ECCENTRICITY AND STATUS BASED TOPOLOGICAL INDICES OF SOME MOLECULAR GRAPHS

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By

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CERTIFICATE

This is to certify that **R. ROHINI** has prepared this thesis entitled **"ECCENTRICITY AND STATUS BASED TOPOLOGICAL INDICES OF SOME MOLECULAR GRAPHS"** submitted to **BHARATHIDASAN UNIVERSITY** for the award of the degree of **Doctor of Philosophy in Mathematics** under my guidance and supervision. This is a bonafide record of the independent and original work done by her under my guidance in the PG and Research Department of Mathematics, Government Arts College, Tiruchirappalli - 620 022, from April 2018 to March 2023. This is also to certify that the thesis represents her independent original work and has not previously formed the basis for award of any Degree, Diploma, Fellowship or any other similar title in any Institution or University.

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DECLARATION

I, **R. Rohini,** here by declare that this thesis entitled **"ECCENTRICITY AND STATUS BASED TOPOLOGICAL INDICES OF SOME MOLECULAR GRAPHS"** submitted to the Bharathidasan University for the award of the degree of **Doctor of Philosophy in Mathematics,** is a record of independent research work carried out by me during the period from April 2018 to March 2023 under the supervision of **Dr. G. Srividhya,** Assistant Professor, PG and Research Department of Mathematics, Government Arts College, Tiruchirappalli-22. This work has not formed the basis for the award of any Degree, Diploma, Associateship or Fellowship, or any other similar title in any Institution or University to my knowledge.

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R. ROHINI

A STUDY ON TOPOLOGICAL INDICES OF GRAPHS

ABSTRACT

In this era of rapid technological development, chemical and pharmaceutical techniques in recent years a large number of new nano materials, crystalline materials, and drugs emerge every year. To determine the chemical properties of such a large number of new compounds and new drugs requires a large amount of chemical experiments.

The topological indices correlate certain physicochemical properties such as boiling point, stability of chemical compounds. The significance of topological indices is usually associated with quantitative structures property relationship (QSPR) and quantitative structure activity relationship (QSAR).

Here we have introduced 21 new indices namely the first and second Zagreb polynomials, Augmented Zagreb index, Harmonic Eccentric index, first and second K-Eccentric indices, first and second K Hyper - Eccentric indices, Multiplicative first and second K-Eccentric indices, the Multiplicative first and second K-Hyper - Eccentric indices, the edge versions of the Randic Index, Sum connectivity index, F-index, the Multiplicative edge versions of the Randic Index, Sum connectivity index, SK, SK1 and SK2 Indices and Nano-Zagreb, Sum Nano-Zagreb indices, Square Reverse Index, F-Reverse index, Reduced Second and hyper Zagreb index and general Reduced second Zagreb index.

By using those new indices we calculated the above topological indices for some special graphs and also for six chemical structures namely (i) The Circumcoronene homologous Series of Benzenoid H_k $(k \geq 1)$ with edges (ii) The general representation of line graph of Circumcoronene Series of Benzenoid H_k $(k \geq 1)$ (iii) 2*D*-sheet of boron triangular nanotube $BT[p, q]$ (iv) 2*D*-sheet of boron-α nanotube $BA[p, q]$ (v) Perfect binary tree (m = 2) (vi) $D_1[n]$ and $D_2[n]$ nanostar dendrimer.

PREFACE

“Graph Theory” is an important branch of Mathematics. It has grown rapidly in recent times with a lot of research activities.

The blossoming of a new branch of study in the field of Chemistry, “Chemical graph theory” is yet another proof of the importance and role of graph theory. In physics, graph theory is applied in continuum statistical mechanics and discrete statistical mechanics. The role of graph theory in Computer science is everywhere.

In this era of rapid technological development, chemical and pharmaceutical techniques in recent years have been rapidly evolved, and thus a large number of new nano materials, crystalline materials, and drugs emerge every year. To determine the chemical properties of such a large number of new compounds and new drugs requires a large amount of chemical experiments, thereby greatly increasing the workload of the chemical and pharmaceutical researchers. Fortunately, the chemical based experiments found that there was strong connection between topology molecular structures and their physical behaviors, chemical characteristics, and biological features, such as melting point, boiling point, and toxicity of drugs.

The concept of “topological index” was first proposed by Hosoya for characterizing the topological nature of a graph. Topological indices are the mathematical measures which correspond to the structures of any simple finite graph. The topological indices correlate certain physicochemical properties such as boiling point, stability of chemical compounds. They are invariant under the graph isomorphism. The significance of topological indices is usually associated with quantitative structures property relationship (QSPR) and quantitative structure activity relationship (QSAR).

Chapter 1 contains review of literature, and basic definitions which are required for the subsequent chapters.

In Chapter 2, we calculate the K-eccentric, K-hyper eccentric, Modified eccentric, Some Connectivity eccentric and K-eccentric types of Polynomial indices of circumcoronene series of Benzenoid H_k system. Also, determined the multiplicative eccentric indices of circumcoronene series Benzenoid H_k system.

In Chapter 3, we introduced the five new indices named the edge version of First, Second and hyper zagreb eccentric indices and modified first and second eccentric indices. Also, we determine the multiplicative first, second and hyper zagreb eccentric indices and modified multiplicative first and second eccentric indices of Circumcronone series Benzenoid graph L(G).

In Chapter 4, we introduced the six new indices named the Milan Randic eccentric boron triangular nanotubes index, inverse Randic eccentric boron triangular nanotubes index, reciprocal Randic eccentric boron triangular nanotubes index, Reduced reciprocal Randic eccentric boron triangular nanotubes index, reduced second Zagreb eccentric boron triangular nanotubes index and Forgotten eccentric boron triangular nanotubes we calculate the Harmonic eccentric index, Zagreb eccentric index and Eccentric connectivity index of structures namely molecular graphs of eccentricity of boron triangular nanotubes by *EBTN[m, n]* respectively.

In Chapter 5, we introduced the five new indices the Milan Randic eccentric boron-α nanotubes index, inverse Randic eccentric boron-α nanotubes index, reciprocal Randic eccentric boron-α nanotubes index, Reduced reciprocal Randic eccentric boron-α nanotubes index, reduced second Zagreb eccentric boron-α nanotubes index and Forgotten eccentric boron-α nanotubes we calculate the Harmonic eccentric index, Zagreb eccentric index and Eccentric connectivity index of structures namely molecular graphs of eccentricity of boron-α nanotubes by *EBAN[m, n]* respectively.

In Chapter 6, we introduced the five new indices the sum, product connectivity status indices, F_l-status index, first and second status Gourava indices, Gourava (a, b)-status indices were investigated. Also, we calculated reciprocal connectivity status indices, ABC, AGS, GAS and ASI status indices, Sum connectivity index, SK, SK1 and SK2 status indices and Nano-Zagreb, Sum Nano-Zagreb status indices, Square Reverse status index, F-Reverse status index, Reduced Second and hyper Zagreb status index and general Reduced second Zagreb status index of perfect binary tree graphs.

In Chapter 7, we introduced four new the distance based topological index the first, second and hyper cutting number-eccentricity indices of nanostar dendrimer $D_1[n]$. Also, introduced the the first, second and hyper cutting number-eccentricity indices of nanostar dendrimer $D_2[n]$.

CHAPTER 1

INTRODUCTION

In this chapter we give basic definitions and ideas regarding the thesis.

1.1 Introduction and Preliminaries

The field of mathematics plays vital role in various fields. One of the important areas in mathematics is graph theory which is used in structural models. The field graph theory started its journey from the problem of Konigsberg Bridge in 1735. Graphs are used in the field of chemistry to model chemical compounds. The following are the basic definitions of graphs.

A **graph** $G = (V, E)$ [13, 55] consists of a set V of vertices (also called nodes) and a set E of edges. If an edge connects to a vertex we say the edge is incident to the vertex and say the vertex is an **endpoint** of the edge. If an edge has only one end vertex then it is called a **loop** edge. And an edge with distinct end vertices is called as **link.** If two or more edges have the same end vertices then they are called **multiple or parallel edges**. Two vertices that are joined by an edge are called **adjacent vertices.** A graph H is a **sub graph** of a graph G if all vertices and edges in H are also in G. A graph is **finite** if both its end vertex set and edge set are finite. A graph with just one vertex is called as trivial and all other graphs are **nontrivial.**

A **walk** in a graph G is a sequence of alternating vertices and edges $v_1e_1v_2e_2$ ... $v_ne_nv_{n+1}$ with $n \geq 0$. The vertices $v_1$and v_{n+1} are called the **origin** and **terminus** respectively and$v_1, v_2, v_3 \ldots . . v_{n+1}$. If $v_1 = v_{n+1}$ then the walk is **closed** otherwise it is **open**. The **length** of the walk is the number of edges in the walk. A walk of length zero is a **trivial walk**. A **trail** is a walk with no repeated edges. A **path** is a walk with no repeated vertices. A **circuit** is a closed trail and a trivial circuit has a single vertex and no edges. A trail or circuit is **Eulerian** if it uses every edge in the graph. A graph is called **Eulerian** if it contains an Eulerian circuit. A path that contains every vertex of G is called a **Hamilton path** of G; similarly a **Hamilton cycle** of G is a cycle that contains every vertex of G. A graph is **Hamilton** if it contains **Hamilton cycle.**

Simple graph is a graph with no loop edges or multiple edges. Edges in a simple graph may be specified by a set {v_i, v_j} of the two vertices that the edge makes adjacent. A graph with more than one edge between a pair of vertices is called a **multigraph.** The **degree** of a vertex is the number of edges incident to the vertex and is denoted by deg (v) or (v), each loop counting as two edges. (G) and $\Delta(G)$ are denoted as the **minimum and maximum degrees** of G respectively.

A **directed graph** is a graph in which the edges may only be traversed in one direction. Edges in a simple directed graph may be specified by an ordered pair (v_i, v_j) of the two vertices that the edge connects. We say that v_i is adjacent to v_j. In a directed graph, the **in-degree** of a vertex is the number of edges incident to the vertex and the **out-degree** of a vertex is the number of edges incident from the vertex.

Simple graphs G and H are called **isomorphic** if there is a bijection from the nodes of G to the nodes of H such that $\{v, w\}$ is an edge in G if and only if $\{(v), (w)\}$ is an edge of H. The function f is called an **isomorphism.** A graph is **connected** if there is a walk between every pair of distinct vertices in the graph. A **connected component** of G is a connected sub-graph H of G such that no other connected sub graph of G contains H.

The **complete graph** on n vertices, denoted K_n, is the simple graph with vertices {1,2,3 ... n} and an edge between every pair of distinct vertices. A graph is called **bipartite** if its set of vertices can be partitioned into two disjoint sets S_1 and S_2 so that every edge in the graph has one end vertex in S_1 and one end vertex in S_2. The **complete bipartite** graph on n, m vertices, denoted $K_{n, m}$ is the simple bipartite graph with nodes $S_1 = \{a_1, a_2, a_3 ... a_n\}$ and $S_2 = \{b_1, b_2, b_3 ... b_m\}$ and with edges connecting each vertex in S_1 to every node in S_2.

A **weighted graph** is a graph G = (V, E) along with a function $w: E \to \mathbb{R}$ that associates a numerical weight to each edge. If G is a weighted graph, then T is a **minimal spanning tree** of G if it is a spanning tree and no other spanning tree of G has smaller total weight.

A **tree** is a connected, simple graph that has no cycles with n-1 edges. Vertices of degree 1 in a tree are called the leaves of the tree. Let G be a simple, connected graph. The sub graph T is a **spanning tree** of G if T is a tree and every vertex in G is a vertex in T. A **forest** is collection of trees. A tree is called a **rooted tree** if one vertex has been designated the **root**, in which case the edges have a natural orientation, towards or away from the root. The tree-order is the partial ordering on the vertices of a tree with $u \leq v$ if and only if the unique path from the root to v passes through.

A rooted tree which is a sub graph of some graph G is a **normal tree** if the ends of every edge in G are comparable in this tree-order whenever those ends are vertices of the tree. In a rooted tree, the **parent** of a vertex is the vertex connected to it on the path to the root; every vertex except the root has a unique parent. A **child** of a vertex v is a vertex of which v is the parent. In a rooted tree and all vertices have at most one parent. A **binary tree** is said to be perfect if all the internal nodes have strictly two children, and every external or leaf node is at the same level or same depth within a tree. A perfect **m-aray** tree with height h, the upper bound for the maximum number of leaves is m^h. The **status**, denoted by σ(u) of a vertex u in G is the sum of distances of all other vertices from u in G.

A graph is said to be **embeddable** in the plane, or **planar**, if it can be drawn in the plane so that its edges intersect only at their ends. Such a drawing of a planar graph G is called as a **planar embedding** of G.

Let $G = (V, E)$ be a graph with $V = S_1 \cup S_2 \cup S_3 ... \cup S_t \cup T$ where each S_i is as set of vertices having atleast two vertices and having the same degree and $T = V - \cup S_i$. The degree **splitting graph** of G is denoted by $D_s(G)$ is obtained from G by adding vertices $w_1, w_2, w_3 ... w_t$ and joining w_i to each vertex of $S_i (1 \leq i \leq t)$.

Let G be a loop less graph. We construct a graph L(G) in the following way: The vertex set of L (G) is in 1-1 correspondence with the edge set of G and two vertices of L(G) are joined by an edge if and only if the corresponding edges of G are adjacent in G. The graph L(G) (which is always a simple graph) is called the **line graph** or the edge graph of G.

The **middle graph** M(G) of a graph G is defined as follows: The vertex set of M (G) is V(G) , the edge set E(G). Two vertices x, y in the vertex set of M(G) are adjacent in M(G) if either (i) x, y are in E(G) and x, y are adjacent in G or (ii) x is in V(G), y is in E(G) and x, y are incident in G. In other words, M (G) is obtained by subdividing each edge of G exactly once and joining all these newly added middle vertices of adjacent edges of G.

The **graph distance** between two vertices and of a finite graph is the minimum length of the paths connecting them. The length of a graph geodesic, too. A **geodesic** is a shortest path between two graph vertices of a graph. The **diameter** diam(G) is the largest distance d(u, v) between any two vertices of a connected graph. The **eccentricity** of a graph vertex in a connected graph is the maximum graph distance between v and any other vertex u of G. For a disconnected graph, all vertices are defined to have **infinite eccentricity**. A vertex v of a graph G is called a **cutvertex** of G if its removal increases the number of components. The **vertex connectivity or simply connectivity** κ(G) of a graph G is the minimum number of vertices whose removal from G results in a disconnected or trivial graph. A graph G is n-connected, n ≥ 1 if κ(G) ≥ n. A graph G is 2-connected if and only if G is nontrivial, connected and contains no cut vertices. A **cutting number** c(v) of a vertex v ∈V(G) in a connected graph G is the number of pairs of vertices {v, w} such that v and w are in different components of G − v. A **perfect m-ary tree** is a full **m-ary tree** in which all leaf nodes are at the same depth. This tree is a perfect binary tree as all internal nodes has exactly two children and all leaf nodes are on the same level.

1.2 Chemical graph theory [77, 82]

A molecular graph is a simple graph such that its vertices correspond to the atoms and the edges to the bonds. Chemical graph theory is a branch of mathematical chemistry which has an important effect on the development of the chemical sciences. In chemical science, the physico-chemical properties of chemical compounds are often modeled by means of a molecular graph based structure descriptors, which are referred to as topological indices. The concept of “topological index” was first proposed by Hosoya for characterizing the topological nature of a graph.

Topological indices [77] are the mathematical measures which correspond to the structures of any simple finite graph. The topological indices correlate certain physicochemical properties such as boiling point, stability of chemical compounds. They are invariant under the graph isomorphism. The significance of topological indices is usually associated with quantitative structures property relationship (QSPR) and quantitative structure activity relationship (QSAR). They are therefore useful descriptors in QSARs and QSPRs that are used for predictive purposes, such as prediction of the toxicity of a chemical or the potency of a drug for future release in the market.

Topological indices are also being used in other fields such as proteomics and DNA sequencing and molecular similarity. The latter has potential for database characterization (Cummins et al. 1996) and combinatorial library design (Zheng et al. 1998). It therefore seems that the future of topological indices and their application to chemistry, biochemistry, biology and medicine are assured for the foreseeable future. Let G be a simple graph, with vertex set $V(G)$ and edge set $E(G)$. The degree d_u of a vertex u is the number of edges that are incident to it. The Wiener index is the first topological index introduced by chemist Wiener in 1947 [37], and it is defined as

$$W(G) = \sum_{\{u,v\} \subset V(G)} d(u, v) \qquad (1.1)$$

A large number of such indices depend only on vertex degree of the molecular graph. One of the oldest and well known topological indices is the first and second Zagreb indices, was first introduced by Gutman et al. in 1972 [33], and it is defined as

$$M_1(G) = \sum_{uv \in E(G)} d_G(v)^2 = \sum_{uv \in E(G)} [d_G(u) + d_G(v)] \qquad (1.2)$$

and $$M_2(G) = \sum_{uv \in E(G)} [d_G(u) \times d_G(v)] \qquad (1.3)$$

The connectivity index introduced in 1975 by Milan Randi´c [61], is defined as

$$R(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_G(u) d_G(v)}} \qquad (1.4)$$

Recently, a closely related variant of the Randi´c connectivity index called the sum-connectivity index was introduced by Zhou and Trinajsti´c [80] in 2008. For a connected graph G, its sum-connectivity index X(G) is defined as

$$X(G) = \sum_{uv \in E(G)} \frac{1}{d_G(u) + d_G(v)} \qquad (1.5)$$

In 2014, Jianxi Li and Chee Shiu introduced another variant of the Randi´c index named the Harmonic index which first appeared in [40]. For a graph G, the harmonic index H(G)is defined as

$$H(G) = \sum_{uv \in E(G)} \frac{2}{d_G(u) d_G(v)} \qquad (1.6)$$

Followed by the first and second Zagreb indices, in 2015 Furtula and Gutman [27] introduced forgotten topological index (also called *F*-index) which was defined as

$$F(G) = \sum_{uv \in E(G)} [d_G(v)]^3 \quad (1.7)$$

The fourth Atom bond connectivity index, $ABC_4(G)$ index was introduced by Ghorbaniet al. [21] in 2011. It is defined as

$$ABC_4(G) = \sum_{uv \in E(G)} \sqrt{\frac{S_u + S_v - 2}{S_u S_v}} . \quad (1.8)$$

The fifth Geometric-arithmetic index, $GA_5(G)$ was introduced by Graovac et al. [31] in 2011 which is defined as

$$GA_5(G) = \sum_{uv \in E(G)} \frac{2\sqrt{S_u S_v}}{S_u + S_v - 2} . \quad (1.9)$$

In 2004, Milicevi, Nikoli, Trinajstic [65], introduced the modified first and second Zagreb indices are respectively defined as

$${}^{m}M_1(G) = \sum_{uv \in E(G)} \frac{1}{d_G(u)^2} , \quad (1.10)$$

$${}^{m}M_2(G) = \sum_{uv \in E(G)} \frac{1}{d_G(u) d_G(v)} \quad (1.11)$$

In 2013, Shirdel et al. [74] introduced the first hyper-Zagreb index of a graph G, which is defined as

$$HM_1(G) = \sum_{uv \in E(G)} [d_G(u) + d_G(v)]^2 \quad (1.12)$$

$$HM_2(G) = \sum_{uv \in E(G)} [d_G(u) d_G(v)]^2 \quad (1.13)$$

In 1998, Estrada et al. [18] introduced the atom-bond connectivity (ABC) index, defined as

$$ABC(G) = \sum_{uv \in E(G)} \sqrt{\frac{d_G(u) + d_G(v) - 2}{d_G(u) d_G(v)}} \quad (1.14)$$

In 2009, Vukicevic and Furtula. [80] introduced the Geometric-arithmetic index, GA(G) index defined as

$$GA = \sum_{uv \in E(G)} \frac{2\sqrt{d_G(u) d_G(v)}}{d_G(u) + d_G(v)} \quad (1.15)$$

Inspired by work on the ABC index, Furtula et al. in 2010 [28] proposed the following modified version of the ABC index and called it as augmented Zagreb index (AZI)

$$AZI(G) = \sum_{uv \in E(G)} \left(\frac{d_G(u)\ d_G(v)}{d_G(u) + d_G(v) - 2}\right)^3 \quad (1.16)$$

In 2017, Farahani, Pradeep Kumar, and Rajesh Kanna introduced the first and second Zagreb polynomial [20] is defined as

$$M_1(G, x) = \sum_{uv \in E(G)} x^{d_u + d_v} \quad (1.17)$$

$$M_2(G, x) = \sum_{uv \in E(G)} x^{d_u \times d_v} \quad (1.18)$$

The properties of $M_1(G, x)$, $M_2(G, x)$ polynomials for some chemical structures can be seen in the work of Gutman [34]. G.H. Fath-Tabar et al. [25]. They defined the third Zagreb index

$$M_3(G) = \sum_{uv \in E(G)} \left(d_u - d_v\right) \quad (1.19)$$

The polynomial $M_3(G, x) = \sum_{uv \in E(G)} x^{|d_u - d_v|}$ (1.20)

In the year 2016 [34], following Zagreb type polynomials were defined as

$$M_4(G, x) = \sum_{uv\in E(G)} x^{d_u(d_u+d_v)} \quad (1.21)$$

$$M_5(G, x) = \sum_{uv\in E(G)} x^{d_v(d_u+d_v)} \quad (1.22)$$

$$M_{a,b}(G, x) = \sum_{uv\in E(G)} x^{ad_u+bd_v} \quad (1.23)$$

$$M_{a,b}(G, x) = \sum_{uv\in E(G)} x^{(d_u+a)(d_v+b)} \quad (1.24)$$

Ranjini et al. [12] redefined first, second and third Zagreb indices of graph G as

$$ReZG_1(G) = \sum_{uv\in E(G)} \frac{d_u + d_v}{d_u \times d_v} \quad (1.25)$$

$$ReZG_2(G) = \sum_{uv\in E(G)} \frac{d_u \times d_v}{d_u + d_v} \quad (1.26)$$

$$ReZG_3(G) = \sum_{uv\in E(G)} (d_u \times d_v)(d_u + d_v) \quad (1.27)$$

B.Furtula et al. [27] defined forgotten (topological) index as

$$F(G) = \sum_{e=uv\in E(G)} [d_G(u)^2 + d_G(v)^2] \quad (1.28)$$

Kulli [56] introduced the first and second Gourava indices of a graph G is defined as

$$GO_1(G) = \sum_{uv\in E(G)} ((d_G(u) + d_G(v)) + d_G(u)d_G(v)) \quad (1.29)$$

$$GO_2(G) = \sum_{uv\in E(G)} (d_G(u) + d_G(v))\,(d_G(u)d_G(v)) \quad (1.30)$$

Kulli [54] introduced the first and second hyper - Gourava indices of a molecular graph G

$$HGO_1(G) = \sum_{uv\in E(G)} [(d_G(u) + d_G(v)) + d_G(u)d_G(v)]^2 \quad (1.31)$$

$$HGO_2(G) = \sum_{uv\in E(G)} [(d_G(u) + d_G(v))\,(d_G(u)d_G(v)]^2 \quad (1.32)$$

In [10], Bhanumathi and Easu Julia Rani introduced the first *K*-Eccentric index $B_1E(G)$ and the second *K*- Eccentric index $B_2E(G)$ of a graph G as

$$B_1E(G) = \sum_{ue}\left[e_G(u) + e_{L(G)}(e)\right] \quad (1.33)$$

$$and \quad B_2E(G) = \sum_{ue}\left[e_G(u)e_{L(G)}(e)\right] \quad (1.34)$$

Similarly In [10], Bhanumathi and Easu Julia Rani defined the first *K*- Hyper eccentric index $HB_1E(G)$ and the second *K*- Hyper eccentric index $HB_2E(G)$ of a graph G as

$$HB_1E(G) = \sum_{ue}\left[e_G(u) + e_{L(G)}(e)\right]^2 \quad (1.35)$$

and $$HB_2E(G) = \sum_{ue}\left[e_G(u)e_{L(G)}(e)\right]^2 \quad (1.36)$$

where in all the cases *ue* means that the vertex *u* and edge *e* are incident in *G* and $e_{L(G)}(e)$ is the eccentricity of e in the line graph $L(G)$ of G.

In [73] V.S. shegehalli and R. kanabur introduced new degree based topological indices (SK indices) as follows

$$SK(G) = \sum_{uv\in E(G)} \frac{d_G(u)+d_G(v)}{2} \quad (1.37)$$

$$SK_1(G) = \sum_{uv\in E(G)} \frac{d_G(u)d_G(v)}{2} \quad (1.38)$$

$$SK_2(G) = \sum_{uv\in E(G)} \left(\frac{d_G(u)d_G(v)}{2}\right)^2 \quad (1.39)$$

Recently Furtula et al. in [27] proposed the reduced second Zagreb index, defined as

$$RM_2(G) = \sum_{uv\in E(G)} (d_G(u) - 1)\,(d_G(v) - 1) \quad (1.40)$$

In [27], kulli introduced the reduced second hyper-Zagreb index, defined as

$$RHM_2(G) = \sum_{uv\in E(G)} [(d_G(u) - 1)\,(d_G(v) - 1)]^2 \quad (1.41)$$

In [32], Sombor index of a graph G is defined as

$$SO(G) = \sum_{uv\in E(G)} \sqrt{d_G(u)^2 + d_G(v)^2} \quad (1.42)$$

In [51], the first and second (a,b)-KA index of a graph was introduced and defined as

$$KA^1_{a,b}(G) = \sum_{uv\in E(G)} [\, d_G(u)^a + d_G(v)^a \,]^b \quad (1.43)$$

$$KA^2_{a,b}(G) = \sum_{uv\in E(G)} [d_G(u)^a d_G(v)^a]^b \quad (1.44)$$

The reduced Sombor index was defined as [51]

$$RSO(G) = \sum_{uv\in E(G)} \sqrt{(d_G(u) - 1)^2 + (d_G(v) - 1)^2} \quad (1.45)$$

The reduced modified Sombor index of a graph G, and it is defined as

$$^{m}RSO(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{(d_G(u)-1)^2+(d_G(v)-1)^2}} \quad (1.46)$$

The first, second status connectivity indices of a graph G are introduced by Ramane et al.in [69], defined as

$$S_1(G) = \sum_{uv\in E(G)}[\sigma(u) + \sigma(v)] \,, \quad (1.47)$$

$$S_2(G) = \sum_{uv\in E(G)}[\sigma(u)\,\sigma(v)] \quad (1.48)$$

[42] Kulli introduced the first and second hyper status indices of a graph G, defined as

$$HS_1(G) = \sum_{uv\in E(G)}[\sigma(u) + \sigma(v)]^2 \,, \quad (1.49)$$

$$HS_2(G) = \sum_{uv\in E(G)}[\sigma(u)\,\sigma(v)]^2 \,. \quad (1.50)$$

Also, [58] Kulli introduced the connectivity status indices as follows:

The sum connectivity status index of graph G is defined as

$$SS(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{\sigma(u)+\sigma(v)}} . \quad (1.51)$$

The product connectivity status index of graph G is defined as

$$\mathrm{PS}(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{\sigma(u)\,\sigma(v)}} . \quad (1.52)$$

The reciprocal product connectivity status index of a graph G is defined as

$$\mathrm{RPS}(G) = \sum_{uv \in E(G)} \sqrt{\sigma(u)\,\sigma(v)} . \quad (1.53)$$

The general first and second status indices of a graph G are defined as

$$S_1^{\,a}(G) = \sum_{uv \in E(G)} [\sigma(u) + \sigma(v)]^a , \quad (1.54)$$

$$S_2^{\,a}(G) = \sum_{uv \in E(G)} [\sigma(u)\,\sigma(v)]^a \quad (1.55)$$

where a is a real number.

[43] Kulli introduced the connectivity status indices as follows:

The F_1-status index of a graph defined as

$$F_1S(G) = \sum_{uv \in E(G)} [\sigma(u)^2 + \sigma(v)^2] \quad (1.56)$$

First and second status Gourava indices, (a, b)-status index of a graph and also symmetric division status index defined as

$$SGO_1(G) = \sum_{uv \in E(G)} [\sigma(u) + \sigma(v) + \sigma(u)\,\sigma(v)] \text{ and} \quad (1.57)$$

$$SGO_2(G) = \sum_{uv \in E(G)} \sigma(u)\,\sigma(v)[\sigma(u) + \sigma(v)] . \quad (1.58)$$

The (a, b)-status index of a graph G is defined as

$$S_{a,b}(G) = \sum_{uv \in E(G)} [\sigma(u)^a \sigma(v)^b + \sigma(u)^b \sigma(v)^a]. \quad (1.59)$$

The symmetric division status index of a graph G is defined as

$$SDS(G) = \sum_{uv \in E(G)} [\frac{\sigma(u)}{\sigma(v)} + \frac{\sigma(v)}{\sigma(u)}]. \qquad (1.60)$$

[57] Kulli introduced the connectivity status indices as follows:

The atom bond connectivity status index of a graph G is defined as

$$ABCS(G) = \sum_{uv \in E(G)} \sqrt{\frac{\sigma(u)+\sigma(v) - 2}{\sigma(u)\, \sigma(v)}}. \qquad (1.61)$$

The arithmetic-geometric status index of a graph G is defined as

$$AGS(G) = \sum_{uv \in E(G)} \frac{\sigma(u)+\sigma(v)}{2\sqrt{\sigma(u)\sigma(v)}}. \qquad (1.62)$$

The geometric arithmetic status index of a connected graph G defined as

$$GAS(G) = \sum_{uv \in E(G)} \frac{2\sqrt{\sigma(u)\, \sigma(u)}}{\sigma(u) + \sigma(u)}. \qquad (1.63)$$

The augmented status index of a graph G is defined as

$$ASI(G) = \sum_{uv \in E(G)} \left(\frac{\sigma(u)\sigma(v)}{\sigma(u)+\sigma(v)-2}\right)^{3}. \qquad (1.64)$$

The harmonic status index defined as HS(G) = $\sum_{\boldsymbol{uv \in E(G)}} \frac{2}{\sigma(u)+\sigma(v)}$. (1.65)

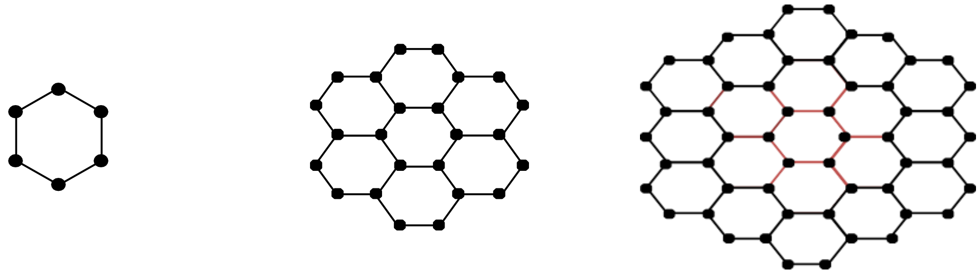
1.3 Structures used:

Mathematical chemistry is a branch of theoretical chemistry for discussion and prediction of the molecular structure using mathematical methods without necessarily referring to quantum mechanics [15, 78]. Chemical graph theory is a branch of mathematical chemistry which applies graph theory to mathematical modeling of chemical aspects [79]. A chemical structure determination includes a chemist's specifying the molecular geometry. Theories of chemical structure were first developed by August Kekule, Archibald Scott Couper, and Aleksandr Butlerov, among others, from about 1858. These theories were first to state that chemical compounds are not a random cluster of atoms and functional groups, but rather had a definite order defined by the valence of the atoms composing the molecule, giving the molecules a three dimensional structure that could be determined or solved. The following are the chemical structures we used. In mathematical chemistry, numbers encoding certain structural features of organic molecules and derived from the corresponding molecular graph, are called graph invariants or more commonly topological indices.

(1) Circumcoronene series of benzenoid system:

Benzenoid system form one of the most important classes of chemical graphs. The benzenoid system is composed of a hexagonal mesh. Many studies have been conducted on benzenoid systems. Several topological indices are calculated for the family of benzenoid system [83, 84].

The circumcoronene series of benzenoid is family of molecular graph, which consist several copy of benzene C_6 on circumference. The first terms of this series are H_1 = benzene, H_2 = coronene, H_3 = circumcoronene H_4 = circumcircum coronene. Consider the circumcoronene series of benzenoid H_k for all integer number $k \geq 1$. In the following Figure1.3.2, all edges belong to E_4, E_5 and E_6 marked by red, green and black colors, respectively. From the structure of H_k (Fig 1.3.2) one can see that the number of vertices / atoms in this benzenoid molecular graph is equal to $|V_k| = 6k^2$ and the number of edges/bonds is equal to $|E_k| = 9k^2 - 3k$ for the structure of H_k.



H_1 = Benzene H_2 = Coronene H_3 = Circumcoronene

Fig.1.3.1 The graphs H_1 H_2 and H_3 from the Circumcoronene Series

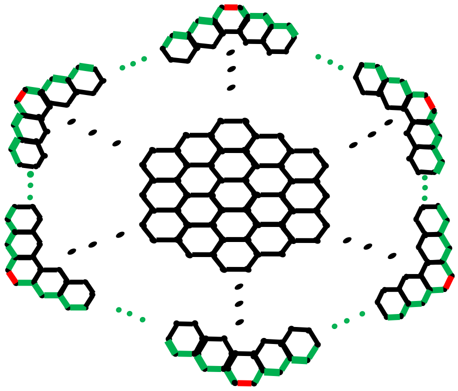


Fig.1.3.2 The Circumcoronene homologous Series of
Benzenoid H_k $(k \geq 1)$ with edges

(2) Line graph of circumcoronene series of benzenoid system:

For any positive integer number k, let $L(G) = L(H_k)$ be the general form of line graph of circumcoronene series of benzenoid system and no of vertices in $L(H_k)$ = number of edges in $H_k = 9k^2 - 3k$ and number of edges in $L(H_k)$ = sum of degrees of the edges of $H_k = 18k^2 - 12k$.

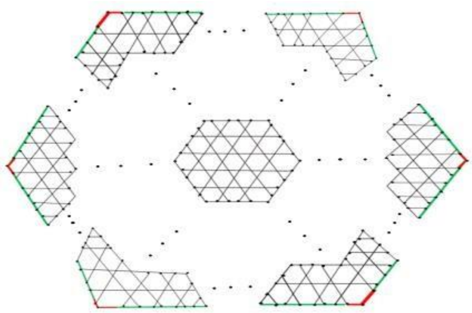
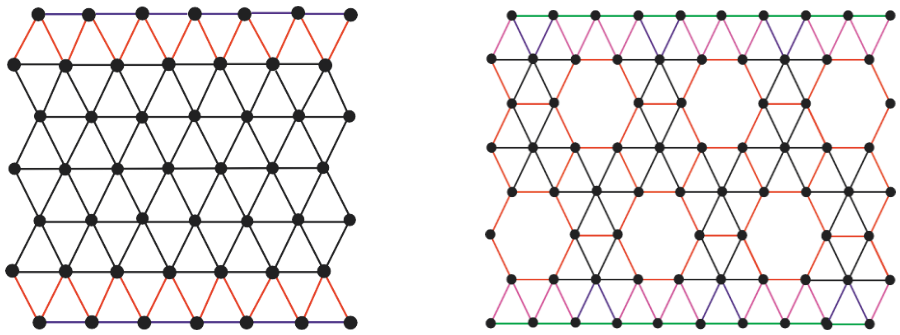


Fig 1.3.3: The general representation of line graph of
circumcoronene series of Benzenoid H_k $(k \geq 1)$

(3) Boron nanotubes:

Recently Jia-Bao, HaniShakerandetal [30] worked on topological aspects of boron nanotubes. Motivated from these works, we compute the third Zagreb index, harmonic index, forgotten index, inverse sum index, modified Zagreb index and symmetric division deign by applying sub division and semi total point graph for boron triangular and boron-α nanotubes.

In last 20 years, various type of boron containing nanomaterials. Boron nanomaterials have been considered as excellent material for enhancing the characteristics of optoelectronic nanodevices because of their broad elastic modulus, high melting point, excessive conductivity.



(a) (b)

Fig1.3.4 : (a) The edge partitions of BT[7, 4] nanotube.
(b) The edge partitions of BA(X) [8, 6] nanotube.

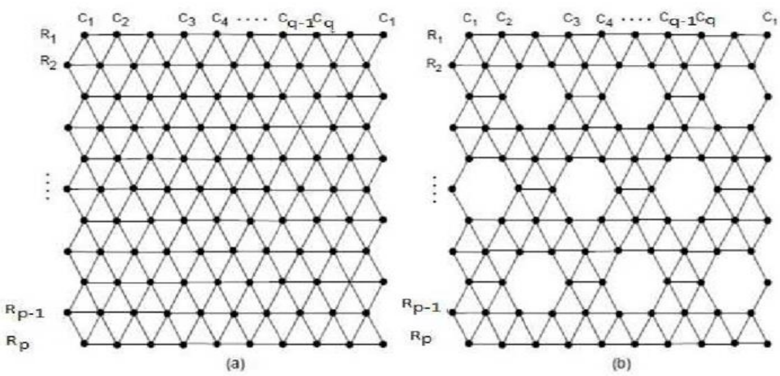


Fig.1.3.5 (a) 2*D*-sheet of boron triangular nanotube *BT*[*p, q*],
(b) 2*D*-sheet of boron-α nanotube *BA*[*p, q*].

These materials can carry excessive emission current, which recommends that they may have great prospective applications in fielde emission area [59]. Boron nano materials some best properties such as excessive resist an cetooxidation at high temperatures, great chemical stability and are stable broad band-gap semi conductor [6, 63]. Moreover, the extensive range of boron nanomaterials themselves could be the building blocks for combining with other existing nanomaterial to designe and create materials with new properties. The boron triangular nanotube was created in 2004 [59] and obtained from a carbon hexagonal nanotube by adding an extra atom to the centre of each hexagon.

Also, a special boron nanotube was fabricated from a carbon hexagonal nanotube in 2008, by adding an extra atom to the centre of certain hexagons [60, 75]. This nanotube is designed by generating a mixture of hexagons and triangles called boron-α nanotube. These nanotubes are important materials for optical, electronic, bio and chemical sensing applications. The comparison study about some computational aspects of boron triangular and boron-α nanotubes has been investigated in [86]. The 3D perceptions of boron triangular and boron-α nanotube are presented in the Fig.1.3.6

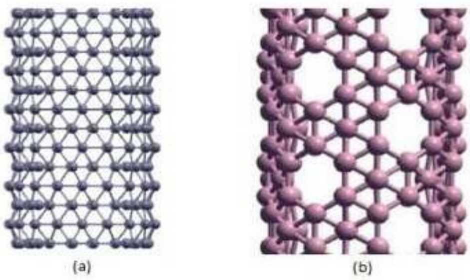


Fig.1.3.6 (a) 3*D*-perception of boron triangular nanotube,
(b) 3*D*-perception of boron-α nanotube.

The molecular graphs of boron triangular and boron-α nanotubes by BT[m, n] and BA[m, n] respectively, where m is the number of rows and n is the number of columns in a 2D sheet of BT[m, n] or BA[m, n] as shown in

Molecular graph	**Order**	**Size**
BT[m, n]	3mn/2	3n(3m - 2)/2
BA(X)[m, n]	n(4m + 1)/3	n(7m - 2)/2
BA(Y)[m, n]	4mn/3	n(7m - 4)/2

Table 1.1.1

Fig. 1.3.5. We categorize the boron-α nanotubes into two classes with respect to m. We denote these classes as BA(X)[m, n], and BA(Y)[m, n]for m ≡ 2 mod 3 and m ≡ 0 mod 3, respectively. The order and size of BA(X)[m, n], and BA(Y)[m, n] are given in Table 1.1.1.

(4) Perfect binary tree:

Perfect binary trees and complete binary trees. We will see that a perfect binary tree of height h has $2^{h+1} - 1$ nodes, the height is $\Theta(\ln(n))$, and the number of leaf nodes is 2^h or $(n + 1)/2$.

A perfect binary tree of height h is a binary tree where:

- All leaf nodes have the same depth, h, and
- All other nodes are full nodes.

A recursive definition of a perfect binary tree is a single node with no children is a perfect binary tree of height $h = 0$, A perfect binary tree with height $h > 0$ is a node where both sub-trees are non-overlapping perfect binary trees of height $h - 1$.

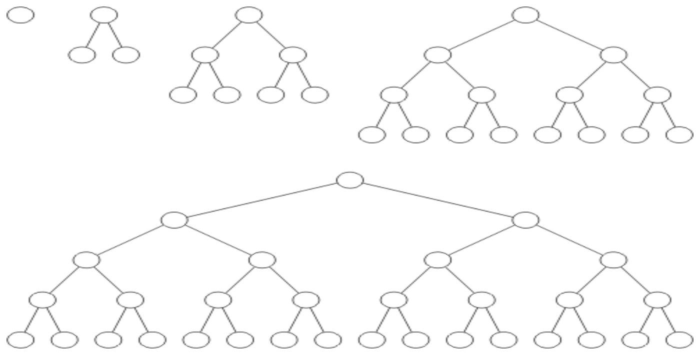


Figure 1.3.7: Perfect binary trees of height $h = 0, 1, 2, 3$, and 4.

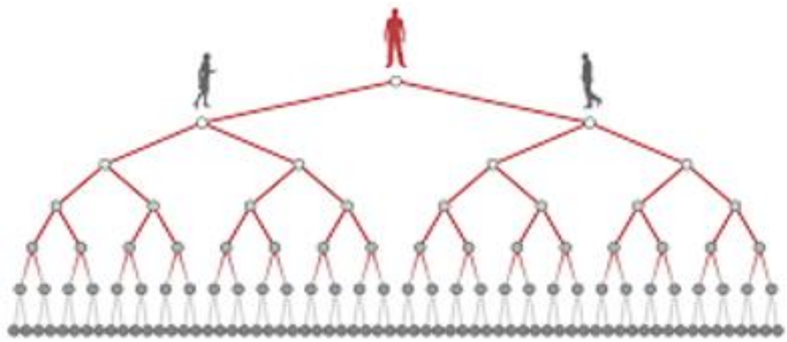


Figure 1.3.8: The general representation of Perfect binary tree graphs

The result of a perfect binary tree of height h has $2^{h+1} - 1$ nodes and perfect binary tree with n nodes has height $\lg(n + 1) - 1 = \Theta(\ln(n))$.

(5) $D_1[n]$ Nanostar Dendrimer

A vertex v of a graph G is called a cutvertex of G if its removal increases the number of components. The vertex connectivity or simply connectivity $\kappa(G)$ of a graph G is the minimum number of vertices whose removal from G results in a disconnected or trivial graph. A cutting number $c(v)$ of a vertex $v \in V(G)$ in a connected graph G is the number of pairs of vertices $\{v, w\}$ such that v and w are in different components of $G - v$.

The cutting number based on topological indices of graphs is introduced in this paper. For two - connected graphs, cutting number of each vertex is zero. So, we define these indices, for graphs with cutvertices only.

The first type of nanostar dendrimers is $D_1[n]$ shown in Fig. 1.3.9. The order and size of $D_1[n]$ nanostar dendrimers are 24+36(n-1) and 27+42(n-1), respectively.

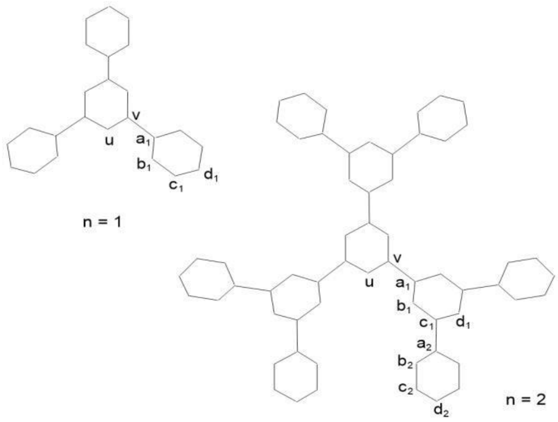


Figure 1.3.9: $D_1[n]$ with n=1 and 2

The second type of nanostar dendrimers is $D_2[n]$ and is shown in Fig. 1.3.10. The order and size of $D_2[n]$ are $120 \times 2^n - 108$ and $140 \times 2^n - 127$ nanostar dendrimers are respectively.

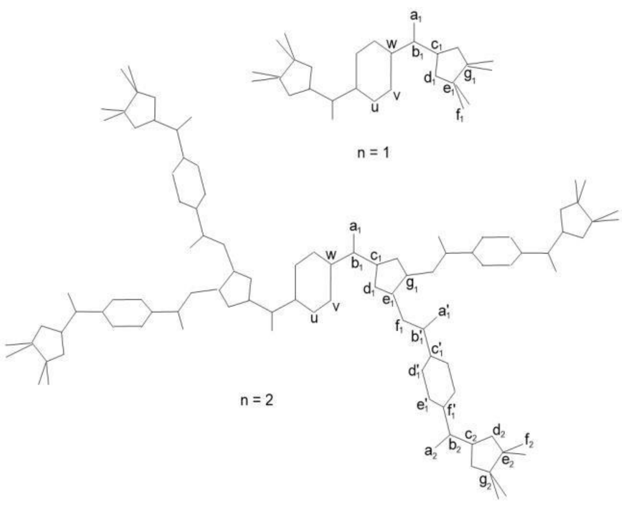


Figure 1.3.10: $D_2[n]$ with n=1 and 2

1.4 Overview of the thesis

In chapter 1 gives the preliminaries and basic definitions.

In chapter 2, we calculate the K-eccentric, K-hyper eccentric, Modified eccentric, Some Connectivity eccentric and K-eccentric types of Polynomial indices of circumcoronene series Benzenoid H_k system. Also, determined the multiplicative eccentric indices of circumcoronene series Benzenoid H_k system.

We introduce the eccentricity based on six indices namely

The connectivity eccentric index, defined as

$$\chi E(G) = \sum_{ue} \frac{1}{\sqrt{e_G(u)\, e_{L(G)}(e)}}$$

The sum and product connectivity eccentric index, defined as

$$XE(G) = \sum_{ue} \frac{1}{\sqrt{e_G(u) + e_{L(G)}(e)}} \text{ and } \chi_p E(G) = \prod_{ue} \frac{1}{\sqrt{e_G(u)\, e_{L(G)}(e)}}$$

The multiplicative sum connectivity eccentric index, defined as

$$X_p E(G) = \prod_{ue} \frac{1}{\sqrt{e_G(u) + e_{L(G)}(e)}}$$

The sum and product line connectivity eccentric index, defined as

$$SLCEII = \sum_{ue} \sqrt{\frac{e_{L(G)}(e)}{e_G(u) + e_G(v)}} \text{ and } PLCEII = \prod_{ue} \sqrt{\frac{e_{L(G)}(e)}{e_G(u) + e_G(v)}}$$

The forgotten (topological) eccentric index, defined as

$$FE(G) = \sum_{e=uv \in E(G)} [e_G(u)^2 + e_G(v)^2].$$

The K-eccentric types of polynomials, defined as

$B_1E(G, x) = \sum_{ue}[x^{e_G(u)+e_{L(G)}(e)}]$,

$B_2E(G, x) = \sum_{ue}[x^{e_G(u)\times e_{L(G)}(e)}$,

$B_3E(G, x) = \sum_{ue} x^{\left|e_{H_k}(u)-e_{L(H_k)}(e)\right|}$,

$B_4E(G, x) = \sum_{ue} x^{e_G(u)(e_G(u)+e_{L(G)}(e))}$,

$B_5E(G, x) = \sum_{ue} x^{e_{L(G)}(e)((e_G(u)+e_{L(G)}(e))}$,

$B_{a,b}E(G, x) = \sum_{ue} x^{a(e_G(u))+b(e_{L(G)}(e))}$ and

$Ɓ_{a,b}E(G, x) = \sum_{ue} x^{(e_G(u)+a)+(e_{L(G)}(e)+b)}$.

The redefine first, second and third K-eccentric indices, defined as

$ReBG_1E(G) = \sum_{ue}\frac{e_G(u)+e_{L(G)}(e)}{e_G(u)\,.\,e_{L(G)}(e)}$

$ReBG_2E(G) = \sum_{ue}\frac{e_G(u)\,.\,e_{L(G)}(e)}{e_G(u)+e_{L(G)}(e)}$

$ReBG_3E(G) = \sum_{ue}(e_G(u) + e_{L(G)}(e))(e_G(u)\,.\,e_{L(G)}(e))$

where in all the cases ue means that the vertex u and edge e are incident in G and $e_{L(G)}(e)$ is the eccentricity of e in the line graph $L(G)$ of G.

In this chapter 3, we calculate the edge version of First, Second and hyper zagreb eccentric indices and modified first and second eccentric indices. Also, we determine the edge version of multiplicative first, second and hyper zagreb eccentric indices and edge version of modified multiplicative first and second eccentric indices of Circumcronone series Benzenoid graph L(G).

We calculate the edge version of first, second and hyper zagreb indices as

$$M_1E(L(G)) = \sum_{ef\in E(L(G))}[e_{L(G)}(e) + e_{L(G)}(f)],$$

$$M_2E(L(G)) = \sum_{ef\in E(L(G))}[e_{L(G)}(e)\times e_{L(G)}(f)] \text{ and}$$

$$HM_1E(L(G)) = \sum_{ef\in E(L(G))}[e_{L(G)}(e) + e_{L(G)}(f)]^2,$$

$$HM_2E(L(G)) = \sum_{ef\in E(L(G))}[e_{L(G)}(e)\times e_{L(G)}(f)]^2 .$$

Also, we calculate the edge version of multiplicative first, second and hyper zagreb indices as

$$M\prod_1 E(L(G)) = \prod_{ef\in E(L(G))}[e_{L(G)}(e) + e_{L(G)}(f)]$$

$$M\prod_2 E(L(G)) = \prod_{ef\in E(L(G))}[e_{L(G)}(e)\times e_{L(G)}(f)] \text{ and}$$

$$HM\prod_1 E(L(G)) = \prod_{ef\in E(L(G))}[e_{L(G)}(e) + e_{L(G)}(f)]^2$$

$$HM\prod_2 E(L(G)) = \prod_{ef\in E(L(G))}[e_{L(G)}(e)\times e_{L(G)}(f)]^2 .$$

We define the edge version of modified eccentric indices as

$${}^mM_1E(L(G)) = \sum_{ef\in E(L(G))}\frac{1}{e_{L(G)}(e)+e_{L(G)}(f)},$$

$${}^mM_2E(L(G)) = \sum_{ef\in E(L(G))}\frac{1}{e_{L(G)}(e)\times e_{L(G)}(f)}$$

and the edge version of harmonic eccentric index as

$$H_bE(L(G)) = \sum_{ef\in E(L(G))}\frac{2}{e_{L(G)}(e)+e_{L(G)}(f)} .$$

Also, we define the edge version of modified multiplicative eccentric indices as

$${}^mM_1\Pi E(L(G)) = \prod_{ef\in E(L(G))}\frac{1}{e_{L(G)}(e)+e_{L(G)}(f)},$$

$${}^mM_2\Pi E(L(G)) = \prod_{ef\in E(L(G))}\frac{1}{e_{L(G)}(e)\times e_{L(G)}(f)} \text{and}$$

the edge version of multiplicative harmonic eccentric index as

$$H_b\Pi E(L(G)) = \prod_{ef \in E(L(G))} \frac{2}{e_{L(G)}(e)+e_{L(G)}(f)} \ .$$

Where in all the cases *ef* means that the edges in L(G), we have $e_{L(G)}(e)$ is the eccentricity of e in the line graph $L(G)$ of G.

If G is a (p, q) graph whose vertices have degrees d_i, then L(G) has q vertices and q_L edges, where $q_L = -q + \frac{1}{2}\sum d_i^{\ 2}$.

In chapter 4, we calculate the Milan Randic eccentric, inverse Randic eccentric, Reduced reciprocal Randic eccentric boron triangular nanotubes index, reduced second Zagreb eccentric, sum line connectivity eccentric boron triangular nanotubes index Eccentric indices of Boron Triangular Nanotubes.

We introduced the Milan Randic eccentric boron triangular nanotubes index, defined as

$$R_{-1/2}\,(\mathrm{EBTN[m,n]}) = \sum_{\boldsymbol{ue})} \frac{1}{\sqrt{e_{(G)}(\mathrm{u}) + e_{(\mathrm{G})}(v)}}.$$

For general detains about $R_{-1/2}(G)$ and its generalized Randic eccentric boron triangular nanotubes index, defined as

$$R_{\alpha}\,(\mathrm{EBTN[m,n]}) = \sum_{\boldsymbol{ue}} \frac{1}{(e_{(G)}(\mathrm{u}) + e_{(\mathrm{G})}(v))^{2}}$$

We introduced the inverse Randic eccentric boron triangular nanotubes index, defined as

$$R\mathrm{R}\alpha\,(\mathrm{EBTN[m,n]}) = \sum_{\boldsymbol{ue}} (e_{(G)}(\mathrm{u}) + e_{(\mathrm{G})}(v))^{\alpha}.$$

We introduced the reciprocal Randic eccentric boron triangular nanotubes index, defined as

$$RR(EBTN[m,n]) = \sum_{uv\in E(G)} \sqrt{e_{(G)}(u)\times e_{(G)}(v)}.$$

We introduced the Reduced reciprocal Randic eccentric boron triangular nanotubes index is defined as

$$RRR(EBTN[m,n]) = \sum_{uv\in E(G)} \sqrt{(e_{(G)}(u)-1)(e_{(G)}(v)-1)}$$

Also, we introduced two indices, defined as

$$M_1(EBTN[m,n]) = \sum_{ue}(e_{(G)}(u) + e_{(G)}(v))^{\alpha} \quad and$$

$$M_2(G) = \sum_{ue}(e_{(G)}(u)\times e_{(G)}(v))^{\alpha}.$$

We introduced the reduced second Zagreb eccentric boron triangular nanotubes index, defined as

$$RM_2(EBTN[m,n]) = \sum_{uv\in E(G)}(e_{(G)}(u)-1)(e_{(G)}(v)-1).$$

We introduced the Forgotten eccentric boron triangular nanotubes index, defined as

$$F(EBTN[m,n]) = \sum_{uv\in E(G)}(e_{(G)}(u))^2 + (e_{(G)}(v))^2).$$

We introduced the first & second modified Zagreb eccentric boron triangular nanotubes index, defined as

$${}^{m}B_1(EBTN[m,n]) = \sum_{ue}\frac{1}{e_{(G)}(u)+e_{(G)}(v)} \text{ and}$$

$${}^{m}B_2(EBTN[m,n]) = \sum_{ue}\frac{1}{e_{(G)}(u)\times e_{(G)}(v)}$$

We introduced the harmonic eccentric boron triangular nanotubes index defined as

$$\mathrm{H(EBTN[m,n])} = \sum_{\boldsymbol{uv \in EG})} \frac{2}{e_{(G)}(\mathrm{u}) + e_{(\mathrm{G})}(v)}$$

We introduced inverse sum eccentric boron triangular nanotubes index, defined as

$$\mathrm{I(EBTN[m,n])} = \sum_{\boldsymbol{ue}} \frac{e_{(G)}(\mathrm{u}) \times e_{(\mathrm{G})}(v)}{e_{(G)}(\mathrm{u}) + e_{(\mathrm{G})}(v)}$$

We introduced the augmented zagreb eccentric boron triangular nanotubes index, defined as

$$\mathrm{A(EBTN[m,n])} = \sum_{\boldsymbol{ue}} \{\frac{e_{(G)}(\mathrm{u}) \times e_{(\mathrm{G})}(v)}{e_{(G)}(\mathrm{u}) + e_{(\mathrm{G})}(v) - 2}\}.$$

We introduced the Randic connectivity eccentric index called the geometric-

arithmetic eccentric boron triangular nanotubes index, defined as

$$\mathrm{GA(EBTN[m,n])} = \sum_{\boldsymbol{ue}} \frac{2\sqrt{(e_{(G)}(\mathrm{u}) \times e_{(\mathrm{G})}(v))}}{(e_{(\mathrm{G})}(u) + e_{(\mathrm{G})}(v))}$$

We introduced the eccentricity based connectivity eccentric boron triangular nanotubes index, defined as

$$\chi\mathrm{E(EBTN[m,n])} = \sum_{ue} \frac{1}{\sqrt{e_G(u)\, e_{L(G)}(e)}}.$$

We introduced the sum connectivity eccentric boron triangular nanotubes index, defined as

$$XE\mathrm{(EBTN[m,n])} = \sum_{ue} \frac{1}{\sqrt{e_G(u) + e_{L(G)}(e)}}.$$

We introduced the sum line connectivity eccentric boron triangular nanotubes index, defined as $SLCEII$ (EBTN[m, n]) $=\sum_{ue}\sqrt{\frac{e_{L(G)}(e)}{e_G(u)+e_G(v)}}$.

In chapter 5, we calculate the Milan Randic eccentric boron-α nanotubes index, inverse Randic eccentric boron-α nanotubes index, Reduced reciprocal Randic eccentric boron-α nanotubes index, reduced second Zagreb eccentric boron-α nanotubes index and sum line connectivity eccentric boron-α nanotubes indices of boron-α Nanotubes.

We introduced the Milan Randic eccentric boron-α nanotubes index, defined as

$$R_{-1/2}(\text{EBAN}[m,n]) = \sum_{\boldsymbol{ue})}\frac{1}{\sqrt{e_{(G)}(u)+e_{(G)}(v)}}.$$

We introduced the reciprocal Randic eccentric boron-α nanotubes index, defined as

$$RR(\text{EBAN}[m,n]) = \sum_{\boldsymbol{uv}\in\boldsymbol{E}(\boldsymbol{G})}\sqrt{e_{(G)}(u)\times e_{(G)}(v)}.$$

We introduced the Reduced reciprocal Randic eccentric boron-α nanotubes index is defined as

$$RRR(\text{EBAN}[m,n]) = \sum_{\boldsymbol{uv}\in\boldsymbol{E}(\boldsymbol{G})}\sqrt{(e_{(G)}(u)-1)(e_{(G)}(v)-1)}$$

Also, we introduced two indices, defined as

$M_1(\text{EBAN}[m,n]) = \sum_{\boldsymbol{ue}}(e_{(G)}(u)+e_{(G)}(v))^{\alpha}$ *and*

$M_2(\text{EBAN}[m,n])) = \sum_{\boldsymbol{ue}}(e_{(G)}(u)\times e_{(G)}(v))^{\alpha}$.

We introduced the reduced second Zagreb eccentric boron-α nanotubes index, defined as

$$\mathrm{RM}_2(\mathrm{EBAN[m,n]}) = \sum_{\boldsymbol{uv\in E(G)}} (e_{(G)}(\mathrm{u})-1)(e_{(\mathrm{G})}(v)-1).$$

We introduced the Forgotten eccentric boron-α nanotubes index, defined as

$$\mathrm{F(EBAN[m,n])} = \sum_{\boldsymbol{uv\in E(G)}} (e_{(\mathrm{G})}(u))^2 + (e_{(\mathrm{G})}(v))^2).$$

We introduced the first & second modified Zagreb eccentric boron-α nanotubes index, defined as

$${}^{m}B_1(\mathrm{EBAN[m,n]}) = \sum_{\boldsymbol{ue}} \frac{1}{e_{(G)}(\mathrm{u}) + e_{(\mathrm{G})}(v)}$$ and

$${}^{m}B_2(\mathrm{EBAN[m,n]}) = \sum_{\boldsymbol{ue}} \frac{1}{e_{(G)}(\mathrm{u}) \times e_{(\mathrm{G})}(v)}$$

We introduced the harmonic eccentric boron-α nanotubes index defined as

$$\mathrm{H(EBAN[m,n])} = \sum_{\boldsymbol{uv\in EG)}} \frac{2}{e_{(G)}(\mathrm{u}) + e_{(\mathrm{G})}(v)}$$

In chapter 6, we calculate the first, second and hyper status indices, also the sum, product connectivity status indices, Fl-status index, first and second status Gourava indices, Gourava (a, b)-status indices. Also, calculate sum and product connectivity status indices, reciprocal connectivity status indices, ABC, AGS, GAS and ASI status indices of perfect binary tree graphs.

The first, second status connectivity indices of a graph G are introduced by Ramane et al.in [69], defined as

$$S_1(G) = \sum_{uv\in E(G)} [\sigma(u) + \sigma(v)],$$

$$S_2(G) = \sum_{uv\in E(G)} [\sigma(u)\, \sigma(v)]$$

[69] Kulli introduced the first and second hyper status indices of a graph G, defined as

$$HS_1(G) = \sum_{uv \in E(G)} [\sigma(u) + \sigma(v)]^2,$$

$$HS_2(G) = \sum_{uv \in E(G)} [\sigma(u)\, \sigma(v)]^2.$$

Also, [69] Kulli introduced the connectivity status indices as follows:

The sum connectivity status index of graph G is defined as

$$SS(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{\sigma(u)+\sigma(v)}}.$$

The product connectivity status index of graph G is defined as

$$\mathrm{PS}(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{\sigma(u)\, \sigma(v)}}.$$

The reciprocal product connectivity status index of a graph G is defined as

$$\mathrm{RPS}(G) = \sum_{uv \in E(G)} \sqrt{\sigma(u)\, \sigma(v)}.$$

The general first and second status indices of a graph G are defined as

$$S_1^{a}(G) = \sum_{uv \in E(G)} [\sigma(u) + \sigma(v)]^{a},$$

$$S_2^{a}(G) = \sum_{uv \in E(G)} [\sigma(u)\, \sigma(v)]^{a}$$

where a is a real number.

[43] Kulli introduced the connectivity status indices as follows:

The F_1-status index of a graph defined as

$$F_1S(G) = \sum_{uv \in E(G)} [\sigma(u)^2 + \sigma(v)^2]$$

first and second status Gourava indices, (a, b)-status index of a graph and also symmetric division status index defined as

$$SGO_1(G) = \sum_{uv \in E(G)} [\sigma(u) + \sigma(v) + \sigma(u)\,\sigma(v)] \text{ and}$$

$$SGO_2(G) = \sum_{uv \in E(G)} \sigma(u)\,\sigma(v)[\sigma(u) + \sigma(v)] .$$

The (a, b)-status index of a graph G is defined as

$$S_{a,b}(G) = \sum_{uv \in E(G)} [\sigma(u)^a \sigma(v)^b + \sigma(u)^b \sigma(v)^a].$$

The symmetric division status index of a graph G is defined as

$$SDS(G) = \sum_{uv \in E(G)} [\frac{\sigma(u)}{\sigma(v)} + \frac{\sigma(v)}{\sigma(u)}].$$

[57] Kulli introduced the connectivity status indices as follows:

The atom bond connectivity status index of a graph G is defined as

$$ABCS(G) = \sum_{uv \in E(G)} \sqrt{\frac{\sigma(u)+\sigma(v)-2}{\sigma(u)\,\sigma(v)}} .$$

The arithmetic-geometric status index of a graph G is defined as

$$AGS(G) = \sum_{uv \in E(G)} \frac{\sigma(u)+\sigma(v)}{2\sqrt{\sigma(u)\sigma(v)}}.$$

The geometric arithmetic status index of a connected graph G defined as

$$GAS(G) = \sum_{uv \in E(G)} \frac{2\sqrt{\sigma(u)\,\sigma(u)}}{\sigma(u) + \sigma(u)} .$$

The augmented status index of a graph G is defined as

$$ASI(G) = \sum_{uv \in E(G)} \left(\frac{\sigma(u)\sigma(v)}{\sigma(u)+\sigma(v)-2}\right)^3 .$$

The harmonic status index defined as HS(G) = $\sum_{uv\in E(G)} \frac{2}{\sigma(u)+\sigma(v)}$.

In chapter 7, we introduced the distance based topological index of a graph G is the first, second and hyper cutting number-eccentricity indices CƐ(G) of nanostar dendrimer $D_1[n]$ is defined as

$$C^{*}\mathcal{E}(G) = \sum_{u\in V(G)}[c(u) + \varepsilon(u)],$$

$$C^{**}\mathcal{E}(G) = \sum_{u\in V(G)}[c(u)\varepsilon(u)] \text{ and}$$

$$HC^{*}\mathcal{E}(G) = \sum_{u\in V(G)}[c(u)^{2} + \varepsilon(u)^{2}],$$

$$HC^{**}\mathcal{E}(G) = \sum_{u\in V(G)}[c(u)^{2}\varepsilon(u)^{2}]$$

Also, introduced the distance based topological index of a graph G is the multiplicative first, second and hyper cutting number-eccentricity indices CƐ(G) of nanostar dendrimer $D_1[n]$ and $D_2[n]$ is defined as

$$C^{*}\mathcal{E}\prod_1(G)\ ) = \prod_{u\in V(G)}[c(u) + \varepsilon(u)]$$

$$C^{**}\mathcal{E}\prod_2\ (G)\ ) = \prod_{u\in V(G)}[c(u)\varepsilon(u)]$$

$$HC^{*}\mathcal{E}\prod_1(G)\ ) = \prod_{u\in V(G)}[c(u)^{2} + \varepsilon(u)^{2}]$$

$$HC^{**}\mathcal{E}\prod_2(G)\ ) = \prod_{u\in V(G)}[c(u)^{2}\varepsilon(u)^{2}]$$

CHAPTER - 2

K-ECCENTRIC, K- HYPER ECCENTRIC, MODIFIED ECCENTRIC AND CONNECTIVITY ECCENTRIC INDICES OF CIRCUMCORONENE SERIES OF BENZENOID H_K SYSTEM

In this chapter, we calculate the K-eccentric, K-hyper eccentric, Modified eccentric, Some Connectivity eccentric and K-eccentric Polynomial indices of Circumcoronene series Benzenoid H_k system. Also, determined the multiplicative eccentric indices of Circumcoronene series Benzenoid H_k system. We introduce the eccentricity based on seven indices namely

The connectivity eccentric index, defined as

$$\chi E(G) = \sum_{ue} \frac{1}{\sqrt{e_G(u)\, e_{L(G)}(e)}} \tag{2.1}$$

The sum and product connectivity eccentric indices, defined as

$$XE(G) = \sum_{ue} \frac{1}{\sqrt{e_G(u) + e_{L(G)}(e)}} \tag{2.2}$$

and $$\chi_p E(G) = \prod_{ue} \frac{1}{\sqrt{e_G(u)\, e_{L(G)}(e)}} \tag{2.3}$$

The multiplicative sum connectivity eccentric indices, defined as

$$X_p E(G) = \prod_{ue} \frac{1}{\sqrt{e_G(u) + e_{L(G)}(e)}} \tag{2.4}$$

The sum and product line connectivity eccentric index, defined as

$$SLCEII(G) = \sum_{ue} \sqrt{\frac{e_{L(G)}(e)}{e_G(u) + e_G(v)}} \tag{2.5}$$

and $$PLCEII(G) = \prod_{ue} \sqrt{\frac{e_{L(G)}(e)}{e_G(u) + e_G(v)}} \tag{2.6}$$

The forgotten (topological) eccentric index, defined as

$$FE(G) = \sum_{e=uv\in E(G)}[e_G(u)^2 + e_G(v)^2] \tag{2.7}$$

The K-eccentric indices of types of polynomials, defined as

$$B_1E(G, x) = \sum_{ue}[x^{e_G(u)+e_{L(G)}(e)}], \tag{2.8}$$

$$B_2E(G, x) = \sum_{ue}[x^{e_G(u)\times e_{L(G)}(e)}, \tag{2.9}$$

$$B_3E(G, x) = \sum_{ue} x^{|e_{H_k}(u)-e_{L(H_k)}(e)|}, \tag{2.10}$$

$$B_4E(G, x) = \sum_{ue} x^{e_G(u)(e_G(u)+e_{L(G)}(e))}, \tag{2.11}$$

$$B_5E(G, x) = \sum_{ue} x^{e_{L(G)}(e)((e_G(u)+e_{L(G)}(e))}, \tag{2.12}$$

$$B_{a,b}E(G, x) = \sum_{ue} x^{a(e_G(u))+b(e_{L(G)}(e))} \tag{2.13}$$

and $$\bar{B}_{a,b}E(G, x) = \sum_{ue} x^{(e_G(u)+a)+(e_{L(G)}(e)+b)}. \tag{2.14}$$

The Redefine first, second and third K-eccentric indices, defined as

$$ReBG_1E(G) = \sum_{ue}\frac{e_G(u)+e_{L(G)}(e)}{e_G(u)\,.\,e_{L(G)}(e)} \tag{2.15}$$

$$ReBG_2E(G) = \sum_{ue}\frac{e_G(u)\,.\,e_{L(G)}(e)}{e_G(u)+e_{L(G)}(e)} \tag{2.16}$$

$$ReBG_3E(G) = \sum_{ue}(e_G(u) + e_{L(G)}(e))(e_G(u)\,.\,e_{L(G)}(e)) \tag{2.17}$$

where in all the cases *ue* means that the vertex *u* and edge *e* are incident in *G* and $e_{L(G)}(e)$ is the eccentricity of e in the line graph $L(G)$ of G.

2.1 K-eccentric, K- hyper eccentric and Multiplicative K-eccentric, K-hyper eccentric indices of Circumcoronene series of Benzenoid H_k system:

In this section, we calculate the first, second K-eccentric and hyper K-eccentric indices and multiplicative first, second K-eccentric and K-hyper eccentric indices of Circumcoronene series of Benzenoid H_k system. The chemical structure of Circumcoronene series of Benzenoid H_k system, are as shown in Fig 1.3.2.

Now, we shall calculate the K-eccentric and K- hyper eccentric indices of Benzenoid H_k system. We obtain a closed formula of this index for a famous molecular graph that is circumcoronene series of Benzenoid H_k. The circumcoronene homologous series of Benzenoid is family of molecular graph, which consist several copy of benzene C_6 on circumference. The first term of this series are H_1 = benzene, H_2 = coronene, H_3 = circumcoronene and H_4 = circumcircumcoronene, see fig 1.3.1 and 1.3.2, where they are shown.

Let G be a graph with vertex set V(G) and edge set E(G). The eccentricities of u, v$\in$ V(G) are denoted by e_u, e_v. For e = uv $\in$ E(G), denote the eccentricities of the end vertices of e by (e_u, e_v). Shown in the figure 1.3.2 red color indicates eccentricities of the end vertices in G and black color indicates eccentricities of the vertices in L(G). The eccentricities of u, v $\in$ V(G) are denoted by e(u), e(v) respectively and for e = uv $\in$ E(G), denote the eccentricities of the end vertices of the edge e by (e(u), e(v)).

Lemma 2.1.1: Let the first terms of this series are H_1 = benzene graph respectively, then

(i) $B_1E(H_1)$ = 72, (ii) $B_2E(H_1)$ = 108, (iii) $HB_1E(H_1)$ = 432 and

(iv) $HB_2E(H_1)$ = 972.

Proof: Consider the first terms of this series are H_1 = benzene graph.

Let V_1 be the vertex set and E_1 be the edge set in H_1 = Benzene, then

$|V_1| = 6\ and |E_1| = 6.$

Also the number of edges with eccentricities of end vertices of H_1 and $L(H_1)$ are given as follows:

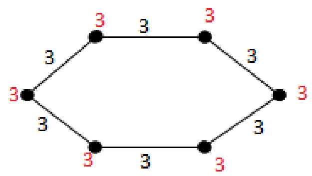


Fig.2.1.1 H_1 = Benzene

Edge set	No. of edges e = vu	Eccentricity of end vertices (e(u), e(v))	Eccentricity of e in $L(H_1)$, $e_{L(H_1)}(e)$
E_1	6	(3, 3)	3

Table 2.1.1

Let $E = \{e \in E(G) / \ e = uv,\ u, v \in V(G),\ e_{H_1}(u) = 3, e_{H_1}(v) = 3\}$ for $e \in E$, $e_{L(H_1)}(e) = 3$.

Hence we have

(i) $B_1E(H_1) = \sum_{ue}[e_{H_1}(u) + e_{L(H_1)}(e)]$

$= \sum_{e=uv\in E(H_1)}[e_{H_1}(u) + e_{L(H_1)}(e) + e_{H_1}(v) + e_{L(H_1)}(e)]$

$= 6[(3+3) + (3+3)] = 72$

(ii) $B_2E(H_1) = \sum_{ue}[e_{H_1}(u) \times e_{L(H_1)}(e)]$

$= \sum_{e=uv\in E(H_1)}[e_{H_1}(u)e_{L(H_1)}(e) + e_{H_1}(v)e_{L(H_1)}(e)]$

$= 6[(3\times3) + (3\times3)] = 108$

(iii) $HB_1E(H_1) = \sum_{ue}\left[e_{H_1}(u) + e_{L(H_1)}(e)\right]^2$

$= \sum_{e=uv\in E(H_1)}\left[[e_{H_1}(u) + e_{L(H_1)}(e)]^2 + [e_{H_1}(v) + e_{L(H_1)}(e)]^2\right]$

$= 6[(3+3)^2 + (3+3)^2] = 432$

(iv) $HB_2E(H_1) = \sum_{ue}\left[e_{H_1}(u) \times e_{L(H_1)}(e)\right]^2$

$= \sum_{e=uv\in E(H_1)}\left[[e_{H_1}(u) \times e_{L(H_1)}(e)]^2 + [e_{H_1}(v) \times e_{L(H_1)}(e)]^2\right]$

$= 6[(3\times3)^2 + (3\times3)^2] = 972$

Lemma 2.1.2: Let the second term of this series are H_2 = Coronene graph respectively, then

(i) $B_1E(H_2) = 714$, (ii) $B_2E(H_2) = 2154$, (iii) $HB_1E(H_2) = 8634$

(ii) and $HB_2E(H_2) = 82182$.

Proof: Consider the H_2 - Coronene graph.

Let V_2 be the vertex set and E_2 be the edge set in H_2 = Coronene, then $|V_2| = 24\ and |E_2| = 30$.

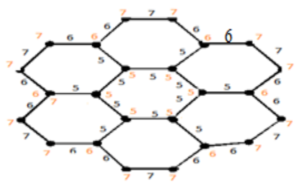


Fig.2.1.2 H_2 = Coronene

Also the number of edges with eccentricities of end vertices of H_2 and $L(H_2)$ are given as follows:

Edge set	No. of edges e = uv	Eccentricity of end vertices (e(u), e(v))	Eccentricity of e in $L(H_2)$, $e_{L(H_2)}(e)$
E_1	6	(5, 5)	5
E_2	6	(5, 6)	5
E_3	12	(6, 7)	6
E_4	6	(7, 7)	7

Table 2.1.2

Hence we have

(i) $B_1E(H_2) = \sum_{ue}\left[e_{H_2}(u) + e_{L(H_2)}(e)\right]$

$$=\sum_{e=uv\in E_1(H_2)}\left[(e_{H_2}(u) + e_{L(H_2)}(e)) + (e_{H_2}(v) + e_{L(H_2)}(e))\right]+\ldots+$$

$$\sum_{e=uv\in E_4(H_2)}\left[(e_{H_2}(u) + e_{L(H_2)}(e)) + (e_{H_2}(v) + e_{L(H_2)}(e))\right]$$

= 6[(5+5) + (5+5)] +……+ 6[(7+7) + (7+7)] = 714

(ii) $B_2E(H_2) = \sum_{ue}\left[e_{H_2}(u) \times e_{L(H_2)}(e)\right]$

$$= \sum_{e=uv\in E_1(H_2)}\left[(e_{H_2}(u)e_{L(H_2)}(e)) + (e_{H_2}(v)\, e_{L(H_2)}(e))\right]+\ldots+$$

$$\sum_{e=uv\in E_4(H_2)}\left[(e_{H_2}(u)e_{L(H_2)}(e)) + (e_{H_2}(v)e_{L(H_2)}(e))\right]$$

= 6[(5×5) + (5×5)] +……+ 6[(7×7) + (7×7)] = 2154

(iii) $HB_1E(H_2) = \sum_{ue}\left[e_{H_2}(u) + e_{L(H_2)}(e)\right]^2$

$$=\sum_{e=uv\in E_1(H_2)}\left[[e_{H_2}(u) + e_{L(H_2)}(e)]^2 + [e_{H_2}(v) + e_{L(H_2)}(e)]^2\right]+\ldots+$$

$$\sum_{e=uv\in E_4(H_2)}\left[[e_{H_2}(u) + e_{L(H_2)}(e)]^2 + [e_{H_2}(v) + e_{L(H_2)}(e)]^2\right]$$

= 6[(5+5)^2 + (5+5)^2] +……+ 6[(7+7)^2 + (7+7)^2] = 8634.

(iv) $HB_2E(H_2) = \sum_{ue}\left[e_{H_2}(u) \times e_{L(H_2)}(e)\right]^2$

$$=\sum_{e=uv\in E_1(H_2)}\left[[e_{H_2}(u) \times e_{L(H_2)}(e)]^2 + [e_{H_2}(v) \times e_{L(H_2)}(e)]^2\right] + \ldots +$$

$$\sum_{e=uv\in E_4(H_2)}\left[[e_{H_2}(u) \times e_{L(H_2)}(e)]^2 + [e_{H_2}(v) \times e_{L(H_2)}(e)]^2\right]$$

$$= 6[(5\times5)^2 + (5\times5)^2] + \ldots\ldots + 6[(7\times7)^2 + (7\times7)^2] = 82182$$

Lemma 2.1.3: Let the third term of this series are H_3 = Circumcoronene graph respectively, then

(i) $B_1E(H_3) = 2646$, (ii) $B_2E(H_3) = 12366$,

(iii) $HB_1E(H_3) = 49770$ and (iv) $HB_2E(H_3) = 1134150$

Proof: Consider the H_3 - Circumecoronene graph.

Let V_3 be the vertex set and E_3 be the edge set in H_3 = Circumcoronene, then $|V_3| = 54\ and |E_3| = 72$

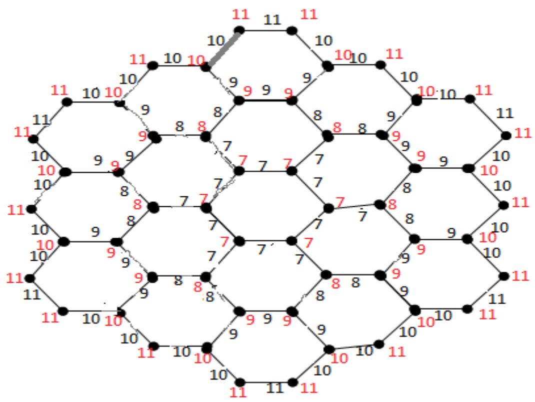


Fig.2.1.3 H_3 = Circumcoronene

Also the number of edges with eccentricities of end vertices of H_3 and $L(H_3)$ are given as follows:

Edge set	No. of edges e = uv	Eccentricity of end vertices (e(u), e(v))	Eccentricity of e in $L(H_3)$, $e_{L(H_3)}(e)$
E_1	6	(7, 7)	7
E_2	6	(7, 8)	7
E_3	12	(8, 9)	8
E_4	6	(9, 9)	9
E_5	12	(9, 10)	9
E_6	24	(10, 11)	10
E_7	6	(11, 11)	11

Table 2.1.3

Hence we have

$(i)\ B_1E(H_3) = \sum_{ue}\left[e_{H_3}(u) + e_{L(H_3)}(e)\right]$

$= \sum_{e=uv\in E_1(H_3)}\left[e_{H_3}(u) + e_{L(H_3)}(e) + e_{H_3}(v) + e_{L(H_3)}(e)\right]$+….+

$\sum_{e=uv\in E_7(H_3)}\left[e_{H_3}(u) + e_{L(H_3)}(e) + e_{H_3}(v) + e_{L(H_3)}(e)\right]$

= 6[(7+7) + (7+7)]+…..+ 6[(11+11) + (11+11)] = 2646

$(ii)\ B_2E(H_3) = \sum_{ue}\left[e_{H_3}(u) \times e_{L(H_3)}(e)\right]$

$= \sum_{e=uv\in E_1(H_3)}\left[e_{H_3}(u)e_{L(H_3)}(e) + e_{H_3}(v)e_{L(H_3)}(e)\right]$+….+

$\sum_{e=uv\in E_7(H_3)}\left[e_{H_3}(u)e_{L(H_3)}(e) + e_{H_3}(v)e_{L(H_3)}(e)\right]$

= 6[(7×7) + (7×7)]+…..+ 6[(11×11) + (11×11)] = 12366

$(iii)\ HB_1E(H_3) = \sum_{ue}\left[e_{H_3}(u) + e_{L(H_3)}(e)\right]^2$

$=\sum_{e=uv\in E_1(H_3)}\left[[e_{H_3}(u) + e_{L(H_3)}(e)]^2 + [e_{H_3}(v) + e_{L(H_3)}(e)]^2\right]$+…+

$\sum_{e=uv\in E_7(H_3)}\left[[e_{H_3}(u) + e_{L(H_3)}(e)]^2 + [e_{H_3}(v) + e_{L(H_3)}(e)]^2\right]$

= $6[(7+7)^2 + (7+7)^2]$+…..+ $6[(11+11)^2 + (11+11)^2]$ = 49770

$(iv)\ HB_2E(H_3) = \sum_{ue}\left[e_{H_3}(u) \times e_{L(H_3)}(e)\right]^2$

$= \sum_{e=uv\in E_1(H_3)}\left[[e_{H_3}(u) \times e_{L(H_3)}(e)]^2 + [e_{H_3}(v) \times e_{L(H_3)}(e)]^2\right] + \ldots +$

$\sum_{e=uv\in E_7(H_3)}\left[[e_{H_3}(u) \times e_{L(H_3)}(e)]^2 + [e_{H_3}(v) \times e_{L(H_3)}(e)]^2\right]$

$= 6[(7\times7)^2 + (7\times7)^2] + \ldots.. + 6[(11\times11)^2 + (11\times11)^2] = 1134150$

Lemma 2.1.4: Let the fourth term of this series are H_4 = Circumcircumcoronene graph respectively, then

(i) $B_1E(H_4) = 6588$, (ii) $B_2E(H_4) = 41868$, (iii) $HB_1E(H_4) = 167580$ and (iv) $HB_2E(H_4) = 7105236$

Proof: Consider the H_4-Circumcircumcoronene graph.

Let V_4 be the vertex set and E_4 be the edge set in H_4 = Circumcircumcoronene, then $|V_4| = 96\ and |E_4| = 132$.

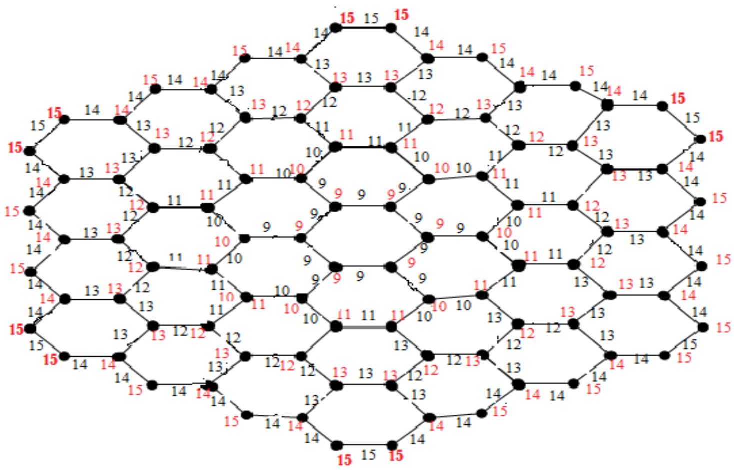


Fig.1.3.4 H_4 = Circumcircumcoronene

Also the number of edges with eccentricities of end vertices of H_4 and $L(H_4)$ are given as follows:

We have

Edge set	No. of edges $e = uv$	Eccentricity of end vertices $(e(u), e(v))$	Eccentricity of e in $L(H_4)$, $e_{L(H_4)}(e)$
E_1	6	(9, 9)	9
E_2	6	(9, 10)	9
E_3	12	(10, 11)	10
E_4	6	(11, 11)	11
E_5	12	(11, 12)	11
E_6	24	(12, 13)	12
E_7	6	(13, 13)	13
E_8	18	(13, 14)	13
E_9	36	(14. 15)	14
E_{10}	6	(15, 15)	15

Table 2.1.4

$(i)\ B_1E(H_4) = \sum_{ue}\left[e_{H_4}(u) + e_{L(H_4)}(e)\right]$

$= \sum_{e=uv\in E_1(H_4)}\left[e_{H_4}(u) + e_{L(H_4)}(e) + e_{H_4}(v) + e_{L(H_4)}(e)\right]+\ldots.+$

$\sum_{e=uv\in E_{10}(H_4)}\left[e_{H_4}(u) + e_{L(H_4)}(e) + e_{H_4}(v) + e_{L(H_4)}(e)\right]$

= 6[(9+9) + (9+9)]+……+6[(15+15) + (15+15)] = 6588

$(ii)\ B_2E(H_4) = \sum_{ue}\left[e_{H_4}(u) \times\ e_{L(H_4)}(e)\right]$

$= \sum_{e=uv\in E_1(H_4)}\left[e_{H_4}(u)e_{L(H_4)}(e) + e_{H_4}(v)e_{L(H_4)}(e)\right]+\ldots.+$

$\sum_{e=uv\in E_{10}(H_4)}\left[e_{H_4}(u)e_{L(H_4)}(e) + e_{H_4}(v)e_{L(H_4)}(e)\right]$

= 6[(9×9) + (9×9)]+……+6[(15×15) + (15×15)] = 41868

$(iii)\ HB_1E(H_4) = \sum_{ue}\left[e_{H_4}(u) + e_{L(H_4)}(e)\right]^2$

$= \sum_{e=uv\in E_1(H_4)}\left[[e_{H_4}(u) + e_{L(H_4)}(e)]^2 + [e_{H_4}(v) + e_{L(H_4)}(e)]^2\right]+\ldots+$

$\sum_{e=uv\in E_{10}(H_4)}\left[[e_{H_4}(u) + e_{L(H_4)}(e)]^2 + [e_{H_4}(v) + e_{L(H_4)}(e)]^2\right]$

$= 6[(9+9)^2 + (9+9)^2]+\ldots\ldots+6[(15+15)^2 + (15+15)^2] = 167580$

$(iv)\ HB_2E(H_4) = \sum_{ue}\left[e_{H_4}(u) \times e_{L(H_4)}(e)\right]^2$

$= \sum_{e=uv\in E_1(H_4)}\left[[e_{H_4}(u)e_{L(H_4)}(e)]^2 + [e_{H_4}(v)e_{L(H_4)}(e)]^2\right]+\ldots+$

$\sum_{e=uv\in E_{10}(H_4)}\left[[e_{H_4}(u)e_{L(H_4)}(e)]^2 + [e_{H_4}(v)e_{L(H_4)}(e)]^2\right]$

$= 6[(9\times9)^2 + (9\times9)^2]+\ldots\ldots+6[(15\times15)^2 + (15\times15)^2]$

$= 7105236.$

Theorem 2.1.5: Let H_k be the general form of circumcoronene series of benzenoid system respectively, then

(i) $B_1E(H_k) = 72k^3 - 27k^2 + 51k - 24$

(ii) $B_2E(H_k) = 216k^4 - 234k^3 + 351k^2 - 141k - 84$

(iii) $HB_1E(H_k) = 816k^4 - 1112k^3 + 789k^2 - 205k + 144$

(iv) $HB_2E(H_k) = 246k^5 - 192k^4 + 296k^3 + 490k^2 - 128k + 260$

Proof: Consider the general form of H_k - Circumcoronene series of Benzenoid system.

Let V_k be the vertex set of H_k and E_k be the edge set in H_k, then $|V_k| = 6k^2$ and $|E_k| = 9k^2 - 3k$ for the structure of H_k.

First, we shall determine the number of edges e = uv with the eccentricity of the end vertices e(u), e(v) and eccentricity of the edge e in $L(H_k)$.

Also the number of edges with eccentricities of end vertices of H_k and $L(H_k)$ are given as follows:

Edge set	No. of edges e = uv	Eccentricity of end vertices (e(u), e(v))	Eccentricity of e in L(G), $e_{L(G)}$ (e)
E_1	6	(2k+1, 2k+1)	2k+1
E_2	6	(2k+1, 2k+2)	2k+1
E_3	12	(2k+2, 2k+3)	2k+2
E_4	6	(2k+3, 2k+3)	2k+3
E_5	12	(2k+3, 2k+4)	2k+3
E_6	24	(2k+4, 2k+5)	2k+4
E_7	6	(2k+5, 2k+5)	2k+5
E_8	18	(2k+5, 2k+6)	2k+5
E_9	36	(2k+6, 2k+7)	2k+6
.	.	.	.
.	.	.	.
.	.	.	.
$E_{3(k-2)-2}$	6	(2k+2(k–2) –1, 2k+2(k–2) –1)	2k+2(k–2) –1
$E_{3(k-2)-1}$	6(k-2)	(2k+2(k–2) –1, 2k+2(k–2))	2k+2(k–2) –1
$E_{3(k-2)}$	12(k-2)	(2k+2(k–2), 2k+2(k–1) –1)	2k+2(k–2)
$E_{3(k-1)-2}$	6	(2k+2(k–1) –1, 2k+2(k–1) –1)	2k+2(k–1) –1
$E_{3(k-1)-1}$	6(k-1)	(2k+2(k–1) –1, 2k+2(k–1))	2k+2(k–1) –1
$E_{3(k-1)}$	12(k-1)	(2k+2(k–1), 2k+2(k–1)+1)	2k+2(k–1)
$E_{3(k-1)+1}$	6	(2k+2(k–1)+1, 2k+2(k–1)+1)	2k+2(k–1)+1

Table 2.1.5

We give these values in the above Table 2.1.5.

Hence we have

$$(i)\ B_1E(H_k) = \sum_{ue}\left[e_{H_k}(u) + e_{L(H_k)}(e)\right]$$

$$= \sum_{e=uv\in E_1(H_k)}\left[e_{H_k}(u) + e_{L(H_k)}(e) + e_{H_k}(v) + e_{L(H_k)}(e)\right] + \ldots +$$

$$\sum_{e=uv\in E_{3(k-1)+1}(H_k)}\left[e_{H_k}(u) + e_{L(H_k)}(e) + e_{H_k}(v) + e_{L(H_k)}(e)\right]$$

Therefore

$$= 6\sum_{i=1}^{k}[8k + 4(2i - 1)] + 6\sum_{i=1}^{k-1} i[8k + 4(2i - 1) + 1] +$$
$$12\sum_{i=1}^{k-1} i[8k + 4(2i) + 1]$$

After simplification, we get

$$B_1E(H_k) = 72k^3 – 27k^2 + 51k – 24$$

(ii) $B_2E(H_k) = \sum_{ue}\left[e_{H_k}(u) \times e_{L(H_k)}(e)\right]$

$$= \sum_{e=uv\in E_1(H_k)}\left[e_{H_k}(u)e_{L(H_k)}(e) + e_{H_k}(v)e_{L(H_k)}(e)\right] + \ldots +$$
$$\sum_{e=uv\in E_{3(k-1)+1}(H_k)}\left[e_{H_k}(u)e_{L(H_k)}(e) + e_{H_k}(v)e_{L(H_k)}(e)\right]$$

Therefore

$$= 6\sum_{i=1}^{k}[(2\,(2k + 2i - 1))^2] +$$
$$6\sum_{i=1}^{k-1} i[(2k + 2i - 1)^2 + (2k + 2i)(2k + 2i - 1)] +$$
$$12\sum_{i=1}^{k-1} i[(2k + 2i)^2 + (2k + 2i)(2k + 2i - 1)]$$

After simplification, we get

$$B_2E(H_k) = 216k^4 – 234k^3 + 351k^2 – 141k – 84$$

(iii) $HB_1E(H_k) = \sum_{ue}\left[e_{H_k}(u) + e_{L(H_k)}(e)\right]^2$

$$= \sum_{e=uv\in E_1(H_k)}\left[[e_{H_k}(u) + e_{L(H_k)}(e)]^2 + [e_{H_k}(v) + e_{L(H_k)}(e)]^2\right] + \ldots +$$
$$\sum_{e=uv\in E_{3(k-1)+1}(H_k)}\left[[e_{H_k}(u) + e_{L(H_k)}(e)]^2 + [e_{H_k}(v) + e_{L(H_k)}(e)]^2\right]$$

Therefore

$$= 6\sum_{i=1}^{k}\left[\left(2\,(2k + 2i - 1)^2\right) + \left(2\,(2k + 2i - 1)\right)^2\right] +$$
$$6\sum_{i=1}^{k-1} i[(2(2k + 2i - 1))^2 + ((2k + 2i) + (2k + 2i - 1))^2] +$$
$$12\sum_{i=1}^{k-1} i[(2(2k + 2i))^2 + ((2k + 2i + 1) + (2k + 2i))^2]$$

After simplification, we get

$$HB_1E(H_k) = 816k^4 - 1112k^3 + 789k^2 - 205k + 144$$

(iv) $HB_2E(H_k) = \sum_{ue}\left[e_{H_k}(u) \times e_{L(H_k)}(e)\right]^2$

$$= \sum_{e=uv\in E_1(H_k)}\left[[e_{H_k}(u) \times e_{L(H_k)}(e)]^2 + [e_{H_k}(v) \times e_{L(H_k)}(e)]^2\right] + .. +$$

$$\sum_{e=uv\in E_{3(k-1)+1}(H_k)}\left[[e_{H_k}(u) + e_{L(H_k)}(e)]^2 + [e_{H_k}(v) + e_{L(H_k)}(e)]^2\right]$$

Therefore

$$= 6\sum_{i=1}^{k}\left[((2k+2i-1)^2) + ((2k+2i-1)^2)^2\right] +$$

$$6\sum_{i=1}^{k-1} i[((2k+2i-1)^2)^2 + ((2k+2i)(2k+2i-1))^2] +$$

$$12\sum_{i=1}^{k-1} i[((2k+2i)^2)^2 + ((2k+2i+1)(2k+2i))^2]$$

After simplification, we get

$$HB_2E(H_k) = 986\,k^6 - 246k^5 + 192k^4 - 296k^3 + 490k^2 - 348k + 260$$

Theorem 2.1.6: For any positive integer number k, let H_k be the general form of circumcoronene series of benzenoid system, then

$$(i)\ B\prod_1 E(H_k) = 6\prod_{i=1}^{k}\left[4(2k+2i-1)^2\right] \times$$

$$6\prod_{i=1}^{k-1} i[2(2k+2i-1)(4k+4i+1) \times$$

$$12\prod_{i=1}^{k-1} i[2(2k+2i)(4k+4i+1]$$

$$(ii)\ B\prod_2 E(H_k) = 6\prod_{i=1}^{k}\left[(2k+2i-1)^4\right] \times$$

$$6\prod_{i=1}^{k-1} i[(2k+2i-1)^3\,(2k+2i) \times$$

$$12\prod_{i=1}^{k-1} i[(2k+2i)^3\,(2k+2i+1)$$

$(iii)\ HB\prod_1 \mathrm{E}(H_k) = 6\prod_{i=1}^{k}\left[16\ (2k+2i-1)^4\right] \times$

$6\prod_{i=1}^{k-1} i[\left(4(2k+2i-1)\right)^2] \times [((4k+4i-1)^2]\times$

$12\prod_{i=1}^{k-1} i[4(2k+2i)^2] + [((4k+4i+1))$

$(iv) HB\prod_2 \mathrm{E}(H_k) = 6\prod_{i=1}^{k}\left[16\ (2k+2i-1)^8\right] \times$

$6\prod_{i=1}^{k-1} i[\left((2k+2i-1)\right)^6 \times (2k+2i)] \times$

$12\prod_{i=1}^{k-1} i[\left((2k+2i)\right)^6 (2k+2i+1)]$

Proof: Consider the General form of $\mathrm{H_k}$ - circumcoronene series of Benzenoid system, we obtain the following

$(i)\ \mathrm{B}\prod_1 \mathrm{E}(H_\mathrm{k}) = \prod_{\mathrm{ue}}\left[e_{H_\mathrm{k}}(u) + e_{L(H_\mathrm{k})}(e)\right]$

$= \prod_{uv\in E_1(H_\mathrm{k})}\left[(e_{H_\mathrm{k}}(u) + e_{L(H_\mathrm{k})}(e)) \times (e_{H_\mathrm{k}}(v) + e_{L(H_\mathrm{k})}(e)\right] \times \ldots\ldots \times$

$\prod_{uv\in E_{uv\in E_{3(\mathrm{k}-1)+1}(H_\mathrm{k})}}\left[(e_{H_\mathrm{k}}(u) + e_{L(H_\mathrm{k})}(e)) \times (e_{H_\mathrm{k}}(v) + e_{L(H_\mathrm{k})}(e))\right]$

$= 6\prod_{i=1}^{k}\left[4(2k+2i-1)^2\right] \times$

$6\prod_{i=1}^{k-1} i[2(2k+2i-1)(4k+4i+1)]\times$

$12\prod_{i=1}^{k-1} i[2(2k+2i)(4k+4i+1)]$

$(ii)\ \mathrm{B}\prod_2 \mathrm{E}(H_\mathrm{k}) = \prod_{\mathrm{ue}}\left[e_{H_\mathrm{k}}(u) \times e_{L(H_\mathrm{k})}(e)\right]$

$= \prod_{uv\in E_1(H_\mathrm{k})}\left[(e_{H_\mathrm{k}}(u) \times e_{L(H_\mathrm{k})}(e)) \times (e_{H_\mathrm{k}}(v) \times e_{L(H_\mathrm{k})}(e))\right] \times \ldots\ldots \times$

$\prod_{uv\in E_{uv\in E_{3(\mathrm{k}-1)+1}(H_\mathrm{k})}}\left[(e_{H_\mathrm{k}}(u) \times e_{L(H_\mathrm{k})}(e)) \times (e_{H_\mathrm{k}}(v) \times e_{L(H_\mathrm{k})}(e))\right]$

$= 6\prod_{i=1}^{k}[(2k+2i-1)^4] \times 6\prod_{i=1}^{k-1} i[(2k+2i-1)^3\ (2k+2i)\times$

$12\prod_{i=1}^{k-1} i[(2k+2i)^3\ (2k+2i+1)]$

(iii) HB $\prod_1$ E(H_k) $= \prod_{ue}\left[e_{H_k}(u) + e_{L(H_k)}(e)\right]^2$

$$=\prod_{uv\in E_1(H_k)}\left[[e_{H_k}(u) + e_{L(H_k)}(e)]^2 \times [\, e_{H_k}(v) + e_{L(H_k)}(e)\right]^2]\times\ldots\times$$

$$\prod_{uv\in E_{uv\in E_{3(k-1)+1}(H_k)}}[[\, e_{H_k}(u) + e_{L(H_k)}(e)\,]^2 \times [\, e_{H_k}(v) + e_{L(H_k)}(e)]^2\,]$$

$$= 6\prod_{i=1}^{k}\left[16\,(2k + 2i - 1)^4\right]$$

$$6\prod_{i=1}^{k-1} i[\left(4(2k + 2i - 1)\right)^2] \times [((4k + 4i - 1)^2]\times$$

$$12\prod_{i=1}^{k-1} i[4(2k + 2i)^2]+[\left((4k + 4i + 1)\right)]$$

(iv) HB $\prod_2$ E(H_k) $= \prod_{ue}\left[e_{H_k}(u)\times e_{L(H_k)}(e)\right]^2$

$$= \prod_{uv\in E_1(H_k)}\left[[e_{H_k}(u)\times e_{L(H_k)}(e)]^2 \times [\, e_{H_k}(v)\times e_{L(H_k)}(e)]^2\right]\times\ldots\times$$

$$\prod_{uv\in E_{uv\in E_{3(k-1)+1}(H_k)}}\left[[e_{H_k}(u)\times e_{L(H_k)}(e)\right]^2 \times \left[e_{H_k}(u)\times e_{L(H_k)}(e)\right]^2\right]$$

$$= 6\prod_{i=1}^{k}[16\,(2k + 2i - 1)^8] \times$$

$$6\prod_{i=1}^{k-1} i[\left((2k + 2i - 1)\right)^6\,(2k + 2i)]\times$$

$$12\prod_{i=1}^{k-1} i[\left((2k + 2i)\right)^6 (2k + 2i + 1)]$$

Using MATLAB programme, we have calculated these indices for H_1, H_2 and H_3. Those values are given below corollaries.

Corollary 2.1.7: H_1 be the first terms of this Benzene in circumcoronene series of Benzenoid H_k system, then

(i) B $\prod_1$ E(H_1) =2176782336, (ii)B $\prod_2$ E(H_1) = 2.824295365×10^{11}

(iii) HB $\prod_1$ E(H_1) = 4.738381338×10^{18} and

(iv) HB $\prod_2$ E(H_1) = 7.976644308×10^{22}

Corollary 2.1.8: H_2 be the second terms of this Coronene in circumcoronene series of Benzenoid H_k system, then

(i) $B\prod_1 E(H_2) = 2.086352657\times10^{64}$, (ii) $B\prod_2 E(H_2) = 2.901497086\times10^{92}$,

(iii) $HB\prod_1 E(H_2) = 3.1023e^{+040}$ and (iv) $HB\prod_2 E(H_2) = 8.4187e^{+18}$

Corollary 2.1.9: H_3 be the third terms of this Circumcoronene in circumcoronene series of Benzenoid H_k system, then

(i) $B\prod_1 E(H_3) = 1.3789e^{+093}$, (ii) $B\prod_2 E(H_3) = 7.3558e^{+128}\times 1.0328e^{+234}$

(iii) $HB\prod_1 E(H_3) = 1.9013e^{+186}$ and

(iv) $HB\prod_2 E(H_3) = 5.4107e^{+257}\times 5.7517e^{+151}\times 6.7910e^{+251}\times 2.7308e^{+064}$

2.2 Modified eccentric indices, multiplicative modified eccentric indices and connectivity eccentric indices of Circumcoronene series of Benzenoid H_k system:

In this section, we calculate the modified eccentric indices, multiplicative modified eccentric indices and connectivity eccentric indices of Circumcoronene series of Benzenoid H_k system.

Theorem 2.2.1: For any positive integer number k, let H_k be the general form of circumcoronene series of Benzenoid system, then

(i) $^{m}B_1(H_k) = \frac{6}{(4k-1)} + \frac{96k^2-162k+66}{64k^2-88k+30} + \frac{192k^2-276k+84}{64k^2-56k+12}$

(ii) $^{m}B_2(H_k) = \frac{12}{(4k-1)^2} + \frac{48k^3-96k^2-60k-12}{(4k^2-4k+1)^2} + \frac{144k^3-264k^2+144k-24}{32k^4-56k^3+36k^2-10k+1}$

(iii) $H_b(H_k) = \frac{12}{(4k-1)} + \frac{192k^2-324k+132}{(64k^2-88k+30)} + \frac{384k^2-552k+168}{64k^2-56k+12}$

Proof: Consider the general form of H_k - Circumcoronene graph. We obtain the following

(i) $^{m}B_1(H_k) = \sum_{uv\in E(H_k)}[\frac{1}{e_{H_k}(u)+e_{L(H_k)}(e)}]$

$$= \sum_{uv\in E_1(H_k)}[\frac{1}{e_{H_k}(u)+e_{L(H_k)}(e)}+\frac{1}{e_{H_k}(v)+e_{L(H_k)}(e)}] +\ldots+$$

$$\sum_{uv\in E_{3(k-1)+1}(H_k)}[\frac{1}{e_{H_k}(u)+e_{L(H_k)}(e)}+\frac{1}{e_{H_k}(v)+e_{L(H_k)}(e)}]$$

Therefore

$$= 6\sum_{r=1}^{k}\left[\frac{1}{2k+2(r-1)+1}\right] + 6\sum_{r=1}^{k-1}\left[\frac{r}{4k+4r-2}+\frac{r}{4k+4r-1}\right] +$$

$$12\sum_{r=1}^{k-1}\left[\frac{r}{4k+4r}+\frac{r}{4k+4r+1}\right]$$

After simplification, we get

$$^{m}B_1(H_k) = \frac{6}{(4k-1)} + \frac{96k^2-162k+66}{64k^2-88k+30} + \frac{192k^2-276k+84}{64k^2-56k+12}$$

(ii) $^{m}B_2(H_k) = \sum_{uv\in E(H_k)}[\frac{1}{e_{H_k}(u)e_{L(H_k)}(e)}]$

$$=\sum_{uv\in E_1(H_k)}[\frac{1}{e_{H_k}(u)\times e_{L(H_k)}(e)}+\frac{1}{e_{H_k}(v)\times e_{L(H_k)}(e)}] + \cdots +$$

$$\sum_{uv\in E_{3(k-1)+1}(H_k)}[\frac{1}{e_{H_k}(u)\times e_{L(H_k)}(e)}+\frac{1}{e_{H_k}(v)\times e_{L(H_k)}(e)}]$$

Therefore

$$= 6\sum_{r=1}^{k}\left[\frac{2}{(2k+2(r-1)+1)^2}\right] +$$

$$6\sum_{r=1}^{k-1}\left[\frac{r}{(2k+2(r-1)+1)^2}+\frac{r}{(2k+2r)(2k+2(r-1)+1)}\right] +$$

$$12\sum_{r=1}^{k-1}\left[\frac{r}{(2k+2r)^2}+\frac{r}{(2k+2r+1)(2k+2r)}\right]$$

After simplification, we get

$$^{m}B_1(H_k) = \frac{12}{(4k-1)^2} + \frac{48k^3-96k^2-60k-12}{(4k^2-4k+1)^2} + \frac{144k^3-264k^2+144k-24}{32k^4-56k^3+36k^2-10k+1}$$

(iii) $H_b(H_k) = \sum_{uv\in E(H_k)}[\frac{2}{e_{H_k}(u)+e_{L(H_k)}(e)}]$

$$= \sum_{uv\in E_1(H_k)}[\frac{2}{e_{H_k}(u)+e_{L(H_k)}(e)} + \frac{2}{e_{H_k}(v)+e_{L(H_k)}(e)}] +$$

$$\sum_{uv\in E_{3(k-1)+1}(H_k)}[\frac{2}{e_{H_k}(u)+e_{L(H_k)}(e)} + \frac{2}{e_{H_k}(v)+e_{L(H_k)}(e)}$$

Therefore

$$= 12\sum_{r=1}^{k}\left[\frac{1}{2k+2(r-1)+1}\right] + 12\sum_{r=1}^{k-1}\left[\frac{r}{4k+4r-2} + \frac{r}{4k+4r-1}\right] +$$

$$24\sum_{r=1}^{k-1}\left[\frac{r}{4k+4r} + \frac{r}{4k+4r+1}\right]$$

After simplification, we get

$$H_b(H_k) = \frac{12}{(4k-1)} + \frac{192k^2-324k+132}{(64k^2-88k+30)} + \frac{384k^2-552k+168}{64k^2-56k+12}$$

For example,

If k = 1 then $^{m}B_1(H_1) = 2$, $^{m}B_2(H_1) = 1.3333$, $H_b(H_1) = 4$.

If k = 2 then $^{m}B_1(H_2) = 5.1257$, $^{m}B_2(H_2) = 1.7839$, $H_b(H_2) = 10.2513$.

If k = 3 then $^{m}B_1(H_3) = 7.9948$, $^{m}B_2(H_3) = 1.8156$, $H_b(H_3) = 15.9896$ and so on.

Theorem 2.2.2: For any positive integer number k, let H_k be the general form of circumcoronene series of Benzenoid system, then

(i) $^{m}B_1\Pi(H_k) = 6\prod_{r=1}^{k}[\frac{1}{4(2k+2r-1)^2}] \times 6\prod_{r=1}^{k-1} r^2 [\frac{1}{(4k+4r-1)^2}] \times$

$$12\prod_{r=1}^{k-1} r^2 [\frac{1}{(4k+4r)^2} \times \frac{1}{4k+4r+1}]$$

(ii) $\quad {}^{m}B_2\Pi(H_k) = 6\prod_{r=1}^{k}[\frac{1}{(2k+2r-1)^4}] \times$

$$6\prod_{r=1}^{k-1} r^2\ [\frac{1}{(2k+2r-1)^2}\times\frac{1}{(2k+2r)(2k+2r-1)}] \times$$

$$12\prod_{r=1}^{k-1} r^2\,[\frac{1}{(2k+2r)^2}\times\frac{1}{(2k+(2r+1))(2k+2r)}]$$

(iii) $\quad H_b\Pi(H_k) = 12\prod_{r=1}^{k}[\frac{1}{4(2k+2r-1)^2}] \times 12\prod_{r=1}^{k-1} r^2\ [\frac{1}{(4k+4r-1)^2}] \times$

$$24\prod_{r=1}^{k-1} r^2\,[\frac{1}{2(2k+2r)^2}\times\frac{1}{4k+4r+1}]$$

Proof: Consider the general form of H_k - Circumcoronene graph. We obtain the following

(i) $\quad {}^{m}B_1\Pi(H_k) = \prod_{uv\in E(H_k)}[\frac{\mathbf{1}}{e_{H_k}(u)+e_{L(H_k)}(e)}]$

$$= \prod_{\boldsymbol{uv}\in E_1(H_k)}[\frac{\mathbf{1}}{e_{H_k}(u)e_{L(H_k)}(e)} + \frac{\mathbf{1}}{e_{H_k}(v)e_{L(H_k)}(e)}] \times \mathbf{\ldots}\times$$

$$\prod_{\boldsymbol{uv}\in E_{3(k-1)+1}(H_k)}[\frac{\mathbf{1}}{e_{H_k}(u)e_{L(H_k)}(e)} + \frac{\mathbf{1}}{e_{H_k}(v)e_{L(H_k)}(e)}]$$

$$= 6\prod_{r=1}^{k}[\frac{1}{4(2k+2r-1)^2}] \times 6\prod_{r=1}^{k-1} r^2\ [\frac{1}{(4k+4r-1)^2}] \times$$

$$12\prod_{r=1}^{k-1} r^2\,[\frac{1}{2(2k+2r)^2}\times\frac{1}{4k+4r+1}]$$

(ii) $\quad {}^{m}B_2\Pi(H_k) = \prod_{uv\in E(H_k)}[\frac{\mathbf{1}}{e_{H_k}(u)\times e_{L(H_k)}(e)}]$

$$= \prod_{\boldsymbol{uv}\in E_1(H_k)}[\frac{\mathbf{1}}{e_{H_k}(u)e_{L(H_k)}(e)} \times \frac{\mathbf{1}}{e_{H_k}(v)e_{L(H_k)}(e)}]\times\mathbf{\ldots}\times$$

$$\prod_{\boldsymbol{uv}\in E_{3(k-1)+1}(H_k)}[\frac{\mathbf{1}}{e_{H_k}(u)e_{L(H_k)}(e)} \times \frac{\mathbf{1}}{e_{H_k}(v)e_{L(H_k)}(e)}]$$

$$= 6\prod_{r=1}^{k}[\frac{1}{(2k+2r-1)^4}] \times$$

$$6\prod_{r=1}^{k-1} r^2\ [\frac{1}{(2k+2r-1)^2}\times\frac{1}{(2k+2r)(2k+2r-1)}] \times$$

$$12\prod_{r=1}^{k-1} r^2\,[\frac{1}{(2k+2r)^2}\times\frac{1}{(2k+(2r+1))(2k+2r)}]$$

(iv) $H_b\Pi(H_k) = \prod_{uv\in E(H_k)}[\frac{2}{e_{H_k}(u)\times e_{L(H_k)}(e)}]$

$$= \prod_{uv\in E_1(H_k)}[\frac{2}{e_{H_k}(u)e_{L(H_k)}(e)} + \frac{2}{e_{H_k}(v)e_{L(H_k)}(e)}] \times \ldots\times$$

$$\prod_{uv\in E_{3(k-1)+1}(H_k)}[\frac{2}{e_{H_k}(u)e_{L(H_k)}(e)} + \frac{2}{e_{H_k}(v)e_{L(H_k)}(e)}]$$

$$= 12\prod_{r=1}^{k}[\frac{1}{4(2k+2r-1)^2}] \times 12\prod_{r=1}^{k-1} r^2\ [\frac{1}{(4k+4r-1)^2}] \times$$

$$24\prod_{r=1}^{k-1} r^2\,[\frac{1}{2(2k+2r)^2} + \frac{1}{4k+4r+1}]$$

For example,

If k = 1 then ${}^{m}B_1\Pi(H_1) = 0.1667$, ${}^{m}B_2\Pi(H_1) = 0.0741$ and

$H_b\Pi(H_1) = 0.3333$.

If k = 2 then ${}^{m}B_1\Pi(H_2) = 7.7066e^{-006}$, ${}^{m}B_2\Pi(H_2) = 1.5232e^{-009}$ and

$H_b\Pi(H_2) = 1.2331e^{-004}$.

If k = 3 then ${}^{m}B_1\Pi(H_3) = 1.7761e^{-011}$, ${}^{m}B_2\Pi(H_3) = 1.9153e^{-020}$ and

$H_b\Pi(H_3) = 2.2734e^{-009}$ so on.

Theorem 2.2.3: Let G be a Circumcoronene Series of Benzenoid H_k ($k \geq 1$). The connectivity eccentric index of G is given by

$$\chi E(H_k) = \left[\frac{12}{4k-1}\right] + \left[\frac{((6k-6)\sqrt{16k^2-20k+6}\,)+4k^2-7k+3}{(4k-3)\sqrt{16k^2-20k+6}}\right] +$$

$$\left[\frac{((12k-12)\sqrt{16k^2-12k+2}\,)+4k^2-6k+2}{(4k-2)\sqrt{16k^2-12k+2}}\right]$$

Proof: Let G be a Circumcoronene Series of Benzenoid H_k, By using the definition of connectivity eccentric index, Hence we have,

$$\chi E(H_k) = \sum_{ue} \frac{1}{\sqrt{e_G(u)e_{L(\mathrm{G})}(e)}}$$

$$= \sum_{uv\in E(H_k)}[\frac{1}{\sqrt{e_{H_\mathrm{k}}(u)e_{L(H_k)}(e)}}+\frac{1}{\sqrt{e_{H_\mathrm{k}}(v)e_{L(H_k)}(e)}}]$$

$$= \sum_{uv\in E_1(H_k)}[\frac{1}{\sqrt{e_{H_\mathrm{k}}(u)e_{L(H_k)}(e)}} + \frac{1}{\sqrt{e_{H_\mathrm{k}}(v)e_{L(H_k)}(e)}}]+\ldots\ldots+$$

$$\sum_{uv\in E_{3(\mathrm{k}-1)+1}(H_k)}[\frac{1}{\sqrt{e_{H_\mathrm{k}}(u)e_{L(H_k)}(e)}} + \frac{1}{\sqrt{e_{H_\mathrm{k}}(v)e_{L(H_k)}(e)}}]$$

$$= [6(\frac{1}{4k^2+4k+1} + \frac{1}{\sqrt{4k^2+4k+1}}) + 6(\frac{1}{\sqrt{4k^2+4k+1}} + \frac{1}{\sqrt{4k^2+6k+2}})+\ldots+$$

$$6(\frac{1}{\sqrt{(4k-1)(4k-1)}} + \frac{1}{\sqrt{(4k-1)(4k-1)}})]$$

Therefore

$$= 6\sum_{r=1}^{k} \left[\frac{2}{2k+2r-1}\right] + 6\sum_{r=1}^{k-1} \left[\frac{r}{2k+2r-1} + \frac{r}{\sqrt{(2k+2r)(2k+2r-1)}}\right] +$$

$$12\sum_{r=1}^{k-1} \left[\frac{r}{2k+2r} + \frac{r}{\sqrt{(2k+2r+1)(2k+2r)}}\right]$$

After simplification, we get

$$\chi E(H_\mathrm{k}) = \left[\frac{12}{4k-1}\right]+ \left[\frac{((6k-6)\sqrt{16k^2-20k+6}\,) +4k^2 -7k+3}{(4k-3)\sqrt{16k^2-20k+6}}\right]+$$

$$\left[\frac{((12k-12)\sqrt{16k^2-12k+2}\,)+4k^2-6k+2}{(4k-2)\sqrt{16k^2-12k+2}}\right]$$

Corollary 2.2.4: Eccentric based connectivity index of H_1, H_2 and H_3 are given by $\chi E(H_1) = 4$, $\chi E(H_2) = 10.2614$ and $\chi E(H_3) = 15.998$.

Theorem 2.2.5: Let G be a Circumcoronene Series of Benzenoid H_k ($k \geq 1$). The product connectivity eccentric index of G is given by

$$\chi_p E(H_k) = 6[\prod_{r=1}^{k}[\frac{1}{(2k+1)}]^2 \times \prod_{r=1}^{k-1}[\frac{r}{(4k+4r-1)} \times \frac{r}{\sqrt{(2k+2r)(2k+2r-1)}}] \times$$

$$12\prod_{r=1}^{k-1}[\frac{r}{(2k+2r)} \times \frac{r}{\sqrt{(2k+2r+1)^2}}]]$$

Proof: Let G be a Circumcoronene Series of Benzenoid H_k, By using the definition of multiplicative product connectivity eccentric index,

Hence we have,

$$\chi_p E(H_k) = \prod_{ue} \frac{1}{\sqrt{e_{H_k}(u)\, e_{L(H_k)}(e)}}$$

$$= \prod_{uv\in E(H_k)}[\frac{1}{\sqrt{e_{H_k}(u)e_{L(H_k)}(e)}} \times \frac{1}{\sqrt{e_{H_k}(v)e_{L(H_k)}(e)}}]$$

$$= \prod_{uv\in E_1(H_k)}[\frac{1}{\sqrt{e_{H_k}(u)e_{L(H_k)}(e)}} \times \frac{1}{\sqrt{e_{H_k}(v)e_{L(H_k)}(e)}}] \times \ldots\ldots \times$$

$$\prod_{uv\in E_{3(k-1)+1}(H_k)}[\frac{1}{\sqrt{e_{H_k}(u)e_{L(H_k)}(e)}} \times \frac{1}{\sqrt{e_{H_k}(v)e_{L(H_k)}(e)}}]$$

$$= 6[\prod_{r=1}^{k}[\frac{1}{(2k+1)}]^2 \times \prod_{r=1}^{k-1}[\frac{r}{(4k+4r-1)} \times \frac{r}{\sqrt{(2k+2r)(2k+2r-1)}}] \times$$

$$12\prod_{r=1}^{k-1}[\frac{r}{(2k+2r)} \times \frac{r}{\sqrt{(2k+2r+1)^2}}]]$$

Corollary 2.2.6: Eccentric based connectivity index of H_1, H_2 and H_3 are given by $\chi_p E(H_1) = 0.6667$, $\chi_p E(H_2) = 0.0020$ and $\chi_p E(H_3) = 2.9289e^{-007}$.

Theorem 2.2.7: Let G be a Circumcoronene Series of Benzenoid H_k ($k \geq 1$). The sum connectivity eccentric index of G is given by

$$XE(H_k) = 6\sum_{r=1}^{k}\left[\frac{1}{2\sqrt{(2k+2r-1)}}\right] + 6\sum_{r=1}^{k-1}\left[\frac{r}{\sqrt{4k+4r-2}} + \frac{r}{\sqrt{4k+4r-1}}\right] +$$

$$12\sum_{r=1}^{k-1}\left[\frac{r}{\sqrt{4k+4r}} + \frac{r}{\sqrt{4k+4r+1}}\right]$$

Proof: Let G be a Circumcoronene Series of Benzenoid H_k, By using the definition of sum connectivity eccentric index,

Hence we have,

$$XE(H_k) = \sum_{ue} \frac{1}{\sqrt{e_{H_k}(u) + e_{L(H_k)}(e)}}$$

$$= \sum_{uv\in E_1(H_k)}\left[\frac{1}{\sqrt{e_{H_k}(u)+e_{L(H_k)}(e)}} + \frac{1}{\sqrt{e_{H_k}(v)+e_{L(H_k)}(e)}}\right] + \cdots +$$

$$\sum_{uv\in E_{3(k-1)+1}(H_k)}\left[\frac{1}{\sqrt{e_{H_k}(u)+e_{L(H_k)}(e)}} + \frac{1}{\sqrt{e_{H_k}(v)+e_{L(H_k)}(e)}}\right]$$

Therefore

$$XE(H_k) = 6\sum_{r=1}^{k}\left[\frac{1}{2\sqrt{(2k+2r-1)}}\right] + 6\sum_{r=1}^{k-1}\left[\frac{r}{\sqrt{4k+4r-2}} + \frac{r}{\sqrt{4k+4r-1}}\right] +$$

$$12\sum_{r=1}^{k-1}\left[\frac{r}{\sqrt{4k+4r}} + \frac{r}{\sqrt{4k+4r+1}}\right]$$

Corollary 2.2.8: Eccentric based connectivity index of H_1, H_2 and H_3 are given by $XE(H_1) = 4.8990$, $XE(H_2) = 17.5006$ and $XE(H_3) = 30.1729$.

Theorem 2.2.9: Let G be a Circumcoronene Series of Benzenoid H_k ($k \geq 1$). The sum connectivity eccentric index of G is given by

$$X_p E(H_k) = 6\left[\prod_{r=1}^{k}\left[\frac{2}{(2k+2r-1)}\right] \times \prod_{r=1}^{k-1}\left[\frac{r}{(2k+2r-1)} + \frac{r}{\sqrt{(2k+2r)(2k+2r-1)}}\right] \times\right.$$

$$\left.2\prod_{r=1}^{k-1}\left[\frac{r}{(2k+2r)} + \frac{r}{\sqrt{(2k+2r+1)^2}}\right]\right]$$

Proof: Let G be a Circumcoronene Series of Benzenoid H_k. By using the definition of multiplicative sum connectivity eccentric index, we have,

$$X_p E(H_k) = \prod_{ue} \frac{1}{\sqrt{e_{H_k}(u) + e_{L(H_k)}(e)}}$$

$$= \prod_{uv\in E(H_k)}\left[\frac{1}{\sqrt{e_{H_k}(u)+e_{L(H_k)}(e)}} \times \frac{1}{\sqrt{e_{H_k}(v)+e_{L(H_k)}(e)}}\right]$$

$$= \prod_{uv\in E_1(H_k)}\left[\frac{1}{\sqrt{e_{H_k}(u)+e_{L(H_k)}(e)}} \times \frac{1}{\sqrt{e_{H_k}(v)+e_{L(H_k)}(e)}}\right] \times \ldots . \times$$

$$\prod_{uv\in E_{3(k-1)+1}(H_k)}\left[\frac{1}{\sqrt{e_{H_k}(u)+e_{L(H_k)}(e)}} \times \frac{1}{\sqrt{e_{H_k}(v)+e_{L(H_k)}(e)}}\right]$$

Therefore

$$= 6\left[\prod_{r=1}^{k}\left[\frac{2}{(2k+2r-1)}\right] \times \prod_{r=1}^{k-1} \left[\frac{r}{(2k+2r-1)} + \frac{r}{\sqrt{(2k+2r)(2k+2r-1)}}\right]\right.$$
$$\left. \times\, 2 \prod_{r=1}^{k-1} \left[\frac{r}{(2k+2r)} + \frac{r}{\sqrt{(2k+2r+1)^2}}\right] \right]$$

Corollary 2.2.10: Eccentric based multiplicative sum connectivity index of H_1, H_2 and H_3 are given by $X_p E(H_1) = 4$, $X_p E(H_2) = 36.\ 3753$ and $X_p E(H_3) = 146.8437$.

Theorem 2.2.11: Let G be a Circumcoronene Series of Benzenoid H_k ($k \geq 1$). The sum line connectivity eccentric index of G is given by

$$SLCEII(H_k) = 6\sum_{r=1}^{k}\left[\frac{1}{2}\right] + 6 \sum_{r=1}^{k-1} r\left[\sqrt{\frac{2k+2r-1}{4k+4r-1}}\right] + 12\sum_{r=1}^{k-1} r\left[\sqrt{\frac{2k+2r}{4k+4r-1}}\right]$$

Proof: Let G be a Circumcoronene Series of Benzenoid H_k, By using the definition of sum line connectivity eccentric index, we have,

$$SLCEII(H_k) = \sum_{uv\in E(H_k)}\sqrt{\frac{e_{L(H_k)}(e)}{e_{H_k}(u)+e_{H_k}(v)}}$$

$$= 6\sqrt{\frac{(2k+1)}{4k^2+4k+1}} + 6\sqrt{\frac{(2k+1)}{4k^2+4k+1}} + \ldots\ldots + 6\sqrt{\frac{1}{(4k-1)}}$$

Therefore

$$= 6\sum_{r=1}^{k}\left[\sqrt{\frac{1}{2}}\right] + 6 \sum_{r=1}^{k-1} r\left[\sqrt{\frac{2k+2r-1}{4k+4r-1}}\right] +$$

$$12\sum_{r=1}^{k-1} r\left[\sqrt{\frac{2k+2r}{4k+4r-1}}\right]$$

Corollary 2.2.12: Eccentric based connectivity index of H_1, H_2 and H_3 are given by SLCEII(H_1) = 4.2426, SLCEII(H_2) = 20.6829 and SLCEII(H_3) = 49.8749 .

Theorem 2.2.13: Let G be a Circumcoronene Series of Benzenoid H_k ($k \geq 1$). The product line connectivity eccentric index of G is given by

$$PLCEII(H_k) = 6\prod_{r=1}^{k}\left[\sqrt{\frac{1}{2}}\right] \times 6\prod_{r=1}^{k-1} r\left[\sqrt{\frac{2k+2r-1}{4k+4r-1}}\right] \times 12\prod_{r=1}^{k-1} r\left[\sqrt{\frac{2k+2r}{4k+4r-1}}\right]$$

Proof: Let G be a Circumcoronene Series of Benzenoid H_k, By using the definition of product line connectivity eccentric index, we have,

$$PLCEII(H_k) = \prod_{uv \in E(H_k)} \sqrt{\frac{e_{L(H_k)}(e)}{e_{H_k}(u) + e_{H_k}(v)}}$$

$$= 6\sqrt{\frac{(2k+1)}{(4k+2)}} \times 6\sqrt{\frac{(2k+1)}{(4k+3)}} \times \ldots\ldots \times 6\sqrt{\frac{1}{(4k-1)}}$$

$$= 6\prod_{r=1}^{k}\left[\sqrt{\frac{1}{2}}\right] \times 6\prod_{r=1}^{k-1} r\left[\sqrt{\frac{2k+2r-1}{4k+4r-1}}\right] \times 12\prod_{r=1}^{k-1} r\left[\sqrt{\frac{2k+2r}{4k+4r-1}}\right]$$

Corollary 2.2.14: Eccentric based connectivity index of H_1, H_2 and H_3 are given by PLCEII(H_1) = 4.2426, PLCEII(H_2) = 593.6051and PLCEII(H_3) = $3.5245e^{+005}$

2.3 K-eccentric types of Polynomial, Redefine eccentric indices and Multiplicative of K-eccentric types of polynomial indices of Benzenoid H_k system:

In this section, we calculate the eccentricity based on forgotten (topological) eccentric index, K-eccentric types of polynomials and redefine first, second and third K-eccentric indices and multiplicative K-eccentric polynomials indices of Circumcoronene series of Benzenoid H_k system.

Theorem 2.3.1: For any positive integer number k, let H_k be the general form of circumcoronene series of Benzenoid system of G, then

(i) $B_1E(H_k, x) = 6\sum_{i=1}^{k} x^{8k + 4(2i - 1)} + 6\sum_{i=1}^{k-1} x^{i[8k+4(2i-1)+1]} + 12\sum_{i=1}^{k-1} x^{i[8k+4(2i)+1]}$

(ii) $B_2E(H_k, x) = 6\sum_{i=1}^{k} x^{2(2k+2i-1)^2} + 6\sum_{i=1}^{k-1} x^{i[(2k+2i-1)^2+(2k+2i)(2k+2i-1)]} +$

$12\sum_{i=1}^{k-1} x^{i[(2k+2i)^2+(2k+2i)(2k+2i-1)]}$

(iii) $HB_1E(H_k,x) = 6\sum_{i=1}^{k} x^{2(2k+2i-1)^2+(2(2k+2i-1))^2} +$

$6\sum_{i=1}^{k-1} x^{i[(2(2k+2i-1)^2+((2k+2i)+(2k+2i-1))^2]} +$

$12\sum_{i=1}^{k-1} x^{i[(2(2k+2i))^2+((2k+2i+1)+(2k+2i))^2]}$

(iv) $HB_2E(H_k,x) = 6\sum_{i=1}^{k} x^{(2k+2i-1)^{2})^2+((2k+2i-2)^2)^2} +$

$6\sum_{i=1}^{k-1} x^{i[(2(2k+2i-1)^2)^2+((2k+2i)+(2k+2i-1))^2]} +$

$12\sum_{i=1}^{k-1} x^{i[((2k+2i)^2)^2+((2k+2i+1)(2k+2i))^2]}$

Proof: Consider the General form of H_k - Circumcoronene series of Benzenoid system.

Hence we have

(i) $B_1E(H_k, x) = \sum_{ue} x^{e_{H_k}(u)+e_{L(H_k)}(e)}$

$= \sum_{uv\in E(H_k)} x^{[e_{H_k}(u)+e_{L(H_k)}(e)]+[e_{H_k}(v)+e_{L(H_k)}(e)]}$

$= 6\sum_{i=1}^{k} x^{8k + 4(2i - 1)} + 6\sum_{i=1}^{k-1} x^{i[8k+4(2i-1)+1]} + 12\sum_{i=1}^{k-1} x^{i[8k+4(2i)+1]}$

(ii) $B_2E(H_k, x) = \sum_{ue} x^{e_{H_k}(u)\times e_{L(H_k)}(e)}$

$= \sum_{uv\in E(H_k)} x^{[e_{H_k}(u)+e_{L(H_k)}(e)]\times [e_{H_k}(v)+e_{L(H_k)}(e)]}$

$= 6\sum_{i=1}^{k} x^{2(2k+2i-1)^2} + 6\sum_{i=1}^{k-1} x^{i[(2k+2i-1)^2+(2k+2i)(2k+2i-1)]} +$

$12\sum_{i=1}^{k-1} x^{i[(2k+2i)^2+(2k+2i)(2k+2i-1)]}$

(iii) $HB_1E(H_k, x) = \sum_{ue} x^{[e_{H_k}(u)+e_{L(H_k)}(e)]^2}$

$$= \sum_{uv\in E(H_k)} x^{[e_{H_k}(u)+e_{L(H_k)}(e)]^2 + [e_{H_k}(v)+e_{L(H_k)}(e)]^2}$$

$$= 6\sum_{i=1}^{k} x^{2(2k+2i-1)^2+(2(2k+2i-1))^2} + 6\sum_{i=1}^{k-1} x^{i[(2(2k+2i-1)^2+((2k+2i)+(2k+2i-1))^2]} +$$

$$12\sum_{i=1}^{k-1} x^{i[(2(2k+2i))^2+((2k+2i+1)+(2k+2i))^2]}$$

(iv) $HB_2E(H_k, x) = \sum_{ue} x^{[e_{H_k}(u)\times e_{L(H_k)}(e)]^2}$

$$= \sum_{uv\in E(H_k)} x^{[e_{H_k}(u)+e_{L(H_k)}(e)]^2\times [e_{H_k}(v)+e_{L(H_k)}(e)]^2}$$

$$= 6\sum_{i=1}^{k} x^{(2k+2i-1)^2)^2+((2k+2i-2)^2)^2} + 6\sum_{i=1}^{k-1} x^{i[(2(2k+2i-1)^2)^2+((2k+2i)+(2k+2i-1))^2]} +$$

$$12\sum_{i=1}^{k-1} x^{i[((2k+2i)^2)^2+((2k+2i+1)(2k+2i))^2]}$$

Corollary 2.3.2: H_1 be the first terms of this Benzene in circumcoronene series of Benzenoid H_k system, then

(i) $B_1E(H_1, x) = 6x^{12}$,

(ii) $B_2E(H_1, x) = 6x^{36}$,

(iii) $HB_1E(H_1, x) = 6x^{72}$ and

(iv) $HB_2E(H_1, x) = 6x^{1296}$.

Corollary 2.3.3: H_2 be the second terms of this Coronene in circumcoronene series of Benzenoid H_k system, then (i) $B_1E(H_2, x) = 6x^{69}+12x^{25}$, (ii) $B_2E(H_2, x) = 6x^{406}+12x^{156}$, (iii) $HB_1E(H_2, x) = 6x^{813}+12x^{313}$ and (iv) $HB_2E(H_2) = 6x^{60516}+12x^{24336}$

Corollary 2.3.4: H_3 be the third terms of this Circumcoronene in circumcoronene series of Benzenoid H_k system, then

(i) $B_1E(H_3) = 6x^{137}+12x^{70}+24x^{41}$, (ii) $B_2E(H_3) = 6x^{1214}+12x^{614}+24x^{420}$,

(iii) $HB_1E(H_3) = 6x^{2429}+12x^{1231}+24x^{841}$ and

(iv) $HB_2E(H_3) = 6x^{421748}+12x^{190948}+24x^{176400}$

Theorem 2.3.5: For any positive integer number k, let H_k be the general form of circumcoronene series of Benzenoid system, then

(i) $B_3E(H_k, x) = 18i$

(ii) $B_3E(H_k, x) = x^{0-6i+24i}$

(iii) $B_4E(H_k, x) = 6\sum_{i=1}^{k} x^{2(2k+2(i-1)+1)^2} + 6\sum_{i=1}^{k-1} x^{i[(2k+2(i-1)(4k+4i-1)]} + 12\sum_{i=1}^{k-1} x^{i[(2k+2i)(4k+4i+1)]}$

(iv) $B_5E(H_k, x) = 6\sum_{i=1}^{k} x^{2(2k+2(i-1)+1)^2} + 6\sum_{i=1}^{k-1} x^{i[(2k+2i)[2(2k+2(i-1)+1]} + 12\sum_{i=1}^{k-1} x^{i[(2k+2i+1)(2(2k+2r)]}$

(v) $B_{a,b}E(H_k, x) = 6\sum_{i=1}^{k} x^{a(4k+4i-2)+b(4k+4i-2)} + 6\sum_{i=1}^{k-1} x^{i[a(4k+4i-2)+b(4k+4i-1)]} + 12\sum_{i=1}^{k-1} x^{i[a(4k+4i)+b(4k+4i+1)]}$

(vi) $B_{a,b}E(H_k, x) = 6\sum_{i=1}^{k} x^{[(4k+4i-2)+a]+[(4k+4i-2)+b]} + 6\sum_{i=1}^{k-1} x^{i[(4k+4i-2)+a]+[(4k+4i-1)+b]} + 12\sum_{i=1}^{k-1} x^{i[(4k+4i)+a)+(4k+4i+1)+b)]}$

Proof: Consider the General form of circumcoronene series of Benzenoid H_k system.

(i) $B_3E(H_k, x) = \sum_{ue}[e_{H_k}(u) - e_{L(H_k)}(e)]$

$= \sum_{uv\in E(H_k)}\big[(e_{H_k}(u) - e_{L(H_k)}(e)) - (e_{H_k}(v) - e_{L(H_k)}(e))\big]$

$= \sum_{uv\in E_1(H_k)}\big[(e_{H_k}(u) - e_{L(H_k)}(e)) - ((e_{H_k}(v) - e_{L(H_k)}(e))\big] + \ldots +$

$\sum_{uv\in E_{3(k-1)+1}(H_k)}\big[(e_{H_k}(u) - e_{L(H_k)}(e)) - ((e_{H_k}(v) - e_{L(H_k)}(e))\big]$

= 18i

(ii) $B_3E(H_k, x) = \sum_{ue} x^{\left|e_{H_k}(u) - e_{L(H_k)}(e)\right|}$

$= x^{0-6i+24i}$

(iii) $B_4E(H_k, x) = \sum_{ue} x^{e_G(u)(e_G(u)+e_{L(G)}(e))}$

$= 6\sum_{i=1}^{k} x^{2(2k+2(i-1)+1)^2} + 6\sum_{i=1}^{k-1} x^{i[(2k+2(i-1)(4k+4i-1)]} +$

$12\sum_{i=1}^{k-1} x^{i[(2k+2i)(4k+4i+1)]}$

(iv) $B_5E(H_k, x) = \sum_{ue} x^{e_{L(G)}(e)((e_G(u)+e_{L(G)}(e))}$

$= 6\sum_{i=1}^{k} x^{2(2k+2(i-1)+1)^2} + 6\sum_{i=1}^{k-1} x^{i[(2k+2i)[2(2k+2(i-1)+1]} +$

$12\sum_{i=1}^{k-1} x^{i[(2k+2i+1)(2(2k+2r)]}$

(v) $B_{a,b}E(H_k, x) = \sum_{ue} x^{a(e_G(u)+e_{L(G)}(e))+b(e_G(u)+e_{L(G)}(e))}$

$= 6\sum_{i=1}^{k} x^{a(4k+4i-2)+b(4k+4i-2)} + 6\sum_{i=1}^{k-1} x^{i[a(4k+4i-2)+b(4k+4i-1)]} +$

$12\sum_{i=1}^{k-1} x^{i[a(4k+4i)+b(4k+4i+1)]}$

(vi) $B_{a,b}E(H_k, x) = \sum_{ue} x^{(e_G(u)+e_{L(G)}(e)+a)+(e_G(v)+e_{L(G)}(e)+b)}$

$= 6\sum_{i=1}^{k} x^{[(4k+4i-2)+a]+[(4k+4i-2)+b]} +$

$$6\sum_{i=1}^{k-1} x^{\,i[(4k+4i-2)+a]+[(4k+4i-1)+b]} +$$

$$12\sum_{i=1}^{k-1} x^{\,i[(4k+4i)+a)+(4k+4i+1)+b)]}$$

Theorem 2.3.6: For any positive integer number k, let H_k be the general form of circumcoronene series of Benzenoid system graph G, then

(i) $ReBG_1E(H_k) = \left[\frac{36k^2+108k-36}{4k^3+24k^2-4k+3}\right] + \left[\frac{4k^3+27k^2-31k}{4k^3+2k^2-16k+3}\right] + \left[\frac{76k^3-78k^2+2k}{8k^3+2k^2+2k}\right]$

(ii) $ReBG_2E(H_k) = \left[\frac{4k^3+24k^2-4k+3}{36k^2+108k-36}\right] + \left[\frac{4k^3+2k^2-16k+3}{4k^3+27k^2-31k}\right] + \left[\frac{8k^3+2k^2+2k}{76k^3-78k^2+2k}\right]$

(iii) $ReBG_3E(H_k) = 1808k^7 - 3188k^6 + 324k^5 + 2362k^4 - 936k^3 - 134k^2 - 28k - 12$

Proof: Consider the General form of circumcoronene series of Benzenoid H_k system.

Hence we have

(i) $ReBG_1E(H_k) = \sum_{ue}\left[\frac{e_{H_k}(u)+e_{L(H_k)}(e)}{e_{H_k}(u).\, e_{L(H_k)}(e)}\right]$

$$= 6\left[\frac{(4k+1)}{(4k^2+4k+1)}\right] + 6\left[\frac{(4k+1)}{(4k^2+4k+1)}\right] + \ldots +$$

$$6\left[\frac{(2k+2(k-1)+1)+(2k+2(k-1)+1)}{(2k+2(k-1)+1).(2k+2(k-1)+1)}\right]$$

Therefore

$$= 6\sum_{i=1}^{k}\left[\frac{(4k+4i-2)}{(2k+2(i-1)+1)^2}\right] + 6\sum_{i=1}^{k-1} i\left[\frac{(4k+4i-2)}{(2k+2(i-1)+1)^2}\right] +$$

$$12\sum_{i=1}^{k-1} i\left[\frac{(4k+4i)}{(2k+2i)^2}\right]$$

After simplification, we get

$$ReBG_1E(H_k) = \left[\frac{36k^2+108k-36}{4k^3+24k^2-4k+3}\right] + \left[\frac{4k^3+27k^2-31k}{4k^3+2k^2-16k+3}\right] + \left[\frac{76k^3-78k^2+2k}{8k^3+2k^2+2k}\right]$$

(ii) ReBG$_2$E(H$_k$) = $\sum_{ue}[\frac{e_{H_k}(u).\ e_{L(H_k)}(e)}{e_{H_k}(u)+e_{L(H_k)}(e)}]$

$$= 6[\frac{(2k+1).(2k+1)}{(2k+1)+(2k+1)}]+ 6[\frac{(2k+1).(2k+1)}{(2k+1)+(2k+1)}]+\ldots+$$

$$[\frac{(2k+2(k-1)+1).(2k+2(k-1)+1}{(2k+2(k-1)+1)+(2k+2(k-1)+1}]$$

Therefore

$$= 6\sum_{i=1}^{k}\left[\frac{(2k+2(i-1)+1)^{2}}{(4k+4i-2)}\right] + 6\sum_{i=1}^{k-1} i\left[\frac{(2k+2(i-1)+1)^{2}}{(4k+4i-2)}\right]+$$

$$12\sum_{i=1}^{k-1} i\left[\frac{(2k+2i)^{2}}{(4k+4i)}\right]$$

After simplification, we get

ReBG$_2$E(H$_k$) = $\left[\frac{4k^3+24k^2-4k+3}{36k^2+108k-36}\right] + \left[\frac{4k^3+2k^2-16k+3}{4k^3+27k^2-31k}\right] + \left[\frac{8k^3+2k^2+2k}{76k^3-78k^2+2k}\right]$

(iii) ReBG$_3$E(H$_k$) = $\sum_{ue}(e_{H_k}(u) + e_{L(H_k)}(e))(e_{H_k}(u).\ e_{L(H_k)}(e))$

= 6[((2k+1)+(2k+1)) ((2k+1).(2k+1))]+6[((2k+1)+(2k+1))

((2k+1).(2k+1))]+… .+ 6[((2k+2(k–1)+1)+(2k+2(k–1)+1))

((2k+2(k–1)+1).(2k+2(k–1)+1)]

Therefore

$$= 6\sum_{i=1}^{k}[(4k + 4i - 2)(2k + 2(i - 1) + 1)^{2}] +$$

$$6\sum_{i=1}^{k-1} i[(4k + 4i - 2)(2k + 2(i - 1) + 1)^{2}] +$$

$$12\sum_{i=1}^{k-1} i[(4k + 4i)(2k + 2i)^{2}]$$

After simplification, we get

ReBG$_3$E(H$_k$) = $1808k^7 - 3188k^6 + 324k^5 + 2362k^4 - 936k^3$

$$-134k^2 - 28k - 12$$

Theorem 2.3.7: For any positive integer number k, let H_k be the general form of circumcoronene series of Benzenoid system graph G, then

(i) $B_1\prod E(H_k, x) = 6\prod_{i=1}^{k} x^{4(2k+2i-1)^2} \times 6\prod_{i=1}^{k-1} x^{i[(2(2k+2i-1)(4k+4i-1)]} \times$

$12\prod_{i=1}^{k-1} x^{i[(2(2k+2i)(4k+4i+1)]}$

(ii) $B_2\prod E(H_k, x)$

$= 6\prod_{i=1}^{k} x^{(2k+2i-1)^4} \times 6\prod_{i=1}^{k-1} x^{i[(2k+2i-1)^3(2k+2i)]} \times 12\prod_{i=1}^{k-1} x^{i[(2k+2i)^3(2k+2i+1)]}$

(iii) $HB_1\prod E(H_k, x) = 6\prod_{i=1}^{k} x^{4(2k+2i-1)^2} \times 6\prod_{i=1}^{k-1} x^{i[(4(2k+2i-1)^2 \times ((2k+2i)+(2k+2i-1))^2]} \times$

$12\prod_{i=1}^{k-1} x^{i[(4(2k+2i)^2)+((2k+2i+1)+(2k+2i))^2]}$

(iv) $HB_2\prod E(H_k, x) = 6\prod_{i=1}^{k} x^{(2k+2i-1)^8} \times 6\prod_{i=1}^{k-1} x^{i[(2k+2i-1)^6(2k+2i)]} \times$

$12\prod_{i=1}^{k-1} x^{i[(2k+2i)^6(2k+2i+1)]}$

Proof: Consider the General form of circumcoronene series of Benzenoid H_k system. Using Table 2.1.5, we obtain the following:

(i) $B_1\prod E(H_k, x) = \prod_{ue} x^{e_{H_k}(u)+e_{L(H_k)}(e)}$

$= \prod_{uv\in E(H_k)} x^{[e_{H_k}(u)+e_{L(H_k)}(e)]\times [e_{H_k}(v)+e_{L(H_k)}(e)]}$

$= 6\prod_{i=1}^{k} x^{4(2k+2i-1)^2} \times 6\prod_{i=1}^{k-1} x^{i[(2(2k+2i-1)(4k+4i-1)]} \times$

$12\prod_{i=1}^{k-1} x^{i[(2(2k+2i)(4k+4i+1)]}$

(ii) $B_2\prod E(H_k, x) = \prod_{ue} x^{e_{H_k}(u)\times e_{L(H_k)}(e)}$

$= \prod_{uv\in E(H_k)} x^{[e_{H_k}(u)\times e_{L(H_k)}(e)]\times [e_{H_k}(v)\times e_{L(H_k)}(e)]}$

$= 6\prod_{i=1}^{k} x^{(2k+2i-1)^4} \times 6\prod_{i=1}^{k-1} x^{i[(2k+2i-1)^3(2k+2i)]} \times 12\prod_{i=1}^{k-1} x^{i[(2k+2i)^3(2k+2i+1)]}$

(iii) $HB_1\prod E(H_k, x) = \prod_{ue} x^{[e_{H_k}(u)+e_{L(H_k)}(e)]^2}$

$$= \prod_{e=uv\in E(H_k)} x^{[e_{H_k}(u)+e_{L(H_k)}(e)]^2\times [e_{H_k}(v)+e_{L(H_k)}(e)]^2}$$

$$= 6\prod_{i=1}^{k} x^{4(2k+2i-1)^2} \times 6\prod_{i=1}^{k-1} x^{i[4(2k+2i-1)^2\times((2k+2i)+(2k+2i-1))^2]} \times$$

$$12\prod_{i=1}^{k-1} x^{i[4(2k+2i)^2)+((2k+2i+1)+(2k+2i))^2]}$$

(iv) $HB_2\prod E(H_k, x) = \prod_{ue} x^{[e_{H_k}(u)\times e_{L(H_k)}(e)]^2}$

$$= \prod_{uv\in E(H_k)} x^{[e_{H_k}(u)\times e_{L(H_k)}(e)]^2\times [e_{H_k}(v)\times e_{L(H_k)}(e)]^2}$$

$$= 6\prod_{i=1}^{k} x^{(2k+2i-1)^8} \times 6\prod_{i=1}^{k-1} x^{i[(2k+2i-1)^6(2k+2i)]}\times$$

$$12\prod_{i=1}^{k-1} x^{i[(2k+2i)^6(2k+2i+1)]}$$

Corollary 2.3.8: H_1 be the first terms of this Benzene in circumcoronene series of Benzenoid H_k system, then (i) $B_1\prod E(H_1, x) = 6x^{36}$, (ii) $B_2\prod E(H_1, x) = 6x^{81}$, (iii) $HB_1\prod E(H_1, x) = 6x^{1296}$ and (iv) $HB_2\prod E(H_1, x) = 6x^{6561}$.

Corollary 2.3.9: H_2 be the second terms of this Coronene in circumcoronene series of Benzenoid H_k system, then

(i) $B_1\prod E(H_2, x) = 6^3x^{406}\times 12x^{156}$,

(ii) $B_2\prod E(H_2, x) = 6^3x^{3776}\times 12x^{1512}$,

(iii) $HB_1\prod E(H_2, x) = 6^3x^{60516}\times 12x^{24336}$ and

(iv) $B_2\prod E(H_2, x) = 6^3x^{6717926}\times 12x^{2286144}$

Corollary 2.3.10: H_3 be the third terms of this Circumcoronene in circumcoronene series of Benzenoid H_k system, then

(i) $B_1\prod E(H_3, x) = 6^4x^{1214}\times 12^2x^{614}\times 24x^{441}$,

(ii) $B_2\prod E(H_3, x) = 6^4x^{26347}\times 12^2x^{11898}\times 24x^{11000}$,

(iii) $HB_1\prod E(H_3, x) = 6^4x^{381973}\times 12^2x^{190948}\times 24x^{176400}$ and

(iv) $HB_2\prod E(H_3, x) = 6^4x^{270699939}\times 12^2x^{74377764}\times 24x^{121000000}$.

Theorem 2.3.11: For any positive integer number k, let H_k be the general form of circumcoronene series of Benzenoid system, then

(i) $B_3\prod E(H_k, x) = 18i$

(ii) $B_3\prod E(H_k, x) = x^{0-6i+24i}$

(iii) $B_4\prod E(H_k, x) = 6\prod_{i=1}^{k} x^{2(2k+2(i-1)+1)^2} \times 6\prod_{i=1}^{k-1} x^{i[(2k+2(i-1)(4k+4i-1)]} \times 12\prod_{i=1}^{k-1} x^{i[(2k+2i)(4k+4i+1)]}$

(iv) $B_5\prod E(H_k, x) = 6\prod_{i=1}^{k} x^{2(2k+2(i-1)+1)^2} \times 6\prod_{i=1}^{k-1} x^{i[(2k+2i)[2(2k+2(i-1)+1]} \times 12\prod_{i=1}^{k-1} x^{i[(2k+2i+1)[2(2k+2i)]}$

(v) $B_{a,b}\prod E(H_k,x) = 6\prod_{i=1}^{k} x^{a(4k+4i-2)+b(4k+4i-2)} \times 6\prod_{i=1}^{k-1} x^{i[a(4k+4i-2)+b(4k+4i-1)]} \times 12\prod_{i=1}^{k-1} x^{i[a(4k+4i)+b(4k+4i+1)]}$

(vi) $B_{a,b}\prod E(H_k, x) = 6\prod_{i=1}^{k} x^{[(4k+4i-2)+a]+[(4k+4i-2)+b]} \times 6\prod_{i=1}^{k-1} x^{i[(4k+4i-2)+a]+[(4k+4i-1)+b]} \times 12\prod_{i=1}^{k-1} x^{i[(4k+4i)+a)+(4k+4i+1)+b)]}$

Proof: Consider the General form of H_k - circumcoronene series of Benzenoid system.

(i) $B_3\prod E(H_k, x) = \prod_{ue}[e_{H_k}(u) - e_{L(H_k)}(e)]$

.
$$= \prod_{uv \in E(H_k)}\left[e_{H_k}(u) - e_{L(H_k)}(e)\right] - \left[e_{H_k}(v) - e_{L(H_k)}(e)\right]$$

$$= \prod_{uv \in E_1(H_k)}\left[(e_{H_k}(u) - e_{L(H_k)}(e)) - (e_{H_k}(v) - e_{L(H_k)}(e))\right] \times \ldots\ldots \times$$

$$\prod_{uv \in E_{3(k-1)+1}(H_k)}\left[(e_{H_k}(u) - e_{L(H_k)}(e)) - (e_{H_k}(v) - e_{L(H_k)}(e))\right]$$

$$= 18i$$

(iii) $B_3\prod E(H_k, x) = \prod_{ue} x^{\left|e_{H_k}(u) + e_{L(H_k)}(e)\right|} = x^{0-6i+24i}$

(iii) $B_4\prod E(H_k, x) = \prod_{ue} x^{e_G(u)(e_G(u)+e_{L(G)}(e))}$

$$= 6\prod_{i=1}^{k} x^{2(2k+2(i-1)+1)^2} \times 6\prod_{i=1}^{k-1} x^{i[(2k+2(i-1)(4k+4i-1)]} \times$$

$$12\prod_{i=1}^{k-1} x^{i[(2k+2i)(4k+4i+1)]}$$

(iv) $B_5\prod E(H_k, x) = \prod_{ue} x^{e_{L(G)}(e)((e_G(u)+e_{L(G)}(e))}$

$$= 6\prod_{i=1}^{k} x^{2(2k+2(i-1)+1)^2} \times 6\prod_{i=1}^{k-1} x^{i[(2k+2i)[2(2k+2(i-1)+1]} \times$$

$$12\prod_{i=1}^{k-1} x^{i[(2k+2i+1)(2(2k+2i)]}$$

(v) $B_{a,b}\prod E(H_k, x) = \prod_{ue} x^{a(e_G(u)+e_{L(G)}(e))+b(e_G(u)+e_{L(G)}(e))}$

$$= 6\prod_{i=1}^{k} x^{a(4k+4i-2)+b(4k+4i-2)} \times$$

$$6\prod_{i=1}^{k-1} x^{i[a(4k+4i-2)+b(4k+4i-1)]} \times 12\prod_{i=1}^{k-1} x^{i[a(4k+4i)+b(4k+4i+1)]}$$

(vi) $B_{a,b}\prod E(H_k, x) = \prod_{ue} x^{(e_G(u)+e_{L(G)}(e)+a)+(e_G(v)+e_{L(G)}(e)+b)}$

$$= 6\prod_{i=1}^{k} x^{[(4k+4i-2)+a]+[(4k+4i-2)+b]} \times 6\prod_{i=1}^{k-1} x^{i[(4k+4i-2)+a]+[(4k+4i-1)+b]} \times$$

$$12\prod_{i=1}^{k-1} x^{i[(4k+4i)+a)+(4k+4i+1)+b)]}$$

Theorem 2.3.12: For any positive integer number k, let H_k be the general form of circumcoronene series of Benzenoid system graph G, then

(i) $ReBG_1\prod E(H_k) = 6\prod_{i=1}^{k}\left[\frac{(4k+4i-2)}{(2k+2i-1)^2}\right]\times 6\prod_{i=1}^{k-1} i\left[\frac{(4k+4i-2)}{(2k+2i-1)^2}\right]\times$
$12\prod_{i=1}^{k-1} i\left[\frac{(4k+4i)}{(2k+2i)^2}\right]$

(ii) $ReBG_2\prod E(H_k) = 6\prod_{i=1}^{k}\left[\frac{(2k+2(i-1)+1)^2}{(4k+4i-2)}\right]\times 6\prod_{i=1}^{k-1} i\left[\frac{(2k+2(i-1)+1)^2}{(4k+4i-2)}\right]\times$
$12\prod_{i=1}^{k-1} i\left[\frac{(2k+2i)^2}{(4k+4i)}\right]$

(iii) $ReBG_3\prod E(H_k) = 6\prod_{i=1}^{k}[(4k+4i-2)(2k+2i-1)^2]\times$
$6\prod_{i=1}^{k-1} i[(4k+4i-2)(2k+2i-1)^2]\times$
$12\prod_{i=1}^{k-1} i[(4k+4i)(2k+2i)^2]$

Proof: Consider the General form of H_k - circumcoronene series of Benzenoid system,.

(i) $ReBG_1\prod E(H_k) = \prod_{ue}[\frac{e_{H_k}(u)+e_{L(H_k)}(e)}{e_{H_k}(u).\ e_{L(H_k)}(e)}]$

$= \prod_{uv\in E_1(H_k)}[\frac{e_{H_k}(u)+e_{L(H_k)}(e)+e_{H_k}(v)+e_{L(H_k)}(e)}{e_{H_k}(u).e_{L(H_k)}(e)\times e_{H_k}(v).e_{L(H_k)}(e)}]\times\ldots.\times$

$\prod_{uv\in E_{3(k-1)+1}(H_k)}[\frac{e_{H_k}(u)+e_{L(H_k)}(e)+e_{H_k}(v)+e_{L(H_k)}(e)}{e_{H_k}(u).e_{L(H_k)}(e)\times e_{H_k}(v).e_{L(H_k)}(e)}]$

$= 6[\frac{(4k+2)}{(4k^2+4k+1)}]\times 6[\frac{(4k+2)}{(4k^2+4k+1)}]\times\ldots.\times 6[\frac{(8k-2)}{(16k^2-8k+1)}]$

Therefore

$ReBG_1\prod E(H_k) = 6\prod_{i=1}^{k}\left[\frac{(4k+4i-2)}{(2k+2(i-1)+1)^2}\right]\times 6\prod_{i=1}^{k-1} i\left[\frac{(4k+4i-2)}{(2k+2(i-1)+1)^2}\right]\times$
$12\prod_{i=1}^{k-1} i\left[\frac{(4k+4i)}{(2k+2i)^2}\right]$

(ii) $ReBG_2\prod E(H_k) = \prod_{ue}[\frac{e_{H_k}(u)\,.\; e_{L(H_k)}(e)}{e_{H_k}(u)+ e_{L(H_k)}(e)}]$

$$= \prod_{uv\in E_1(H_k)}[\frac{e_{H_k}(u).e_{L(H_k)}(e).e_{H_k}(v).e_{L(H_k)}(e)}{e_{H_k}(u)+ e_{L(H_k)}(e)+ e_{H_k}(v)+ e_{L(H_k)}(e)}]\times....\times$$

$$\prod_{uv\in E_{3(\mathrm{k}-1)+1}(H_k)}[\frac{e_{H_k}(u).e_{L(H_k)}(e).e_{H_k}(v).e_{L(H_k)}(e)}{e_{H_k}(u)+ e_{L(H_k)}(e)+e_{H_k}(v)+ e_{L(H_k)}(e)}]$$

Therefore

$$ReBG_2\prod E(H_k) = 6[\frac{(4k^2+4k+1)}{(4k+2)}]\times 6[\frac{(4k^2+4k+1)}{(4k+2)}]\times \ldots\times 6[\frac{(16k^2-8k+1)}{(8k-2)}]$$

$$= 6\prod_{i=1}^{k}\left[\frac{(2k+2(i-1)+1)^{2}}{(4k+4i-2)}\right]\times 6\prod_{i=1}^{k-1} i\left[\frac{(2k+2(i-1)+1)^{2}}{(4k+4i-2)}\right]\times$$

$$12\prod_{i=1}^{k-1} i\left[\frac{(2k+2i)^{2}}{(4k+4i)}\right]$$

(iii) $ReBG_3\prod E(H_k) = \prod_{ue}(e_{H_k}(u) + e_{L(H_k)}(e))(e_{H_k}(u).\; e_{L(H_k)}(e))$

$$= \prod_{uv\in E_1(H_k)}[(e_{H_k}(u) + e_{L(H_k)}(e) + e_{H_k}(v) +$$

$$e_{L(H_k)}(e))(e_{H_k}(u).e_{L(H_k)}(e).e_{H_k}(v).e_{L(H_k)}(e))]]\times....\times$$

$$\prod_{uv\in E_{3(\mathrm{k}-1)+1}(H_k)}[(e_{H_k}(u) + e_{L(H_k)}(e) + e_{H_k}(v) +$$

$$e_{L(H_k)}(e))(e_{H_k}(u).e_{L(H_k)}(e).e_{H_k}(v).e_{L(H_k)}(e))]$$

Therefore

$$ReBG_3\prod E(H_k) = 6\prod_{i=1}^{k}[(4k+4i-2)(2k+2(i-1)+1)^{2}]\times$$

$$6\prod_{i=1}^{k-1} i[(4k+4i-2)(2k+2(i-1)+1)^{2}]\times$$

$$12\prod_{i=1}^{k-1} i[(4k+4i)(2k+2i)^{2}]$$

CHAPTER 3

EDGE VERSION OF FIRST, SECOND AND HYPER ZAGREB ECCENTRIC, MODIFIED ECCENTRIC INDICES AND MULTIPLICATIVE ECCENTRIC INDICES OF GRAPH L(G) = L(H_K)

In this chapter, we calculate the edge version of First, Second and hyper zagreb eccentric indices and modified first and second eccentric indices. Also, we determine the multiplicative first, second and hyper zagreb eccentric indices and modified multiplicative first and second eccentric indices of Circumcronone series Benzenoid graph L(G).

we calculate the edge version of first, second and hyper zagreb indices as

$$M_1E(L(G)) = \sum_{ef \in E(L(G))}\left[e_{L(G)}(e) + e_{L(G)}(f)\right], \tag{3.1}$$

$$M_2E\big(L(G)\big) = \sum_{ef \in E(L(G))}\left[e_{L(G)}(e) \times e_{L(G)}(f)\right] \text{ and} \tag{3.2}$$

$$HM_1E(L(G)) = \sum_{ef \in E(L(G))}\left[e_{L(G)}(e) + e_{L(G)}(f)\right]^2, \tag{3.3}$$

$$HM_2E(L(G)) = \sum_{ef \in E(L(G))}\left[e_{L(G)}(e) \times e_{L(G)}(f)\right]^2 . \tag{3.4}$$

Also, we calculate the edge version of multiplicative first, second and hyper zagreb indices as

$$M\prod\nolimits_1 E(L(G)) = \prod_{ef \in E(L(G))}[e_{L(G)}(e) + e_{L(G)}(f)] \tag{3.5}$$

$$M\prod\nolimits_2 E(L(G)) = \prod_{ef \in E(L(G))}[e_{L(G)}(e) \times e_{L(G)}(f)] \text{ and} \tag{3.6}$$

$$HM\prod\nolimits_1 E(L(G)) = \prod_{ef \in E(L(G))}[e_{L(G)}(e) + e_{L(G)}(f)]^2 \tag{3.7}$$

$$HM\prod\nolimits_2 E(L(G)) = \prod_{ef \in E(L(G))}[e_{L(G)}(e) \times e_{L(G)}(f)]^2 . \tag{3.8}$$

We define the edge version of modified eccentric indices as

$$^{m}M_1E(L(G)) = \sum_{ef \in E(L(G))} \frac{1}{e_{L(G)}(e)+e_{L(G)}(f)}, \quad (3.9)$$

$$^{m}M_2E(L(G)) = \sum_{ef \in E(L(G))} \frac{1}{e_{L(G)}(e)\times e_{L(G)}(f)} \quad (3.10)$$

and the edge version of harmonic eccentric index as

$$H_bE(L(G)) = \sum_{ef \in E(L(G))} \frac{2}{e_{L(G)}(e)+e_{L(G)}(f)} . \quad (3.11)$$

Also, we define the edge version of modified Multiplicative eccentric indices as

$$^{m}M_1\Pi E(L(G)) = \prod_{ef \in E(L(G))} \frac{1}{e_{L(G)}(e)+e_{L(G)}(f)} , \quad (3.12)$$

$$^{m}M_2\Pi E(L(G)) = \prod_{ef \in E(L(G))} \frac{1}{e_{L(G)}(e)\times e_{L(G)}(f)} \text{ and} \quad (3.13)$$

the edge version of multiplicative harmonic eccentric index as

$$H_b\Pi E(L(G)) = \prod_{ef \in E(L(G))} \frac{2}{e_{L(G)}(e)+e_{L(G)}(f)} . \quad (3.14)$$

Where in all the cases *ef* means that the edges in L(G), we have $e_{L(G)}(e)$ is the eccentricity of e in the line graph $L(G)$ of G.

If G is a (p, q) graph whose vertices have degrees d_i, then L(G) has q vertices and q_L edges, where $q_L = -q + \frac{1}{2}\sum d_i^{\,2}$.

From the general representation of line graph of circumcoronene series of Benzenoid $L(H_k)$ ($k \geq 1$) with edges marking (Fig. 1.3.3)

Let V_k be the vertex set of $L(H_k)$ and E_k be the edge set in $L(H_k)$, then $|V_k| = 9k^2 - 3k$ and $|E_k| = 18k^2 - 12k$ for the structure of $L(H_k)$.

We give these values in the follows:

No. of Vertices In L(G)	Eccentricity of e in L(G)	No. of edge set (e(u), e(v))	$(e_{L(G)}(e), e_{L(G)}(f))$
6	2k+1	6 x 3	(2k+1, 2k+1)
6	2k+1	6 x 2	(2k+1, 2k+2)
12	2k+2	6 x 3	(2k+2, 2k+3)
6	2k+3	6 x 4	(2k+3, 2k+3)
12	2k+3	6 x 4	(2k+3, 2k+4)
24	2k+4	6 x 5	(2k+4, 2k+5)
6	2k+5	6 x 6	(2k+5, 2k+5)
18	2k+5	6 x 6	(2k+5, 2k+6)
36	2k+6	6 x 7	(2k+6, 2k+7)
.	.	.	.
.	.	.	.
.	.	.	.
6	2k+2(k–2) –1	6×(2k-4)	(2k+2(k–2) –1, 2k+2(k–2) –1)
6(k-2)	2k+2(k–2) –1	6×(2k-4)	(2k+2(k–2) –1, 2k+2(k–2))
12(k-2)	2k+2(k–2)	6×(2k-3)	(2k+2(k–2), 2k+2(k–1) –1)
6	2k+2(k–1) –1	6×(2k-2)	(2k+2(k–1) –1, 2k+2(k–1) –1)
6(k-1)	2k+2(k–1) –1	6×(2k-2)	(2k+2(k–1) –1, 2k+2(k–1))
12(k-1)	2k+2(k–1)	6×(2k-3)	(2k+2(k–1), 2k+2(k–1)+1)
6	2k+2(k–1)+1	6×2	(2k+2(k–1)+1, 2k+2(k–1)+1)

Table 3.1.1

3.1 First, Second, Hyper Zagreb eccentric indices and multiplicative First, Second, Hyper Zagreb eccentric indices of graph $L(H_k)$:

In this section we calculate the edge version of First, Second, hyper zagreb eccentric indices and edge version of multiplicative first, second eccentric indices of Circumcronone series Benzenoid graph $L(H_k)$.

Theorem 3.1.1: For any positive integer number k, let L(G) = L(H_k) be the general form of line graph of circumcoronene series of benzenoid system, then

(i) $M_1E(L(G)) = 12k^4+32k^3+102k^2-382k + 524$

(ii) $M_2E(L(G)) = 176k^4 + 56k^3 - 1060k^2 + 1126k - 122$

(iii) $HM_1E(L(G)) = 660k^4 + 725k^3 - 4665k^2 + 4787k - 509$

(iv) $HM_2E(L(G)) = 544k^6+7808k^5-31080k^4+37360k^3-21904k^2+6380k-270$

Proof: Consider the General form of line graph L(H_k) - Circumcoronene series.

Hence we have

$$(i)\ M_1E(L(G)) = \sum_{ef\in E(L(G))}[e_{L(G)}(e) + e_{L(G)}(f)]$$

$$= \sum_{ef\in E_1(L(G))}[e_{L(G)}(e) + e_{L(G)}(f)]+\ldots\ldots+$$

$$\sum_{ef\in E_{3(k-1)+1}(L(G))}[e_{L(G)}(e) + e_{L(G)}(f)]$$

Therefore

$$= 6\times 3[4k + 2] + 6\times 2[4k + 3] + 6\times(2i + 1)\sum_{i=1}^{k-2}(4k + 4i)$$

$$6\times(2i + 2)\sum_{i=1}^{k-2}(4k + 4i + 1)+$$

$$6\times(2i + 2)\sum_{i=1}^{k-2}(4k + 4i + 3) +$$

$$6\times 3[(2k - 3)(8k - 4)] + 6\times 2[(8k - 3)]$$

After simplification, we get

$$M_1E(L(G)) = 12k^4+32k^3+102k^2-382k + 524$$

$$(ii)\ M_2E(L(G)) = \sum_{ef\in E(L(G))}[e_{L(G)}(e)\times e_{L(G)}(f)]$$

$$= \sum_{ef\in E_1(L(G))}[e_{L(G)}(e)\times e_{L(G)}(f)]+\ldots..+$$

$$\sum_{ef\in E_{3(k-1)+1}(L(G))}[e_{L(G)}(e)\times e_{L(G)}(f)]$$

Therefore

$$= 6 \times 3[(2k+1)^2] + 6\times 2[(2k+1)(2k+2)] +$$

$$6 \times (2i+1)\sum_{i=1}^{k-2}(2k+1)^2 +$$

$$6 \times (2i+2)\sum_{i=1}^{k-2}(2k+2i)(2k+2i+1)+$$

$$6 \times (2i+2)\sum_{i=1}^{k-2}(2k+2i+1)(2k+2i+2) +$$

$$6\times 3(2k-3)[(2k+2(k-1)^2] +$$

$$6\times 2[(2k+2(k-1))(2k+2(k-1)+1)]$$

After simplification, we get

$$M_2E(L(G)) = 176k^4 + 56k^3 - 1060k^2 + 1126\text{k} - 122$$

$$(iii)\ HM_1E(L(G)) = \sum_{ef\in E(L(G))}\left[e_{L(G)}(e) + e_{L(G)}(f)\right]^2$$

$$= \sum_{ef\in E_1(L(G))}\left[e_{L(G)}(e) + e_{L(G)}(f)\right]^2 + \ldots\ldots +$$

$$\sum_{ef\in E_{3(\text{k}-1)+1}(L(G))}\left[e_{L(G)}(e) + e_{L(G)}(f)\right]^2$$

Therefore

$$= 6\times 3[4k+2]^2 + 6\times 2[4k+3]^2 +$$

$$6 \times (2i+1)\sum_{i=1}^{k-2}[4k+4i]^2 +$$

$$6 \times (2i+2)\sum_{i=1}^{k-2}[(4k+4i+1)]^2 +$$

$$6 \times (2i+2)\sum_{i=1}^{k-2}[(4k+4i+3)]^2 +$$

$$6 \times 3(2k-3)[8k-4]^2 + 6\times 2[(8k-3)]^2$$

After simplification, we get

$$HM_1E(L(G)) = 660k^4 + 725k^3 - 4665k^2 + 4787\text{k} - 509$$

$(iv)\ HM_2E(L(G)) = \sum_{ef \in E(L(G))}\left[e_{L(G)}(e) \times e_{L(G)}(f)\right]^2$

$$= \sum_{ef \in E_1(L(G))}\left[e_{L(G)}(e) \times e_{L(G)}(f)\right]^2 + \ldots\ldots +$$

$$\sum_{ef \in E_{3(k-1)+1}(L(G))}\left[e_{L(G)}(e) \times e_{L(G)}(f)\right]^2$$

Therefore

$$= 6 \times 3[(2k+1)^2]^2 + 6 \times 2[(2k+1)(2k+2)]^2 +$$

$$6 \times (2i+1)\sum_{i=1}^{k-2}[(2k+2i)^2]^2 +$$

$$6 \times (2i+2)\sum_{i=1}^{k-2}[(2k+2i)(2k+2i+1)]^2 +$$

$$6 \times (2i+2)\sum_{i=1}^{k-2}[(2k+2i+1)\,(2k+2i+2)]^2 +$$

$$6 \times 3(2k-3)[(2k+2(k-1))^2]^2 +$$

$$6 \times 2[(2k+2(k-1))(2k+2(k-1)+1)]^2$$

After simplification, we get

$$HM_2E(L(G)) = 544k^6 + 7808k^5 - 31080k^4 + 37360k^3 - 21904k^2 + 6380k - 270$$

For example, let us evaluate these indices for $L(H_4)$.

Consider the $L(H_4)$ - line graph of Circumcircumcoronene .

Let V_4 be the vertex set and E_4 be the edge set in $L(H_4)$ then $|V_4| = 132$ and $|E_4| = 240$.

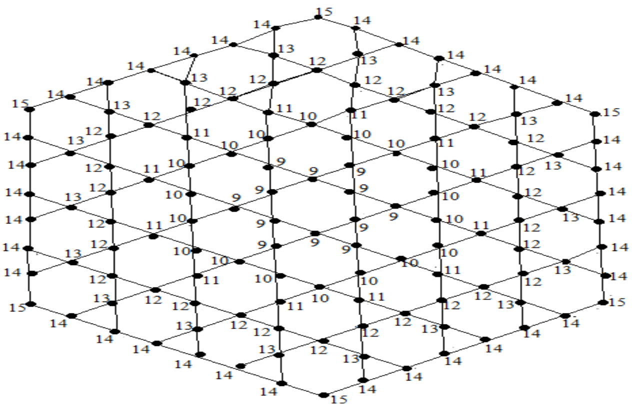


Fig. 3.1.1: $L(H_4)$-Circumcircumcoronene

Also, the number of vertices of L(G), the eccentricity of e, $e_{L(G)}(e)$ in the line graph $L(G)$ are given as follows:

No. of Vertices In L(G)	Eccentricity of e in L(G)	No. of edge set (e(u), e(v))	($e_{L(G)}(e)$, $e_{L(G)}(f)$)
6	9	18	(9, 9)
6	9	12	(9, 10)
12	10	18	(10, 10)
6	10	24	(10, 11)
12	11	24	(11, 12)
24	12	30	(12, 12)
6	12	36	(12, 13)
18	13	36	(13, 14)
36	14	30	(14. 14)
6	14	12	(14, 15)

Table 3.1.2

Thus we have,

(i) $M_1E(L(\mathrm{H}_4)) = \sum_{ef \in E(L(G))}[e_{L(G)}(e) + e_{L(G)}(f)]$ = 5748

(ii) $M_2E(L(\mathrm{H}_4)) = \sum_{ef \in E(L(G))}[e_{L(G)}(e) \times e_{L(G)}(f)]$ = 35034

(iii) $HM_1E(L(\mathrm{H}_4)) = \sum_{ef \in E(L(G))}[e_{L(G)}(e) + e_{L(G)}(f)]^2$ = 3945168

(iv) $HM_2E(L(\mathrm{H}_4)) = \sum_{ef \in E(L(G))}[e_{L(G)}(e) \times e_{L(G)}(f)]^2$ = 157593348

Corollary 3.1.2: $L(H_1)$ be the first terms of the line graph of Circumcoronene series of Benzenoid $L(H_k)$. Then,

(i) $M_1E(L(\mathrm{H}_1))$ = 36, (ii) $M_2E(L(\mathrm{H}_1))$ = 54,

(iii) $HM_1E(L(\mathrm{H}_1))$ = 216 and (iv) $HM_2E(L(\mathrm{H}_1))$ = 486.

Corollary 3.1.3: $L(H_2)$be the second terms of the line graph of Circumcoronene series of Benzenoid $L(H_k)$. Then

(i) $M_1E(L(\mathrm{H}_2))$ = 540, (ii) $M_2E(L(\mathrm{H}_2))$ = 1530,

(iii) $HM_1E(L(\mathrm{H}_2))$ = 6144 and (iv) $HM_2E(L(\mathrm{H}_2))$ = 50994.

Corollary 3.1.4: $L(H_3)$be the third terms of the line graph of Circumcoronene series of Benzenoid $L(H_k)$. Then (i) $M_1E(L(\mathrm{H}_3))$ = 2196, (ii) $M_2E(L(\mathrm{H}_3))$ = 9714, (iii) $HM_1E(L(\mathrm{H}_3))$ = 38928 and (iv) $HM_2E(L(\mathrm{H}_3))$ = 798594.

Theorem 3.1.5: For any positive integer number k, let $L(H_k)$ be the general form of line graph of circumcoronene series of benzenoid system, then

(i) $M\prod_1 E(L(G))$

$$= 6\times 3[4k + 2] + 6\times 2[4k + 3] + 6\times(2i + 1)\prod_{i=1}^{k-2}(4k + 4i)$$

$$6 \times (2i + 2)\prod_{i=1}^{k-2}(4k + 4i + 3)+$$

$6 \times (2i + 2) \prod_{i=1}^{k-2}(4k + 4i + 1) +$

$6 \times 3[(2k - 3)(8k - 4)] + 6\times 2[(8k - 3)]$

(ii) $M \prod_2 E(L(G)) = 6\times 3[(2k + 1)^2] \times 6\times 2[(2k + 1)(2k + 2)]\times$

$6\times (2i + 1) \prod_{i=1}^{k-2}(2k + 1)^2 \times$

$6 \times (2i + 2) \prod_{i=1}^{k-2}((2k + 2i)(2k + 2i + 1)) \times$

$6 \times (2i + 2) \prod_{i=1}^{k-2}(2k + 2i + 1)(2k + 2i + 2)\times$

$6 \times 3(2k - 3)[(2k + 2(k - 1)^2]\times$

$6\times 2[(2k + 2(k - 1))(2k + 2(k - 1) + 1)]$

(iii) $HM \prod_1 E(L(G)) = 6\times 3[4k + 2]^2 + 6\times 2[4k + 3]^2 +$

$6 \times (2i + 1) \prod_{i=1}^{k-2}(4k + 4i)^2 +$

$6 \times (2i + 2) \prod_{i=1}^{k-2}(4k + 4i + 1)^2 +$

$6 \times (2i + 2) \prod_{i=1}^{k-2}(4k + 4i + 3)^2 +$

$6 \times 3[(2k - 3)(8k - 4)]^2 + 6\times 2[(8k - 3)]^2$

(iv) $HM \prod_2 E(L(G)) = 6\times 3[(2k + 1)^2]^2 \times 6\times 2[(2k + 1)(2k + 2)]^2 \times$

$6 \times (2i + 1) \prod_{i=1}^{k-2}[(2k + 1)^2]^2 \times$

$6 \times (2i + 2) \prod_{i=1}^{k-2}((2k + 2i)(2k + 2i + 1))^2 \times$

$6 \times (2i + 2) \prod_{i=1}^{k-2}[(2k + 2i + 1)(2k + 2i + 2)]^2 \times$

$6 \times 3(2k - 3)[(2k + 2(k - 1)^2]^2 \times$

$6 \times 2[(2k + 2(k - 1))(2k + 2(k - 1) + 1)]^2$

Proof: Consider the General form of line graph $L(H_k)$- Circumcoronene series. Hence we have

(i) $M\prod_1 E(L(G)) = \prod_{ef\in E(L(G))}[e_{L(G)}(e)+e_{L(G)}(f)]$

$= \prod_{ef\in E_1(L(G))}[e_{L(G)}(e)+ e_{L(G)}(f)]\times\ldots\ldots\times$

$\prod_{uv\in E_{3(k-1)+1}(G)}[e_{L(G)}(e)+e_{L(G)}(f)]$

$= 6\times 3[4k+2]+6\times 2[4k+3]+$

$6\times(2i+1)\prod_{i=1}^{k-2}(4k+4i)\times$

$6\times(2i+2)\prod_{i=1}^{k-2}(4k+4i+3)\times$

$6\times(2i+2)\prod_{i=1}^{k-2}(4k+4i+1)\times$

$6\times 3[(2k-3)(8k-4)]+6\times 2[(8k-3)]$

(ii) $M\prod_2 E(L(G)) = \prod_{ef\in E(L(G))}[e_{L(G)}(e)\times e_{L(G)}(f)]$

$= \prod_{ef\in E_1(L(G))}[e_{L(G)}(e)\times e_{L(G)}(f)]\times\ldots\times$

$\prod_{ef\in E_{3(k-1)+1}(L(G))}[e_{L(G)}(e)\times e_{L(G)}(f)]$

$= 6\times 3[(2k+1)^2]\times 6\times 2[(2k+1)(2k+2)]\times$

$6\times(2i+1)\prod_{i=1}^{k-2}(2k+1)^2\times$

$6\times(2i+2)\prod_{i=1}^{k-2}((2k+2i)(2k+2i+1))\times$

$6\times(2i+2)\prod_{i=1}^{k-2}(2k+2i+1)\times(2k+2i+2)\times$

$6\times 3(2k-3)[(2k+2(k-1)^2]\times$

$6\times 2[(2k+2(k-1))(2k+2(k-1)+1)]$

(iii) $HM\prod_1 E(L(G)) = \prod_{ef\in E(L(G))}\left[e_{L(G)}(e)+e_{L(G)}(f)\right]^2$

$$= \prod_{ef\in E_1(L(G))}\left[e_{L(G)}(e)+e_{L(G)}(f)\right]^2 \times \ldots\ldots \times$$

$$\prod_{ef\in E_{3(\mathrm{k}-1)+1}(L(G))}\left[e_{L(G)}(e)+e_{L(G)}(f)\right]^2$$

$$= 6\times 3[4k+2]^2 + 6\times 2[4k+3]^2 +$$

$$6\times(2i+1)\prod_{i=1}^{k-2}(4k+4i)^2 +$$

$$6\times(2i+2)\prod_{i=1}^{k-2}(4k+4i+1)^2 +$$

$$6\times(2i+2)\prod_{i=1}^{k-2}(4k+4i+3)^2 +$$

$$6\times 3[(2k-3)(8k-4)]^2 + 6\times 2[(8k-3)]^2$$

(iv) $HM\prod_2 E(L(G)) = \prod_{ef\in E(L(G))}\left[e_{L(G)}(e)\times e_{L(G)}(f)\right]^2$

$$= \prod_{ef\in E_1(L(G))}\left[e_{L(G)}(e)\times e_{L(G)}(f)\right]^2 \times \ldots \times$$

$$\prod_{ef\in E_{3(\mathrm{k}-1)+1}(L(G))}\left[e_{L(G)}(e)\times e_{L(G)}(f)\right]^2$$

$$= 6\times 3[(2k+1)^2]^2 \times 6\times 2[(2k+1)(2k+2)]^2 \times$$

$$6\times(2i+1)\prod_{i=1}^{k-2}[(2k+1)^2]^2 \times$$

$$6\times(2i+2)\prod_{i=1}^{k-2}((2k+2i)(2k+2i+1))^2 \times$$

$$6\times(2i+2)\prod_{i=1}^{k-2}[(2k+2i+1)(2k+2i+2)]^2 \times$$

$$6\times 3(2k-3)[(2k+2(k-1)^2]^2 \times$$

$$6\times 2[(2k+2(k-1))(2k+2(k-1)+1)]^2$$

Using MATLAB programme, we have calculated these indices for $L(H_1)$, $L(H_2)$ and $L(H_3)$. Those values are given below corollaries.

Corollary 3.1.6: $L(H_1)$be the first terms of the line graph of Circumcoronene series of Benzene $L(H_k)$. Then, $(i)\ M\prod_1 E(L(H_1)) = 36$, (ii) $M\prod_2 E(L(H_1)) = 54$, (iii) $HM\prod_1 E(L(H_1)) = 216$ and (iv) $HM\prod_2 E(L(H_1)) = 486$.

Corollary 3.1.7: $L(H_2)$be the second terms of the line graph of Circumcoronene series of Benzene $L(H_k)$. Then

$(i)\ M\prod_1 E(L(H_2)) = 266872320$, (ii) $M\prod_2 E(L(H_2)) = 1.7636e^{+01}$,

(iii) $HM\prod_1 E(L(H_2)) = 4.5795e^{+012}$ and

$(iv)\ HM\prod_2 E(L(H_2)) = 1.9999e^{+016}$

Corollary 3.1.8: $L(H_3)$be the third terms of the line graph of Circumcoronene series of Benzene $L(H_k)$. Then

(i) $M\prod_1 E(L(H_3)) = 2.2049e^{+017}$,

$(ii)\ M\prod_2 E(L(H_3)) = 6.0553e^{+021}$,

(iii) $HM\prod_1 E(L(H_3)) = 1.0050e^{+026}$ and

$(iv)\ HM\prod_2 E(L(H_3)) = 7.5799e^{+034}$.

3.2 Modified first, second eccentric indices and multiplicative modified first, second eccentric indices of graph $L(H_k)$:

In this section we calculate the edge version of modified first, second zagreb eccentric indices and multiplicative modified first, second eccentric indices of Circumcronone series Benzenoid graph L(G).

Theorem 3.2.1: For any positive integer number k, let $L(H_k)$ be the general form of line graph circumcoronene series of benzenoid system, then

(i) $^{m}M_1E(L(G)) = \frac{192k^6-96k^5-60k^4+348k^3+168k^2-66k-36}{32k^6+40k^5-4k^4-20k^3-6k^2} + \frac{6k^2-18k+18}{2k^2-2k+5}$

(ii) ${}^{m}M_2E(L(G)) = \frac{4608k^6-21888k^5+43200k^4-41184k^3+20016k^2-4752k+432}{864k^5+648k^4-16416k^3+12960k^2-3672k-9072}$

(iii) $H_bE(L(G)) = \frac{960k^5+1656k^4-7452k^3+504k^2+5208k-858}{128k^5-68k^4-316k^3+185k^2-89k-33} +$

$$\frac{1008k^4-2952k^3+4788k^2-3672k+786}{64k^4-120k^3+228k^2-152k+30}$$

Proof: Consider the General form of $L(H_k)$- Circumcoronene graph. Hence we have

(i) ${}^{m}M_1E(L(G)) = \sum_{ef\in E(L(G))} \frac{1}{e_{L(G)}(e)+e_{L(G)}(f)}$

$$=\sum_{ef\in E_1(L(G))}\left[\frac{1}{e_{L(G)}(e)+e_{L(G)}(f)}\right]+\ldots\ldots+$$

$$\sum_{ef\in E_{3(k-1)+1}(L(G))}\left[\frac{1}{e_{L(G)}(e)+e_{L(G)}(f)}\right]$$

Therefore

$$= 6\times 3\left[\frac{1}{2(2k+1)}\right]+6\times 2\left[\frac{1}{(4k+3)}\right]+6\times(2i+1)\sum_{i=1}^{k-2}\left[\frac{1}{2(2k+2i)}\right]+$$

$$6\times(2i+2)\sum_{i=1}^{k-2}\left[\frac{1}{4k+4i+1}\right]+6\times(2i+2)\sum_{i=1}^{k-2}\left[\frac{1}{4k+4i+3}\right]+$$

$$6\times 3(2k-3)\left[\frac{1}{2(2k+2(k-1))}\right]+6\times 2\left[\frac{1}{(4k+4(k-1)+1)}\right]$$

After simplification, we get

$${}^{m}M_1E(L(G)) = \frac{192k^6-96k^5-60k^4+348k^3+168k^2-66k-36}{32k^6+40k^5-4k^4-20k^3-6k^2} + \frac{6k^2-18k+18}{2k^2-2k+5}$$

(ii) ${}^{m}M_2E(L(G)) = \sum_{ef\in E(L(G))} \frac{1}{e_{L(G)}(e)\times e_{L(G)}(f)}$

$$=\sum_{ef\in E_1(L(G))}\left[\frac{1}{e_{L(G)}(e)\times e_{L(G)}(f)}\right]+\ldots\ldots+$$

$$\sum_{ef\in E_{3(k-1)+1}(L(G))}\left[\frac{1}{e_{L(G)}(e)\times e_{L(G)}(f)}\right]$$

Therefore

$$= 6\times 3[\frac{1}{(2k+1)^{2}}] + 6\times 2[\frac{1}{(2k+1)(2k+2)}] +$$

$$6\times(2i+1)\ \sum_{i=1}^{k-2}[\frac{1}{(2k+2i)^{2}}]\ +$$

$$6\times(2i+2)\sum_{i=1}^{k-2}[\frac{1}{(2k+2i)(2k+2i+1)}]+$$

$$6\times(2i+2)\sum_{i=1}^{k-2}[\frac{1}{(2k+2i+1)(2k+2i+2)}]+$$

$$6\times 2[\frac{1}{(2k+2(k-1))(2k+2(k-1)+1)}]$$

$$6\times 3(2k-3)[\frac{1}{(2k+2(k-1))^{2}}]$$

After simplification, we get

$$^{m}M_2E(L(G)) \quad = \frac{4608k^6-21888k^5+43200k^4-41184k^3+20016k^2-4752k+432}{864k^5+648k^4-16416k^3+12960k^2-3672k-9072}$$

(iii) $H_bE(L(G)) \quad = \sum_{ef\in E(L(G))}\frac{2}{e_{L(G)}(e)+e_{L(G)}(f)}$

$$=\sum_{ef\in E_1(L(G))}[\ \frac{2}{e_{L(G)}(e)+e_{L(G)}(f)}\ ] + \ldots\ldots.. +$$

$$\sum_{ef\in E_{3(k-1)+1}(L(G))}[\ \frac{2}{e_{L(G)}(e)+e_{L(G)}(f)}\ ]$$

Therefore

$$= 6\times 3\left[\frac{1}{(2k+1)}\right] + 6\times 2\left[\frac{2}{2k+3}\right] + 6\times(2i+1)\ \sum_{i=1}^{k-2}[\frac{1}{(2k+2i)}]+$$

$$6\times(2i+2)\sum_{i=1}^{k-2}[\frac{2}{4k+4i+1}]+ 6\times(2i+2)\sum_{i=1}^{k-2}\left[\frac{2}{4k+4i+3}\right] +$$

$$6\times 3(2k-3)[\frac{1}{(2k+2(k-1))}] + 6\times 2[\frac{2}{4k+4(k-1)+1}]$$

After simplification, we get

$$H_bE(L(G)) = \frac{960k^5+1656k^4-7452k^3+504k^2+5208k-858}{128k^5-68k^4-316k^3+185k^2-89k-33} + \frac{1008k^4-2952k^3+4788k^2-3672k+786}{64k^4-120k^3+228k^2-152k+30}$$

Corollary 3.2.2: $L(H_1)$be the first terms of the line graph of Circumcoronene series of Benzene $L(H_k)$.Then

(i) $^{m}M_1E(L(H_1)) = 0.1111$, (ii) $^{m}M_2E(L(H_1)) = 0.0031$ and

(iii) $H_bE(L(H_1)) = 0.2222$.

Corollary 3.2.3: $L(H_2)$be the second terms of the line graph of Circumcoronene series of Coronene $L(H_k)$.Then

(i) $^{m}M_1E(L(H_2)) = 0.1341$, (ii) $^{m}M_2E(L(H_2)) = 3.6460e^{-015}$ and

(iii) $H_bE(L(H_2)) = 0.2683$.

Corollary 3.2.4: $L(H_3)$be the third terms of the line graph of Circumcoronene series of Circumcoronene $L(H_k)$. Then

(i) $^{m}M_1E(L(H_3)) = 0.0977$, (ii) $^{m}M_2E(L(H_3)) = 3.7262e^{-004}$, and

(iii) $H_bE(L(H_3)) = 0.0015$.

Theorem 3.2.5: For any positive integer number k, let $L(H_k)$ be the general form of line graph circumcoronene series of benzenoid system, then

(i) $$^{m}M_1\Pi E(L(G)) = 6\times 3\left[\frac{1}{2(2k+1)}\right] + 6\times 2\left[\frac{1}{(4k+3)}\right] + 6\times(2i+1)\prod_{i=1}^{k-2}\left[\frac{1}{2(2k+2i)}\right] + 6\times(2i+2)\prod_{i=1}^{k-2}\left[\frac{1}{4k+4i+1}\right] + 6\times(2i+2)\prod_{i=1}^{k-2}\left[\frac{1}{4k+4i+3}\right] + 6\times 3(2k-3)\left[\frac{1}{2(2k+2(k-1))}\right] + 6\times 2\left[\frac{1}{(4k+4(k-1)+1)}\right]$$

(ii) $$^{m}M_2\Pi E(L(G)) = 6\times 3\left[\frac{1}{(2k+1)^2}\right]\times 6\times 2\left[\frac{1}{(2k+1)(2k+2)}\right]\times 6\times(2i+2)\prod_{i=1}^{k-2}\left[\frac{1}{(2k+2i)(2k+2i+1)}\right]\times 6\times(2i+2)\prod_{i=1}^{k-2}\left[\frac{1}{(2k+2i+1)(2k+2i+2)}\right]\times 6\times 3(2k-3)\left[\frac{1}{(2k+2(k-1))^2}\right]\times 6\times 2\left[\frac{1}{(2k+2(k-1))(2k+2(k-1)+1)}\right]$$

(iii) $H_b\Pi E(L(G)) = 6\times 3\left[\frac{1}{(2k+1)}\right] + 6\times 2\left[\frac{2}{2k+3}\right] + 6\times (2i+1)\prod_{i=1}^{k-2}\left[\frac{1}{(2k+2i)}\right] +$

$$6\times(2i+2)\prod_{i=1}^{k-2}\left[\frac{2}{4k+4i+1}\right]+6\times(2i+2)\prod_{i=1}^{k-2}\left[\frac{2}{4k+4i+3}\right]+$$

$$6\times 3(2k-3)\left[\frac{1}{(2k+2(k-1))}\right] + 6\times 2\left[\frac{2}{4k+4(k-1)+1}\right]$$

Proof: Consider the General form of $L(H_k)$- Circumcoronene graph G.

Hence we have

(i) ${}^mM_1\Pi E(L(G)) = \prod_{\boldsymbol{ef}\in \boldsymbol{E(L(G))}}[\frac{1}{e_{L(G)}(e)+e_{L(G)}(f)}]$

$$= \prod_{ef\in E_1(L(G))}[\frac{1}{e_{L(G)}(e)+e_{L(G)}(f)}]\times\ldots\ldots\ldots\times$$

$$\prod_{ef\in E_{3(k-1)+1}(L(G))}[\frac{1}{e_{L(G)}(e)+e_{L(G)}(f)}]$$

$$= 6\times 3\left[\frac{1}{2(2k+1)}\right] + 6\times 2\left[\frac{1}{(4k+3)}\right] + 6\times (2i+1)\prod_{i=1}^{k-2}\left[\frac{1}{2(2k+2i)}\right] +$$

$$6\times(2i+2)\prod_{i=1}^{k-2}\left[\frac{1}{4k+4i+1}\right] + 6\times(2i+2)\prod_{i=1}^{k-2}\left[\frac{1}{4k+4i+3}\right]+$$

$$6\times 3(2k-3)\left[\frac{1}{2(2k+2(k-1))}\right] + 6\times 2\left[\frac{1}{(4k+4(k-1)+1)}\right]$$

(ii) ${}^mM_2\Pi E(L(G)) = \prod_{\boldsymbol{ef}\in \boldsymbol{E(L(G))}}[\frac{1}{e_{L(G)}(e)\times e_{L(G)}(f)}]$

$$= \prod_{ef\in E_1(L(G))}[\frac{1}{e_{L(G)}(e)\times e_{L(G)}(f)}]\times\ldots\ldots\ldots\times$$

$$\prod_{ef\in E_{3(k-1)+1}(L(G))}[\frac{1}{e_{L(G)}(e)\times e_{L(G)}(f)}]$$

$$= 6\times 3\left[\frac{1}{(2k+1)^2}\right]\times 6\times 2\left[\frac{1}{(2k+1)(2k+2)}\right]\times$$

$$6\times(2i+2)\prod_{i=1}^{k-2}\left[\frac{1}{(2k+2i)(2k+2i+1)}\right]\times$$

$$6\times(2i+2)\prod_{i=1}^{k-2}\left[\frac{1}{(2k+2i+1)(2k+2i+2)}\right]\times$$

$$6\times 3(2k-3)\left[\frac{1}{(2k+2(k-1))^2}\right]\times$$

$$6\times 2\left[\frac{1}{(2k+2(k-1))(2k+2(k-1)+1)}\right]$$

(iii) $H_b\Pi E(L(G)) = \prod_{ef\in E(L(G))}[\frac{2}{e_{L(G)}(e)+e_{L(G)}(f)}]$

$$= \prod_{ef\in E_1(L(G))}[\frac{2}{e_{L(G)}(e)+e_{L(G)}(f)}] \times\ldots\ldots\times$$

$$\prod_{ef\in E_{3(k-1)+1}(L(G))}[\frac{2}{e_{L(G)}(e)+e_{L(G)}(f)}]$$

$$= 6\times 3[\frac{1}{(2k+1)}] + 6\times 2[\frac{2}{2k+3}] + 6\times(2i+1)\prod_{i=1}^{k-2}[\frac{1}{(2k+2i)}] +$$

$$6\times(2i+2)\prod_{i=1}^{k-2}[\frac{2}{4k+4i+1}] + 6\times(2i+2)\prod_{i=1}^{k-2}[\frac{2}{4k+4i+3}] +$$

$$6\times 3(2k-3)[\frac{1}{(2k+2(k-1))}] + 6\times 2[\frac{2}{4k+4(k-1)+1}]$$

Using MATLAB programme, we have calculated these indices for $L(H_1)$, $L(H_2)$ and $L(H_3)$. Those values are given below as corollaries.

Corollary 3.2.6: $L(H_1)$be the first terms of the line graph of Circumcoronene series of Benzene $L(H_k)$. Then, (i) ${}^mM_1\Pi E(L(H_1)) = 0.0031$,
(ii) ${}^mM_2\Pi E(L(H_1)) = 0.0031$and (iii) $H_b\Pi E(L(H_1)) = 0.0123$.

Corollary 3.2.7: $L(H_2)$ be the second terms of the line graph of Circumcoronene series of Coronene $L(H_k)$.Then,

(i) ${}^mM_1\Pi E(L(H_2)) = 9.7301e^{-007}$, (ii) ${}^mM_2\Pi E(L(H_2)) = 3.6460e^{-015}$
and (iii) $H_b\Pi E(L(H_2)) = 1.5568e^{-005}$

Corollary 3.2.8: $L(H_3)$ be the third terms of the line graph of Circumcoronene series of Circumcoronene $L(H_k)$.Then,

(i) ${}^mM_1\Pi E(L(H_3)) = 7.5276e^{-014}$, (ii) ${}^mM_2\Pi E(L(H_3)) = 3.4140e^{-031}$
and (iii) $H_b\Pi E(L(H_3)) = 9.6353e^{-012}$.

3.3 Eccentricity based on Zagreb indices, Redefined Zagreb indices and Gourava indices and hyper Gourava indices of polynomials for $L(H_k)$:

In this section, we calculate the eccentricity based on Zagreb indices, Redefined Zagreb indices and Gourava indices and hyper Gourava indices of polynomials for $L(H_k)$.

Theorem 3.3.1: For any positive integer number k, let $L(H_k)$ be the general form of line graph of circumcoronene series of benzenoid system, then we calculate the following indices of

(i) FE(L(G)), (ii) $B_1E(L(G, x))$, (iii) $B_2E(L(G, x))$, (iv) $B_3E(L(G))$,

(v) $B_3E(L(G, x))$,(vi) $B_4E(L(G, x))$, (vii) $B_5E(L(G, x))$, (viii) $B_{a,b}E(L(G, x))$

and (ix) $Ƀ_{a,b}E(L(G, x))$ for the graph (H_k).

Proof: Consider the General form of $L(H_k)$- Circumcoronene graph G.

Hence we have

(i) $FE(L(G)) = \sum_{ef\in E(L(G))}[e_{L(G)}(e)^2 + e_{L(G)}(f)^2]$

$$= 6\times 3[2(2k+1)^2] + 6\times 2[(2k+1)^2 + (2k+2)^2] +$$
$$6\times(2i+1)\sum_{i=1}^{k-2}[(2(2k+1))^2]+$$
$$6\times(2i+2)\sum_{i=1}^{k-2}[(2k+2i)^2 + (2k+2i+1)^2]+$$
$$6\times(2i+2)\sum_{i=1}^{k-2}[(2k+2i+1)^2 + (2k+2i+2)^2]+$$
$$6\times 3[(2k-3)][2\big(2k+2(k-1)\big)]^2+$$
$$6\times 2[\big(2k+2(k-1)\big)^2 + (2k+2(k-1)+1)^2]$$

(ii) $B_1E(L(G, x)) = \sum_{ef\in E(L(G))} x^{e_{L(G)}(e)+e_{L(G)}(f)}$

$$= 6\times 3\,x^{2(2k+1)} + 6\times 2\,x^{4k+3} + 6\times(2i+1)\sum_{i=1}^{k-2} x^{4k+4i} +$$
$$6\times(2i+2)\sum_{i=1}^{k-2} x^{4k+4i+1} + 6\times(2i+2)\sum_{i=1}^{k-2} x^{4k+4i+3} +$$
$$6\times 3(2k-3)x^{(8k-4)} + 6\times 2\,x^{8k-3}$$

(iii) $B_2E(L(G,x)) = \sum_{ef \in E(L(G))} x^{e_{L(G)}(e) \times e_{L(G)}(f)}$

$$= 6 \times 3\, x^{(2k+1)^2} + 6 \times 2\, x^{(2k+1)(2k+2)} +$$

$$6 \times (2i+1) \sum_{i=1}^{k-2} x^{(2k+1)^2} +$$

$$6 \times (2i+2) \sum_{i=1}^{k-2} x^{(2k+2i)(2k+2i+1)} +$$

$$6 \times 3(2k-3) x^{(2k+2(k-1)^2} + 6 \times 2\, x^{(2k+2(k-1))(2k+2(k-1)+1)}$$

(iv) $B_3E(L(G)) = \sum_{ef \in E(L(G))} |e_{L(G)}(e) - e_{L(G)}(f)|$

$$= 0 + 6 \times 2(-1) + 0 + 6 \times (2i+2) \sum_{i=1}^{k-2} (-1) +$$

$$6 \times (2i+2) \sum_{i=1}^{k-2} (-1) + 0 + 6 \times 2\,(-1)$$

$$= -24 + 6 \times (2i+2) \sum_{i=1}^{k-2} (-1) + 6 \times (2i+2) \sum_{i=1}^{k-2} (-1)$$

(v) $B_3E(L(G,x)) = \sum_{ef \in E(L(G))} x^{|e_{L(G)}(e) - e_{L(G)}(f)|}$

$$= -24 + 6 \times (2i+2) \sum_{i=1}^{k-2} x^{(-1)} + 6 \times (2i+2) \sum_{i=1}^{k-2} x^{(-1)}$$

(vi) $B_4E(L(G,x)) = \sum_{ef \in E(L(G))} x^{e_{L(G)}(e)((e_{L(G)}(e) + e_{L(G)}(f))}$

$$= 6 \times 3\, x^{2(2k+1)^2} + 6 \times 2\, x^{(2k+1)[(2k+1)+(2k+2)]}$$

$$6 \times (2i+1) \sum_{i=1}^{k-2} x^{(2k+2i)(4k+4i)} +$$

$$6 \times (2i+2) \sum_{i=1}^{k-2} x^{(2k+2i)(4k+4i+1)} +$$

$$6 \times (2i+2) \sum_{i=1}^{k-2} x^{(2k+2i+1)(4k+4i+3)} +$$

$$6 \times 3(2k-3) x^{(2k+2(k-1))(8k-4)} + 6 \times 2\, x^{(2k+2(k-1))(8k-3)}$$

(vii) $B_5E(L(G, x)) = \sum_{ef \in E(L(G))} x^{e_{L(G)}(f)((e_{L(G)}(e)+e_{L(G)}(f))}$

$= 6 \times 3\, x^{2(2k+1)^2} + 6\times 2\, x^{(2k+2)[(4k+3)]} +$

$6 \times (2i+1) \sum_{i=1}^{k-2} x^{2(2k+2i)^2} +$

$6 \times (2i+2) \sum_{i=1}^{k-2} x^{(2k+2i+1)(4k+4i+1)} +$

$6 \times (2i+2) \sum_{i=1}^{k-2} x^{(2k+2i+2)(4k+4i+3)} +$

$6 \times 3(2k-3) x^{2(2k+2(k-1))^2} + 6\times 2\, x^{(2k+2(k-1)+1)(4k+4(k-1)+1)}$

(viii) $B_{a,b}E(L(G, x)) = \sum_{ef \in E(L(G))} x^{a(e_{L(G)}(e)) + b(e_{L(G)}(f))}$

$= 6 \times 3\, x^{[a(2k+1)+b(2k+1)]} + 6\times 2\, x^{[a(2k+1)+b(2k+2)]} +$

$6 \times (2i+1) \sum_{i=1}^{k-2} x^{[a(2k+2i)+b(2k+2i)]} +$

$6 \times (2i+2) \sum_{i=1}^{k-2} x^{[a(2k+2i)+b(2k+2i+2)]} +$

$6 \times (2i+2) \sum_{i=1}^{k-2} x^{[a(2k+2i+1)+b(2k+2i+2)]} +$

$6 \times 3(2k-3) x^{[a(2k+2(k-1))+b(2k+2(k-1))]} + 6\times 2\, x^{[a(2k+2(k-1))+b(2k+2(k-1)+1)]}$

(ix) $B_{a,b}E(L(G, x)) = \sum_{ef \in E(L(G))} x^{(e_{L(G)}(e)+a)+(e_{L(G)}(f)+b)}$

$= 6 \times 3\, x^{[(2k+1)+a+(2k+1)+b)]} + 6\times 2\, x^{[(2k+1)+a+(2k+2)+b)]} +$

$6 \times (2i+1) \sum_{i=1}^{k-2} x^{[(2k+2i)+a)+(2k+2i)+b)]} +$

$6 \times (2i+2) \sum_{i=1}^{k-2} x^{[(2k+2i)+a)+(2k+2i+2)+b)]} +$

$6 \times (2i+2) \sum_{i=1}^{k-2} x^{[(2k+2i+1)+a)+(2k+2i+2)+b)]} +$

$6 \times 3(2k-3) x^{[(2k+2(k-1+a)+(2k+2(k-1)+b)]} +$

$6 \times 2\, x^{[(2k+2(k-1+a)+(2k+2(k-1)+1+b)]}$

3.3.2 Redefined Zagreb indices for $L(H_k)$

For any positive integer number k, let $L(H_k)$ be the general form of circumcoronene series of benzenoid system graph G, then

(i) $ReBG_1E(L(G))$

$$= 6\times3[\frac{4k+2}{(2k+1)^2}]+6\times2[\frac{4k+3}{(2k+1).(2k+2)}]+6\times(2i+1)\sum_{i=1}^{k-2}[\frac{4k+4i}{(2k+2i)^2}]+$$

$$6\times(2i+2)\sum_{i=1}^{k-2}[\frac{4k+4i+1}{(2k+2i).(2k+2i+1)}]+6\times(2i+2)\sum_{i=1}^{k-2}[\frac{4k+4i+3}{(2k+2i+1).(2k+2i+2)}]+$$

$$6\times3(2k-3)[\frac{8k-4}{(2k+2(k-1))^2}]+6\times2[\frac{8k-3}{((2k+2(k-1).(2k+2(k-1)+1))}$$

(ii) $ReBG_2E(L(G))$

$$= 6\times3[\frac{(2k+1)^2}{4k+2}]+6\times2[\frac{(2k+1).(2k+2)}{4k+3}]+6\times(2i+1)\sum_{i=1}^{k-2}[\frac{(2k+2i)^2}{4k+4i}]+$$

$$6\times(2i+2)\sum_{i=1}^{k-2}[\frac{(2k+2i).(2k+2i+1)}{4k+4i+1}]+6\times(2i+2)\sum_{i=1}^{k-2}[\frac{(2k+2i+1).(2k+2i+2)}{4k+4i+3}]+$$

$$6\times3(2k-3)[\frac{(2k+2(k-1))^2}{8k-4}]+6\times2[\frac{((2k+2(k-1).(2k+2(k-1)+1))}{8k-3}]$$

Proof: Consider the General form of H_k - Circumcoronene graph G.

Hence we have

(i) $ReBG_1E(L(G)) = \sum_{ef\in E(L(G))}\frac{e_{L(G)}(e)+e_{L(G)}(f)}{e_{L(G)}(e)\,.\,e_{L(G)}(f)}$

$$= 6\times3[\frac{(2k+1)+(2k+1)}{(2k+1).(2k+1)}]+6\times2[\frac{(2k+1)+(2k+2)}{(2k+1).(2k+2)}]+$$

$$6\times(2i+1)\sum_{i=1}^{k-2}[\frac{(2k+2i)+(2k+2i)}{(2k+2i).(2k+2i)}]+$$

$$6\times(2i+2)\sum_{i=1}^{k-2}[\frac{(2k+2i)+(2k+2i+1)}{(2k+2i).(2k+2i+1)}]+$$

$$6\times(2i+2)\sum_{i=1}^{k-2}[\frac{(2k+2i+1)+(2k+2i+2)}{(2k+2i+1).(2k+2i+2)}]+$$

$$6 \times 3(2k-3)[\frac{((2k+2(k-1)+(2k+2(k-1))}{((2k+2(k-1).(2k+2(k-1))}] +$$

$$6 \times 2[\frac{((2k+2(k-1)+(2k+2(k-1)+1))}{((2k+2(k-1).(2k+2(k-1)+1))}]$$

Therefore

$$= \quad 6\times3[\frac{4k+2}{(2k+1)^{2}}]+6\times2[\frac{4k+3}{(2k+1).(2k+2)}]+ 6\times (2i+1) \sum_{i=1}^{k-2}[\frac{4k+4i}{(2k+2i)^{2}}] +$$

$$6\times(2i+2)\sum_{i=1}^{k-2}[\frac{4k+4i+1}{(2k+2i).(2k+2i+1)}]+6\times(2i+2)\sum_{i=1}^{k-2}[\frac{4k+4i+3}{(2k+2i+1).(2k+2i+2)}]+$$

$$6\times3(2k-3)[\frac{8k-4}{(2k+2(k-1))^{2}}]+6\times 2[\frac{8k-3}{((2k+2(k-1).(2k+2(k-1)+1))}$$

(ii) $\quad ReBG_2E(L(G)) = \sum_{ef\in E(L(G))} \frac{e_{L(G)}(e)\,.\,e_{L(G)}(f)}{e_{L(G)}(e)+e_{L(G)}(f)}$

Therefore

$$= 6 \times 3[\frac{(2k+1)^{2}}{4k+2}] + 6 \times 2[\frac{(2k+1).(2k+2)}{4k+3}]+$$

$$6 \times (2i+1) \sum_{i=1}^{k-2}[\frac{(2k+2i)^{2}}{4k+4i}] + 6 \times (2i+2)\sum_{i=1}^{k-2}[\frac{(2k+2i).(2k+2i+1)}{4k+4i+1}] +$$

$$6 \times (2i+2)\sum_{i=1}^{k-2}[\frac{(2k+2i+1).(2k+2i+2)}{4k+4i+3}] + 6 \times 3(2k-3)[\frac{(2k+2(k-1))^{2}}{8k-4}] +$$

$$6 \times 2[\frac{((2k+2(k-1).(2k+2(k-1)+1))}{8k-3}]$$

3.3.3 Gourava indices and Hyper - Gourava indices of $L(H_k)$

For any positive integer number k, let $L(H_k)$ be the general form of circumcoronene series of benzenoid system graph G. Here, we determine the Gourava indices and Hyper-Gourava indices of $L(H_k)$.

Hence we have

(i) $GO_1E(L(G)) = \sum_{ef \in E(L(G))} (e_{L(G)}(e) + e_{L(G)}(f)) + (e_{L(G)}(e) e_{L(G)}(f))$

$$= 6 \times 3[3(2k+1)^3] + 6 \times 2\big[(2k+1) + (2k+2) + \big((2k+1)(2k+2)\big)\big] +$$

$$6 \times (2i+1) \sum_{i=1}^{k-2} [\big(3(2k+2i)\big)^3] +$$

$$6 \times (2i+2) \sum_{i=1}^{k-2} \begin{matrix}[(2k+2i) + (2k+2i+1) + \\ ((2k+2i)(2k+2i+1))]\end{matrix}$$

$$6 \times (2i+2) \sum_{i=1}^{k-2} \begin{matrix}[(2k+2i+1) + (2k+2i+2) \\ +((2k+2i+1)(2k+2i+2)]\end{matrix} +$$

$$6 \times 3[(2k-3)][(3\big(2k+2(k-1)\big)^3] +$$

$$6 \times 2[(2k+2(k-1) + (2k+2(k-1)+1)) +$$

$$((2k+2(k-1))(2k+2(k-1)+1)]$$

(ii) $GO_2E(L(G)) = \sum_{ef \in E(L(G))} (e_{L(G)}(e) + e_{L(G)}(f))\,(e_{L(G)}(e) e_{L(G)}(f)$

$$= 6 \times 3[2(2k+1)^3] + 6 \times 2[(4k+3)(2k+1) + (2k+2)] +$$

$$6 \times (2i+1) \sum_{i=1}^{k-2} [\big(2(2k+2i)\big)^3 (2k+2i)] +$$

$$6 \times (2i+2) \sum_{i=1}^{k-2} [(4k+4i+1)(2k+2i) + (2k+2i+1)] +$$

$$6 \times (2i+2) \sum_{i=1}^{k-2} [(4k+4i+3)(2k+2i+1) + (2k+2i+2)] +$$

$$6 \times 3[(2k-3)][(2\big(2k+2(k-1)\big)^3] +$$

$$6 \times 2[\big(2k+2(k-1)\big)^2 (2k+2(k-1)+1) +$$

$$(2k+2(k-1)+1)^2 (2k+2(k-1))]$$

(iii) $HGO_1E(L(G)) = \sum_{ef \in E(L(G))} [(e_{L(G)}(e) + e_{L(G)}(f)) + (e_{L(G)}(e) e_{L(G)}(f))]^2$

$$= 6 \times 3\big[3(2k+1)^3\big]^2$$

$$6 \times 2\big[(2k+1) + (2k+2) + \big((2k+1)(2k+2)\big)\big]^2 +$$

$6 \times (2i+1) \sum_{i=1}^{k-2} [\left(3(2k+2i)\right)^3]^2 +$

$6 \times (2i+2) \sum_{i=1}^{k-2} [(2k+2i) + (2k+2i+1) +$

$((2k+2i)(2k+2i+1))]^2 +$

$6 \times (2i+2) \sum_{i=1}^{k-2} [(2k+2i+1) + (2k+2i+2) +$

$((2k+2i+1)(2k+2i+2)]^2 +$

$6 \times 3[(2k-3)][(3\left(2k+2(k-1)\right)^3]^2 +$

$6 \times 2[(2k+2(k-1) + (2k+2(k-1)+1))^2 +$

$((2k+2(k-1))(2k+2(k-1)+1)]$

(iii) $HGO_2E(L(G)) = \sum_{ef \in E(L(G))} [(e_{L(G)}(e) + e_{L(G)}(f))\,(e_{L(G)}(e) e_{L(G)}(f))]^2$

$= 6 \times 3[2(2k+1)^3]^2 + 6 \times 2[(4k+3)(2k+1) + (2k+2)]^2 +$

$6 \times (2i+1) \sum_{i=1}^{k-2} [\left(2(2k+2i)\right)^3 (2k+2i)]^2 +$

$6 \times (2i+2) \sum_{i=1}^{k-2} [(4k+4i+1)(2k+2i) + (2k+2i+1)]^2 +$

$6 \times (2i+2) \sum_{i=1}^{k-2} [(4k+4i+3)(2k+2i+1) + (2k+2i+2)]^2 +$

$6 \times 3[(2k-3)][(2\left(2k+2(k-1)\right)^3]^2 +$

$6 \times 2[\left(2k+2(k-1)\right)^2 (2k+2(k-1)+1) +$

$(2k+2(k-1)+1)^2 (2k+2(k-1))]^2$

CHAPTER 4

ECCENTRIC INDICES OF BORON TRIANGULAR NANOTUBES

In this chapter, we calculate the Milan Randic eccentric, inverse Randic eccentric, Reduced reciprocal Randic eccentric boron triangular nanotubes index, reduced second Zagreb eccentric, sum line connectivity eccentric boron triangular nanotubes index Eccentric indices of Boron Triangular Nanotubes.

We introduced the Milan Randic eccentric boron triangular nanotubes index, defined as

$$R_{-1/2}\,(\mathrm{EBTN}[m,n]) = \sum_{\boldsymbol{ue})} \frac{1}{\sqrt{e_{(G)}(\mathrm{u}) + e_{(\mathrm{G})}(v)}}. \tag{4.1}$$

For general detains about $R_{-1/2}(G)$ and its generalized Randic eccentric boron triangular nanotubes index, defined as

$$R_{\alpha}\,(\mathrm{EBTN}[m,n]) = \sum_{\boldsymbol{ue}} \frac{1}{(e_{(G)}(\mathrm{u}) + e_{(\mathrm{G})}(v))^{2}} \tag{4.2}$$

We introduced the inverse Randic eccentric boron triangular nanotubes index, defined as

$$R\mathrm{R}\alpha\;(\mathrm{EBTN}[m,n]) = \sum_{\boldsymbol{ue}}(e_{(G)}(\mathrm{u}) + \; e_{(\mathrm{G})}(v))^{\alpha}. \tag{4.3}$$

We introduced the reciprocal Randic eccentric boron triangular nanotubes index, defined as

$$\mathrm{RR}(\mathrm{EBTN}[m,n]) = \sum_{\boldsymbol{uv}\in\boldsymbol{E(G)}} \sqrt{e_{(G)}(\mathrm{u}) \times e_{(\mathrm{G})}(v)}. \tag{4.4}$$

We introduced the Reduced reciprocal Randic eccentric boron triangular nanotubes index is defined as

$$\mathrm{RRR(EBTN[m,n])} = \sum_{uv\in E(G)} \sqrt{(e_{(G)}(\mathrm{u})-1)(e_{(G)}(\mathrm{v})-1)} \qquad (4.5)$$

Also, we introduced two indices, defined as

$$\mathrm{M_1(EBTN[m,n])} = \sum_{ue}(e_{(G)}(\mathrm{u}) + e_{(\mathrm{G})}(v))^{\alpha} \quad and \qquad (4.6)$$

$$\mathrm{M_2(G)} = \sum_{ue}(e_{(G)}(\mathrm{u}) \times e_{(\mathrm{G})}(v))^{\alpha}. \qquad (4.7)$$

We introduced the reduced second Zagreb eccentric boron triangular nanotubes index, defined as

$$\mathrm{RM_2(EBTN[m,n])} = \sum_{uv\in E(G)}(e_{(G)}(\mathrm{u})-1)(e_{(\mathrm{G})}(v)-1). \qquad (4.8)$$

We introduced the Forgotten eccentric boron triangular nanotubes index, defined as

$$\mathrm{F(EBTN[m,n])} = \sum_{uv\in E(G)}(e_{(\mathrm{G})}(u))^2 + (e_{(\mathrm{G})}(v))^2). \qquad (4.9)$$

We introduced the first & second modified Zagreb eccentric boron triangular nanotubes index, defined as

$$^{m}\mathrm{B_1(EBTN[m,n])} = \sum_{ue} \frac{1}{e_{(G)}(\mathrm{u}) + e_{(\mathrm{G})}(v)} \text{ and} \qquad (4.10)$$

$$^{m}\mathrm{B_2(EBTN[m,n])} = \sum_{ue} \frac{1}{e_{(G)}(\mathrm{u}) \times e_{(\mathrm{G})}(v)} \qquad (4.11)$$

We introduced the harmonic eccentric boron triangular nanotubes index defined as

$$\mathrm{H(EBTN[m,n])} = \sum_{uv\in EG)} \frac{2}{e_{(G)}(\mathrm{u}) + e_{(\mathrm{G})}(v)} \qquad (4.12)$$

We introduced inverse sum eccentric boron triangular nanotubes index, defined as

$$I(EBTN[m,n]) = \sum_{ue} \frac{e_{(G)}(u) \times e_{(G)}(v)}{e_{(G)}(u) + e_{(G)}(v)} \quad (4.13)$$

We introduced the augmented zagreb eccentric boron triangular nanotubes index,

defined as $$A(EBTN[m,n]) = \sum_{ue} \{\frac{e_{(G)}(u) \times e_{(G)}(v)}{e_{(G)}(u) + e_{(G)}(v) - 2}\} \quad (4.14)$$

We introduced the Randic connectivity eccentric index called the geometric-arithmetic eccentric boron triangular nanotubes index, defined as

$$GA(EBTN[m,n]) = \sum_{ue} \frac{2\sqrt{(e_{(G)}(u) \times e_{(G)}(v))}}{(e_{(G)}(u) + e_{(G)}(v))} \quad (4.15)$$

We introduced the eccentricity based connectivity eccentric boron triangular nanotubes index, defined as

$$\chi E(EBTN[m,n]) = \sum_{ue} \frac{1}{\sqrt{e_G(u)\, e_{L(G)}(e)}} . \quad (4.16)$$

We introduced the sum line connectivity eccentric boron triangular nanotubes index, defined as

$$SLCEII\,(EBTN[m,n]) = \sum_{ue} \sqrt{\frac{e_{L(G)}(e)}{e_G(u) + e_G(v)}}. \quad (4.17)$$

4.1. Some eccentric indices of Boron Triangular Nanotubes:

In this section, we introduced the molecular graphs of eccentricity of boron triangular nanotubes by *EBTN[m, n]* respectively, where *m* is the number of rows and *n* is the number of columns in a *EBTN[m, n]*as shown in order *3mn/2* and size *3n(3m - 2)/2.*

Molecular graph	**Order**	**Size**
EBTN[m, n]	*3mn/2*	*3n(3m - 2)/2*

Table 4.1.1

Also the number of edges with eccentricities of end vertices of G and L(G) are given as follows:

Edge set	No. of edges e = uv	Eccentricity of end vertices (e(u), e(v))	Eccentricity of e in L(G), $e_{L(G)}$ (e)
E_1	2(m-1)	(m-1, m-1)	m
E_2	4(m-1)	(m-1, m-2)	m-1
E_3	2(m-1)	(m-2, m-2)	m-1
E_4	4(m-1)	(m-2, m-3)	m-2
E_5	2(m-1)	(m-3, m-3)	m-2
E_6	4(m-1)	(m-3, m-4)	m-3
E_7	2(m-1)	(m-4, m-4)	m-3
E_8	4(m-1)	(m-4, m-5)	m-4
E_9	2(m-1)	(m-5, m-5)	m-4
E_{10}	4(m-1)	(m-5, m-6)	m-5
.	.	.	.
.	.	.	.
E_{m-5}	2(m-1)	((m+5)/2, (m+3)/2))	(m+5)/2
E_{m-4}	4(m-1)	((m+3)/2, (m+3)/2))	(m+5)/2
E_{m-3}	2(m-1)	((m+3)/2, (m+1)/2))	(m+3)/2
E_{m-2}	4(m-1)	((m+1)/2, (m+1)/2))	(m+3)/2
E_{m-1}	2(m-1)	((m+1)/2, (m+1)/2))	(m+1)/2
E_m	(m-1)	((m+1)/2, (m+1)/2))	(m+1)/2

Table 4.1.2

Theorem 4.1.1: Let the graph G = EBTN[m, n] be a eccentricity of boron triangular nanotubes respectively, then

(i) $EBTNB_1[m, n] = 26m^2 + 8mn - 26m - 8n - 4$

(ii) $EBTNB_2[m, n] = (m-1)[m(4m-4) - n(n^3 - 2n^2 - n + 2)] + (\frac{m-1}{2})[m(4m - 4n^2 + 4n) + n(n^3 - 2n^2 + n)$

(iii) $EBTNHB_1[m, n] = 48m^2 n - 32m^2 - 48m^2 n^2 + 12mn^4 - 14mn^3 + 52mn^2 - 40mn - 12n^4 + 24n^3 - 4n^2 + 34m^3 - 2m^2 + 4m - 8n - 4$

(iv) $EBTNHB_2[m, n] = (m-1)[4m^2 + 4mn^2 - 4m - n^4 + 2n^3 + n^2 - 2n]^2 + (\frac{m-1}{8})[36m + 14n^2 - 14n - 32]$

Proof: Consider the eccentricity of boron triangular nanotube EBTN[m, n] nanotube. From fig.1.3.5 EBTN [m, n] nanotube

Let $V_{m,\ n}$ be the vertex set and $E_{m,\ n}$ be the edge set in EBTN [m, n], then

$$|V_{m,\ n}| = \frac{3mn}{2}\ and|E_2| = \frac{3n(3m\ -\ 2)}{2}.$$

Also the number of edges with eccentricities of end vertices of G and L(G) are given

(i) $EBTNB_1[m, n] = \sum_{ue}[e_G(u) + e_{L(G)}(e)]$

$$= \sum_{e=uv\in E(G)}[e_G(u) + e_{L(G)}(e) + e_G(v) + e_{L(G)}(e)]$$

$$= \sum_{uv\in E_1(G)}[e_G(u) + e_{L(G)}(e) + e_G(v) + e_{L(G)}(e)] + \ldots\ldots + \sum_{uv\in E_m(G)}[e_G(u) + e_{L(G)}(e) + e_G(v) + e_{L(G)}(e)]$$

Therefore

$$= 2(m-1)\ [\ \sum_{i=1}^{n-1}(4m + (2 - 4i)\ ] +$$

$$4(m-1)\ [\ \sum_{i=1}^{n-1}(4m - (1 + 4i)\ ] + 2m\ (m-1)$$

After simplification, we get

$EBTNB_1\,[m, n] = 26\,m^2 + 8mn - 26m - 8n - 4$

(ii) $EBTNB_2\,[m, n] = \sum_{ue}\left[e_G(u) \times e_{L(G)}(e)\right]$

$$= \sum_{uv\in E_1(G)}\left[(e_G(u) \times e_{L(G)}(e)) + (e_G(v) \times e_{L(G)}(e))\right] + \ldots\ldots .. +$$

$$\sum_{uv\in E_m(G)}[(e_G(u) \times e_{L(G)}(e)) + (e_G(v) \times e_{L(G)}(e))]$$

Therefore

$$= 2(m-1)\ [2\sum_{i=1}^{n-1}(m - i)(m - 1 + i)\ ] +$$

$$4(m-1)\ [\ \sum_{i=1}^{n-1}(m - i)^2\ + (m - (1 + i)(m - i)\ ] +$$

$$\frac{1}{2}\ (m-1)\ (m^2 - 1)$$

After simplification, we get

$$EBTNB_2\,[m, n] = (m - 1)[m(4m-4) - n(n^3 - 2n^2\ - n + 2)] +$$

$$(\frac{m-1}{2})\,[m\ (4m - 4n^2 + 4n) + n(n^3 - 2n^2\ + n)$$

(iii) $EBTNHB_1\,[m, n] = \sum_{ue}\left[e_G(u) + e_{L(G)}(e)\right]^2$

$$= \sum_{uv\in E_1(G)}\left[e_G(u) + e_{L(G)}(e)\right]^2 + \left[e_G(v) + e_{L(G)}(e)\right]^2] + \ldots .. +$$

$$\sum_{uv\in E_m(G)}\left[\left[e_G(u) + e_{L(G)}(e)\right]^2 + \left[e_G(v) + e_{L(G)}(e)\right]^2\right]$$

Therefore

$$= 2(m-1)\sum_{i=1}^{n-1}[2(2m-(2i-1))^{2}\ ] +$$

$$4(m-1)\ \sum_{i=1}^{n-1}[2(m-i)]^{2} + [(2m-(2i+1)]^{2} + (m-1)\ (2m^{2})$$

After simplification, we get

$$EBTNHB_{1}[m,n] = 48m^{2}n - 32m^{2} - 48m^{2}n^{2} + 12mn^{4} - 14mn^{3} + 52mn^{2} -$$

$$40mn - 12n^{4} + 24n^{3} - 4n^{2} + 34m^{3} - 2m^{2} + 4m - 8n - 4$$

(iv) $EBTNHB_{2}[m,n] = \sum_{ue}\left[e_{G}(u) \times e_{L(G)}(e)\right]^{2}$

$$= \sum_{e=uv\in E_1(G)}[\left[e_G(u)\times e_{L(G)}(e)\right]^{2} + \left[e_G(v)\times e_{L(G)}(e)\right]^{2}] + \ldots .. +$$

$$\sum_{e=uv\in E_m(G)}[\left[e_G(u)\times e_{L(G)}(e)\right]^{2} + \left[e_G(v)\times e_{L(G)}(e)\right]^{2}]$$

Therefore

$$= 2(m-1)\sum_{i=1}^{n-1} 2[((m-i)(m-1+i))^{2}] +$$

$$4(m-1)\sum_{i=1}^{n-1}[(m-i)^{4} + (m-(1+i)(m-i)]^{2} +$$

$$(m-1)\ [2(\frac{m^{2}-1}{4})^{2}]$$

After simplification, we get

$$EBTNHB_{2}[m,n] = (m-1)[4m^{2} + 4mn^{2} - 4m - n^{4} + 2n^{3} + n^{2} - 2n]^{2} +$$

$$(\frac{m-1}{8})[36m + 14n^{2} - 14n - 32]$$

Theorem 4.1.2: Let the graph G = EBTN [m, n] be a eccentric indices of boron triangular nanotubes respectively, then

(i) $R_{-1/2}(\text{EBTN}[m,n]) = \frac{1}{\sqrt{26m^2 + 8mn - 26m - 8n - 4}}$

(ii) $R_{\alpha}(\text{EBTN}[m,n]) = \frac{1}{\left(26m^2 + 8mn - 26m - 8n - 4\right)^2}$

(iii) $RR\alpha\ (\text{EBTN}[m,n]) = \left(26m^2 + 8mn - 26m - 8n - 4\right)^{\alpha}$

(iv) $M_1(\text{EBTN}[m,n]) = \left(26m^2 + 8mn - 26m - 8n - 4\right)^{\alpha}$

(v) $M_2(\text{EBTN}[m,n]) = (m-1)[m(4m-4) - n(n^3 - 2n^2 - n + 2)] +$

$$\left(\frac{m-1}{2}\right)[m(4m - 4n^2 + 4n) + n(n^3 - 2n^2 + n)$$

Proof: Consider the eccentricity of boron triangular nanotubes G = EBTN [m, n].

Hence we have

(i) $R_{-1/2}(\text{EBTN}[m,n]) = \sum_{ue} \frac{1}{\sqrt{e_{(G)}(u) + e_{(G)}(v)}}$

$$= \sum_{uv \in E_1(G)} \frac{1}{\sqrt{e_G(u) + e_{L(G)}(e) + e_G(v) + e_{L(G)}(e)}} + \ldots.. +$$

$$\sum_{uv \in E_m(G)} \frac{1}{\sqrt{e_G(u) + e_{L(G)}(e) + e_G(v) + e_{L(G)}(e)}}$$

$$= \frac{1}{\sqrt{2(m-1)[\sum_{i=1}^{n-1}(4m + (2-4i) + 4(m-1)[\sum_{i=1}^{n-1}(4m - (1+4i)] + 2m(m-1)}}$$

After simplification, we get

$$R_{-1/2}(\text{EBTN}[m,n]) = \frac{1}{\sqrt{26m^2 + 8mn - 26m - 8n - 4}}$$

(ii) $R_{\alpha}(\text{EBTN}[m,n]) = \sum_{ue} \frac{1}{(e_{(G)}(u) + e_{(G)}(v))^2}$

$$= \sum_{uv \in E_1(G)} \frac{1}{(e_G(u)+e_{L(G)}(e)+e_G(v)+e_{L(G)}(e))^2} + \ldots.. +$$

$$\sum_{uv \in E_m(G)} \frac{1}{(e_G(u)+e_{L(G)}(e)+e_G(v)+e_{L(G)}(e))^2}$$

$$= \frac{1}{\left(2(m-1)[\sum_{i=1}^{n-1}(4m+(2-4i)+4(m-1)+[\sum_{i=1}^{n-1}(4m-(1+4i)]+2m(m-1)\right)^2}$$

After simplification, we get

$$R_{\alpha}(\text{EBTN}[m,n]) = \frac{1}{\left(26m^2 + 8mn - 26m - 8n - 4\right)^2}$$

(iii) $RR\alpha(\text{EBTN}[m,n]) = \sum_{ue}(e_{(G)}(u) + e_{(G)}(v))^{\alpha}$

$$= \sum_{uv \in E_1(G)}(e_G(u) + e_{L(G)}(e) + e_G(v) + e_{L(G)}(e))^{\alpha} + \ldots\ldots +$$

$$\sum_{uv \in E_m(G)}(e_G(u) + e_{L(G)}(e) + e_G(v) + e_{L(G)}(e))^{\alpha}$$

$$= [2(m-1)[\sum_{i=1}^{n-1}(4m + (2-4i)] +$$

$$4(m-1)[\sum_{i=1}^{n-1}(4m - (1+4i)] + 2m(m-1)]^{\alpha}$$

After simplification, we get

$$RR\alpha(\text{EBTN}[m,n]) = \left(26m^2 + 8mn - 26m - 8n - 4\right)^{\alpha}$$

(iv) $M_1(\text{EBTN}[m,n]) = \sum_{ue}(e_{(G)}(u) + e_{(G)}(v))^{\alpha}$

$$= \sum_{uv \in E_1(G)}(e_G(u) + e_{L(G)}(e) + e_G(v) + e_{L(G)}(e))^{\alpha} + \ldots\ldots +$$

$$\sum_{uv \in E_m(G)}(e_G(u) + e_{L(G)}(e) + e_G(v) + e_{L(G)}(e))^{\alpha}$$

$$= [2(m-1)[\sum_{i=1}^{n-1}(4m + (2-4i)] +$$

$$4(m-1)[\sum_{i=1}^{n-1}(4m - (1+4i)] + 2m(m-1)]^{\alpha}$$

After simplification, we get

$$M_1(EBTN[m,n]) = \left(26m^2 + 8mn - 26m - 8n - 4\right)^{\alpha}$$

(v) $M_2(EBTN[m,n]) = \sum_{ue}(e_{(G)}(u) \times e_{(G)}(v))^{\alpha}$

$$= \sum_{uv\in E_1(G)}(((e_G(u)\times e_{L(G)}(e)) + (e_G(v)\times e_{L(G)}(e)))^{\alpha} + \ldots\ldots +$$

$$\sum_{uv\in E_m(G)}(((e_G(u)\times e_{L(G)}(e)) + (e_G(v)\times e_{L(G)}(e)))^{\alpha}$$

$$= [2(m-1)\left[2\sum_{i=1}^{n-1}(m-i)(m-1+i)\right] +$$

$$4(m-1)\left[\sum_{i=1}^{n-1}(m-i)^2 + (m-(1+i)(m-i)\right] +$$

$$\frac{1}{2}(m-1)(m^2-1)]^{\alpha}$$

After simplification, we get

$$M_2(EBTN[m,n]) = (m-1)[m(4m-4) - n(n^3 - 2n^2 - n + 2)] +$$

$$\left(\frac{m-1}{2}\right)[m(4m - 4n^2 + 4n) + n(n^3 - 2n^2 + n)$$

Theorem 4.1.3: Let the graph G = EBTN [m, n] be a eccentricity of boron triangular nanotubes respectively, then

(i) ${}^{m}B_1(EBTN[m,n]) = 4(m-1)\left[\frac{2m-n^2-n-1}{16m^2-8mn^2+4n^4-8n^3+8mn+4n^2}\right] +$

$$\left[\frac{4m-2n^2+2n-1}{4m^2-4mn^2+6mn+2m+n^4-3n^3+n^2+2n}\right] + (m-1)\frac{2}{m}$$

(ii) $\quad {}^{m}B_2(EBTN[m,n]) = 2(m-1)\left[\frac{16m^2-16mn^2+16mn+16m+4n^4-8n^3+4n^2+8n}{(4m^2-4mn^2+4mn+4m+n^4-2n^3-n^2+2)^2}\right]+$

$$\frac{13}{4}(m-1)\left[\frac{(4m^2-4mn^2+4mn-2m+n^4-2n^3+2n^2-3n)^2}{8m^3-12m^2n^2-4m^2+12m^2n-4mn+6mn^4-16mn^3+10mn^2-n^6-3n^5-6n^4+5n^3-n^2}\right]+\frac{8}{m+1}$$

(iii) $\quad H_b(EBTN[m,n]) = 2(m-1)\left[\frac{2m-n^2+n-1}{8m^2-8mn^2+8mn-8m+n^4-2n^3+3n^2-2n+1}\right]+$

$$4(m-1)\left[\frac{4m+2n^2+2n+1}{4m^2-4mn^2+4mn+2m+n^4-2n^3+n}\right]+(m-1)\left(\frac{4}{m}\right)$$

Proof: Consider the boron triangular nanotube G = EBTN[m, n].

Hence we have

(i) $\quad {}^{m}B_1(EBTN[m,n]) \quad = \sum_{ue}\frac{1}{e_{(G)}(u)+e_{(G)}(v)}$

$$=\sum_{uv\in E_1(G)}\left[\frac{1}{e_G(u)+e_{L(G)}(e)}+\frac{1}{e_G(v)+e_{L(G)}(e)}\right]+\ldots\ldots+$$

$$\sum_{uv\in E_m(G)}\left[\frac{1}{e_G(u)+e_{L(G)}(e)}+\frac{1}{e_G(v)+e_{L(G)}(e)}\right]$$

Therefore

$$=2(m-1)\sum_{i=1}^{n-1}\left[\frac{4m-(4i-2)}{((m-i)+(m-1-i))^2}+\right.$$

$$\left.4(m-1)\sum_{i=1}^{n-1}\frac{4m-(4i+1)}{(2m-2i)(2m-(2i+1))}\right]+(m-1)\frac{2}{m}$$

After simplification, we get

$${}^{m}B_1(EBTN[m,n]) = 4(m-1)\left[\frac{2m-n^2-n-1}{16m^2-8mn^2+4n^4-8n^3+8mn+4n^2}\right]+$$

$$\left[\frac{4m-2n^2+2n-1}{4m^2-4mn^2+6mn+2m+n^4-3n^3+n^2+2n}\right]+(m-1)\frac{2}{m}$$

(ii) $\quad {}^{m}B_2(EBTN[m,n]) = \sum_{uv\in EG)} \frac{1}{e_G(u)\times e_{L(G)}(e)}$

$$= \sum_{uv\in E_1(G)}\left[\frac{1}{e_G(u)\times e_{L(G)}(e)} + \frac{1}{e_G(v)\times e_{L(G)}(e)}\right] + \ldots +$$

$$\sum_{uv\in E_m(G)}\left[\frac{1}{e_G(u)\times e_{L(G)}(e)} + \frac{1}{e_G(v)\times e_{L(G)}(e)}\right]$$

$$= 2(m-1)\sum_{i=1}^{n-1}\left[\frac{2(m-i)(m-i+1)}{((m-i)(m-i+1))^2} + \right.$$

$$\left. 4(m-1)\sum_{i=1}^{n-1}\frac{(m-(1+i))(m-i)+(m-i)^2}{(m-i)^3(m-(1+i)))}\right] + (m-1)\left(\frac{8}{m^2-1}\right)$$

After simplification, we get

$${}^{m}B_2(EBTN[m,n]) = 2(m-1)\left[\frac{16m^2-16mn^2+16mn+16m+4n^4-8n^3+4n^2+8n}{(4m^2-4mn^2+4mn+4m+n^4-2n^3-n^2+2)^2}\right]+$$

$$\frac{13}{4}(m-1)\left[\frac{(4m^2-4mn^2+4mn-2m+n^4-2n^3+2n^2-3n)^2}{8m^3-12m^2n^2-4m^2+12m^2n-4mn+6mn^4-16mn^3+10mn^2-n^6-3n^5-6n^4+5n^3-n^2}\right]+\frac{8}{m+1}$$

(iii) $\quad H_b(EBTN[m,n]) = \sum_{uv\in EG)} \frac{2}{e_G(u)+e_{L(G)}(e)}$

$$= \sum_{uv\in E_1(G)}\left[\frac{2}{e_G(u)+e_{L(G)}(e)} + \frac{2}{e_G(v)+e_{L(G)}(e)}\right] + \ldots\ldots +$$

$$\sum_{uv\in E_m(G)}\left[\frac{2}{e_G(u)+e_{L(G)}(e)} + \frac{2}{e_G(v)+e_{L(G)}(e)}\right]$$

$$= 2(m-1)\sum_{i=1}^{n-1}\left[\frac{8m-(8i-4)}{((m-i)+(m-1-i))^2} + \right.$$

$$4(m-1)\sum_{i=1}^{n-1}\frac{(4m-(4i+1)}{(m-i)(2m-(2i+1))} + (m-1)\left(\frac{4}{m}\right)$$

After simplification, we get

$$H_b(EBTN[m,n]) = 2(m-1)\left[\frac{2m-n^2+n-1}{8m^2-8mn^2+8mn-8m+n^4-2n^3+3n^2-2n+1}\right]+$$

$$4(m-1)\left[\frac{4m+2n^2+2n+1}{4m^2-4mn^2+4mn+2m+n^4-2n^3+n}\right] + (m-1)\left(\frac{4}{m}\right)$$

Theorem 4.1.4: Let the graph G = EBTN [m, n] be a eccentricity of boron triangular nanotubes respectively, then

(i) $I(\mathrm{EBTN}[m,n]) = \left[\frac{(m-1)[m(4m-4) - n(n^3 - 2n^2 - n+2)] + (\frac{m-1}{2})[m(4m - 4n^2 + 4n) + n(n^3 - 2n^2 + n)}{26m^2 + 8mn - 26m - 8n - 4}\right]$

(ii) $A(\mathrm{EBTN}[m,n]) = \left[\frac{(m-1)[m(4m-4) - n(n^3 - 2n^2 - n+2)] + (\frac{m-1}{2})[m(4m - 4n^2 + 4n) + n(n^3 - 2n^2 + n)}{26m^2 + 8mn - 26m - 8n - 6}\right]$

(iii) $GA(\mathrm{EBTN}[m,n]) = \frac{2\sqrt{(m-1)[m(4m-4) - n(n^3 - 2n^2 - n+2)] + (\frac{m-1}{2})[m(4m - 4n^2 + 4n) + n(n^3 - 2n^2 + n)}}{26m^2 + 8mn - 26m - 8n - 4}$

Proof: Consider the eccentricity of boron triangular nanotube G = EBTN[m, n]. Hence we have

(i) $I(\mathrm{EBTN}[m,n]) = \sum_{ue} \frac{e_{(G)}(u) \times e_{(G)}(v)}{e_{(G)}(u) + e_{(G)}(v)}$

$$= \sum_{uv \in E_1(G)} \left[\frac{(e_{(G)}(u) \times e_{L(G)}(e)) + (e_{(G)}(v) \times e_{L(G)}(e))}{(e_{(G)}(u) + e_{L(G)}(e)) + (e_{(G)}(v) + e_{L(G)}(e))}\right] + \dots +$$

$$\sum_{uv \in E_m(G)} \left[\frac{(e_{(G)}(u) \times e_{L(G)}(e)) + (e_{(G)}(v) \times e_{L(G)}(e))}{(e_{(G)}(u) + e_{L(G)}(e)) + (e_{(G)}(v) + e_{L(G)}(e))}\right]$$

$$= \left[\frac{2(m-1)\,[2\sum_{i=1}^{n-1}(m-i)(m-1+i)] + 4(m-1)\,[\sum_{i=1}^{n-1}(m-i)^2 + (m-(1+i)(m-i)] + \frac{1}{2}(m-1)(m^2-1)}{2(m-1)\,[\sum_{i=1}^{n-1}(4m+(2-4i)] + 4(m-1)\,[\sum_{i=1}^{n-1}(4m-(1+4i)] + 2m(m-1)}\right]$$

After simplification, we get

$$= \left[\frac{(m-1)[m(4m-4) - n(n^3 - 2n^2 - n + 2)] + (\frac{m-1}{2})\,[m\,(4m - 4n^2 + 4n) + n(n^3 - 2n^2 + n)}{26\,m^2 + 8mn - 26m - 8n - 4} \right]$$

(ii) $A(EBTN[m,n]) = \sum_{ue} \{ \frac{e_{(G)}(u) \times e_{(G)}(v)}{e_{(G)}(u) + e_{(G)}(v) - 2} \}$

$$= \sum_{uv \in E_1(G)} [\frac{(e_{(G)}(u) \times e_{L(G)}(e)) + (e_{(G)}(v) \times e_{L(G)}(e))}{(e_{(G)}(u) + e_{L(G)}(e)) - 2) + (e_{(G)}(v) + e_{L(G)}(e)) - 2)}] + \ldots +$$

$$\sum_{uv \in E_m(G)} [\frac{(e_{(G)}(u) \times e_{L(G)}(e)) + (e_{(G)}(v) \times e_{L(G)}(e))}{(e_{(G)}(u) + e_{L(G)}(e)) - 2) + (e_{(G)}(v) + e_{L(G)}(e)) - 2)}]$$

$$= \left[\frac{2(m-1)\,[2 \sum_{i=1}^{n-1} (m-i)(m-1+i)\,] + 4(m-1)\,[\sum_{i=1}^{n-1} (m-i)^2 + (m-(1+i)(m-i)\,] + \frac{1}{2}\,(m-1)\,(m^2 - 1)}{(2(m-1)[\sum_{i=1}^{n-1} (4m + (2-4i)\,] + 4(m-1)[\sum_{i=1}^{n-1} (4m - (1+4i)\,] + 2m\,(m-1)) - 2} \right]$$

After simplification, we get

$$= \left[\frac{(m-1)[m(4m-4) - n(n^3 - 2n^2 - n + 2)] + (\frac{m-1}{2})\,[m\,(4m - 4n^2 + 4n) + n(n^3 - 2n^2 + n)}{26\,m^2 + 8mn - 26m - 8n - 6} \right]$$

(iii) $GA(EBTN[m,n]) = \sum_{ue} \frac{2\sqrt{(e_{(G)}(u) \times e_{(G)}(v))}}{(e_{(G)}(u) + e_{(G)}(v))}$

$$= \sum_{uv \in E_1(G)} \frac{2\sqrt{(e_{(G)}(u) \times e_{(G)}(e)) + (e_{(G)}(v) \times e_{(G)}(e)}}{((e_{(G)}(u) + e_{(G)}(e)) + ((e_{(G)}(v) + e_{L(G)}(e))} + \ldots +$$

$$\sum_{uv \in E_m(G)} \frac{2\sqrt{(e_{(G)}(u) \times e_{(G)}(e)) + (e_{(G)}(v) \times e_{(G)}(e)}}{((e_{(G)}(u) + e_{(G)}(e)) + ((e_{(G)}(v) + e_{L(G)}(e))}$$

$$= \frac{2\sqrt{2(m-1)\,[2\sum_{i=1}^{n-1}(m-i)(m-1+i)\,]+4(m-1)\,[\sum_{i=1}^{n-1}(m-i)^{2}+(m-(1+i)(m-i)\,]+\frac{1}{2}\,(m-1)\,(m^{2}-1)}}{2(m-1)\,[\sum_{i=1}^{n-1}(4m+(2-4i)\,]+4(m-1)\,[\sum_{i=1}^{n-1}(4m-(1+4i)\,]+2m\,(m-1)}$$

After simplification, we get

$$= \frac{2\sqrt{(m-1)[m(4m-4)-n(n^{3}-2n^{2}-n+2)]+(\frac{m-1}{2})\,[m\,(4m-4n^{2}+4n)+n(n^{3}-2n^{2}+n)}}{26\,m^{2}+8mn-26m-8n-4}$$

Theorem 4.1.5: Let the graph G = EBTN[m, n] be a eccentricity of boron triangular nanotubes respectively, then

$(i)\ \ \chi E(\text{EBTN}[m,n])$

$$= \frac{1}{\sqrt{(m-1)[m(4m-4)-n(n^{3}-2n^{2}-n+2)]+(\frac{m-1}{2})\,[m\,(4m-4n^{2}+4n)+n(n^{3}-2n^{2}+n)}}$$

$$(ii)\ \ XE(\text{EBTN}[m,n]) = \frac{1}{\sqrt{26\,m^{2}+8mn-26m-8n-4}}$$

(iii) SLCEII(EBTN[m, n]

$$= \sqrt{(m-1)\left[\frac{2m-n^{2}+n+2}{2m-n^{2}+n}\right]+2(m-1)\left[\frac{2m-n^{2}+n}{2m-n^{2}+n+1}\right]+\left[\frac{m+1}{2}\right]}$$

Proof: Consider the eccentricity of boron triangular nanotube G = EBTN[m, n].

Hence we have

$$(i)\ \ \chi E(\text{EBTN}[m,n]) = \sum_{ue}\frac{1}{\sqrt{e_G(u)e_{L(G)}(e)}}$$

$$= \sum_{uv\in E_1(G)} \frac{1}{\sqrt{(e_G(u)e_{L(G)}(e))+(e_G(u)e_{L(G)}(e))}} +\ldots\ldots+$$

$$\sum_{uv\in E_{\mathrm{m}}(G)} \frac{1}{\sqrt{(e_G(u)e_{L(G)}(e))+(e_G(u)e_{L(G)}(e))}}$$

$$= \frac{1}{\sqrt{2(\mathrm{m}-1)\,[2\sum_{i=1}^{n-1}(m-i)(m-1+i)\,]+4(\mathrm{m}-1)\,[\sum_{i=1}^{n-1}(m-i)^2+(m-(1+i)(m-i)\,]+\frac{1}{2}\,(\mathrm{m}-1)\,(\mathrm{m}^2-1)}}$$

After simplification, we get

$\chi E(\mathrm{EBTN}[\mathrm{m},\mathrm{n}])$

$$= \frac{1}{\sqrt{(\mathrm{m}-1)[\mathrm{m}(4\mathrm{m}-4)-\mathrm{n}(\mathrm{n}^3-2\mathrm{n}^2-n+2)]+(\frac{m-1}{2})\,[\mathrm{m}\,(4\mathrm{m}-4\mathrm{n}^2+4\mathrm{n})+\mathrm{n}(\mathrm{n}^3-2\mathrm{n}^2+n)}}$$

$$(ii)\ X(\mathrm{EBTN}[\mathrm{m},\mathrm{n}]) = \sum_{ue} \frac{1}{\sqrt{e_G(u)+e_{L(G)}(e)}}$$

$$= \sum_{uv\in E_1(G)} \frac{1}{\sqrt{(e_G(u)+e_{L(G)}(e))+(e_G(u)+e_{L(G)}(e))}} +\ldots..+$$

$$\sum_{uv\in E_{\mathrm{m}}(G)} \frac{1}{\sqrt{(e_G(u)+e_{L(G)}(e))+(e_G(u)+e_{L(G)}(e))}}$$

$$= \frac{1}{\sqrt{2(\mathrm{m}-1)\,[\sum_{i=1}^{n-1}(4m+(2-4i)\,]+4(\mathrm{m}-1)\,[\sum_{i=1}^{n-1}(4m-(1+4i)\,]+2\mathrm{m}\,(\mathrm{m}-1)}}$$

After simplification, we get

$$XE(\mathrm{EBTN}[\mathrm{m},\mathrm{n}]) = \frac{1}{\sqrt{26\,\mathrm{m}^2+8\mathrm{mn}-26\mathrm{m}-8\mathrm{n}-4}}$$

(iii) *SLCEII* (EBTN[m, n]) = $\sum_{ue}\sqrt{\frac{e_{L(G)}(e)}{e_G(u)+e_G(v)}}$.

$$= \sum_{uv\in E_1(G)}\sqrt{\frac{e_{L(G)}(e)}{e_G(u)+e_G(v)}} + \ldots\ldots + \sum_{uv\in E_m(G)}\sqrt{\frac{e_{L(G)}(e)}{e_G(u)+e_G(v)}}$$

$$= \sqrt{2(\mathrm{m}-1)\sum_{i=1}^{n-1}[\frac{m-i+1)}{2m-2i} + 4(\mathrm{m}-1)\sum_{i=1}^{n-1}\frac{m-i}{2m-(2i+1)} + \frac{m+1}{2}}$$

After simplification, we get

$$SLCEII\ (\mathrm{EBTN}[\mathrm{m},\mathrm{n}]) = \sqrt{(\mathrm{m}-1)\left[\frac{2\mathrm{m}-\mathrm{n}^2+\mathrm{n}+2}{2\mathrm{m}-\mathrm{n}^2+\mathrm{n}}\right] + 2(\mathrm{m}-1)\left[\frac{2\mathrm{m}-\mathrm{n}^2+\mathrm{n}}{2\mathrm{m}-\mathrm{n}^2+\mathrm{n}+1}\right] + \left[\frac{m+1}{2}\right]}$$

CHAPTER 5

ECCENTRIC INDICES OF BORON-α NANOTUBES EBAN[M, N]

In this chapter, we calculate the Milan Randic eccentric, inverse Randic eccentric, Reduced reciprocal Randic eccentric boron-α nanotubes index, reduced second Zagreb eccentric, sum line connectivity eccentric boron-α nanotubes index Eccentric indices of boron-α Nanotubes

We introduced the Milan Randic eccentric boron-α nanotubes index, defined as

$$R_{-1/2}\,(\mathrm{EBAN}[m,n]) = \sum_{\boldsymbol{ue})} \frac{1}{\sqrt{e_{(G)}(u) + e_{(G)}(v)}}. \tag{5.1}$$

We introduced the reciprocal Randic eccentric boron-α nanotubes index, defined as

$$\mathrm{RR}(\mathrm{EBAN}[m,n]) = \sum_{\boldsymbol{uv}\in\boldsymbol{E}(\boldsymbol{G})} \sqrt{e_{(G)}(u) \times e_{(G)}(v)}. \tag{5.2}$$

We introduced the Reduced reciprocal Randic eccentric boron-α nanotubes index is defined as

$$\mathrm{RRR}(\mathrm{EBAN}[m,n]) = \sum_{\boldsymbol{uv}\in\boldsymbol{E}(\boldsymbol{G})} \sqrt{(e_{(G)}(u) - 1)(e_{(G)}(v) - 1)} \tag{5.3}$$

Also, we introduced two indices, defined as

$$M_1(\mathrm{EBAN}[m,n]) = \sum_{\boldsymbol{ue}} (e_{(G)}(u) + e_{(G)}(v))^{\alpha} \quad and \tag{5.4}$$

$$M_2(\mathrm{EBAN}[m,n])) = \sum_{\boldsymbol{ue}} (e_{(G)}(u) \times e_{(G)}(v))^{\alpha}. \tag{5.5}$$

We introduced the reduced second Zagreb eccentric boron-α nanotubes index, defined as

$$RM_2(EBAN[m,n]) = \sum_{uv\in E(G)}(e_{(G)}(u) - 1)(e_{(G)}(v) - 1). \quad (5.6)$$

We introduced the Forgotten eccentric boron-α nanotubes index, defined as

$$F(EBAN[m,n]) = \sum_{uv\in E(G)}(e_{(G)}(u))^2 + (e_{(G)}(v))^2). \quad (5.7)$$

We introduced the first & second modified Zagreb eccentric boron-α nanotubes index, defined as

$${}^{m}B_1(EBAN[m,n]) = \sum_{ue}\frac{1}{e_{(G)}(u)+ e_{(G)}(v)} \text{ and} \quad (5.8)$$

$${}^{m}B_2(EBAN[m,n]) = \sum_{ue}\frac{1}{e_{(G)}(u)\times e_{(G)}(v)} \quad (5.9)$$

We introduced the harmonic eccentric boron-α nanotubes index defined as

$$H(EBAN[m,n]) = \sum_{uv\in EG)}\frac{2}{e_{(G)}(u)+ e_{(G)}(v)} \quad (5.10)$$

5.1 Eccentricity of Boron-α Nanotubes EBAN[m,n]

In this section, we introduced the molecular graphs of eccentricity of boron-α nanotubes by *EBAN[m, n]*,respectively, where *m* is the number of rows and *n* is the number of columns in a *EBAN[m, n]*as shown in Fig. 1.3.5

Molecular graph	**Order**	**Size**
EBAN(X)[m, n]	*n(4m + 1)/3*	*n(7m - 2)/2*
EBAN(Y)[m, n]	*4mn/3*	*n(7m - 4)/2*

Table 5.1.1

We categorize the boron-α nanotubes into two classes with respect to m.

We denote these classes as *EBAN(X)[m, n]* and *EBAN(Y)[m, n]* for $m \equiv 2 \bmod 3$ and $m \equiv 0 \bmod 3$, respectively.

In the following theorem, we compute the some indices for *EBAN[m, n]* .

Consider the eccentricity of boron-α nanotube EBAN(X) [m, n] nanotube.

Let $V_{m,\ n}$ be the vertex set and $E_{m,\ n}$ be the edge set in EBAN(X) [m, n], then $|V_{m,\ n}| = \frac{n(4m+1)}{3}$ and $|E_{m,\ n}| = \frac{n(7m-2)}{2}$.

Also the number of edges with eccentricities of end vertices of G and L(G) are given as follows:

Edge set	No. of edges e = uv	Eccentricity of end vertices (e(u), e(v))	Eccentricity of e in L(G), $e_{L(G)}$ (e)
E_1	18	(m-1, m-1)	m
E_2	36	(m-1, m-2)	m-1
E_3	18	(m-2, m-2)	m-1
E_4	24	(m-2, m-3)	m-2
E_5	6	(m-3, m-3)	m-2
E_6	24	(m-3, m-4)	m-3
E_7	18	(m-4, m-4)	m-3
.	.	.	.
.	.	.	.
E_{m-5}	18(m-2)	((m+4)/2, (m+4)/2))	(m+8)/2
E_{m-4}	24(m-2)	((m+4)/2, (m+2)/2))	(m+4)/2
E_{m-3}	6	((m+2)/2, (m+2)/2))	(m+4)/2
E_{m-2}	24(m-1)	((m+2)/2, m/2)	(m+2)/2
E_{m-1}	18(m-1)	(m/2, m/2)	(m+2)/2
E_m	18m	(m/2, m/2)	m/2

Table 5.1.2

5.2 Some eccentric indices of boron-α nanotubes EBAN[m, n]

In this section we calculate the K-eccentric and K- hyper eccentric indices of boron-α nanotubes.

Theorem 5.2.1: Consider the graph G = EBAN(X) [m, n] be a eccentricity of boron-α nanotubes respectively, then

$(i)\ E\mathrm{B}_1\mathrm{AN(X)[m,n]} = 320m - 256n^2 + 256n + 266$

$$(ii)\ E\mathrm{B}_2\mathrm{AN(X)\ [m,n]} = 288m^2 - 600mn^2 + 600mn + 312m + 393n^4 - 768n^3 - 288n^2 + 672n + 576$$

$$(iii)\ HE\mathrm{B}_1\mathrm{AN(X)[m,\ n]} = 744m^2 - 1344mn^2 + 1344mn + 772m + 840n^4 - 1680n^3 - 576n^2 + 1416n + 1296$$

$$(iv)\ HE\mathrm{B}_2\mathrm{AN(X)\ [m,n]} = 9(4m^2 - 4mn^2 + 4mn + 4m + n^4 - 2n^3 - n^2 + 2n)^2 + \frac{45}{4}[(4m^2 - 12mn^2 + 12mn + 16m + 9n^4 - 18n^3 - 15n^2 + 24n + 16) + (4m^2 - 12mn^2 + 12mn + 12m + 9n^4 - 18n^3 - 9n^2 + 18n + 8)^2] + \frac{15}{2}[(4m^2 - 4mn^2 + 4mn - 8m + n^4 - 2n^3 - 5n^2 - 4n + 4)(4m^2 - 4mn^2 + 4mn - 12m + n^4 - 2n^3 + 7n^2 - 6n + 8)^2] + 3(4m^2 - 12mn^2 + 12mn + 4m + 9n^4 - 18n^3 + 3n^2 + 6n)^2 + \frac{9}{4}(m^4 + 4m^3 + 4m^2) + \frac{9}{4}(m^4)$$

Proof: Consider the eccentricity of boron-α nanotube EBAN(X) [m, n] nanotube.

Let V_k be the vertex set of EBAN(X) [m, n]and E_k be the edge set in

EBAN(X) [m, n], then $\left|V_{m,\ n}\right| = \frac{n(4m+1)}{3}\ and \left|E_{m,\ n}\right| = \frac{n(7m-2)}{2}$ for the structure

of EBAN(X) [m, n].

Also the number of edges with eccentricities of end vertices of G and L(G).

We give these values in the above Table 5.1.2

(i) $EB_1AN(X)\ [m, n] = \sum_{ue}\left[e_G(u) + e_{L(G)}(e)\right]$

$= \sum_{e=uv\in E_1(G)}\left[e_G(u) + e_{L(G)}(e) + e_G(v) + e_{L(G)}(e)\right] + \ldots +$

$\sum_{e=uv\in E_m(G)}\left[e_G(u) + e_{L(G)}(e) + e_G(v) + e_{L(G)}(e)\right]$

$= 18\sum_{i=1}^{n-1}[4m-2] + 36\sum_{i=1}^{n-1}[(4m-12i+7] +$

$24\sum_{i=1}^{n-1}[4m-4i-5] + 6\sum_{i=1}^{n-1}[4m-12i-2] +$

$18(2m+2)+18(2m)$

After simplification, we get

$$= 320m - 256n^2 + 256n + 266$$

(ii) $EB_2AN(X)\ [m, n] = \sum_{ue}\left[e_G(u)\times e_{L(G)}(e)\right]$

$= \sum_{e=uv\in E_1(G)}\left[(e_G(u)\times e_{L(G)}(e)) + (e_G(v)\times e_{L(G)}(e))\right] + \ldots +$

$\sum_{e=uv\in E_m(G)}\left[(e_G(u)\times e_{L(G)}(e)) + (e_G(v)\times e_{L(G)}(e))\right]$

$= 18\sum_{i=1}^{n-1} 2[(m-i)\times(m-i+1)] +$

$36\sum_{i=1}^{n-1}[(m-3i+2)^2 + ((m-3i+1)\times(m-3i+2)] +$

$$24\sum_{i=1}^{n-1}[(m-i-1)^2+((m-i-2)\times(m-i-1)]+$$

$$6\sum_{i=1}^{n-1}2[(m-3i)\times(m-3i+1)]+$$

$$18\left(\frac{m(m+2)}{2}\right)+18\left(\frac{m^2}{2}\right)$$

After simplification, we get

$$=288m^2-600mn^2+600mn+312m+393n^4-768n^3$$

$$-288n^2+672n+576$$

(iii) $HEB_1AN(X)\,[m,n]=\sum_{ue}\left[e_G(u)+e_{L(G)}(e)\right]^2$

$$=\sum_{e=uv\in E_1(G)}\left[[e_G(u)+e_{L(G)}(e)]^2+[e_G(v)+e_{L(G)}(e)]^2\right]+\ldots+$$

$$\sum_{e=uv\in E_m(G)}\left[[e_G(u)+e_{L(G)}(e)]^2+[e_G(v)+e_{L(G)}(e)]^2\right]$$

$$=18\sum_{i=1}^{n-1}2\left[(2m-2i+1)^2\right]+$$

$$36\sum_{i=1}^{n-1}[(2m-6i+4)^2+(2m-6i+3)^2]+$$

$$24\sum_{i=1}^{n-1}[(2m-2i-2)^2+(2m-2i-3)^2]+$$

$$6\sum_{i=1}^{n-1}2\left[(2m-6i+1)^2\right]+18\,(2(m+1)^2)+18\,(2m^2)$$

After simplification, we get

$$=744m^2-1344mn^2+1344mn+772m+840n^4-1680n^3$$

$$-576n^2+1416n+1296$$

(iv) $HEB_2AN(X)\,[m,n]=\sum_{ue}\left[e_G(u)\times e_{L(G)}(e)\right]^2$

$$=\sum_{e=uv\in E_1(G)}\left[[e_G(u)\times e_{L(G)}(e)]^2+[e_G(v)\times e_{L(G)}(e)]^2\right]+\ldots+$$

$$\sum_{e=uv\in E_m(G)}\left[[e_G(u)\times e_{L(G)}(e)]^2+[e_{H_k}(v)\times e_{L(H_k)}(e)]^2\right]$$

$= 18\sum_{i=1}^{n-1} 2[(m-i)\times(m-i+1)]^2 +$

$36\sum_{i=1}^{n-1}[(m-3i+2)^2 + ((m-3i+1)\times(m-3i+2))^2]+$

$24\sum_{i=1}^{n-1}[(m-i-1)^2 + ((m-i-2)\times(m-i-1))^2]$

$6\sum_{i=1}^{n-1} 2[(m-3i)\times(m-3i+1)]^2 + 18\left(\frac{(m(m+2))^2}{8}\right) + 18\left(\frac{m^4}{8}\right)$

After simplification, we get

$= 9(4m^2 - 4mn^2 + 4mn + 4m + n^4 - 2n^3 - n^2 + 2n)^2 +$

$\frac{45}{4}[(4m^2 - 12mn^2 + 12mn + 16m + 9n^4 - 18n^3 - 15n^2 +$

$24n + 16) + (4m^2 - 12mn^2 + 12mn + 12m + 9n^4 - 18n^3 - 9n^2 +$

$18n + 8)^2] + \frac{15}{2}[(4m^2 - 4mn^2 + 4mn - 8m + n^4 - 2n^3 - 5n^2 -$

$4n + 4)(4m^2 - 4mn^2 + 4mn - 12m + n^4 - 2n^3 + 7n^2 - 6n + 8)^2] +$

$3(4m^2 - 12mn^2 + 12mn + 4m + 9n^4 - 18n^3 + 3n^2 + 6n)^2 +$

$\frac{9}{4}(m^4 + 4m^3 + 4m^2) + \frac{9}{4}(m^4)$

Lemma 5.2.1: Let the edge partitions of G = EBAN(X)[8, 6] nanotube graph respectively, then (i) EB_1AN(X)$[8,6]$ = 3732, (ii) EB_2AN(X)$[8,6]$ = 11124,

(iii) HEB_1AN(X)$[8,6]$ = 44700 and (iv) HEB_2AN(X)$[8,6]$ = 438060

Proof: Consider the edge partitions of EBAN [8, 6] nanotube graph.

Let V be the vertex set and E be the edge set in EBAN [8, 6], then $|V| = 66\ and |E| = 162$.

Also the number of edges with eccentricities of end vertices of G and L(G) are given as follows

Edge set	No. of edges e = uv	Eccentricity of end vertices (e(u), e(v))	Eccentricity of e in L(G), $e_{L(G)}$ (e)
E_1	18	(7, 7)	8
E_2	36	(7, 6)	7
E_3	18	(6, 6)	7
E_4	24	(6, 5)	6
E_5	6	(5, 5)	6
E_6	24	(5, 4)	5
E_7	18	(4, 4)	5
E_8	18	(4, 4)	4

Table 5.2.1

(i) EBAN$B_1[8,6] = \sum_{ue}\left[e_G(u) + e_{L(G)}(e)\right]$ = 3732

(ii) EBAN$B_2[8,6] = \sum_{ue}\left[e_G(u) \times e_{L(G)}(e)\right]$ =11124

(iii)EBAN$HB_1[8,6] = \sum_{ue}\left[e_G(u) + e_{L(G)}(e)\right]^2$= 44700

(iv)EBAN$HB_2[8,6] = \sum_{ue}\left[e_G(u) \times e_{L(G)}(e)\right]^2$= 438060

Theorem 5.2.5: Let the graph G = EBAN [m, n] be a eccentricity of boron-α nanotubes respectively, then

(i) $R_{-1/2}$ (EBAN[m, n]) = $\frac{1}{\sqrt{320m-256n^2+256n+266}}$

(ii) M_1(EBAN[m, n]) = $(320m - 256n^2 + 256n + 266)^{\alpha}$

(iii) M_2(EBAN[m, n]) = $(288m^2 - 600mn^2 + 600mn + 312m + 393n^4 - 768n^3 - 288n^2 + 672n + 576)^{\alpha}$

(iv) RR(EBAN[m, n]) =

$$\sqrt{288m^2 - 600mn^2 + 600mn + 312m + 393n^4 - 768n^3 - 288n^2 + 672n + 576}$$

Proof: Consider the eccentricity of boron-α nanotubes G = EBAN [m, n]. Hence we have

(i) $R_{-1/2}(\text{EBAN}[m,n]) = \sum_{ue} \frac{1}{\sqrt{e_{(G)}(u) + e_{(G)}(v)}}$

$= \frac{1}{\sqrt{18\sum_{i=1}^{n-1}[4m-2]+36\sum_{i=1}^{n-1}[(4m-12i+7]+24\sum_{i=1}^{n-1}[4m-4i-5]+6\sum_{i=1}^{n-1}[4m-12i-2]+18(2m+2)+18(2m)}}$

After simplification, we get

$$R_{-1/2}(\text{EBAN}[m,n]) = \frac{1}{\sqrt{320m-256n^2+256n+266}}$$

(ii) $M_1(\text{EBAN}[m,n]) = \sum_{ue}(e_{(G)}(u) + e_{(G)}(v))^{\alpha}$

Therefore

$$= [18\sum_{i=1}^{n-1}\left[4m - 2\times 3^{i-1}\right] + 36\sum_{i=1}^{n-1}[(4m - 12i + 7] +$$

$$24\sum_{i=1}^{n-1}[4m - 4i - 5] + 6\sum_{i=1}^{n-1}[4m - 12i - 2] + 18(2m+2)+18(2m)\,]^{\alpha}$$

After simplification, we get

$$M_1(\text{EBAN}[m,n]) = (320m - 256n^2 + 256n + 266)^{\alpha}$$

(iii) $M_2(\text{EBAN}[m,n]) = \sum_{ue}(e_{(G)}(u)\times e_{(G)}(v))^{\alpha}$

Therefore

$$= 18\sum_{i=1}^{n-1} 2[(m-i)\times(m-i+1)]$$

$$36\sum_{i=1}^{n-1}[(m-3i+2)^2 + ((m-3i+1)\times(m-3i+2)] +$$

$$24\sum_{i=1}^{n-1}[(m-i-1)^2 + ((m-i-2)\times(m-i-1)] +$$

$$6\sum_{i=1}^{n-1} 2[(m-3i)\times(m-3i+1)] +$$

$$18\left(\frac{m(m+2)}{2}\right) + 18\left(\frac{m^2}{2}\right)]^{\alpha}$$

After simplification, we get

$$M_2(EBAN[m,n]) = (288m^2 - 600mn^2 + 600mn + 312m + 393n^4 - 768n^3 - 288n^2 + 672n + 576)^{\alpha}$$

(iv) $RR(EBAN[m,n]) = \sum_{uv \in E(G)} \sqrt{e_{(G)}(u) \times e_{(G)}(v)}$

Therefore

$$= \sqrt{18\sum_{i=1}^{n-1} 2[(m-i)\times(m-i+1)] + 36\sum_{i=1}^{n-1}[(m-3i+2)^2 + ((m-3i+1)\times(m-3i+2)] + 24\sum_{i=1}^{n-1}[(m-i-1)^2 + ((m-i-2)\times(m-i-1)] + 6\sum_{i=1}^{n-1} 2[(m-3i)\times(m-3i+1)] + 18\left(\frac{m(m+2)}{2}\right) + 18\left(\frac{m^2}{2}\right)}$$

After simplification, we get

$$= \sqrt{288m^2 - 600mn^2 + 600mn + 312m + 393n^4 - 768n^3 - 288n^2 + 672n + 576}$$

Theorem 5.2.6: Let the graph G = EBAN [m, n] be a eccentric indices of boron-α nanotubes respectively, then

(i) $^{m}B_1(EBAN[m,n]) = \sum_{ue} \frac{1}{e_{(G)}(u) + e_{(G)}(v)}$

Therefore

$$= 4(m-1)\left[\frac{2m-n^2-n-1}{16m^2-8mn^2+4n^4-8n^3+4n^2+8mn}\right] + \left[\frac{4m-2n^2+2n-1}{4m^2-4mn^2+6mn+2m+n^4-3n^3+n^2+2n}\right] + (m-1)\left[\frac{2}{m}\right]$$

(ii) $\quad {}^{m}B_2(EBAN[m,n]) = \sum_{ue} \frac{1}{e_{(G)}(u) \times e_{(G)}(v)}$

Therefore

$$= (m\text{-}1)\left[\frac{16m^2-16mn^2+16mn+16m+4n^4-8n^3+4n^2+8n}{(4m^2-4mn^2+4mn+4m+n^4-2n^3-n^2+2n)^2}\right]+$$

$$\frac{13}{4}(m\text{-}1)\left[\frac{(4m^2-4mn^2+4mn-2m+n^4-2n^3-2n^2-2n)^2}{8m^3-12m^2n^2-4m^2+12m^2n-4mn+6mn^4-16mn^3+10mn^2-n-3n}\right]+\quad \frac{8}{m+1}$$

(iii) $\quad H_b(EBAN[m,n]) = \sum_{uv\in EG)} \frac{2}{e_{(G)}(u)+e_{(G)}(v)}$

Therefore

$$= \left[\frac{152}{(4m^2-4mn^2+4mn+4m+n^4-2n^3-n^2+2n)^2}\right]$$

$$\left[\frac{\frac{(1872m^2-5184mn^2+5184mn+5760m+3888n^4-7776n^3-4752n^2+8640n+4608)^2}{\left(4m^2-12mn^2+12mn+16m+9n^4-18n^3-15n^2+24n+16\right)}}{(4m^2-12mn^2+12mn+12mn+12m+9n^4-18n^3-9n^2+18n+8)}\right]+$$

$$\left[\frac{\frac{1536m^2-1536mn^2+1536mn-3840m+384n^4-768n^3+2304n^2-1872n+2304}{\left(4m^2-4mn^2+4mn-8m+n^4-2n^3+5n^2-4n+4\right)}}{(4m^2-4mn^2+4mn-12m+n^4-2n^3+7n^2-6n+8)}\right]+$$

$$\left[\frac{24}{(4m^2-12mn^2-12mn+4m+9n^4-18n^3+3n^2+6n)}\right]+\left[\frac{576m^2+576m}{m^4+2n^3}\right]$$

Theorem 5.2.7: Let the graph G = EBAN [m, n] be a eccentricity of boron-α nanotubes respectively, then

(i) $I\text{EBAN}[\mathrm{m,n}]) = \sum_{ue} \frac{e_{(G)}(\mathrm{u}) \times e_{(\mathrm{G})}(v)}{e_{(G)}(\mathrm{u}) + e_{(\mathrm{G})}(v)}$

Therefore

$$= \left[\frac{18\sum_{i=1}^{n-1}2[(m-i)\times(m-i+1)]+36\sum_{i=1}^{n-1}[(m-3i+2)^2+((m-3i+1)\times(m-3i+2)]+ 24\sum_{i=1}^{n-1}[(m-i-1)^2+((m-i-2)\times(m-i-1)]+6\sum_{i=1}^{n-1}2[(m-3i)\times(m-3i+1)]+ 18\left(\frac{m(m+2)}{2}\right)+18\left(\frac{m^2}{2}\right)}{18\sum_{i=1}^{n-1}(4m-2)+36\sum_{i=1}^{n-1}[(4m-12i+7]+24\sum_{i=1}^{n-1}[4m-4i-5]+6\sum_{i=1}^{n-1}[4m-12i-2]}\right]$$

$$= \left[\frac{288m^2-600mn^2+600mn+312m+393n^4-768n^3-288n^2+672n+576}{320m-256n^2+256n+266}\right]$$

(ii) $A(\text{EBAN}[\mathrm{m,n}]) = \sum_{ue}\left[\frac{e_{(G)}(\mathrm{u})\times e_{(\mathrm{G})}(v)}{e_{(G)}(\mathrm{u})+e_{(\mathrm{G})}(v)-2}\right]$

Therefore

$$= \left[\frac{18\sum_{i=1}^{n-1}2[(m-i)\times(m-i+1)]+36\sum_{i=1}^{n-1}[(m-3i+2)^2+((m-3i+1)\times(m-3i+2)]+ 24\sum_{i=1}^{n-1}[(m-i-1)^2+((m-i-2)\times(m-i-1)]+6\sum_{i=1}^{n-1}2[(m-3i)\times(m-3i+1)]+ 18\left(\frac{m(m+2)}{2}\right)+18\left(\frac{m^2}{2}\right)}{[18\sum_{i=1}^{n-1}(4m-2)+36\sum_{i=1}^{n-1}[(4m-12i+7]+24\sum_{i=1}^{n-1}[4m-4i-5]+6\sum_{i=1}^{n-1}[4m-12i-2]\,]-2}\right]$$

$$= \left[\frac{288m^2-600mn^2+600mn+312m+393n^4-768n^3-288n^2+672n+576}{320m-256n^2+256n+264}\right]$$

(iii) $GA(\text{EBAN}[\mathrm{m,n}]) = \sum_{ue} \frac{2\sqrt{(e_{(G)}(\mathrm{u})\times e_{(\mathrm{G})}(v))}}{(e_{(G)}(\mathrm{u})+e_{(\mathrm{G})}(v))}$

Therefore

$$= \frac{2\sqrt{18\sum_{i=1}^{n-1}2[(m-i)\times(m-i+1)]+36\sum_{i=1}^{n-1}[(m-3i+2)^2+((m-3i+1)\times(m-3i+2)]+ 24\sum_{i=1}^{n-1}[(m-i-1)^2+((m-i-2)\times(m-i-1)]+6\sum_{i=1}^{n-1}2[(m-3i)\times(m-3i+1)]+ 18\left(\frac{m(m+2)}{2}\right)+18\left(\frac{m^2}{2}\right)}}{18\sum_{i=1}^{n-1}(4m-2)+36\sum_{i=1}^{n-1}[(4m-12i+7]+24\sum_{i=1}^{n-1}[4m-4i-5]+6\sum_{i=1}^{n-1}[4m-12i-2]}$$

$$= \frac{2\sqrt{288m^2 - 600mn^2 + 600mn + 312m + 393n^4 - 768n^3 - 288n^2 + 672n + 576}}{320m - 256n^2 + 256n + 264}$$

Theorem 5.2.8: Let the graph G = EBAN[m, n] be a eccentricity of boron triangular nanotubes respectively, then

(i) $\chi E(\text{EBAN}[m,n]) = \sum_{ue} \frac{1}{\sqrt{e_G(u) e_{L(G)}(e)}}$

$$= \left[\frac{1}{\sqrt{\begin{array}{c} 18\sum_{i=1}^{n-1} 2[(m-i)\times(m-i+1)] + 36\sum_{i=1}^{n-1}[(m-3i+2)^2 + ((m-3i+1)\times(m-3i+2)] + \\ 24\sum_{i=1}^{n-1}[(m-i-1)^2 + ((m-i-2)\times(m-i-1)] + 6\sum_{i=1}^{n-1} 2[(m-3i)\times(m-3i+1)] + \\ 18\left(\frac{m(m+2)}{2}\right) + 18\left(\frac{m^2}{2}\right)\end{array}}}\right]$$

$$= \left[\frac{1}{\sqrt{288m^2 - 600mn^2 + 600mn + 312m + 393n^4 - 768n^3}}\right]$$

(ii) $XE(\text{EBAN}[m,n]) = \sum_{ue} \frac{1}{\sqrt{e_G(u) + e_{L(G)}(e)}}$

$$= \left[\frac{1}{\sqrt{18\sum_{i=1}^{n-1}[4m-2] + 36\sum_{i=1}^{n-1}[(4m-12i+7] + 24\sum_{i=1}^{n-1}[4m-4i-5] + 6\sum_{i=1}^{n-1}[4m-12i-2]}}\right]$$

$$= \left[\frac{1}{\sqrt{320m - 256n^2 + 256n + 266}}\right]$$

(iii) $SLCEII(\text{EBAN}[m,n]) = \sum_{ue} \sqrt{\frac{e_{L(G)}(e)}{e_G(u) + e_G(v)}}.$

$$= 18\sqrt{\frac{m-i+1}{2m-2i}} + 36\sqrt{\frac{m-3i+2}{2m-6i+3}} + 24\sqrt{\frac{m-i-1}{2m-2i-3}} + 6\sqrt{\frac{m-3i+1}{2m-6i}} + 18\sqrt{\frac{m+2}{2m}} + 18\sqrt{\frac{1}{2}}$$

CHAPTER 6

STATUS INDICES OF PERFECT BINARY TREE GRAPHS

In this chapter, we calculate the first, second and hyper status indices, Sum and Product reciprocal connectivity status indices, F_l-status index, Gourava indices, (a, b)-status index, ABC, AG and Augmented status indices of perfect binary tree graphs.

6.1 Results for status of perfect binary tree graphs

In this section, some status of vertices of perfect binary tree graphs was analyzed. A binary tree is said to be perfect if all the internal nodes have strictly two children, and every external or leaf node is at the same level or same depth within a tree. A perfect m-array tree with height h, the upper bound for the maximum number of leaves is m^h. If there is a zero-index level, the number of nodes on the h level is exactly 2^h.

Level 0: 2^0 nodes, Level 1: 2^1 nodes, Level 2: 2^2 nodes, Level 3: 2^3 nodes and so on.

So total number of node in a perfect binary tree with height h is

$$n = 2^0 + 2^1 + 2^2 + 2^3 + \ldots 2^{h-1} + 2^h = 2^{h+1} - 1.$$

The status, denoted by $\sigma(u)$ of a vertex u in G the sum of distance of all other vertices from u to all other vertices in G.

Let G be a binary tree with height h.

Let u_0 denote the central vertex, u_1 denotes a vertex on level 1, u_2 denotes a vertex on level 2,….,u_h denotes a vertex on level h.

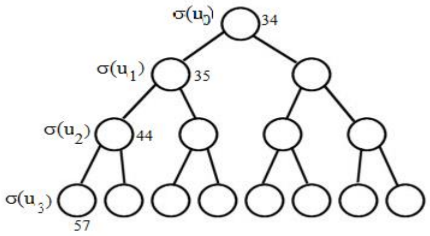


Fig. 6.1.1 Perfect binary tree with status

Now, the status values of vertices of a binary tree are given in table 6.1.1. and so on.

Height of perfect binary tree	$\sigma(u_0)$	$\sigma(u_1)$	$\sigma(u_2)$	$\sigma(u_3)$	$\sigma(u_4)$	
0	0					
1	2	3				
2	10	11	16			
3	34	35	44	57		
4	98	99	116	141	170	
5	258	259	292	341	398	459
.						
.	.	.	.	.	.	.

Table 6.1.1

Next, We shall calculate the status vales of vertices of a perfect binary tree with height n.

$\sigma(u_0) = 2^{n+1}(n-1) + 2$, $\sigma(u_1) = 2^{n+1}(2^0(n-2)+1) + 3$,

$\sigma(u_2) = 2^n(2^1(n-1)+1) + 4$, $\sigma(u_3) = 2^{n-1}(2^2(n)+1) + 5$,

$\sigma(u_4) = 2^{n-2}(2^3(n+1)+1) + 6$, $\sigma(u_5) = 2^{n-3}(2^4(n+2)+1) + 7$,

…………………………………

$\sigma(u_{i-i}) = 2^{n-(i-3)}\{(2^{i-2}[n+(i-1)+3)+1]+(i-1)+2\}$,

$\sigma(u_i) = 2^{n-(i-2)}\{(2^{i-1}[n+(i-3)+1]+i+2\}$,

…………………

$\sigma(u_n) = (2n-1)\,2^{n+1} - 2^{n+2} + n + 5$

Now, we compute $\sigma(u_i) + \sigma(u_{i-i})$ and $\sigma(u_i)\ \sigma(u_{i-i})$

$$\sigma(u_i) + \sigma(u_{i-i}) = 2^{n+1}(2n+2i-7)+3\times 2^{n-i+2}+2i+3 \qquad \text{---- (1)}$$

$$\sigma(u_i)\ \sigma(u_{i-i}) = (4n^2 + 4i^2 + 8ni - 28n - 28i + 48)2^{2n} +$$

$$(4i^2 + 4ni + 6n - 8i - 22)2^n + (24n + 24i - 80)2^{2n-i} +$$

$$(12i+20)2^{n-i} + 32\times 2^{2n-2i} + i^2 + 3i + 2 \qquad \text{---- (2)}$$

Therefore, from (1) & (2)

$$\sum_{i=1}^{n} 2^i(\sigma(u_i) + \sigma(u_{i-1}))$$

$$= \sum_{i=1}^{n} 2^i[(2^{n+1}(n+i-3) + 2^{n-i+2} + i + 2) +$$

$$(2^{n+1}(n+i-4) + 2^{n-i+3} + i + 1)]$$

$$= \sum_{i=1}^{n} 2^i[2^{n+1}(2n+2i-7)+3\times 2^{n-i+2}+2i+3] \qquad \text{---- (3)}$$

$\sum_{i=1}^{n} 2^{i} (\sigma(u_i) + \sigma(u_{i-1})$

$$= \sum_{i=1}^{n} 2^{i} [(4n^2 + 4i^2 + 8ni - 28n - 28i + 48)2^{2n} + (4i^2 + 4ni + 6n - 8i - 22)2^{n} + (24n + 24i - 80)2^{2n-i} + (12i+20)2^{n-i} + 32 \times 2^{2n-2i} + i^2 + 3i + 2] \quad \text{----- (4)}$$

6.2: Status based indices of perfect binary tree graphs

In this section we calculate the first, second and hyper status indices, also the sum, product connectivity status indices, F_l-status index, first and second status Gourava indices, Gourava (a, b)-status indices were investigated. Moreover, it was calculated sum and product connectivity status indices, reciprocal connectivity status indices, ABC, AGS, GAS and ASI status indices of perfect binary tree graphs.

Theorem 6.2.1: Let G be the perfect binary tree graphs. Then

(i) $S_1(G) = (16n - 36)\, 2^{2n} + (8n+38)2^{n} - 2$

(ii) $S_2(G) = (40n^2 - 152n + 176)\, 2^{3n} + (24n^2 - 4n - 148)\, 2^{2n} + (2n^2 - 30n - 24)\, 2^{n} - 4$

(iii) $S_3(G) = 2^{2n+1} - 2^{n}$

(iv) $S_4(G) = (68n^2 - 256n + 239)\, 2^{3n} + (44n^2 + 76n + 30)\, 2^{2n} + (4n^2 - 35n + 89)\, 2^{n} - 35$

(v) $S_5(G) = (68n^2 - 64n + 224)\, 2^{3n} + (36n^2 + 72n - 196)\, 2^{2n} + (4n^2 - 26n + 109)\, 2^{n} + 9$

(vi) $HS_1(G) = (128n^2 - 576n + 712)2^{3n} + (128n^2 - 64n - 576)2^{2n} + (8n^2 + 112n - 118)2^{n} - 18$

(vii) $HS_2(G) = \sum_{i=1}^{n} 2^i [(4n^2 + 4i^2 + 8ni - 28n - 28i + 48)2^{2n} +$
$(4i^2 + 4ni + 6n - 8i - 22)2^n + (24n+24i - 80)2^{2n-i} +$
$(12i+20)2^{n-i} + 32 \times 2^{2n-2i} + i^2 + 3i + 2] \times$
$[(4n^2 + 4i^2 + 8ni - 28n - 28i + 48)2^{2n} + (4i^2 + 4ni +$
$6n - 8i - 22)2^n + (24n+24i - 80)2^{2n-i} + (12i+20)2^{n-i} +$
$32 \times 2^{2n-2i} + i^2 + 3i + 2]^2$

(viii) $S_1^a(G) = \sum_{i=1}^{n} 2^i [(4n + 4i - 14)2^n + 12 \times 2^{n-i} + 2i + 3]^a$

(ix) $S_2^a(G) = \sum_{i=1}^{n} 2^i [(4n^2 + 4i^2 + 8ni - 28n - 28i + 48)2^{2n} +$
$(4i^2 + 4ni + 6n - 8i - 22)2^n + (24n + 24i - 80)2^{2n-i} +$
$(12i+20)2^{n-i} + 32 \times 2^{2n-2i} + i^2 + 3i + 2]^a$

Proof: By using the above definition and values, we deduce

(i) $S_1(G) = \sum_{uv \in E(G)} [\sigma(u) + \sigma(v)] = \sum_{i=1}^{n} 2^i (\sigma(u_i) + \sigma(u_{i-1}))$

$= \sum_{i=1}^{n} 2^i [(2^{n+1}(n + i - 3) + 2^{n-i+2} + i + 2) +$

$(2^{n+1}(n + i - 4) + 2^{n-i+3} + i + 1)]$ from (1) & (2)

Therefore

$= \sum_{i=1}^{n} 2^i [2^{n+1}(2n+2i-7) + 3 \times 2^{n-i+2} + 2i + 3]$

After simplification, we get

$S_1(G) = (16n - 36) 2^{2n} + (8n + 38) 2^n - 2$

(ii) $S_2(G) = \sum_{uv \in E(G)} [\sigma(u)\, \sigma(v)]$

$= \sum_{i=1}^{n} 2^i (\sigma(u_i) \times \sigma(u_{i-1}))$

Therefore

$$= \sum_{i=1}^{n} 2^{i} [(2^{n+1}(n+i-3) + 2^{n-i+2} + i + 2) \times (2^{n+1}(n+i-4) + 2^{n-i+3} + i + 1)]$$

$$= [(4n^2 - 28n + 48)2^{2n} + (30n - 102)2^{n} + 54] (2^{n} - 1) + [(8n - 24)2^{2n} + (4n+16)+15] ((n-1) 2^{n+1} + 2) + [(4\times 2^{n} + 1)] ((n^2 - 2n + 3) 2^{n+1} - 6)$$

After simplification, we get

$$S_2(G) = (40n^2 - 152n+176) 2^{3n} + (24n^2 - 4n - 148) 2^{2n} + (2n^2 - 30n - 24) 2^{n} - 4$$

(iii) $S_3(G) = \sum_{uv\in E(G)} |\sigma(u) - \sigma(v)|$

$$= \sum_{i=1}^{n} 2^{i} |(\sigma(u_i) - \sigma(u_{i-1})|$$

Therefore

$$= \sum_{i=1}^{n} 2^{i} |(2^{n+1}(n+i-3) + 2^{n-i+2} + i + 2) - (2^{n+1}(n+i-4) + 2^{n-i+3} + i + 1)|$$

After simplification, we get

$$S_3(G) = 2^{2n+1} - 2^{n}$$

(iv) $S_4(G) = \sum_{uv\in E(G)} \sigma(u)[\sigma(u) + \sigma(v)]$

$$= \sum_{i=1}^{n} 2^{i} \sigma(u)[(\sigma(u_i) + \sigma(u_{i-1}))]$$

Therefore

$$= \sum_{i=1}^{n} 2^{i} [(2^{n+1}(n+i-3) + 2^{n-i+2} + i + 2) + [(2^{n+1}(n+i-3) + 2^{n-i+2} + i + 2) (2^{n+1}(n+i-4) + 2^{n-i+3} + i + 1)]]$$

After simplification, we get

$S_4(G) = (68n^2 - 256n + 239)\, 2^{3n} + (44n^2 + 76n + 30)\, 2^{2n} + (4n^2 - 35n + 89)\, 2^n - 35$

(v) $S_5(G) = \sum_{uv\in E(G)} \sigma(v)[\sigma(u) + \sigma(v)]$

$= \sum_{i=1}^{n} 2^i \sigma(v)[(\sigma(u_i) + \sigma(u_{i-1}))]$

Therefore

$=\sum_{i=1}^{n} 2^i (2^{n+1}(n+i-4) + 2^{n-i+3} + i + 1)$

$[(2^{n+1}(n+i-3) + 2^{n-i+2} + i + 2) +$

$(2^{n+1}(n+i-4) + 2^{n-i+3} + i + 1)]$

After simplification, we get

$S_5(G) = (68n^2 - 64n + 224)\, 2^{3n} + (36n^2 + 72n - 196)\, 2^{2n} + (4n^2 - 26n + 109)\, 2^n + 9$

(vi) $HS_1(G) = \sum_{uv\in E(G)}[\sigma(u) + \sigma(v)]^2$

$= \sum_{i=1}^{n} 2^i [(\sigma(u_i) + \sigma(u_{i-1})]^2$

$= \sum_{i=1}^{n} 2^i [(2^{n+1}(n+i-3) + 2^{n-i+2} + i + 2) +$

$(2^{n+1}(n+i-4) + 2^{n-i+3} + i + 1)]^2$

Therefore

$= \sum_{i=1}^{n} 2^i [2^{n+1}(2n+2i-7) + 3\times 2^{n-i+2} + 2i + 3]^2$

After simplification, we get

$$HS_1(G) = (128n^2 - 576n + 712)2^{3n} + (128n^2 - 64n - 576)\, 2^{2n} + (8n^2 + 112n - 118)2^n - 18$$

(vii) $HS_2(G) = \sum_{uv\in E(G)}[\sigma(u)\times \sigma(v)]^2$

$= \sum_{i=1}^{n}[2^i\,(\sigma(u_i)\times\sigma(u_{i-1})]^2$

$= \sum_{i=1}^{n} 2^i\,[(2^{n+1}(n+i-3) + 2^{n-i+2} + i + 2) \times$

$(2^{n+1}(n+i-4) + 2^{n-i+3} + i + 1)]^2$

Therefore

$HS_2(G) = \sum_{i=1}^{n} 2^i\,[(4n^2 + 4i^2 + 8ni - 28n - 28i + 48)2^{2n} +$

$\left(4i^2 + 4ni + 6n - 8i - 22\right)2^n +$

$(24n+24i - 80)2^{2n-i} + (12i+20)2^{n-i} + 32\times 2^{2n-2i} + i^2 + 3i + 2] \times$

$[\left(4n^2 + 4i^2 + 8ni - 28n - 28i + 48\right)2^{2n} +$

$\left(4i^2 + 4ni + 6n - 8i - 22\right)2^n +$

$(24n+24i - 80)\,2^{2n-i} + (12i+20)\,2^{n-i} + 32\times 2^{2n-2i} + i^2 + 3i + 2]^2$

(viii) $S_1^{\,a}(G) = \sum_{uv\in E(G)}[\sigma(u) + \sigma(v)]^a = \sum_{i=1}^{n} 2^i\,[(\sigma(u_i) + \sigma(u_{i-1})]^a$

$= \sum_{i=1}^{n} 2^i\,[(2^{n+1}(n+i-3) + 2^{n-i+2} + i + 2) +$

$(2^{n+1}(n+i-4) + 2^{n-i+3} + i + 1)]^a$

Therefore

$S_1^{\,a}(G) = \sum_{i=1}^{n} 2^i\,[(4n + 4i - 14)2^n + 12\times 2^{n-i} + 2i + 3\ ]^a$

(ix) $S_2^{\,a}(G) = \sum_{uv\in E(G)}[\sigma(u)\times \sigma(v)]^a = \sum_{i=1}^{n}[2^i\,(\sigma(u_i)\times\sigma(u_{i-1})]^a$

$= \sum_{i=1}^{n} 2^i\,[(2^{n+1}(n+i-3) + 2^{n-i+2} + i + 2) \times$

$(2^{n+1}(n+i-4) + 2^{n-i+3} + i + 1)]^a$

Therefore

$$S_2^{\,a}(G) = \sum_{i=1}^{n} 2^{i}\, [(4n^2 + 4i^2 + 8ni - 28n - 28i + 48)2^{2n} +$$
$$(4i^2 + 4ni + 6n - 8i - 22)2^{n} +$$
$$(24n + 24i - 80)\, 2^{2n-i} + (12i + 20)\, 2^{n-i} + 32 \times 2^{2n-2i} + i^2 + 3i + 2\,]^{a}$$

Theorem 6.2.2: Let G be the perfect binary tree graph. Then

(i) $$F_1 S(G) = (64n^2 - 288n + 360)\, 2^{3n} + (64n^2 - 40n - 280)\, 2^{2n} +$$
$$(4n^2 + 52n - 70)\, 2^{n} - 10$$

(ii) $$SGO_1(G) = (40n^2 - 152n + 176)\, 2^{3n} + (24n^2 - 12n - 284)\, 2^{2n} +$$
$$(2n^2 - 22n + 14)\, 2^{n} - 6$$

(iii) $$SGO_2(G) = [(40n^2 - 152n + 176)\, 2^{3n} + (24n^2 - 4n - 148)\, 2^{2n} +$$
$$(2n^2 - 30n - 24)\, 2^{n} - 4] \times$$
$$[(16n - 36)\, 2^{2n} + (8n + 38)\, 2^{n} - 2]$$

(iv) $$S_{a,b}(G) = \sum_{i=1}^{n} 2^{i}\, [(2^{n+1}(n + i - 3) + 2^{n-i+2} + i + 2)^{a} \times$$
$$(2^{n+1}(n + i - 4) + 2^{n-i+3} + i + 1)^{b} +$$
$$\left(2^{n+1}(n + i - 3) + 2^{n-i+2} + i + 2\right)^{b} \times$$
$$(2^{n+1}(n + i - 4) + 2^{n-i+3} + i + 1)^{a}\,]$$

(v) $$SDS(G) = \sum_{i=1}^{n} 2^{i}$$

$$\left[\frac{\begin{gathered}[(4n^2 + 4i^2 + 8ni - 32n + 64)2^{2n} + (4i^2 + 4ni + 4n - 12i - 16)2^{n} + (32n + 32i - 128)2^{2n-i} + \\ (16i + 16)2^{n-i} + 64 \times 2^{2n-2i} + i^2 + 2i + 1] + [4n^2 + (4i^2 + 8ni - 24n - 24i + 36)2^{2n} \\ +(4i^2 + 4ni + 8n - 2i - 24)2^{n}\ (16n + 16i - 48)2^{2n-i} + + (8i + 16)2^{n-i} + 16 \times 2^{2n-2i} + i^2 + 4i + 4]\end{gathered}}{\begin{gathered}(4n^2 + 4i^2 + 8ni - 28n - 28i + 48)2^{2n} + (4i^2 + 4ni + 6n - 8i - 22)2^{n} + + (24n + 24i - 80)\, 2^{2n-i} + (12i + 20)\, 2^{n-i} \\ (4n^2 + 4i^2 + 8ni - 28n - 28i + 48)2^{2n} + +32 \times 2^{2n-2i} + i^2 + 3i + 2\end{gathered}}\right]$$

Proof: By using the above definition and values, we deduce

(i) $F_1 S(G) = \sum_{uv\in E(G)}[\sigma(u)^2 + \sigma(v)^2]$

$$= \sum_{i=1}^{n} 2^i [\sigma(u_i)^2 + \sigma(u_{i-1})^2]$$

Therefore

$$= \sum_{i=1}^{n} 2^i [(2^{n+1}(n+i-3) + 2^{n-i+2} + i + 2)^2 + (2^{n+1}(n+i-4) + 2^{n-i+3} + i + 1)^2]$$

After simplification, we get

$$F_1 S(G) = (64n^2 - 288n + 360)\, 2^{3n} + (64n^2 - 40n - 280)\, 2^{2n} + (4n^2 + 52n - 70)\, 2^n - 10$$

(ii) $SGO_1(G) = \sum_{uv\in E(G)}[\sigma(u) + \sigma(v) + \sigma(u)\,\sigma(v)]$

$$= \sum_{i=1}^{n} 2^i [\sigma(u_i) + \sigma(u_{i-1}) + \sigma(u_i)\,\sigma(u_{i-1})]$$

Therefore

$$= \sum_{i=1}^{n} 2^i [(2^{n+1}(2n + 2i - 7) + 3\times 2^{n-i+2} + 2i + 3) + (((2^{n+1}(n+2i-1) + 2^{1-i}) \times (2^{n+1}(n+2i-3) + 2^{2-i}))]$$

After simplification, we get

$$SGO_1(G) = (32n^2 - 144n + 176)\, 2^{3n} + (32n^2 + 4n - 184)\, 2^{2n} + (2n^2 + 38n + 14)\, 2^n - 6$$

(iii) $SGO_2(G) = \sum_{uv\in E(G)}[\sigma(u)\sigma(v)(\sigma(u) + \sigma(v))]$

$$= \sum_{i=1}^{n} 2^i [(\sigma(u_i)\sigma(u_{i-1}))[\sigma(u_i) + \sigma(u_{i-1})]]$$

After simplification, we get

$$SGO_2(G) = [(40n^2 - 152n+176)\, 2^{3n} + (24n^2 - 4n - 148)\, 2^{2n} + (2n^2 - 30n - 24)\, 2^n - 4] \times [(16n - 36)\, 2^{2n} + (8n + 38)\, 2^n - 2]$$

(iv) $S_{a,b}(G) = \sum_{uv \in E(G)} [\sigma(u)^a \sigma(v)^b + \sigma(u)^b \sigma(v)^a]$

$$= \sum_{i=1}^{n} 2^i\, [(\sigma(u_i)^a \sigma(u_{i-1}))^b + (\sigma(u_i)^b \sigma(u_{i-1})^b)$$

Therefore

$$S_{a,b}(G) = \sum_{i=1}^{n} 2^i\, [(2^{n+1}(n+i-3) + 2^{n-i+2} + i + 2)^a \times (2^{n+1}(n+i-4) + 2^{n-i+3} + i + 1)^b + \left(2^{n+1}(n+i-3) + 2^{n-i+2} + i + 2\right)^b \times (2^{n+1}(n+i-4) + 2^{n-i+3} + i + 1)^a]$$

(v) $SDS(G) = \sum_{uv \in E(G)} \left[\frac{\sigma(u)}{\sigma(v)} + \frac{\sigma(v)}{\sigma(u)}\right]$

$$= \sum_{i=1}^{n} 2^i \left[\frac{\sigma(u_i)}{\sigma(u_{i-1})} + \frac{\sigma(u_{i-1})}{\sigma(u_i)}\right]$$

Therefore

$$SDS(G) = \sum_{i=1}^{n} 2^i$$

$$\left[[(4n^2 + 4i^2 + 8ni - 32n + 64)2^{2n} + (4i^2 + 4ni + 4n - 12i - 16)2^n + (32n + 32i - 128)2^{2n-i} + (16i + 16)2^{n-i} + 64 \times 2^{2n-2i} + i^2 + 2i + 1] + [4n^2 + (4i^2 + 8ni - 24n - 24i + 36)2^{2n} \frac{+(4i^2 + 4ni + 8n - 2i - 24)2^n (16n + 16i - 48)2^{2n-i} + + (8i+16)2^{n-i} + 16 \times 2^{2n-2i} + i^2 + 4i + 4]}{(4n^2 + 4i^2 + 8ni - 28n - 28i + 48)2^{2n} + (4i^2 + 4ni + 6n - 8i - 22)2^n + + (24n + 24i - 80)\, 2^{2n-i} + (12i+20)\, 2^{n-i} \; (4n^2 + 4i^2 + 8ni - 28n - 28i + 48)2^{2n} + + 32 \times 2^{2n-2i} + i^2 + 3i + 2} \right]$$

Theorem 6.2.3: Let G be the perfect binary tree graph. Then

(i) $$SS(G) = \frac{1}{\sqrt{(16n-36)\,2^{2n} + (8n+38)\,2^{n} - 2}}$$

(ii) $$PS(G) = \frac{1}{\sqrt{(40n^{2}-152n+176)\,2^{3n} + (24n^{2}-4n-148)\,2^{2n} + (2n^{2}-30n-24)\,2^{n} - 4}}$$

(iii) $$RPS(G) = \sqrt{\begin{array}{c}(40n^{2}-152n+176)2^{3n} + (24n^{2}-4n-148)2^{2n} \\ +(2n^{2}-30n-24)\,2^{n} - 4\end{array}}$$

Proof: By using the above definition and values, we deduce

(i) $$SS(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{\sigma(u)+\sigma(v)}}$$

$$= \sum_{i=1}^{n} 2^{i}\left[\frac{1}{\sqrt{\sigma(u_{i})+\sigma(u_{i-1})}}\right]$$

$$= \sum_{i=1}^{n} 2^{i}\left[\frac{1}{\sqrt{(2^{n+1}(n+i-3)+2^{n-i+2}+i+2)+(2^{n+1}(n+i-4)+2^{n-i+3}+i+1)}}\right]$$

Therefore

$$= \sum_{i=1}^{n} 2^{i}\left[\frac{1}{\sqrt{2^{n+1}(2n+2i-7)+3\times 2^{n-i+2}+2i+3}}\right]$$

After simplification, we get

$$SS(G) = \frac{1}{\sqrt{(16n-36)\,2^{2n} + (8n+38)\,2^{n} - 2}}$$

(ii) $PS(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{\sigma(u)\,\sigma(v)}}$

$$= \sum_{i=1}^{n} 2^{i} \left[\frac{1}{\sqrt{\sigma(u_i)\,\sigma(u_{i-1})}} \right]$$

Therefore

$$= \sum_{i=1}^{n} 2^{i} \left[\frac{1}{\sqrt{(2^{n+1}(n+i-3)+2^{n-i+2}+i+2) \times (2^{n+1}(n+i-4)+2^{n-i+3}+i+1)}} \right]$$

After simplification, we get

$$PS(G) = \frac{1}{\sqrt{(40n^2 - 152n+176)\,2^{3n} + (24n^2 - 4n - 148)\,2^{2n} + (2n^2 - 30n - 24)\,2^{n} - 4}}$$

(iii) $RPS(G) = \sum_{uv \in E(G)} \sqrt{\sigma(u)\,\sigma(v)}$

$$= \sum_{i=1}^{n} 2^{i} \sqrt{\sigma(u_i)\,\sigma(u_{i-1})}$$

After simplification, we get

$$RPS(G) = \sqrt{(40n^2 - 152n + 176)\,2^{3n} + (24n^2 - 4n - 148)\,2^{2n} + (2n^2 - 30n - 24)\,2^{n} - 4}$$

Theorem 6.2.4: Let G be the perfect binary tree graph. Then

(i) $ABCS(G) = \sqrt{\frac{[(16n - 36)\,2^{2n} + (8n + 38)\,2^{n} - 2] - 2}{(40n^2 - 152n+176)\,2^{3n} + (24n^2 - 4n - 148)\,2^{2n} + (2n^2 - 30n - 24)\,2^{n} - 4}}$

(ii) $AGS(G) = \frac{(16n - 36)\,2^{2n} + (8n + 38)\,2^{n} - 2}{2\sqrt{(40n^2 - 152n+176)\,2^{3n} + (24n^2 - 4n - 148)\,2^{2n} + (2n^2 - 30n - 24)\,2^{n} - 4}}$

(iii) $GAS(G) = \frac{2\sqrt{(40n^2 - 152n+176)\,2^{3n} + (24n^2 - 4n - 148)\,2^{2n} + (2n^2 - 30n - 24)\,2^n - 4}}{(16n - 36)\,2^{2n} + (8n+38)\,2^n - 2}$

(iv) $ASI(G) = \left(\frac{(40n^2 - 152n+176)\,2^{3n} + (24n^2 - 4n - 148)\,2^{2n} + (2n^2 - 30n - 24)\,2^n - 4}{[(16n - 36)\,2^{2n} + (8n+38)\,2^n - 2] - 2}\right)^3$

Proof: By using the above definition and values, we deduce

(i) $ABCS(G) = \sum_{uv\in E(G)} \sqrt{\frac{\sigma(u)+\sigma(v) - 2}{\sigma(u)\,\sigma(v)}}$

Therefore

$$= \sum_{i=1}^{n} 2^i \left[\sqrt{\frac{\sigma(u_i) + \sigma(u_{i-1}) - 2}{\sigma(u_i)\,\sigma(u_{i-1})}}\right]$$

After simplification, we get

$$ABCS(G) = \sqrt{\frac{[(16n - 36)\,2^{2n} + (8n+38)\,2^n - 2] - 2}{(40n^2 - 152n+176)\,2^{3n} + (24n^2 - 4n - 148)\,2^{2n} + (2n^2 - 30n - 24)\,2^n - 4}}$$

(ii) $AGS(G) = \sum_{uv\in E(G)} \frac{\sigma(u)+\sigma(v)}{2\sqrt{\sigma(u)\sigma(v)}}$

$$= \sum_{i=1}^{n} 2^i \left[\frac{\sigma(u_i) + \sigma(u_{i-1})}{2\sqrt{\sigma(u_i)\,\sigma(u_{i-1})}}\right]$$

Therefore

$$= \sum_{i=1}^{n} 2^i \left[\frac{2^{n+1}(2n+2i-7) + 3\times 2^{n-i+2} + 2i + 3}{2\sqrt{(2^{n+1}(n+i-3) + 2^{n-i+2} + i + 2)\times(2^{n+1}(n+i-4) + 2^{n-i+3} + i + 1)}}\right]$$

After simplification, we get

$$AGS(G) = \frac{(16n - 36)\,2^{2n} + (8n+38)\,2^n - 2}{2\sqrt{(40n^2 - 152n+176)\,2^{3n} + (24n^2 - 4n - 148)\,2^{2n} + (2n^2 - 30n - 24)\,2^n - 4}}$$

(iii) $GAS(G) = \sum_{uv\in E(G)} \frac{2\sqrt{\sigma(u)\,\sigma(v)}}{\sigma(u)+\sigma(v)}$

$$= \sum_{i=1}^{n} 2^{i} \left[\frac{2\sqrt{\sigma(u_i)\,\sigma(u_{i-1})}}{\sigma(u_i)+\sigma(u_{i-1})}\right]$$

Therefore

$$= \sum_{i=1}^{n} 2^{i} \left[\frac{2\sqrt{(2^{n+1}(n+i-3)+2^{n-i+2}+i+2)\times(2^{n+1}(n+i-4)+2^{n-i+3}+i+1)}}{2^{n+1}(2n+2i-7)+3\times 2^{n-i+2}+2i+3}\right]$$

After simplification, we get

$$GAS(G) = \frac{2\sqrt{(40n^2-152n+176)\,2^{3n}+(24n^2-4n-148)\,2^{2n}+(2n^2-30n-24)\,2^{n}-4}}{(16n-36)\,2^{2n}+(8n+38)\,2^{n}-2}$$

(iv) $ASI(G) = \sum_{uv\in E(G)} \left(\frac{\sigma(u)\sigma(v)}{\sigma(u)+\sigma(v)-2}\right)^3$

$$= \sum_{i=1}^{n} 2^{i} \left(\frac{\sigma(u_i)\,\sigma(u_{i-1})}{\sigma(u_i)+\sigma(u_{i-1})-2}\right)^3$$

Therefore

$$= \sum_{i=1}^{n} 2^{i} \left(\frac{(2^{n+1}(n+i-3)+2^{n-i+2}+i+2)\times(2^{n+1}(n+i-4)+2^{n-i+3}+i+1)}{[2^{n+1}(2n+2i-7)+3\times 2^{n-i+2}+2i+3]-2}\right)^3$$

After simplification, we get

$$ASI(G) = \left(\frac{(40n^2-152n+176)\,2^{3n}+(24n^2-4n-148)\,2^{2n}+(2n^2-30n-24)\,2^{n}-4}{[(16n-36)\,2^{2n}+(8n+38)\,2^{n}-2]-2}\right)^3$$

Theorem 6.2.5: Let G be the perfect binary tree graph. Then

(i) $SK(G) = \left[\frac{(16n-36)\, 2^{2n} + (8n+38)\, 2^{n} - 2}{2}\right]$

(ii) $SK_1(G) = \left[\frac{(40n^2 - 152n + 176)\, 2^{3n} + (24n^2 - 4n - 148)\, 2^{2n} + (2n^2 - 30n - 24)\, 2^{n} - 4}{2}\right]$

(iii) $SK_2(G) = \left[\frac{(40n^2 - 152n + 176)\, 2^{3n} + (24n^2 - 4n - 148)\, 2^{2n} + (2n^2 - 30n - 24)\, 2^{n} - 4}{2}\right]^2$

Proof: By using the above definition and values, we deduce

(i) $SK(G) = \sum_{uv \in E(G)} \left[\frac{\sigma(u)+\sigma(v)}{2}\right]$

Therefore

$$= \sum_{uv \in E(G)} \left[\frac{(\sigma(u_i)+\sigma(u_{i-1}))}{2}\right]$$

After simplification, we get

$$SK(G) = \left[\frac{(16n-36)\, 2^{2n} + (8n+38)\, 2^{n} - 2}{2}\right]$$

(ii) $SK_1(G) = \sum_{uv \in E(G)} \left[\frac{\sigma(u)\, \sigma(v)}{2}\right]$

Therfore

$$= \sum_{uv \in E(G)} \left[\frac{(\sigma(u_i)\, \sigma(u_{i-1}))}{2}\right]$$

After simplification, we get

$$SK_1(G) = \left[\frac{(40n^2 - 152n + 176)\, 2^{3n} + (24n^2 - 4n - 148)\, 2^{2n} + (2n^2 - 30n - 24)\, 2^{n} - 4}{2}\right]$$

(iii) $SK_2(G) = \sum_{uv \in E(G)} \left[\frac{\sigma(u)\,\sigma(v)}{2}\right]^2$

Therefore

$$= \sum_{uv \in E(G)} \left[\frac{(\sigma(u_i)\,\sigma(u_{i-1}))}{2}\right]^2$$

After simplification, we get

$$SK_2(G) = \left[\frac{(40n^2 - 152n + 176)\,2^{3n} + (24n^2 - 4n - 148)\,2^{2n} + (2n^2 - 30n - 24)\,2^n - 4}{2}\right]^2$$

Theorem 6.2.6: Let G be the perfect binary tree graph. Then

(i) $N(G) = (16n + 15)\,2^{3n} + (8n^2 + 4n + 226)\,2^{2n} + (-9n - 20)\,2^n - 44$

(ii) $\chi\alpha N(G) = [(16n + 15)\,2^{3n} + (8n^2 + 4n + 226)\,2^{2n} + (-9n - 20)\,2^n - 44]^{\alpha}$

Proof: By using the above definition and values, we deduce

(i) $N(G) = \sum_{uv \in E(G)} [\sigma^2(u) - \sigma^2(v)]$

$$= \sum_{uv \in E(G)} [(\sigma^2(u_i) - \sigma^2(u_{i-1})]$$

Therfore

$$= \sum_{i=1}^{n} 2^i [(2^{n+1}(n + i - 3) + 2^{n-i+2} + i + 2)^2 -$$

$$(2^{n+1}(n + i - 4) + 2^{n-i+3} + i + 1)^2]$$

After simplification, we get

$$N(G) = (16n + 15)\,2^{3n} + (8n^2 + 4n + 226)\,2^{2n} + (-9n - 20)\,2^n - 44$$

(ii) $\chi\alpha N(G) = \sum_{uv\in E(G)} [\sigma^2(u) - \sigma^2(v)]^{\alpha}$

$= \sum_{uv\in E(G)} [(\sigma^2(u_i) - \sigma^2(u_{i-1})]$

Therfore

$$= \sum_{i=1}^{n} 2^{i} [(2^{n+1}(n+i-3) + 2^{n-i+2} + i + 2)^2 - (2^{n+1}(n+i-4) + 2^{n-i+3} + i + 1)^2]^{\alpha}$$

After simplification, we get

$$\chi\alpha N(G) = [(16n + 15)\, 2^{3n} + (8n^2 + 4n + 226)\, 2^{2n} + (-9n - 20)\, 2^{n} - 44]^{\alpha}$$

Theorem 6.2.7: Let G be the perfect binary tree graph. Then

(i) $QC(G) = 14\times 2^{3n} - 26\times 2^{2n} - 13\times 2^{n} + 25$

(ii) $FC(G) = (128n^2 - 576n + 712)2^{3n} + (128n^2 - 64n - 576)2^{2n} + (8n^2 + 112n - 118)2^{n} - 18$

Proof: By using the above definition and values, we deduce

(i) $QC(G) = \sum_{uv\in E(G)} [\sigma(u) - \sigma(v)]^2$

$= \sum_{i=1}^{n} 2^{i} [(\sigma(u_i) - \sigma(u_{i-1})]^2$

Therefore

$$= \sum_{i=1}^{n} 2^{i} [(2^{n+1}(n+i-3) + 2^{n-i+2} + i + 2) - (2^{n+1}(n+i-4) + 2^{n-i+3} + i + 1)]^2$$

After simplification, we get

$$QC(G) = 14\times 2^{3n} - 26\times 2^{2n} - 13\times 2^{n} + 25$$

(ii) $FC(G) = \sum_{uv \in E(G)} [\sigma(u) + \sigma(v)]^2$

$= \sum_{i=1}^{n} 2^i [(\sigma(u_i) + \sigma(u_{i-1}))]$

Therefore

$$= \sum_{i=1}^{n} 2^i [(2^{n+1}(n+i-3) + 2^{n-i+2} + i + 2) + (2^{n+1}(n+i-4) + 2^{n-i+3} + i + 1)]^2$$

After simplification, we get

$$FC(G) = (128n^2 - 576n + 712)2^{3n} + (128n^2 - 64n - 576)2^{2n} + (8n^2 + 112n - 118)2^n - 18$$

Theorem 2.1.4: Let G be the perfect binary tree graph. Then

(i) $RF(G) = (148n - 84)\, 2^{3n} + (12n^2 + 144n - 200)\, 2^{2n} + (n^2 + 51n + 154)\, 2^n - 30$

(ii) $RM_2(G) = (20n^2 - 116n + 64)\, 2^{3n} + (8n^2 + 82n - 112)\, 2^{2n} + (2n^2 - 4n + 48)\, 2^n - 30$

(iii) $$RHM_2(G) = \sum_{i=1}^{n} 2^i [((2^{n+1}(n+i-3) + 2^{n-i+2} + i + 1)(2^{n+1}(n+i-4) + 2^{n-i+3} + i)]^2$$

(iv) $$RHM_2^{\alpha}(G) = \sum_{i=1}^{n} 2^i [((2^{n+1}(n+i-3) + 2^{n-i+2} + i + 1)(2^{n+1}(n+i-4) + 2^{n-i+3} + i)]^{\alpha}$$

Proof: By using the above definition and values, we deduce

(i) $RF(G) = \sum_{uv \in E(G)} [(\sigma(u) - 1)^2 + (\sigma(v) - 1)^2]$

$= \sum_{i=1}^{n} 2^i [(\sigma(u_i) - 1)^2 + (\sigma(u_{i-1}) - 1)^2]$

Therefore

$$= \sum_{i=1}^{n} 2^{i} [((2^{n+1}(n+i-3) + 2^{n-i+2} + i + 2) - 1)^{2} +$$

$$((2^{n+1}(n+i-4) + 2^{n-i+3} + i + 1) - 1)^{2})]$$

After simplification, we get

$$RF(G) = (148n - 84)\, 2^{3n} + (12n^{2} + 144n - 200)\, 2^{2n} + (n^{2} + 51n + 154)\, 2^{n} - 30$$

(ii) $RM_{2}(G) = \sum_{uv \in E(G)} [(\sigma(u) - 1)(\sigma(v) - 1)]$

$$= \sum_{i=1}^{n} 2^{i} [(\sigma(u_{i}) - 1)(\sigma(u_{i-1}) - 1)]$$

Therefore

$$= \sum_{i=1}^{n} 2^{i} [((2^{n+1}(n+i-3) + 2^{n-i+2} + i + 2) - 1)$$

$$((2^{n+1}(n+i-4) + 2^{n-i+3} + i + 1) - 1)]$$

After simplification, we get

$$RM_{2}(G) = (20n^{2} - 116n + 64)\, 2^{3n} + (8n^{2} + 82n - 112)\, 2^{2n} + (2n^{2} - 4n + 48)\, 2^{n} - 30$$

(iii) $RHM_{2}(G) = \sum_{uv \in E(G)} [(\sigma(u) - 1)(\sigma(v) - 1)]^{2}$

$$= \sum_{i=1}^{n} 2^{i} [(\sigma(u_{i}) - 1)(\sigma(u_{i-1}) - 1)]^{2}$$

$$= \sum_{i=1}^{n} 2^{i} [((2^{n+1}(n+i-3) + 2^{n-i+2} + i + 2) - 1)$$

$$((2^{n+1}(n+i-4) + 2^{n-i+3} + i + 1) - 1)]^{2}$$

Therefore

$$RHM_2(G) = \sum_{i=1}^{n} 2^{i}\,[((2^{n+1}(n+i-3)+2^{n-i+2}+i+1)$$

$$(2^{n+1}(n+i-4)+2^{n-i+3}+i)]^{2}$$

(iv) $$RHM_2^{\alpha}(G) = \sum_{uv\in E(G)} [(\sigma(u)-1)(\sigma(v)-1)]^{\alpha}$$

$$= \sum_{i=1}^{n} 2^{i}\,[(\sigma(u_i)-1)(\sigma(u_{i-1})-1)]^{\alpha}$$

$$= \sum_{i=1}^{n} 2^{i}\,[((2^{n+1}(n+i-3)+2^{n-i+2}+i+2)-1)$$

$$((2^{n+1}(n+i-4)+2^{n-i+3}+i+1)-1)]^{\alpha}$$

Therefore

$$RHM_2^{\alpha}(G) = \sum_{i=1}^{n} 2^{i}\,[((2^{n+1}(n+i-3)+2^{n-i+2}+i+1)$$

$$(2^{n+1}(n+i-4)+2^{n-i+3}+i)]^{\alpha}$$

CHAPTER 7

COMPUTE THEIR FIRST, SECOND AND HYPER CUTTING NUMBER- ECCENTRICITY INDICES OF NANOSTAR DENDRIMER D_1[N] AND D_2[N]

In this chapter, we introduced the distance based topological index of a graph G is the first, second and hyper cutting number-eccentricity indices CƐ(G) of nanostar dendrimer $D_1[n]$ and $D_2[n]$ is defined as

$$C^{*}\mathcal{E}(G) = \sum_{u\in V(G)}[c(u) + \varepsilon(u)],$$

$$C^{**}\mathcal{E}(G) = \sum_{u\in V(G)}[c(u)\varepsilon(u)] \text{ and}$$

$$HC^{*}\mathcal{E}(G) = \sum_{u\in V(G)}[c(u)^{2} + \varepsilon(u)^{2}],$$

$$HC^{**}\mathcal{E}(G) = \sum_{u\in V(G)}[c(u)^{2}\varepsilon(u)^{2}]$$

Also, introduced the distance based topological index of a graph G is the multiplicative first, second and hyper cutting number-eccentricity indices of nanostar dendrimer $D_1[n]$ and $D_2[n]$ is defined as

$$C^{*}\mathcal{E}\prod_1(G)\ ) = \prod_{u\in V(G)}[c(u) + \varepsilon(u)]$$

$$C^{**}\mathcal{E}\prod_2\ (G)\ ) = \prod_{u\in V(G)}[c(u)\varepsilon(u)]$$

$$HC^{*}\mathcal{E}\prod_1(G)\ ) = \prod_{u\in V(G)}[c(u)^{2} + \varepsilon(u)^{2}]$$

$$HC^{**}\mathcal{E}\prod_2(G)\ ) = \prod_{u\in V(G)}[c(u)^{2}\varepsilon(u)^{2}]$$

The first type of nanostar dendrimer is $D_1[n]$ with n =1 and 2. The order and size of $D_1[n]$ nanostar dendrimers are 24+36(n-1) and 27+42(n-1). The second type of nanostar dendrimers is $D_2[n]$ and is shown in Fig. 1.3.9. The order and size of $D_2[n]$ are $120 \times 2^n - 108$ and $140 \times 2^n - 127$ nanostar dendrimers are respectively.

7.1 The cutting number-eccentricity indices of nanostar dentrimers $D_1[n]$ with n=1 and n=2

In this section, we compute the cutting number-eccentricity indices of the nanostar dendrimers $D_1[n]$ with n = 1 shown in Fig.1.3.8, respectively.

The edge set of vertices $D_1[n]$ with their no.of edges, cutting no.of vertices and eccentricity of end vertices for $1 \le i \le n$.

Edge set	No. of edges e = uv	Cutting No. of end vertices (c(u), c(v))	Eccentricity of end vertices (ε(u), ε(v))
E_1	$6\times2^{n-1}$	(0, $[((18.2^n)-12)-7]\times 6$)	(3n+4, 3n+3)
E_2	$3\times2^{n-1}$	($[((18.2^n)-12)-6]\times 5$), ($[((18.2^n)-12)-7]\times 6$)	(3n+4, 3n+3)
E_3	$6\times2^{n-1}$	($[((18.2^n)-12)-6]\times 5$, 0)	(3n+3i+1, 3n+3i+2)
E_4	$6\times2^{n-1}$	(0, 0)	(3n+3i+2, 3n+3i+3)
E_5	$6\times2^{n-1}$	(0, 0)	(3n+3i+3, 3n+3i+4)
E_6	$6\times2^{n-1}$	(0, 0)	(3n+3i+2, 3n+3(i+1)+1)

Table 7.1.1

Theorem 7.1.1: The first cutting number-eccentricity index of $D_1[n]$ is given by $C^*\varepsilon(D_1[n]) = 891\times2^{2n} + 36\, n^2\, 2^n + 35\, n\, 2^n - 1467\times 2^{n-1}$

Proof: Consider the first cutting number-eccentricity index of $D_1[n]$ is given by

$C^*\mathcal{E}(D_1[n]) = \sum_{u\in V(G)} c(u) + \varepsilon(u)$

$= \sum_{u\in V(D_1[n])}[(c(u) + c(v)) + (\varepsilon(u)+ \varepsilon(v))]$

$= [0+([((18.2^n)-12)-7]\times 6) +((3n + 4) + (3n + 3))] +$

$[((18.2^n) - 12) - 6]\times 5) + [((18.2^n) - 12) - 7]\times 6) +$

$((3n + 4) + (3n + 3))]+ [(0 + [((18.2^n) - 12) - 6]\times 5) +$

$((3n + 3i + 1) + (3n + 3i + 2)]+ [(0 + 0) +$

$((3n + 3i + 2) + (3n + 3i + 3))]+$

$[(0 + 0) + ((3n + 3i + 3) + (3n + 3i + 4))]$

$[(0 + 0) + ((3n + 3i + 2) + (3n + 3(i + 1) + 1))]$

Therefore

$= \sum_{i=1}^{n}\{6 \times 2^{n-1} [((18.2^n) - 12) - 7] \times 6))] +$

$[((18.2^n) - 12) - 6] \times 5))] + 30n + 24i + 29\}+$

$\sum_{i=1}^{n}\{3 \times 2^{n-1} [((18.2^n) - 12) - 6] \times 5))]+$

$[((18.2^n) - 12) - 7] \times 6))] + 2(3n) + 7\}$

After simplification, we get

$$C^*\mathcal{E}(D_1[n]) = 891\times 2^{2n} +36 n^2 2^n +35 n 2^n - 1467\times 2^{n-1}$$

Theorem 7.1.2: The second cutting number-eccentricity index of $D_1[n]$ is given by

$$C^{**}\mathcal{E}(D_1[n]) = 594\times 2^{2n} +129n^3 2^n +273n^2 2^n +456 n 2^n - 156\times 2^n + 27 n^2 2^{n-1} +63 n 2^{n-1}$$

Proof: Consider the second cutting number-eccentricity index of $D_1[n]$ is given by

$$C^{**}\mathcal{E}(D_1[n]) = \sum_{u\in V(G)} c(u)\varepsilon(u)$$

$$= \sum_{u\in V(D_1[n])}[(c(u)c(v)) + (\varepsilon(u)\,\varepsilon(v))]$$

Therefore

$$= \sum_{i=1}^{n} 6 \times 2^{n-1}[\ 45n^2 + 36i^2 + 62ni + 56i + 87n + 44] +$$

$$\sum_{i=1}^{n} 3 \times 2^{n-1}\{[((18.2^n) - 12) - 6] \times 5))] +$$

$$[((18.2^n) - 12) - 7] \times 6))] + 9n^2 + 21n + 12\}$$

After simplification, we get

$$C^{**}\mathcal{E}(D_1[n]) = 594\times 2^{2n} + 129n^3 2^n + 273n^2 2^n + 456\, n\, 2^n - 156\times 2^n + 27\, n^2\, 2^{n-1} + 63\, n\, 2^{n-1}$$

Theorem 7.1.3: The hyper first cutting number-eccentricity index of $D_1[n]$ is given by

$$HC^{*}\mathcal{E}(D_1[n]) = 352836\times 2^{4n} - 727056\times 2^{3n} + 374679\times 2^{2n} + 576n^3 2^n + 1638n^2 2^n + 1563\, n\, 2^n + 399\times 2^n + 147\times 2^{n-1}$$

Proof: Consider the first hyper cutting number-eccentricity index of $D_1[n]$ is given by

$$HC^{*}\mathcal{E}(D_1[n]) = \sum_{u\in V(G)} c(u)^2 + \varepsilon(u)^2,$$

$$= \sum_{u\in V(D_1[n])}(c(u) + c(v))^2 + (\varepsilon(u) + \varepsilon(v))^2$$

Therefore

$$= \sum_{i=1}^{n}\{[6 \times 2^{n-1}[((18.2^n) - 12) - 7] \times 6)) +$$

$$[((18.2^n) - 12) - 6] \times 5))]^2 + (6n + 7)^2 + (6n + 6i + 3)^2$$

$$+(6n + 6i + 5)^2 + 2(6n + 6i + 7)^2 +$$

$$\sum_{i=1}^{n}\{3 \times 2^{n-1}[((18.2^{n}) - 12) - 6] \times 5)) + [((18.2^{n}) - 12) - 7] \times 6))]^{2} + (6n + 7)^{2}\}$$

After simplification, we get

$$HC^{*}\mathcal{E}(D_1[n]) = 352836\times 2^{4n} - 727056\times 2^{3n} + 374679\times 2^{2n} + 576n^{3}2^{n} + 1638n^{2}2^{n} + 1563\, n\, 2^{n} + 399\times 2^{n} + 147\times 2^{n-1}$$

Theorem 7.1.4: The hyper second cutting number-eccentricity index of $D_1[n]$ is given by

$$HC^{**}\mathcal{E}(D_1[n]) = (129n^{3}2^{n} + 273n^{2}2^{n} + 456\, n\, 2^{n} + 132\times 2^{n})^{2} + [(135\times 2^{2n} - 135\times 2^{n}) \times (162\times 2^{2n} - 171\times 2^{n})]^{2} + [27n^{2}\times 2^{n-1} + 63n2^{n-1} + 36\times 2^{n-1}]^{2}$$

Proof: Consider the hyper second cutting number-eccentricity index of $D_1[n]$ with n =1 is given by

$$HC^{**}\mathcal{E}(D_1[n]) = \sum_{u\in V(G)} c(u)^{2}\,\varepsilon(u)^{2},$$

$$= \sum_{u\in V(D_1[n])}(c(u)\, c(v))^{2} + (\varepsilon(u)\,\varepsilon(v))^{2}$$

Therefore

$$= \sum_{i=1}^{n}[6 \times 2^{n-1}[(45n^{2} + 36i^{2} + 62ni + 56i + 87n + 44)]^{2} + \sum_{i=1}^{n}[3 \times 2^{n-1}\{[((18.2^{n}) - 12) - 6] \times 5))] \times [((18.2^{n}) - 12) - 7] \times 6))]^{2} + [9n^{2} + 21n + 12]^{2}\}$$

After simplification, we get

$$HC^{**}\mathcal{E}(D_1[n]) = (129n^{3}2^{n} + 273n^{2}2^{n} + 456\, n\, 2^{n} + 132\times 2^{n})^{2} + [(135\times 2^{2n} - 135\times 2^{n}) \times (162\times 2^{2n} - 171\times 2^{n})]^{2} + [27n^{2}\times 2^{n-1} + 63n2^{n-1} + 36\times 2^{n-1}]^{2}$$

Corollary 7.1.5: The first cutting number eccentricity index of $D_1[1]$ is given by

$$C^*\mathcal{E}(D_1[1]) = 690+615+630+102+114+114$$

Proof: Using symmetry of the nanostar dendrimer $D_1[1]$

we use one branch of $D_1[1]$ as labeled in Fig.7.1.1

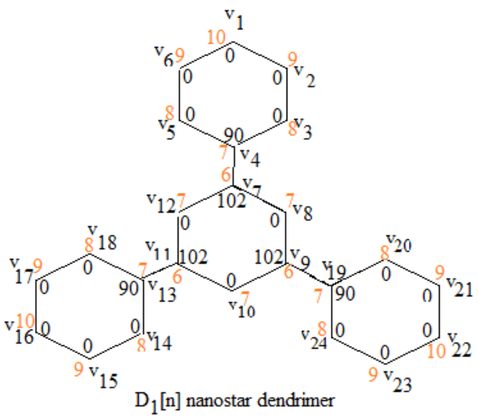


Fig 7.1.1

Edge set	No. of edges e = uv	Cutting No. of end vertices (c(u), c(v))	Eccentricity of end vertices (ε(u), ε(v))
E_1	6	(0, 102)	(7, 6)
E_2	3	(90, 102)	(7, 6)
E_3	6	(0, 0)	(7, 8)
E_4	6	(0, 0)	(8, 9)
E_5	6	(0, 0)	(9, 10)
E_6	6	(0, 0)	(9, 10)

Table 7.1.2

$$C^*\mathcal{E}(D_1[1]) = \sum_{u\in V(D_1[1])} c(u) + \varepsilon(u)$$

$$= 6(0+102+7+6)+3(90+102+7+6)+6(90+0+7+8)+6(0+0+8+9)$$

$$6(0+0+9+10)+6(0+0+9+10)$$

Therefore

$$C^{*}\mathcal{E}(D_1[1]) = 690+615+630+102+114+114$$

Corollary 7.1.6: The second cutting number eccentricity index of $D_1[1]$ is given by

$$C^{**}\mathcal{E}(D_1[1]) = 252+27666+336+432+540+540$$

Proof: Using symmetry of the nanostar dendrimer $D_1[1]$ we use only one branch of $D_1[1]$ as labeled in Fig.7.1.2

$$C^{**}\mathcal{E}(D_1[1]) = \sum_{u \in V(D_1[1])} c(u)\varepsilon(u)$$

$$= 6[(0\times102)+(7\times6)]+3[(90\times102)+(7\times6)]+$$

$$6[(90\times0)+(7\times8)]+6[(0\times0)+(8\times9)]$$

$$6[(0\times0)+(9\times10)]+6[(0\times0)+(9\times10)]$$

Therefore

$$C^{**}\mathcal{E}(D_1[1]) = 252+27666+336+432+540+540$$

Corollary 7.1.7: The hyper first cutting number-eccentricity index of $D_1[1]$ is given by

$$HC^{*}\mathcal{E}(D_1[1]) = 62424+111099+49950+1734+2166+2166$$

Proof: Consider the hyper first cutting number-eccentricity index of $D_1[1]$ is given by

$$HC^{*}\mathcal{E}(D_1[1]) = \sum_{u \in V(D_1[1])} c(u)^2 + \varepsilon(u)^2$$

$$= 6[(0+102)^2+(7+6)^2]+3[(90+102)^2+(7+6)^2]+6[(90+0)^2+(7+8)^2] +$$

$$6[(0+0)^2+(8+9)^2]+6[(0+0)^2+(9+10)^2]+6[(0+0)^2+(9+10)^2]$$

Therefore

$$HC^{*}\mathcal{E}(D_1[1]) = 62424+111099+49950+1734+2166+2166$$

Corollary 7.1.8: The hyper second hyper cutting number-eccentricity index of $D_1[1]$ is given by

$$HC^{**}\mathcal{E}(D_1[1]) =10584+84282984+18816+31104+48600+48600$$

Proof: Consider the hyper second cutting number-eccentricity index of $D_1[n]$ is given by

$$HC^{**}\mathcal{E}(D_1[1]) = \sum_{u\in V(D_1[1])} c(u)^2\, \varepsilon(u)^2,$$

$$= 6[(0\times102)^2+(7\times6)^2]+3[(90\times102)^2+(7\times6)^2]+6[(90\times0)^2+(7\times8)^2] + 6[(0\times0)^2+(8\times9)^2]+6[(0\times0)^2+(9\times10)^2]+6[(0\times0)^2+(9\times10)^2]$$

Therefore

$$HC^{**}\mathcal{E}(D_1[1]) = 10584+84282984+18816+31104+48600+48600$$

Corollary 7.1.9: The first cutting number-eccentricity index of $D_1[2]$ is given by

$$C^{*}\mathcal{E}(D_1[2]) = 4542+3213+4410+2046+2058+3678+3564+348+372$$

Edge set	No. of edges e = uv	Cutting No. of end vertices (c(u), c(v))	Eccentricity of end vertices (ε(u), ε(v))
E_1	6	(0, 738)	(10, 9)
E_2	3	(714, 738)	(10, 9)
E_3	6	(714, 0)	(10, 11)
E_4	6	(0, 318)	(11, 12)
E_5	6	(318, 0)	(12, 13)
E_6	6	(318, 270)	(12, 13)
E_7	12	(270, 0)	(13, 14)
E_8	12	(0, 0)	(14, 15)
E_9	12	(0, 0)	(15, 16)

Table 7.1.3

Proof:

$$C^*\mathcal{E}(D_1[2]) = \sum_{u\in V(G)} c(u) + \varepsilon(u)$$

$$= 6(0+738+10+9)+3(714+738+10+9)+6(714+0+10+11)+$$
$$6(0+318+11+12)+ (318+0+12+13)+6(318+270+12+13)+$$
$$12(270+0+13+14)+12(0+0+14+15)+12(0+0+15+16)$$

Therefore

$$C^*\mathcal{E}(D_1[2]) = 4542+3213+4410+2046+2058+3678+3564+348+372$$

Corollary 7.1.10: The second cutting number eccentricity index of $D_1[2]$ is given by

$$C^{**}\mathcal{E}(D_1[2]) = 540+1581066+660+792+936+516096+2184+ 2520+2880$$

Proof:

$$C^{**}\varepsilon(D_1[2]) = \sum_{u\in V(D_1[2])} c(u)\varepsilon(u)$$

$$= 6[(0\times738)+(10\times9)] +3[(714\times738)+(10\times9)] + 6[(714\times0)+(10\times11)] + 6[(0\times318)+(11\times12)] + 6[(318\times0)+(12\times13)] + 6[(318\times270) +(12\times13)] + 12[(270\times0)+(13\times14)] + 12[(0\times0)+(14\times15)] + 12[(0\times0)+(15\times16)]$$

Therefore

$$C^{**}\varepsilon(D_1[2]) = 540+1581066+660+792+936+516096+2184+2520+2880$$

Corollary 7.1.11: The hyper first cutting number-eccentricity index of $D_1[2]$ is given by

$$HC^{*}\varepsilon(D_1[2]) = 3270030+6325995+3061422+609918+610494+2078214+10092+11532$$

Proof: Using symmetry of the nanostar dendrimer$D_1[2]$

$$HC^{*}\varepsilon(G) = \sum_{u\in V(D_1[2])} c(u)^2 + \varepsilon(u)^2$$

$$= 6[(0+738)^2+(10+9)^2]+3[(714+738)^2+(10+9)^2]+ 6[(714+0)^2+(10+11)^2]+ 6[(0+318)^2+(11+12)^2]+ 6[(318+0)^2+(12+13)^2]+6[(318+270)^2+(12+13)^2]+ 12[(270+0)^2+(13+14)^2]+12[(0+0)^2+(14+15)^2]+ 12[(0+0)^2+(15+16)^2]$$

After simplification, we get

$$HC^{**}\varepsilon(D_1[2]) = 3270030+6325995+3061422+609918+610494+2078214+10092+11532$$

Corollary 7.1.12: The second cutting number eccentricity index of $D_1[2]$ is given by

$$HC^{**}\mathcal{E}(D_1[2]) = 48600+832972022172 +72600+104544+ 146016+7371963936+397488+529200+691200$$

Proof: Using symmetry of the nanostar dendrimer$D_1[2]$

$$HC^{**}\mathcal{E}(D_1[2]) = \sum_{u\in V(D_1[2])} c(u)^2\, \varepsilon(u)^2$$

Therefore

$$= 6[(0\times738)^2+(10\times9)^2]+3[(714\times738)^2+(10\times9)^2]+ 6[(714\times0)^2+(10\times11)^2]+ 6[(0\times318)^2+(11\times12)^2]+ 6[(318\times0)^2+(12\times13)^2]+6[(318\times270)^2+(12\times13)^2]+ 12[(270\times0)^2+(13\times14)^2]+12[(0\times0)^2+(14\times15)^2]+ 12[(0\times0)^2+(15\times16)^2]$$

After simplification, we get

$$HC^{**}\mathcal{E}(D_1[2]) = 48600+832972022172 +72600+104544+ 146016+7371963936+397488+529200+691200$$

7.2 The second type of cutting number-eccentricity index of nanostar dentrimers $D_2[n]$

In this section, we compute the second type of cutting number-eccentricity index of nanostar dendrimers is $D_2[n]$ with shown in Fig.1.3.8, respectively.

Edge set	No. of edges e = uv	Cutting No. of end vertices (c(u), c(v))	Eccentricity of end vertices (ε(u), ε(v))
E_1	$6x2^{n-1}$	$(0, [((18.2^n)-12)-19]x18)$	(10n+10i-10, 10n+10i-11)
E_2	$3x2^{n-1}$	$([((18.2^n)-12)-19]x17),$ $[((18.2^n)-12)-19]x18))$	(10n+10i-10, 10n+10i-11)
E_3	$6x2^{n-1}$	$([((18.2^n)-12)-18]x17, 0)$	(10n+10i-10, 10n+10i-9)
E_4	$6x2^{n-1}$	$(0, [((18.2^n)-12)-7]x6)$	(10n+10i-9, 10n+10i-8)
E_5	$6x2^{n-1}$	$([((18.2^n)-12)-7]x6, 0)$	(10n+10i-8, 10n+10i-7)
E_6	$6x2^{n-1}$	$([((18.2^n)-12)-7]x6),$ $[((18.2^n)-12)-6]x5))$	(10n+10i-8, 10n+10i-7)
E_7	$12x2^{n-1}$	$([((18.2^n)-12)-6]x5, 0)$	(10n+10i-7, 10n+10i-6)
E_8	$12x2^{n-1}$	(0, 0)	(10n+10i-6, 10n+10i-5)
E_9	$12x2^{n-1}$	(0, 0)	(10n+10i-5, 10n+10i-4)
E_{10}	$12x2^{n-1}$	(0, 0)	(10n+10i-4, 10n+10(i+1)-3)

Table 7.2.1

Theorem 7.2.1: The first cutting number-eccentricity index of $D_2[n]$ is given by

$$C^*\mathcal{E}(D_2[n]) = 3672\times2^{2n} + 405\, n^2\, 2^n + 1215\, n\, 2^n - 6195\times2^n$$

Proof: Using symmetry of the nanostar dendrimer $D_2[n]$

Using the data given in above table,

$$C^*\mathcal{E}(D_2[n]) = \sum_{u\in V(G)} c(u) + \varepsilon(u)$$

$$= \sum_{u\in V(G)}[c(u) + c(v) + \varepsilon(u) + \varepsilon(v)]$$

Therefore

$$= \sum_{i=1}^{n} 6 \times 2^{n-1}\{[((18.2^{n}) - 12) - 19] \times 18))] +$$
$$[((18.2^{n}) - 12) - 18] \times 17))] + 3[((18.2^{n}) - 12) - 7] \times 6))] +$$
$$[((18.2^{n}) - 12) - 6] \times 5))] + 10(10n) + 10(10i) - 87\} +$$
$$\sum_{i=1}^{n} 3 \times 2^{n-1}\{[((18.2^{n}) - 12) - 19] \times 17))]+$$
$$[((18.2^{n}) - 12) - 19] \times 18))] + 2(10n) + 2(10i) - 21\} +$$
$$\sum_{i=1}^{n} 12 \times 2^{n-1}\{[((18.2^{n}) - 12) - 6] \times 5))]+$$
$$+8(10n) + 8(10i) - 30\}$$

After simplification, we get

$$C^{*}\mathcal{E}(D_2[n]) = 3672 \times 2^{2n} + 405\, n^2 2^n + 1215\, n\, 2^n - 6195 \times 2^n$$

Which is the required result.

Theorem 7.2.2: The first cutting number-eccentricity index of $D_2[n]$ is given by

$$C^{**}\mathcal{E}(D_2[n]) = 177876 \times 2^{3n} - 572184 \times 2^{2n} + 5450\, n^3 2^n + 5835 n^2 2^n$$
$$- 6230 n\, 2^n + 473580 \times 2^n - 315\, n^2 2^{n-1} - 315\, n\, 2^{n-1}$$

Proof: Consider the first cutting number-eccentricity index of $D_2[n]$ is given by

$$C^{**}\mathcal{E}(D_1[n]) = \sum_{u \in V(G)} c(u)\varepsilon(u)$$
$$= \sum_{u \in V(D_1[n])} [c(u)c(v) + \varepsilon(u)\,\varepsilon(v)]$$

Therefore

$$= \sum_{i=1}^{n} 6 \times 2^{n-1}\{[\big((10n + 10i - 10) \times (10n + 10i - 11)\big) +$$
$$((10n + 10i - 10) \times (10n + 10i - 9)) +$$
$$((10n + 10i - 9) \times (10n + 10i - 8)) +$$

$((10n+10i-8)\times(10n+10i-7))+$

$[[((18.2^n)-12)-7]\times 6))]\times\ [[((18.2^n)-12)-6]\times 5))]+$

$((10n+10i-8)\times(10n+10i-7))]\}+$

$\sum_{i=1}^{n} 3\times 2^{n-1}\{[[((18.2^n)-12)-19]\times 17))]+$

$[[((18.2^n)-12)-19]\times 18))]+$

$(10n+10i-10)\ \times\ (10n+10i-11)\}$

$\sum_{i=1}^{n} 12\times 2^{n-1}\{((10n+10i-7)\times(10n+10i-6))]+$

$((10n+10i-6)\times(10n+10i-5))]+$

$((10n+10i-5)\times(10n+10i-4))]+$

$((10n+10i-4)\times(10n+10(i+1)-3))]\}$

After simplification, we get

$$C^{**}\mathcal{E}(D_2[n]) = 177876\times 2^{3n} - 572184\times 2^{2n} + 5450\, n^3 2^n + 5835 n^2 2^n$$

$$- 6230 n\, 2^n + 473580\times 2^n - 315\, n^2 2^{n-1} - 315\, n\, 2^{n-1}$$

Theorem 7.2.3: The hyper first cutting number-eccentricity index of $D_2[n]$ is given by

$$HC^{*}\mathcal{E}(D_2[n]) = 1311228\times 2^{3n} - 8071704\times 2^{2n} + 39600\, n^3 2^n + 59310 n^2 2^n$$

$$- 15390\, n\, 2^n + 21534504\times 2^n + 1323\times 2^{n-1}$$

Proof: Using symmetry of the nanostar dendrimer $D_2[n]$ we use only one branch of $D_2[n]$ as labeled in Fig 7.2.1

Using the data given in above table, the second type of cutting number-eccentricity index of

$D_2[n]$ for $n \geq 2$, we have

$$HC^{*}\mathcal{E}(D_2[n]) = \sum_{u\in V(G)} c(u)^2 + \varepsilon(u)^2$$

$$= \sum_{u\in V(G)}[(c(u) + c(v))^2 + (\varepsilon(u)+ \varepsilon(v))^2]$$

Therefore

$$= \sum_{i=1}^{n} 6 \times 2^{n-1}\{[((18.2^{n}) - 12) - 19] \times 18))^2] +$$

$$\big((10n + 10i - 10) + (10n + 10i - 11)\big)^2$$

$$[((18.2^{n}) - 12) - 18] \times 17))^2]$$

$$((10n + 10i - 10) + (10n + 10i - 9))^2 +$$

$$[[((18.2^{n}) - 12) - 7] \times 6))^2]$$

$$((10n + 10i - 9) + (10n + 10i - 8))^2 +$$

$$[[((18.2^{n}) - 12) - 7] \times 6))^2]$$

$$((10n + 10i - 8) + (10n + 10i - 7))^2 +$$

$$[[((18.2^{n}) - 12) - 7] \times 6))] + [[((18.2^{n}) - 12) - 6] \times 5))]^2\}$$

$$\sum_{i=1}^{n} 3 \times 2^{n-1}\{[((18.2^{n}) - 12) - 19] \times 17))]+$$

$$[((18.2^{n}) - 12) - 19] \times 18))]^2 +$$

$$\big((10n + 10i - 10) + (10n + 10i - 11)\big)^2\}+$$

$$\sum_{i=1}^{n} 12 \times 2^{n-1}\{[((18.2^{n}) - 12) - 6] \times 5))]^2 +$$

$$((10n + 10i - 7) + (10n + 10i - 6))^2 +$$

$$((10n + 10i - 6) + (10n + 10i - 5))^2 +$$

$$((10n + 10i - 5) + (10n + 10i - 4))^2 +$$

$$((10n + 10i - 4) + (10n + 10(i + 1) - 3))^2\}$$

After simplification, we get

$$C^{*}\mathcal{E}(D_2[n]) = 1311228\times 2^{3n} - 8071704\times 2^{2n} + 39600\, n^{3}\, 2^{n} + 59310 n^{2}\, 2^{n}$$

$$- 15390\, n\, 2^{n} + 21534504\times 2^{n} + 1323\times 2^{n-1}$$

Which is the required result.

Theorem 7.2.4: The hyper second cutting number-eccentricity index of $D_2[n]$ is given by

$$HC^{**}\mathcal{E}(D_2[n]) = 6954238\times 2^{5n} + 94665408\times 2^{4n} - 5530189\times 2^{3n}$$

$$-7336034\times 2^{2n} + 903426\, n^{5} + 348700\, n^{4}\, 2^{n} - 348700\, n^{3}\, 2^{n}$$

$$+400968 n^{2}\, 2^{n} - 228604 n\, 2^{n} + 436658\times 2^{n} + 48854\times 2^{n-1}$$

Proof: Consider the second cutting number-eccentricity index of $D_2[n]$ is given by

$$HC^{**}\mathcal{E}(D_2[n]) = \sum_{u\in V(G)} c(u)^{2}\, \varepsilon(u)^{2}$$

$$= \sum_{u\in V(D_2[n])} [(c(u)c(v))^{2} + (\varepsilon(u)\,\varepsilon(v))^{2}]$$

Therefore

$$=\sum_{i=1}^{n} 6\times 2^{n-1}\{[\big((10n+10i-10)\times (10n+10i-11)\big)^{2} +$$

$$((10n+10i-10)\times(10n+10i-9))^{2} +$$

$$((10n+10i-9)\times(10n+10i-8))^{2} +$$

$$((10n+10i-8)\times(10n+10i-7))^{2} +$$

$$[[((18.2^{n})-12)-7]\times 6))]\times [[((18.2^{n})-12)-6]\times 5))]^{2} +$$

$$((10n+10i-8)\times(10n+10i-7))]^{2}\} +$$

$$\sum_{i=1}^{n} 3\times 2^{n-1}\{[[((18.2^{n})-12)-19]\times 17))]^{2} +$$

$$[[((18.2^{n})-12)-19]\times 18))]^{2} +$$

$$((10n + 10i - 10) \times (10n + 10i - 11))^2 \} +$$

$$\sum_{i=1}^{n} 12 \times 2^{n-1} \{((10n + 10i - 7) \times (10n + 10i - 6))]^2 +$$

$$((10n + 10i - 6) \times (10n + 10i - 5))]^2 +$$

$$((10n + 10i - 5) \times (10n + 10i - 4))]^2 +$$

$$((10n + 10i - 4) \times (10n + 10(i + 1) - 3))]^2 \}$$

After simplification, we get

$$HC^{**}\mathcal{E}(D_2[n]) = 6954238 \times 2^{5n} + 94665408 \times 2^{4n} - 5530189 \times 2^{3n}$$

$$- 7336034 \times 2^{2n} + 903426\, n^5 + 348700\, n^4\, 2^n - 348700\, n^3\, 2^n$$

$$+ 400968 n^2\, 2^n - 228604 n\, 2^n + 436658 \times 2^n + 48854 \times 2^{n-1}$$

Which is the required result.

APPLICATION

During the last few years, numerous graph-theoretic methods have been developed for the analysis and prediction of physiochemical, environmental, and bio medicinal properties of molecules.

Use of topological indices in structure activity relationship studies seems to play an important role in situations where biological activity is determined predominantly by topological architecture of molecular structure. One of the main medical and social problems nowadays is HIV and we are in need of Anti-HIV therapy which is in need of new drugs with less toxic, active against the drug resistant mutants. It was analyzed Weiner index, Zagreb index played a vital role in solving this problem.

It has been considered as the main source of medicines and during the past two decades thousands of compounds and their metabolites with several different type of biological activity such as anti microbial, anti inflammatory, anti-malarial, antioxidant, anti-HIV, and anti Cancer activity. Study of topological indices helps in acquiring that medicine with less toxic.

The constitutional formula of a molecule is in essence, a planar graph where vertices represent the atoms and edges are the covalent bonds. Since such a graph adequately depicts the topology of the molecule, it is not surprising that the graph-theoretic approaches in explaining the physical and biological properties of diverse groups of chemicals.

In this work we have introduced many topological indices with multiplicative version, edge version, and eccentricity. These indices give the nearer values depicting the physical and biological properties of molecules.

CONCLUSION

In this work we have introduced 21 indices namely the multiplicative Harmonic index, Multiplicative ISI index, multiplicative F-index, the multiplicative first and second Zagreb polynomial, the multiplicative modified first and second Zagreb indices, the multiplicative fifth Geometric-arithmetic index, the multiplicative fourth atom bond connectivity index, the multiplicative Augmented Zagreb index, Harmonic Eccentric index, the First and second K- Eccentric indices, First and second K Hyper -Eccentric indices, Multiplicative K Eccentric indices, the first and second Multiplicative K Hyper Eccentric indices, the edge version of the Harmonic index, the edge version of the Randic Index, the edge version of the Sum connectivity index, the edge version of F- index, the Multiplicative edge version of the Randic Index, the inverse Randic eccentric, Reduced reciprocal Randic eccentric boron triangular nanotubes index, reduced second Zagreb eccentric, sum line connectivity eccentric boron triangular nanotubes index Eccentric indices of Boron Triangular Nanotubes. the first, second and hyper status indices, Sum and Product reciprocal connectivity status indices, F_l-status index, Gourava indices, (a, b)-status index, ABC, AG and Augmented status indices, Sum connectivity index, SK, SK1 and SK2 status indices and Nano-Zagreb, Sum Nano-Zagreb status indices, Square Reverse status index, F-Reverse status index, Reduced Second and hyper Zagreb status index and general Reduced second Zagreb status index of perfect binary tree graphs.

These indices may surely give hands to the chemist in finding the more and more appropriate or nearer values in studying about molecules.

In future the higher orders of these indices and also the comparative study of indices can carried out.

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EDGE VERSION OF *K*-ECCENTRIC INDICES OF CIRCUMCORONENE SERIES BENZENOID $L(H_k)$ SYSTEM

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Abstract

In this paper, the edge version of *K*-eccentric indices of a circumcoronene series of Benzenoid $L(H_k)$ system is computed.

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1. Introduction

Let G be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The notation uv represents an edge between two vertices u and v in G. The *distance* $d(u, v)$ between any two vertices u and v is the length of the shortest path which connects u and v. It is defined as the number of edges in a shortest path that connects the vertices u and v. The *eccentricity* $e_G(v)$ of a vertex v in G is the largest distance between v and any other vertex u of G. The *line graph* $L(G)$ of G is the graph whose vertex set corresponds to the edges of G such that two vertices of $L(G)$ are adjacent if the corresponding edges of G are adjacent. Let $e_{L(G)}(e)$ denote the eccentricity of an edge e in $L(G)$, where $L(G)$ is the line graph of G. A *molecular graph* is a finite, simple graph such that its vertices correspond to the atoms and the edges to the bonds. There are several topological indices that have some applications in theoretical chemistry in QSPR/QSAR study [2, 3]. The *Wiener index* is the first graph invariant reported (distance based) topological index and is defined as the half of the sum of the distances between all the pairs of vertices in a graph [1]. Also, the edge version of wiener index based on the distance between edges is introduced by Iranmanesh et al. [2]. These *topological indices* are formulated as follows:

$$W_v(G) = \sum_{\{u, v\} \subset V(G)} d(u, v),$$

$$W_e(G) = \sum_{\{e, f\} \subset E(G)} d(e, f).$$

The *degree* of a vertex v is denoted by d_v, and that of the edge e by d_e. The degree of a vertex v is the number of vertices joining v.

Bhanumathi and Rani [1] introduced the edge versions of the first and second K-eccentric indices, and the first and second K-hyper eccentric indices of a graph G defined as

$$B_1E(G) = \sum_{ue}[e_G(u) + e_{L(G)}(e)],$$

$$B_2E(G) = \sum_{ue}[e_G(u)e_{L(G)}(e)],$$

and

$$HB_1E(G) = \sum_{ue}[e_G(u) + e_{L(G)}(e)]^2,$$

$$HB_2E(G) = \sum_{ue}[e_G(u)e_{L(G)}(e)]^2.$$

Kulli [5] introduced the following indices:

The modified first and second Zagreb indices of a graph G defined as

$${}^{m}M_1(G) = \sum_{u\in VG}\frac{1}{d_G(u)^2},\quad {}^{m}M_2(G) = \sum_{uv\in EG}\frac{1}{d_G(u)d_G(v)}.$$

The edge versions of modified first and second K-eccentric indices of a graph G have been defined as

$${}^{m}B_1(G) = \sum_{ue}\frac{1}{d_G(u) + d_G(e)},\quad {}^{m}B_2(G) = \sum_{ue}\frac{1}{d_G(u)d_G(e)}.$$

Favaron et al. [10] and Zhong [11] introduced the harmonic index of a graph G as

$$H(G) = \sum_{uv\in EG}\frac{2}{d_G(u) + d_G(v)}.$$

In all the cases ue means the vertex u and edge e are incident in G and $e_{L(G)}(e)$ is the eccentricity of e in the line graph $L(G)$ of G. If G is a (p, q) graph whose vertices have degrees d_i, then $L(G)$ has q vertices and q_L edges, where $q_L = -q + \frac{1}{2}\sum d_i^2$.

In this paper, the edge versions of eccentricity of circumcoronene series of Benzenoid $L(H_k)$ graphs are introduced. Furthermore, the values of these indices for circumcoronene series of Benzenoid graphs are determined.

2. Results for Edge Version of Eccentricity of Graph $L(H_k)$

This section is devoted to study the edge version of eccentricity of circumcoronene series of Benzenoid system $L(H_k)$.

Also, we obtain a closed formula for a famous molecular graph, namely, circumcoronene series of Benzenoid graph. The circumcoronene homologous series of Benzenoid graphs is a family of molecular graphs, which consists of several copies of benzene C_6 on its circumference. The terms of this series are represented as, H_1-benzene, H_2-coronene, H_3-circumcoronene and H_4-circumcircumcoronene etc. A Benzenoid system is a connected geometric figure. It is obtained by arranging congruent regular hexagons in a plane. Consequently, two hexagons are either disjoint or have a common edge, see Figure 1 and Figure 2, where these are shown.

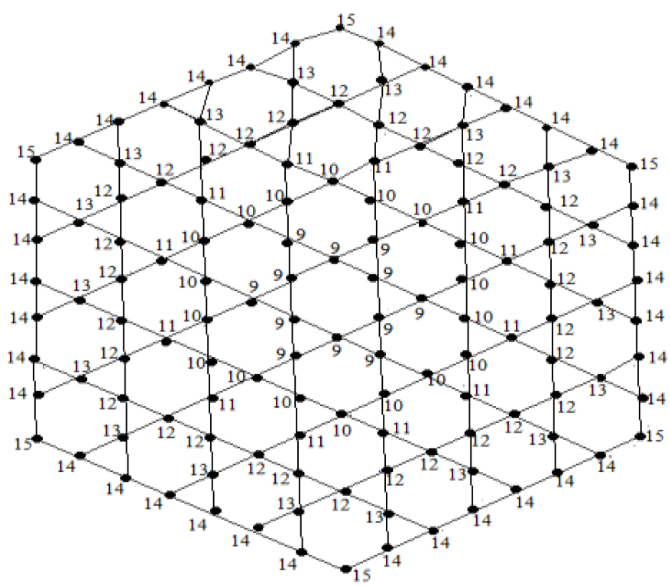

Figure 1. $L(H_4)$.

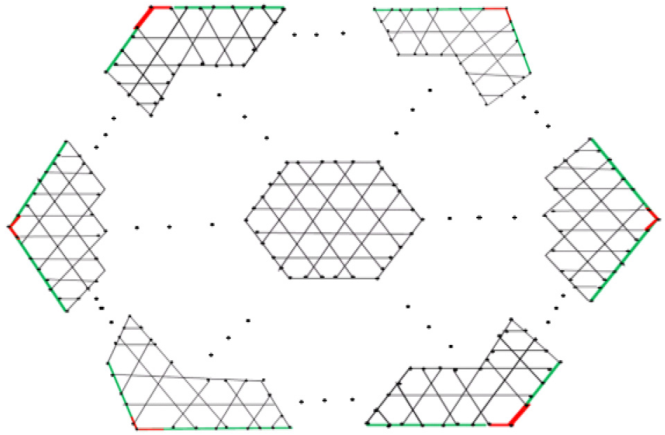

Figure 2. $L(H_k)$.

The line graph of circumcircumcoronene $L(H_4)$ (Figure 1).

The general representation of line graph of circumcoronene series of Benzenoid $L(H_k)(k \geq 1)$ with edges marked (Figure 2).

Consider the circumcoronene series of Benzenoid $L(H_k)$, where H_k is the defining parameter as illustrated in Figures 1 and 2.

The number of vertices in circumcoronene series of Benzenoid $L(H_k)$ is equal to $|V(H_k)| = 9k^2 - 3k$ and the number of edges is $|E(H_k)| = 18k^2 - 12k$.

To evaluate the edge version of topological indices of this circumcoronene series of Benzenoid system $L(H_k)$, the required number of vertices, eccentricity of *e* in $L(H_k)$, the number of edges *ef*, and the number of end vertices are given in Table 2.1.

Table 2.1

No. of vertices in $L(H_k)$	Eccentricity of e in $L(H_k)$	No. of edge set $(e(u), e(v))$	$(e_{L(G)}(e), e_{L(G)}(f))$
6	$2k+1$	6×3	$(2k+1, 2k+1)$
6	$2k+1$	6×2	$(2k+1, 2k+2)$
12	$2k+2$	6×3	$(2k+2, 2k+3)$
6	$2k+3$	6×4	$(2k+3, 2k+3)$
12	$2k+3$	6×4	$(2k+3, 2k+4)$
24	$2k+4$	6×5	$(2k+4, 2k+5)$
6	$2k+5$	6×6	$(2k+5, 2k+5)$
18	$2k+5$	6×6	$(2k+5, 2k+6)$
36	$2k+6$	6×7	$(2k+6, 2k+7)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
6	$2k+2(k-2)-1$	$6\times(2k-4)$	$(2k+2(k-2)-1, 2k+2(k-2)-1)$
$6(k-2)$	$2k+2(k-2)-1$	$6\times(2k-4)$	$(2k+2(k-2)-1, 2k+2(k-2))$
$12(k-2)$	$2k+2(k-2)$	$6\times(2k-3)$	$(2k+2(k-2), 2k+2(k-1)-1)$
6	$2k+2(k-1)-1$	$6\times(2k-2)$	$(2k+2(k-1)-1, 2k+2(k-1)-1)$
$6(k-1)$	$2k+2(k-1)-1$	$6\times(2k-2)$	$(2k+2(k-1)-1, 2k+2(k-1))$
$12(k-1)$	$2k+2(k-1)$	$6\times(2k-3)$	$(2k+2(k-1), 2k+2(k-1)+1)$
6	$2k+2(k-1)+1$	6×2	$(2k+2(k-1)+1, 2k+2(k-1)+1)$

Here, we define the edge version of the first and second K-eccentric indices of a graph as

$$M_1E(L(H_k)) = \sum_{ef\in E(L(G))}[e_{L(H_k)}(e) + e_{L(H_k)}(f)],$$

$$M_2E(L(H_k)) = \sum_{ef\in E(L(G))}[e_{L(H_k)}(e) \times e_{L(H_k)}(f)].$$

Also, we define edge version of the first and second K-hyper eccentric indices of a graph as

$$HM_1E(L(H_k)) = \sum_{ef \in E(L(H_k))} [e_{L(H_k)}(e) + e_{L(H_k)}(f)]^2,$$

$$HM_2E(L(H_k)) = \sum_{ef \in E(L(H_k))} [e_{L(H_k)}(e) \times e_{L(H_k)}(f)]^2.$$

Furthermore, we define the edge version of modified first and second K-eccentric indices of a graph as

$$^m M_1E(L(H_k)) = \sum_{ef \in E(L(H_k))} \frac{1}{e_{L(H_k)}(e) + e_{L(H_k)}(f)},$$

$$^m M_2E(L(H_k)) = \sum_{ef \in E(L(H_k))} \frac{1}{e_{L(H_k)}(e) \times e_{L(H_k)}(f)}.$$

The edge version of harmonic eccentric index of a graph is defined as

$$HE(L(H_k)) = \sum_{ef \in E(L(H_k))} \frac{2}{e_{L(H_k)}(e) + e_{L(H_k)}(f)}.$$

In all the cases, *ef* means that for the edges in $L(H_k)$, we have that $e_{L(H_k)}(e)$ is the eccentricity of *e* in the line graph $L(H_k)$ of *G*.

3. The EDGE Version of Eccentric Indices of Graph $L(H_k)$

The edge versions of the first and second K-eccentric indices, the first and second K-hyper eccentric indices are investigated. Moreover, the edge versions of modified first and second K-eccentric indices and harmonic eccentric index of circumcoronene series of Benzenoid $L(H_k)$ graphs have been calculated.

Theorem 3.1. *For any positive integer k, let* $L(H_k)$ *be the general form of circumcoronene series of Benzenoid system. Then*

(i) $M_1E(L(H_k)) = 12k^4 + 32k^3 + 102k^2 - 382k + 524,$

(ii) $M_2E(L(H_k)) = 176k^4 + 56k^3 - 1060k^2 + 1126k - 1150,$

(iii) $HM_1E(L(H_k)) = 660k^4 + 725k^3 - 4665k^2 + 4787k - 509,$

(iv) $HM_2E(L(H_k)) = 544k^6 + 7808k^5 - 31080k^4 + 37360k^3$

$$- 21904k^2 + 6380k - 270.$$

Proof. By using the above definition and table values (2.1), we deduce

(i)

$$M_1E(L(H_k))$$

$$= \sum_{ef \in E(L(H_k))} [e_{L(H_k)}(e) + e_{L(H_k)}(f)]$$

$$= \sum_{ef \in E_1(L(H_k))} [e_{L(H_k)}(e) + e_{L(H_k)}(f)]$$

$$+ \cdots + \sum_{ef \in E_{3(k-1)+1}(L(G))} [e_{L(H_k)}(e) + e_{L(H_k)}(f)]$$

$$= 6 \times 3[4k + 2] + 6 \times 2[4k + 3] + 6 \times (2i + 1)\sum_{i=1}^{k-2} (4k + 4i)$$

$$+ 6 \times (2i + 2)\sum_{i=1}^{k-2} (4k + 4i + 1) + 6 \times (2i + 2)\sum_{i=1}^{k-2} (4k + 4i + 3)$$

$$+ 6 \times 3[(2k - 3)(8k - 4)] + 6 \times 2[(8k - 3)].$$

After simplification, we get

$$= 12k^4 + 32k^3 + 102k^2 - 382k + 524$$

(ii)

$$M_2E(L(H_k))$$

$$= \sum_{ef \in E(L(G))} [e_{L(H_k)}(e) \times e_{L(H_k)}(f)]$$

$$= \sum_{ef \in E_1(L(G))} [e_{L(H_k)}(e) \times e_{L(H_k)}(f)]$$

$$+ \cdots + \sum_{ef \in E_{3(k-1)+1}(L(H_k))} [e_{L(H_k)}(e) \times e_{L(H_k)}(f)]$$

$$= 6 \times 3[(2k+1)^2] + 6 \times 2[(2k+1)(2k+2)] + 6 \times (2i+1)$$

$$\times \sum_{i=1}^{k-2} (2k+1)^2 + 6 \times (2i+2) \sum_{i=1}^{k-2} (2k+2i)(2k+2i+1)$$

$$+ 6 \times (2i+2) \sum_{i=1}^{k-2} (2k+2i+1)(2k+2i+2)$$

$$+ 6 \times 3(2k-3)[(2k+2(k-1))^2]$$

$$+ 6 \times 2[(2k+2(k-1))(2k+2(k-1)+1)].$$

After simplification, we get

$$= 176k^4 + 56k^3 - 1060k^2 + 1126k - 1150.$$

(iii)

$$HM_1E(L(H_k))$$

$$= \sum_{ef \in E(L(H_k))} [e_{L(H_k)}(e) \times e_{L(H_k)}(f)]^2$$

$$= \sum_{ef \in E_1(L(G))} [e_{L(H_k)}(e) + e_{L(H_k)}(f)]^2$$

$$+ \cdots + \sum_{ef \in E_{3(k-1)+1}(L(H_k))} [e_{L(H_k)}(e) + e_{L(H_k)}(f)]^2$$

$$= 6 \times 3[4k+2]^2 + 6 \times 2[4k+3]^2 + 6 \times (2i+1)$$

$$\times \sum_{i=1}^{k-2} [4k+4i]^2 + 6 \times (2i+2) \sum_{i=1}^{k-2} [(4k+4i+1)]^2$$

$$+ 6 \times (2i+2) \sum_{i=1}^{k-2} [(4k+4i+3)]^2$$

$$+ 6 \times 3(2k-3)[8k-4]^2 + 6 \times 2[(8k-3)]^2.$$

After simplification, we get

$$= 660k^4 + 725k^3 - 4665k^2 + 4787k - 509.$$

(iv)

$$HM_2E(L(H_k))$$

$$= \sum_{ef \in E(L(H_k))} [e_{L(H_k)}(e) \times e_{L(H_k)}(f)]^2$$

$$= \sum_{ef \in E_1(L(H_k))} [e_{L(H_k)}(e) \times e_{L(H_k)}(f)]^2$$

$$+ \cdots + \sum_{ef \in E_{3(k-1)+1}(L(H_k))} [e_{L(H_k)}(e) \times e_{L(H_k)}(f)]^2$$

$$= 6 \times 3[(2k+1)^2]^2 + 6 \times 2[(2k+1)(2k+2)]^2 + 6 \times (2i+1)$$

$$\times \sum_{i=1}^{k-2} [(2k+2i)^2]^2 + 6 \times (2i+2) \sum_{i=1}^{k-2} [(2k+2i)(2k+2i+1)]^2$$

$$+ 6 \times (2i+2) \sum_{i=1}^{k-2} [(2k+2i+1)(2k+2i+2)]^2$$

$$+ 6 \times 3A(2k-3)[(2k+2(k-1))^2]^2$$

$$+ 6 \times 2[(2k+2(k-1))(2k+2(k-1)+1)]^2.$$

After simplification, we get

$$= 544k^6 + 7808k^5 - 31080k^4 + 37360k^3 - 21904k^2 + 6380k - 270.$$

Theorem 3.2. *For any positive integer k, let* $L(H_k)$ *be the general form of line graph of circumcoronene series of Benzenoid system. Then*

(i)

$$^{m}M_1E(L(H_k)) = \frac{192k^6 - 96k^5 - 60k^4 + 348k^3 + 168k^2 - 66k - 36}{32k^6 + 40k^5 - 4k^4 - 20k^3 - 6k^2}$$

$$+ \frac{6k^2 - 18k + 18}{2k^2 - 2k + 5}$$

(ii)

$$^{m}M_2E(L(H_k))$$

$$= \frac{4608k^6 - 21888k^5 + 43200k^4 - 41184k^3 + 20016k^2 - 4752k + 432}{864k^5 + 648k^4 - 16416k^3 + 12960k^2 - 3672k - 9072}$$

(iii)

$$H_bE(L(H_k)) = \frac{960k^5 + 1656k^4 - 7452k^3 + 504k^2 + 5208k - 858}{128k^5 - 68k^4 - 316k^3 + 185k^2 - 89k - 33}$$

$$+ \frac{1008k^4 - 2952k^3 + 4788k^2 - 3672k + 786}{64k^4 - 120k^3 + 228k^2 - 152k + 30}.$$

Proof. By using the above definition and table values (2.1), we deduce

(i)

$$^{m}M_1E(L(H_k))$$

$$= \sum_{ef \in E(L(H_k))} \frac{1}{e_{L(H_k)}(e) + e_{L(H_k)}(f)}$$

$$= \sum_{ef \in E_1(L(H_k))} \left[\frac{1}{e_{L(H_k)}(e) + e_{L(H_k)}(f)}\right]$$

$$+ \cdots + \sum_{ef \in E_{3(k-1)+1}(L(G))} \left[\frac{1}{e_{L(H_k)}(e) + e_{L(H_k)}(f)}\right]$$

$$= 6 \times 3\left[\frac{1}{2(2k+1)}\right] + 6 \times 2\left[\frac{1}{(4k+3)}\right] + 6 \times (2i+1)$$

$$\times \sum_{i=1}^{k-2} \left[\frac{1}{2(2k+2i)}\right] + 6 \times (2i+2) \sum_{i=1}^{k-2} \left[\frac{1}{4k+4i+1}\right]$$

$$+ 6 \times (2i+2) \sum_{i=1}^{k-2} \left[\frac{1}{4k+4i+3}\right]$$

$$+ 6 \times 3(2k-3)\left[\frac{1}{2(2k+2(k-1))}\right] + 6 \times 2\left[\frac{1}{(4k+4(k-1)+1)}\right].$$

After simplification, we get

$$= \frac{192k^6 - 96k^5 - 60k^4 + 348k^3 + 168k^2 - 66k - 36}{32k^6 + 40k^5 - 4k^4 - 20k^3 - 6k^2} + \frac{6k^2 - 18k + 18}{2k^2 - 2k + 5}$$

(ii)

$${}^m M_2 E(L(H_k))$$

$$= \sum_{ef \in E(L(H_k))} \frac{1}{e_{L(H_k)}(e) \times e_{L(H_k)}(f)}$$

$$= \sum_{ef \in E_1(L(G))} \left[\frac{1}{e_{L(H_k)}(e) \times e_{L(H_k)}(f)} \right]$$

$$+ \cdots + \sum_{ef \in E_{3(k-1)+1}(L(G))} \left[\frac{1}{e_{L(H_k)}(e) \times e_{L(H_k)}(f)} \right]$$

$$= 6 \times 3 \left[\frac{1}{(2k+1)^2} \right] + 6 \times 2 \left[\frac{1}{(2k+1)(2k+2)} \right] + 6 \times (2i+1)$$

$$\times \sum_{i=1}^{k-2} \left[\frac{1}{(2k+2i)^2} \right] + 6 \times (2i+2) \sum_{i=1}^{k-2} \left[\frac{1}{(2k+2i)(2k+2i+1)} \right]$$

$$+ 6 \times (2i+2) \sum_{i=1}^{k-2} \left[\frac{1}{(2k+2i+1)(2k+2i+2)} \right]$$

$$+ 6 \times 3(2k-3) \left[\frac{1}{(2k+2(k-1))^2} \right]$$

$$+ 6 \times 2 \left[\frac{1}{(2k+2(k-1))(2k+2(k-1)+1)} \right].$$

After simplification, we get

$$= \frac{4608k^6 - 21888k^5 + 43200k^4 - 41184k^3 + 20016k^2 - 4752k + 432}{864k^5 + 648k^4 - 16416k^3 + 12960k^2 - 3672k - 9072}.$$

(iii)

$$HE(L(H_k))$$

$$= \sum_{ef \in E(L(G))} \frac{2}{e_{L(H_k)}(e) + e_{L(H_k)}(f)}$$

$$= \sum_{ef \in E_1(L(H_k))} \left[\frac{2}{e_{L(H_k)}(e) + e_{L(H_k)}(f)}\right]$$

$$+ \cdots + \sum_{ef \in E_{3(k-1)+1}(L(G))} \left[\frac{2}{e_{L(H_k)}(e) + e_{L(H_k)}(f)}\right]$$

$$= 6 \times 3\left[\frac{1}{(2k+1)}\right] + 6 \times 2\left[\frac{2}{2k+3}\right] + 6 \times (2i+1)$$

$$\times \sum_{i=1}^{k-2} \left[\frac{1}{(2k+2i)}\right] + 6 \times (2i+2) \sum_{i=1}^{k-2} \left[\frac{2}{4k+4i+1}\right]$$

$$+ 6 \times (2i+2) \sum_{i=1}^{k-2} \left[\frac{2}{4k+4i+3}\right]$$

$$+ 6 \times 3(2k-3)\left[\frac{1}{(2k+2(k-1))}\right] + 6 \times 2\left[\frac{2}{4k+4(k-1)+1}\right].$$

After simplification, we get

$$= \frac{960k^5 + 1656k^4 - 7452k^3 + 504k^2 + 5208k - 858}{128k^5 - 68k^4 - 316k^3 + 185k^2 - 89k - 33}$$

$$+ \frac{1008k^4 - 2952k^3 + 4788k^2 - 3672k + 786}{64k^4 - 120k^3 + 228k^2 - 152k + 30}.$$

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SOME STATUS INDICES OF BINARY TREE GRAPHS

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Abstract

In this paper, the status of binary tree and status based some indices of binary tree graphs are computed.

1. Introduction

Let G be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The notation uv represents an edge between two vertices u

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and v in G. The distance $d(u, v)$ between any two vertices u and v is the length of the shortest path which connects u and v.

A molecular graph is a graph such that its vertices correspond to the atoms and their edges to the bonds. Chemical graph theory is a branch of mathematical chemistry which has an important effect on the development of the chemical sciences. A single number that can be used to characterize some property of the graph of a molecular is called a *topological index* for that graph. Numerous topological descriptors have found some applications in theoretical chemistry especially in QSPR/QSAR research.

The status [8] of a vertex $u \in V(G)$ is defined as the sum of its distance from every other vertex in $V(G)$ and is denoted by $\sigma(u)$. That is,

$$\sigma(u) = \sum_{u \in V(G)} d(u, v).$$

The first and second status connectivity indices of a graph G are introduced by Ramane and Yalnaik in [1], defined as

$$S_1(G) = \sum_{uv \in E(G)} [\sigma(u) + \sigma(v)], \quad S_2(G) = \sum_{uv \in E(G)} [\sigma(u)\sigma(v)].$$

In [2], Kulli introduced the first and second hyper status indices of a graph G, defined as

$$HS_1(G) = \sum_{uv \in E(G)} [\sigma(u) + \sigma(v)]^2, \quad HS_2(G) = \sum_{uv \in E(G)} [\sigma(u)\sigma(v)]^2.$$

Also, in [2], Kulli introduced the following connectivity status indices.

The sum connectivity status index of a graph G is defined as

$$SS(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{\sigma(u) + \sigma(v)}}.$$

The product connectivity status index of a graph G is defined as

$$PS(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{\sigma(u)\sigma(v)}}.$$

The reciprocal product connectivity status index of a graph G is defined as

$$RPS(G) = \sum_{uv \in E(G)} \sqrt{\sigma(u)\sigma(v)}.$$

The general first and second status indices of a graph G are defined as

$$S_1^a(G) = \sum_{uv \in E(G)} [\sigma(u) + \sigma(v)]^a, \quad S_2^a(G) = \sum_{uv \in E(G)} [\sigma(u)\sigma(v)]^a,$$

where a is called as a *real number*.

In [6], Kulli introduced the following connectivity status indices.

The F_1-status index of a graph is defined as

$$F_1S(G) = \sum_{uv \in E(G)} [\sigma(u)^2 + \sigma(v)^2].$$

First and second Gourava indices, (a, b)-status index of a graph. Also, the symmetric division status index is defined as

$$SGO_1(G) = \sum_{uv \in E(G)} [\sigma(u) + \sigma(v) + \sigma(u)\sigma(v)]$$

and

$$SGO_2(G) = \sum_{uv \in E(G)} \sigma(u)\sigma(v)[\sigma(u) + \sigma(v)].$$

The (a, b)-status index of a graph G is defined as

$$S_{a,b}(G) = \sum_{uv \in E(G)} [\sigma(u)^a \sigma(v)^b + \sigma(u)^b \sigma(v)^a].$$

The symmetric division status index of a graph G is defined as

$$SDS(G) = \sum_{uv \in E(G)} \left[\frac{\sigma(u)}{\sigma(v)} + \frac{\sigma(v)}{\sigma(u)}\right].$$

In [3], Kulli introduced the following connectivity status indices.

The atom bond connectivity status index of a graph G is defined as

$$ABCS(G) = \sum_{uv \in E(G)} \sqrt{\frac{\sigma(u) + \sigma(v) - 2}{\sigma(u)\sigma(v)}}.$$

The arithmetic-geometric status index of a graph G is defined as

$$AGS(G) = \sum_{uv\in E(G)} \frac{\sigma(u) + \sigma(v)}{2\sqrt{\sigma(u)\sigma(v)}}.$$

The geometric-arithmetic status index of a connected graph G defined as

$$GAS(G) = \sum_{uv\in E(G)} \frac{2\sqrt{\sigma(u)\sigma(v)}}{\sigma(u) + \sigma(v)}.$$

The augmented status index of a graph G is defined as

$$ASI(G) = \sum_{uv\in E(G)} \left(\frac{\sigma(u)\sigma(v)}{\sigma(u) + \sigma(v)}\right)^3.$$

In [9], Ramane et al. introduced the harmonic status index defined as

$$HS(G) = \sum_{uv\in E(G)} \frac{2}{\sigma(u) + \sigma(v)}.$$

In this paper, some statuses of perfect binary tree graphs were introduced. Furthermore, the value of these indices for perfect binary tree graphs was determined.

2. Results for Status of Perfect Binary Tree Graphs

In this section, some status of vertices of perfect binary tree graphs was analyzed. A binary tree is said to be *perfect* if all the internal nodes have strictly two children, and every external or leaf node is at the same level or same depth within a tree. For a perfect m-array tree with height h, the upper bound for the maximum number of leaves is m^h. If there is a zero-index level, then the number of nodes on the h level is exactly 2^h.

Level 0: 2^0 nodes, Level 1: 2^1 nodes, Level 2: 2^2 nodes, Level 3: 2^3 nodes and so on.

So the total number of nodes in a perfect binary tree with height h is

$$n = 2^0 + 2^1 + 2^2 + 2^3 + \cdots + 2^{h-1} + 2^h = 2^{h+1} - 1.$$

The status, denoted by $\sigma(u)$ of a vertex u in G is the sum of distances of all other vertices from u to all other vertices in G.

Let G be a binary tree with height h.

Let u_0 denote the central vertex, u_1 denote a vertex on level 1, u_2 denote a vertex on level 2, $\ldots$, u_h denote a vertex on level h.

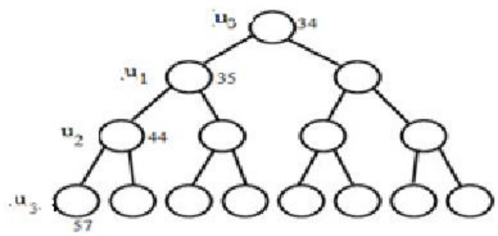

Figure 1. Perfect binary tree $(m = 2)$ with status.

Now, the status values of vertices of a binary tree are given in Table 1.

Height of perfect binary tree	$\sigma(u_0)$	$\sigma(u_1)$	$\sigma(u_2)$	$\sigma(u_3)$	$\sigma(u_4)$	$\sigma(u_5)$
0	0					
1	2	3				
2	10	11	16			
3	34	35	44	57		
4	98	99	116	141	170	
5	258	259	292	341	398	459

and so on.

Next, we calculate the status vales of vertices of a perfect binary tree with height n:

$$\sigma(u_0) = 2^{n+1}(n-1) + 2,\ \sigma(u_1) = 2^{n+1}(2^0(n-2)+1) + 3,$$

$$\sigma(u_2) = 2^n(2^1(n-1)+1) + 4,$$

$$\sigma(u_3) = 2^{n-1}(2^2(n)+1) + 5,\ \sigma(u_4) = 2^{n-2}(2^3(n+1)+1) + 6,$$

$$\sigma(u_5) = 2^{n-3}(2^4(n+2)+1) + 7,$$

…

$$\sigma(u_{i-i}) = 2^{n-(i-3)}\{2^{i-2}[(n+(i-1)+3)+1]+(i-1)+2\},$$

$$\sigma(u_i) = 2^{n-(i-2)}\{2^{i-1}[n+(i+3)+1]+i+2\},$$

…

$$\sigma(u_n) = (2n-1)2^{n+1} - 2^{n+2} + n + 5.$$

Now, we compute $\sigma(u_i)+\sigma(u_{i-i})$ and $\sigma(u_i)\sigma(u_{i-i})$:

$$\sigma(u_i)+\sigma(u_{i-i}) = 2^{n+1}(2n+2i-7)+3\times 2^{n-i+2}+2i+3, \tag{1}$$

$$\begin{aligned}\sigma(u_i)\sigma(u_{i-i}) = {} & (4n^2+4i^2+8ni-28n-28i+48)2^{2n} \\ & + (4i^2+4ni+6n-8i-22)2^n \\ & + (24n+24i-80)2^{2n-i} \\ & + (12i+20)2^{n-i}+32\times 2^{2n-2i}+i^2+3i+2.\end{aligned} \tag{2}$$

Therefore, from (1) and (2),

$$\begin{aligned}\sum_{i=1}^{n} 2^i(\sigma(u_i)+\sigma(u_{i-1})) = {} & \sum_{i=1}^{n} 2^i[(2^{n+1}(n+i-3)+2^{n-i+2}+i+2) \\ & + (2^{n+1}(n+i-4)+2^{n-i+3}+i+1)] \\ = {} & \sum_{i=1}^{n} 2^i[2^{n+1}(2n+2i-7)+3\times 2^{n-i+2}+2i+3],\end{aligned} \tag{3}$$

$$\begin{aligned}\sum_{i=1}^{n} 2^i(\sigma(u_i)+\sigma(u_{i-1})) = {} & \sum_{i=1}^{n} 2^i[(4n^2+4i^2+8ni-28n-28i+48)2^{2n} \\ & + (4i^2+4ni+6n-8i-22)2^n \\ & + (24n+24i-80)2^{2n-i}+(12i+20)2^{n-i} \\ & + 32\times 2^{2n-2i}+i^2+3i+2].\end{aligned} \tag{4}$$

2.1. Status based indices of binary tree graphs

The first, second and hyper status indices, also the sum, product connectivity status indices, F_1-status index, first and second status Gourava indices, Gourava (a, b)-status indices were investigated. Moreover, it was calculated sum and product connectivity status indices, reciprocal connectivity status indices, ABC, AGS, GAS and ASI status indices of perfect binary tree graphs.

Theorem 2.1.1. *Let G be the perfect binary tree graph. Then*

(i) $S_1(G) = (16n - 36)2^{2n} + (8n + 38)2^n - 2.$

(ii)

$$S_2(G) = (40n^2 - 152n + 176)2^{3n} + (24n^2 - 4n - 148)2^{2n}$$
$$+ (2n^2 - 30n - 24)2^n - 4.$$

(iii)

$$HS_1(G) = (128n^2 - 576n + 712)2^{3n} + (128n^2 - 64n - 576)2^{2n}$$
$$+ (8n^2 + 112 - 118)2^n - 18.$$

(iv)

$$HS_2(G) = \sum_{i=1}^{n} 2^i [(4n^2 + 4i^2 + 8ni - 28n - 28i + 48)2^{2n}$$
$$+ (4i^2 + 4ni + 6n - 8i - 22)2^n + (24n + 24i - 80)2^{2n-i}$$
$$+ (12i + 20)2^{n-i} + 32 \times 2^{2n-2i} + i^2 + 3i + 2]$$
$$\times [(4n^2 + 4i + 8ni - 28n - 28i + 48)2^n$$
$$+ (4i^2 + 4ni + 6n - 8i - 22)2^n + (24n + 24i - 80)2^{n-i}$$
$$+ (12i + 20)2^{n-i} + 32 \times 2^{2n-2i} + i^2 + 3i + 2]^2.$$

(v) $S_1^a(G) = \sum_{i=1}^{n} 2^i[(4n + 4i - 14)2^n + 12 \times 2^{n-i} + 2i + 3]^a.$

(vi)

$$S_2^a(G) = \sum_{i=1}^{n} 2^i[(4n^2 + 4i^2 + 8ni - 28n - 28i + 48)2^{2n}$$

$$+ (4i^2 + 4ni + 6n - 8i - 22)2^n + (24n + 24i - 80)2^{2n-i}$$

$$+ (12i + 20)2^{n-i} + 32 \times 2^{2n-2i} + i^2 + 3i + 2]^a.$$

Proof. By using the above definition and values, we deduce

(i)

$$S_1(G) = \sum_{uv \in E(G)} [\sigma(u) + \sigma(v)] = \sum_{i=1}^{n} 2^i(\sigma(u_i) + \sigma(u_{i-1}))$$

$$= \sum_{i=1}^{n} 2^i[(2^{n+1}(n + i - 3) + 2^{n-i+2} + i + 2)$$

$$+ (2^{n+1}(n + i - 4) + 2^{n-i+3} + i + 1)] \text{ from (1) and (2)}$$

$$= \sum_{i=1}^{n} 2^i[2^{n+1}(2n + 2i - 7) + 3 \times 2^{n-i+2} + 2i + 3]$$

$$= (16n - 36)2^{2n} + (8n + 38)2^n - 2.$$

(ii)

$$S_2(G) = \sum_{uv \in E(G)} [\sigma(u)\sigma(v)] = \sum_{i=1}^{n} 2^i(\sigma(u_i) \times \sigma(u_{i-1}))$$

$$= \sum_{i=1}^{n} 2^i[(2^{n+1}(n + i - 3) + 2^{n-i+2} + i + 2)$$

$$\times (2^{n+1}(n + i - 4) + 2^{n-i+3} + i + 1)]$$

$$= [(4n^2 - 28n + 48)2^{2n} + (30n - 102)2^n + 54](2^n - 1)$$

$$+ [(8n - 24)2^{2n} + (4n + 16) + 15]((n - 1)2^{n+1} + 2)$$

$$+ [(4 \times 2^{n} + 1)]((n^{2} - 2n + 3)2^{n+1} - 6)$$

$$= (40n^{2} - 152n + 176)2^{3n} + (24n^{2} - 4n - 148)2^{2n}$$

$$+ (2n^{2} - 30n - 24)2^{n} - 4.$$

(iii)

$$HS_1(G) = \sum_{uv \in E(G)} [\sigma(u) + \sigma(v)]^2 = \sum_{i=1}^{n} 2^i [(\sigma(u_i) + \sigma(u_{i-1}))]^2$$

$$= \sum_{i=1}^{n} 2^i [(2^{n+1}(n + i - 3) + 2^{n-i+2} + i + 2)$$

$$+ (2^{n+1}(n + i - 4) + 2^{n-i+3} + i + 1)]^2$$

$$= \sum_{i=1}^{n} 2^i [2^{n+1}(2n + 2i - 7) + 3 \times 2^{n-i+2} + 2i + 3]^2$$

$$= (128n^{2} - 576n + 712)2^{3n} + (128n^{2} - 64n - 576)2^{2n}$$

$$+ (8n^{2} + 112n - 118)2^{n} - 18.$$

(iv)

$$HS_2(G) = \sum_{uv \in E(G)} [\sigma(u) \times \sigma(v)]^2 = \sum_{i=1}^{n} [2^i (\sigma(u_i)\sigma(u_{i-1}))]^2$$

$$= \sum_{i=1}^{n} 2^i [(2^{n+1}(n + i - 3) + 2^{n-i+2} + i + 2)$$

$$\times (2^{n+1}(n + i - 4) + 2^{n-i+3} + i + 1)]^2$$

$$= \sum_{i=1}^{n} 2^i [(4n^{2} + 4i^{2} + 8ni - 28n - 28i + 48)2^{2n}$$

$$+ (4i^{2} + 4ni + 6n - 8i - 22)2^{n}$$

$$+ (24n + 24i - 80)2^{2n-i} + (12i + 20)2^{n-i}$$

$$+ 32 \times 2^{2n-2i} + i^{2} + 3i + 2]$$

$$\times [(4n^2 + 4i^2 + 8ni - 28n - 28i + 48)2^{2n}$$

$$+ (4i^2 + 4ni + 6n - 8i - 22)2^n$$

$$+ (24n + 24i - 80)2^{2n-i} + (12i + 20)2^{n-i}$$

$$+ 32 \times 2^{2n-2i} + i^2 + 3i + 2]^2.$$

(v)

$$S_1^a(G) = \sum_{uv\in E(G)} [\sigma(u) + \sigma(v)]^a = \sum_{i=1}^{n} 2^i [(\sigma(u_i) + \sigma(u_{i-1}))]^a$$

$$= \sum_{i=1}^{n} 2^i [(2^{n+1}(n + i - 3) + 2^{n-i+2} + i + 2)$$

$$+ (2^{n+1}(n + i - 4) + 2^{n-i+3} + i + 1)]^a$$

$$= \sum_{i=1}^{n} 2^i [(4n + 4i - 14)2^n + 12 \times 2^{n-i} + 2i + 3]^a.$$

(vi)

$$S_2^a(G) = \sum_{uv\in E(G)} [\sigma(u) \times \sigma(v)]^a = \sum_{i=1}^{n} [2^i (\sigma(u_i) \times \sigma(u_{i-1}))]^a$$

$$= \sum_{i=1}^{n} 2^i [(2^{n+1}(n + i - 3) + 2^{n-i+2} + i + 2)$$

$$\times (2^{n+1}(n + i - 4) + 2^{n-i+3} + i + 1]^a$$

$$= \sum_{i=1}^{n} 2^i [(4n^2 + 4i^2 + 8ni - 28n - 28i + 48)2^{2n}$$

$$+ (4i^2 + 4ni + 6n - 8i - 22)2^n$$

$$+ (24n + 24i - 80)2^{2n-i} + (12i + 20)2^{n-i}$$

$$+ 32 \times 2^{2n-2i} + i^2 + 3i + 2]^a.$$

Theorem 2.1.2. *Let G be the perfect binary tree graph. Then*

(i)

$$F_1S(G) = (64n^2 - 288n + 360)2^{3n} + (64n^2 - 40n - 280)2^{2n} + (4n^2 + 52n - 70)2^n - 10.$$

(ii)

$$SGO_1(G) = (40n^2 - 152n + 176)2^{3n} + (24n^2 - 12n - 284)2^{2n} + (2n^2 - 22n + 14)2^n - 6.$$

(iii)

$$SGO_2(G) = [(40n^2 - 152n + 176)2^{3n} + (24n^2 - 4n - 148)2^{2n} + (2n^2 - 30n - 24)2^n - 4] \times [(16n - 36)2^{2n} + (8n + 38)2^n - 2].$$

(iv)

$$S_{a,b}(G) = \sum_{i=1}^{n} 2^i [(2^{n+1}(n+i-3) + 2^{n-i+2} + i + 2)^a \times (2^{n+1}(n+i-4) + 2^{n-i+3} + i + 1)^b + (2^{n+1}(n+i-3) + 2^{n-i+2} + i + 2)^b \times (2^{n+1}(n+i-4) + 2^{n-i+3} + i + 1)^a].$$

(v)

$SDS(G)$

$$= \sum_{i=1}^{n} 2^i \left[\frac{\begin{array}{l} [(4n^2 + 4i^2 + 8ni - 32n + 64)2^{2n} + (4i^2 + 4ni + 4n - 12i - 16)2^n \\ + (32n + 32i - 128)2^{2n-i} + (16i + 16)2^{n-i} + 64 \times 2^{2n-2i} + i^2 + 2i + 1] \\ + [4n^2 + (4i^2 + 8ni - 24n - 24i + 36)2^{2n} + (4i^2 + 4ni + 8n - 2i - 24)2^n \\ \times (16n + 16i - 48)2^{2n-i} + (8i + 16)2^{n-i} + 16 \times 2^{2n-2i} + i^2 + 4i + 4] \end{array}}{\begin{array}{l} (4n^2 + 4i^2 + 8ni - 28n - 28i + 48)2^{2n} + (4i^2 + 4ni + 6n - 8i - 22)2^n \\ + (24n + 24i - 80)2^{2n-i} + (12i + 20)2^{n-i}(4n^2 + 4i^2 \\ + 8ni - 28n - 28i + 48)2^{2n} + 32 \times 2^{2n-2i} + i^2 + 3i + 2 \end{array}} \right].$$

Proof. By using the above definition and values, we deduce

(i)

$$F_1S(G) = \sum_{uv\in E(G)} [\sigma(u)^2 + \sigma(v)^2] = \sum_{i=1}^{n} 2^i [\sigma(u_i)^2 + \sigma(u_{i-1})^2]$$

$$= \sum_{i=1}^{n} 2^i [(2^{n+1}(n+i-3) + 2^{n-i+2} + i + 2)^2$$

$$+ (2^{n+1}(n+i-4) + 2^{n-i+3} + i + 1)^2]$$

$$= (64n^2 - 288n + 360)2^{3n} + (64n^2 - 40n - 280)2^{2n}$$

$$+ (4n^2 + 52n - 70)2^n - 10.$$

(ii)

$$SGO_1(G) = \sum_{uv\in E(G)} [\sigma(u) + \sigma(v) + \sigma(u)\sigma(v)]$$

$$= \sum_{i=1}^{n} 2^i [\sigma(u_i) + \sigma(u_{i-1}) + \sigma(u_i)\sigma(u_{i-1})]$$

$$= \sum_{i=1}^{n} 2^i [(2^{n+1}(2n+2i-7) + 3\times 2^{n-i+2} + 2i + 3)$$

$$+ ((2^{n+1}(n+2i-1) + 2^{1-i}) \times (2^{n+1}(n+2i-3) + 2^{2-i}))]$$

$$= (32n^2 - 144n + 176)2^{3n} + (32n^2 + 4n - 184)2^{2n}$$

$$+ (2n^2 + 38n + 14)2^n - 6.$$

(iii)

$$SGO_2(G) = \sum_{uv\in E(G)} [\sigma(u) + \sigma(v) + \sigma(u)\sigma(v)]$$

$$= \sum_{i=1}^{n} 2^i [(\sigma(u_i)\sigma(u_{i-1}))[\sigma(u_i) + \sigma(u_{i-1})]]$$

$$= [(40n^2 - 152n + 176)2^{3n} + (24n^2 - 4n - 148)2^{2n}$$

$$+ (2n^2 - 30n - 24)2^n - 4]$$

$$\times [(16n - 36)2^{2n} + (8n + 38)2^n - 2].$$

(iv)

$$S_{a,b}(G) = \sum_{uv \in E(G)} [\sigma(u)^a \sigma(v)^b + \sigma(u)^b \sigma(v)^a]$$

$$= \sum_{i=1}^{n} 2^i [(\sigma(u_i)^a \sigma(u_{i-1}))^b + (\sigma(u_i)^b \sigma(u_{i-1})^b)$$

$$= \sum_{i=1}^{n} 2^i [(2^{n+1}(n+i-3) + 2^{n-i+2} + i + 2)^a$$

$$\times (2^{n+1}(n + i - 4) + 2^{n-i+3} + i + 1)^b$$

$$+ (2^{n+1}(n + i - 3) + 2^{n-i+2} + i + 2)^b$$

$$\times (2^{n+1}(n + i - 4) + 2^{n-i+3} + i + 1)^a].$$

(v)

$SDS(G)$

$$= \sum_{uv \in E(G)} \left[\frac{\sigma(u)}{\sigma(v)} + \frac{\sigma(v)}{\sigma(u)}\right] = \sum_{i=1}^{n} 2^i \left[\frac{\sigma(u_i)}{\sigma(u_{i-1})} + \frac{\sigma(u_{i-1})}{\sigma(u_i)}\right]$$

$$= \sum_{i=1}^{n} 2^i \left[\frac{\begin{array}{l}[(4n^2 + 4i^2 + 8ni - 32n + 64)2^{2n} + (4i^2 + 4ni + 4n - 12i - 16)2^n \\ + (32n + 32i - 128)2^{2n-i} + (16i + 16)2^{n-i} + 64 \times 2^{2n-2i} + i^2 + 2i + 1] \\ + [4n^2 + (4i^2 + 8ni - 24n - 24i + 36)2^{2n} + (4i^2 + 4ni + 8n - 2i - 24)2^n \\ \times (16n + 16i - 48)2^{2n-i} + (8i + 16)2^{n-i} + 16 \times 2^{2n-2i} + i^2 + 4i + 4]\end{array}}{\begin{array}{l}(4n^2 + 4i^2 + 8ni - 28n - 28i + 48)2^{2n} + (4i^2 + 4ni + 6n - 8i - 22)2^n \\ + (24n + 24i - 80)2^{2n-i} + (12i + 20)2^{n-i}(4n^2 + 4i^2 \\ + 8ni - 28n - 28i + 48)2^{2n} + 32 \times 2^{2n-2i} + i^2 + 3i + 2\end{array}}\right].$$

Theorem 2.1.3. *Let G be the perfect binary tree graph. Then*

(i) $$SS(G) = \frac{1}{\sqrt{(16n-36)2^{2n} + (8n+38)2^{n} - 2}}.$$

(ii) $$PS(G) = \frac{1}{\sqrt{(40n^2 - 152n + 176)2^{3n} + (24n^2 - 4n - 148)2^{2n} + (2n^2 - 30n - 24)2^{n} - 4}}.$$

(iii) $$RPS(G) = \sqrt{(40n^2 - 152n + 176)2^{3n} + (24n^2 - 4n - 148)2^{2n} + (2n^2 - 30n - 24)2^{n} - 4}.$$

Proof. By using the above definition and values, we deduce

(i)

$$SS(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{\sigma(u)+\sigma(v)}} = \sum_{i=1}^{n} 2^{i}\left[\frac{1}{\sqrt{\sigma(u_i)+\sigma(u_{i-1})}}\right]$$

$$= \sum_{i=0}^{n} 2^{i}\left[\frac{1}{\sqrt{(2^{n+1}(n+i-3) + 2^{n-i+2} + i + 2) + (2^{n+1}(n+i-4) + 2^{n-i+3} + i + 1)}}\right]$$

$$= \sum_{i=1}^{n} 2^{i}\left[\frac{1}{\sqrt{2^{n+1}(2n+2i-7) + 3\times 2^{n-i+2} + 2i + 3}}\right]$$

$$= \frac{1}{\sqrt{(16n-36)2^{2n} + (8n+38)2^{n} - 2}}.$$

(ii)

$$PS(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{\sigma(u)\sigma(v)}} = \sum_{i=1}^{n} 2^{i}\left[\frac{1}{\sqrt{\sigma(u_i)\sigma(u_{i-1})}}\right]$$

$$= \sum_{i=1}^{n} 2^{i} \left[\frac{1}{\sqrt{\begin{array}{l}(2^{n+1}(n+i-3) + 2^{n-i+2} + i + 2) \\ \times (2^{n+1}(n+i-4) + 2^{n-i+3} + i + 1)\end{array}}} \right]$$

$$= \frac{1}{\sqrt{\begin{array}{l}(40n^2 - 152n + 176)2^{3n} + (24n^2 - 4n - 148)2^{2n} \\ + (2n^2 - 30n - 24)2^{n} - 4\end{array}}}.$$

(iii)

$$RPS(G) = \sum_{uv \in E(G)} \sqrt{\sigma(u)\sigma(v)} = \sum_{i=1}^{n} 2^{i} \sqrt{\sigma(u_i)\sigma(u_{i-1})}$$

$$= \sqrt{\begin{array}{l}(40n^2 - 152n + 176)2^{3n} + (24n^2 - 4n - 148)2^{2n} \\ + (2n^2 - 30n - 24)2^{n} - 4\end{array}}.$$

Theorem 2.1.4. *Let G be the perfect binary tree graph. Then*

(i) $$ABCS(G) = \sqrt{\frac{[(16n - 36)2^{2n} + (8n + 38)2^{n} - 2] - 2}{\begin{array}{l}(40n^2 - 152n + 176)2^{3n} + (24n^2 - 4n - 148)2^{2n} \\ + (2n^2 - 30n - 24)2^{n} - 4\end{array}}}.$$

(ii) $$AGS(G) = \frac{(16n - 36)2^{2n} + (8n + 38)2^{n} - 2 - 2}{2\sqrt{\begin{array}{l}(40n^2 - 152n + 176)2^{3n} + (24n^2 - 4n - 148)2^{2n} \\ + (2n^2 - 30n - 24)2^{n} - 4\end{array}}}.$$

(iii) $$GAS(G) = \frac{2\sqrt{\begin{array}{l}(40n^2 - 152n + 176)2^{3n} + (24n^2 - 4n - 148)2^{2n} \\ + (2n^2 - 30n - 24)2^{n} - 4\end{array}}}{(16n - 36)2^{2n} + (8n + 38)2^{n} - 2}.$$

(iv) $$ASI(G) = \left(\frac{(40n^2 - 152n + 176)2^{3n} + (24n^2 - 4n - 148)2^{2n} + (2n^2 - 30n - 24)2^n - 4}{[(16n - 36)2^{2n} + (8n + 38)2^n - 2] - 2} \right)^3.$$

Proof. By using the above definition and values, we deduce

(i)

$$ABCS(G) = \sum_{uv \in E(G)} \sqrt{\frac{\sigma(u) + \sigma(v) - 2}{\sigma(u)\sigma(v)}}$$

$$= \sum_{i=1}^{n} 2^i \left[\sqrt{\frac{\sigma(u_i) + \sigma(u_{i-1}) - 2}{\sigma(u_i)\sigma(u_{i-1})}} \right]$$

$$= \sqrt{\frac{[(16n - 36)2^{2n} + (8n + 38)2^n - 2] - 2}{(40n^2 - 152n + 176)2^{3n} + (24n^2 - 4n - 148)2^{2n} + (2n^2 - 30n - 24)2^n - 4}}.$$

(ii)

$$AGS(G) = \sum_{uv \in E(G)} \frac{\sigma(u) + \sigma(v)}{2\sqrt{\sigma(u)\sigma(v)}} = \sum_{i=1}^{n} 2^i \left[\frac{\sigma(u_i) + \sigma(u_{i-1})}{2\sqrt{\sigma(u_i)\sigma(u_{i-1})}} \right]$$

$$= \sum_{i=1}^{n} 2^i \left[\frac{2^{n+1}(2n + 2i - 7) + 3 \times 2^{n-i+2} + 2i + 3}{2\sqrt{(2^{n+1}(n + i - 3) + 2^{n-i+2} + i + 2) \times (2^{n+1}(n + i - 4) + 2^{n-i+3} + i + 1)}} \right]$$

$$= \frac{(16n - 36)2^{2n} + (8n + 38)2^n - 2}{2\sqrt{(40n^2 - 152n + 176)2^{3n} + (24n^2 - 4n - 148)2^{2n} + (2n^2 - 30n - 24)2^n - 4}}.$$

(iii)

$$GAS(G) = \sum_{uv\in E(G)} \frac{2\sqrt{\sigma(u)\sigma(v)}}{\sigma(u)+\sigma(v)}$$

$$= \sum_{i=1}^{n} 2^i \left[\frac{2\sqrt{\sigma(u_i)\sigma(u_{i-1})}}{\sigma(u_i)+\sigma(u_{i-1})}\right]$$

$$= \sum_{i=1}^{n} 2^i \left[\frac{2\sqrt{\begin{array}{l}(2^{n+1}(n+i-3)+2^{n-i+2}+i+2)\\ \times(2^{n+1}(n+i-4)+2^{n-i+3}+i+1)\end{array}}}{2^{n+1}(2n+2i-7)+3\times 2^{n-i+2}+2i+3}\right]$$

$$= \frac{2\sqrt{\begin{array}{l}(40n^2-152n+176)2^{3n}+(24n^2-4n-148)2^{2n}\\ +(2n^2-30n-24)2^n-4\end{array}}}{(16n-36)2^{2n}+(8n+38)2^n-2}.$$

(iv)

$$ASI(G) = \sum_{uv\in E(G)} \left(\frac{\sigma(u)\sigma(v)}{\sigma(u)+\sigma(v)-2}\right)^3$$

$$= \sum_{i=1}^{n} 2^i \left(\frac{\sigma(u_i)\sigma(u_{i-1})}{\sigma(u_i)+\sigma(u_{i-1})-2}\right)^3$$

$$= \sum_{i=1}^{n} 2^i \left(\frac{\begin{array}{l}(2^{n+1}(n+i-3)+2^{n-i+2}+i+2)\times(2^{n+1}(n+i-4)\\ +2^{n-i+3}+i+1)\end{array}}{[2^{n+1}(2n+2i-7)+3\times 2^{n-i+2}+2i+3]-2}\right)^3$$

$$= \left(\frac{\begin{array}{l}(40n^2-152n+176)2^{3n}+(24n^2-4n-148)2^{2n}\\ +(2n^2-30n-24)2^n-4\end{array}}{[(16n-36)2^{2n}+(8n+38)2^n-2]-2}\right)^3.$$

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