leq \max \{f_a [\emptyset](x), f_a [\emptyset](x)\}, \text{ since } A(x^{-1}) = A(x)$$

$$\leq f_a [\emptyset](x)$$

$$f_a [[\emptyset] (m(x^{-1})n] \leq f_a [\emptyset](x)$$

Hence $f_a [\emptyset]$ is an M – N anti fuzzy soft group of G.

4.2 M – N Anti Fuzzy Normal Soft Subgroups

In this Section, we have discussed the concept of M-N Anti fuzzy normal soft subgroups based on the concept of fuzzy normal soft group [1, 8, 35 and 62]. We defined the M-N Anti fuzzy normal soft set and also defined the intersection of two and three variables of M-N level subsets of the anti fuzzy normal soft subgroup and some of its elementary properties.

Definition 4.2.1

Let G be an M – N group and (F, A) be an anti-fuzzy soft subgroup of G if

$$(1) F \{m(x y) n\} \leq \max \{F(x), F(y)\}$$

$$(2) F \{(m x^{-1}) n\} \leq F(x) \text{ hold for any } x, y \in G, m \in M, n \in N, \text{ then } (F, A) \text{ is said}$$

be an M – N anti fuzzy soft subgroup of G. Here $F: A \rightarrow P(G)$

Definition 4.2.2

Let G be an M – N group and (F, A) be an anti-fuzzy soft subgroup of G if

$$(1) F(m x) \leq F(x)$$

$$(2) F(x n) \leq F(x) \text{ hold for any } x \in G, m \in M, \text{ and } n \in N, \text{ then } (F, A) \text{ is said be}$$

an M – N anti fuzzy soft subgroup of G.

Definition 4.2.3

Let G be an M – N group. (F, A) is said be an M – N anti fuzzy normal soft subgroup of G if (F, A) is not only an M – N anti fuzzy soft subgroup of G, but also an anti-fuzzy normal soft subgroup of G.

Theorem 4.2.4

Let G be an $M - N$ group, A and B both be $M - N$ anti fuzzy subgroup of G . Then $A \cup B$ is an $M - N$ anti fuzzy subgroup of G .

Proof:

$$(1) \quad (A \cup B)(m(xy)n) \leq \max \{(A \cup B)(x), (A \cup B)(y)\}$$

$$(2) \quad (A \cup B)(mx^{-1}n) = (A \cup B)(x)$$

$$(1) \Rightarrow (A \cup B)(m(xy)n) = \max \{A(m(xy)n), B(m(xy)n)\}$$

$$\leq \max \{\max \{A(mx), A(y)n\}, \max \{B(mx), B(yn)\}\}$$

$$\leq \max \{\max \{A(x), B(x)\}, \max \{A(y), B(y)\}\}, \quad A(mx) \leq A(x)$$

$$(A \cup B)(m(xy)n) \leq \max \{(A \cup B)(x), (A \cup B)(y)\}$$

$$(2) \Rightarrow (A \cup B)(mx^{-1}n) = (A \cup B)(mxx^{-1}xn)$$

$$(A \cup B)(mx^{-1}n) = \max \{A(mxx^{-1}), B(xn)\}$$

$$\leq \max \{\max \{A(mx), A(x^{-1})\}, B(x)\}$$

$$\leq \max \{\max \{A(x), A(x^{-1})\}, B(x)\}$$

$$\leq \max \{A(x), B(x)\}$$

$$= (A \cup B)(x)$$

$$(A \cup B)(mx^{-1}n) = (A \cup B)(x)$$

Hence (1) and (2) is proved.

Theorem 4.2.5

If α and β are the two $M - N$ anti fuzzy soft subgroup of G , then $\alpha \cup \beta$ is an $M - N$ anti fuzzy soft subgroup of G

Proof:

Let α and β be two $M - N$ anti fuzzy soft subgroup of G .

$$(3) (\alpha \cup \beta)(m(xy^{-1})n) \leq \max \{ (\alpha \cup \beta)(mx), (\alpha \cup \beta)(y^{-1}n) \}$$

$$\begin{aligned} (\alpha \cup \beta)(m(xy^{-1})n) &= \max \{ \alpha(m(xy^{-1})n), \beta(m(xy^{-1})n) \} \\ &\leq \max \{ \max \{ (\alpha(mx), (\alpha(y^{-1}n)) \}, \max \{ \beta(mx), \beta(y^{-1}n) \} \} \\ &\leq \max \{ \max \{ (\alpha(mx), \beta(mx)) \}, \max \{ (\alpha(y^{-1}n), \beta(y^{-1}n)) \} \} \\ &\leq \max \{ (\alpha \cup \beta)(mx), (\alpha \cup \beta)(y^{-1}n) \} \end{aligned}$$

$$\text{Therefore } (\alpha \cup \beta)(m(xy^{-1})n) \leq \max \{ (\alpha \cup \beta)(mx), (\alpha \cup \beta)(y^{-1}n) \}$$

$$(4) (\alpha \cup \beta)(mxn) = (\alpha \cup \beta)(mx^{-1}n)$$

$$\begin{aligned} (\alpha \cup \beta)(mxn) &= \max \{ \alpha(mxn), \beta(mxn) \} \\ &= \max \{ \alpha(mx^{-1}n), \beta(mx^{-1}n) \} \\ &= (\alpha \cup \beta)(mx^{-1}n) \end{aligned}$$

Hence $\alpha \cup \beta$ is an M – N anti fuzzy soft subgroup of G.

Theorem 4.2.6

The union of any two M – N anti fuzzy normal soft subgroup of G is also an M – N anti fuzzy normal soft subgroup G.

Proof:

Let α , and β be the M – N anti fuzzy normal soft subgroup of G.

By the previous theorem we know that, $\alpha \cup \beta$ is an M – N anti fuzzy soft subgroup of G.

Let $x, y \in G$, $m \in M$, and $n \in N$

To prove that $(\alpha \cup \beta)(m(yxy^{-1})n) = (\alpha \cup \beta)(mxn)$

$$\begin{aligned} \text{Now } (\alpha \cup \beta)(m(yxy^{-1})n) &= \max \{ \alpha(m(yxy^{-1})n), \beta(m(yxy^{-1})n) \} \\ &= \max \{ [\alpha(mxn), \beta(mxn)] \} \\ &= (\alpha \cup \beta)(mxn) \end{aligned}$$

Hence $(\alpha \cup \beta)(m(yxy^{-1})n) = (\alpha \cup \beta)(m(x)n)$.

Hence $\alpha \cup \beta$ is an M – N anti fuzzy normal soft subgroup of G.

Note 4.2.7

If $(\alpha \cup \beta)_i, i \in \Delta$ are M – N anti fuzzy normal soft subgroup of G, then $\bigcup_{i \in \Delta} (\alpha \cup \beta)_i$ is a M – N anti fuzzy normal soft subgroup of G.

Definition 4.2.8

Let G be a group, α is a M – N anti-fuzzy soft subgroup of G is said be a M – N anti fuzzy normal soft subgroup if

$$\alpha(m(xy x^{-1})n) = \alpha(m(y)n) \text{ (or)}$$

$$\alpha(m(xy)n) \leq \alpha(m(yx)n) \text{ for all } x, y \in G, m \in M, \text{ and } n \in N$$

Theorem 4.2.9

Let α is an M – N anti fuzzy normal soft subgroup of G, then for any $y \in G$ we have $\alpha(m(y^{-1}xy)n) = \alpha(m(yxy^{-1})n)$.

Proof:

Let α be an M- N anti fuzzy normal soft subgroup G, then for any $y \in G$.

$$\text{Now } \alpha(m(y^{-1}xy)n) = \alpha(m(xy^{-1}y)n)$$

$$= \alpha(m(x)n)$$

$$= \alpha(m(yy^{-1}x)n)$$

$$= \alpha(m(yxy^{-1})n)$$

$$\text{Therefore } \alpha(m(y^{-1}xy)n) = \alpha(m(yxy^{-1})n)$$

Hence the theorem.

Theorem 4.2.10

If α is an M- N anti fuzzy normal soft subgroup of G, then $g\alpha g^{-1}$ is also M- N anti fuzzy normal soft subgroup of G, for all $g \in G$.

Proof:

Let α be an M- N anti fuzzy normal soft subgroup of G, then $g\alpha g^{-1}$ is an M- N anti fuzzy subgroup of G, for all $g \in G$

$$\begin{aligned} \text{Now } g\alpha g^{-1} (m(yxy^{-1})n) &= \alpha(g^{-1}m(yxy^{-1})n)g \\ &= \alpha(m(yxy^{-1})n) \\ &= \alpha(mxn) \\ &= \alpha(g(mxn))g^{-1} \\ &= g\alpha g^{-1} (mxn) \end{aligned}$$

$$\text{Therefore } g\alpha g^{-1} (m(yxy^{-1})n) = g\alpha g^{-1} (mxn)$$

Hence the theorem

Theorem 4.2.11

If $\alpha \cup \beta$ is an M- N anti fuzzy normal soft subgroup of G, then $g(\alpha \cup \beta)g^{-1}$ is also an M- N anti fuzzy normal soft subgroup of G, for all $g \in G$.

Proof:

If $\alpha \cup \beta$ is an M- N anti fuzzy normal soft subgroup of G, then $g(\alpha \cup \beta)g^{-1}$ is also an M- N anti fuzzy normal soft subgroup of G, for all $g \in G$.

$$\text{To prove that } g(\alpha \cup \beta)g^{-1} (m(yxy^{-1})n) = g(\alpha \cup \beta)g^{-1} (mxn)$$

$$\begin{aligned} \text{Now } g(\alpha \cup \beta)g^{-1} (m(yxy^{-1})n) &= (\alpha \cup \beta)(g^{-1}(m(yxy^{-1})n)g) \\ &= (\alpha \cup \beta)(m(yxy^{-1})n) \end{aligned}$$

$$= (\alpha \cup \beta)(m \times n)$$

$$= (\alpha \cup \beta)(g(m \times n)g^{-1})$$

$$= g(\alpha \cup \beta)g^{-1}(m \times n)$$

$$\text{Therefore } g(\alpha \cup \beta)g^{-1}(m(y \times y^{-1})n) = g(\alpha \cup \beta)g^{-1}(m \times n)$$

Hence the theorem

Definition 4.2.12

Let $\alpha \cup \beta$ be an M – N anti fuzzy soft subgroup of a group G. For any $t \in [0, 1]$, we define the M – N anti level subset of $\alpha \cup \beta$ is the set

$$(\alpha \cup \beta)_t = \{x \in G / (\alpha \cup \beta)(m \times x) \leq t, (\alpha \cup \beta)(x \times n) \leq t \text{ for all } m \in M, n \in N\}$$

Theorem 4.2.13

Let G be a group and $\alpha \cup \beta$ be an anti fuzzy subset of G. Then $\alpha \cup \beta$ is an M – N anti fuzzy normal soft subgroup of G iff the anti-level subset $(\alpha \cup \beta)_t, t \in [0, 1]$ are M- N anti fuzzy subgroup of G.

Proof:

Let $\alpha \cup \beta$ be an M – N anti fuzzy normal soft subgroup of G and the anti-level subset

$$(\alpha \cup \beta)_t = \{x \in G / (\alpha \cup \beta)(m \times x) \leq t, (\alpha \cup \beta)(x \times n) \leq t, t \in [0, 1] m \in M, n \in N\}$$

Let $x, y \in (\alpha \cup \beta)_t$, then $(\alpha \cup \beta)(m \times x) \leq t$ and $(\alpha \cup \beta)(x \times n) \leq t$

$$\text{Now } (\alpha \cup \beta)(m(xy^{-1})n) \leq \max \{(\alpha \cup \beta)(m \times x), (\alpha \cup \beta)(y^{-1}n)\}$$

$$= \max \{(\alpha \cup \beta)(m \times x), (\alpha \cup \beta)(y \times n)\}$$

$$\leq \max \{t, t\}$$

$$(\alpha \cup \beta)(m(xy^{-1})n) \leq t$$

$$m(xy^{-1})n \in (\alpha \cup \beta)_t$$

Therefore $(\alpha \cup \beta)_t$ is an M – N anti fuzzy subgroup of G.

Conversely,

Let us assume that $(\alpha \cup \beta)_t$ is an M – N anti fuzzy subgroup G.

Let $x, y \in (\alpha \cup \beta)_t$ then $(\alpha \cup \beta)(m x) \leq t$ and $(\alpha \cup \beta)(x n) \leq t$

Also $(\alpha \cup \beta)(m (xy^{-1}) n) \leq t$

$$\begin{aligned} \text{Since } m (xy^{-1}) n \in (\alpha \cup \beta)_t &= \max \{t, t\} \\ &= \max \{(\alpha \cup \beta)(m x), (\alpha \cup \beta)(y n)\} \end{aligned}$$

Therefore $(\alpha \cup \beta)(m (xy^{-1}) n) \leq \max \{(\alpha \cup \beta)(m x), (\alpha \cup \beta)(y n)\}$

Hence $\alpha \cup \beta$ is an M – N anti fuzzy normal soft subgroup of G.

Definition 4.2.14

Let G be a group and $\alpha \cup \beta$ be an M – N anti-fuzzy normal soft subgroup of G.

Let $N(\alpha \cup \beta) = \{y \in G / (\alpha \cup \beta)(m (y x y^{-1}) n) = (\alpha \cup \beta)(m x n) \text{ for all } x \in G, m \in M, n \in N\}$, then $N(\alpha \cup \beta)$ is called the M – N anti fuzzy soft Normalize of $\alpha \cup \beta$.

Theorem 4.2.15

Let G be a group and $\alpha \cup \beta$ be an anti fuzzy subset of G. Then $\alpha \cup \beta$ is an M – N anti fuzzy normal soft subgroup of G iff the anti-level subset $(\alpha \cup \beta)_t, t \in [0, 1]$ are M- N anti fuzzy normal subgroup of G.

Proof:

Let $\alpha \cup \beta$ be a M – N anti fuzzy normal soft subgroup of G and the anti-level subset $(\alpha \cup \beta)_t, t \in [0, 1]$

Let $x \in G$ and $y \in (\alpha \cup \beta)_t$, then $(\alpha \cup \beta)(m y n) \leq t$, for all $m \in M, n \in N$

Now $(\alpha \cup \beta)(m (x y x^{-1}) n) = (\alpha \cup \beta)(m y n) \leq t$

Since $\alpha \cup \beta$ is an M – N anti fuzzy normal soft subgroup of G.

That is $(\alpha \cup \beta) (m (x y x^{-1}) n) \leq t$

Therefore $(m (x y x^{-1}) n \in (\alpha \cup \beta)_t$

Hence $(\alpha \cup \beta)_t$ is an M – N anti fuzzy normal subgroup of G.

4.3 M-N Anti Homomorphism of fuzzy soft subgroups

In this section, we have defined the M-N anti homomorphism of fuzzy soft subgroup and the M-N anti level subsets of a fuzzy soft subgroup. We have also discussed the concept of M-N anti homomorphism of fuzzy soft groups and some of its elementary properties.

Definition 4.3.1

Let G and G^1 be any two M – N groups. If (f, A) is an fuzzy soft subgroup of an M – N group G, then the function $f_a: G \rightarrow G^1$ is said be an M – N anti homomorphism of fuzzy soft subgroup if

- (1) $f_a(xy) = f_a(x) f_a(y)$ for all $x, y \in G, a \in A$
- (2) $f_a(mx) = m f_a(x)$, for all $x \in G, a \in A$ and $m \in M$
- (3) $f_a(y)n = f_a(y)n$, for all $y \in G, a \in A$ and $n \in N$

Note 4.3.2

If λ is a constant and $\ker f_a$ is an M – N anti fuzzy soft subgroup, then

- (1) $f_a(\lambda) f_a(mx) = \lambda(mx) = \lambda(x)$, for all $x \in G, a \in A$ and $m \in M$.
- (2) $f_a(\lambda) f_a(xn) = \lambda(xn) = \lambda(x)$, for all $x \in G, a \in A$ and $n \in N$,

Theorem 4.3.3

Let f_a be an M – N anti homomorphism of fuzzy soft subgroup from an M – N group G onto an M – N group G^1 . If λ is an M – N fuzzy subgroup of G and λ is an f_a – soft invariant, then $f_a(\lambda)$, the image of λ under f_a is an M – N anti fuzzy soft subgroup of G^1 .

Proof:

We know that λ is a constant and $\ker f_a$ is an M – N anti fuzzy soft subgroup.

Now $f_a(\lambda) (f_a(x) f_a(y)) = f_a(\lambda) (f_a(xy))$, for all $x, y \in G, a \in A$

$$= \lambda(xy), \text{ since by note}$$

$$\leq \max \{ \lambda(x), \lambda(y) \}$$

$$\leq \max \{ f_a(\lambda) (f_a(x)), f_a(\lambda) (f_a(y)) \}$$

Therefore $f_a(\lambda) (f_a(x) f_a(y)) \leq \max \{ f_a(\lambda) (f_a(x)), f_a(\lambda) (f_a(y)) \}$

Clearly $f_a(\lambda)$ is an anti fuzzy soft subgroup of G^1 .

To prove that $f_a(\lambda)$ is an M – N anti fuzzy soft subgroup of G^1

Let $f_a(\lambda) \in G^1$, then

$$\begin{aligned} (2) \quad f_a(\lambda) (m f_a(x)) &= f_a(\lambda) (f_a(mx)) \\ &= \lambda(mx) \\ &\leq \lambda(x), \text{ by the definition } A(mx) \leq A(x) \\ &= f_a(\lambda) f_a(x) \end{aligned}$$

Therefore $f_a(\lambda) (m f_a(x)) = f_a(\lambda) f_a(x)$

$$\begin{aligned} (2) \quad f_a(\lambda) (f_a(x)n) &= f_a(\lambda) (f_a(xn)) \\ &= \lambda(xn) \\ &\leq \lambda(x), \text{ by the definition } A(xn) \leq A(x) \\ &= f_a(\lambda) f_a(x) \end{aligned}$$

Therefore $f_a(\lambda) (f_a(x)n) = f_a(\lambda) f_a(x)$

Hence $f_a(\lambda)$ is an M – N anti fuzzy soft subgroup of G^1 .

Theorem 4.3.4

The $M - N$ anti homomorphic pre- image of an $M - N$ anti fuzzy soft subgroup of an $M - N$ group G^1 is an $M - N$ anti fuzzy soft subgroup of an $M - N$ group G .

Proof:

Let $f_a: G \rightarrow G^1$ is said be an $M - N$ anti homomorphism of fuzzy soft subgroup

Let μ be an fuzzy set on the $M - N$ anti fuzzy subgroup of G^1 .

Now $\lambda(xy) = \mu(f_a(xy))$ for all $a \in A$ and $x, y \in G$

$$= \mu(f_a(x) f_a(y)), \text{ since } f_a \text{ is an anti-homomorphism}$$

$$\leq \max \{ \mu f_a(x), \mu f_a(y) \}, \text{ since } \mu \text{ is an anti-fuzzy subgroup of } G^1$$

$$= \max \{ \lambda(x), \lambda(y) \}$$

That is, $\lambda(xy) \leq \max \{ \lambda(x), \lambda(y) \}$

Let $x \in G$,

$$\lambda(x^{-1}) = \mu(f_a(x^{-1}))$$

$$= \mu(f_a(x)^{-1}), \text{ since } f_a \text{ is an anti-homomorphism of fuzzy soft subgroup}$$

$$= \mu(f_a(x)), \text{ since } \mu \text{ is an } M - N \text{ anti fuzzy subgroup of } G^1$$

$$= \lambda(x)$$

$$\lambda(x^{-1}) \leq \lambda(x)$$

Clearly $\lambda(mx) = \mu(f_a(mx))$ for some $m \in M$ and $x \in G$

$$= \mu(mf_a(x)), \text{ since } f_a \text{ is an } M - N \text{ anti homomorphism of an fuzzy soft group}$$

$$\leq \mu f_a(x), \text{ since } \mu \text{ } M - N \text{ anti fuzzy subgroup of } G$$

$$= \lambda(x)$$

That is, $\lambda(mx) \leq \lambda(x)$

Next, $\lambda(xn) = \mu(f_a(xn))$, for some $n \in N$ and $x \in G$

$$\begin{aligned} &= \mu(f_a(x)n), \text{ since } f_a \text{ is an } M - N \text{ anti homomorphism of an fuzzy soft group} \\ &\leq \mu f_a(x), \text{ since } \mu \text{ } M - N \text{ anti fuzzy subgroup of } G \\ &= \lambda(x) \end{aligned}$$

That is, $\lambda(xn) \leq \lambda(x)$

Hence λ is an $M - N$ anti fuzzy subgroup of G .

Theorem 4.3.5

If $f_a: G \rightarrow G^1$ is an $M - N$ anti homomorphism of an fuzzy soft subgroup of a group G , then,

$$(1) f_a(e) = e^1, \text{ where } e^1 \text{ is the unity element of } G^1$$

$$(2) f_a(x^{-1}) = f_a(x)^{-1} \text{ for all } x \in G.$$

Proof:

Given that $f_a: G \rightarrow G^1$ is an $M - N$ anti homomorphism of an fuzzy soft subgroup of a group G ,

$$(1) \Rightarrow \text{Suppose } f_a(mx) e^1 = f_a(mx) = f_a(x), \text{ for some } m \in M, a \in A \text{ and } x \in G$$

$$= f_a(xe), \text{ since } e \text{ is an identity element in } G$$

$$= f_a(x) f_a(e), \text{ since } f_a \text{ is an anti-homomorphism of an fuzzy soft subgroup}$$

$$f_a(x) e^1 = f_a(x) f_a(e), \text{ by left cancellation law}$$

$$\text{Therefore } f_a(e) = e^1$$

Similarly, we can prove that $f_a(xn) e^1 = f_a(xn) = f_a(x)$, for some $n \in N, a \in A$ and $x \in G$

That implies $f_a(e) = e^1$

$$(2) \Rightarrow \text{We know that } e^1 = f_a(me), \text{ since } A(mx) \leq A(x)$$

$$\begin{aligned}
 &= f_a(e) \\
 &= f_a(xx^{-1}) \\
 &= f_a(x) f_a(x^{-1}) \text{ since } f_a \text{ is an anti-homomorphism of an fuzzy soft}
 \end{aligned}$$

subgroup

$$\begin{aligned}
 e^1(f_a(x))^{-1} &= f_a(x^{-1}) \\
 (f_a(x))^{-1} &= f_a(x^{-1})
 \end{aligned}$$

Similarly, we can prove that $e^1 = f_a(en)$, since $A(xn) \leq A(x)$

That implies, $(f_a(x))^{-1} = f_a(x^{-1})$

Hence the proof.

Definition 4.3.6

Let μ be an M – N anti-fuzzy subgroup of an M – N group G. Then the M – N subgroup μ_t for $t \in [0, 1]$ and $t \leq \mu(e)$, are called anti level M – N subgroup of μ .

Theorem 4.3.7

The M – N anit homomorphic image of a level M – N subgroup of an M – N fuzzy subgroup μ of an M – N group G is a level M – N subgroup of an M – N fuzzy soft subgroup $f_a(\mu)$ is an M – N soft subgroup G^1 , where μ is f_a – soft invariant.

Proof:

Let G and G^1 be any two M – N group.

Let $f_a: G \rightarrow G^1$ be an M – N anti homomorphism of an fuzzy soft subgroup of a group G.

Let μ be an M – N fuzzy subgroup of G.

Clearly $f_a(\mu)$ is an M – N fuzzy soft subgroup of G^1 .

Let μ_t be a level M – N subgroup of an M – N fuzzy subgroup μ of G.

Since f_a is an M – N anti homomorphism fuzzy soft subgroup,

$f_a(\mu)$ is an M – N soft subgroup $f_a(\mu)$ of G^1 and $f_a(\mu_t) = (f_a(\mu))_t$

Hence $(f_a(\mu))_t$ is a level M – N soft subgroup $f_a(\mu)$ of G^1 .

Theorem 4.3.8

The M – N anti homomorphism pre – image of a level M – N soft subgroup of an M – N fuzzy subgroup μ of an M – N group G^1 is a level M – N subgroup of an M- N fuzzy soft subgroup $f_a^{-1}(\mu)$ of an M – N group G.

Proof:

Let $a \in A$ $m \in M$ and $n \in N$

Let G and G^1 be any two M – N group.

Let $f_a: G \rightarrow G^1$ be an M – N anti homomorphism of an fuzzy soft subgroup of a group G.

Let μ be an M – N fuzzy subgroup of G^1 .

Clearly $f_a^{-1}(m\mu) = f_a^{-1}(\mu)$ and $f_a^{-1}(\mu n) = f_a^{-1}(\mu)$ is an M – N fuzzy soft subgroup of G.

Let μ_t be a level M – N subgroup of an M – N fuzzy subgroup μ of G^1 .

Since f_a is an M – N anti homomorphism fuzzy soft subgroup,

$f_a^{-1}(\mu_t)$ is an M – N soft subgroup of $f_a^{-1}(\mu)$ of G and $f_a^{-1}(\mu_t) = (f_a^{-1}(\mu))_t$, is an M – N soft subgroup of an M – N fuzzy soft subgroup $f_a^{-1}(\mu)$ of G.

That is $(f_a^{-1}(\mu))_t$ is a level M – N subgroup of an M – N fuzzy soft subgroup $f_a^{-1}(\mu)$ of G.

Hence the proof.

CHAPTER – 5

CONJUGATE OF M-N ANTI FUZZY SOFT SUBGROUPS OF A GROUP

5. CONJUGATE OF M-N ANTI FUZZY SOFT SUBGROUPS OF A GROUP

In this chapter, we have intersected into two sections to explore the concept of conjugate of M-N anti-fuzzy soft subgroups of a group. The intersected sections are as follows,

5.1 Conjugate of M-N anti-fuzzy soft subgroups

In this section, we have defined the conjugate of M-N anti-fuzzy soft subgroups based on the concept of fuzzy soft group [2, 6 and 34], and some elementary properties are discussed.

Definition 5.1.1

Let G be an $M - N$ anti group and (μ, A) be a conjugate of fuzzy soft subgroup of G if

$$(1) \mu_a(m x) \leq \mu_a(x) = \lambda_a(g^{-1} x g)$$

$$(2) \mu_a(y n) \leq \mu_a(y) = \lambda_a(g^{-1} y g) \text{ hold for any } x, y \in G, a \in A, m \in M, \text{ and } n$$

$\in N$, then (μ, A) is said be a **conjugate of M – N anti fuzzy soft subgroup** of G .

Theorem 5.1.2

Let μ_a and λ_a be any $M - N$ anti fuzzy soft subgroup of the group G , then μ_a and λ_a are conjugate of $M - N$ anti fuzzy soft subgroup of G iff $\mu_a = \lambda_a$

Proof

Given that μ_a and λ_a are conjugate of $M - N$ anti fuzzy soft subgroup of the group G

To prove that $\mu_a = \lambda_a$

Since μ_a and λ_a are conjugate of $M - N$ anti fuzzy soft subgroup of the group G

By the definition there exists $g \in G$, such that

$$, \quad \mu_a(mx) = \lambda_a(g^{-1} x g) \text{ for every } x \in G, m \in M \text{ and } a \in A$$

$$\mu_a(xn) = \lambda_a(g^{-1} x g) \text{ for every } x \in G, n \in N \text{ and } a \in A$$

Let $mx = gmx$ for all $g, x \in G, m \in M$, then

$$\mu_a(gmx) = \lambda_a(g^{-1}gxg)$$

$$\mu_a(gmx) = \lambda_a(xg)$$

$$\mu_a(gx) = \lambda_a(xg), \text{ since } \mu(mx) \leq \mu(x)$$

And let $xn = gxn$ for all $g, x \in G, n \in N$, then

$$\mu_a(gxn) = \lambda_a(g^{-1}gxg)$$

$$\mu_a(gxn) = \lambda_a(xg)$$

$$\mu_a(gx) = \lambda_a(xg), \text{ since } \mu(xn) \leq \mu(x)$$

For some $g = e \in G, m \in M$

We have $\mu_a(mex) = \lambda_a(mxe)$

$$\mu_a(mx) = \lambda_a(mx), \text{ since } \mu(xm) \leq \mu(x)$$

$$\mu_a(x) = \lambda_a(x)$$

$$\mu_a = \lambda_a$$

Similarly we can prove that $\mu_a(exn) = \lambda_a(xen)$

$$\text{Hence } \mu_a = \lambda_a$$

Conversely,

To prove that, μ_a and λ_a are conjugate of M – N anti fuzzy soft subgroup of G

$$\text{Let } \mu_a = \lambda_a$$

$$\mu_a(mx) = \lambda_a(mx)$$

$$\mu_a(mx) = \mu_a(x) = \lambda_a(x), \text{ since } \mu(xm) \leq \mu(x)$$

$$\text{By the definition, } \mu_a(mx) = \lambda_a(e^{-1}xe)$$

$$\text{Similarly we can prove that } \mu_a(xn) = \lambda_a(e^{-1}xe)$$

Hence μ_a and λ_a are conjugate of M – N anti fuzzy soft subgroup of G.

Theorem 5.1.3

Let λ_a be an M – N anti fuzzy soft subgroup of a group G, and μ_a be a fuzzy soft subset of G. If μ_a and λ_a are conjugate of M – N anti fuzzy soft subgroup of the group G, then μ_a is an M – N anti fuzzy soft subgroup of a group G.

Proof

Let e be an identity element of the group G.

If μ_a and λ_a are conjugate of M – N anti fuzzy soft subgroup of the group G

By the definition, since there exists an element $g \in G$, such that

$$\begin{aligned}\lambda_a(x) &\geq \lambda_a(mx) = \mu_a(g^{-1}xg) \text{ for all } x \in G \\ \Rightarrow \lambda_a(x) &= \mu_a(g^{-1}xg) \text{ and} \\ \lambda_a(x) &\geq \lambda_a(xn) = \mu_a(g^{-1}xg) \text{ for all } x \in G, m \in M, a \in A \text{ and } n \in N, \\ \Rightarrow \lambda_a(x) &= \mu_a(g^{-1}xg)\end{aligned}$$

Also $\mu_a(x) \geq \mu_a(mx) = \mu_a(exe)$

$$\begin{aligned}\mu_a(x) &= \mu_a(g^{-1}gxg^{-1}g) \\ &= \lambda_a(gxg^{-1})\end{aligned}$$

Therefore $\mu_a(x) = \lambda_a(gxg^{-1})$

Similarly we can prove that $\mu_a(x) \geq \mu_a(xn) = \mu_a(gxg^{-1})$

To prove that, μ_a is an M – N anti fuzzy soft subgroup of a group G.

Since λ_a be an M – N anti fuzzy soft subgroup of a group G,

Now $\mu_a(xy) \leq \mu_a(mxyn) = \mu_a(emxeyne)$

$$\begin{aligned}\mu_a(mxyn) &= \mu_a(g^{-1}gmxg^{-1}gyn g^{-1}g) \\ &= \lambda_a(gmxg^{-1}gyn g^{-1}g) \\ &\leq \max \{ \lambda_a(gmxg^{-1}), \lambda_a(gyn g^{-1}) \}, \text{ since } \mu(xm) \leq \mu(x) \\ \mu_a(mxyn) &\leq \max \{ \lambda_a(gxg^{-1}), \lambda_a(gyg^{-1}) \}\end{aligned}$$

$$\mu_a(xy) \leq \max \{ \mu_a(x), \mu_a(y) \}$$

$$\text{Also } \mu_a(mx^{-1}n) = \lambda_a(gmxy^{-1}g^{-1}gnyg^{-1})$$

$$\leq \max \{ \lambda_a(gmxg^{-1}gy^{-1}g^{-1}), \lambda_a(gynyg^{-1}) \},$$

$$\leq \max \{ \max \{ \lambda_a(gmxg^{-1}) \lambda_a(gy^{-1}g^{-1}) \}, \lambda_a(gynyg^{-1}) \}, \text{ since } \mu(xm) \leq \mu(x)$$

$$\leq \max \{ \max \{ \lambda_a(gxg^{-1}) \lambda_a(gy^{-1}g^{-1}) \}, \lambda_a(gygy^{-1}) \},$$

$$\leq \max \{ \{ \lambda_a(gxg^{-1}), \lambda_a(gygy^{-1}) \}, \text{ since } \mu(y) = \mu(y^{-1})$$

$$\leq \max \{ \lambda_a(gxg^{-1}), \lambda_a(gy^{-1}g^{-1}) \}$$

$$= \lambda_a(gxg^{-1})$$

$$\mu_a(mx^{-1}n) = \lambda_a(gxg^{-1})$$

Hence μ_a is an M – N anti fuzzy soft subgroup of a group G.

5.2 M-N Anti Fuzzy Soft Middle Coset

Definition 5.2.1

Let λ_a be an M- N anti fuzzy soft subgroup of a group G. then for any s, t $\in$ G the **M – N anti fuzzy soft middle coset** $s \lambda_a t$ of the G is defined by $(s \lambda_a t)(mxn) = \lambda_a(s^{-1}xt^{-1})$ for all $x \in G$ and $a \in A$.

Theorem 5.2.2

If λ_a is an M – N anti fuzzy soft subgroup of a group G, then for any s $\in$ G, the M – N anti fuzzy soft middle coset $s \lambda_a s^{-1}$ of the group G is also a M – N anti fuzzy soft subgroup of the group G.

Proof

Let λ_a be an M – N anti fuzzy soft subgroup of a group G and s $\in$ G

Let x, y $\in$ G, m $\in$ M, a $\in$ A and n $\in$ N, then

$$\begin{aligned} (s \lambda_a s^{-1})(mxy^{-1}n) &= \lambda_a(ms^{-1}xy^{-1}sn) \\ &= \lambda_a(ms^{-1}xss^{-1}y^{-1}sn) \end{aligned}$$

$$\begin{aligned}
 &= \lambda_a (m (s^{-1} xs) (s^{-1} y^{-1} s) n) \\
 &\leq \max \{ \lambda_a (m(s^{-1}xs)), \lambda_a (s^{-1}y^{-1} s)n) \}, \text{ since } A(mx) \leq A(x), A(yn) \leq A(y) \\
 &\leq \max \{ \lambda_a ((s^{-1}xs), \lambda_a (s^{-1}y^{-1} s)) \},
 \end{aligned}$$

Since λ_a is an M – N anti fuzzy soft subgroup of a group G

Therefore $(s \lambda_a s^{-1}) (m xy^{-1}n) \leq \max \{ \lambda_a ((s^{-1}xs), \lambda_a (s^{-1}y^{-1}s)) \}$

Hence $s \lambda_a s^{-1}$ is an M – N anti fuzzy soft subgroup of the group G

Theorem 5.2.3

Let λ_a be any M – N anti fuzzy soft subgroup of a group G and $s \lambda_a s^{-1}$ be an M – N anti fuzzy soft middle coset of G, then $o(s \lambda_a s^{-1}) = o(\lambda_a)$ for any $s \in G$.

Proof:

Let λ_a be an M – N anti fuzzy soft subgroup of a group G and $s \in G$

By the above theorem (that is theorem 5.2.2)

$s \lambda_a s^{-1}$ is an M – N anti fuzzy soft subgroup of the group G

Thus $(s \lambda_a s^{-1}) (m xn) = \lambda_a (m s^{-1} x sn)$, for all $x \in G$, $m \in M$, $a \in A$ and $n \in N$

Therefore λ_a and $s \lambda_a s^{-1}$ are conjugate of M – N anti fuzzy soft subgroup of G

We know that the theorem if λ_a and μ_a are conjugate of M – N anti fuzzy soft subgroup of a group G, then $o(\lambda_a) = o(\mu_a)$

Hence $o(s \lambda_a s^{-1}) = o(\lambda_a)$ for any $s \in G$ and $a \in A$.

Definition 5.2.4

Let λ_a and μ_a be an M – N anti fuzzy soft subgroup of the group G, and f be a positive fuzzy set, then for $s \in G$ we define the **M – N anti positive double fuzzy soft coset**

$$(\lambda_a s \mu_a)^f \text{ by } (\lambda_a s \mu_a)^f = \max \{ (s \lambda_a)^f, (s \mu_a)^f \} \text{ for some } a \in A$$

Theorem 5.2.5

The $M - N$ anti positive double fuzzy soft coset $(\lambda_a \text{ s } \mu_a)^f$ is $M - N$ anti fuzzy soft subgroup of the group G , when λ_a, μ_a are $M - N$ anti fuzzy soft subgroup of G

Proof

Let $x, y \in G$, $m \in M$, $a \in A$ and $n \in N$

$$\begin{aligned} \text{Now } (\lambda_a \text{ s } \mu_a)^f (mxy^{-1}n) &= \max \{ (s\lambda_a)^f (mxy^{-1}n), (s\mu_a)^f (mxy^{-1}n) \} \\ &= \max \{ f(s)\lambda_a (mxy^{-1}n), f(s)\mu_a (mxy^{-1}n) \} \\ &\leq f(s) \max \{ \max \{ \lambda_a (mx), \lambda_a (y^{-1}n) \}, \max \{ \mu_a (mx), \mu_a (y^{-1}n) \} \} \end{aligned}$$

Since $\lambda (mx) \leq \lambda (x)$, $\lambda (x^{-1}) = \lambda (x)$ and $\mu (xn) \leq \mu (x)$, $\mu (x^{-1}) = \mu (x)$

$$\begin{aligned} &\leq f(s) \max \{ \max \{ \lambda_a (x), \lambda_a (y) \}, \max \{ \mu_a (x), \mu_a (y) \} \} \\ &\leq f(s) \max \{ \max \{ \lambda_a (x), \mu_a (x) \}, \max \{ \lambda_a (y), \mu_a (y) \} \} \\ &= \max \{ f(s) \max \{ \lambda_a (x), \mu_a (x) \}, f(s) \max \{ \lambda_a (y), \mu_a (y) \} \} \\ &= \max \{ (\lambda_a \text{ s } \mu_a)^f (x), (\lambda_a \text{ s } \mu_a)^f (y) \} \end{aligned}$$

Therefore $(\lambda_a \text{ s } \mu_a)^f (mxy^{-1}n) \leq \max \{ (\lambda_a \text{ s } \mu_a)^f (x), (\lambda_a \text{ s } \mu_a)^f (y) \}$

Hence $(\lambda_a \text{ s } \mu_a)^f$ is $M - N$ anti fuzzy soft subgroup of the group G

Theorem 5.2.6

If $x^{-1} f_a x$ is an soft middle coset of a group G , for some $x \in G$ and $a \in A$, then for each x form the normalize $N(f_a)$ of $M - N$ anti fuzzy soft subgroup f_a of G iff f_a is $M - N$ anti fuzzy normal soft subgroup.

Proof: It is trivial.

Proposition 5.2.7

Let f_a be an $M - N$ anti fuzzy normal soft subgroup of a group G by x and y , then every fuzzy middle soft coset $xf_a y$ coincides with some left and right cosets $s(f_a)$ and $f_a(s)$ respectively, where s^{-1} is the product of $y^{-1} x^{-1}$

Theorem 5.2.8

Let G be an $M - N$ group of order 2 and f_a is an $M - N$ anti fuzzy normal soft subgroup of G , then for some $x, y \in G$ the middle coset x of y coincides with fuzzy subgroup f_a

Proof: It is trivial.

CHAPTER – 6

CONCLUSION

6. CONCLUSION

In this thesis, we have defined the concept of M-N fuzzy soft subgroups based on the concept of fuzzy group. We have also defined the M-N fuzzy soft set and M-N level subsets of a fuzzy soft subgroup. Further, the M-N normal fuzzy soft set are defined and discussed the intersection of two and three variables of M-N level subsets of normal fuzzy soft subgroups with some of its elementary properties. Here, the M-N homomorphism of fuzzy soft subgroup and the M-N Homomorphism of level subsets of fuzzy soft subgroups are also defined.

Subsequently, we have discussed the Conjugate of M-N fuzzy soft subgroups based on the concept of fuzzy soft group. The conjugate of M-N fuzzy soft subgroups and some elementary properties are also deliberated. We have defined and discussed the concept of M-N fuzzy soft middle coset with some related results.

Furthermore, we have defined the M-N anti-fuzzy soft set and described the M-N level subsets of Anti-fuzzy soft subgroups. We have also discussed the concept of M-N Anti-fuzzy soft subgroups with some of its elementary properties. In addition, we defined the M-N Anti-fuzzy normal soft set and the intersection of two and three variables of M-N level subsets of anti-fuzzy normal soft subgroup. Moreover, we have defined the M-N anti-homomorphism of fuzzy soft subgroup and the M-N Homomorphism of anti-level subsets of fuzzy soft subgroups. We discussed the Conjugate of M-N anti-fuzzy soft subgroups based on the concept of fuzzy soft group. In addition, we defined the conjugate of M-N anti-fuzzy soft subgroups and some of the elementary properties are also discussed. Finally, we defined the concept of M- N anti-fuzzy soft middle coset and discussed about some of the related results.

LIST OF PUBLICATIONS

LIST OF PUBLICATIONS

1. M. Kaliraja and S. Rumenaka, M-N Fuzzy Normal Soft Groups, International Journal of Fuzzy Mathematical Archive, Vol. 13, No. 2, 2017, 159-165.
2. M. Kaliraja and S. Rumenaka, M-N Anti Fuzzy Normal Soft Groups, International Journal of Mathematics And its Applications, 6(1-E)(2018), 1035–1042.
3. M. Kaliraja and S. Rumenaka, Some Results on M-N fuzzy soft groups, Journal of Applied Science and Computations, Volume V, Issue XII, December/2018, Page No: 2429, ISSN NO: 1076-5131. **(UGC APPROVED LIST NO.: 10765131)**
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5. M. Kaliraja and S. Rumenaka, Conjugate of M-N Fuzzy Soft Subgroups. Advances in Mathematics: Scientific Journal, 8 (2019), no.3, 568–573. **(UGC CARE LIST: SCOPUS)**
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PRESENTATIONS

- ❖ Presented a paper entitled “M-N Anti fuzzy soft groups” International Conference on Emerging Trends in Mathematical Physical and Chemical Sciences (ETMPC – 2019)”, organized by the Faculty of Science and Humanities, Noorul Islam Centre for Higher Education, Kumaracoil, held on 22 February 2019.
- ❖ Presented a paper entitled ” Conjugate of M-N Fuzzy Soft Subgroups”, *International Conference on Recent Advances In Pure and Applied Mathematics*, Raja Doraisingam Government Arts College, Sivagangai, 27th -28th August 2019.

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M-N Fuzzy Normal Soft Groups

M. Kaliraja* and S. Rumenaka

PG and Research Department of Mathematics
H.H. The Rajah's College, Pudukkottai-622001, India
mkr.maths009@gmail.com; rumenaka@gmail.com

*Corresponding author.

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Abstract. In this paper, we have discussed the concept of M-N fuzzy normal soft group, we then define the M-N level subsets of a fuzzy normal soft subgroup and its some elementary properties are also discussed. The presented method in this manuscript is more sensible and also reliable in solving the problems. This method can solve the decision making problems.

Keywords: Fuzzy group, M-N fuzzy group, M-N fuzzy soft subgroup, M-N level subset, M-N fuzzy soft normalize.

AMS Mathematics Subject Classification (2010): 20N25, 06D72, 20E15

1. Introduction

There are various types of uncertainties in the real world, but few classical mathematical tools may not be suitable to model these uncertainties. Many intricate problems in economics, social science, engineering, medical science and many other fields involve undefined data. These problems which one comes face to face with in life cannot be solved using classical mathematic methods. In classical mathematics, a mathematical model of an object is devised and the concept of the exact solution of this model is not yet determined. Since, the classical mathematical model is too complex, the exact solution cannot be found. There are several well-renowned theories available to describe uncertainty. For instance, Rosenfeld [7] introduced the concept of fuzzy subgroup in 1971 and the fuzzy normal subgroup was revealed by Wu [10] during 1981. Further, the theory of fuzzy sets was inspired by Zadoh [11] in addition to this, Molodtsov [5] have introduced the concept of softsets in 1999. Furthermore, Maji et al., [4] as well introduction the concept of fuzzy soft sets in 2001 and Jacobson [3] introduced the concept of m-group M-subgroup.

In 2015, Patel et al. [6] were developed three variables on normal fuzzy soft subgroup. Sarala and Suganya [8] beunraveled the three variables on normal fuzzy soft subgroup in 2004. In addition, Kandasamy [9] have introduced the fuzzy algebra during 2003. An introduction to the new definition of Soft sets and soft groups depending on inclusion relation and intersection of sets were exposed by Akta and Cagman [1]. In 1981, Das [2] studied the Fuzzy groups and level subgroups. Moreover, Maij et al. [4] were introduced the fuzzy soft set in 2001.

M-N Fuzzy Normal Soft Groups

In the present manuscript, we have discussed the concept of M-N fuzzy normal soft group based on the concept of Normal fuzzy soft group [6, 8]. In section 2, we presented the basic definition, notations on M-N fuzzy normal soft group and required results on fuzzy normal soft group. In section 3, we define the M-N fuzzy soft set, normal fuzzy soft set and also define the M-N level subsets of a normal fuzzy soft subgroup. We have also discussed the concept of M-N fuzzy normal soft group and some of its elementary properties.

2. Preliminaries

In this section, some basic definitions and results needed are given. For the sake of convenience we set out the former concepts which will be used in this paper.

Definition 2.1. Let G be any non-empty set. A mapping $\alpha : G \rightarrow [0, 1]$ is called fuzzy set in G .

Definition 2.2. Let x be a non-empty set. A fuzzy subset α of X is a function

$$\alpha : X \rightarrow [0, 1].$$

Definition 2.3. Let G be a group. A fuzzy subset α of G is called a fuzzy subgroup if for $x, y \in G$

- (1) $\alpha(xy) \geq \min\{\alpha(x), \alpha(y)\}$
- (2) $\alpha(x^{-1}) = \alpha(x)$

Definition 2.4. A pair (F, A) is called a soft set over U , where F is a mapping given by $F: A \rightarrow P(U)$

Definition 2.5. Let (F, A) be a soft set over G . Then (F, A) is called a soft group over G if $F(\alpha)$ is a group G for all $\alpha \in A$.

Definition 2.6. A pair (F, A) is called a fuzzy soft set over U , where $F: A \rightarrow I^U$ is a mapping $I = [0, 1]$, $F(\alpha)$ is a fuzzy subset of U for all $\alpha \in A$.

Definition 2.7. Let (F, A) be a fuzzy soft set over G . Then (F, A) is called a fuzzy soft group if $F(\alpha)$ is a fuzzy subgroup G for all $\alpha \in A$.

Definition 2.8. Let (F, A) and (G, B) be two fuzzy soft set over U . Then (F, A) is called a fuzzy soft subset of (G, B) denoted by $(F, A) \subseteq (G, B)$ if

- (1) $A \subseteq B$
- (2) $F(\alpha)$ is a fuzzy subset of $G(\alpha)$ for each $\alpha \in A$.

Definition 2.9. A fuzzy set α is called a fuzzy soft subgroup of a group G , if for $x, y \in G$

- (1) $\alpha(xy) \geq \min\{\alpha(x), \alpha(y)\}$
- (2) $\alpha(x^{-1}) \geq \alpha(x)$

Definition 2.10. A subgroup H of G is called a normal subgroup of a G if $aH = Ha$ for all $a \in G$

Definition 2.11. Let G be a group. A fuzzy subgroup α of G is said be normal if for all $x, y \in G$, $\alpha(Xyx^{-1}) = \alpha(y)$ (or) $\alpha(xy) \geq \alpha(yx)$

Definition 2.12. Let G be a group. A fuzzy soft subgroup α of G is said be fuzzy normal soft subgroup, if for all $x, y \in G$ and $\alpha(xyx^{-1}) = \alpha(y)$ (or) $\alpha(xy) \geq \alpha(yx)$

Definition 2.13. Let $\alpha \cap \beta$ be a fuzzy soft subgroup of a group G , for any $t \in [0, 1]$, we define the level subset of $\alpha \cap \beta$ is the set

$$(\alpha \cap \beta)_t = \{x \in X / (\alpha \cap \beta)(x) \geq t\}$$

Definition 2.14. Let G be a group and $\alpha \cap \beta$ be a fuzzy normal soft subgroup of G .

Let $N(\alpha \cap \beta) = \{y \in G / (\alpha \cap \beta)(y x y^{-1}) = (\alpha \cap \beta)(x) \text{ for all } x \in G\}$, then $N(\alpha \cap \beta)$ is called the fuzzy soft Normalize of $\alpha \cap \beta$.

Definition 2.15. Let M, N be left and right operator sets of group G respectively if $(m x)n = m(x n)$ for all $x \in G$, $m \in M, n \in N$. Then G is said be an $M - N$ group.

Definition 2.16. If α is an $M - N$ fuzzy subgroup of an $M - N$ group G . Then the following statement holds for all $x, y \in G, m \in M$, and $n \in N$

- (1) $\alpha(m(x y)n) \geq \min\{\alpha(x), \alpha(y)\}$
- (2) $\alpha(m x^{-1} n) \geq \alpha(x)$

Definition 2.17. Let G be an $M - N$ group. α is said be an $M - N$ fuzzy normal subgroup of G if α is not only an $M - N$ fuzzy subgroup of G , but also fuzzy normal subgroup of G .

3. M-N fuzzy normal soft group

In this section, we shall define $M - N$ fuzzy soft group, fuzzy normal soft group, discussed the concept of $M - N$ fuzzy normal soft group based on the concept of fuzzy normal soft group [6, 8], also define the $M - N$ level subsets of a fuzzy normal soft subgroup and its some elementary properties are discussed.

Definition 3.1. Let G be an $M - N$ group and (F, A) be a fuzzy soft subgroup of G if

$$(1) F\{m(x y)n\} \geq \min\{F(x), F(y)\}$$

(2) $F\{(m x^{-1})n\} \geq F(x)$ hold for any $x, y \in G, m \in M, n \in N$, then (F, A) is said be an $M - N$ fuzzy soft subgroup of G . Here $F: A \rightarrow P(G)$

Definition 3.2. Let G be an $M - N$ group and (F, A) be a fuzzy soft subgroup of G if

$$(1) F(m x) \geq F(x)$$

(2) $F(x n) \geq F(x)$ hold for any $x \in G, m \in M$, and $n \in N$, then (F, A) is said be an $M - N$ fuzzy soft subgroup of G .

Definition 3.3. Let G be an $M - N$ group. (F, A) is said be an $M - N$ fuzzy normal soft subgroup of G if (F, A) is not only an $M - N$ soft fuzzy subgroup of G , but also fuzzy normal soft subgroup of G .

M-N Fuzzy Normal Soft Groups

Theorem 3.4. If α , β and γ are three M – N fuzzy soft subgroup of G, then $\alpha \cap \beta \cap \gamma$ is a M- N fuzzy soft subgroup of G

Proof: Let α , β and γ be three M - N fuzzy soft subgroup of G.

$$\begin{aligned}
 (1) \quad & (\alpha \cap \beta \cap \gamma) (m(xy^{-1})n) \geq \min \{ (\alpha \cap \beta \cap \gamma) (m x), (\alpha \cap \beta \cap \gamma) (y^{-1}n) \} \\
 & (\alpha \cap \beta \cap \gamma) (m (xy^{-1}) n) \geq \min \{ (\alpha \cap \beta) (m (xy^{-1}) n), \gamma (m (xy^{-1}) n) \} \\
 & \geq \min \{ \min \{ (\alpha \cap \beta) (m x), (\alpha \cap \beta) (y^{-1}n) \}, \min \{ \gamma (m x), \gamma (y^{-1}n) \} \} \\
 & \geq \min \{ \min \{ (\alpha \cap \beta) (m x), \gamma (m x) \}, \min \{ (\alpha \cap \beta) (y^{-1}n), \gamma (y^{-1}n) \} \} \\
 & \geq \min \{ (\alpha \cap \beta \cap \gamma) (m x), (\alpha \cap \beta \cap \gamma) (y^{-1}n) \}. \\
 (2) \quad & (\alpha \cap \beta \cap \gamma) (m x n) = (\alpha \cap \beta \cap \gamma) (m x^{-1}n) \\
 & (\alpha \cap \beta \cap \gamma) (m x n) = \{ (\alpha \cap \beta) (m x n), \gamma (m x n) \} \\
 & = \{ [\alpha (m x n), \beta (m x n)], \gamma (m x n) \} \\
 & = \{ [\alpha (m x^{-1}n), \beta (m x n)], \gamma (m x n) \} \\
 & = \{ (\alpha \cap \beta) (m x^{-1}n), \gamma (m x^{-1}n) \} \\
 & = (\alpha \cap \beta \cap \gamma) (m x^{-1}n)
 \end{aligned}$$

Hence, $\alpha \cap \beta \cap \gamma$ is an M – N fuzzy soft subgroup of G.

Theorem 3.5. The intersection of any three M – N fuzzy normal soft subgroup of G is also an M – N fuzzy normal soft subgroup G.

Proof: Let α , β and γ are three M – N fuzzy normal soft subgroup of G.

By the previous theorem we know that, $\alpha \cap \beta \cap \gamma$ is an M – N fuzzy soft subgroup of G.

Let $x, y \in G, m \in M$, and $n \in N$

To prove that $(\alpha \cap \beta \cap \gamma) (m y x y^{-1} n) = (\alpha \cap \beta \cap \gamma) (m x n)$

$$\begin{aligned}
 \text{Now } (\alpha \cap \beta \cap \gamma) (m y x y^{-1} n) &= \min \{ (\alpha \cap \beta) (m (y x y^{-1}) n), \gamma (m (y x y^{-1}) n) \} \\
 &= \min \{ [\alpha (m (y x y^{-1}) n), \beta (m (y x y^{-1}) n)], \gamma (m (y x y^{-1}) n) \} \\
 &= \min \{ [\alpha (m x n), \beta (m x n)], \gamma (m x n) \} \\
 &= \min \{ (\alpha \cap \beta) (m x n), \gamma (m x n) \} \\
 &= (\alpha \cap \beta \cap \gamma) (m x n)
 \end{aligned}$$

Therefore, $(\alpha \cap \beta \cap \gamma) (m y x y^{-1} n) = (\alpha \cap \beta \cap \gamma) (m x n)$.

Hence $\alpha \cap \beta \cap \gamma$ is an M – N fuzzy normal soft subgroup G.

Note 3.6. If $(\alpha \cap \beta)_i, i \in \Delta$ are M – N fuzzy normal soft subgroup of G, then $\bigcap_{i \in \Delta} (\alpha \cap \beta)_i$ is a M – N fuzzy normal soft subgroup of G.

Definition 3.7. Let G be a group, α is a M – N fuzzy soft subgroup of G is said be a M – N fuzzy normal soft subgroup if

$$\begin{aligned}
 & \alpha (m (x y x^{-1}) n) = \alpha (m y n) \text{ (or)} \\
 & \alpha (m (x y) n) \geq \alpha (m (y x) n) \text{ for all } x, y \in G, m \in M, \text{ and } n \in N.
 \end{aligned}$$

Theorem 3.8. Let α is an M – N fuzzy normal soft subgroup of G, then for any $y \in G$ we have $\alpha (m (y^{-1} x y) n) = \alpha (m (x y) n)$

Proof: Let α is an M- N fuzzy normal soft subgroup G, then for any $y \in G$.

$$\begin{aligned}
 \text{Now } \alpha (m (y^{-1} x y) n) &= \alpha (m (x y^{-1} y) n) \\
 &= \alpha (m (x) n) \\
 &= \alpha (m (y y^{-1} x) n) \\
 &= \alpha (m (y x y^{-1}) n)
 \end{aligned}$$

M. Kaliraja and S. Rumenaka

Therefore $\alpha(m(y^{-1}xy)n) = \alpha(m(yxy^{-1})n)$.
Hence the theorem.

Theorem 3.9. If α is an M- N fuzzy normal soft subgroup G, then $g\alpha g^{-1}$ is also M- N fuzzy normal soft subgroup G, for all $g \in G$.

Proof: Let α be an M- N fuzzy normal soft subgroup G, then $g\alpha g^{-1}$ is an M- N subgroup G, for all $g \in G$.

$$\begin{aligned} \text{Now } g\alpha g^{-1}(m(yxy^{-1})n) &= \alpha(g^{-1}m(yxy^{-1})ng) \\ &= \alpha(m(yxy^{-1})n) \\ &= \alpha(mxn) \\ &= \alpha(g(mxn))g^{-1} \\ &= g\alpha g^{-1}(mxn) \end{aligned}$$

Therefore, $g\alpha g^{-1}(m(yxy^{-1})n) = g\alpha g^{-1}(mxn)$.
Hence the theorem.

Theorem 3.10. Let $\alpha \cap \beta$ is a M – N fuzzy normal soft subgroup of G, then

$$(\alpha \cap \beta)(m(y^{-1}xy)n) = (\alpha \cap \beta)(mxn).$$

Proof: Let $\alpha \cap \beta$ is an M – N fuzzy normal soft subgroup of G.

Let x, y $\in$ G.

$$\begin{aligned} \text{Now } (\alpha \cap \beta)(m(y^{-1}xy)n) &= (\alpha \cap \beta)(m(xy^{-1}y)n) \\ &= (\alpha \cap \beta)(m(x)n) \\ &= (\alpha \cap \beta)(m(yy^{-1}x)n) \\ &= (\alpha \cap \beta)(m(yxy^{-1})n) \end{aligned}$$

$$(\alpha \cap \beta)(m(y^{-1}xy)n) = (\alpha \cap \beta)(m(yxy^{-1})n).$$

Hence the proof.

Theorem 3.11. If $\alpha \cap \beta$ is a M- N fuzzy normal soft subgroup G, then $g(\alpha \cap \beta)g^{-1}$ is also a M- N fuzzy normal soft subgroup G, for all $g \in G$.

Proof: If $\alpha \cap \beta$ is a M- N fuzzy normal soft subgroup G, then $g(\alpha \cap \beta)g^{-1}$ is also a M- N fuzzy normal soft subgroup G, for all $g \in G$.

$$\begin{aligned} \text{To prove that } g(\alpha \cap \beta)g^{-1}(m(yxy^{-1})n) &= g(\alpha \cap \beta)g^{-1}(mxn). \\ \text{Now } g(\alpha \cap \beta)g^{-1}(m(yxy^{-1})n) &= (\alpha \cap \beta)(g^{-1}m(yxy^{-1})ng) \\ &= (\alpha \cap \beta)(m(yxy^{-1})n) \\ &= (\alpha \cap \beta)(mxn) \\ &= (\alpha \cap \beta)(g(mxn))g^{-1} \\ &= g(\alpha \cap \beta)g^{-1}(mxn). \end{aligned}$$

Therefore, $g(\alpha \cap \beta)g^{-1}(m(yxy^{-1})n) = g(\alpha \cap \beta)g^{-1}(mxn)$.
Hence the theorem.

Definition 3.12. Let $\alpha \cap \beta$ be an M – N fuzzy soft subgroup of a group G. For any $t \in [0, 1]$, we define the M – N level subset of $\alpha \cap \beta$ is the set

$$(\alpha \cap \beta)_t = \{x \in G / (\alpha \cap \beta)(mx) \geq t, (\alpha \cap \beta)(xn) \geq t \text{ for all } m \in M, n \in N\}$$

Theorem 3.13. Let G be a group and $\alpha \cap \beta$ be a fuzzy subset of G. Then $\alpha \cap \beta$ is a M – N fuzzy normal soft subgroup of G iff the level subset $(\alpha \cap \beta)_t, t \in [0, 1]$ are M- N subgroup of G.

M-N Fuzzy Normal Soft Groups

Proof: Let $\alpha \cap \beta$ be an M – N fuzzy normal soft subgroup of G and the level subset $(\alpha \cap \beta)_t = \{x \in G / (\alpha \cap \beta)(mx) \geq t, (\alpha \cap \beta)(xn) \geq t, t \in [0,1] m \in M, n \in N\}$

Let $x, y \in (\alpha \cap \beta)_t$ then $(\alpha \cap \beta)(mx) \geq t$ and $(\alpha \cap \beta)(xn) \geq t$

$$\begin{aligned} \text{Now } (\alpha \cap \beta)(mxy^{-1}n) &\geq \min \{(\alpha \cap \beta)(mx), (\alpha \cap \beta)(y^{-1}n)\} \\ &= \min \{(\alpha \cap \beta)(mx), (\alpha \cap \beta)(yn)\} \\ &\geq \min \{t, t\} \\ (\alpha \cap \beta)(mxy^{-1}n) &\geq t \\ mxy^{-1}n &\in (\alpha \cap \beta)_t \end{aligned}$$

Therefore $(\alpha \cap \beta)_t$ is a M – N subgroup of G.

Conversely, let us assume that $(\alpha \cap \beta)_t$ is an M – N subgroup G.

Let $x, y \in (\alpha \cap \beta)_t$ then $(\alpha \cap \beta)(mx) \geq t$ and $(\alpha \cap \beta)(xn) \geq t$

Also $(\alpha \cap \beta)(m(xy^{-1})n) \geq t$.

$$\begin{aligned} \text{Since } m(xy^{-1})n \in (\alpha \cap \beta)_t &= \min \{t, t\} \\ &= \min \{(\alpha \cap \beta)(mx), (\alpha \cap \beta)(yn)\}. \end{aligned}$$

Therefore $(\alpha \cap \beta)(m(xy^{-1})n) \geq \min \{(\alpha \cap \beta)(mx), (\alpha \cap \beta)(yn)\}$.

Hence $\alpha \cap \beta$ is an M – N fuzzy normal soft subgroup of G.

Definition 3.14. Let G be a group and $\alpha \cap \beta$ be an M – N fuzzy normal soft subgroup of G.

Let $N(\alpha \cap \beta) = \{y \in G / (\alpha \cap \beta)(m(yxy^{-1})n) = (\alpha \cap \beta)(mxn) \text{ for all } x \in G, m \in M, n \in N\}$, then $N(\alpha \cap \beta)$ is called the M – N fuzzy soft normalizer of $\alpha \cap \beta$.

Theorem 3.15. Let G be a group and $\alpha \cap \beta$ be a fuzzy subset of G. Then $\alpha \cap \beta$ is a M – N fuzzy normal soft subgroup of G iff the level subset $(\alpha \cap \beta)_t, t \in [0, 1]$ are M- N normal subgroup of G.

Proof: Let $\alpha \cap \beta$ be a M – N fuzzy normal soft subgroup of G and level subset $(\alpha \cap \beta)_t, t \in [0,1]$.

Let $x \in G$ and $y \in (\alpha \cap \beta)_t$, then $(\alpha \cap \beta)(my n) \geq t$, for all $m \in M, n \in N$.

Now $(\alpha \cap \beta)(m(xy x^{-1})n) = (\alpha \cap \beta)(my n) \geq t$.

Since $\alpha \cap \beta$ is an M – N fuzzy normal soft subgroup of G.

That is $(\alpha \cap \beta)(m(xy x^{-1})n) \geq t$.

Therefore $m(xy x^{-1})n \in (\alpha \cap \beta)_t$.

Hence $(\alpha \cap \beta)_t$ is an M – N normal subgroup of G.

4. Conclusion

The main results in the present manuscript are based on the concept of fuzzy normal soft group [6 and 8]. We have also defined the M-N level subsets of a fuzzy normal soft subgroup and its some elementary properties are discussed.

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M. Kaliraja and S. Rumenaka

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M-N Anti Fuzzy Normal Soft Groups

M. Kaliraja^{1,*} and S. Rumenaka¹

¹ PG and Research Department of Mathematics, H.H.The Rajah's college, Pudukkottai, Tamilnadu, India.

Abstract: In this paper, we have discussed the concept of M-N anti fuzzy normal soft group, we then define the M-N anti level subsets of a normal fuzzy soft subgroup and its some elementary properties are also discussed.

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Keywords: Fuzzy group, M-N anti fuzzy group, M-N anti fuzzy soft subgroup, M-N anti level subset, M-N anti fuzzy soft normalize.
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1. Introduction

There are various types of uncertainties in the real world, but few classical mathematical tools may not be suitable to model these uncertainties. Many intricate problems in economics, social science, engineering, medical science and many other fields involve undefined data. These problems which one comes face to face with in life cannot be solved using classical mathematic methods. In classical mathematics, a mathematical model of an object is devised and the concept of the exact solution of this model is not yet determined. Since, the classical mathematical model is too complex, the exact solution cannot be found. There are several well-renowned theories available to describe uncertainty. For instance, Rosenfeld [12] introduced the concept of fuzzy subgroup in 1971 and the fuzzy normal subgroup was revealed by Wu [16] during 1981. Further, the theory of fuzzy sets was inspired by Zadeh [18] in addition to this, Molodtsov [9] have introduced the concept of soft sets in 1999. Furthermore, Majiet. al., [8] as well introduction the concept of fuzzy soft sets in 2001 and Jacobson [6].

In 2015, R. Patel, Ramakant Bhardmal, Sanjay Choudhary, Sunil [11] were developed three variables on normal fuzzy soft subgroup. Sarala and Suganya [13] be unraveled the three variables on normal fuzzy soft subgroup in 2004. In addition, Vasantha Kandasamy and Smarandache [15] have introduced the Fuzzy Algebra during 2003. An introduction to the new definition of Soft sets and soft groups depending on inclusion relation and intersection of sets were exposed by Akta and Cagman [1]. In 1981, Das [2] studied the Fuzzy groups and level subgroups. Moreover, Maij, Biswas and Ray [8] were introduced the fuzzy soft set in 2001.

In [4] Biswas introduced the concept of anti-fuzzy subgroup of groups. Shen researched anti-fuzzy subgroups in [14] and Dong [3] studied the product of anti- fuzzy subgroups. Feng and Yao [5] studied the concept of (λ, μ) anti-fuzzy subgroups. Pandiamml et al, (2010) defined a new algebraic structure of anti L-fuzzy normal M-subgroups. Wang Sheng-hai [17] further obtained some basic properties of anti fuzzy subgroups and anti fuzzy normal fuzzy subgroups of group. Mourad Oqla Massa'deh [10] have discussed The M-N-homomorphism and M-N-anti homomorphism over M-N-fuzzy subgroups in 2012,

* E-mail: mkr.maths009@gmail.com

In our earlier work have discussed the concept of M-N fuzzy normal soft groups [7].

In the present manuscript, we have discussed the concept of M-N anti fuzzy normal soft group based on the concept of Normal fuzzy soft group [2, 7, 10, 13]. In section 2, we presented the basic definition; notations on M-N fuzzy normal soft group and M-N anti fuzzy normal soft subgroup are required results on fuzzy normal soft group. In section 3, we define the M-N anti fuzzy soft group, normal fuzzy soft group and also define the M-N anti level subsets of a normal fuzzy soft subgroup. We have also discussed the concept of M-N anti fuzzy normal soft group and some of its elementary properties.

2. Preliminaries

In this section, some basic definitions and results needed are given. For the sake of convenience we set out the former concepts which will be used in this paper.

Definition 2.1. Let G be any non-empty set. A mapping $\alpha : G \rightarrow [0, 1]$ is called fuzzy set in G .

Definition 2.2. Let x be a non-empty set. A fuzzy subset α of X is a function $\alpha : X \rightarrow [0, 1]$.

Definition 2.3. Let G be a group. A fuzzy subset α of G is called an anti fuzzy subgroup if for $x, y \in G$

$$(1). \alpha(xy) \leq \max \{ \alpha(x), \alpha(y) \}.$$

$$(2). \alpha(x^{-1}) = \alpha(x).$$

Definition 2.4. A pair (F, A) is called a soft set over U , where F is a mapping given by $F : A \rightarrow P(U)$.

Definition 2.5. Let (F, A) be a soft set over G . Then (F, A) is called a soft group over G if $F(\alpha)$ is a group G for all $\alpha \in A$.

Definition 2.6. A pair (F, A) is called a fuzzy soft set over U , where $F : A \rightarrow I^U$ is a mapping $I = [0, 1]$, $F(\alpha)$ is a fuzzy subset of U for all $\alpha \in A$.

Definition 2.7. Let (F, A) be a fuzzy soft set over G . Then (F, A) is called a fuzzy soft group if $F(\alpha)$ is a fuzzy subgroup G for all $\alpha \in A$.

Definition 2.8. Let (F, A) and (G, B) be two fuzzy soft set over U . Then (F, A) is called a fuzzy soft subset of (G, B) denoted by $(F, A) \subseteq (G, B)$ if

$$(1). A \subseteq B.$$

$$(2). F(\alpha) \text{ is a fuzzy subset of } G(\alpha), \text{ for each } \alpha \in A.$$

Definition 2.9. A fuzzy set α is called an anti fuzzy soft subgroup of a group G , if for $x, y \in G$

$$(1). \alpha(xy) \leq \max \{ \alpha(x), \alpha(y) \}.$$

$$(2). \alpha(x^{-1}) \leq \alpha(x).$$

Definition 2.10. Let G be a group. A fuzzy soft subgroup α of G is said be an anti fuzzy normal soft subgroup, if for all $x, y \in G$ and $\alpha(xy x^{-1}) = \alpha(y)$ (or) $\alpha(xy) \leq \alpha(yx)$.

Definition 2.11. Let $\alpha \cup \beta$ be an anti fuzzy soft subgroup of a group G , for any $t \in [0, 1]$, we define the anti level subset of $\alpha \cup \beta$ is the set $(\alpha \cup \beta)_t = \{x \in X / (\alpha \cup \beta)(x) \leq t\}$.

Definition 2.12. Let G be a group and $\alpha \cup \beta$ be an anti fuzzy normal soft subgroup of G . Let $N(\alpha \cup \beta) = \{y \in G / (\alpha \cup \beta)(xy^{-1}) = (\alpha \cup \beta)(x) \text{ for all } x \in G\}$, then $N(\alpha \cup \beta)$ is called an anti fuzzy soft Normalizer of $\alpha \cup \beta$.

Definition 2.13. Let M, N be left and right operator sets of group G respectively, if $(mx)n = m(xn)$ for all $x \in G, m \in M, n \in N$. Then G is said to be an $M - N$ group.

Definition 2.14. If α is an $M - N$ fuzzy subgroup of an MN group G . Then the following statement holds for all $x, y \in G, m \in M$, and $n \in N$

$$(1). \alpha(m(xy)n) \geq \min\{\alpha(x), \alpha(y)\}.$$

$$(2). \alpha(mx^{-1}n) \geq \alpha(x).$$

Definition 2.15. Let G be an $M - N$ group. α is said to be an $M - N$ normal fuzzy subgroup of G if α is not only an $M - N$ fuzzy subgroup of G , but also normal fuzzy subgroup of G .

Definition 2.16. Let G be an $M - N$ group and (F, A) be a fuzzy soft subgroup of G if

$$(1). F\{m(xy)n\} \geq \min\{F(x), F(y)\}.$$

$$(2). F\{(mx^{-1})n\} \geq F(x) \text{ hold for any } x, y \in G, m \in M, n \in N, \text{ then } (F, A) \text{ is said to be an } M - N \text{ fuzzy soft subgroup of } G.$$

Here $F : A \rightarrow P(G)$.

Definition 2.17. Let G be an $M - N$ group and (F, A) be a fuzzy soft subgroup of G if

$$(1). F(mx) \geq F(x).$$

$$(2). F(xn) \geq F(x) \text{ hold for any } x \in G, m \in M, \text{ and } n \in N, \text{ then } (F, A) \text{ is said to be an } M - N \text{ fuzzy soft subgroup of } G.$$

Definition 2.18. Let G be an $M - N$ group. (F, A) is said to be an $M - N$ fuzzy normal soft subgroup of G if (F, A) is not only an $M - N$ fuzzy soft subgroup of G , but also normal fuzzy soft subgroup of G .

Definition 2.19. Let G be a group, α is a $M - N$ fuzzy soft subgroup of G is said to be a $M - N$ fuzzy normal soft subgroup if $\alpha(mxyx^{-1}n) = \alpha(myn)$ (or) $\alpha(m(xy)n) \geq \alpha(m(yx)n)$ for all $x, y \in G, m \in M$, and $n \in N$.

Definition 2.20. Let $\alpha \cap \beta$ be an $M - N$ fuzzy soft subgroup of a group G . For any $t \in [0, 1]$, we define the $M - N$ level subset of $\alpha \cap \beta$ is the set $(\alpha \cap \beta)_t = \{x \in G / (\alpha \cap \beta)(mx) \geq t, (\alpha \cap \beta)(xn) \geq t \text{ for all } m \in M, n \in N\}$.

Definition 2.21. Let G be a group and $\alpha \cap \beta$ be an $M - N$ fuzzy normal soft subgroup of G . Let $N(\alpha \cap \beta) = \{y \in G / (\alpha \cap \beta)(m(xy^{-1})n) = (\alpha \cap \beta)(mxn) \text{ for all } x \in G, m \in M, n \in N\}$, then $N(\alpha \cap \beta)$ is called the $M - N$ fuzzy soft Normalizer of $\alpha \cap \beta$.

Definition 2.22. If α is an $M - N$ anti fuzzy subgroup of an $M - N$ group G . Then the following statement holds for all $x, y \in G, m \in M$, and $n \in N$

$$(1). \alpha(m(xy)n) \leq \max\{\alpha(x), \alpha(y)\}.$$

$$(2). \alpha(mx^{-1}n) \leq \alpha(x).$$

Definition 2.23. Let G be an $M - N$ group. α is said to be an $M - N$ anti fuzzy normal subgroup of G if α is not only an $M - N$ anti fuzzy subgroup of G , but also an anti fuzzy normal subgroup of G .

3. M-N Anti Fuzzy Normal Soft Groups

In this section, we shall define M-N anti fuzzy soft group, anti fuzzy normal soft subgroup, discussed the concept of M-N anti fuzzy normal soft group based on the concept of fuzzy Normal soft group [2, 7, 10, 13], also define the M-N anti level subsets of a fuzzy Normal soft subgroup and its some elementary properties are discussed.

Definition 3.1. Let G be an $M - N$ group and (F, A) be an anti fuzzy soft subgroup of G if

- (1). $F\{m(xy)n\} \leq \max\{F(x), F(y)\}$
- (2). $F\{(mx^{-1})n\} \leq F(x)$ hold for any $x, y \in G, m \in M, n \in N$, then (F, A) is said be an $M - N$ anti fuzzy soft subgroup of G . Here $F : A \rightarrow P(G)$.

Definition 3.2. Let G be an $M - N$ group and (F, A) be an anti fuzzy soft subgroup of G if

- (1). $F(mx) \leq F(x)$
- (2). $F(xn) \leq F(x)$ hold for any $x \in G, m \in M$, and $n \in N$, then (F, A) is said be an $M - N$ anti fuzzy soft subgroup of G .

Definition 3.3. Let G be an $M - N$ group. (F, A) is said be an $M - N$ anti fuzzy normal soft subgroup of G if (F, A) is not only an $M - N$ anti fuzzy soft subgroup of G , but also an anti fuzzy normal soft subgroup of G .

Theorem 3.4. Let G be an $M - N$ group, A and B both be $M - N$ anti fuzzy subgroup of G . Then $A \cup B$ is an $M - N$ anti fuzzy subgroup of G .

Proof.

- (1). $(A \cup B)(m(xy)n) \leq \max\{(A \cup B)(x), (A \cup B)(y)\}$.
 - (2). $(A \cup B)(mx^{-1}n) = (A \cup B)(x)$.
- $$\begin{aligned} (1) \Rightarrow (A \cup B)(m(xy)n) &= \max\{A(m(xy)n), B(m(xy)n)\} \\ &\leq \max\{\max\{A(mx), A(ny)\}, \max\{B(mx), B(ny)\}\} \\ &\leq \max\{\max\{A(x), B(x)\}, \max\{A(y), B(y)\}\}, A(mx) \leq A(x) \\ (A \cup B)(m(xy)n) &\leq \max\{(A \cup B)(x), (A \cup B)(y)\} \\ (2) \Rightarrow (A \cup B)(mx^{-1}n) &= (A \cup B)(mxx^{-1}xn) \\ (A \cup B)(mx^{-1}n) &= \max\{A(mxx^{-1}), B(xn)\} \\ &\leq \max\{\max\{A(mx), A(x^{-1})\}, B(x)\} \\ &\leq \max\{\max\{A(x), A(x^{-1})\}, B(x)\} \\ &\leq \max\{A(x), B(x)\} \\ &= (A \cup B)(x) \\ (A \cup B)(mx^{-1}n) &= (A \cup B)(x) \end{aligned}$$

Hence (1) and (2) is proved. □

Theorem 3.5. If α and β are the two $M - N$ anti fuzzy soft subgroup of G , then $\alpha \cup \beta$ is an $M - N$ anti fuzzy soft subgroup of G .

Proof. Let α and β be two $M-N$ anti fuzzy soft subgroup of G .

$$\begin{aligned}
 (1). \quad & (\alpha \cup \beta)(m(xy^{-1})n) \leq \max\{(\alpha \cup \beta)(mx), (\alpha \cup \beta)(y^{-1}n)\} \\
 & (\alpha \cup \beta)(m(xy^{-1})n) = \max\{\alpha(m(xy^{-1})n), \beta(m(xy^{-1})n)\} \\
 & \leq \max\{\max\{(\alpha(mx), (\alpha(y^{-1}n)\}, \max\{\beta(mx), \beta(y^{-1}n)\}\} \\
 & \leq \max\{\max\{(\alpha(mx), \beta(mx)\}, \max\{(\alpha(y^{-1}n), \beta(y^{-1}n)\}\} \\
 & \leq \max\{(\alpha \cup \beta)(mx), (\alpha \cup \beta)(y^{-1}n)\}
 \end{aligned}$$

Therefore $(\alpha \cup \beta)(m(xy^{-1})n) \leq \max\{(\alpha \cup \beta)(mx), (\alpha \cup \beta)(y^{-1}n)\}$.

$$\begin{aligned}
 (2). \quad & (\alpha \cup \beta)(mxn) = (\alpha \cup \beta)(mx^{-1}n) \\
 & (\alpha \cup \beta)(mxn) = \max\{\alpha(mxn), \beta(mxn)\} \\
 & = \max\{\alpha(mx^{-1}n), \beta(mx^{-1}n)\} \\
 & = (\alpha \cup \beta)(mx^{-1}n)
 \end{aligned}$$

Hence $\alpha \cup \beta$ is an $M - N$ anti fuzzy soft subgroup of G . $\square$

Theorem 3.6. The union of any two $M - N$ anti fuzzy normal soft subgroup of G is also an $M - N$ anti fuzzy normal soft subgroup G .

Proof. Let α , and β be the $M - N$ anti fuzzy normal soft subgroup of G . By the previous theorem we know that, $\alpha \cup \beta$ is an $M - N$ anti fuzzy soft subgroup of G . Let $x, y \in G, m \in M$, and $n \in N$. To prove that $(\alpha \cup \beta)(m(yxy^{-1})n) = (\alpha \cup \beta)(mxn)$.
Now

$$\begin{aligned}
 (\alpha \cup \beta)(m(yxy^{-1})n) &= \max\{\alpha(m(yxy^{-1})n), \beta(m(yxy^{-1})n)\} \\
 &= \max\{[\alpha(mxn), \beta(mxn)]\} \\
 &= (\alpha \cup \beta)(mxn)
 \end{aligned}$$

Hence $(\alpha \cup \beta)(m(yxy^{-1})n) = (\alpha \cup \beta)(mxn)$. Hence $\alpha \cup \beta$ is an $M - N$ anti fuzzy normal soft subgroup of G . $\square$

Note 3.7. If $(\alpha \cup \beta)_i, i \in \Delta$ are $M - N$ anti fuzzy normal soft subgroup of G , then $U_{i \in \Delta}(\alpha \cup \beta)_i$ is a $M - N$ anti fuzzy normal soft subgroup of G .

Definition 3.8. Let G be a group, α is a $M - N$ anti fuzzy soft subgroup of G is said be a $M - N$ anti fuzzy normal soft subgroup if $\alpha(m(xy^{-1})n) = \alpha(myn)$ (or) $\alpha(m(xy)n) \leq \alpha(m(yx)n)$ for all $x, y \in G, m \in M$, and $n \in N$.

Theorem 3.9. Let α be an $M - N$ anti fuzzy normal soft subgroup of G , then for any $y \in G$ we have $\alpha(m(y^{-1}xy)n) = \alpha(m(yxy^{-1})n)$.

Proof. Let α be an $M - N$ anti fuzzy normal soft subgroup G , then for any $y \in G$. Now

$$\begin{aligned}
 \alpha(m(y^{-1}xy)n) &= \alpha(m(xy^{-1}y)n) \\
 &= \alpha(m(x)n) \\
 &= \alpha(m(yy^{-1}x)n) \\
 &= \alpha(m(yxy^{-1})n)
 \end{aligned}$$

Therefore $\alpha(m(y^{-1}xy)n) = \alpha(m(yxy^{-1})n)$. $\square$

Theorem 3.10. *If α is an $M - N$ anti fuzzy normal soft subgroup of G , then $g\alpha g^{-1}$ is also $M - N$ anti fuzzy normal soft subgroup of G , for all $g \in G$.*

Proof. Let α be an $M - N$ anti fuzzy normal soft subgroup of G , then $g\alpha g^{-1}$ is an $M - N$ anti fuzzy subgroup of G , for all $g \in G$. Now

$$\begin{aligned} g\alpha g^{-1}(m(xy^{-1})n) &= \alpha(g^{-1}m(xy^{-1})n)g \\ &= \alpha(m(xy^{-1})n) \\ &= \alpha(mxn) \\ &= \alpha(g(mxn))g^{-1} \\ &= g\alpha g^{-1}(mxn) \end{aligned}$$

Therefore $g\alpha g^{-1}(m(xy^{-1})n) = g\alpha g^{-1}(mxn)$. □

Theorem 3.11. *If $\alpha \cup \beta$ is an $M - N$ anti fuzzy normal soft subgroup of G , then $g(\alpha \cup \beta)g^{-1}$ is also an $M - N$ anti fuzzy normal soft subgroup of G , for all $g \in G$.*

Proof. If $\alpha \cup \beta$ is an $M - N$ anti fuzzy normal soft subgroup of G , then $g(\alpha \cup \beta)g^{-1}$ is also an $M - N$ anti fuzzy normal soft subgroup of G , for all $g \in G$.

To prove that $g(\alpha \cup \beta)g^{-1}(m(xy^{-1})n) = g(\alpha \cup \beta)g^{-1}(mxn)$. Now

$$\begin{aligned} g(\alpha \cup \beta)g^{-1}(m(xy^{-1})n) &= (\alpha \cup \beta)(g^{-1}(m(xy^{-1})n)g) \\ &= (\alpha \cup \beta)(m(xy^{-1})n) \\ &= (\alpha \cup \beta)(mxn) \\ &= (\alpha \cup \beta)(g(mxn))g^{-1} \\ &= g(\alpha \cup \beta)g^{-1}(mxn) \end{aligned}$$

Therefore $g(\alpha \cup \beta)g^{-1}(m(xy^{-1})n) = g(\alpha \cup \beta)g^{-1}(mxn)$. □

Definition 3.12. *Let $\alpha \cup \beta$ be an $M - N$ anti fuzzy soft subgroup of a group G . For any $t \in [0, 1]$, we define the $M - N$ anti level subset of $\alpha \cup \beta$ is the set $(\alpha \cup \beta)_t = \{x \in G / (\alpha \cup \beta)(mx) \leq t, (\alpha \cup \beta)(xn) \leq t \text{ for all } m \in M, n \in N\}$.*

Theorem 3.13. *Let G be a group and $\alpha \cup \beta$ be an anti fuzzy subset of G . Then $\alpha \cup \beta$ is an $M - N$ anti fuzzy normal soft subgroup of G iff the anti level subset $(\alpha \cup \beta)_t, t \in [0, 1]$ are $M - N$ anti fuzzy subgroup of G .*

Proof. Let $\alpha \cup \beta$ be an $M - N$ anti fuzzy normal soft subgroup of G and the anti level subset

$$(\alpha \cup \beta)_t = \{x \in G / (\alpha \cup \beta)(mx) \leq t, (\alpha \cup \beta)(xn) \leq t, t \in [0, 1] m \in M, n \in N\}$$

Let $x, y \in (\alpha \cup \beta)_t$, then $(\alpha \cup \beta)(mx) \leq t$ and $(\alpha \cup \beta)(xn) \leq t$. Now

$$\begin{aligned} (\alpha \cup \beta)(m(xy^{-1})n) &\leq \max\{(\alpha \cup \beta)(mx), (\alpha \cup \beta)(y^{-1}n)\} \\ &= \max\{(\alpha \cup \beta)(mx), (\alpha \cup \beta)(yn)\} \\ &\leq \max\{t, t\} \end{aligned}$$

$$(\alpha \cup \beta)(m(xy^{-1})n) \leq t$$

$m(xy^{-1})n \in (\alpha \cup \beta)_t$. Therefore $(\alpha \cup \beta)_t$ is an $M - N$ anti fuzzy subgroup of G .

Conversely, let us assume that $(\alpha \cup \beta)_t$ is an $M - N$ anti fuzzy subgroup G . Let $x, y \in (\alpha \cup \beta)_t$ then $(\alpha \cup \beta)(mx) \leq t$ and $(\alpha \cup \beta)(xn) \leq t$. Also $(\alpha \cup \beta)(m(xy^{-1})n) \leq t$. Since $m(xy^{-1})n \in (\alpha \cup \beta)_t = \max\{t, t\} = \max\{(\alpha \cup \beta)(mx), (\alpha \cup \beta)(yn)\}$. Therefore $(\alpha \cup \beta)(m(xy^{-1})n) \leq \max\{(\alpha \cup \beta)(mx), (\alpha \cup \beta)(yn)\}$. Hence $\alpha \cup \beta$ is an $M - N$ anti fuzzy normal soft subgroup of G . $\square$

Definition 3.14. Let G be a group and $\alpha \cup \beta$ be an $M - N$ anti fuzzy normal soft subgroup of G . Let $N(\alpha \cup \beta) = \{y \in G / (\alpha \cup \beta)(m(yxy^{-1})n) = (\alpha \cup \beta)(mxn) \text{ for all } x \in G, m \in M, n \in N\}$, then $N(\alpha \cup \beta)$ is called the $M - N$ anti fuzzy soft Normalize of $\alpha \cup \beta$.

Theorem 3.15. Let G be a group and $\alpha \cup \beta$ be an anti fuzzy subset of G . Then $\alpha \cup \beta$ is an $M - N$ anti fuzzy normal soft subgroup of G iff the anti level subset $(\alpha \cup \beta)_t, t \in [0, 1]$ are $M - N$ anti fuzzy normal subgroup of G .

Proof. Let $\alpha \cup \beta$ be a $M - N$ anti fuzzy normal soft subgroup of G and the anti level subset $(\alpha \cup \beta)_t, t \in [0, 1]$. Let $x \in G$ and $y \in (\alpha \cup \beta)_t$, then $(\alpha \cup \beta)(myn) \leq t$, for all $m \in M, n \in N$. Now $(\alpha \cup \beta)(m(xyx^{-1})n) = (\alpha \cup \beta)(myn) \leq t$. Since $\alpha \cup \beta$ is an $M - N$ anti fuzzy normal softsubgroup of G . That is $(\alpha \cup \beta)(m(xyx^{-1})n) \leq t$. Therefore $(m(xyx^{-1})n) \leq (\alpha \cup \beta)_t$. Hence $(\alpha \cup \beta)_t$ is an $M - N$ anti fuzzy normal subgroup of G . $\square$

4. Conclusion

The main results in the present manuscript are based on the concept of Anti fuzzy normal soft group [2, 7, 10, 13]. We have also defined the M-N anti level subsets of a fuzzy normal soft subgroup and its some elementary properties are discussed.

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Some Results on M-N fuzzy soft groups

M. Kaliraja* and S. Rumenaka

Assistant Professor, PG and Research Department of Mathematics,
H.H. The Rajah's college, Pudukkottai-622001, India.
mkr.maths009@gmail.com & rumenaka@gmail.com

Abstract

In this paper, we have discussed the concept of M-N fuzzy soft group, then we define the M-N level subsets of a fuzzy soft subgroup and its some elementary properties are also discussed. The presented method in this manuscript is more sensible and also reliable in solving the problems. This method can solve the decision making problems.

Key words: Fuzzy group, M-N fuzzy group, M-N fuzzy soft subgroup, M-N level subset, M-N fuzzy soft homomorphism.

1. Introduction

There are various types of uncertainties in the real world, but few classical mathematical tools may not be suitable to model these uncertainties. Many intricate problems in economics, social science, engineering, medical science and many other fields involve undefined data. These problems which one comes face to face with in life cannot be solved using classical mathematic methods. In classical mathematics, a mathematical model of an object is devised and the concept of the exact solution of this model is not yet determined. Since, the classical mathematical model is too complex, the exact solution cannot be found. There are several well-renowned theories available to describe uncertainty. For instance, Rosenfeld [8] introduced the concept of fuzzy subgroup in 1971 and the theory of fuzzy sets was inspired by Zadeh [11] in addition to this, Molodtsov [6] have introduced the concept of soft sets in 1999. Furthermore, Maji et. al., [5] as well introduction the concept of fuzzy soft sets in 2001 and Jacobson [3] introduced the concept of M-group M-subgroup.

Sarala and Suganya [9] be unraveled some properties of fuzzy soft groups in 2014 In addition, Vasantha Kandasamy and Smarandache [10] have introduced the Fuzzy Algebra during 2003. An introduction to the new definition of Soft sets and soft groups depending on inclusion relation and intersection of sets were exposed by Akta and Cagman [1]. In 1981, Das [2] studied the Fuzzy groups and level subgroups. Moreover, Maji, Biswas and Ray [5] were introduced the fuzzy soft set in 2001. In our earlier work we have discussed the concept of M-N fuzzy normal soft group in [4].

In the present manuscript, we have discussed the concept of M-N fuzzy soft group based on the concept of fuzzy soft group [4, 7 and 9]. In section 2, we presented the basic definition, notations on M-N fuzzy soft group and required results on fuzzy soft group. In section 3, we define the M-N fuzzy soft set and define the M-N level subsets of a fuzzy soft subgroup. We have also discussed the concept of M-N fuzzy soft group and some of its elementary properties.

2. Preliminaries

In this section, some basic definitions and results needed are given. For the sake of convenience we set out the former concepts which will be used in this paper.

Definition 2.1

Let G be any non-empty set. A mapping $f: G \rightarrow [0, 1]$ is called fuzzy set in G .

Definition 2.2

Let x be a non-empty set. A fuzzy subset f of X is a function $f: X \rightarrow [0, 1]$

Definition 2.3

Let G be a group. A fuzzy subset f of G is called a fuzzy subgroup if for $x, y \in G$

- (1) $f(xy) \geq \min \{ f(x), f(y) \}$
- (2) $f(x^{-1}) = f(x)$

Definition 2.4

A pair (F, A) is called a soft set over U , where F is a mapping given by $F: A \rightarrow P(U)$

Definition 2.5

Let (F, A) be a soft set over G . Then (F, A) is called a soft group over G if $F(a)$ is a group G for all $a \in A$.

Definition 2.6

A pair (F, A) is called a fuzzy soft set over U , where $F: A \rightarrow I^U$ is a mapping $I = [0, 1]$, $F(a)$ is a fuzzy subset of U for all $a \in A$.

Definition 2.7

Let (F, A) be a fuzzy soft set over G . Then (F, A) is called a fuzzy soft group if $F(a)$ is a fuzzy subgroup G for all $a \in A$.

Definition 2.8

Let (F, A) and (G, B) be two fuzzy soft set over U . Then (F, A) is called a fuzzy soft subset of (G, B) denoted by $(F, A) \subseteq (G, B)$ if

- (1) $A \subseteq B$
- (2) $F(a)$ is a fuzzy subset of $G(a)$ for each $a \in A$.

Definition 2.9

A fuzzy set f is called a fuzzy soft subgroup of a group G , if for $x, y \in G$

- (1) $f(xy) \geq \min\{f(x), f(y)\}$
- (2) $f(x^{-1}) \geq f(x)$

Definition 2.10 [4]

Let M, N be left and right operator sets of group G respectively if $(m x) n = m (x n)$ for all $x \in G, m \in M, n \in N$. Then G is said to be an $M - N$ group.

Definition 2.11[4]

Let G be an $M - N$ group and (F, A) be a fuzzy soft subgroup of G if

- (1) $F\{m(x y) n\} \geq \min\{F(x), F(y)\}$
- (2) $F\{(m x^{-1}) n\} \geq F(x)$ hold for any $x, y \in G, m \in M, n \in N$, then (F, A) is said to be an $M - N$ fuzzy soft subgroup of G .

Here $F: A \rightarrow P(G)$

Definition 2.12 [4]

Let G be an $M - N$ group and (F, A) be a fuzzy soft subgroup of G and A is a parameters of the set if

- (1) $F(mx) \geq F(x)$
- (2) $F(xn) \geq F(x)$ hold for any $x \in G, m \in M$, and $n \in N$, then (F, A) is said to be an $M - N$ fuzzy soft subgroup of G .

3. M-N fuzzy soft group

In this section, we shall define M-N fuzzy soft group, discussed the concept of M-N fuzzy soft group based on the concept of fuzzy soft group [4, 7 and 9], and give some elementary properties are discussed.

Definition 3:1

Let G be a group and (f, A) be an $M - N$ fuzzy soft set over G . Then (f, A) is said to be a $M - N$ fuzzy soft group over G iff for each $a \in A$ and $x, y \in G$,

- (1) $f_a\{m(x y) n\} \geq \min\{f_a(x), f_a(y)\}$
- (2) $f_a\{(m x^{-1}) n\} \geq f_a(x)$ hold for each $a \in A, m \in M, n \in N$, f_a is a $M - N$ fuzzy soft subgroup of a group G .

Theorem 3:2

Let G be a group and (f, A) be an $M - N$ fuzzy soft set over G . Then (f, A) is said to be an $M - N$ fuzzy soft subgroup on a group G iff for each $a \in A$ and $x, y \in G$

Proof:

Assume that (f, A) is a $M - N$ fuzzy soft subgroup of a group G

To prove that $f_a(m(xy^{-1})n) \geq \min \{f_a(x), f_a(y)\}$

Let $x, y \in G$ and $a \in A$

$$\begin{aligned} \text{We have } f_a(m(xy^{-1})n) &\geq \min \{f_a(mx), f_a(y^{-1}n)\} \\ &\geq \min \{f_a(x), f_a(y^{-1})\} \text{ by (definition 2:12)} \\ &\geq \min \{f_a(x), f_a(y)\} \text{ by } A(x) = A(x^{-1}) \end{aligned}$$

Therefore $f_a(m(xy^{-1})n) \geq \min \{f_a(x), f_a(y)\}$

Conversely,

Assume that $f_a(m(xy^{-1})n) \geq \min \{f_a(x), f_a(y)\}$

To prove that (f_a, A) is an $M - N$ fuzzy subgroup of a group G

Now $f_a(men) = f_a(mxx^{-1}n)$

$$\begin{aligned} &\geq \min \{f_a(mx), f_a(x^{-1}n)\} \\ &\geq \min \{f_a(x), f_a(x^{-1})\} \\ &\geq \min \{f_a(x), f_a(x)\} \\ &= f_a(x) \end{aligned}$$

Therefore $f_a(men) = f_a(x)$ where e is the identity element of G .

Next $f_a(m x^{-1}n) = f_a(m e x^{-1}n)$

$$\begin{aligned} &\geq \min \{f_a(me), f_a(x^{-1}n)\} \\ &\geq \min \{f_a(e), f_a(x^{-1})\} \\ &\geq \min \{f_a(e), f_a(x)\} \end{aligned}$$

$$f_a(m x^{-1}n) = f_a(x)$$

On the other hand, for each $a \in A$ and $x, y \in G$

$$\begin{aligned} f_a(m xyn) &= f_a(m x (y^{-1})^{-1}n) \\ &\geq \min \{f_a(mx), f_a((y^{-1})^{-1}n)\} \\ &\geq \min \{f_a(mx), f_a(y^{-1}n)\} \\ &\geq \min \{f_a(mx), f_a(y n)\} \end{aligned}$$

$$f_a(m xyn) \geq \min \{f_a(x), f_a(y)\}$$

Hence the proof

Theorem 3:3

If (f, A) is an $M - N$ fuzzy soft set and e is the unit element of G . Then for each $a \in A$ and for each $x \in X$,

$$(1) \quad f_a(x^{-1}) \geq f_a(x)$$

$$(2) \quad f_a(e) = f_a(x)$$

Theorem 3.4

Let f_a and g_a be two $M - N$ fuzzy soft group of G , then $f_a \cap g_a$ is an $M - N$ fuzzy soft group of G .

Proof

To prove that (1) $(f_a \cap g_a)(mxyn) \geq \min \{(f_a \cap g_a)(x), (f_a \cap g_a)(y)\}$

$$(2) \quad (f_a \cap g_a)(mx^{-1}n) = (f_a \cap g_a)(x)$$

$$\begin{aligned} (1) \Rightarrow (f_a \cap g_a)(mxyn) &= \min \{f_a(mxyn), g_a(mxyn)\} \\ &\geq \min \{\min \{f_a(mx), f_a(yn)\}, \min \{g_a(mx), g_a(yn)\}\} \\ &\geq \min \{\min \{f_a(x), f_a(y)\}, \min \{g_a(x), g_a(y)\}\} \end{aligned}$$

$$\begin{aligned}
& \geq \min \{ \min \{ f_a(x), g_a(x) \}, \min \{ f_a(y), g_a(y) \} \} \\
(f_a \cap g_a)(mxyn) & \geq \min \{ (f_a \cap g_a)(x), (f_a \cap g_a)(y) \} \\
(2) \Rightarrow (f_a \cap g_a)(mx^{-1}n) &= (f_a \cap g_a)(mx^{-1}xx^{-1}n) \\
& \geq \min \{ (f_a \cap g_a)(mx^{-1}x), (f_a \cap g_a)(x^{-1}n) \} \\
& \geq \min \{ \min \{ (f_a \cap g_a)(mx^{-1}), (f_a \cap g_a)(x) \}, (f_a \cap g_a)(x^{-1}n) \} \\
& \geq \min \{ \min \{ (f_a \cap g_a)(x^{-1}), (f_a \cap g_a)(x) \}, (f_a \cap g_a)(x^{-1}) \} \\
& \geq \min \{ \min \{ (f_a \cap g_a)(x), (f_a \cap g_a)(x) \}, (f_a \cap g_a)(x) \} \\
& \geq \min \{ (f_a \cap g_a)(x), (f_a \cap g_a)(x) \} \\
& = (f_a \cap g_a)(x) \\
f_a \cap g_a(x^{-1}) &= (f_a \cap g_a)(x) \\
& \text{Hence the proof}
\end{aligned}$$

Proposition 3:5

If f_a and g_a are two $M - N$ fuzzy soft group of G , then the following statements holds for all $x, y \in G, m \in M, n \in N$

$$\begin{aligned}
(1) (f_a \wedge g_a)(mxyn) & \geq \min \{ (f_a \wedge g_a)(x), (f_a \wedge g_a)(y) \} \\
(2) (f_a \wedge g_a)(mx^{-1}n) &= (f_a \wedge g_a)(x)
\end{aligned}$$

Proof

Straight forward.

Proposition 3:6

If f_a is an $M - N$ fuzzy soft subgroup of a group over G , then $f_a(m(xy)n)^2 = f_a(x^2y^2)$ is an $M - N$ fuzzy soft subgroup of a group G .

Proof:

Let $x, y \in G, m \in M, n \in N$

$$\begin{aligned}
(1) f_a(m(xy)n)^2 &= f_a((m(xy)n), (m(xy)n)) \\
& \geq \min \{ f_a(m(xy)n), f_a(m(xy)n) \} \\
& \geq \min \{ \min \{ f_a(mx), f_a(yn) \}, \min \{ f_a(mx), f_a(yn) \} \} \\
& \geq \min \{ \min \{ f_a(x), f_a(y) \}, \min \{ f_a(x), f_a(y) \} \} \\
& \geq \min \{ \min \{ f_a(x), f_a(x) \}, \min \{ f_a(y), f_a(y) \} \} \\
& \geq \min \{ f_a(x \cdot x), f_a(y \cdot y) \} \\
& \geq \min \{ f_a(x^2), f_a(y^2) \} \\
& = f_a(x^2y^2) \\
f_a(m(xy)n)^2 &= f_a(x^2y^2) \\
(2) f_a(m(xy)^{-1}n) &= f_a(my^{-1}x^{-1}n) \\
& \geq \min \{ f_a(my^{-1}), f_a(x^{-1}n) \} \\
& \geq \min \{ f_a(y^{-1}), f_a(x^{-1}) \} \\
& \geq f_a(yx) \\
& \geq f_a(xy) \\
f_a(m(xy)^{-1}n) &= f_a(xy)
\end{aligned}$$

Proposition 3:7

If $\{f_{ai}\}_{i \in I}$ is an $M - N$ fuzzy soft group of G , then $\bigcup f_{ai}$ is an $M - N$ fuzzy soft group G whose element $\bigcup f_{ai} = \{x, \forall f_{ai}(x) / x \in G\}$, where $i \in I$.

Proof:

Let $x, y \in G, m \in M, n \in N$ and $i \in I$.

$$\begin{aligned}
(1) \bigcup f_{ai}(m(xy)n) & \geq \min \{ \bigcup f_{ai}(x), \bigcup f_{ai}(y) \} \\
\bigcup f_{ai}(m(xy)n) &= \bigvee f_{ai}(m(xy)n)
\end{aligned}$$

$$\begin{aligned}
& \geq \bigvee \min \{ f_{ai}(mx), f_{ai}(yn) \} \\
& \geq \bigvee \min \{ f_{ai}(x), f_{ai}(y) \} \quad A(mx) \geq A(x) \\
& \geq \min \{ \bigvee f_{ai}(x), \bigvee f_{ai}(y) \} \\
& \geq \min \{ \bigvee f_{ai}(x), \bigvee f_{ai}(y) \} \\
& \bigvee f_{ai}(m(xy)n) \geq \min \{ \bigvee f_{ai}(x), \bigvee f_{ai}(y) \} \\
(2) \quad & \bigvee f_{ai}(m(x^{-1})n) \geq \bigvee f_{ai}(x) \\
& \bigvee f_{ai}(m(x^{-1})n) = \bigvee f_{ai}(m(x^{-1}xx^{-1})n) \\
& \geq \bigvee f_{ai}(m(x^{-1}xx^{-1})n) \\
& \geq \bigvee \min \{ f_{ai}(mx^{-1}x), f_{ai}(x^{-1}n) \} \\
& \geq \bigvee \min \{ \min \{ f_{ai}(mx^{-1}), f_{ai}(x) \}, f_{ai}(x^{-1}n) \} \\
& \geq \bigvee \min \{ \min \{ f_{ai}(x^{-1}), f_{ai}(x) \}, f_{ai}(x^{-1}) \} \quad A(mx) \geq A(x) \\
& \geq \bigvee \min \{ \min \{ f_{ai}(x), f_{ai}(x) \}, f_{ai}(x) \} \quad A(x) = A(x^{-1}) \\
& \geq \bigvee \min \{ f_{ai}(x), f_{ai}(x) \} \\
& \geq \bigvee f_{ai}(x) \\
& \geq (\bigvee f_{ai})(x) \\
& \bigvee f_{ai}(m(x^{-1})n) \geq \bigvee f_{ai}(x)
\end{aligned}$$

Hence $\bigvee f_{ai}(x)$ is an $M - N$ fuzzy soft group of G .

Proposition 3:8

Let G_1 and G_2 both be $M - N$ group and ϕ be a soft homomorphism from G_1 and G_2 . If f_a is an $M - N$ fuzzy soft group of G_2 , then the pre - image $\phi^{-1}(f_a)$ is an $M - N$ fuzzy soft group of G_1

Proposition 3:9

Let $\phi : G_1 \rightarrow G_2$ be an epimorphism and f_a be an $M - N$ fuzzy soft set in G_2 . If $\phi^{-1}(f_a)$ is an $M - N$ fuzzy soft group of G_1 , f_a is an $M - N$ fuzzy soft group of G_2 .

Proposition 3:10

Let f_a be an $M - N$ fuzzy soft groups over G , and ϕ is endomorphism of G , then $f_a[\phi]$ is an $M - N$ fuzzy soft group of G .

Proof:

Let $x, y \in G$, $m \in M$, and $n \in N$

$$\begin{aligned}
(1) \quad & f_a[[\phi](m(xy)n)] = f_a[\phi](m(xy)n) \\
& \geq \min \{ f_a(\phi(mx)), f_a(\phi(yn)) \} \\
& \geq \min \{ f_a(\phi(x)), f_a(\phi(y)) \}, \quad A(mx) \geq A(x) \\
& \geq \min \{ f_a(\phi(x)), f_a(\phi(y)) \} \\
& \geq \min \{ f_a[\phi](x), f_a[\phi](y) \} \\
& f_a[[\phi](m(xy)n)] \geq \min A(mx) \geq A(x) \\
(2) \quad & f_a[[\phi](m(x^{-1})n)] = f_a[\phi](m(x^{-1}xx^{-1})n) \\
& \geq \min \{ f_a[\phi](mx^{-1}x), f_a[\phi](x^{-1}n) \} \\
& \geq \min \{ \min \{ f_a[\phi](mx^{-1}), f_a[\phi](x) \}, f_a[\phi](x^{-1}n) \} \\
& \geq \min \{ \min \{ f_a[\phi](x^{-1}), f_a[\phi](x) \}, f_a[\phi](x^{-1}) \} \\
& \geq \min \{ f_a[\phi](x^{-1}), f_a[\phi](x^{-1}) \} \\
& \geq \min \{ f_a[\phi](x), f_a[\phi](x) \}, \quad A(x^{-1}) = A(x) \\
& \geq f_a[\phi](x) \\
& f_a[[\phi](m(x^{-1})n)] \geq f_a[\phi](x)
\end{aligned}$$

Hence $f_a[\phi]$ is an $M - N$ fuzzy soft group of G .

4. Conclusion

The main results in the present manuscript are based on the concept of $M-N$ fuzzy soft group [4, 7 and 9]. We have also defined the $M-N$ level subsets of a fuzzy soft subgroup and its some elementary properties are discussed.

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M-N Anti Fuzzy Soft Groups

M. Kaliraja*

Assistant Professor,

PG and Research Department of Mathematics,

H.H. The Rajah's College, Pudukkottai, India.

S. Rumenaka

Assistant Professor,

PG and Research Department of Mathematics,

H.H. The Rajah's college, Pudukkottai, India.

E-mail: rumenaka@gmail.com

**Corresponding author E-mail: mkr.maths009@gmail.com*

Abstract

In this paper, we have discussed the concept of M-N anti fuzzy soft group, then we define the M-N anti level subsets of a fuzzy soft subgroup and its some elementary properties are also discussed. The presented method in this manuscript is more sensible and also reliable in solving the problems. This method can solve the decision making problems.

Key words: Fuzzy Group, M-N Anti Fuzzy Group, M-N Anti Fuzzy Soft Subgroup, M-N Anti Level Subset, M-N Anti Fuzzy Soft Homomorphism.

1. Introduction

There are various types of uncertainties in the real world, but few classical mathematical tools may not be suitable to model these uncertainties. Many intricate problems in economics, social science, engineering, medical science and many other fields involve undefined data. These problems which one comes face to face with in life cannot be solved using classical mathematic methods. In classical mathematics, a mathematical model of an object is devised and the concept of the exact solution of this model is not yet determined. Since, the classical mathematical model is too complex, the exact solution cannot be found. There are several well-renowned theories available to describe uncertainty. For instance, Rosenfeld [9] introduced the concept of fuzzy subgroup in 1971 and the theory of fuzzy sets was inspired by Zadeh [12] in addition to this, Molodtsov [7] have introduced the concept of soft sets in 1999. Furthermore, Majiet. al., [6] as well introduction the concept of fuzzy soft sets in 2001 and Jacobson [3] introduced the concept of M-group M-subgroup.

Sarala and Suganya[10] be unraveled some properties of fuzzy soft groups in 2014. In addition, Vasantha Kandasamy and Smarandache [11] have introduced the Fuzzy Algebra during 2003. An introduction to the new definition of Soft sets and soft groups depending on inclusion relation and intersection of sets were exposed by Akta and Cagman [1]. In 1981, Das [2] studied the Fuzzy groups and level subgroups. Moreover, Maji, Biswas and Ray [6W] were introduced the fuzzy soft set in 2001.

In the present manuscript, we have discussed the concept of M-N anti fuzzy soft group based on the concept of M – N fuzzy soft group[4 and 6]. In section 2, we presented the basic definition, notations on

M-N anti fuzzy soft group and required results on fuzzy soft group. In section 3, we define the M-N anti fuzzy soft set and define the M-N anti level subsets of a fuzzy soft subgroup. We have also discussed the concept of M-N anti fuzzy soft group and some of its elementary properties.

2. Preliminaries

In this section, some basic definitions and results needed are given. For the sake of convenience we set out the former concepts which will be used in this paper.

Definition 2.1

Let G be any non-empty set. A mapping $f: G \rightarrow [0, 1]$ is called fuzzy set in G .

Definition 2.2

Let x be a non-empty set. A fuzzy subset f of X is a function $f: X \rightarrow [0, 1]$

Definition 2.3[5]

Let G be a group. A fuzzy subset f of G is called an anti fuzzy subgroup if for $x, y \in G$

$$f(xy) \leq \max \{f(x), f(y)\}$$

$$f(x^{-1}) = f(x)$$

Definition 2.4[5]

Let f be an M- N anti fuzzy subgroup of a set G . For $t \in [0, 1]$, the anti level subset of f is the set $f_t = \{x \in G / f(mx) \leq t, f(xn) \leq t, m \in M, n \in N\}$. This is called a M – N anti level subset of f .

Definition 2.5

A pair (F, A) is called a soft set over U , where F is a mapping given by $F: A \rightarrow P(U)$

Definition 2.6

Let (F, A) be a soft set over G . Then (F, A) is called a soft group over G if $F(a)$ is a group G for all $a \in A$.

Definition 2.7[10]

A pair (F, A) is called a fuzzy soft set over U , where $F: A \rightarrow I^U$ is a mapping $I = [0, 1]$, $F(a)$ is a fuzzy subset of U for all $a \in A$.

Definition 2.8[10]

Let (F, A) be a fuzzy soft set over G . Then (F, A) is called a fuzzy soft group if $F(a)$ is a fuzzy subgroup G for all $a \in A$.

Definition 2.9 [10]

Let (F, A) and (G, B) be two fuzzy soft set over U . Then (F, A) is called a fuzzy soft subset of (G, B) denoted by $(F, A) \subseteq (G, B)$ if

$$(1) A \subseteq B$$

$$(2) F(a) \text{ is a fuzzy subset of } G(a) \text{ for each } a \in A.$$

Definition 2.10

A fuzzy set f is called an anti fuzzy soft subgroup of a group G , if for $x, y \in G$ $f(xy) \leq \max \{f(x), f(y)\}$ $f(x^{-1}) \leq f(x)$

Definition 2.11 [4]

Let M, N be left and right operator sets of group G respectively if $(m x) n = m (x n)$ for all $x \in G, m \in M, n \in N$. Then G is said to be an $M - N$ group.

Definition 2:12 [5]

Let G be an $M - N$ anti group and (F, A) be a fuzzy soft subgroup of G if

$$(1) F \{m(x y) n\} \leq \max \{F(x), F(y)\}$$

(2) $F \{(m x^{-1}) n\} \leq F(x)$ hold for any $x, y \in G, m \in M, n \in N$, then (F, A) is said to be an $M - N$ anti fuzzy soft subgroup of G . Here $F: A \rightarrow P(G)$

Definition 2:13 [5]

Let G be an $M - N$ anti group and (F, A) be a fuzzy soft subgroup of G and A is a parameters of the set if

$$F(m x) \leq F(x)$$

$F(x n) \leq F(x)$ hold for any $x \in G, m \in M$, and $n \in N$, then (F, A) is said to be an $M - N$ anti fuzzy soft subgroup of G .

Example 2: 13

Let F be a fuzzy soft subgroup of an $M - N$ group G . A is the parameters of the set, then F is defined by

$$F(x) = \begin{cases} 0.1 & \text{if } x \in G \\ 0.9 \text{ and above} & \text{if } x \notin G \end{cases}$$

Where $x = \{1, 2, 3, 4, 5, 6\}$ $F(1) = 0.1, F(2) = 0.03, F(3) = 0.06, F(4) = 0.6, M = \{1, 2, 3\}$ and $N = \{1, 3, 5\}$, here $N \subset A$ and $M \subset A$ where A is a natural numbers.

3. M-N Anti Fuzzy Soft Group

In this section, we shall define $M - N$ anti fuzzy soft group, discussed the concept of $M - N$ antifuzzy soft group based on the concept of fuzzy soft group [6 and 7], and give some elementary properties are discussed.

Definition 3:1 [5]

Let G be a group and (f, A) be an $M - N$ anti fuzzy soft set over G . Then (f, A) is said to be a $M - N$ anti fuzzy soft group over G iff for each $a \in A$ and $x, y \in G$,

$$(1) f_a \{m(x y) n\} \leq \max \{f_a(x), f_a(y)\}$$

(2) $f_a \{(m x^{-1}) n\} \leq f_a(x)$ hold for each $a \in A, m \in M, n \in N, f_a$ is a $M - N$ anti fuzzy soft subgroup of a group G .

Theorem 3:2

Let G be a group and (f, A) be an $M - N$ anti fuzzy soft set over G . Then (f, A) is said to be an $M - N$ anti fuzzy soft subgroup on a group G iff for each $a \in A$ and $x, y \in G$

Proof:

Assume that (f, A) is a $M - N$ anti fuzzy soft subgroup of a group G

To prove that $f_a(m(x y^{-1}) n) \leq \max \{f_a(x), f_a(y)\}$

Let $x, y \in G$ and $a \in A$

We have $f_a(m(x y^{-1}) n) \leq \max \{f_a(m x), f_a(y^{-1} n)\}$

$$\leq \max \{f_a(x), f_a(y^{-1})\} \text{ by (definition 2:12)}$$

$$\leq \max \{f_a(x), f_a(y)\} \text{ by } A(x) = A(x^{-1})$$

$$\text{Therefore } f_a(m(xy^{-1})n) \leq \max \{f_a(x), f_a(y)\}$$

Conversely,

$$\text{Assume that } f_a(m(xy^{-1})n) \leq \max \{f_a(x), f_a(y)\}$$

To prove that (f_a, A) is an $M - N$ anti fuzzy subgroup of a group G

$$\text{Now } f_a(men) = f_a(mxx^{-1}n)$$

$$\leq \max \{f_a(mx), f_a(x^{-1}n)\}$$

$$\leq \max \{f_a(x), f_a(x^{-1})\}$$

$$\leq \max \{f_a(x), f_a(x)\}$$

$$= f_a(x)$$

Therefore $f_a(me) = f_a(x)$ where e is the identity element of G .

$$\text{Next } f_a(mx^{-1}n) = f_a(mex^{-1}n)$$

$$\leq \max \{f_a(me), f_a(x^{-1}n)\}$$

$$\leq \max \{f_a(e), f_a(x^{-1})\}$$

$$\leq \max \{f_a(e), f_a(x)\}$$

$$f_a(mx^{-1}n) = f_a(x)$$

On the other hand, for each $a \in A$ and $x, y \in G$

$$f_a(mxyn) = f_a(mx(y^{-1})^{-1}n)$$

$$\leq \max \{f_a(mx), f_a((y^{-1})^{-1}n)\}$$

$$\leq \max \{f_a(mx), f_a(y^{-1}n)\}$$

$$\leq \max \{f_a(mx), f_a(yn)\}$$

$$f_a(mxyn) \leq \max \{f_a(x), f_a(y)\}$$

Hence the proof

Theorem 3:3

If (f, A) is an $M - N$ anti fuzzy soft set and e is the unit element of G . Then for each $a \in A$ and for each $x \in X$,

$$f_a(x^{-1}) \leq f_a(x)$$

$$f_a(e) = f_a(x)$$

Theorem 3.4

Let f_a and g_a be two $M - N$ anti fuzzy soft group of G , then $f_a \cap g_a$ is an $M - N$ anti fuzzy soft group of G .

Proof

$$\text{To prove that (1) } (f_a \cap g_a)(mxyn) \leq \max \{(f_a \cap g_a)(x), (f_a \cap g_a)(y)\}$$

$$(2) (f_a \cap g_a)(mx^{-1}n) = (f_a \cap g_a)(x)$$

$$(1) \Rightarrow (f_a \cap g_a)(mxyn) = \max \{f_a(mxyn), g_a(mxyn)\}$$

$$\leq \max \{\max \{f_a(mx), f_a(yn)\}, \max \{g_a(mx), g_a(yn)\}\}$$

$$\begin{aligned}
&\leq \max \{ \max \{ f_a(x), f_a(y) \}, \max \{ g_a(x), g_a(y) \} \} \\
&\leq \max \{ \max \{ f_a(x), g_a(x) \}, \max \{ f_a(y), g_a(y) \} \} \\
&(f_a \cap g_a)(mxyn) \leq \max \{ (f_a \cap g_a)(x), (f_a \cap g_a)(y) \} \\
&\text{(2)} \Rightarrow (f_a \cap g_a)(mx^{-1}n) = (f_a \cap g_a)(mx^{-1}xx^{-1}n) \\
&\leq \max \{ (f_a \cap g_a)(mx^{-1}x), (f_a \cap g_a)(x^{-1}n) \} \\
&\leq \max \{ \max \{ (f_a \cap g_a)(mx^{-1}), (f_a \cap g_a)(x) \}, (f_a \cap g_a)(x^{-1}n) \} \\
&\leq \max \{ \max \{ (f_a \cap g_a)(x^{-1}), (f_a \cap g_a)(x) \}, (f_a \cap g_a)(x^{-1}) \} \\
&\leq \max \{ \max \{ (f_a \cap g_a)(x), (f_a \cap g_a)(x) \}, (f_a \cap g_a)(x) \} \\
&\leq \max \{ (f_a \cap g_a)(x), (f_a \cap g_a)(x) \} \\
&= (f_a \cap g_a)(x) \\
&(f_a \cap g_a)(x^{-1}) = (f_a \cap g_a)(x)
\end{aligned}$$

Hence the proof

Proposition 3:5

If f_a and g_a are two $M - N$ anti fuzzy soft group of G , then the following statements holds for all $x, y \in G, m \in M, n \in N$

$$\begin{aligned}
&(1) (f_a \wedge g_a)(mxyn) \leq \max \{ (f_a \wedge g_a)(x), (f_a \wedge g_a)(y) \} \\
&(2) (f_a \wedge g_a)(mx^{-1}n) = (f_a \wedge g_a)(x)
\end{aligned}$$

Proof

Straight forward.

Proposition 3:6

If f_a is an $M - N$ anti fuzzy soft subgroup of a group over G , then $f_a(m(xy)n)^2 = f_a(x^2y^2)$ is an $M - N$ anti fuzzy soft subgroup of a group G .

Proof:

Let $x, y \in G, m \in M, n \in N$

$$\begin{aligned}
&(1) f_a(m(xy)n)^2 = f_a((m(xy)n), (m(xy)n)) \\
&\leq \max \{ f_a(m(xy)n), f_a(m(xy)n) \} \leq \max \{ \max \{ f_a(mx), f_a(yn) \}, \max \{ f_a(mx), f_a(yn) \} \} \\
&\leq \max \{ \max \{ f_a(x), f_a(y) \}, \max \{ f_a(x), f_a(y) \} \} \\
&\leq \max \{ \max \{ f_a(x), f_a(x) \}, \max \{ f_a(y), f_a(y) \} \} \\
&\leq \max \{ f_a(x \cdot x), f_a(y \cdot y) \} \leq \max \{ f_a(x^2), f_a(y^2) \} \\
&= f_a(x^2y^2) f_a(m(xy)n)^2 = f_a(x^2y^2) \\
&(2) f_a(m(xy)^{-1}n) = f_a(my^{-1}x^{-1}n) \leq \max \{ f_a(my^{-1}), f_a(x^{-1}n) \} \\
&\leq \max \{ f_a(y^{-1}), f_a(x^{-1}) \} \\
&\leq f_a(yx) \leq f_a(xy) f_a(m(xy)^{-1}n) = f_a(xy)
\end{aligned}$$

Proposition 3:7

If $\{f_{ai}\}_{i \in I}$ is an $M - N$ anti fuzzy soft group of G , then Uf_{ai} is an $M - N$ anti fuzzy soft group G whose element $U f_{ai} = \{x, \forall f_{ai}(x) / x \in G\}$, where $i \in I$.

Proof:

Let $x, y \in G, m \in M, n \in N$ and $i \in f_a$.

$$(1) \quad U_{f_{ai}}(m(xy)n) \leq \max \{U_{f_{ai}}(x), U_{f_{ai}}(y)\}$$

$$U_{f_{ai}}(m(xy)n) = V_{f_{ai}}(m(xy)n)$$

$$\leq V \max \{f_{ai}(mx), f_{ai}(yn)\}$$

$$\leq V \max \{f_{ai}(x), f_{ai}(y)\}, \text{ since } A(mx) \leq A(x)$$

$$\leq \max \{V_{f_{ai}}(x), V_{f_{ai}}(y)\}$$

$$\leq \max \{U_{f_{ai}}(x), U_{f_{ai}}(y)\}$$

$$U_{f_{ai}}(m(xy)n) \leq \max \{U_{f_{ai}}(x), U_{f_{ai}}(y)\}$$

$$(2) \quad U_{f_{ai}}(m(x^{-1})n) \leq U_{f_{ai}}(x)$$

$$U_{f_{ai}}(m(x^{-1})n) = U_{f_{ai}}(m(x^{-1}xx^{-1})n)$$

$$\leq V_{f_{ai}}(m(x^{-1}xx^{-1})n)$$

$$\leq V \max \{f_{ai}(mx^{-1}x), f_{ai}(x^{-1}n)\}$$

$$\leq V \max \{\max \{f_{ai}(mx^{-1}), f_{ai}(x)\}, f_{ai}(x^{-1}n)\}$$

$$\leq V \max \{\max \{f_{ai}(x^{-1}), f_{ai}(x)\}, f_{ai}(x^{-1})\}, \text{ since } A(mx) \leq A(x)$$

$$\leq V \max \{\max \{f_{ai}(x), f_{ai}(x)\}, f_{ai}(x)\} \quad A(x) = A(x^{-1})$$

$$\leq V \max \{f_{ai}(x), f_{ai}(x)\} \leq V_{f_{ai}}(x) \leq (U_{f_{ai}})(x)$$

$$U_{f_{ai}}(m(x^{-1})n) \leq U_{f_{ai}}(x)$$

Hence $U_{f_{ai}}(x)$ is an $M-N$ anti fuzzy soft group of G .

Definition 3.8[5]

Let $f_a \cap g_a$ be an $M-N$ anti fuzzy soft subgroup of a group G . For any $t \in [0, 1]$, we define the $M-N$ anti level subset of $f_a \cap g_a$ is the set

$$(f_a \cap g_a)_t = \{x \in G / (f_a \cap g_a)(mx) \leq t, (f_a \cap g_a)(xn) \leq t \text{ for all } m \in M, n \in N\}$$

Theorem 3.9

Let G be a group and $f_a \cap g_a$ be an $M-N$ anti fuzzy soft subgroup of G . Then the anti level subset $(f_a \cap g_a)_t, t \in [0, 1]$ is an $M-N$ anti fuzzy soft subgroup of G .

Proof:

Let $f_a \cap g_a$ be an $M-N$ anti fuzzy soft subgroup of G and the anti level subset

$$(f_a \cap g_a)_t = \{x \in G / (f_a \cap g_a)(mx) \leq t, (f_a \cap g_a)(xn) \leq t, t \in [0, 1] \text{ } m \in M, n \in N\}$$

Let $x, y \in (f_a \cap g_a)_t$, then $(f_a \cap g_a)(mx) \leq t$ and $(f_a \cap g_a)(xn) \leq t$

$$\text{Now } (f_a \cap g_a)(mxy^{-1}n) \leq \max \{(f_a \cap g_a)(mx), (f_a \cap g_a)(y^{-1}n)\}$$

$$= \max \{(f_a \cap g_a)(mx), (f_a \cap g_a)(yn)\}$$

$$\leq \max \{t, t\}$$

$$(f_a \cap g_a)(mxy^{-1}n) \leq t$$

$$mxy^{-1}n \in (f_a \cap g_a)_t$$

Therefore $(f_a \cap g_a)_t$ is an $M - N$ anti fuzzy soft subgroup of G .

Proposition 3:10

Let G_1 and G_2 both be $M - N$ anti group and ϕ be a soft homomorphism from G_1 and G_2 . If f_a is an $M - N$ anti fuzzy soft group of G_2 , then the pre - image $\phi^{-1}(f_a)$ is an $M - N$ anti fuzzy soft group of G_1 .

Proposition 3:11

Let $\phi : G_1 \rightarrow G_2$ be an epimorphism and f_a be an $M - N$ anti fuzzy soft set in G_2 . If $\phi^{-1}(f_a)$ is an $M - N$ anti fuzzy soft group of G_1 , f_a is an $M - N$ anti fuzzy soft group of G_2 .

Proposition 3:12

Let f_a be an $M - N$ anti fuzzy soft groups over G , and ϕ is endomorphism of G , then $f_a[\phi]$ is an $M - N$ anti fuzzy soft group of G .

Proof:

Let $x, y \in G$, $m \in M$, and $n \in N$

$$(1) f_a[[\phi](m(xy)n)] = f_a[\phi](m(xy)n)$$

$$\leq \max \{f_a[\phi](mx), f_a[\phi](ny)\}$$

$$\leq \max \{f_a(\phi(x)), f_a(\phi(y))\}, \text{ since } A(mx) \leq A(x)$$

$$\leq \max \{f_a(\phi(x)), f_a(\phi(y))\}$$

$$\leq \max \{f_a[\phi](x), f_a[\phi](y)\}$$

$$f_a[[\phi](m(xy)n)] \leq \max A(mx) \geq A(x)$$

$$(2) f_a[[\phi](m(x^{-1})n)] = f_a[\phi](m(x^{-1})n)$$

$$\leq \max \{f_a[\phi](mx^{-1}x), f_a[\phi](x^{-1}n)\}$$

$$\leq \max \{\max \{f_a[\phi](mx^{-1}), f_a[\phi](x)\}, f_a[\phi](x^{-1}n)\}$$

$$\leq \max \{\max \{f_a[\phi](x^{-1}), f_a[\phi](x)\}, f_a[\phi](x^{-1})\}$$

$$\leq \max \{f_a[\phi](x^{-1}), f_a[\phi](x)\}$$

$$\leq \max \{f_a[\phi](x), f_a[\phi](x)\}, \text{ since } A(x^{-1}) = A(x)$$

$$\leq f_a[\phi](x)$$

$$f_a[[\phi](m(x^{-1})n)] \leq f_a[\phi](x)$$

Hence $f_a[\phi]$ is an $M - N$ anti fuzzy soft group of G .

4. Conclusion

The main results in the present manuscript are based on the concept of $M - N$ antifuzzy soft group [4 and 5]. We have also defined the $M - N$ anti level subsets of a fuzzy soft subgroup and its some elementary properties are discussed.

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Conjugate of M-N Anti Fuzzy Soft Subgroups

M. Kaliraja^{1*} and S. Rumenaka²

^{*1,2}Assistant Professor, PG and Research Department of Mathematics,
H.H. The Rajah's College, Pudukottai- 622 001, Affiliated to Bharathidasan
University, Tiruchirappalli, Tamilnadu, India.

^{1*}Email: mkr.maths009@gmail.com

²Email: rumenaka@gmail.com

Abstract

In this paper, we have discussed the concept of a conjugate of M-N anti fuzzy soft subgroup and define the M-N anti fuzzy soft middle cosets. It is some elementary properties are discussed.

Keywords: Fuzzy group; M-N anti fuzzy soft subgroup; Conjugate of M-N anti fuzzy soft subgroups; M – N anti fuzzy soft middle cosets.

1. Introduction

In [13], Rosenfeld introduced the concept of fuzzy subgroup in 1971 and the theory of fuzzy sets was inspired by Zadeh [17] in addition to this, Molodtsov [10] have introduced the concept of soft sets in 1999. Furthermore, In 2009, Maji et. al., introduced the concept of fuzzy soft sets in [9] and Jacobson introduced the concept of M-group M-subgroup in [3]. In [18], Wen-Xiang Gu et.al., have discussed the concept of Fuzzy groups with operators in the year of 1994.

Sarala and Suganya [14] be unraveled some properties of fuzzy soft groups in 2014 in addition, Vasantha Kandasamy and Smarandache [16] have introduced the Fuzzy Algebra during 2003. An introduction to the new definition of Soft sets and soft groups depending on inclusion relation and intersection of sets were exposed by Akta and Cagman [1]. In 1981, Das [2] studied the Fuzzy groups and level subgroups. Moreover, Maij, Biswas and Ray [9] were introduced the fuzzy soft set in 2001. In [15], Shobha Shukla have studied the conjugate fuzzy subgroup in 2013. Mourad Oqla Massa'deh[9] studied M- fuzzy cosets, M –conjugate of M – fuzzy subgroups. In[6], our earlier work we have discussed the concept of a conjugate of M – N fuzzy soft subgroups.

In the present manuscript, we have discussed the concept of conjugate of M-N anti fuzzy soft subgroups based on the concept of M - N anti fuzzy soft groups [6, 7 and 12]. In section 2, we presented the basic definition, notations on conjugate of M-N anti fuzzy soft group and required results on anti fuzzy soft group. In section 3, we define the conjugate of M-N anti fuzzy soft subgroups and define the M-N anti fuzzy soft middle cosets.

2. Preliminaries

In this section, some basic definitions and results needed are given. For the sake of convenience we set out the former concepts which will be used in this paper.

Definition 2.1[11]

Let μ be an M- N anti fuzzy subgroup of a set G. For $t \in [0, 1]$, the level subset of μ is the set $\mu_t = \{x \in G / \mu(mx) \leq t, \mu(xn) \leq t, m \in M, n \in N\}$. This is called a M – N anti level subset of μ

Definition 2.2[11]

Let M, N be left and right operator sets of group G respectively if $(m x) n = m (x n)$ for all $x \in G, m \in M, n \in N$. Then G is said be an M – N group.

Definition 2:3 [7]

Let G be an $M - N$ anti group and (μ, A) be a fuzzy soft subgroup of G if

$$(1) \mu \{m(x y) n\} \leq \max \{ \mu (x), \mu (y) \}$$

$$(2) \mu \{(m x^{-1}) n\} = \mu (x) \text{ hold for any } x, y \in G, m \in M, n \in N, \text{ then } (\mu, A) \text{ is}$$

said be an $M - N$ anti fuzzy soft subgroup of G . Here $\mu: A \rightarrow P(G)$

Definition 2:4 [7]

Let G be an $M - N$ anti group and (μ, A) be a fuzzy soft subgroup of G if

$$(1) \mu (m x) \leq \mu (x)$$

$$(2) \mu (x n) \leq \mu (x) \text{ hold for any } x \in G, m \in M, \text{ and } n \in N, \text{ then } (\mu, A) \text{ is}$$

said be an $M - N$ anti fuzzy soft subgroup of G .

Definition 2.5 [8]

Let μ and λ be two fuzzy subgroup of G , then μ and λ are said to be conjugate fuzzy subgroup of G if for some $g \in G$, $\mu (x) = \lambda (g^{-1} x g)$ for every $x \in G$.

Definition 2.6 [8]

Let μ and λ be two $M - N$ fuzzy soft subgroup of G , then μ and λ are said to be conjugate of $M - N$ fuzzy soft subgroup of G if for some $g \in G$,

$$(1) \mu (mx) = \lambda (g^{-1} x g) \text{ for every } x \in G, m \in M$$

$$(2) \mu (xn) = \lambda (g^{-1} x g) \text{ for every } x \in G, n \in N$$

3. Conjugate of M-N anti fuzzy soft subgroup

In this section, we shall define conjugate of $M-N$ anti fuzzy soft subgroups based on the concept of fuzzy soft group [6, 7 and 12], and give some elementary properties are discussed.

Definition 3.1

Let G be an $M - N$ anti group and (μ, A) be a conjugate of fuzzy soft subgroup of G if

$$(1) \mu (m x) \leq \mu (x) = \lambda (g^{-1} x g)$$

$$(2) \mu (y n) \leq \mu (y) = \lambda (g^{-1} y g) \text{ hold for any } x, y \in G, m \in M, \text{ and } n \in N, \text{ then } (\mu, A) \text{ is said be a conjugate of } M - N \text{ anti fuzzy soft subgroup of } G.$$

Theorem 3:2

Let μ and λ be any $M - N$ anti fuzzy soft subgroup of the group G , then μ and λ are conjugate of $M - N$ anti fuzzy soft subgroup of G iff $\mu = \lambda$

Proof:

Given that μ and λ are conjugate of $M - N$ anti fuzzy soft subgroup of the group G . To prove that $\mu = \lambda$

Since μ and λ are conjugate of $M - N$ anti fuzzy soft subgroup of the group G

By the definition there exists $g \in G$, such that

$$, \mu (mx) = \lambda (g^{-1} x g) \text{ for every } x \in G, m \in M$$

$$\mu (xn) = \lambda (g^{-1} x g) \text{ for every } x \in G, n \in N$$

Let $mx = gmx$ for all $g, x \in G, m \in M$, then

$$\mu (gmx) = \lambda (g^{-1} gx g)$$

$$\mu (gmx) = \lambda (x g)$$

$$\mu (gx) = \lambda (x g), \text{ since } \mu (mx) \leq \mu (x)$$

And let $xn = gxn$ for all $g, x \in G, n \in N$, then

$$\mu (gxn) = \lambda (g^{-1} gx g)$$

$$\mu(gxn) = \lambda(xg)$$

$$\mu(gx) = \lambda(xg), \quad \text{since } \mu(xn) \leq \mu(x)$$

For some $g = e \in G, m \in M$

We have $\mu(mex) = \lambda(mxe)$

$$\mu(mx) = \lambda(mx), \text{ since } \mu(xm) \leq \mu(x)$$

$$\mu(x) = \lambda(x)$$

$$\mu = \lambda$$

Similarly, $\mu(exn) = \lambda(xen)$

$$\text{Hence } \mu = \lambda$$

Conversely,

To prove that, μ and λ are conjugate of $M - N$ anti fuzzy soft subgroup of G

Let $\mu = \lambda$

$$\mu(mx) = \lambda(mx)$$

$$\mu(mx) = \mu(x) = \lambda(x), \text{ since } \mu(xm) \leq \mu(x)$$

By the definition, $\mu(mx) = \lambda(e^{-1}xe)$

Similarly we can prove that $\mu(xn) = \lambda(e^{-1}xe)$

Hence μ and λ are conjugate of $M - N$ anti fuzzy soft subgroup of G

Theorem 3:3

Let λ be an $M - N$ anti fuzzy soft subgroup of a group G , and μ be a fuzzy soft subset of G . If μ and λ are conjugate of $M - N$ anti fuzzy soft subgroup of the group G , then μ is an $M - N$ anti fuzzy soft subgroup of a group G .

Proof

Let e be an identity element of the group G .

If μ and λ are conjugate of $M - N$ anti fuzzy soft subgroup of the group G

By the definition, since there exists an element $g \in G$, such that

$$\lambda(x) \geq \lambda(mx) = \mu(g^{-1}xg) \text{ for all } x \in G$$

$$\Rightarrow \lambda(x) = \mu(g^{-1}xg) \text{ and}$$

$$\lambda(x) \geq \lambda(xn) = \mu(g^{-1}xg) \quad \forall x \in G, m \in M, \text{ and } n \in N,$$

$$\Rightarrow \lambda(x) = \mu(g^{-1}xg)$$

$$\text{Also } \mu(x) \geq \mu(mx) = \mu(exe)$$

$$\mu(x) = \mu(g^{-1}gxg^{-1}g)$$

$$= \lambda(gxg^{-1})$$

$$\text{Therefore } \mu(x) = \lambda(gxg^{-1})$$

Similarly we can prove that $\mu(x) \geq \mu(xn) = \mu(gxg^{-1})$

To prove that, μ is an $M - N$ anti fuzzy soft subgroup of a group G .

Since λ be an $M - N$ anti fuzzy soft subgroup of a group G ,

Now $\mu(xy) \geq \mu(mxyn) = \mu(emxeyne)$

$$\mu(mxyn) = \mu(g^{-1}gmxg^{-1}gyn g^{-1})$$

$$= \lambda(gmxg^{-1}gyn g^{-1})$$

$$\leq \max \{ \lambda(gmxg^{-1}), \lambda(gyn g^{-1}) \}, \text{ since } \mu(xm) \leq \mu(x)$$

$$\mu(mxyn) \leq \max \{ \lambda(gxg^{-1}), \lambda(gy g^{-1}) \}$$

$$\mu(xy) \leq \max \{ \mu(x), \mu(y) \}$$

$$\begin{aligned}
 \text{Also } \mu(mx^{-1}n) &= \lambda(gmxy^{-1}g^{-1}gnyg^{-1}) \\
 &\leq \max \{ \lambda(gmxg^{-1}gy^{-1}g^{-1}), \lambda(gyn g^{-1}) \}, \\
 &\leq \max \{ \max \{ \lambda(gmxg^{-1}) \lambda(gy^{-1}g^{-1}) \}, \lambda(gyn g^{-1}) \}, \text{ since } \mu(xm) \leq \mu(x) \\
 &\leq \max \{ \max \{ \lambda(gxg^{-1}) \lambda(gy^{-1}g^{-1}) \}, \lambda(gyg^{-1}) \}, \\
 &\leq \max \{ \lambda(gxg^{-1}), \lambda(gyg^{-1}) \}, \text{ since } \mu(y) = \mu(y^{-1}) \\
 &\leq \max \{ \lambda(gxg^{-1}), \lambda(gy^{-1}g^{-1}) \} \\
 &= \lambda(gxg^{-1}) \\
 \mu(mx^{-1}n) &= \lambda(gxg^{-1})
 \end{aligned}$$

Hence μ is an M – N anti fuzzy soft subgroup of a group G.

4. M-N Anti Fuzzy Soft Middle Coset

Definition 4.1

Let λ be an M- N anti fuzzy soft subgroup of a group G. then for any $a, b \in G$ the M– N anti fuzzy soft middle coset $a\lambda b$ of the G is defined by $(a\lambda b)(mxn) = \lambda(a^{-1}x b^{-1})$ for all $x \in G$.

Theorem 4.2

If λ is an M – N anti fuzzy soft subgroup of a group G, then for any $a \in G$, the M – N anti fuzzy soft middle coset $a\lambda a^{-1}$ of the group G is also a M – N anti fuzzy soft subgroup of the group G.

Proof:

$$\begin{aligned}
 &\text{Let } \lambda \text{ be an M – N anti fuzzy soft subgroup of a group G and } a \in G \\
 &\text{Let } x, y \in G, m \in M \text{ and } n \in N, \text{ then} \\
 (a\lambda a^{-1})(mxy^{-1}n) &= \lambda(ma^{-1}xy^{-1}an) \\
 &= \lambda(ma^{-1}xaa^{-1}y^{-1}an) \\
 &= \lambda(m(a^{-1}xa)(a^{-1}y^{-1}a)n) \\
 &\leq \max \{ \lambda(m(a^{-1}xa)), \lambda(a^{-1}y^{-1}a)n \}, \text{ since } A(mx) \leq A(x), A(yn) \leq A(y) \\
 &\leq \max \{ \lambda((a^{-1}xa)), \lambda(a^{-1}y^{-1}a) \},
 \end{aligned}$$

Since λ is an M – N anti fuzzy soft subgroup of a group G
Therefore $(a\lambda a^{-1})(mxy^{-1}n) \leq \max \{ \lambda((a^{-1}xa)), \lambda(a^{-1}y^{-1}a) \}$
Hence $a\lambda a^{-1}$ is an M – N anti fuzzy soft subgroup of the group G

Theorem 4.3

Let λ be any M – N anti fuzzy soft subgroup of a group G and $a\lambda a^{-1}$ be an M – N anti fuzzy soft middle coset of G, then $o(a\lambda a^{-1}) = o(\lambda)$ for any $a \in G$.

Proof:

Let λ be an M – N anti fuzzy soft subgroup of a group G and $a \in G$
By the above theorem (that is theorem 4.2), $a\lambda a^{-1}$ is an M – N anti fuzzy soft subgroup of the group. Thus $(a\lambda a^{-1})(mxn) = \lambda(ma^{-1}xan)$, for all $x \in G, m \in M$ and $n \in N$.
Therefore λ and $a\lambda a^{-1}$ are conjugate of M – N anti fuzzy soft subgroup of G. We know that, if λ and μ are conjugate of M – N anti fuzzy soft subgroup of the group G, then $o(\lambda) = o(\mu)$

$$\text{Hence } o(a\lambda a^{-1}) = o(\lambda) \text{ for any } a \in G.$$

Definition 4.4

Let λ and μ be an M – N anti fuzzy soft subgroup of the group G, and f be a positive fuzzy set, then for $a \in G$ we define the M – N anti positive double fuzzy soft coset

$$(\lambda a \mu)^f \text{ by } (\lambda a \mu)^f = \min \{ (a\lambda)^f, (a\mu)^f \}$$

Theorem 4.5

The $M - N$ anti positive double fuzzy soft coset $(\lambda \text{ a } \mu)^f$ is $M - N$ anti fuzzy soft subgroup of the group G , when λ, μ are $M - N$ anti fuzzy soft subgroup of G

Proof:

Let $x, y \in G, m \in M$ and $n \in N$

$$\begin{aligned} \text{Now } (\lambda \text{ a } \mu)^f (mxy^{-1}n) &= \max \{ (a \lambda)^f (mxy^{-1}n), (a \mu)^f (mxy^{-1}n) \} \\ &= \max \{ f(a)\lambda(mxy^{-1}n), f(a)\mu(mxy^{-1}n) \} \\ &\leq f(a) \max \{ \max \{ \lambda(mx), \lambda(y^{-1}n) \}, \max \{ \mu(mx), \mu(y^{-1}n) \} \} \end{aligned}$$

$$\begin{aligned} \text{Since } \lambda(mx) &\leq \lambda(x), \lambda(x^{-1}) = \lambda(x), (xn) \leq \mu(x), \mu(x^{-1}) = \mu(x) \\ &\leq f(a) \max \{ \max \{ \lambda(x), \lambda(y) \}, \max \{ \mu(x), \mu(y) \} \} \\ &\leq f(a) \max \{ \max \{ \lambda(x), \mu(x) \}, \max \{ \lambda(y), \mu(y) \} \} \\ &= \max \{ f(a) \max \{ \lambda(x), \mu(x) \}, f(a) \max \{ \lambda(y), \mu(y) \} \} \\ &= \max \{ (\lambda \text{ a } \mu)^f (x), (\lambda \text{ a } \mu)^f (y) \} \end{aligned}$$

$$\text{Therefore } (\lambda \text{ a } \mu)^f (mxy^{-1}n) \leq \max \{ (\lambda \text{ a } \mu)^f (x), (\lambda \text{ a } \mu)^f (y) \}$$

Hence $(\lambda \text{ a } \mu)^f$ is $M - N$ anti fuzzy soft subgroup of the group G .

4. Conclusion

The main results of this manuscript, a conjugate of $M - N$ anti fuzzy soft subgroup based on [6 and 12]. We have also defined the $M - N$ anti fuzzy soft middle coset and its some elementary properties are discussed.

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A CONJUGATE OF M-N FUZZY SOFT SUBGROUPS

M. Kaliraja^{1*} and S. Rumenaka²

¹PG and Research Department of Mathematics,

H.H.The Rajah's College, Pudukottai, Affiliated to Bharathidasan University,
Tiruchirappalli, Tamilnadu, India. E-mail: mkr.maths009@gmail.com

²Research Scholar, PG and Research Department of Mathematics, H.H.The Rajah's
College, Pudukottai, Affiliated to Bharathidasan University, Tiruchirappalli, Tamilnadu, India. Email
rumenaka@gmail.com

Abstract. In this paper, we have discussed the concept of a conjugate of M-N fuzzy soft subgroup of a group and define the M-N fuzzy soft middle co-sets. Also its some elementary properties are discussed. The aim of the paper is to investigate conjugate of M –N fuzzy soft subgroup of a group from a general point of view.

Key words: Fuzzy subgroup; M-N fuzzy subgroup; M-N fuzzy soft subgroup; fuzzy soft middle co-sets.

AMS Mathematics Subject Classification (2010): 20N25, 06D72, 20E15.

1. Introduction

There are various types of uncertainties in the real world, but few classical mathematical tools may not be suitable to model these uncertainties. Many intricate problems in economics, social science, engineering, medical science and many other fields involve undefined data. These problems which one comes face to face with in life cannot be solved using classical mathematic methods. In classical mathematics, a mathematical model of an object is devised and the concept of the exact solution of this model is not yet determined. Since, the classical mathematical model is too complex, the exact solution cannot be found. There are several well-renowned theories available to describe uncertainty. For instance, Rosenfeld [10] introduced the concept of fuzzy subgroup in 1971 and the theory of fuzzy sets was inspired by Zadeh [14] in addition to this, Molodtsov [7] have introduced the concept of soft sets in 1999. Furthermore, Majiet. al., [6] as well introduction the concept of fuzzy soft sets in 2001 and Jacobson [3] introduced the concept of M-group M-subgroup.

Sarala and Suganya [11] unravelled some properties of fuzzy soft groups in 2014. In addition, Vasantha Kandasamy and Smarandache [13] have introduced the Fuzzy Algebra during 2003. An introduction to the new definition of Soft sets and soft groups depending on inclusion relation and intersection of sets were exposed by Akta and Cagman [1]. In 1981, Das [2] studied the Fuzzy groups

and level subgroups. Moreover, Maji, Biswas and Ray [6] were introduced the fuzzy soft set in 2001. In [9] the notion of a conjugate fuzzy subgroup of a fuzzy group was introduced and studied. ShobhaShukla [12] studied the conjugate fuzzy subgroup in 2013. Mourad Oqla Massa'deh [9] studied M- fuzzy co-sets, M –conjugate of M –fuzzy subgroups.

In the present manuscript, we have discussed the concept of conjugate of M-N fuzzy soft group based on the concept of M - N fuzzy soft group [5 and 9]. In section 2, we presented the basic definition, notations on conjugate of M-N fuzzy soft group and required results on fuzzy soft group. In section 3, we define the conjugate of M-N fuzzy soft subgroups, define the M-N fuzzy soft middle co-sets and related results are discussed.

2. Preliminaries

In this section, some basic definitions and results needed are given. For the sake of convenience we set out the former concepts which will be used in this paper.

Definition 2.1

Let G be any non-empty set. A mapping $\mu : G \rightarrow [0, 1]$ is called fuzzy set in G .

Definition 2.2

Let x be a non-empty set. A fuzzy subset μ of X is a function $\mu : X \rightarrow [0, 1]$

Definition 2.3

Let G be a group. A fuzzy subset μ of G is called a fuzzy subgroup if for $x, y \in G$

$$(1) \mu(xy) \geq \min \{\mu(x), \mu(y)\}$$

$$(2) \mu(x^{-1}) = \mu(x)$$

Definition 2.4

Let μ be an M- N fuzzy subgroup of a set G . For $t \in [0, 1]$, the level subset of μ is the set $\mu_t = \{x \in G / \mu(mx) \geq t, \mu(xn) \geq t, m \in M, n \in N\}$. This is called a M – N level subset of μ .

Definition 2.5

A pair (μ, A) is called a soft set over U , where μ is a mapping given by $\mu: A \rightarrow P(U)$

Definition 2.6

Let (μ, A) be a soft set over G . Then (μ, A) is called a soft group over G if $\mu(a)$ is a group G for all $a \in A$.

Definition 2.7

A pair (μ, A) is called a fuzzy soft set over U , where $\mu: A \rightarrow I^U$ is a mapping $I = [0, 1]$, $\mu(a)$ is a fuzzy subset of U for all $a \in A$.

Definition 2.8 [5]

Let (μ, A) be a fuzzy soft set over G . Then (μ, A) is called a fuzzy soft group if $\mu(a)$ is a fuzzy subgroup G for all $a \in A$.

Definition 2.9 [4]

Let (μ, A) and (λ, B) be two fuzzy soft set over U . Then (μ, A) is called a fuzzy soft subset of (λ, B) denoted by $(\mu, A) \subseteq (\lambda, B)$ if

$$(1) A \subseteq B$$

$$(2) \mu(a) \text{ is a fuzzy subset of } \lambda(a) \text{ for each } a \in A.$$

Definition 2.10

A fuzzy set μ is called a fuzzy soft subgroup of a group G , if for $x, y \in G$

- (1) $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$
- (2) $\mu(x^{-1}) \geq \mu(x)$

Definition 2.11 [8]

Let M, N be left and right operator sets of group G respectively if $(m x) n = m (x n)$ for all $x \in G$, $m \in M, n \in N$. Then G is said to be an $M - N$ group.

Definition 2.12 [4]

Let G be an $M - N$ group and (μ, A) be a fuzzy soft subgroup of G if

$$(1) \mu \{m(x y) n\} \geq \min \{ \mu (x), \mu (y) \}$$

(2) $\mu \{(m x^{-1}) n\} \geq \mu(x)$ hold for any $x, y \in G, m \in M, n \in N$, then (μ, A) is said to be an $M - N$ fuzzy soft subgroup of G . Here $\mu: A \rightarrow P(G)$

Definition 2.13 [5]

Let G be an $M - N$ group and (μ, A) be a fuzzy soft subgroup of G if

$$(1) \mu (m x) \geq \mu (x)$$

(2) $\mu (x n) \geq \mu (x)$ hold for any $x \in G, m \in M$, and $n \in N$, then (μ, A) is said to be an $M - N$ fuzzy soft subgroup of G .

Definition 2.14 [12]

Let μ and λ be two fuzzy subgroup of G , then μ and λ are said to be conjugate fuzzy subgroup of G if for some $g \in G$, $\mu(x) = \lambda(g^{-1} x g)$ for every $x \in G$.

3. Conjugate of M-N fuzzy soft subgroup

In this section, we shall define the conjugate of M-N fuzzy soft subgroup, discussed the Conjugate of M-N fuzzy soft group based on the concept of fuzzy soft group [5 and 9], and give some elementary properties are discussed.

Definition 3.1

Let μ and λ be two $M - N$ fuzzy soft subgroup of G , then μ and λ are said to be conjugate of $M - N$ fuzzy soft subgroup of G if for some $g \in G$,

$$(1) \mu (mx) = \lambda (g^{-1} x g) \text{ for every } x \in G, m \in M$$

$$(2) \mu (xn) = \lambda (g^{-1} x g) \text{ for every } x \in G, n \in N$$

Definition 3.2

Let G be an $M - N$ group and (μ, A) be a conjugate of fuzzy soft subgroup of G if

$$(1) \mu (m x) \geq \mu (x) = \lambda (g^{-1} x g)$$

(2) $\mu (y n) \geq \mu (y) = \lambda (g^{-1} y g)$ hold for any $x, y \in G, m \in M$, and $n \in N$, then (μ, A) is said to be a conjugate of $M - N$ fuzzy soft subgroup of G .

Theorem 3.3

Let μ and λ be any $M - N$ fuzzy soft subgroup of the group G , then μ and λ are conjugate of $M - N$ fuzzy soft subgroup of G iff $\mu = \lambda$.

Proof: Given that μ and λ are conjugate of $M - N$ fuzzy soft subgroup of the group G . We have to prove that $\mu = \lambda$. Since μ and λ are conjugate of $M - N$ fuzzy soft subgroup of the group G , by the definition there exists $g \in G$, such that, $\mu(mx) = \lambda(g^{-1} x g)$ for every $x \in G, m \in M$, $\mu(xn) = \lambda(g^{-1} x g)$ for every $x \in G, n \in N$. Let $mx = gmx$ for all $x \in G, m \in M$, then:

$$\mu(gmx) = \lambda(g^{-1} g x g)$$

$$\mu(gmx) = \lambda(xg)$$

$$\mu(gx) = \lambda(xg), \quad \text{since } \mu(mx) \geq \mu(x).$$

And, let $xn = gxn$ for all $g, x \in G, n \in N$, then:

$$\mu(gxn) = \lambda(g^{-1}gxg)$$

$$\mu(gxn) = \lambda(xg)$$

$$\mu(gx) = \lambda(xg), \quad \text{since } \mu(xn) \geq \mu(x).$$

For some $g = e \in G, m \in M$, we have $\mu(mex) = \lambda(mxe)$

$$\mu(mx) = \lambda(mx), \text{ since } \mu(xm) \geq \mu(x)$$

$$\mu(x) = \lambda(x), \mu = \lambda.$$

Similarly we can prove that $\mu(exn) = \lambda(xen)$, hence $\mu = \lambda$.

Conversely, we have to prove that, μ and λ are conjugate of $M - N$ fuzzy soft subgroup of G .

Let $\mu = \lambda$, $\mu(mx) = \lambda(mx)$, $\mu(mx) = \mu(x) = \lambda(x)$, since $\mu(xm) \geq \mu(x)$. By the definition, $\mu(mx) = \lambda(e^{-1}xe)$. Similarly we can prove that $\mu(xn) = \lambda(e^{-1}xe)$. Hence μ and λ are conjugate of $M - N$ fuzzy soft subgroup of G .

Theorem 3.4

Let λ be an $M - N$ fuzzy soft subgroup of a group G , and μ be a fuzzy soft subset of G . If μ and λ are conjugate of $M - N$ fuzzy soft subgroup of the group G , then μ is an $M - N$ fuzzy soft subgroup of a group G .

Proof: Let e be an identity element of the group G . If μ and λ are conjugate of $M - N$ fuzzy soft subgroup of the group G , by the definition, since there exists an element $g \in G$, such that:

$$\lambda(x) \leq \lambda(mx) = \mu(g^{-1}xg) \text{ for all } x \in G$$

$$\Rightarrow \lambda(x) = \mu(g^{-1}xg) \text{ and}$$

$$\lambda(x) \leq \lambda(xn) = \mu(g^{-1}xg) \text{ for all } x \in G, m \in M, \text{ and } n \in N,$$

$$\Rightarrow \lambda(x) = \mu(g^{-1}xg).$$

$$\text{Also, } \mu(x) \leq \mu(mx) = \mu(exe)$$

$$\mu(x) = \mu(g^{-1}gxg^{-1}g)$$

$$= \lambda(gxg^{-1}).$$

Therefore, $\mu(x) = \lambda(gxg^{-1})$. Similarly we can prove that $\mu(x) \leq \mu(xn) = \mu(gxg^{-1})$.

We have to prove that, μ is an $M - N$ fuzzy soft subgroup of a group G . Since λ be an $M - N$ fuzzy soft subgroup of a group G , now:

$$\mu(xy) \leq \mu(mxyn) = \mu(emxeyne)$$

$$\mu(mxyn) = \mu(g^{-1}gmxg^{-1}gyn g^{-1})$$

$$= \lambda(gmxg^{-1}gyn g^{-1})$$

$$\geq \min \{ \lambda(gmxg^{-1}), \lambda(gyn g^{-1}) \}, \text{ since } \mu(xm) \geq \mu(x)$$

$$\mu(mxyn) \geq \min \{ \lambda(gxg^{-1}), \lambda(gyg^{-1}) \}$$

$$\mu(xy) \geq \min \{ \mu(x), \mu(y) \}$$

Also,

$$\mu(mx^{-1}n) = \lambda(gmxy^{-1}g^{-1}gn y g^{-1})$$

$$\geq \min \{ \lambda(gmxg^{-1}g y^{-1}g^{-1}), \lambda(gyn g^{-1}) \},$$

$$\geq \min \{ \min \{ \lambda(gmxg^{-1}) \lambda(g y^{-1}g^{-1}) \}, \lambda(gyn g^{-1}) \}, \text{ since } \mu(xm) \geq \mu(x)$$

$$\geq \min \{ \min \{ \lambda(gxg^{-1}) \lambda(g y^{-1}g^{-1}) \}, \lambda(gyg^{-1}) \},$$

$$\geq \min \{ \{ \lambda(gxg^{-1}), \lambda(gyg^{-1}) \}, \mu(y) = \mu(y^{-1}) \}$$

$$\geq \min \{ \lambda(gxg^{-1}), \lambda(gy^{-1}g^{-1}) \}$$

$$= \lambda(gxg^{-1}),$$

$$\mu(mx^{-1}n) = \lambda(gxg^{-1}).$$

Hence μ is an $M - N$ fuzzy soft subgroup of a group G .

4. M-N Fuzzy Soft Middle Co-set

Definition 4.1

Let λ be an $M - N$ fuzzy soft subgroup of a group G . then for any $a, b \in G$ the $M - N$ fuzzy soft middle coset $a\lambda b$ of the G is defined by $(a\lambda b)(mxn) = \lambda(a^{-1}xb^{-1})$ for all $x \in G$.

Theorem 4.2

If λ is an $M - N$ fuzzy soft subgroup of a group G , then for any $a \in G$, the $M - N$ fuzzy soft middle coset $a\lambda a^{-1}$ of the group G is also a $M - N$ fuzzy soft subgroup of the group G .

Proof: Let λ be an $M - N$ fuzzy soft subgroup of a group G and $a \in G$. Let $x, y \in G$, $m \in M$ and $n \in N$, then:

$$\begin{aligned} (a \lambda a^{-1})(m x y^{-1} n) &= \lambda(m a^{-1} x y^{-1} a n) = \lambda(m a^{-1} x a a^{-1} y^{-1} a n) = \lambda(m (a^{-1} x a) (a^{-1} y^{-1} a) n) \\ &\geq \min \{ \lambda(m (a^{-1} x a)), \lambda(a^{-1} y^{-1} a n) \}, \text{ since } A(mx) \geq A(x), A(yn) \geq A(y) \\ &\geq \min \{ \lambda((a^{-1} x a)), \lambda(a^{-1} y^{-1} a) \}, \end{aligned}$$

since λ is an $M - N$ fuzzy soft subgroup of a group G . Therefore,

$$(a \lambda a^{-1})(m x y^{-1} n) \geq \min \{ \lambda((a^{-1} x a)), \lambda(a^{-1} y^{-1} a) \}.$$

Hence $a\lambda a^{-1}$ is an $M - N$ fuzzy soft subgroup of the group G .

Theorem 4.3

Let λ be any $M - N$ fuzzy soft subgroup of a group G and $a\lambda a^{-1}$ be an $M - N$ fuzzy soft middle coset of G , then $o(a\lambda a^{-1}) = o(\lambda)$ for any $a \in G$.

Proof: Let λ be an $M - N$ fuzzy soft subgroup of a group G and $a \in G$. By the theorem 3.6 $a\lambda a^{-1}$ is an $M - N$ fuzzy soft subgroup of the group G . Thus $(a \lambda a^{-1})(m x n) = \lambda(m a^{-1} x a n)$, for all $x \in G, m \in M$ and $n \in N$. Therefore, λ and $a\lambda a^{-1}$ are conjugate of $M - N$ fuzzy soft subgroup of G .

We know that the theorem if λ and μ are conjugate of $M - N$ fuzzy soft subgroup of the group G , then $o(\lambda) = o(\mu)$. Hence $o(a\lambda a^{-1}) = o(\lambda)$ for any $a \in G$.

Definition 4.4

Let λ and μ be an $M - N$ fuzzy soft subgroup of the group G , and f be a positive fuzzy set, then for $a \in G$ we define the $M - N$ positive double fuzzy soft coset

$$(\lambda \mu)^f \text{ by } (\lambda \mu)^f = \min \{ (a \lambda)^f, (a \mu)^f \}.$$

Theorem 4.5

The $M - N$ positive double fuzzy soft co-set $(\lambda \mu)^f$ is $M - N$ fuzzy soft subgroup of the group G , when λ, μ are $M - N$ fuzzy soft subgroup of G

Proof: Let $x, y \in G, m \in M$ and $n \in N$. Now,

$$\begin{aligned} (\lambda \mu)^f(m x y^{-1} n) &= \min \{ (a \lambda)^f(m x y^{-1} n), (a \mu)^f(m x y^{-1} n) \} \\ &= \min \{ f(a) \lambda(m x y^{-1} n), f(a) \mu(m x y^{-1} n) \} \\ &\geq f(a) \min \{ \min \{ \lambda(m x), \lambda(y^{-1} n), \min \{ \mu(m x), \mu(y^{-1} n) \} \} \}. \end{aligned}$$

Since, $\lambda(m x) \geq \lambda(x), \lambda(x^{-1}) = \lambda(x), (x n) \geq \mu(x), \mu(x^{-1}) = \mu(x)$

$$\begin{aligned} &\geq f(a) \min \{ \min \{ \lambda(x), \lambda(y), \min \{ \mu(x), \mu(y) \} \} \} \\ &\geq f(a) \min \{ \min \{ \lambda(x), \mu(x) \}, \min \{ \lambda(y), \mu(y) \} \} \\ &= \min \{ f(a) \min \{ \lambda(x), \mu(x) \}, f(a) \min \{ \lambda(y), \mu(y) \} \} \\ &= \min \{ (\lambda \mu)^f(x), (\lambda \mu)^f(y) \}. \end{aligned}$$

Therefore, $(\lambda \mu)^f(m x y^{-1} n) \geq \min \{ (\lambda \mu)^f(x), (\lambda \mu)^f(y) \}$. Hence $(\lambda \mu)^f$ is $M - N$ fuzzy soft subgroup of the group G .

4. Conclusion

The main results in the present manuscript are based on the concept conjugate of $M - N$ fuzzy soft group [5 and 12]. We have also defined the $M - N$ fuzzy soft middle co-set and its some elementary properties are discussed.

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