A STUDY ON TOPOLOGICAL INDICES OF GRAPHS USING DISTANCE IN GRAPHS

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By

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CERTIFICATE

This is to certify that **P.GLADYIS** has prepared this thesis entitled **"A STUDY ON TOPOLOGICAL INDICES OF GRAPHS USING DISTANCE IN GRAPHS "** submitted to **BHARATHIDASAN UNIVERSITY** for the award of the degree of **Doctor of Philosophy in Mathematics** under my guidance and supervision. This is a bonafide record of the independent and original work done by her under my guidance in the PG and Research Department of Mathematics, Government Arts College, Tiruchirappalli -620 022, from April 2018 to March 2023. This is also to certify that the thesis represents her independent original work and has not previously formed the basis for award of any Degree, Diploma, Fellowship or any other similar title in any Institution or University.

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DECLARATION

I, **P. Gladyis,** hereby declare that this thesis entitled **"A STUDY ON TOPOLOGICAL INDICES OF GRAPHS USING DISTANCE IN GRAPHS"** submitted to the Bharathidasan University for the award of the degree of **Doctor of Philosophy in Mathematics**, is a record of independent research work carried out by me during the period from April 2018 to March 2023 under the supervision of **Dr. G. Srividhya**, Assistant Professor, PG and Research Department of Mathematics, Government Arts College, Tiruchirappalli-22. This work has not formed the basis for the award of any Degree, Diploma, Associateship or Fellowship, or any other similar title in any Institution or University to my knowledge.

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SA	**Thesis checkup.docx** Document Thesis checkup.docx (D19258346)	1
SA	**Chapter 1.doc** Document Chapter 1.doc (D19255525)	1
SA	**COMPUTING DEGREE BASED TOPOLOGICAL INDEICES.pdf** Document COMPUTING DEGREE BASED TOPOLOGICAL INDEICES.pdf (D37321299)	6
SA	**K. Julietraja-18147991114-publication 2.pdf** Document K. Julietraja-18147991114-publication 2.pdf (D94568023)	1
SA	**Basava and hyper-Basava indices of graphs.pdf** Document Basava and hyper-Basava indices of graphs.pdf (D111629257)	3
W	URL: https://math.stackexchange.com/questions/997420/prove-that-sum-sum-k-1n-1k-n-1-choose-k-1-2n-k-1-2k-is-zer Fetched: 2021-10-30T11:45:02.4730000	1

CONTENTS

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A STUDY ON TOPOLOGICAL INDICES OF GRAPHS USING DISTANCE IN GRAPHS

Abstract

In this era of rapid technological development, chemical and pharmaceutical techniques in recent years a large number of new nanomaterials, crystalline materials, and drugs emerge every year. To determine the chemical properties of such a large number of new compounds and new drugs requires a large amount of chemical experiments.

The topological indices correlate certain physicochemical properties such as boiling point, stability of chemical compounds. The significance of topological indices is usually associated with quantitative structures property relationship (QSPR) and quantitative structure activity relationship(QSAR).

Here we have introduced 213 new indices namely Arithmetic, Harmonic, Geometric Arithmetic, Arithmetic Geometric, sum and product connectivity, ABC, Inverse sum, Augumented Zagreb, Hyper First, second and Third Zagreb, F, Reduced Forgotten , Reduced second Zagreb, Reduced second Hyper Zagreb, General Reduced Forgotten, General Reduced second Zagreb, General Reduced Hyper second Zagreb cutting number index. Also we define multiplicative and polynomial indices of a graph G. First Zagreb vertex cutting number index, F-vertex cutting number index, Y-index vertex cutting number index, Inverse Degree vertex cutting number index, Modified Zagreb vertex cutting number index, Zeroth-order General Randic vertex cutting number index, The Reduced First Zagreb vertex cutting number index, The Reduced F-index and the Reduced Modified first Zagreb vertex cutting number indices of G. Also we define multiplicative and polynomial indices of a graph G. First, Second, Third, Fourth and Fifth cutting number indices,

SK, SK_1 and SK_2 cutting number indices and Nano-Zagreb, Sum Nano-Zagreb cutting number indices of a graph G. Also we define multiplicative and polynomial indices of a graph G. eccentricity based Redefined First, Second, and Third Zagreb indices, Modified First, Second, Third, Fourth and Fifth Zagreb indices, Randic, Reciprocal Randic, Reduced Reciprocal Randic and Generalized Randic indices. Also we define multiplicative and polynomial indices of a graph G. eccentricity based Sombor, modified Sombor, Reduced Sombor, Reduced modified Sombor, The first and second (a, b) – KA indices and the first and second Reduced (a, b) – KA indices , Square Reduced Sum,Product, Arithmetic, Geometric and Harmonic indices, First F, Second F, The Minus F and Square F indices. Also we define multiplicative and polynomial indices of Nanostar Dendrimer D(n). General first and second status index, , General first and second Gourava status index, General modified first and second Gourava status index also find the corresponding polynomial of a Molecular graph of Dendrimer D(n).

By using those new indices we calculated the above topological indices for 4 chemical structures namely (i) Nanostar Dendrimer NS[n] (ii) Nanostar Dendrimer D_n (iii) Nanostar Dendrimer D(n) (iv) Hexogonal core Dendrimer $D_{3,n-1}$

PREFACE

"Graph Theory" is an important branch of Mathematics. It has grown rapidly in recent times with a lot of research activities.

The blossoming of a new branch of study in the field of Chemistry, "Chemical graph theory" is yet another proof of the importance and role of graph theory. In physics, graph theory is applied in continuum statistical mechanics and discrete statistical mechanics. The role of graph theory in Computer science is everywhere.

In this era of rapid technological development, chemical and pharmaceutical techniques in recent years have been rapidly evolved, and thus a large number of new nano materials, crystalline materials, and drugs emerge every year. To determine the chemical properties of such a large number of new compounds and new drugs requires a large amount of chemical experiments, thereby greatly increasing the workload of the chemical and pharmaceutical researchers. Fortunately, the chemical based experiments found that there was strong connection between topology molecular structures and their physical behaviors, chemical characteristics, and biological features, such as melting point, boiling point, and toxicity of drugs.

The concept of "topological index" was first proposed by Hosoya for characterizing the topological nature of a graph. Topological indices are the mathematical measures which correspond to the structures of any simple finite graph. The topological indices correlate certain physicochemical properties such as boiling point, stability of chemical compounds. They are invariant under the graph isomorphism. The significance of topological indices is usually associated with quantitative structures property relationship (QSPR) and quantitative structure activity relationship (QSAR).

Chapter 1 contains review of literature, and basic definitions which are required for the subsequent chapters.

In Chapter 2, we introduced 57 new indices named Arithmetic, Harmonic, Geometric Arithmetic, Arithmetic Geometric, Sum and Product connectivity, ABC, Inverse sum, Augumented Zagreb Index, Hyper First, second and Third Zagreb, F , Reduced Forgotten, Reduced second Zagreb, Reduced second Hyper Zagreb, General Reduced Forgotten, General Reduced second Zagreb, General Reduced Hyper second Zagreb cutting number Index. Also we define multiplicative and polynomial Indices of a chemical structure Nanostar Dendrimer NS[n].

In Chapter 3, we introduced 27 new indices on namely, First Zagreb vertex cutting number Index, F-vertex cutting number Index, Y-Index vertex cutting number index, Inverse Degree vertex cutting number index, Modified Zagreb vertex cutting number index, Zeroth-order General Randic vertex cutting number index, The Reduced First Zagreb vertex cutting number index, The Reduced F-index and the Reduced Modified first Zagreb vertex cutting number indices of G. Also we define multiplicative and polynomial indices of a graph NS[n].

In Chapter 4, we derived ten new indices First, Second, Third, Fourth and Fifth cutting number indices, SK, SK_1 and SK_2 cutting number indices, Nano-Zagreb and Sum Nano-Zagreb cutting number indices of a Nanostar Dendrimer D_n.

In Chapter 5, we define polynomial of First, Second, Third, Fourth and Fifth cutting number indices, polynomial of SK, SK_1 and SK_2 cutting number indices, polynomial of Nano-Zagreb and Sum Nano-Zagreb cutting number indices of a Nanostar Dendrimer D_n.

In Chapter 6, we introduced ten new indices Multiplicative of First, Second, Third, Fourth and Fifth cutting number Indices, Multiplicative of SK, SK_1 and SK_2 cutting number Indices, Multiplicative of Nano-Zagreb and sum Nano-Zagreb cutting number indices of a Nanostar Dendrimer D_n.

In Chapter 7, we calculate eccentricity based Redefined First, Second and Third Zagreb indices, Modified First, Second, Third, Fourth andFifth Zagreb indices, Randic, Reciprocal Randic, Reduced Reciprocal Randic and Generalized Randic indices. Also we define multiplicative and polynomial indices of Hexoconal core Dendrimer $D_{3,n-1}$.

In Chapter 8, we obtained eccentricity based Sombor, modified Sombor, Reduced Sombor, Reduced modified Sombor, The first and second (a, b) – KA indices and the first and second Reduced (a, b) – KA indices , Square Reduced sum, product, Arithmetic, Geometric and Harmonic indices, First F, Second F, The Minus F and Square F indices. Also we define multiplicative and polynomial indices of Nanostar Dendrimer D(n).

In Chapter 9, we derive General first and second status index, General first and second Gourava status index, General modified first and second Gourava status index also find the corresponding polynomial of a molecular graph of Dendrimer D(n).

CHAPTER 1

INTRODUCTION

In this chapter we give basic definitions and ideas regarding the thesis.

1.1 BASIC CONCEPTS IN GRAPH THEORY

A **graph** $G = (V, E)$ consists of a set V of vertices (also called nodes) and a set E of edges. If an edge connects to a vertex we say the edge is incident to the vertex and say the vertex is an **endpoint** of the edge. If an edge has only one end vertex then it is called a **loop** edge and an edge with distinct end vertices is called as **link.** If two or more edges have the same end vertices then they are called **multiple or parallel edges**. Two vertices that are joined by an edge are called **adjacent vertices.** A graph H is a **subgraph** of a graph G if all vertices and edges in H are also in G. A graph is **finite** if both its end vertex set and edge set are finite. A graph with just one vertex is called as trivial and all other graphs are **nontrivial.**

A **walk** in a graph G is a sequence of alternating vertices and edges $v_1e_1v_2e_2 \dots v_ne_nv_{n+1}$ with $n \geq 0$. The vertices v_1 and v_{n+1} are called the **origin** and **terminus** respectively and $v_1, v_2, v_3 \dots\dots v_{n+1}$. If $v_1 = v_{n+1}$ then the walk is **closed** otherwise it is **open**. The **length** of the walk is the number of edges in the walk. A walk of length zero is a **trivial walk**. A **trail** is a walk with no repeated edges. A **path** is a walk with no repeated vertices. A **circuit** is a closed trail and a trivial circuit has a single vertex and no edges. A trail or circuit is **Eulerian** if it uses every edge in the graph. A graph is called **Eulerian** if it contains an Eulerian circuit. A path that contains every vertex of G is called a **Hamilton path** of G; similarly a **Hamilton cycle** of G is a cycle that contains every vertex of G. A graph is **Hamilton** if it contains **Hamilton cycle.**

Simple graph is a graph with no loop edges or multiple edges. Edges in a simple graph may be specified by a set { v_i, v_j}of the two vertices that the edge makes adjacent. A graph with more than one edge between a pair of vertices is called a **multigraph.** The **degree** of a vertex is the number of edges incident to the vertex and is denoted by deg (v) or (v), each loop counting as two edges. (G) and $\Delta(G)$ are denoted as the **minimum and maximum degrees** of G respectively.

A **directed graph** is a graph in which the edges may only be traversed in one direction. Edges in a simple directed graph may be specified by an ordered pair (v_i, v_j) of the two vertices that the edge connects. We say that v_i is adjacent to v_j.In a directed graph, the **in-degree** of a vertex is the number of edges incident to the vertex and the **out-degree** of a vertex is the number of edges incident from the vertex.

Simple graphs G and H are called **isomorphic** if there is a bijection f from the nodes of G to the nodes of H such that $\{v, w\}$is an edge in G if and only if$\{(v), (w)\}$ is an edge of H. The function f is called an **isomorphism.** A graph is **connected** if there is a walk between every pair of distinct vertices in the graph. A **connected component** of G is a connected sub-graph H of G such that no other connected sub graph of G contains H.

The **complete graph** on n vertices, denoted K_n, is the simple graph with vertices{1,2,3 ... n}and an edge between every pair of distinct vertices. A graph is called **bipartite** if its set of vertices can be partitioned into two disjoint sets S_1 and S_2 so that every edge in the graph has one end vertex in S_1 and one endvertex in S_1. The **complete bipartite** graph on n, m vertices, denoted $K_{n,m}$ is the simple bipartite graph with nodes $S_1 = \{a_1, a_2, a_3 ... a_n\}$ and $S_2 = \{b_1, b_2, b_3 ... b_m\}$ and with edges connecting each vertex in S_1 to every node in S_2.

A **weighted graph** is a graph G = (V, E) along with a function w: $E \rightarrow \mathbb{R}$ that associates a numerical weight to each edge. If G is a weighted graph, then T is a **minimal spanning tree** of G if it is a spanning tree and no other spanning tree of G has smaller total weight.

A **tree** is a connected, simple graph that has no cycles with n-1 edges. Vertices of degree 1 in a tree are called the leaves of the tree. Let G be a simple, connected graph. The sub graph T is a **spanning tree** of G if T is a tree and every vertex in G is a vertex in T. A **forest** is collection of trees. A tree is called a **rooted tree** if one vertex has been designated the **root**, in which case the edges have a natural orientation, towards or away from the root. The tree-order is the partial ordering on the vertices of a tree with u ≤ v if and only if the unique path from the root to v passes through u.

A rooted tree which is a sub graph of some graph G is a **normal tree** if the ends of every edge in G are comparable in this tree-order whenever those ends are vertices of the tree. In a rooted tree, the **parent** of a vertex is the vertex connected to it on the path to the root; every vertex except the root has a unique parent. A **child** of a vertex v is a vertex of which v is the parent. In a rooted tree and all vertices have at most one parent.

A graph is said to be **embeddable** in the plane, or **planar**, if it can be drawn in the plane so that its edges intersect only at their ends. Such a drawing of a planar graph G is called as a **planar embedding** of G.

Let $G = (V, E)$ be a graph with $V = S_1 \cup S_2 \cup S_3 \ldots \cup S_t \cup T$ where each S_i is a set of vertices having at least two vertices and having the same degree and$T = V - \cup S_i$. The degree **splitting graph** of G is denoted by $D_s(G)$ is obtained from G by adding vertices $w_1, w_2, w_3 \ldots w_t$ and joining w_i to each vertex of $S_i (1 \leq i \leq t)$.

Let G be a loop less graph. We construct a graph L(G) in the following way: The vertex set of L (G) is in 1-1 correspondence with the edge set of G and two vertices of L(G) are joined by an edge if and only if the corresponding edges of G are adjacent in G. The graph L(G) (which is always a simple graph) is called the **line graph** or the edge graph of G.

The **middle graph** M(G) of a graph G is defined as follows: The vertex set of M (G) is V(G) , the edge set E(G). Two vertices x, y in the vertex set of M(G) areadjacent in M(G) if either (i) x, y are in E(G) and x, y are adjacent in G or (ii) x is in V(G), y is in E(G) and x, y are incident in G. In other words, M (G) is obtained by subdividing each edge of G exactly once and joining all these newly added middle vertices of adjacent edges of G.

The **total graph** T(G) of a graph G is defined as a graph with vertex set V(G), the edge set E(G) and two vertices x, y of T(G) are adjacent in T(G) if either (i) x, y are in V(G) and x is adjacent to y in G or (ii) x, y are in E(G) and x, y are adjacent in G or (iii) x is in V(G), y is in E(G) and x, y are incident in G.

The **graph distance** between two vertices and of a finite graph is the minimum length of the paths connecting them. The length of a graph geodesic, too. A **geodesic** is a shortest path between two graph vertices of a graph.The **diameter** diam(G) is the largest distance d(u,v) between any two vertices of a connected graph. The **eccentricity** of a graph vertex in a connected graph is the maximum graph distance between v and any other vertex u of G. For a disconnected graph, all vertices are defined to have **infinite eccentricity**. A vertex v of a graph G is called a **cutvertex** of G if its removal increases the number of components. The **vertex connectivity** or **simply connectivity** $\kappa(G)$ of a graph is the minimum number of vertices whose removal from G results in a disconnected or trival graph. A graph G is **n-connected**, $n \geq 1$if $\kappa(G) \geq n$. A graph G is **2-connected** if and only if G is nontrival, connected and contains no cutvertices. A **cutting number** c(v) of a vertex

$v \in V(G)$ in a connected graph G is the number of pairs of vertices {v,w} such that v and w are in different components of G-v. The **status** of a vertex $u \in V(G)$ is defined as the sum of its distance from every other vertex in V(G) and is denoted by $\sigma(u)$.

1.2 GENERAL SURVEY ON TOPOLOGICAL INDICES

At the beginning of the twenty first century a large number of topological indices have been defined in literature. However, only a small fraction of these indices have been extensively used in studies of structure – activity relationships. The role of these descriptors for instance in drug discovery have been well established and several successful applications have been reported. Estrada (2001) introduces a method to generalize some of the most well known topological indices, called "Classical topological indices". This approach permits the generalization of several of these classical indices as well as their optimization for a better description of the properties under study. A two-volume monograph by Trinajstic also offers an excellent introduction to the subject. Dobrynin et al (2001) characterized methods for computation of W and combinatorial expressions for W, for various classes of trees, few conjectures and open problems were mentioned. Fishermann et al (2002) calculated the Winer Inedx versus maximum degree in trees. Danial Bonchev et al (2002) derived a formula for Wiener index of thorn trees, stars, rings and rods. Xiaoying Wu et al (2003) constructed a graph with its Wiener number less than some integer, among all graphs with n vertices and k cut vertices. (Senpeng Eu & Bo-yin yang 2005) calculated the explicit formulae for generalized Wiener indices which hold on hexagonal chains. (MehdiEliasi & BijanTaeri 2008) introduced the four new operations on graphs and studied the Wiener indices of the resulting graph. Dankelmann et al (2009) introduced the edge Wiener index of a graph which is the sum of the distances between all pairs of edges of G and proved the sharp upper bound for graphs of order n. Ivan Gutman (2009) introduced a simple formula for computing TW. Mehdi Eliasi et al (2012) determined the Wiener index of graphs which were constructed by some graph operations such as Myciclski's construction and generalized hierarchical product of graphs. Ivan Gutman et al (2014) characterized eulerian graphs with smallest and greatest Wiener indices. Collections of research papers in the area of chemical graph theory have been edited by Balaban et al (1975)

1.3 Chemical graph theory

A molecular graph is a simple graph such that its vertices correspond to the atoms and the edges to the bonds. Chemical graph theory is a branch of mathematical chemistry which has an important effect on the development of the chemical ciences. In chemical science, the physico-chemical properties of chemical compounds are often modeled by means of a molecular graph based structure descriptors, which are referred to as topological indices. The concept of " topological index" was first proposed by Hosoya for characterizing the topological nature of a graph.

Topological indices are also being used in other fields such as proteomics and DNA sequencing and molecular similarity. The latter has potential for database characterization (Cummins et al. 1996) and combinatorial library design (Zheng et al.1998). It therefore seems that the future of topological indices and their application to chemistry, biochemistry, biology and medicine are assured for the foreseeable future. Let G be a simple graph, with vertex set $V(G)$ and edge set $E(G)$.The degree $d_u$of a vertex u is the number of edges that are incident to it. The Wiener index is the first topological index introduced by chemist Wiener in 1947[33], and it is defined as

$$W(G) = \sum_{\{u,v\}\subset V(G)} d(u, v) \qquad (1.1)$$

A large number of such indices depend only on vertex degree of the molecular graph. One of the oldest and well known topological indices is the first and second Zagreb indices, was first introduced by Gutman et al. in 1972 [26], and it is defined as

$$M_1(G) = \sum_{v\in V} d_G(v)^2 = \sum_{uv\in E(G)}[d_G(u) + d_G(v)] \quad (1.2)$$

$$M_2(G) = \sum_{uv\in E(G)} d_G(u)d_G(v) \quad (1.3)$$

The third Zagreb index was first introduced by Fath – Tabar (2011). This index is defined as follows:

$$M_3(G) = \sum_{uv\in E(G)} | d_u - d_v | \quad (1.4)$$

and the polynomial

$$M_3(G,x) = \sum_{uv\in E(G)} x^{| d_u - d_v |} \quad (1.5)$$

Randic index was proposed by the chemist Milan Randic [85] in 1975, is defined as,

$$R(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{d_u d_v}} \quad (1.6)$$

In 1998 Bollobas and Erdos [9] introduced the general randic index is defined as

$$R_\alpha(G) = \sum_{uv\in E(G)}(d_u d_v)^\alpha \quad (1.7)$$

The Zeroth-order general Randic index, $0\ R_\alpha(G)$ was defined in [34] as

$$0\ R_\alpha(G) = \sum_{u\in V(G)} d(u)^\alpha \quad (1.8)$$

The new/old topological indices studied by I. Gutman et. al. are the following [31,32]:

The reciprocal Randic index is defined as

$$RR(G) = \sum_{uv\in E(G)} \sqrt{d(u)d(v)} \quad (1.9)$$

The reciprocal Randic index is defined as

$$RRR(G) = \sum_{uv\in E(G)} \sqrt{(d(u)-1)(d(v)-1)} \quad (1.10)$$

In [85] B. Zhou and N. Trinajstic et. al. developed sum connectivity index. The sum connectivity index is defined as

$$\chi(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{d_u + d_v}} \quad (1.11)$$

In 2014, Jianxi Li and Chee Shiu introduced another variant of the Randi'c index named the Harmonic index which first appeared in [39]. For a graph G, the harmonic index H(G) is defined as

$$H(G) = \sum_{uv\in E(G)} \frac{2}{d_u+d_v} \tag{1.12}$$

Discrete Adriatic indices have been defined by Vukičević and Gašperov in 2010 [101], as a way of generalizing well-known molecular descriptors defined as the sum of individual bond contributions, such as the second Zagreb index or the Randic index. Among the 148 discrete Adriatic indices analyzed by Vukicevic and Gasperov on the benchmark datasets of the International Academy of Mathematical Chemistry. 20 indices were selected as significant predictors of physic chemical properties. One among such indices named as the inverse sum indeg index, and is defined as

$$ISI(G) = \sum_{uv\in E(G)} \frac{d_u d_v}{d_u+d_v} \tag{1.13}$$

Followed by the first and second Zagreb indices, in 2015 Furtula and Gutman [24] introduced Forgotten topological index (also called F-index) Which was defined as

$$F(G) = \sum_{v\in V(G)}[d_G(v)]^3 = \sum_{uv\in E(G)}[d_G(u) + d_G(v)]^2 \tag{1.14}$$

The F-polynomial of graph G is also defined as:

$$F(G,x) = \sum_{uv\in E(G)} x^{[d_G(u)+d_G(v)]^2} \tag{1.15}$$

Milicevi, Nikoli, Trinajstic introduced in 2004 the modified first and second Zagreb indices [79] are respectively defined as

$$m_{M_1}(G) = \sum_{u\in V(G)} \frac{1}{d_G(u)^2} \tag{1.16}$$

$$m_{M_2}(G) = \sum_{uv\in E(G)} \frac{1}{d_G(u)d_G(v)} \tag{1.17}$$

In 2013, Shirdel et al. [93] introduced the first hyper-Zagreb index of a graph G, which is defined as

$$HM_1(G) = \sum_{uv\in E(G)}[d_G(u)+d_G(v)]^2 \tag{1.18}$$

In 2016, the second hyper-Zagreb index [12], of a graph G is defined as

$$HM_2(G) = \sum_{uv\in E(G)}[d_G(u)d_G(v)]^2 \tag{1.19}$$

In 2017, Chaluvaraju et al. defined first and second hyper-Zagreb polynomials as

$$HM_1(G, x) = \sum_{uv\in E(G)} x^{[d_G(u)+d_G(v)]^2} \tag{1.20}$$

$$HM_2(G, x) = \sum_{uv\in E(G)} x^{[d_G(u)d_G(v)]^2} \tag{1.21}$$

The concept of Atom-bond connectivity index was introduced in the Chemical graph theory by Estrada et al. (1998). The Atom-bond Connectivity index of a graph G is defined as follows:

$$ABC(G) = \sum_{uv\in E(G)} \sqrt{\frac{d_u+d_v-2}{d_u d_v}} \tag{1.22}$$

In 2009, the Geometric-Arithemetic index, GA(G) index of a graph G was introduced by Vukičević and Furtula [100] and it is defined as

$$GA(G) = \sum_{uv\in E(G)} \frac{2\sqrt{d_u d_v}}{d_u+d_v} \tag{1.23}$$

Shigehalli and kanabur [88] introduced following new degree-based topological indices. Arithmetic – Geometric (AG_1) index

$$AG_1(G) = \sum_{uv\in E(G)} \frac{d_u+d_v}{2\sqrt{d_u d_v}} \tag{1.24}$$

Inspired by work on the ABC index, Furtula et al. in 2010 [22] Proposed the following modified version of the ABC index and called it as Augmented Zagreb index (AZI):

$$\mathrm{AZI} = \sum_{uv \in E(G)} \left(\frac{d_u d_v}{d_u + d_v - 2}\right)^3 \tag{1.25}$$

Recently in 2013, Ranjini et al. redefined the Zagreb indices ie, the redefined first, second and third Zagreb indices of a graph G are defined as

$$\mathrm{ReZG}_1(G) = \sum_{uv \in E(G)} \left(\frac{d_u + d_v}{d_u d_v}\right) \tag{1.26}$$

$$\mathrm{ReZG}_2(G) = \sum_{uv \in E(G)} \left(\frac{d_u d_v}{d_u + d_v}\right) \tag{1.27}$$

$$\mathrm{ReZG}_3(G) = \sum_{uv \in E(G)} (d_u d_v)(d_u + d_v) \tag{1.28}$$

In [88] V.S. shegehalli and R. Kanabur introduced new degree based Topological indices (SK indices) as follows

$$SK(G) = \sum_{uv \in E(G)} \frac{d_G(u) + d_G(v)}{2} \tag{1.29}$$

$$SK_1(G) = \sum_{uv \in E(G)} \frac{d_G(u) d_G(v)}{2} \tag{1.30}$$

$$SK_2(G) = \sum_{uv \in E(G)} \left(\frac{d_G(u) + d_G(v)}{2}\right)^2 \tag{1.31}$$

Recently Furtula et al. in [23] proposed the reduced second Zagreb index, defined as

$$RM_2(G) = \sum_{uv \in E(G)} (d_G(u) - 1)\,(d_G(v) - 1) \tag{1.32}$$

The reduced second Zagreb index, Kulli in [45] defined the reduced second Zagreb polynomial as

$$RM_2(G, x) = \sum_{uv \in E(G)} x^{(d_G(u) - 1)\,(d_G(v) - 1)} \tag{1.33}$$

In [45], kulli introduced the reduced second hyper-Zagreb index, defined as

$$RHM_2(G) = \sum_{uv\in E(G)} [(d_G(u)-1)\,(d_G(v)-1)]^2 \quad (1.34)$$

Considering the reduced second hyper-Zagreb index, Kulli in [45] defined the reduced second hyper-Zagreb polynomial of a graph G as

$$RHM_2(G, x) = \sum_{uv\in E(G)} x^{[(d_G(u)-1)\,(d_G(v)-1)]^2} \quad (1.35)$$

We now introduce the general reduced second Zagreb index of a graph, defined as

$$RM_2{}^{a}(G) = \sum_{uv\in E(G)} [(d(u)-1)(d(v)-1)]^{a} \quad (1.36)$$

In [96] A.Subhashini and J.Basker Babujee introduced Reduced forgotten topological index as

$$RF(G) = \sum_{uv\in E(G)} [\,(d_G(u)-1)^2 + (d_G(v)-1)^2] \quad (1.37)$$

RF – polynomial of graph G is also defined as

$$RF(G, x) = \sum_{uv\in E(G)} x^{[(d_G(u)-1)^2+(d_G(v)-1)^2]} \quad (1.38)$$

In 2004, Gutman & Das [29] defined the first and second Zagreb polynomial in the following way:

$$M_1(G, x) = \sum_{uv\in E(G)} x^{(d_u+d_v)} \quad (1.39)$$

$$M_2(G, x) = \sum_{uv\in E(G)} x^{(d_u d_v)} \quad (1.40)$$

In the year 2016, Gutman. I. introduced Zagreb type polynomials were defined

$$M_4(G, x) = \sum_{uv\in E(G)} x^{d_u(d_u+d_v)} \quad (1.41)$$

$$M_5(G, x) = \sum_{uv\in E(G)} x^{d_v(d_u+d_v)} \quad (1.42)$$

V.R.Kulli and I.Gutman introduced the following indices:

In [69], Sombor index of a graph G is defined as

$$SO(G) = \sum_{uv\in E(G)} \sqrt{d_G(u)^2 + d_G(v)^2} \tag{1.43}$$

The modified Sombor index of a graph G as

$$^{m}SO(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{d_G(u)^2+d_G(v)^2}} \tag{1.44}$$

In [70], the first and second (a,b)-KA index of a graph was introduced and defined as

$$KA^1_{a,b}(G) = \sum_{uv\in E(G)} [\, d_G(u)^a + d_G(v)^a \,]^b \tag{1.45}$$

$$KA^2_{a,b}(G) = \sum_{uv\in E(G)} [d_G(u)^a d_G(v)^a]^b \tag{1.46}$$

The reduced Sombor index was defined as [70]

$$RSO(G) = \sum_{uv\in E(G)} \sqrt{(d_G(u)-1)^{2} + (d_G(v)-1)^2} \tag{1.47}$$

The reduced modified Sombor index of a graph G, and it is defined as

$$m_{RSO(G)} = \sum_{uv\in E(G)} \frac{1}{\sqrt{(d_G(u)-1)^{2}+(d_G(v)-1)^2}} \tag{1.48}$$

The first and second reduced (a,b)-KA indices of a graph G, defined as

$$RKA^1_{a,b}(G)=\sum_{uv\in E(G)} [(d_G(u)-1)^a + (d_G(v)-1)^a]^b \tag{1.49}$$

$$RKA^2_{a,b}(G) = \sum_{uv\in E(G)} [(d_G(u)-1)^a (d_G(v)-1)^a]^b \tag{1.50}$$

In [24&49], Kulli introduced first and second F-index of a graph G are defined respectively as

$$F_1(G) = \sum_{i=1}^{n}[d_G(u)^2 + d_G(v)^2] \tag{1.51}$$

$$F_2(G) = \sum_{i=1}^{n}[d_G(u)^2\, d_G(v)^2] \tag{1.52}$$

The minus F-index was introduced by Kulli in [66] defined as

$$MF(G) = \sum_{i=1}^{n} |d_G(u)^2 - d_G(v)^2| \tag{1.53}$$

In [66], Kulli introduced the square F-index of a graph G, defined as

$$QF(G) = \sum_{i=1}^{n} |d_G(u)^2 - d_G(v)^2|^2 \tag{1.54}$$

The minus and square F-polynomial of a graph were introduced by Kulli in [66], and they are defined as

$$MF(G, x) = \sum_{i=1}^{n} x^{|d_G(u)^2 - d_G(v)^2|} \tag{1.55}$$

$$QF(G, x) = \sum_{i=1}^{n} x^{|d_G(u)^2 - d_G(v)^2|^2} \tag{1.56}$$

General first and second and Gourava first and second status indices of a graph were introduced by Kulli in [56,71], and they are defined as

$$S_1^{\,a}(G) = \sum_{uv\in E(G)} [\sigma(u) + \sigma(v)]^{a} \tag{1.57}$$

$$S_2^{\,a}(G) = \sum_{uv\in E(G)} [\sigma(u)\,\sigma(v)]^{a} \tag{1.58}$$

$$SG_1^{\,a}(G) = \sum_{uv\in E(G)} [\sigma(u) + \sigma(v) + \sigma(u)\sigma(v)]^{a} \tag{1.59}$$

$$SG_2^{\,a}(G) = \sum_{uv\in E(G)} [(\sigma(u) + \sigma(v))(\sigma(u)\,\sigma(v))]^{a} \tag{1.60}$$

Nano Zagreb index [37] is defined as and sum Nano Zagreb index

$$\mathcal{NZc}(G) = \sum_{uv\in E(G)} [d^2(u) - d^2(v)] \tag{1.61}$$

and sum Nano Zagreb index [18] is defined as

$$\chi_\alpha\mathcal{NZc}(G) = \sum_{uv\in E(G)} [d^2(u) - d^2(v)]^{\alpha} \tag{1.62}$$

Motivated by the definition of the product connectivity index, the multiplicative sum connectivity index, multiplicative product connectivity index, the multiplicative Atom-bond connectivity index and the multiplicative Geometric-Arithmetic index was proposed by Kulli in 2016 [64]. They are defined as follows:

(a) The multiplicative sum connectivity index of a graph G is defined as

$$\chi\prod(\mathrm{G}) = \prod_{uv\in \mathrm{E(G)}} \frac{1}{\sqrt{\mathrm{d_u+d_v}}} \tag{1.63}$$

(b) The multiplicative Randic connectivity index of a graph G is defined as

$$\mathrm{R}\prod(\mathrm{G}) = \prod_{uv\in \mathrm{E(G)}} \frac{1}{\sqrt{\mathrm{d_u d_v}}} \tag{1.64}$$

(c) The multiplicative Harmonic index of a graph G is defined as

$$\mathrm{H}\prod(\mathrm{G}) = \prod_{uv\in \mathrm{E(G)}} \frac{2}{\mathrm{d_u+d_v}} \tag{1.65}$$

(d) The multiplicative Atom-bond connectivity index of a graph G is defined as

$$\mathrm{ABC}\prod(\mathrm{G}) =\prod_{uv\in \mathrm{E(G)}} \sqrt{\frac{\mathrm{d_u+d_v-2}}{\mathrm{d_u d_v}}} \tag{1.66}$$

(e) The multiplicative Geometric-Arithmetic connectivity index of a graph G is defined as

$$\mathrm{GA}\prod(\mathrm{G}) = \prod_{uv\in \mathrm{E(G)}} \frac{2\sqrt{\mathrm{d_u d_v}}}{\mathrm{d_u+d_v}} \tag{1.67}$$

Inspired by work on Sombor indices, kulli introduced the multiplicative Sombor index, multiplicative modified Sombor index are defined as follows:

(f) The multiplicative Sombor index of a graph G is defined as

$$\mathrm{SO}\prod(\mathrm{G}) = \prod_{uv\in E(\mathrm{G})} \sqrt{\mathrm{d_G(u)^2 + d_G(v)^2}} \tag{1.68}$$

(g) The multiplicative modified Sombor index of a graph G is defined as

$$m_{\mathrm{SO}} \prod(\mathrm{G})= \prod_{uv\in \mathrm{E(G)}} \frac{1}{\sqrt{\mathrm{d_G(u)^2+d_G(v)^2}}} \tag{1.69}$$

(h) The multiplicative first and second (a,b)-KA indices of a graph G were defined by Kulli in [70], as

$$\mathrm{KA^1}_{\mathrm{a,b}}\prod(\mathrm{G}) = \prod_{uv\in E(\mathrm{G})}[\mathrm{d_G(u)}^a + \mathrm{d_G(v)}^a]^b \tag{1.70}$$

$$\mathrm{KA^2}_{\mathrm{a,b}} \prod(\mathrm{G})= \prod_{uv\in E(\mathrm{G})}[\mathrm{d_G(u)}^a \mathrm{d_G(v)}^a]^b \tag{1.71}$$

(i) The multiplicative reduced first and second (a,b)-KA indices of a graph G are defined as

$$\mathrm{RKA}^{1}{}_{a,b}\prod(G)=\prod_{uv\in E(G)}[(d_G(u)-1)^{a}+(d_G(v)-1)^{a}]^{b} \quad (1.72)$$

$$\mathrm{RKA}^{2}{}_{a,b}\prod(G)=\prod_{uv\in E(G)}[(d_G(u)-1)^{a}(d_G(v)-1)^{a}]^{b} \quad (1.73)$$

In [90] V.S. shegehalli and R. kanabur introduced new degree based Topological indices (SK indices) as follows

$$\mathrm{SK}\prod(G)=\prod_{uv\in E(G)}\frac{d_G(u)+d_G(v)}{2} \quad (1.74)$$

$$\mathrm{SK}_1\prod(G)=\prod_{uv\in E(G)}\frac{d_G(u)d_G(v)}{2} \quad (1.75)$$

$$\mathrm{SK}_2\prod(G)=\prod_{uv\in E(G)}\left(\frac{d_G(u)+d_G(v)}{2}\right)^{2} \quad (1.76)$$

1.4 CHAPTER ORGANIZATION

The thesis is organized into nine chapters as follows:

Chapter 1 is an introductory chapter in which the basic concepts of graph theory are outlined. Also in light of our work, the definitions, the literature survey of topological indices and some results related and have been listed.

In **chapter 2,** we have studied the Arithmetic cutting number index, Harmonic cutting number index, Geometric Arithmetic cutting number index, Arithmetic Geometric cutting number index, sum connectivity cutting number index, product connectivity cutting number index, ABC cutting number index, Inverse sum cutting number index, Augumented cutting number Zagreb index, Hyper First, second and Third Zagreb cutting number index, F cutting number index, Reduced forgotten cutting number index, Reduced second Zagreb cutting number index, Reduced second hyper Zagreb cutting number index, General Reduced forgotten cutting number index, General Reduced second Zagreb cutting number index, General Reduced hyper second Zagreb cutting number index. Also we define multiplicative and polynomial indices of a graph G and we found these indices of a chemical structure Nanostar Dendrimer NS[n]. Here we introduced 57 new indices namely

(i) Arithmetic, Harmonic, Geometric Arithmetic, Arithmetic Geometric, sum and product connectivity, ABC, Inverse sum and Augumented Zagreb cutting number indices as,

$$A_C(G) = \sum_{uv\in E(G)} \left(\frac{c(u) + c(v)}{2}\right)$$

$$H_C(G) = \sum_{uv\in E(G)} \left(\frac{2}{c(u) + c(v)}\right)$$

$$GA_C(G) = \sum_{uv\in E(G)} \left(\frac{2\sqrt{c(u)c(v)}}{c(u) + c(v)}\right)$$

$$AG_C(G) = \sum_{uv\in E(G)} \left(\frac{c(u) + c(v)}{2\sqrt{c(u)c(v)}}\right)$$

$$S_C\,(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{c(u) + c(v)}}$$

$$P_C\,(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{c(u)c(v)}}$$

$$ABC_C(G) = \sum_{uv\in E(G)} \sqrt{\frac{c(u) + c(v) - 2}{c(u) + c(v)}}$$

$$IS_C(G) = \sum_{uv\in E(G)} \left[\frac{c(u)c(v)}{c(u)+c(v)}\right]$$

$$AZ_C(G) = \sum_{uv\in E(G)} \left[\frac{c(u)c(v)}{c(u)+c(v)-2}\right]^3$$

(ii) Hyper first, second and third Zagreb, F, Reduced forgotten, Reduced second Zagreb and Reduced hyper second Zagreb cutting number indices are defined as

$$HM_{1C}(G) = \sum_{uv\in E(G)} [c(u) + c(v)]^2$$

$$HM_{2C}(G) = \sum_{uv\in E(G)} [c(u)c(v)]^2$$

$$HM_{3C}(G) = \sum_{uv\in E(G)} |c(u) - c(v)|^2$$

$$F_C(G) = \sum_{uv\in E(G)} [c(u)^2 + c(v)^2]$$

$$RF_C(G) = \sum_{uv\in E(G)} [(c(u)-1)^2 + (c(v)-1)^2]$$

$$RM_{2C}(G) = \sum_{uv\in E(G)} [(c(u)-1)(c(v)-1)]$$

$$RHM_{2C}(G) = \sum_{uv\in E(G)} [(c(u)-1)(c(v)-1)]^2$$

(iii) General Reduced forgotten, General Reduced second Zagreb and General Reduced hyper second Zagreb cutting number indices are defined as

$$RF_C{}^a(G) = \sum_{uv\in E(G)} [(c(u)-1)^2 + (c(v)-1)^2]^a$$

$$RM_{2C}{}^a(G) = \sum_{uv\in E(G)} [(c(u)-1)(c(v)-1)]^a$$

$$RHM_{2C}{}^a(G) = \sum_{uv\in E(G)} [(c(u)-1)(c(v)-1)]^{2a}$$

and also we define multiplicative and polynomial indices of Nanostar Dendrimer NS[n].

In **chapter 3**, we introduced 27 new indices on namely, First Zagreb vertex cutting number index, F-vertex cutting number index, Y-Index vertex cutting number index, Inverse Degree vertex cutting number index, Modified Zagreb vertex cutting number index, zeroth-order General Randic vertex cutting number index, The Reduced first Zagreb vertex cutting number index, The Reduced F-Index and the Reduced Modified first Zagreb vertex cutting number indices of G. Also we define multiplicative and polynomial indices of a Nanostar Dendrimer D_n. Here we introduced new indices on cutting number namely

(i) First zagreb, F-Index and Y-Index vertex cutting number indices of G, as

$$M_{1VC}(G) = \sum_{u\in V(G)} [c(u)]^2$$

$$F_{VC}(G) = \sum_{u\in V(G)} [c(u)]^3$$

$$Y_{VC}(G) = \sum_{u\in V(G)} [c(u)]^4$$

(ii) Inverse Degree Index, Modified Zagreb and zeroth-order General Randic vertex cutting number indices of G are defined as

$$ID_{VC}(G) = \sum_{u\in V(G)} \left[\frac{1}{c(u)}\right]$$

$$^{m}M_{1VC}(G) = \sum_{u\in V(G)} \left[\frac{1}{c(u)^2}\right]$$

$$^{O}R_{\alpha VC}(G) = \sum_{u\in V(G)} [c(u)]^{\alpha}$$

(iii) Reduced first Zagreb, Reduced F-Index and Reduced modified first Zagreb vertex cutting nmber indices of G are defined as

$$RM_{1VC}(G) = \sum_{u\in V(G)} (c(u)-1)^2$$

$$RF_{VC}(G) = \sum_{u\in V(G)} (c(u)-1)^3$$

$$^{m}RM_{1VC}(G) = \sum_{u\in V(G)} \left[\frac{1}{(c(u)-1)^2}\right]$$

and also we define vertex based multiplicative and polynomial indices of Nanostar Dendrimer NS[n].

In **Chapter 4,** we introduced ten new indices First, Second, Third, Fourth and Fifth Zagreb cutting number indices, SK, SK_1 and SK_2 cutting number indices, Nano-Zagreb and sum Nano-Zagreb cutting number indices of a Nanostar Dendrimer D_n. Here we introduced new indices on cutting number namely

(i) First, Second, Third, Fourth and Fifth Zagreb cutting number indices as,

$$M_{1C}(G) = \sum_{uv\in E(G)} [\, c(u) + c(v)]$$

$$M_{2C}(G) = \sum_{uv\in E(G)} [\, c(u)c(v)]$$

$$M_{3C}(G) = \sum_{uv\in E(G)} |\, c(u) - c(v)\, |$$

$$M_{4C}(G) = \sum_{uv\in E(G)} c(u)[(c(u) + c(v)]$$

$$M_{5C}(G) = \sum_{uv\in E(G)} c(v)[c(u) + c(v)]$$

(ii) SK, SK_1 and SK_2 cutting number indices are defined as,

$$SK_C(G) = \sum_{uv\in E(G)} \left[\frac{c(u)+c(v)}{2}\right]$$

$$SK_{1C}(G) = \sum_{uv\in E(G)} \left[\frac{c(u)c(v)}{2}\right]$$

$$SK_{2C}(G) = \sum_{uv\in E(G)} \left[\frac{c(u)+c(v)}{2}\right]^2$$

(iii) Nano-Zagreb and sum Nano-Zagreb cutting number indices are defined as,

$$\mathcal{N}Z_C(G) = \sum_{uv\in E(G)} [c^2(u) - c^2(v)]$$

$$\chi_\alpha \mathcal{N}Z_C(G) = \sum_{uv\in E(G)} [c^2(u) - c^2(v)]^\alpha$$

In **Chapter 5,** we define polynomial of First, Second, Third, Fourth and Fifth Zagreb cutting number indices, polynomial of SK, SK_1 and SK_2 cutting number indices, polynomial of Nano-Zagreb and sum Nano-Zagreb cutting number indices of a Nanostar Dendrimer D_n. Here we introduced new polynomial indices of cutting number namely

(i) polynomial of First, Second, Third, Fourth and Fifth Zagreb cutting number Indices as,

$$M_{1C}(G, x) = \sum_{uv\in E(G)} x^{(c(u)+c(v))}$$

$$M_{2C}(G, x) = \sum_{uv\in E(G)} x^{(c(u)\, c(v))}$$

$$M_{3C}(G, x) = \sum_{uv\in E(G)} x^{|c(u) - c(v)|}$$

$$M_{4C}(G, x) = \sum_{uv\in E(G)} x^{c(u)[(c(u)+c(v)]}$$

$$M_{5C}(G, x) = \sum_{uv\in E(G)} x^{c(v)[c(u)+c(v)]}$$

(ii) polynomial of SK, SK_1 and SK_2 cutting number indices are defined as,

$$SK_C(G, x) = \sum_{uv\in E(G)} x^{\left[\frac{C(u)+C(v)}{2}\right]}$$

$$SK_{1C}(G, x) = \sum_{uv\in E(G)} x^{\left[\frac{C(u)\times C(v)}{2}\right]}$$

$$SK_{2C}(G, x) = \sum_{uv\in E(G)} x^{\left[\frac{C(u)+C(v)}{2}\right]^2}$$

(iii) polynomial of Nano-Zagreb and sum Nano-Zagreb cutting number indices are defined as,

$$\mathcal{N}Z_C(\mathrm{G}, \mathrm{x}) = \sum_{uv\in E(\mathrm{G})} x^{(c^2(u) - c^2(v))}$$

$$\chi_\alpha \mathcal{N}Z_C(\mathrm{G}, \mathrm{x}) = \sum_{uv\in E(\mathrm{G})} x^{(c^2(u) - c^2(v))^\alpha}$$

In **Chapter 6,** we introduced ten indices, Multiplicative of First, Second, Third, Fourth and Fifth Zagreb cutting number indices, Multiplicative of SK, SK_1 and SK_2 cutting number indices, Multiplicative of Nano-Zagreb and sum Nano-Zagreb cutting number indices of a Nanostar Dendrimer D_n. Here we introduced multiplicative of new indices on cutting number namely

(i) Multiplicative of First, Second, Third, Fourth and Fifth zagreb cutting number indices as,

$$M_{1C}\prod(\mathrm{G}) = \prod_{uv\in E(\mathrm{G})}(c(u) + c(v))$$

$$M_{2C}\prod(\mathrm{G}) = \prod_{uv\in E(\mathrm{G})}(c(u)c(v))$$

$$M_{3C}\prod(\mathrm{G}) = \prod_{uv\in E(\mathrm{G})} |\, c(u) - c(v)\, |$$

$$M_{4C}\prod(\mathrm{G}) = \prod_{uv\in E(\mathrm{G})} c(u)[(c(u) + c(v)]$$

$$M_{5C}\prod(\mathrm{G}) = \prod_{uv\in E(\mathrm{G})} c(v)[(c(u) + c(v)]$$

(ii) Multiplicative of SK, SK_1 and SK_2 cutting number Indices are defined as,

$$SK_C\prod(\mathrm{G}) = \prod_{uv\in E(\mathrm{G})}[\frac{c(u)+c(v)}{2}]$$

$$SK_{1C}\prod(\mathrm{G}) = \prod_{uv\in E(\mathrm{G})}[\frac{c(u)c(v)}{2}]$$

$$SK_{2C}\prod(\mathrm{G}) = \prod_{uv\in E(\mathrm{G})} [\frac{c(u)+c(v)}{2}]^2$$

(iii) Multiplicative of Nano-Zagreb and sum Nano-Zagreb cutting number indices are defined as,

$$\mathcal{N}Z_C\prod(\mathrm{G}) = \prod_{uv\in E(\mathrm{G})}(c^2(u) - c^2(v))$$

$$\chi_\alpha \mathcal{N}Z_C\prod(\mathrm{G}) = \prod_{uv\in E(\mathrm{G})} (c^2(u) - c^2(v))^\alpha$$

In **Chapter 7,** we calculate eccentricity based Redefined First, Second and Third Zagreb indices, Modified First, Second, Third, Fourth and Fifth Zagreb indices, Randic, Reciprocal Randic, Reduced Reciprocal Randic and Generalized Randic indices. Also we define multiplicative and polynomial indices of Hexoconal core Dendrimer $D_{3,n-1}$.

(i) Redefined First, Second and Third Zagreb indices of G, as

$$ReZG_1(G) = \sum_{uv \in E(G)} \left(\frac{e(u)+e(v)}{e(u)\, e(v)}\right)$$

$$ReZG_2(G) = \sum_{uv \in E(G)} \left(\frac{e(u)e(v)}{e(u)+e(v)}\right)$$

$$ReZG_3(G) = \sum_{uv \in E(G)} [e(u)e(v)\,(e(u)+e(v)]$$

(ii) Modified First, Second, Third, Fourth and Fifth Zagreb indices of G, are defined

$$m_{M_1}(G) = \sum_{uv \in E(G)} [\frac{1}{e(u)+e(v)}]$$

$$m_{M_2}(G) = \sum_{uv \in E(G)} \left[\frac{1}{e(u)e(v)}\right]$$

$$m_{M_3}(G) = \sum_{uv \in E(G)} [\frac{1}{|e(u)-e(v)|}]$$

$$m_{M_4}(G) = \sum_{uv \in E(G)} \left[\frac{1}{e(u)(e(u)+e(v))}\right]$$

$$m\, _{M_5}(G) = \sum_{uv \in E(G)} [\frac{1}{e(v)(e(u)+e(v))}]$$

(iii) Randic, Reciprocal Randic, Reduced Reciprocal Randic and Generalized Randic indices of G, are defined

$$R(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{e(u)e(v)}}$$

$$RR(G) = \sum_{uv \in E(G)} \sqrt{e(u)e(v)}$$

$$RRR(G) = \sum_{uv \in E(G)} \sqrt{(e(u)-1)(e(v)-1)}$$

$$R_\alpha(G) = \sum_{uv \in E(G)} [e(u)e(v)]^\alpha$$

In **Chapter 8,** we obtained eccentricity based Sombor, modified Sombor, Reduced Sombor, Reduced modified Sombor, The first and second (a,b) – KA indices and the first and second Reduced (a,b) – KA indices. Square Reduced sum, product, Arithmetic, Geometric and Harmonic indices. First F, Second F, The Minus F and Square F indices. Also we define multiplicative and polynomial indices of Nanostar Dendrimer D(n).

(i) Sombor, Modified Sombor, Reduced Sombor and Reduced Modified Sombor indices of G, as

$$\mathrm{SO(G)} = \sum_{uv\in E(\mathrm{G})} \sqrt{e(u)^2 + e(v)^2}$$

$$m_{\mathrm{SO}}(\mathrm{G}) = \sum_{uv\in E(\mathrm{G})} \frac{1}{\sqrt{e(u)^2+e(v)^2}}$$

$$\mathrm{RSO(G)} = \sum_{uv\in E(\mathrm{G})} \sqrt{(e(u)-1)^2 + (e(v)-1)^2}$$

$$m_{\mathrm{RSO}}(\mathrm{G}) = \sum_{uv\in E(\mathrm{G})} \frac{1}{\sqrt{(e(u)-1)^2+(e(v)-1)^2}}$$

(ii) The First and second (a, b) - KA indices and the first and second Reduced (a, b) - KA indices of G are defined

$$\mathrm{KA}^1{}_{a,b}(\mathrm{G}) = \sum_{uv\in E(\mathrm{G})} [e(u)^a + e(v)^a]^b$$

$$\mathrm{KA}^2{}_{a,b}(\mathrm{G}) = \sum_{uv\in E(\mathrm{G})} [e(u)^a e(v)^a]^b$$

$$\mathrm{RKA}^1{}_{a,b}(\mathrm{G}) = \sum_{uv\in E(\mathrm{G})} [(e(u)-1)^a + (e(v)-1)^a]^b$$

$$\mathrm{RKA}^2{}_{a,b}(\mathrm{G}) = \sum_{uv\in E(\mathrm{G})} [(e(u)-1)^a (e(v)-1)^a]^b$$

(iii) Square Reduced sum, Square Reduced product, Square Reduced Arithmetic, Square Reduced Geometric and Square Reduced Harmonic indices of G, are defined

$$\mathrm{SRS(G)} = \sum_{uv\in E(\mathrm{G})} [(e(u)-1)^2 + (e(v)-1)^2]$$

$$\mathrm{SRP(G)} = \sum_{uv\in E(\mathrm{G})} [(e(u)-1)^2 ((e(v)-1)^2]$$

$$\mathrm{SRA(G)} = \sum_{uv\in E(\mathrm{G})} [\frac{[(e(u)-1)^2 + (e(v)-1)^2]}{2}]$$

$$SRH(G) = \sum_{uv\in E(G)} \left[\frac{2}{[(e(u)-1)^2 + (e(v)-1)^2]}\right]$$

$$SRG(G) = \sum_{uv\in E(G)} \sqrt{[(e(u)-1)^2\,(e(v)-1)^2]}$$

(iv) First F, Second F, The Minus F, Square F indices of G are defined

$$F_1(G) = \sum_{i=1}^{n} [e(u)^2 + e(v)^2]$$

$$F_2(G) = \sum_{i=1}^{n} [e(u)^2\, e(v)^2]$$

$$MF(G) = \sum_{i=1}^{n} |e(u)^2 - e(v)^2|$$

$$QF(G) = \sum_{i=1}^{n} |e(u)^2 - e(v)^2|^2$$

In **Chapter 9,** we derive General first and second status index, General first and second Gourava status index, General modified first and second Gourava status index and also find the corresponding polynomial of a molecular graph of Dendrimer D(n).

$$S_1^{\,a}(G) = \sum_{uv\in E(G)} [\sigma(u) + \sigma(v)]^{a}$$

$$S_2^{\,a}(G) = \sum_{uv\in E(G)} [\sigma(u) \times \sigma(v)]^{a}$$

$$SG_1^{\,a}(G) = \sum_{uv\in E(G)} [\sigma(u) + \sigma(v) + \sigma(u)\sigma(v)]^{a}$$

$$SG_2^{\,a}(G) = \sum_{uv\in E(G)} [(\sigma(u) + \sigma(v))(\sigma(u)\,\sigma(v))]^{a}$$

$$m_{SG_1{}^{a}(G)} = \sum_{uv\in E(G)} \left[\frac{1}{(\sigma(u)+\sigma(v)+\sigma(u)\sigma(v))^{a}}\right]$$

$$m_{SG_2{}^{a}(G)} = \sum_{uv\in E(G)} \left[\frac{1}{(\sigma(u)+\sigma(v))(\sigma(u)\sigma(v))^{a}}\right]$$

CHAPTER 2

EDGE BASED CUTTING NUMBER TOPOLOGICAL INDICES OF NANOSTAR DENDRIMER NS[n]

In this chapter, we define Arithmetic cutting number index, Harmonic cutting number index, Geometric Arithmetic cutting number index, Arithmetic Geometric cutting number index, sum and product connectivity cutting number index, ABC cutting number index, Inverse sum cutting number index, Augumented cutting number Zagreb index, Hyper first, second and third Zagreb cutting number indices, F cutting number index, Reduced forgotten cutting number index, Reduced second Zagreb cutting number index, Reduced second hyper Zagreb cutting number index, General reduced forgotten cutting number index, General reduced second Zagreb cutting number index, General reduced hyper second Zagreb cutting number index. Also we define multiplicative and polynomial indices of a graph G. Here we introduced new indices on cutting number namely

(i) Arithmetic, Harmonic, Geometric Arithmetic, Arithmetic Geometric cutting sum and product connectivity, ABC, Inverse sum and Augumented Zagreb cutting number indices are defined as,

$$A_C(G) = \sum_{uv\in E(G)} \left(\frac{c(u) + c(v)}{2}\right) \tag{2.1}$$

$$H_C(G) = \sum_{uv\in E(G)} \left(\frac{2}{c(u) + c(v)}\right) \tag{2.2}$$

$$GA_C(G) = \sum_{uv\in E(G)} \left(\frac{2\sqrt{c(u)c(v)}}{c(u) + c(v)}\right) \tag{2.3}$$

$$AG_C(G) = \sum_{uv\in E(G)} \left(\frac{c(u) + c(v)}{2\sqrt{c(u)c(v)}}\right) \tag{2.4}$$

$$S_C (G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{c(u) + c(v)}} \tag{2.5}$$

$$P_C (G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{c(u)c(v)}} \tag{2.6}$$

$$\mathrm{ABCc(G)} = \sum_{uv\in E(\mathrm{G})} \sqrt{\frac{c(u) + c(v) - 2}{c(u) + c(v)}} \tag{2.7}$$

$$\mathrm{ISc(G)} = \sum_{uv\in E(\mathrm{G})} \left[\frac{c(u)c(v)}{c(u)+c(v)}\right] \tag{2.8}$$

$$\mathrm{AZ_C(G)} = \sum_{uv\in E(\mathrm{G})} \left[\frac{c(u)c(v)}{c(u)+c(v)-2}\right]^3 \tag{2.9}$$

(ii) Hyper first, second and third Zagreb indices, F, Reduced forgotten, Reduced second Zagreb and Reduced hyper second Zagreb cutting number indices are defined as

$$\mathrm{HM_{1C}(G)} = \sum_{uv\in E(\mathrm{G})} [c(u) + c(v)]^2 \tag{2.10}$$

$$\mathrm{HM_{2C}(G)} = \sum_{uv\in E(\mathrm{G})} [c(u)c(v)]^2 \tag{2.11}$$

$$\mathrm{HM_{3C}(G)} = \sum_{uv\in E(\mathrm{G})} |c(u) - c(v)|^2 \tag{2.12}$$

$$\mathrm{F_C(G)} = \sum_{uv\in E(G)} [c(u)^2 + c(v)^2] \tag{2.13}$$

$$\mathrm{RF_C(G)} = \sum_{uv\in E(G)} [(c(u) - 1)^2 + (c(v) - 1)^2] \tag{2.14}$$

$$\mathrm{RM_{2C}(G)} = \sum_{uv\in E(G)} [(c(u) - 1)(c(v) - 1)] \tag{2.15}$$

$$\mathrm{RHM_{2C}(G)} = \sum_{uv\in E(\mathrm{G})} [(c(u) - 1)(c(v) - 1)]^2 \tag{2.16}$$

(iii) General reduced forgotten, General reduced second Zagreb and General reduced hyper second Zagreb cutting number indices are defined as

$$\mathrm{RF_C}^{a}\mathrm{(G)} = \sum_{uv\in E(G)} [(c(u) - 1)^2 + (c(v) - 1)^2]^{a} \tag{2.17}$$

$$\mathrm{RM_{2C}}^{a}\mathrm{(G)} = \sum_{uv\in E(G)} [(c(u) - 1)(c(v) - 1)]^{a} \tag{2.18}$$

$$\mathrm{RHM_{2C}}^{a}\mathrm{(G)} = \sum_{uv\in E(\mathrm{G})} [(c(u) - 1)(c(v) - 1)]^{2a} \tag{2.19}$$

and also we define multiplicative and polynomial indices of Nanostar Dendrimer NS[n].

Dendrimers are highly ordered branched macromolecules Nanostar Dendrimer NS[n], where n is the defining parameter as illustrated in Fig. 1 and 2. The node and edge cardinalities for graph of this Nanostar Dendrimer are $18 \times 2^n - 12$ and $21 \times 2^n - 15$ respectively.

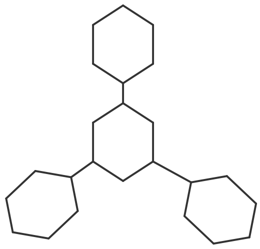


Fig. 1 Nanostar Dendrimer NS[1]

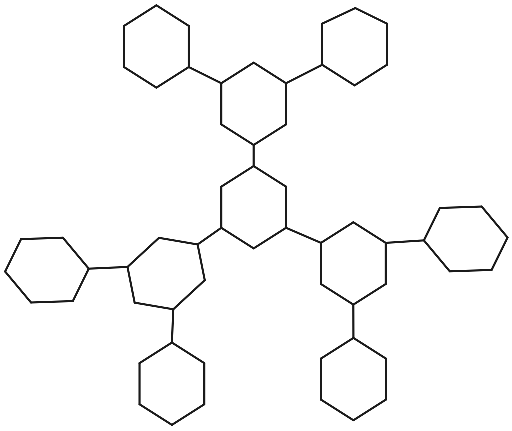


Fig. 2 Nanostar Dendrimer NS[2]

Nanostar Dendrimer NS[n]

Number of edges e = uv	cutting number of end vertices (c(u), c(v))
$12 \times 2^{n-1}$	(0, 0)
$6 \times 2^{n-1}$ $3 \times 2^{n-1}$ $6 \times 2^{n-1}$	$[\{((18 \times 2^n) - 12) - 6\} \times 5, 0]$ $[\{((18 \times 2^n) - 12) - 6\} \times 5, \{((18 \times 2^n) - 12) - 7\} \times 6]$ $[\{((18 \times 2^n) - 12) - 7\} \times 6, 0]$
$6 \times 2^{n-2}$ $3 \times 2^{n-2}$ $6 \times 2^{n-2}$	$[\{((18 \times 2^n) - 12) - 18\} \times 17, 0]$ $[\{((18 \times 2^n) - 12) - 18\} \times 17, \{((18 \times 2^n) - 12) - 19\} \times 18]$ $[\{((18 \times 2^n) - 12) - 19\} \times 18, 0\}$

. . .	
6×2^{i} 3×2^{i} 6×2^{i}	$[\{((18\times 2^{n})-12)-((6\times 2^{n-i})-6)\}\times((6\times 2^{n-i})-7),0]$ $[\{((18\times 2^{n})-12)-((6\times 2^{n-i})-6)\}\times((6\times 2^{n-i})-7),[\{((18\times 2^{n})-12)-((6\times 2^{n-i})-5)\}\times((6\times 2^{n-i})-6)]$ $[\{((18\times 2^{n})-12)-((6\times 2^{n-i})-5)\}\times((6\times 2^{n-i})-6),0]$
. .	
6×2^{2} 3×2^{2} 6×2^{2}	$[\{((18\times 2^{n})-12)-((6\times 2^{n-2})-6)\}\times((6\times 2^{n-2})-7),0]$ $[\{((18\times 2^{n})-12)-((6\times 2^{n-2})-6)\}\times((6\times 2^{n-2})-7),[\{((18\times 2^{n})-12)-((6\times 2^{n-2})-5)\}\times((6\times 2^{n-2})-6)]$ $[\{((18\times 2^{n})-12)-((6\times 2^{n-2})-5)\}\times((6\times 2^{n-2})-6),0]$
6×2^{1} 3×2^{1} 6×2^{1}	$[\{((18\times 2^{n})-12)-((6\times 2^{n-1})-6)\}\times((6\times 2^{n-1})-7),0]$ $[\{((18\times 2^{n})-12)-((6\times 2^{n-1})-6)\}\times((6\times 2^{n-1})-7),[\{((18\times 2^{n})-12)-((6\times 2^{n-1})-5)\}\times((6\times 2^{n-1})-6)]$ $[\{((18\times 2^{n})-12)-((6\times 2^{n-1})-5)\}\times((6\times 2^{n-1})-6),0]$
6×2^{0} 3×2^{0} 6×2^{0}	$[\{((18\times 2^{n})-12)-((6\times 2^{n})-6)\}\times((6\times 2^{n})-7),0]$ $[\{((18\times 2^{n})-12)-((6\times 2^{n})-6)\}\times((6\times 2^{n})-7),[\{((18\times 2^{n})-12)-((6\times 2^{n})-5)\}\times((6\times 2^{n})-6)]$ $[\{((18\times 2^{n})-12)-((6\times 2^{n})-5)\}\times((6\times 2^{n})-6),0]$
Total number of edges	$21\times 2^{n}-15$

Table (1)

2.1 Arithmetic, Harmonic, Geometric Arithmetic, Arithmetic Geometric, sum and product connectivity, ABC, Inverse sum, Augumented Zagreb cutting number topological indices of Nanostar Dendrimer NS[n]

In this section, we compute Arithmetic, Harmonic, Geometric Arithmetic, Arithmetic Geometric, sum and product connectivity, ABC, Inverse sum and Augumented Zagreb cutting number topological indices of Nanostar Dendrimer NS[n] in the following theorems.

Theorem 2.1.1: Let NS[n] be a Nanostar Dendrimer. Then,

$A_C(NS[n]) = 972 \times 2^{2n} - 1053 \times 2^{2n} - 324 \times 2^{2n+1} + 324 \times 2^{n+1} + 1431 \times 2^{n} - 378.$

Proof: $A_C(NS[n]) = \sum_{uv \in E(NS[n])} \left(\frac{c(u) + c(v)}{2}\right)$

$$= \sum_{i=0}^{n-1} (6x2^{i}) \left[\frac{[\{((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 6)\} \times (6 \times 2^{n-i} - 7)] + 0}{2}\right] +$$

$$\sum_{i=0}^{n-1} (3x2^{i}) \left[\frac{[\{((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 6)\} \times (6 \times 2^{n-i} - 7)] + [\{((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 5)\} \times (6 \times 2^{n-i} - 6)]}{2}\right] +$$

$$\sum_{i=0}^{n-1} (6x2^{i}) \left[\frac{[\{((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 5)\} \times (6 \times 2^{n-i} - 6)] + 0}{2}\right]$$

$$= 972 \times 2^{2n} - 1053 \times 2^{2n} - 324 \times 2^{2n+1} + 324 \times 2^{n+1} + 1431 \times 2^{n} - 378.$$

Theorem 2.1.2: Let NS[n] be a Nanostar Dendrimer. Then,

$$H_C(NS[n]) = 6 \sum_{i=0}^{n-1} 2^{i} \left[\frac{2}{108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^{n} + 42}\right] +$$

$$3 \sum_{i=0}^{n-1} 2^{i} \left[\frac{2}{216 \times 2^{2n-i} - 72 \times 2^{2n-2i} - 234 \times 2^{n} + 84}\right] +$$

$$6 \sum_{i=0}^{n-1} 2^{i} \left[\frac{2}{108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^{n} + 42}\right]$$

Proof: $H_C(NS[n]) = \sum_{uv \in E(NS[n])} \left(\frac{2}{c(u) + c(v)}\right)$

$$= \sum_{i=0}^{n-1} (6 \times 2^{i}) \left[\frac{2}{[\{((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 6)\} \times (6 \times 2^{n-i} - 7)] + 0}\right] +$$

$$\sum_{i=0}^{n-1} (3 \times 2^{i}) \left[\frac{2}{[\{((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 6)\} \times (6 \times 2^{n-i} - 7)] + [\{((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 5)\} \times (6 \times 2^{n-i} - 6)]}\right] +$$

$$\sum_{i=0}^{n-1} (6 \times 2^{i}) \left[\frac{2}{[\{((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 5)\} \times (6 \times 2^{n-i} - 6)] + 0}\right]$$

$$= 6 \sum_{i=0}^{n-1} 2^{i} \left[\frac{2}{108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^{n} + 42}\right] +$$

$$3 \sum_{i=0}^{n-1} 2^{i} \left[\frac{2}{216 \times 2^{2n-i} - 72 \times 2^{2n-2i} - 234 \times 2^{n} + 84}\right] +$$

$$6 \sum_{i=0}^{n-1} 2^{i} \left[\frac{2}{108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^{n} + 42}\right]$$

Theorem 2.1.3: Let NS[n] be a Nanostar Dendrimer. Then,

$$GA_C(NS[n]) = 6\sum_{i=0}^{n-1} 2^{i} \left[\frac{\sqrt{\begin{array}{c}11664\times 2^{4n-2i} - 7776\times 2^{4n-3i} + 8424\times 2^{3n-2i} - \\ 25272\times 2^{3n-i} + 9180\times 2^{2n-i} + 1296\times 2^{4n-4i} - \\ 3060\times 2^{2n-2i} + 13608\times 2^{2n} - 9828\times 2^{n} + 1764\end{array}}}{216\times 2^{2n-i} - 72\times 2^{2n-2i} - 234\times 2^{n} + 84}\right]$$

Proof: $GA_C(NS[n]) = \sum_{uv\in E(NS\{n])} \left(\frac{2\sqrt{c(u)c(v)}}{c(u)+c(v)}\right)$

$$= \sum_{i=0}^{n-1} (3x2^{i}) \left[\frac{2\sqrt{\begin{array}{c}\{(((18\times 2^{n}) - 12) - (6\times 2^{n-i} - 6)) \times (6\times 2^{n-i} - 7)\} \times \\ \{(((18\times 2^{n}) - 12) - (6\times 2^{n-i} - 5)) \times (6\times 2^{n-i} - 6)\}\end{array}}}{\begin{array}{c}\{(((18\times 2^{n}) - 12) - (6\times 2^{n-i} - 6)) \times (6\times 2^{n-i} - 7)\} + \\ \{(((18\times 2^{n}) - 12) - (6\times 2^{n-i} - 5)) \times (6\times 2^{n-i} - 6)\}\end{array}}\right]$$

$$= 6\sum_{i=0}^{n-1} 2^{i} \left[\frac{\sqrt{\begin{array}{c}11664\times 2^{4n-2i} - 7776\times 2^{4n-3i} + 8424\times 2^{3n-2i} - \\ 25272\times 2^{3n-i} + 9180\times 2^{2n-i} + 1296\times 2^{4n-4i} - \\ 3060\times 2^{2n-2i} + 13608\times 2^{2n} - 9828\times 2^{n} + 1764\end{array}}}{216\times 2^{2n-i} - 72\times 2^{2n-2i} - 234\times 2^{n} + 84}\right]$$

Theorem 2.1.4: Let NS[n] be a Nanostar Dendrimer. Then,

$$AG_C(NS[n]) = [\tfrac{3}{2}]\sum_{i=0}^{n-1} 2^{i} \left[\frac{216\times 2^{2n-i} - 72\times 2^{2n-2i} - 234\times 2^{n} + 84}{\sqrt{\begin{array}{c}11664\times 2^{4n-2i} - 7776\times 2^{4n-3i} + 8424\times 2^{3n-2i} - \\ 25272\times 2^{3n-i} + 9180\times 2^{2n-i} + 1296\times 2^{4n-4i} - \\ 3060\times 2^{2n-2i} + 13608\times 2^{2n} - 9828\times 2^{n} + 1764\end{array}}}\right]$$

Proof: $AG_C(NS[n]) = \sum_{uv\in E(NS[n])} \left(\frac{c(u)+c(v)}{2\sqrt{c(u)c(v)}}\right)$

$$= \sum_{i=0}^{n-1} (3x2^{i}) \left[\frac{\begin{array}{c}\{(((18\times 2^{n}) - 12) - (6\times 2^{n-i} - 6)) \times (6\times 2^{n-i} - 7)\} + \\ \{(((18\times 2^{n}) - 12) - (6\times 2^{n-i} - 5)) \times (6\times 2^{n-i} - 6)\}\end{array}}{2\sqrt{\begin{array}{c}\{(((18\times 2^{n}) - 12) - (6\times 2^{n-i} - 6)) \times (6\times 2^{n-i} - 7)\} \times \\ \{(((18\times 2^{n}) - 12) - (6\times 2^{n-i} - 5)) \times (6\times 2^{n-i} - 6)\}\end{array}}}\right]$$

$$= [\tfrac{3}{2}]\sum_{i=0}^{n-1} 2^{i} \left[\frac{216\times 2^{2n-i} - 72\times 2^{2n-2i} - 234\times 2^{n} + 84}{\sqrt{\begin{array}{c}11664\times 2^{4n-2i} - 7776\times 2^{4n-3i} + 8424\times 2^{3n-2i} - \\ 25272\times 2^{3n-i} + 9180\times 2^{2n-i} + 1296\times 2^{4n-4i} - \\ 3060\times 2^{2n-2i} + 13608\times 2^{2n} - 9828\times 2^{n} + 1764\end{array}}}\right]$$

Theorem 2.1.5: Let NS[n] be a Nanostar Dendrimer. Then,

$$S_C(NS[n]) = 6\sum_{i=0}^{n-1} 2^{i}\left[\frac{1}{\sqrt{108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n} + 42}}\right] +$$

$$3\sum_{i=0}^{n-1} 2^{i}\left[\frac{1}{\sqrt{216\times 2^{2n-i} - 72\times 2^{2n-2i} - 234\times 2^{n} + 84}}\right] +$$

$$6\sum_{i=0}^{n-1} 2^{i}\left[\frac{1}{\sqrt{108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n} + 42}}\right]$$

Proof: $S_C(NS[n]) = \sum_{uv\in E(NS[n])} \frac{1}{\sqrt{c(u)+c(v)}}$

$$= \sum_{i=0}^{n-1} (6\times 2^{i})\left[\frac{1}{\sqrt{\{(((18\times 2^{n}) - 12)\ -(6\times 2^{n-i} - 6))\times(6\times 2^{n-i} - 7)\} + 0}}\right] +$$

$$\sum_{i=0}^{n-1} (3\times 2^{i})\left[\frac{1}{\sqrt{\begin{array}{c}\{(((18\times 2^{n}) - 12)\ -(6\times 2^{n-i} - 6))\times(6\times 2^{n-i} - 7)\} + \\ \{(((18\times 2^{n}) - 12)\ -(6\times 2^{n-i} - 5))\times(6\times 2^{n-i} - 6)\}\end{array}}}\right] +$$

$$\sum_{i=0}^{n-1} (6\times 2^{i})\left[\frac{1}{\sqrt{[\{((18\times 2^{n}) - 12)\ -(6\times 2^{n-i} - 5)\}\times(6\times 2^{n-i} - 6)] + 0}}\right]$$

$$= 6\sum_{i=0}^{n-1} 2^{i}\left[\frac{1}{\sqrt{108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n} + 42}}\right] +$$

$$3\sum_{i=0}^{n-1} 2^{i}\left[\frac{1}{\sqrt{216\times 2^{2n-i} - 72\times 2^{2n-2i} - 234\times 2^{n} + 84}}\right] +$$

$$6\sum_{i=0}^{n-1} 2^{i}\left[\frac{1}{\sqrt{108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n} + 42}}\right]$$

Theorem 2.1.6: Let NS[n] be a Nanostar Dendrimer. Then,

$$Pc(NS[n]) = 3\sum_{i=0}^{n-1} 2^{i}\left[\frac{1}{\sqrt{\begin{array}{c}11664\times 2^{4n-2i} - 7776\times 2^{4n-3i} + 8424\times 2^{3n-2i} - \\ 25272\times 2^{3n-i} + 9180\times 2^{2n-i} + 1296\times 2^{4n-4i} - \\ 3060\times 2^{2n-2i} + 13608\times 2^{2n} - 9828\times 2^{n} + 1764\end{array}}}\right]$$

Proof: $P_C(NS[n]) = \sum_{uv\in E(NS[n])}\left[\frac{1}{\sqrt{c(u)c(v)}}\right]$

$$= \sum_{i=0}^{n-1} (3\times 2^{i})\left[\frac{1}{\sqrt{\begin{array}{c}\{(((18\times 2^{n}) - 12) - (6\times 2^{n-i} - 6))\times(6\times 2^{n-i} - 7)\}\times \\ \{(((18\times 2^{n}) - 12) - (6\times 2^{n-i} - 5))\times(6\times 2^{n-i} - 6)\}\end{array}}}\right]$$

$$= 3\sum_{i=0}^{n-1} 2^{i}\left[\frac{1}{\sqrt{\begin{array}{c}11664\times 2^{4n-2i}-7776\times 2^{4n-3i}+8424\times 2^{3n-2i}-\\ 25272\times 2^{3n-i}+9180\times 2^{2n-i}+1296\times 2^{4n-4i}-\\ 3060\times 2^{2n-2i}+13608\times 2^{2n}-9828\times 2^{n}+1764\end{array}}}\right]$$

Theorem 2.1.7: Let NS[n] be a Nanostar Dendrimer. Then,

$$ABC_C(NS[n]) = 6\sum_{i=0}^{n-1} 2^{i}\sqrt{\frac{108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+40}{108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+42}} +$$

$$3\sum_{i=0}^{n-1} 2^{i}\sqrt{\frac{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^{n}+82}{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^{n}+84}} +$$

$$6\sum_{i=0}^{n-1} 2^{i}\sqrt{\frac{108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+40}{108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+42}}$$

Proof: $ABCc(NS[n]) = \sum_{uv\in E(NS[n])}\sqrt{\frac{c(u)+c(v)-2}{c(u)+c(v)}}$

$$= \sum_{i=0}^{n-1}(6\times 2^{i})\sqrt{\frac{\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-6))\times(6\times 2^{n-i}-7)\}+0-2}{\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-6))\times(6\times 2^{n-i}-7)\}+0}} +$$

$$\sum_{i=0}^{n-1}(3\times 2^{i})\sqrt{\frac{\begin{array}{c}\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-6))\times(6\times 2^{n-i}-7)\}+\\ \{(((18\times 2^{n})-12)-(6\times 2^{n-i}-5))\times(6\times 2^{n-i}-6)\}-2\end{array}}{\begin{array}{c}\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-6))\times(6\times 2^{n-i}-7)\}+\\ \{(((18\times 2^{n})-12)-(6\times 2^{n-i}-5))\times(6\times 2^{n-i}-6)\}\end{array}}} +$$

$$\sum_{i=0}^{n-1}(6\times 2^{i})\sqrt{\frac{\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-5))\times(6\times 2^{n-i}-6)\}+0-2}{\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-5))\times(6\times 2^{n-i}-6)\}+0}} +$$

$$= 6\sum_{i=0}^{n-1} 2^{i}\sqrt{\frac{108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+40}{108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+42}} +$$

$$3\sum_{i=0}^{n-1} 2^{i}\sqrt{\frac{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^{n}+82}{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^{n}+84}} +$$

$$6\sum_{i=0}^{n-1} 2^{i}\sqrt{\frac{108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+40}{108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+42}}$$

Theorem 2.1.8: Let NS[n] be a Nanostar Dendrimer. Then,

$$IS_C(NS[n]) = 3\sum_{i=0}^{n-1} 2^{i}\left[\frac{\begin{array}{c}11664\times 2^{4n-2i}-7776\times 2^{4n-3i}+8424\times 2^{3n-2i}-\\ 25272\times 2^{3n-i}+9180\times 2^{2n-i}+1296\times 2^{4n-4i}-\\ 3060\times 2^{2n-2i}+13608\times 2^{2n}-9828\times 2^{n}+1764\end{array}}{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^{n}+84}\right]$$

Proof: ISc(NS[n]) $= \sum_{uv \in E(NS[n])} \left[\frac{c(u)c(v)}{c(u)+c(v)}\right]$

$$= \sum_{i=0}^{n-1} (3\text{x}2^{i}) \left[\frac{\{(((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 6)) \times (6 \times 2^{n-i} - 7)\} \times \{(((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 5)) \times (6 \times 2^{n-i} - 6)\}}{\{(((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 6)) \times (6 \times 2^{n-i} - 7)\} + \{(((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 5)) \times (6 \times 2^{n-i} - 6)\}}\right]$$

$$= 3 \sum_{i=0}^{n-1} 2^{i} \left[\frac{11664 \times 2^{4n-2i} - 7776 \times 2^{4n-3i} + 8424 \times 2^{3n-2i} - 25272 \times 2^{3n-i} + 9180 \times 2^{2n-i} + 1296 \times 2^{4n-4i} - 3060 \times 2^{2n-2i} + 13608 \times 2^{2n} - 9828 \times 2^{n} + 1764}{216 \times 2^{2n-i} - 72 \times 2^{2n-2i} - 234 \times 2^{n} + 84}\right]$$

Theorem 2.1.9: Let NS[n] be a Nanostar Dendrimer. Then,

$$AZ_{C}(NS[n]) = 3 \sum_{i=0}^{n-1} 2^{i} \left[\frac{11664 \times 2^{4n-2i} - 7776 \times 2^{4n-3i} + 8424 \times 2^{3n-2i} - 25272 \times 2^{3n-i} + 9180 \times 2^{2n-i} + 1296 \times 2^{4n-4i} - 3060 \times 2^{2n-2i} + 13608 \times 2^{2n} - 9828 \times 2^{n} + 1764}{216 \times 2^{2n-i} - 72 \times 2^{2n-2i} - 234 \times 2^{n} + 82}\right]^{3}$$

Proof: $AZ_{C}(NS[n]) = \sum_{uv \in E(NS[n])} \left[\frac{c(u)c(v)}{c(u)+c(v)-2}\right]^{3}$

$$= \sum_{i=0}^{n-1} (3\text{x}2^{i}) \left[\frac{\{(((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 6)) \times (6 \times 2^{n-i} - 7)\} \times \{(((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 5)) \times (6 \times 2^{n-i} - 6)\}}{\{(((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 6)) \times (6 \times 2^{n-i} - 7)\} + \{(((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 5)) \times (6 \times 2^{n-i} - 6)\} - 2}\right]^{3}$$

$$= 3 \sum_{i=0}^{n-1} 2^{i} \left[\frac{11664 \times 2^{4n-2i} - 7776 \times 2^{4n-3i} + 8424 \times 2^{3n-2i} - 25272 \times 2^{3n-i} + 9180 \times 2^{2n-i} + 1296 \times 2^{4n-4i} - 3060 \times 2^{2n-2i} + 13608 \times 2^{2n} - 9828 \times 2^{n} + 1764}{216 \times 2^{2n-i} - 72 \times 2^{2n-2i} - 234 \times 2^{n} + 82}\right]^{3}$$

2.2 Hyper first, second and third Zagreb indices, F, Reduced forgotten, Reduced second Zagreb and Reduced hyper second Zagreb cutting number topological indices of Nanostar Dendrimer NS[n]

In the following theorems, we find Hyper first, second and third Zagreb indices, F, Reduced forgotten, Reduced second Zagreb and Reduced hyper second Zagreb cutting number topological indices of Nanostar Dendrimer NS[n].

Theorem 2.2.1: Let NS[n] be the Nanostar Dendrimer. Then,

$$HM_{1C}(NS[n]) = \left[\frac{7278336}{21}\right] \times 2^{4n} + 173988 \times 2^{3n} - \left[\frac{10382256}{3}\right] \times 2^{2n} - 606528 \times n \times 2^{3n} + 216432 \times n \times 2^{2n} + \left[\frac{2708640}{7}\right] \times 2^{n} - 42336.$$

Proof: $HM_{1C}(NS[n]) = \sum_{uv \in E(NS[n])} [c(u) + c(v)]^2$

$$= \sum_{i=0}^{n-1} (6 \times 2^{i})[\{(((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 6)) \times (6 \times 2^{n-i} - 7)\} + 0]^2 +$$

$$\sum_{i=0}^{n-1} (3 \times 2^{i}) [\{(((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 6)) \times (6 \times 2^{n-i} - 7)\} +$$

$$\{(((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 5)) \times (6 \times 2^{n-i} - 6)\}]^2 +$$

$$\sum_{i=0}^{n-1} (6 \times 2^{i}) [\{(((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 5)) \times (6 \times 2^{n-i} - 6)\} + 0]^2$$

$$= 6 \sum_{i=0}^{n-1} 2^{i} \left[108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^{n} + 42\right]^2 +$$

$$3 \sum_{i=0}^{n-1} 2^{i} \left[216 \times 2^{2n-i} - 72 \times 2^{2n-2i} - 234 \times 2^{n} + 84\right]^2 +$$

$$6 \sum_{i=0}^{n-1} 2^{i} \left[108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^{n} + 42\right]^2$$

$$HM_{1C}(NS[n]) = \left[\frac{7278336}{21}\right] \times 2^{4n} + 173988 \times 2^{3n} - \left[\frac{10382256}{3}\right] \times 2^{2n} - 606528 \times n \times 2^{3n} + 216432 \times n \times 2^{2n} + \left[\frac{2708640}{7}\right] \times 2^{n} - 42336.$$

Theorem 2.2.2: Let NS[n] be the Nanostar Dendrimer. Then,

$$HM_{2C}(NS[n]) = 3 \sum_{i=0}^{n-1} 2^{i} [11664 \times 2^{4n-2i} - 7776 \times 2^{4n-3i} + 8424 \times 2^{3n-2i} - 25272 \times 2^{3n-i} + 9180 \times 2^{2n-i} + 1296 \times 2^{4n-4i} - 3060 \times 2^{2n-2i} + 13608 \times 2^{2n} - 9828 \times 2^{n} + 1764]^2.$$

Proof: $HM_{2C}(NS[n]) = \sum_{uv\in E(NS[n])} [c(u)c(v)]^2$

$$= \sum_{i=0}^{n-1} (3 \times 2^{i})[\{(((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 6)) \times (6 \times 2^{n-i} - 7)\} \times$$

$$\{(((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 5) \times (6 \times 2^{n-i} - 6)\}]^2$$

$$= 3\sum_{i=0}^{n-1} 2^{i} [11664 \times 2^{4n-2i} - 7776 \times 2^{4n-3i} + 8424 \times 2^{3n-2i} -$$
$$25272 \times 2^{3n-i} + 9180 \times 2^{2n-i} + 1296 \times 2^{4n-4i} -$$
$$3060 \times 2^{2n-2i} + 13608 \times 2^{2n} - 9828 \times 2^{n} + 1764]^2.$$

Theorem 2.2.3: Let NS[n] be the Nanostar Dendrimer. Then,

$$HM_{3C}(NS[n]) = \left[\frac{1213056}{7}\right] \times 2^{4n} + 88452 \times 2^{3n} - 432756 \times 2^{2n} + \left[\frac{1345248}{7}\right] \times$$
$$2^{n} - 303264 \times n \times 2^{3n} + 106272 \times n \times 2^{3n} - 21168.$$

Proof: $HM_{3C}(NS[n]) = \sum_{uv\in E(NS[n])} |c(u) - c(v)|^2$

$$= \sum_{i=0}^{n-1} (6 \times 2^{i}) [\{(((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 6)) \times (6 \times 2^{n-i} - 7)\} - 0]^2 +$$

$$\sum_{i=0}^{n-1} (3 \times 2^{i}) [\{(((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 6)) \times (6 \times 2^{n-i} - 7)\} -$$

$$\{(((18 \times 2^{n}) - (6 \times 2^{n-i} - 5)) \times (6 \times 2^{n-i} - 6)\}]^2 +$$

$$\sum_{i=0}^{n-1} (6 \times 2^{i})[\{(((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 5)) \times (6 \times 2^{n-i} - 6)\} - 0]^2$$

$$HM_{3C}(NS[n]) = \left[\frac{1213056}{7}\right] \times 2^{4n} + 88452 \times 2^{3n} - 432756 \times 2^{2n} + \left[\frac{1345248}{7}\right] \times$$
$$2^{n} - 303264 \times n \times 2^{3n} + 106272 \times n \times 2^{3n} - 21168.$$

Theorem 2.2.4: Let NS[n] be a Nanostar Dendrimer. Then,

$$FR_{C}(NS[n]) = 6\sum_{i=0}^{n-1} 2^{i} [108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^{n} + 42]^2 +$$

$$3\sum_{i=0}^{n-1} 2^{i} [\{108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^{n} + 42\}^2 +$$
$$\{108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^{n} + 42\}^2] +$$
$$6\sum_{i=0}^{n-1} 2^{i} [108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^{n} + 42]^2$$

Proof: $FR_C(NS[n]) = \sum_{uv\in E(NS[n])} [c(u)^2 + c(v)^2]$

$$= \sum_{i=0}^{n-1} (6 \times 2^i) [\{(((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 6)) \times (6 \times 2^{n-i} - 7)\}^2 + 0^2] +$$

$$\sum_{i=0}^{n-1} (3 \times 2^i) [\{(((18 \times 2^n) - 12) - (6\times 2^{n-i} - 6)) \times (6 \times 2^{n-i} - 7)\}^2 +$$

$$\{(((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 5)) \times (6 \times 2^{n-i} - 6)\}^2] +$$

$$\sum_{i=0}^{n-1} (6 \times 2^i) [\{(((18 \times 2^n) - 12) - (6\times 2^{n-i} - 5)) \times (6 \times 2^{n-i} - 6)\}^2 + 0^2]$$

$$= 6 \sum_{i=0}^{n-1} 2^i [108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^n + 42]^2 +$$

$$3 \sum_{i=0}^{n-1} 2^i [\{108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^n + 42\}^2 +$$

$$\{108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^n + 42\}^2] +$$

$$6 \sum_{i=0}^{n-1} 2^i [108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^n + 42]^2$$

Theorem 2.2.5: Let NS[n] be a Nanostar Dendrimer. Then,

$$RF_C(NS[n]) = \left[\frac{1819584}{7}\right] \times 2^{4n} + 131220 \times 2^{3n} - 642168\times 2^{2n} + \left[\frac{1968942}{7}\right] \times 2^n -$$
$$454896 \times n \times 2^{3n} + 157464\times n \times 2^{2n} - 30270$$

Proof: $RF_C(NS[n]) = \sum_{uv\in E(NS[n])}[(c(u) - 1)^2 + (c(v) - 1)^2]$

$$= \sum_{i=0}^{n-1} (6 \times 2^i) [\{\{(((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 6)) \times$$
$$(6 \times 2^{n-i} - 7)\} - 1\}^2 + (0 - 1)^2] +$$

$$\sum_{i=0}^{n-1} (3 \times 2^i) [\{(((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 6)) \times$$
$$(6 \times 2^{n-i} - 7)\} - 1]^2 +$$
$$[\{(((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 5)) \times$$
$$(6 \times 2^{n-i} - 6)\} - 1\}^2] +$$

$$\sum_{i=0}^{n-1} (6 \times 2^i) [\{\{(((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 5)) \times$$
$$(6 \times 2^{n-i} - 6)\} - 1\}^2 + (0 - 1)^2]$$

$$RF_C(NS[n]) = \left[\frac{1819584}{7}\right] \times 2^{4n} + 131220 \times 2^{3n} - 642168 \times 2^{2n} + \left[\frac{1968942}{7}\right] \times$$
$$2^n - 454896 \times n \times 2^{3n} + 157464\times n \times 2^{2n} - 30270$$

Theorem 2.2.6: Let NS[n] be a Nanostar Dendrimer. Then,

$$RM_{2C}(NS[n]) = \left[\frac{303264}{7}\right] \times 2^{4n} + 21384 \times 2^{3n} - 106002 \times 2^{2n} + \left[\frac{311847}{7}\right] \times 2^{n} - 75816 \times n \times 2^{3n} + 26892 \times n \times 2^{3n} - 4551$$

Proof: $RM_{2C}(NS[n]) = \sum_{uv \in E(NS[n])}[(c(u) - 1)(c(v) - 1)]$

$$= \sum_{i=0}^{n-1} (6 \times 2^{i}) [\{\{(((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 6)) \times (6 \times 2^{n-i} - 7)\} - 1\}(0 - 1)] +$$

$$\sum_{i=0}^{n-1} (3 \times 2^{i}) [\{\{(((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 6)) \times (6 \times 2^{n-i} - 7)\} - 1\} \{\{(((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 5)) \times (6 \times 2^{n-i} - 6)\} - 1\}] +$$

$$\sum_{i=0}^{n-1} (6 \times 2^{i}) [\{\{(((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 5)) \times (6 \times 2^{n-i} - 6)\} - 1\}\{0 - 1\}]$$

$$RM_{2C}(NS[n]) = \left[\frac{303264}{7}\right] \times 2^{4n} + 21384 \times 2^{3n} - 106002 \times 2^{2n} + \left[\frac{311847}{7}\right] \times 2^{n} - 75816 \times n \times 2^{3n} + 26892 \times n \times 2^{3n} - 4551$$

Theorem 2.2.7: Let NS[n] be a Nanostar Dendrimer. Then,

$$RHM_{2C}(NS[n]) = 6 \sum_{i=0}^{n-1} 2^{i} [108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n} + 41]^{2} +$$

$$3 \sum_{i=0}^{n-1} 2^{i} [\{108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n} + 41\} \{108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^{n} + 41\}]^{2} +$$

$$6 \sum_{i=0}^{n-1} 2^{i} [108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n} + 41]^{2}$$

Proof: $RHM_{2C}(NS[n]) = \sum_{uv \in E(NS[n])} [(c(u) - 1)(c(v) - 1)]^{2}$

$$= \sum_{i=0}^{n-1} (6 \times 2^{i}) [\{\{(((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 6)) \times (6 \times 2^{n-i} - 7)\} - 1\} \{0 - 1\}]^{2} +$$

$$\sum_{i=0}^{n-1} (3 \times 2^{i}) [\{\{(((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 6)) \times (6 \times 2^{n-i} - 7)\} - 1\} \{\{(((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 5)) \times (6 \times 2^{n-i} - 6)\} - 1\}]^{2} +$$

$$\sum_{i=0}^{n-1} (6 \times 2^{i}) [\{\{(((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 5)) \times (6 \times 2^{n-i} - 6)\} - 1\}\{0 - 1\}]^{2}$$

$$= 6\sum_{i=0}^{n-1} 2^{i} [108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^{n} + 41]^{2} +$$

$$3\sum_{i=0}^{n-1} 2^{i} [\{108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^{n} + 41\} \{108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^{n} + 41\}]^{2} +$$

$$6\sum_{i=0}^{n-1} 2^{i} [108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^{n} + 41]^{2}$$

2.3 General reduced forgotten, General reduced second Zagreb and General reduced hyper second Zagreb cutting number based topological indices of Nanostar Dendrimer NS[n]

Now, we shall calculate General reduced forgotten, General reduced second Zagreb and General reduced hyper second Zagreb cutting number based topological indices of Nanostar Dendrimer NS[n] in the following theorems.

Theorem 2.3.1: Let NS[n] be a Nanostar Dendrimer. Then,

$$RF_C{}^{a}(NS[n]) = 6\sum_{i=0}^{n-1} 2^{i} [(108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^{n} + 41)^{2} + 1]^{a}$$

$$3\sum_{i=0}^{n-1} 2^{i} [\{108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^{n} + 41\}^{2} + \{108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^{n} + 41\}^{2}]^{a} +$$

$$6\sum_{i=0}^{n-1} 2^{i} [(108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^{n} + 41)^{2} + 1]^{a}$$

Proof: $RF_C{}^{a}(NS[n]) = \sum_{uv \in E(NS[n])} [(c(u) - 1)^{2} + (c(v) - 1)^{2}]^{a}$

$$= \sum_{i=0}^{n-1} (6 \times 2^{i}) [\{\{(((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 6)) \times (6 \times 2^{n-i} - 7)\} - 1\}^{2} + (0 - 1)^{2}]^{a} +$$

$$\sum_{i=0}^{n-1} (3 \times 2^{i}) [\{(((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 6)) \times (6 \times 2^{n-i} - 7)\} - 1]^{2} +$$

$$[\{(((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 5)) \times (6 \times 2^{n-i} - 6)\} - 1\}^{2}]^{a} +$$

$$\sum_{i=0}^{n-1} (6 \times 2^{i}) \ [\{\{(((18 \times 2^{n})–12) – (6 \times 2^{n-i} – 5)) \times$$

$$(6 \times 2^{n-i} – 6)\} – 1\}^{2} + (0 – 1)^{2}]^{a}$$

$$= 6\sum_{i=0}^{n-1} 2^{i} [(108 \times 2^{2n-i} –36 \times 2^{2n-2i} +6 \times 2^{n-i} – 126 \times 2^{n} + 41)^{2} + 1]^{a}$$

$$3\sum_{i=0}^{n-1} 2^{i} [(108 \times 2^{2n-i} – 36 \times 2^{2n-2i} + 6 \times 2^{n-i} – 126 \times 2^{n} + 41)^{2} +$$

$$(108 \times 2^{2n-i} – 36 \times 2^{2n-2i} – 6 \times 2^{n-i} – 108 \times 2^{n} + 41)^{2}]^{a} +$$

$$6\sum_{i=0}^{n-1} 2^{i} [(108 \times 2^{2n-i} – 36 \times 2^{2n-2i} – 6 \times 2^{n-i} – 108 \times 2^{n} + 41)^{2} + 1]^{a}$$

Theorem 2.3.2: Let NS[n] be a Nanostar Dendrimer. Then,

$$RM_{2C}^{a}(NS[n]) = 6\sum_{i=0}^{n-1} 2^{i} [108 \times 2^{2n-i} – 36 \times 2^{2n-2i} + 6 \times 2^{n-i} – 126\times 2^{n} + 41]^{a} +$$

$$3\sum_{i=0}^{n-1} 2^{i} [\{108 \times 2^{2n-i} – 36 \times 2^{2n-2i} + 6 \times 2^{n-i} – 126\times 2^{n} + 41\}$$

$$\{108 \times 2^{2n-i} – 36 \times 2^{2n-2i} – 6 \times 2^{n-i} – 108 \times 2^{n} + 41\}]^{a} +$$

$$6\sum_{i=0}^{n-1} 2^{i} [108 \times 2^{2n-i} – 36 \times 2^{2n-2i} – 6\times 2^{n-i} – 108\times 2^{n} + 41]^{a}$$

Proof: $RM_{2C}^{a}(NS[n]) = \sum_{uv\in E(NS[n])} [(c(u) – 1)(c(v) – 1)]^{a}$

$$= \sum_{i=0}^{n-1} (6 \times 2^{i}) [\{\{\{((18 \times 2^{n}) –12) – (6 \times 2^{n-i} – 6)\}\times$$

$$(6 \times 2^{n-i} – 7)\} – 1\}\{0 – 1\}]^{a} +$$

$$\sum_{i=0}^{n-1} (3 \times 2^{i}) [\{\{\{((18 \times 2^{n}) –12) – (6 \times 2^{n-i} – 6)\}\times$$

$$(6 \times 2^{n-i} – 7)\} – 1\}$$

$$\{\{\{((18 \times 2^{n}) – 12) – (6 \times 2^{n-i} – 5)\}\times$$

$$(6 \times 2^{n-i} – 6)\} – 1\}]^{a} +$$

$$\sum_{i=0}^{n-1} (6 \times 2^{i}) [\{\{\{((18 \times 2^{n}) –12) – (6 \times 2^{n-i} – 5)\}\times$$

$$(6 \times 2^{n-i} – 6)\} – 1\}\{0 – 1\}]^{a}$$

$$= 6\sum_{i=0}^{n-1} 2^{i} [108 \times 2^{2n-i} – 36 \times 2^{2n-2i} + 6 \times 2^{n-i} – 126\times 2^{n} + 41]^{a} +$$

$$3\sum_{i=0}^{n-1} 2^{i} [\{108 \times 2^{2n-i} – 36 \times 2^{2n-2i} + 6 \times 2^{n-i} – 126\times 2^{n} + 41\}$$

$$\{108 \times 2^{2n-i} – 36 \times 2^{2n-2i} – 6 \times 2^{n-i} – 108 \times 2^{n} + 41\}]^{a} +$$

$$6\sum_{i=0}^{n-1} 2^{i} [108 \times 2^{2n-i} – 36 \times 2^{2n-2i} – 6\times 2^{n-i} – 108\times 2^{n} + 41]^{a}$$

Theorem 2.3.3: Let NS[n] be a Nanostar Dendrimer. Then,

$$\mathrm{RHM}_{2C}{}^{a}(\mathrm{NS[n]}) = 6\sum_{i=0}^{n-1} 2^{i}\,[108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126\times 2^{n} + 41]^{2a} +$$
$$3\sum_{i=0}^{n-1} 2^{i}\,[\{108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126\times 2^{n} + 41\}$$
$$\{108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^{n} + 41\}]^{2a} +$$
$$6\sum_{i=0}^{n-1} 2^{i}\,[108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n} + 41]^{2a}$$

Proof: $\mathrm{RHM}_{2C}{}^{a}(\mathrm{NS[n]}) = \sum_{uv\in E(\mathrm{NS[n]})} [(c(u)-1)(c(v)-1)]^{2a}$

$$= \sum_{i=0}^{n-1} (6 \times 2^{i})\,[\{\{\{((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 6)\}\times$$
$$(6 \times 2^{n-i} - 7)\} - 1\}\{0 - 1\}]^{2a} +$$
$$\sum_{i=0}^{n-1} (3 \times 2^{i})\,[\{\{\{((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 6)\}\times$$
$$(6 \times 2^{n-i} - 7)\} - 1\}$$
$$\{\{\{((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 5)\}\times$$
$$(6 \times 2^{n-i} - 6)\} - 1\}]^{2a} +$$
$$\sum_{i=0}^{n-1} (6 \times 2^{i})\,[\{\{\{((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 5)\}\times$$
$$(6 \times 2^{n-i} - 6)\} - 1\}\{0 - 1\}]^{2a}$$
$$= 6\sum_{i=0}^{n-1} 2^{i}\,[108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126\times 2^{n} + 41]^{2a} +$$
$$3\sum_{i=0}^{n-1} 2^{i}\,[\{108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126\times 2^{n} + 41\}$$
$$\{108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^{n} + 41\}]^{2a} +$$
$$6\sum_{i=0}^{n-1} 2^{i}\,[108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n} + 41]^{2a}$$

2.4 Polynomial of Arithmetic, Harmonic, Geometric Arithmetic, Arithmetic Geometric, sum and product connectivity, ABC, Inverse sum, Augumented Zagreb cutting number topological indices of Nanostar Dendrimer NS[n]

Next, we compute polynomial of Arithmetic, Harmonic, Geometric Arithmetic, Arithmetic Geometric, sum and product connectivity, ABC, Inverse sum, Augumented Zagreb cutting number based topological indices of Nanostar Dendrimer NS[n] in the following theorems.

Theorem 2.4.1: Let NS[n] be a Nanostar Dendrimer. Then,

$$A_C(NS[n],x) = 6\sum_{i=0}^{n-1} 2^{i}\ x^{\left[\frac{108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+42}{2}\right]} +$$
$$3\sum_{i=0}^{n-1} 2^{i}\ x^{\left[\frac{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^{n}+84}{2}\right]} +$$
$$6\sum_{i=0}^{n-1} 2^{i}\ x^{\left[\frac{108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+42}{2}\right]}$$

Proof: $A_C(NS[n],x) = \sum_{uv\in E(NS[n])} x^{\left(\frac{c(u)+c(v)}{2}\right)}$

$$= \sum_{i=0}^{n-1} (6\times 2^{i})\, x^{\left[\frac{\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-6))\times(6\times 2^{n-i}-7)\}+0}{2}\right]} +$$
$$\sum_{i=0}^{n-1} (3\times 2^{i})\, x^{\left[\frac{\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-6))\times(6\times 2^{n-i}-7)\}+\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-5))\times(6\times 2^{n-i}-6)\}}{2}\right]} +$$
$$\sum_{i=0}^{n-1} (6\times 2^{i})\, x^{\left[\frac{\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-5))\times(6\times 2^{n-i}-6)\}+0}{2}\right]}$$
$$= 6\sum_{i=0}^{n-1} 2^{i}\ x^{\left[\frac{108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+42}{2}\right]} +$$
$$3\sum_{i=0}^{n-1} 2^{i}\ x^{\left[\frac{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^{n}+84}{2}\right]} +$$
$$6\sum_{i=0}^{n-1} 2^{i}\ x^{\left[\frac{108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+42}{2}\right]}$$

Theorem 2.4.2: Let NS[n] be a Nanostar Dendrimer. Then

$$H_C(NS[n],x) = 6\sum_{i=0}^{n-1} 2^{i}\ x^{\left[\frac{2}{108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+42}\right]} +$$
$$3\sum_{i=0}^{n-1} 2^{i}\ x^{\left[\frac{2}{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^{n}+84}\right]} +$$
$$6\sum_{i=0}^{n-1} 2^{i}\ x^{\left[\frac{2}{108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+42}\right]}$$

Proof: $H_C(NS[n],x) = \sum_{uv\in E(NS[n])} x^{\left(\frac{2}{c(u)+c(v)}\right)}$

$$= \sum_{i=0}^{n-1} (6\times 2^{i})\, x^{\left[\frac{2}{\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-6))\times(6\times 2^{n-i}-7)\}+0}\right]} +$$
$$\sum_{i=0}^{n-1} (3\times 2^{i})\, x^{\left[\frac{2}{\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-6))\times(6\times 2^{n-i}-7)\}+\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-5))\times(6\times 2^{n-i}-6)\}}\right]} +$$
$$\sum_{i=0}^{n-1} (6\times 2^{i})\, x^{\left[\frac{2}{\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-5))\times(6\times 2^{n-i}-6)\}+0}\right]}$$
$$= 6\sum_{i=0}^{n-1} 2^{i}\ x^{\left[\frac{2}{108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+42}\right]} +$$

$$3\sum_{i=0}^{n-1} 2^{i}\ x^{\left[\frac{2}{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^{n}+84}\right]}+$$

$$6\sum_{i=0}^{n-1} 2^{i}\ x^{\left[\frac{2}{108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+42}\right]}$$

Theorem 2.4.3: Let NS[n] be a Nanostar Dendrimer. Then,

$$GA_C(NS[n], x) = 6\sum_{i=0}^{n-1} 2^{i}\ x^{\left[\frac{\sqrt{\begin{array}{c}11664\times 2^{4n-2i}-7776\times 2^{4n-3i}+8424\times 2^{3n-2i}-\\ 25272\times 2^{3n-i}+9180\times 2^{2n-i}+1296\times 2^{4n-4i}-\\ 3060\times 2^{2n-2i}+13608\times 2^{2n}-9828\times 2^{n}+1764\end{array}}}{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^{n}+84}\right]}$$

Proof: $GA_C(NS[n], x) = \sum_{uv\in E(NS[n])} x^{\left(\frac{2\sqrt{c(u)c(v)}}{c(u)+c(v)}\right)}$

$$= \sum_{i=0}^{n-1} (3x2^{i})\, x^{\left[\frac{2\sqrt{\begin{array}{c}\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-6))\times(6\times 2^{n-i}-7)\}\times\\ \{(((18\times 2^{n})-12)-(6\times 2^{n-i}-5))\times(6\times 2^{n-i}-6)\}\end{array}}}{\begin{array}{c}\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-6))\times(6\times 2^{n-i}-7)\}+\\ \{(((18\times 2^{n})-12)-(6\times 2^{n-i}-5))\times(6\times 2^{n-i}-6)\}\end{array}}\right]}$$

$$= 6\sum_{i=0}^{n-1} 2^{i}\ x^{\left[\frac{\sqrt{\begin{array}{c}11664\times 2^{4n-2i}-7776\times 2^{4n-3i}+8424\times 2^{3n-2i}-\\ 25272\times 2^{3n-i}+9180\times 2^{2n-i}+1296\times 2^{4n-4i}-\\ 3060\times 2^{2n-2i}+13608\times 2^{2n}-9828\times 2^{n}+1764\end{array}}}{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^{n}+84}\right]}$$

Theorem 2.4.4: Let NS[n] be a Nanostar Dendrimer. Then,

$$AG_C(NS[n], x) = [\tfrac{3}{2}]\sum_{i=0}^{n-1} 2^{i}\ x^{\left[\frac{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^{n}+84}{\sqrt{\begin{array}{c}11664\times 2^{4n-2i}-7776\times 2^{4n-3i}+8424\times 2^{3n-2i}-\\ 25272\times 2^{3n-i}+9180\times 2^{2n-i}+1296\times 2^{4n-4i}-\\ 3060\times 2^{2n-2i}+13608\times 2^{2n}-9828\times 2^{n}+1764\end{array}}}\right]}$$

Proof: $AG_C(NS[n], x) = \sum_{uv\in E(NS[n])} x^{\left(\frac{c(u)+c(v)}{2\sqrt{c(u)c(v)}}\right)}$

$$= \sum_{i=0}^{n-1} (3x2^{i})\, x^{\left[\frac{\begin{array}{c}\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-6))\times(6\times 2^{n-i}-7)\}+\\ \{(((18\times 2^{n})-12)-(6\times 2^{n-i}-5))\times(6\times 2^{n-i}-6)\}\end{array}}{2\sqrt{\begin{array}{c}\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-6))\times(6\times 2^{n-i}-7)\}\times\\ \{(((18\times 2^{n})-12)-(6\times 2^{n-i}-5))\times(6\times 2^{n-i}-6)\}\end{array}}}\right]}$$

$$= [\tfrac{3}{2}]\sum_{i=0}^{n-1} 2^{i}\ x^{\left[\frac{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^{n}+84}{\sqrt{\begin{array}{c}11664\times 2^{4n-2i}-7776\times 2^{4n-3i}+8424\times 2^{3n-2i}-\\ 25272\times 2^{3n-i}+9180\times 2^{2n-i}+1296\times 2^{4n-4i}-\\ 3060\times 2^{2n-2i}+13608\times 2^{2n}-9828\times 2^{n}+1764\end{array}}}\right]}$$

Theorem 2.4.5: Let NS[n] be a Nanostar Dendrimer. Then,

$$S_C(NS[n],x) = 6\sum_{i=0}^{n-1} 2^{i}\ x^{\left[\frac{1}{\sqrt{108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+42}}\right]} +$$
$$3\sum_{i=0}^{n-1} 2^{i}\ x^{\left[\frac{1}{\sqrt{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^{n}+84}}\right]} +$$
$$6\sum_{i=0}^{n-1} 2^{i}\ x^{\left[\frac{1}{\sqrt{108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+42}}\right]}$$

Proof: $S_C(NS[n],x) = \sum_{uv\in E(NS[n])} x^{\left(\frac{1}{\sqrt{c(u)+c(v)}}\right)}$

$$= \sum_{i=0}^{n-1} (6\times 2^{i})\ x^{\left[\frac{1}{\sqrt{\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-6))\times(6\times 2^{n-i}-7)\}+0}}\right]} +$$
$$\sum_{i=0}^{n-1} (3\times 2^{i})\ x^{\left[\frac{1}{\sqrt{\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-6))\times(6\times 2^{n-i}-7)\}+\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-5))\times(6\times 2^{n-i}-6)\}}}\right]} +$$
$$\sum_{i=0}^{n-1} (6\times 2^{i})\ x^{\left[\frac{1}{\sqrt{\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-5))\times(6\times 2^{n-i}-6)\}+0}}\right]}$$
$$= 6\sum_{i=0}^{n-1} 2^{i}\ x^{\left[\frac{1}{\sqrt{108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+42}}\right]} +$$
$$3\sum_{i=0}^{n-1} 2^{i}\ x^{\left[\frac{1}{\sqrt{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^{n}+84}}\right]} +$$
$$6\sum_{i=0}^{n-1} 2^{i}\ x^{\left[\frac{1}{\sqrt{108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+42}}\right]}$$

Theorem 2.4.6: Let NS[n] be a Nanostar Dendrimer. Then,

$$P_C(NS[n],x) = 3\sum_{i=0}^{n-1} 2^{i}\ x^{\left[\frac{1}{\sqrt{\begin{array}{c}11664\times 2^{4n-2i}-7776\times 2^{4n-3i}+8424\times 2^{3n-2i}-\\ 25272\times 2^{3n-i}+9180\times 2^{2n-i}+1296\times 2^{4n-4i}-\\ 3060\times 2^{2n-2i}+13608\times 2^{2n}-9828\times 2^{n}+1764\end{array}}}\right]}$$

Proof: $P_C(NS[n],x) = \sum_{uv\in E(NS[n])} x^{\left[\frac{1}{\sqrt{c(u)c(v)}}\right]}$

$$= \sum_{i=0}^{n-1} (3\times 2^{i})\, x^{\left[\frac{1}{\sqrt{\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-6))\times(6\times 2^{n-i}-7)\}\times\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-5))\times(6\times 2^{n-i}-6)\}}}\right]}$$

$$= 3\sum_{i=0}^{n-1} 2^{i}\, x^{\left[\frac{1}{\sqrt{\begin{array}{c}11664\times 2^{4n-2i} - 7776\times 2^{4n-3i} + 8424\times 2^{3n-2i} - \\ 25272\times 2^{3n-i} + 9180\times 2^{2n-i} + 1296\times 2^{4n-4i} - \\ 3060\times 2^{2n-2i} + 13608\times 2^{2n} - 9828\times 2^{n} + 1764\end{array}}}\right]}$$

Theorem 2.4.7: Let NS[n] be a Nanostar Dendrimer. Then,

$$ABC_C(NS[n], x) = 6\sum_{i=0}^{n-1} 2^{i}\, x^{\sqrt{\frac{108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n} + 40}{108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n} + 42}}} +$$

$$3\sum_{i=0}^{n-1} 2^{i}\, x^{\sqrt{\frac{216\times 2^{2n-i} - 72\times 2^{2n-2i} - 234\times 2^{n} + 82}{216\times 2^{2n-i} - 72\times 2^{2n-2i} - 234\times 2^{n} + 84}}} +$$

$$6\sum_{i=0}^{n-1} 2^{i}\, x^{\sqrt{\frac{108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n} + 40}{108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n} + 42}}} +$$

Proof: $ABC_c(NS[n], x) = \sum_{uv\in E(NS[n])} x^{\sqrt{\frac{c(u)+c(v)-2}{c(u)+c(v)}}}$

$$= \sum_{i=0}^{n-1}(6\times 2^{i})\; x^{\sqrt{\frac{[\{((18\times 2^{n})-12) - (6\times 2^{n-i}-6)\}\times(6\times 2^{n-i}-7)] + 0 - 2}{[\{((18\times 2^{n})-12) - (6\times 2^{n-i}-6)\}\times(6\times 2^{n-i}-7)] + 0}}} +$$

$$\sum_{i=0}^{n-1}(3\times 2^{i})\; x^{\sqrt{\frac{[\{((18\times 2^{n})-12) - (6\times 2^{n-i}-6)\}\times(6\times 2^{n-i}-7)] + [\{((18\times 2^{n})-12) - (6\times 2^{n-i}-5)\}\times(6\times 2^{n-i}-6)] - 2}{[\{((18\times 2^{n})-12) - (6\times 2^{n-i}-6)\}\times(6\times 2^{n-i}-7)] + [\{((18\times 2^{n})-12) - (6\times 2^{n-i}-5)\}\times(6\times 2^{n-i}-6)]}}} +$$

$$\sum_{i=0}^{n-1}(6\times 2^{i})\; x^{\sqrt{\frac{[\{((18\times 2^{n})-12) - (6\times 2^{n-i}-5)\}\times(6\times 2^{n-i}-6)] + 0 - 2}{[\{((18\times 2^{n})-12) - (6\times 2^{n-i}-5)\}\times(6\times 2^{n-i}-6)] + 0}}}$$

$$= 6\sum_{i=0}^{n-1} 2^{i}\, x^{\sqrt{\frac{108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n} + 40}{108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n} + 42}}}$$

$$3\sum_{i=0}^{n-1} 2^{i}\, x^{\sqrt{\frac{216\times 2^{2n-i} - 72\times 2^{2n-2i} - 234\times 2^{n} + 82}{216\times 2^{2n-i} - 72\times 2^{2n-2i} - 234\times 2^{n} + 84}}}$$

$$6\sum_{i=0}^{n-1} 2^{i}\, x^{\sqrt{\frac{108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n} + 40}{108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n} + 42}}}$$

Theorem 2.4.8: Let NS[n] be a Nanostar Dendrimer. Then,

$$IS_C(NS[n], x) = 3\sum_{i=0}^{n-1} 2^{i}\, x^{\left[\frac{\begin{array}{c}11664\times 2^{4n-2i} - 7776\times 2^{4n-3i} + 8424\times 2^{3n-2i} - \\ 25272\times 2^{3n-i} + 9180\times 2^{2n-i} + 1296\times 2^{4n-4i} - \\ 3060\times 2^{2n-2i} + 13608\times 2^{2n} - 9828\times 2^{n} + 1764\end{array}}{216\times 2^{2n-i} - 72\times 2^{2n-2i} - 234\times 2^{n} + 84}\right]}$$

Proof: ISc(NS[n], x) = $\sum_{uv\in E\,[NS(n)]} x^{\left[\frac{c(u)c(v)}{c(u)+c(v)}\right]}$

$$= \sum_{i=0}^{n-1}(3\text{x}2^{i})\, x^{\left[\frac{[\{((18\times 2^{n})-12)\;-(6\times 2^{n-i}-6)\}\times(6\times 2^{n-i}-7)]\times[\{((18\times 2^{n})-12)\;-(6\times 2^{n-i}-5)\}\times(6\times 2^{n-i}-6)]}{[\{((18\times 2^{n})-12)\;-(6\times 2^{n-i}-6)\}\times(6\times 2^{n-i}-7)]+[\{((18\times 2^{n})-12)\;-(6\times 2^{n-i}-5)\}\times(6\times 2^{n-i}-6)]}\right]}$$

$$= 3\sum_{i=0}^{n-1} 2^{i}\; x^{\left[\frac{11664\times 2^{4n-2i}-7776\times 2^{4n-3i}+8424\times 2^{3n-2i}-25272\times 2^{3n-i}+9180\times 2^{2n-i}+1296\times 2^{4n-4i}-3060\times 2^{2n-2i}+13608\times 2^{2n}-9828\times 2^{n}+1764}{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^{n}+84}\right]}$$

Theorem 2.4.9: Let NS[n] be a Nanostar Dendrimer. Then,

$$AZ_C(NS[n], x) = 3\sum_{i=0}^{n-1} 2^{i}\; x^{\left[\frac{11664\times 2^{4n-2i}-7776\times 2^{4n-3i}+8424\times 2^{3n-2i}-25272\times 2^{3n-i}+9180\times 2^{2n-i}+1296\times 2^{4n-4i}-3060\times 2^{2n-2i}+13608\times 2^{2n}-9828\times 2^{n}+1764}{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^{n}+82}\right]^{3}}$$

Proof: AZ$_C$(NS[n], x) = $\sum_{uv\in E(NS[n])} x^{\left[\frac{c(u)c(v)}{c(u)+c(v)-2}\right]^{3}}$

$$= \sum_{i=0}^{n-1}\;(3\text{x}2^{i})\, x^{\left[\frac{[\{((18\times 2^{n})-12)\;-(6\times 2^{n-i}-6)\}\times(6\times 2^{n-i}-7)]\times[\{((18\times 2^{n})-12)\;-(6\times 2^{n-i}-5)\}\times(6\times 2^{n-i}-6)]}{[\{((18\times 2^{n})-12)\;-(6\times 2^{n-i}-6)\}\times(6\times 2^{n-i}-7)]+[\{((18\times 2^{n})-12)\;-(6\times 2^{n-i}-5)\}\times(6\times 2^{n-i}-6)]-2}\right]^{3}}$$

$$= 3\sum_{i=0}^{n-1} 2^{i}\; x^{\left[\frac{11664\times 2^{4n-2i}-7776\times 2^{4n-3i}+8424\times 2^{3n-2i}-25272\times 2^{3n-i}+9180\times 2^{2n-i}+1296\times 2^{4n-4i}-3060\times 2^{2n-2i}+13608\times 2^{2n}-9828\times 2^{n}+1764}{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^{n}+82}\right]^{3}}$$

2.5 Polynomial of Hyper first, second and third Zagreb indices, F, Reduced forgotten, Reduced second Zagreb and Reduced hyper second Zagreb cutting number topological indices of Nanostar Dendrimer NS[n]

In this section, we calculate polynomial of hyper first, second and third Zagreb indices, F, Reduced forgotten, Reduced second Zagreb and Reduced hyper second Zagreb cutting number topological indices of Nanostar Dendrimer NS[n] in the following theorems.

Theorem 2.5.1: Let NS[n] be the Nanostar Dendrimer. Then,

$$HM_{1C}(NS[n], x) = 6\sum_{i=0}^{n-1} 2^{i}\; x^{[108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+42]^{2}} +$$
$$3\sum_{i=0}^{n-1} 2^{i}\; x^{[\,216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^{n}+84\,]^{2}} +$$
$$6\sum_{i=0}^{n-1} 2^{i}\; x^{[108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+42]^{2}}$$

Proof: $HM_{1C}(NS[n], x) = \sum_{uv \in E(NS[n])} x^{[c(u)+c(v)]^2}$

$$= \sum_{i=0}^{n-1} (6 \times 2^{i})\, x^{[\{\{((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 6)\} \times (6 \times 2^{n-i} - 7)\} + 0]^2} +$$

$$\sum_{i=0}^{n-1} (3 \times 2^{i})\, x^{[\{\{((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 6)\} \times (6 \times 2^{n-i} - 7)\} + \{\{((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 5)\} \times (6 \times 2^{n-i} - 6)\}]^2} +$$

$$\sum_{i=0}^{n-1} (6 \times 2^{i})\, x^{[\{\{((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 5)\} \times (6 \times 2^{n-i} - 6)\} + 0]^2}$$

$$= 6 \sum_{i=0}^{n-1} 2^{i}\, x^{[108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^{n} + 42]^2} +$$

$$3 \sum_{i=0}^{n-1} 2^{i}\, x^{[216 \times 2^{2n-i} - 72 \times 2^{2n-2i} - 234 \times 2^{n} + 84]^2} +$$

$$6 \sum_{i=0}^{n-1} 2^{i}\, x^{[108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^{n} + 42]^2}$$

Theorem 2.5.2: Let NS[n] be the Nanostar Dendrimer. Then,

$$HM_{2C}(NS[n], x) = 3 \sum_{i=0}^{n-1} 2^{i}\, x^{[11664 \times 2^{4n-2i} - 7776 \times 2^{4n-3i} + 8424 \times 2^{3n-2i} - 25272 \times 2^{3n-i} + 9180 \times 2^{2n-i} + 1296 \times 2^{4n-4i} - 3060 \times 2^{2n-2i} + 13608 \times 2^{2n} - 9828 \times 2^{n} + 1764]^2}.$$

Proof: $HM_{2C}(NS[n], x) = \sum_{uv \in E(NS[n])} x^{[c(u)c(v)]^2}$

$$= \sum_{i=0}^{n-1} (3 \times 2^{i})\, x^{[\{\{((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 6)\} \times (6 \times 2^{n-i} - 7)\} \times \{\{((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 5) \times (6 \times 2^{n-i} - 6)\}\}]^2}$$

$$= 3 \sum_{i=0}^{n-1} 2^{i}\, x^{[11664 \times 2^{4n-2i} - 7776 \times 2^{4n-3i} + 8424 \times 2^{3n-2i} - 25272 \times 2^{3n-i} + 9180 \times 2^{2n-i} + 1296 \times 2^{4n-4i} - 3060 \times 2^{2n-2i} + 13608 \times 2^{2n} - 9828 \times 2^{n} + 1764]^2}$$

Theorem 2.5.3: Let NS[n] be the Nanostar Dendrimer. Then,

$$HM_{3C}(NS[n], x) = 6 \sum_{i=0}^{n-1} 2^{i}\, x^{[108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^{n} + 42]^2} +$$

$$3 \sum_{i=0}^{n-1} 2^{i}\, x^{[12 \times 2^{n-i} - 18 \times 2^{n}]^2} +$$

$$6 \sum_{i=0}^{n-1} 2^{i}\, x^{[108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^{n} + 42]^2}$$

Proof: $HM_{3C}(NS[n], x) = \sum_{uv \in E(NS[n])} x^{|c(u) - c(v)|^2}$

$$= \sum_{i=0}^{n-1} (6 \times 2^i)\, x^{[\{\{((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 6)] \times (6 \times 2^{n-i} - 7)\} - 0]^2} +$$

$$[\{\{((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 6)\} \times (6 \times 2^{n-i} - 7)\} -$$

$$\sum_{i=0}^{n-1} (3 \times 2^i)\, x^{\{\{((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 5)\} \times (6 \times 2^{n-i} - 6)\}]^2} +$$

$$\sum_{i=0}^{n-1} (6 \times 2^i)\, x^{[\{\{((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 5)\} \times (6 \times 2^{n-i} - 6)\} - 0]^2}$$

$$= 6 \sum_{i=0}^{n-1} 2^i\, x^{[108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^n + 42]^2} +$$

$$3 \sum_{i=0}^{n-1} 2^i\, x^{[12 \times 2^{n-i} - 18 \times 2^n]^2} +$$

$$6 \sum_{i=0}^{n-1} 2^i\, x^{[108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^n + 42]^2}$$

Theorem 2.5.4: Let NS[n] be a Nanostar Dendrimer. Then,

$$F_C(NS[n], x) = 6 \sum_{i=0}^{n-1} 2^i\, x^{[108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^n + 42]^2} +$$

$$[\{108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^n + 42\}^2 +$$

$$3 \sum_{i=0}^{n-1} 2^i\, x^{\{108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^n + 42\}^2\,]} +$$

$$6 \sum_{i=0}^{n-1} 2^i\, x^{[108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^n + 42]^2}$$

Proof: $F_C(NS[n], x) = \sum_{uv \in E(NS[n])} x^{[c(u)^2 + c(v)^2]}$

$$= \sum_{i=0}^{n-1} (6 \times 2^i)\, x^{[\{\{((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 6)\} \times (6 \times 2^{n-i} - 7)\}^2 + 0^2\,]} +$$

$$[\{\{((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 6)\} \times (6 \times 2^{n-i} - 7)\}^2 +$$

$$\sum_{i=0}^{n-1} (3 \times 2^i)\, x^{\{\{((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 5)\} \times (6 \times 2^{n-i} - 6)\}^2\,]}$$

$$\sum_{i=0}^{n-1} (6 \times 2^i)\, x^{\{\{((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 5)\} \times (6 \times 2^{n-i} - 6)\}^2\,]}$$

$$= 6 \sum_{i=0}^{n-1} 2^i\, x^{[108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^n + 42]^2} +$$

$$[\{108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^n + 42\}^2 +$$

$$3 \sum_{i=0}^{n-1} 2^i\, x^{\{108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^n + 42\}^2\,]} +$$

$$6 \sum_{i=0}^{n-1} 2^i\, x^{[108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^n + 42]^2}$$

Theorem 2.5.5: Let NS[n] be a Nanostar Dendrimer. Then,

$$RF_C(NS[n], x) = 6\sum_{i=0}^{n-1} 2^{i}\ x^{[(108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^{n} + 41)^{2} + 1]}$$

$$3\sum_{i=0}^{n-1} 2^{i}\ x^{[\{108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^{n}+41\}^{2} + \{108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n}+41\}^{2}]} +$$

$$6\sum_{i=0}^{n-1} 2^{i}\ x^{[(108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n}+41)^{2} + 1]}$$

Proof: $RF_C(NS[n], x) = \sum_{uv\in E(NS[n])} x^{[(c(u)-1)^{2} + (c(v)-1)^{2}]}$

$$= \sum_{i=0}^{n-1}(6 \times 2^{i})\ x^{[\{\{\{((18\times 2^{n}) - 12) - (6\times 2^{n-i} - 6)\}\times(6\times 2^{n-i} - 7)\} - 1\}^{2} + \{0-1\}^{2}]} +$$

$$\sum_{i=0}^{n-1} (3 \times 2^{i})\ x^{[\{\{\{((18\times 2^{n}) - 12) - (6\times 2^{n-i} - 6)\}\times(6\times 2^{n-i} - 7)\} - 1\}^{2} + \{\{\{((18\times 2^{n}) - 12) - (6\times 2^{n-i} - 5)\}\times(6\times 2^{n-i} - 6)\} - 1\}^{2}]} +$$

$$\sum_{i=0}^{n-1} (6 \times 2^{i})\ x^{[\{\{\{((18\times 2^{n}) - 12) - (6\times 2^{n-i} - 5)\}\times(6\times 2^{n-i} - 6)\} - 1\}^{2} + \{0-1\}^{2}]}$$

$$= 6\sum_{i=0}^{n-1} 2^{i}\ x^{[108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n}+41)^{2} + 1]}$$

$$3\sum_{i=0}^{n-1} 2^{i}\ x^{[\{108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^{n}+41\}^{2} + \{108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n}+41\}^{2}]} +$$

$$6\sum_{i=0}^{n-1} 2^{i}\ x^{[(108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n}+41)^{2} + 1]}$$

Theorem 2.5.6: Let NS[n] be a Nanostar Dendrimer. Then,

$$RM_{2C}(NS[n], x) = 6\sum_{i=0}^{n-1} 2^{i}\ x^{[-(108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n}+41)]} +$$

$$3\sum_{i=0}^{n-1} 2^{i}\ x^{[(108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n}+41)(108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n}+41)]} +$$

$$6\sum_{i=0}^{n-1} 2^{i}\ x^{[-(108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n}+41)]}$$

Proof: $RM_{2C}(NS[n], x) = \sum_{uv\in E(NS[n])} x^{[(c(u)-1)(c(v)-1)]}$

$$= \sum_{i=0}^{n-1}(6 \times 2^{i})\, x^{[\{\{(((18\times 2^{n})-12) - (6\times 2^{n-i}-6))\times(6\times 2^{n-i}-7)\} - 1\}\{0-1\}]} +$$

$$\sum_{i=0}^{n-1}(3 \times 2^{i})\, x^{[\{\{(((18\times 2^{n})-12) - (6\times 2^{n-i}-6))\times(6\times 2^{n-i}-7)\} - 1\}\{\{(((18\times 2^{n})-12) - (6\times 2^{n-i}-5))\times(6\times 2^{n-i}-6)\} - 1\}]} +$$

$$\sum_{i=0}^{n-1}(6 \times 2^{i})\, x^{[\{\{[((18\times 2^{n})-12) - (6\times 2^{n-i}-5)]\times(6\times 2^{n-i}-6)\} - 1\}\{0-1\}]}$$

$$= 6\sum_{i=0}^{n-1} 2^{i}\ x^{[-(108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n}+41)]} +$$

$$3\sum_{i=0}^{n-1} 2^{i}\ x^{[(108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+41)(108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+41)]} +$$

$$6\sum_{i=0}^{n-1} 2^{i}\ x^{[-(108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+41)]}$$

Theorem 2.5.7: Let NS[n] be a Nanostar Dendrimer. Then,

$$RHM_{2C}(NS[n], x) = 6\sum_{i=0}^{n-1} 2^{i}\ x^{[108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+41]^{2}} +$$

$$3\sum_{i=0}^{n-1} 2^{i}\ x^{[(108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+41)(108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+41)]^{2}} +$$

$$6\sum_{i=0}^{n-1} 2^{i}\ x^{[108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+41]^{2}}$$

Proof: $RHM_{2C}(NS[n], x) = \sum_{uv\in E(NS[n])} x^{[(c(u)-1)(c(v)-1)]^{2}}$

$$= \sum_{i=0}^{n-1}(6\times 2^{i})x^{[\{\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-6))\times(6\times 2^{n-i}-7)\}-1\}\{0-1\}]^{2}} +$$

$$\sum_{i=0}^{n-1}(3\times 2^{i})x^{[\{\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-6))\times(6\times 2^{n-i}-7)\}-1\}\{\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-5))\times(6\times 2^{n-i}-6)\}-1\}]^{2}} +$$

$$\sum_{i=0}^{n-1}(6\times 2^{i})x^{[\{\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-5))\times(6\times 2^{n-i}-6)\}-1\}\{0-1\}]^{2}}$$

$$= 6\sum_{i=0}^{n-1} 2^{i}\ x^{[108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+41]^{2}} +$$

$$3\sum_{i=0}^{n-1} 2^{i}\ x^{[\{108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+41\}\{108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+41\}]^{2}} +$$

$$6\sum_{i=0}^{n-1} 2^{i}\ x^{[108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+41]^{2}}$$

2.6 Polynomial of General reduced forgotten, General reduced second Zagreb and General reduced hyper second Zagreb cutting number topological indices of Nanostar Dendrimer NS[n]

Theorem 2.6.1: Let NS[n] be a Nanostar Dendrimer. Then,

$$RF_{C}^{\,a}(NS[n], x) = 6\sum_{i=0}^{n-1} 2^{i}\ x^{[(108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+41)^{2}+1]^{a}}$$

$$3\sum_{i=0}^{n-1} 2^{i}\ x^{[\{108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+41\}^{2}+\{108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+41\}^{2}]^{a}} +$$

$$6\sum_{i=0}^{n-1} 2^{i}\ x^{[(108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+41)^{2}+1]^{a}}$$

Proof: $RF_C{}^a(NS[n], x) = \sum_{uv\in E(NS[n])} x^{[(c(u)-1)^2+(c(v)-1)^2]^a}$

$= \sum_{i=0}^{n-1} (6 \times 2^i)\, x^{[\{\{(((18\times2^n)-12)-(6\times2^{n-i}-6))\times(6\times2^{n-i}-7)\}-1\}^2+(0-1)^2]^a} +$

$[\{(((18\times2^n)-12)-(6\times2^{n-i}-6))\times(6\times2^{n-i}-7)\}-1]^2 +$

$\sum_{i=0}^{n-1} (3 \times 2^i)\, x^{[\{(((18\times2^n)-12)-(6\times2^{n-i}-5))\}^2 \times(6\times2^{n-i}-6)\}-1\}^2]^a} +$

$\sum_{i=0}^{n-1} (6 \times 2^i)\, x^{[\{\{(((18\times2^n)-12)-(6\times2^{n-i}-5))\times(6\times2^{n-i}-6)\}-1\}^2+(0-1)^2]^a}$

$= 6\sum_{i=0}^{n-1} 2^i\, x^{[(108\times2^{2n-i}-36\times2^{2n-2i}+6\times2^{n-i}-126\times2^n+41)^2+1]^a}$

$[\{108\times2^{2n-i}-36\times2^{2n-2i}+6\times2^{n-i}-126\times2^n+41\}^2 +$

$3\sum_{i=0}^{n-1} 2^i\, x^{\{108\times2^{2n-i}-36\times2^{2n-2i}-6\times2^{n-i}-108\times2^n+41\}^2]^a} +$

$6\sum_{i=0}^{n-1} 2^i\, x^{[(108\times2^{2n-i}-36\times2^{2n-2i}-6\times2^{n-i}-108\times2^n+41)^2+1]^a}$

Theorem 2.6.2: Let NS[n] be a Nanostar Dendrimer. Then,

$RM_{2C}{}^a(NS[n], x) = 6\sum_{i=0}^{n-1} 2^i\, x^{[108\times2^{2n-i}-36\times2^{2n-2i}+6\times2^{n-i}-126\times2^n+41]^a} +$

$[\{108\times2^{2n-i}-36\times2^{2n-2i}+6\times2^{n-i}-126\times2^n+41\}$

$3\sum_{i=0}^{n-1} 2^i\, x^{\{108\times2^{2n-i}-36\times2^{2n-2i}-6\times2^{n-i}-108\times2^n+41\}]^a} +$

$6\sum_{i=0}^{n-1} 2^i\, x^{[108\times2^{2n-i}-36\times2^{2n-2i}-6\times2^{n-i}-108\times2^n+41]^a}$

Proof: $RM_{2C}{}^a(NS[n], x) = \sum_{uv\in E(NS[n])} x^{[(c(u)-1)(c(v)-1)]^a}$

$= \sum_{i=0}^{n-1} (6 \times 2^i)\, x^{[\{\{\{((18\times2^n)-12)-(6\times2^{n-i}-6)\}\times(6\times2^{n-i}-7)\}-1\}\{0-1\}]^a} +$

$[\{\{\{((18\times2^n)-12)-(6\times2^{n-i}-6)\}\times(6\times2^{n-i}-7)\}-1\}$

$\sum_{i=0}^{n-1} (3 \times 2^i)\, x^{\{\{\{((18\times2^n)-12)-(6\times2^{n-i}-5)\}\times(6\times2^{n-i}-6)\}-1\}]^a} +$

$\sum_{i=0}^{n-1} (6 \times 2^i)\, x^{[\{\{\{((18\times2^n)-12)-(6\times2^{n-i}-5)\}\times(6\times2^{n-i}-6)\}-1\}\{0-1\}]^a}$

$= 6\sum_{i=0}^{n-1} 2^i\, x^{[108\times2^{2n-i}-36\times2^{2n-2i}+6\times2^{n-i}-126\times2^n+41]^a} +$

$[\{108\times2^{2n-i}-36\times2^{2n-2i}+6\times2^{n-i}-126\times2^n+41\}$

$3\sum_{i=0}^{n-1} 2^i\, x^{\{108\times2^{2n-i}-36\times2^{2n-2i}-6\times2^{n-i}-108\times2^n+41\}]^a} +$

$6\sum_{i=0}^{n-1} 2^i\, x^{[108\times2^{2n-i}-36\times2^{2n-2i}-6\times2^{n-i}-108\times2^n+41]^a}$

Theorem 2.6.3: Let NS[n] be a Nanostar Dendrimer. Then,

$$\mathrm{RHM}_{2C}{}^{a}(NS[n],x) = 6\sum_{i=0}^{n-1} 2^{i}\ x^{[108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n}+41]^{2a}} +$$

$$3\sum_{i=0}^{n-1} 2^{i}\ x^{[\{108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n}+41\}\{108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n}+41\}]^{2a}} +$$

$$6\sum_{i=0}^{n-1} 2^{i}\ x^{[108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n}+41]^{2a}}$$

Proof: $\mathrm{RHM}_{2C}{}^{a}(NS[n], x) = \sum_{uv\in E(NS[n])} x^{[(c(u)-1)(c(v)-1)]^{2a}}$

$$= \sum_{i=0}^{n-1} (6\times 2^{i}) x^{[\{\{\{((18\times 2^{n})-12)-(6\times 2^{n-i}-6)\}\times(6\times 2^{n-i}-7)\}-1\}\{0-1\}]^{2a}} +$$

$$\sum_{i=0}^{n-1} (3\times 2^{i})\ x^{[\{\{\{((18\times 2^{n})-12)-(6\times 2^{n-i}-6)\}\times(6\times 2^{n-i}-7)\}-1\}\{\{\{((18\times 2^{n})-12)-(6\times 2^{n-i}-5)\}\times(6\times 2^{n-i}-6)\}-1\}]^{2a}} +$$

$$\sum_{i=0}^{n-1} (6\times 2^{i})\ x^{[\{\{\{((18\times 2^{n})-12)-(6\times 2^{n-i}-5)\}\times(6\times 2^{n-i}-6)\}-1\}\{0-1\}]^{2a}}$$

$$= 6\sum_{i=0}^{n-1} 2^{i}\,[108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n} + 41]^{2a} +$$

$$3\sum_{i=0}^{n-1} 2^{i}\,[\{108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n} + 41\}$$

$$\{108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n} + 41\}]^{2a} +$$

$$6\sum_{i=0}^{n-1} 2^{i}\,[108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n} + 41]^{2a}$$

2.7 Multiplicative of Arithmetic, Harmonic, Geometric Arithmetic, Arithmetic Geometric, sum and product connectivity, ABC, Inverse sum and Augumented Zagreb cutting number topological indices of Nanostar Dendrimer NS[n]

In this section, we compute multiplicative of Arithmetic, Harmonic, Geometric Arithmetic, Arithmetic Geometic, sum and product connectivity, ABC, Inverse sum and Augumented Zagreb cutting number topological indices of Nanostar Dendrimer NS[n] in the following theorems.

Theorem 2.7.1: Let NS[n] be a Nanostar Dendrimer. Then,

$$A_C\prod[(NS[n]) = 108\{(54n\times 2^{2n} - 63\times 2^{2n} - 18\times 2^{2n+1} + 18\times 2^{n+1} + 3n\times 2^{n} + 84\times 2^{n} - 21)$$

$$(108n\times 2^{2n} - 117\times 2^{2n} - 36\times 2^{2n+1} + 36\times 2^{n+1} + 159\times 2^{n} - 42)$$

$$(54n\times 2^{2n} - 54\times 2^{2n} - 18\times 2^{2n+1} + 18\times 2^{n+1} - 3n\times 2^{n} + 75\times 2^{n} - 21)\}$$

Proof: $A_C\prod(NS[n]) = \prod_{e=uv\in E(NS[n])} \left(\frac{c(u)+c(v)}{2}\right)$

$$= \prod_{i=0}^{n-1} (6\times 2^{i}) \left[\frac{\{(((18\times2^{n})-12)-(6\times2^{n-i}-6))\times(6\times2^{n-i}-7)\}+0}{2}\right]\times$$

$$\prod_{i=0}^{n-1} (3\times 2^{i}) \left[\frac{\{(((18\times2^{n})-12)-(6\times2^{n-i}-6))\times(6\times2^{n-i}-7)\}+\{(((18\times2^{n})-12)-(6\times2^{n-i}-5))\times(6\times2^{n-i}-6)\}}{2}\right]\times$$

$$\prod_{i=0}^{n-1} (6\times 2^{i}) \left[\frac{\{(((18\times2^{n})-12)-(6\times2^{n-i}-5))\times(6\times2^{n-i}-6)\}+0}{2}\right]$$

$$= 108\{(54n\times2^{2n}-63\times2^{2n}-18\times2^{2n+1}+18\times2^{n+1}+3n\times2^{n}+84\times2^{n}-21)$$
$$(108n\times2^{2n}-117\times2^{2n}-36\times2^{2n+1}+36\times2^{n+1}+159\times2^{n}-42)$$
$$(54n\times2^{2n}-54\times2^{2n}-18\times2^{2n+1}+18\times2^{n+1}-3n\times2^{n}+75\times2^{n}-21)\}$$

Theorem 2.7.2: Let NS[n] be a Nanostar Dendrimer. Then,

$$H_C\prod(NS[n]) = 108\prod_{i=0}^{n-1} 2^{3i}\left\{\sqrt{\frac{2}{108\times2^{2n-i}-36\times2^{2n-2i}+6\times2^{n-i}-126\times2^{n}+42}}\times\sqrt{\frac{2}{216\times2^{2n-i}-72\times2^{2n-2i}-234\times2^{n}+84}}\times\sqrt{\frac{2}{108\times2^{2n-i}-36\times2^{2n-2i}-6\times2^{n-i}-108\times2^{n}+42}}\right\}$$

Proof: $H_C\prod(NS[n]) = \prod_{e=uv\in E(NS[n])} \sqrt{\frac{2}{c(u)+c(v)}}$

$$= \prod_{i=0}^{n-1} (6\times 2^{i}) \sqrt{\frac{2}{\{(((18\times2^{n})-12)-(6\times2^{n-i}-6))\times(6\times2^{n-i}-7)\}+0}}\times$$

$$\prod_{i=0}^{n-1} (3\times 2^{i}) \sqrt{\frac{2}{\{(((18\times2^{n})-12)-(6\times2^{n-i}-6))\times(6\times2^{n-i}-7)\}+\{(((18\times2^{n})-12)-(6\times2^{n-i}-5))\times(6\times2^{n-i}-6)\}}}\times$$

$$\prod_{i=0}^{n-1} (6\times 2^{i}) \sqrt{\frac{2}{\{(((18\times2^{n})-12)-(6\times2^{n-i}-5))\times(6\times2^{n-i}-6)\}+0}}$$

$$= 108\prod_{i=0}^{n-1} 2^{3i}\left\{\sqrt{\frac{2}{108\times2^{2n-i}-36\times2^{2n-2i}+6\times2^{n-i}-126\times2^{n}+42}}\times\sqrt{\frac{2}{216\times2^{2n-i}-72\times2^{2n-2i}-234\times2^{n}+84}}\times\sqrt{\frac{2}{108\times2^{2n-i}-36\times2^{2n-2i}-6\times2^{n-i}-108\times2^{n}+42}}\right\}$$

Theorem 2.7.3: Let NS[n] be a Nanostar Dendrimer. Then,

$$GA_C\prod(NS[n]) = 6\prod_{i=0}^{n-1} 2^{\,i}\left[\frac{\sqrt{\begin{matrix}11664\times 2^{4n-2i}-7776\times 2^{4n-3i}+8424\times 2^{3n-2i}-\\ 25272\times 2^{3n-i}+9180\times 2^{2n-i}+1296\times 2^{4n-4i}-\\ 3060\times 2^{2n-2i}+13608\times 2^{2n}-9828\times 2^{n}+1764\end{matrix}}}{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^{n}+84}\right]$$

Proof: $GA_C\prod(NS[n]) = \prod_{e=uv\in E(NS[n])}\left[\frac{2\sqrt{c(u)c(v)}}{c(u)+c(v)}\right]$

$$= \prod_{i=0}^{n-1}(3\times 2^{\,i})\left[\frac{2\sqrt{\begin{matrix}\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-6))\times(6\times 2^{n-i}-7)\}\times\\ \{(((18\times 2^{n})-12)-(6\times 2^{n-i}-5))\times(6\times 2^{n-i}-6)\}\end{matrix}}}{\begin{matrix}\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-6))\times(6\times 2^{n-i}-7)\}+\\ \{(((18\times 2^{n})-12)-(6\times 2^{n-i}-5))\times(6\times 2^{n-i}-6)\}\end{matrix}}\right]$$

$$= 6\prod_{i=0}^{n-1} 2^{\,i}\left[\frac{\sqrt{\begin{matrix}11664\times 2^{4n-2i}-7776\times 2^{4n-3i}+8424\times 2^{3n-2i}-\\ 25272\times 2^{3n-i}+9180\times 2^{2n-i}+1296\times 2^{4n-4i}-\\ 3060\times 2^{2n-2i}+13608\times 2^{2n}-9828\times 2^{n}+1764\end{matrix}}}{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^{n}+84}\right]$$

Theorem 2.7.4: Let NS[n] be a Nanostar Dendrimer. Then,

$$AG_C\prod(NS[n]) = \left[\frac{3}{2}\right]\prod_{i=0}^{n-1} 2^{\,i}\left[\frac{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^{n}+84}{\sqrt{\begin{matrix}11664\times 2^{4n-2i}-7776\times 2^{4n-3i}+8424\times 2^{3n-2i}-\\ 25272\times 2^{3n-i}+9180\times 2^{2n-i}+1296\times 2^{4n-4i}-\\ 3060\times 2^{2n-2i}+13608\times 2^{2n}-9828\times 2^{n}+1764\end{matrix}}}\right]$$

Proof: $AG_C\prod(NS[n]) = \prod_{e=uv\in E(NS[n])}\left(\frac{c(u)+c(v)}{2\sqrt{c(u)c(v)}}\right)$

$$= 3\prod_{i=0}^{n-1} 2^{\,i}\left[\frac{\begin{matrix}\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-6))\times(6\times 2^{n-i}-7)\}+\\ \{(((18\times 2^{n})-12)-(6\times 2^{n-i}-5))\times(6\times 2^{n-i}-6)\}\end{matrix}}{2\sqrt{\begin{matrix}\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-6))\times(6\times 2^{n-i}-7)\}\times\\ \{(((18\times 2^{n})-12)-(6\times 2^{n-i}-5))\times(6\times 2^{n-i}-6)\}\end{matrix}}}\right]$$

$$= \left[\frac{3}{2}\right]\prod_{i=0}^{n-1} 2^{\,i}\left[\frac{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^{n}+84}{\sqrt{\begin{matrix}11664\times 2^{4n-2i}-7776\times 2^{4n-3i}+8424\times 2^{3n-2i}-\\ 25272\times 2^{3n-i}+9180\times 2^{2n-i}+1296\times 2^{4n-4i}-\\ 3060\times 2^{2n-2i}+13608\times 2^{2n}-9828\times 2^{n}+1764\end{matrix}}}\right]$$

Theorem 2.7.5: Let NS[n] be a Nanostar Dendrimer. Then,

$$S_C\prod(NS[n]) = 108\prod_{i=0}^{n-1} 2^{3i}\left\{\left[\frac{1}{\sqrt{108\times2^{2n-i}-36\times2^{2n-2i}+6\times2^{n-i}-126\times2^{n}+42}}\right]\left[\frac{1}{\sqrt{216\times2^{2n-i}-72\times2^{2n-2i}-234\times2^{n}+84}}\right]\left[\frac{1}{\sqrt{108\times2^{2n-i}-36\times2^{2n-2i}-6\times2^{n-i}-108\times2^{n}+42}}\right]\right\}$$

Proof: $S_C\prod(NS[n]) = \prod_{e=uv\in E(NS[n])}\left[\frac{1}{\sqrt{c(u)+c(v)}}\right]$

$$= \prod_{i=0}^{n-1}(6\times2^{i})\left[\frac{1}{\sqrt{\{(((18\times2^{n})-12)-(6\times2^{n-i}-6))\times(6\times2^{n-i}-7)\}+0}}\right]\times$$

$$\prod_{i=0}^{n-1}(3\times2^{i})\left[\frac{1}{\sqrt{\{(((18\times2^{n})-12)-(6\times2^{n-i}-6))\times(6\times2^{n-i}-7)\}+\{(((18\times2^{n})-12)-(6\times2^{n-i}-5))\times(6\times2^{n-i}-6)\}}}\right]\times$$

$$\prod_{i=0}^{n-1}(6\times2^{i})\left[\frac{1}{\sqrt{\{(((18\times2^{n})-12)-(6\times2^{n-i}-5))\times(6\times2^{n-i}-6)\}+0}}\right]$$

$$S_C\prod(NS[n]) = 108\prod_{i=0}^{n-1} 2^{3i}\left\{\left[\frac{1}{\sqrt{108\times2^{2n-i}-36\times2^{2n-2i}+6\times2^{n-i}-126\times2^{n}+42}}\right]\left[\frac{1}{\sqrt{216\times2^{2n-i}-72\times2^{2n-2i}-234\times2^{n}+84}}\right]\left[\frac{1}{\sqrt{108\times2^{2n-i}-36\times2^{2n-2i}-6\times2^{n-i}-108\times2^{n}+42}}\right]\right\}$$

Theorem 2.7.6: Let NS[n] be a Nanostar Dendrimer. Then,

$$P_C\prod(NS[n]) = 3\prod_{i=0}^{n-1}2^{i}\left[\frac{1}{\sqrt{\begin{array}{c}11664\times2^{4n-2i}-7776\times2^{4n-3i}+8424\times2^{3n-2i}-\\25272\times2^{3n-i}+9180\times2^{2n-i}+1296\times2^{4n-4i}-\\3060\times2^{2n-2i}+13608\times2^{2n}-9828\times2^{n}+1764\end{array}}}\right]$$

Proof: $P_C\prod(NS[n]) = \prod_{e=uv\in E(NS[n])}\left[\frac{1}{\sqrt{c(u)c(v)}}\right]$

$$= \prod_{i=0}^{n-1}(3\times2^{i})\left[\frac{1}{\sqrt{\{(((18\times2^{n})-12)-(6\times2^{n-i}-6))\times(6\times2^{n-i}-7)\}\times\{(((18\times2^{n})-12)-(6\times2^{n-i}-5))\times(6\times2^{n-i}-6)\}}}\right]$$

$$= 3\prod_{i=0}^{n-1} 2^i \left[\frac{1}{\sqrt{\begin{array}{c}11664\times2^{4n-2i} - 7776\times2^{4n-3i}+8424\times2^{3n-2i} - \\ 25272\times2^{3n-i}+9180\times2^{2n-i}+1296\times2^{4n-4i} - \\ 3060\times2^{2n-2i}+13608\times2^{2n} - 9828\times2^{n}+1764\end{array}}}\right]$$

Theorem 2.7.7: Let NS[n] be a Nanostar Dendrimer. Then,

$$ABC_C\prod(NS[n]) = 108\prod_{i=0}^{n-1} 2^{3i}\left\{\sqrt{\frac{108\times2^{2n-i}-36\times2^{2n-2i}+6\times2^{n-i}-126\times2^{n}+40}{108\times2^{2n-i}-36\times2^{2n-2i}+6\times2^{n-i}-126\times2^{n}+42}}\right.$$
$$\sqrt{\frac{216\times2^{2n-i}-72\times2^{2n-2i}-234\times2^{n}+82}{216\times2^{2n-i}-72\times2^{2n-2i}-234\times2^{n}+84}}$$
$$\left.\sqrt{\frac{108\times2^{2n-i}-36\times2^{2n-2i}-6\times2^{n-i}-108\times2^{n}+40}{108\times2^{2n-i}-36\times2^{2n-2i}-6\times2^{n-i}-108\times2^{n}+42}}\right\}$$

Proof: $ABC_C\prod NS[n]) = \prod_{e=uv\in E(NS[n])}\sqrt{\frac{c(u)+c(v)-2}{c(u)+c(v)}}$

$$= \prod_{i=0}^{n-1} (6\times 2^{i})\sqrt{\frac{\{(((18\times2^{n})-12)-(6\times2^{n-i}-6))\times(6\times2^{n-i}-7)\}+0-2}{\{(((18\times2^{n})-12)-(6\times2^{n-i}-6))\times(6\times2^{n-i}-7)\}+0}}\times$$

$$\prod_{i=0}^{n-1} (3\times 2^{i})\sqrt{\frac{\begin{array}{c}\{(((18\times2^{n})-12)-(6\times2^{n-i}-6))\times(6\times2^{n-i}-7)\}+\\ \{(((18\times2^{n})-12)-(6\times2^{n-i}-5))\times(6\times2^{n-i}-6)\}-2\end{array}}{\begin{array}{c}\{(((18\times2^{n})-12)-(6\times2^{n-i}-6))\times(6\times2^{n-i}-7)\}+\\ \{(((18\times2^{n})-12)-(6\times2^{n-i}-5))\times(6\times2^{n-i}-6)\}\end{array}}}\times$$

$$\prod_{i=0}^{n-1} (6\times 2^{i})\sqrt{\frac{\{(((18\times2^{n})-12)-(6\times2^{n-i}-5))\times(6\times2^{n-i}-6)\}+0-2}{\{(((18\times2^{n})-12)-(6\times2^{n-i}-5))\times(6\times2^{n-i}-6)\}+0}}\times$$

$$= 108\prod_{i=0}^{n-1} 2^{3i}\left\{\sqrt{\frac{108\times2^{2n-i}-36\times2^{2n-2i}+6\times2^{n-i}-126\times2^{n}+40}{108\times2^{2n-i}-36\times2^{2n-2i}+6\times2^{n-i}-126\times2^{n}+42}}\right.$$
$$\sqrt{\frac{216\times2^{2n-i}-72\times2^{2n-2i}-234\times2^{n}+82}{216\times2^{2n-i}-72\times2^{2n-2i}-234\times2^{n}+84}}$$
$$\left.\sqrt{\frac{108\times2^{2n-i}-36\times2^{2n-2i}-6\times2^{n-i}-108\times2^{n}+40}{108\times2^{2n-i}-36\times2^{2n-2i}-6\times2^{n-i}-108\times2^{n}+42}}\right\}$$

Theorem 2.7.8: Let NS[n] be a Nanostar Dendrimer. Then,

$$IS_C\prod(NS[n]) = 3\prod_{i=0}^{n-1} 2^{i}\left[\frac{\begin{array}{c}11664\times2^{4n-2i}-7776\times2^{4n-3i}+8424\times2^{3n-2i}-\\ 25272\times2^{3n-i}+9180\times2^{2n-i}+1296\times2^{4n-4i}-\\ 3060\times2^{2n-2i}+13608\times2^{2n}-9828\times2^{n}+1764\end{array}}{216\times2^{2n-i}-72\times2^{2n-2i}-234\times2^{n}+84}\right]$$

Proof: $IS_C\prod$ (NS[n]) $= \prod_{e=uv\in E(NS[n])}\left[\frac{c(u)c(v)}{c(u)+c(v)}\right]$

$$= \prod_{i=0}^{n-1}(3\times 2^{i})\left[\frac{\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-6))\times(6\times 2^{n-i}-7)\}\times\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-5))\times(6\times 2^{n-i}-6)\}}{\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-6))\times(6\times 2^{n-i}-7)\}+\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-5))\times(6\times 2^{n-i}-6)\}}\right]$$

$$= 3\prod_{i=0}^{n-1}2^{i}\left[\frac{11664\times 2^{4n-2i}-7776\times 2^{4n-3i}+8424\times 2^{3n-2i}-25272\times 2^{3n-i}+9180\times 2^{2n-i}+1296\times 2^{4n-4i}-3060\times 2^{2n-2i}+13608\times 2^{2n}-9828\times 2^{n}+1764}{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^{n}+84}\right]$$

Theorem 2.7.9: Let NS[n] be a Nanostar Dendrimer. Then,

$$AZc\prod(NS[n]) = 3\prod_{i=0}^{n-1}2^{i}\left[\frac{11664\times 2^{4n-2i}-7776\times 2^{4n-3i}+8424\times 2^{3n-2i}-25272\times 2^{3n-i}+9180\times 2^{2n-i}+1296\times 2^{4n-4i}-3060\times 2^{2n-2i}+13608\times 2^{2n}-9828\times 2^{n}+1764}{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^{n}+82}\right]^{3}$$

Proof: $AZc\prod$ (NS[n]) $= \prod_{e=uv\in E(NS[n])}\left[\frac{c(u)c(v)}{c(u)+c(v)-2}\right]^{3}$

$$= \prod_{i=0}^{n-1}(3\times 2^{i})\left[\frac{\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-6))\times(6\times 2^{n-i}-7)\}\times\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-5))\times(6\times 2^{n-i}-6)\}}{\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-6))\times(6\times 2^{n-i}-7)\}+\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-5))\times(6\times 2^{n-i}-6)\}-2}\right]^{3}$$

$$= 3\prod_{i=0}^{n-1}2^{i}\left[\frac{11664\times 2^{4n-2i}-7776\times 2^{4n-3i}+8424\times 2^{3n-2i}-25272\times 2^{3n-i}+9180\times 2^{2n-i}+1296\times 2^{4n-4i}-3060\times 2^{2n-2i}+13608\times 2^{2n}-9828\times 2^{n}+1764}{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^{n}+82}\right]^{3}$$

2.8 Multiplicative of hyper first, second and third Zagreb indices, F, Reduced forgotten, Reduced second Zagreb and Reduced hyper second Zagreb cutting number topological indices of Nanostar Dendrimer NS[n]

In this section, we calculate Multiplicative of hyper first, second and third Zagreb indices, F, Reduced forgotten, Reduced second Zagreb and Reduced hyper second Zagreb cutting number topological indices of Nanostar Dendrimer NS[n] in the following theorems.

Theorem 2.8.1: Let NS[n] be the Nanostar Dendrimer. Then,

$HM_{1C}\prod(NS[n]) =$

$$108\prod_{i=0}^{n-1} 2^{3i}\begin{bmatrix}\left(108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+42\right)^{2}\\ \left(216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^{n}+84\right)^{2}\\ (108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+42)^{2}\end{bmatrix}$$

Proof: $HM_{1C}\prod(NS[n]) = \prod_{e=uv\in E(NS[n])}[c(u)+c(v)]^{2}$

$$= \prod_{i=0}^{n-1}(6\times 2^{i})\,[\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-6))\times (6\times 2^{n-i}-7)\}+0]^{2}\times$$

$$\prod_{i=0}^{n-1}(3\times 2^{i})\,[\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-6))\times (6\times 2^{n-i}-7)\}+\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-5))\times(6\times 2^{n-i}-6)\}]^{2}\times$$

$$\prod_{i=0}^{n-1}(6\times 2^{i})\,[\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-5))\times (6\times 2^{n-i}-6)\}+0]^{2}$$

$$=108\prod_{i=1}^{n} 2^{3i}\begin{bmatrix}\left(108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+42\right)^{2}\\ \left(12\times 2^{n-i}-18\times 2^{n}\right)^{2}\\ (108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+42)^{2}\end{bmatrix}$$

Theorem 2.8.2: Let NS[n] be the Nanostar Dendrimer. Then,

$$HM_{2C}\prod(NS[n])=3\prod_{i=0}^{n-1} 2^{i}\,[11664\times 2^{4n-2i}-7776\times 2^{4n-3i}+8424\times 2^{3n-2i}-25272\times 2^{3n-i}+9180\times 2^{2n-i}+1296\times 2^{4n-4i}-3060\times 2^{2n-2i}+13608\times 2^{2n}-9828\times 2^{n}+1764]^{2}$$

Proof: $HM_{2C}\prod(NS[n]) = \prod_{e=uv\in E(NS[n])}[c(u)\,c(v)]^{2}$

$$= \prod_{i=0}^{n-1}(3\times 2^{i})[\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-6))\times (6\times 2^{n-i}-7)\}+\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-5))\times(6\times 2^{n-i}-6)\}]^{2}$$

$$= 3\prod_{i=0}^{n-1} 2^{i}\,[11664\times 2^{4n-2i}-7776\times 2^{4n-3i}+8424\times 2^{3n-2i}-25272\times 2^{3n-i}+9180\times 2^{2n-i}+1296\times 2^{4n-4i}-3060\times 2^{2n-2i}+13608\times 2^{2n}-9828\times 2^{n}+1764]^{2}.$$

Theorem 2.8.3: Let NS[n] be the Nanostar Dendrimer. Then,

$HM_{3C}\prod(NS[n])=$

$$108\prod_{i=0}^{n-1}2^{3i}\begin{bmatrix}\left(108\times 2^{2n-I}-36\times 2^{2n-2i}+6\times 2^{n-I}-126\times 2^{n}+42\right)^{2}\\ \left(216\times 2^{2n-I}-72\times 2^{2n-2i}-234\times 2^{n}+84\right)^{2}\\ \left(108\times 2^{2n-I}-36\times 2^{2n-2i}-6\times 2^{n-I}-108\times 2^{n}+42\right)^{2}\end{bmatrix}$$

Proof: $HM_{3C}\prod(NS[n]) = \prod_{e=uv\in E(NS[n])}|c(u)-c(v)|^{2}$

$$=\prod_{i=0}^{n-1}(6\times 2^{i})[\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-6))\times(6\times 2^{n-i}-7)\}-0]^{2}\times$$
$$\prod_{i=0}^{n-1}(3\times 2^{i})[(((18\times 2^{n})-12)-(6\times 2^{n-i}-6))\times(6\times 2^{n-i}-7)\}-$$
$$\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-5))\times(6\times 2^{n-i}-6)\}]^{2}\times$$
$$\prod_{i=0}^{n-1}(6\times 2^{i-1})[\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-5))\times(6\times 2^{n-i}-6)\}-0]^{2}$$
$$=108\prod_{i=0}^{n-1}2^{3i}\{(108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+42)^{2}$$
$$(12\times 2^{n-i}-18\times 2^{n})^{2}$$
$$(108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+42)^{2}\}$$

Theorem 2.8.4: Let NS[n] be a Nanostar Dendrimer. Then,

$$F_{C}\prod(NS[n])=108\prod_{i=0}^{n-1}2^{3i}\{(108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+42]^{2}$$
$$[\{108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+42\}^{2}+$$
$$\{108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+42\}^{2}]$$
$$[\{108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+42\}^{2}]\}$$

Proof: $F_{C}\prod[NS(n)]=\prod_{e=uv\in E[NS(n)]}[c(u)^{2}+c(v)^{2}]$

$$=\prod_{i=0}^{n-1}(6\times 2^{i})[\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-6))\times(6\times 2^{n-i}-7)\}^{2}+0^{2}]\times$$
$$\prod_{i=0}^{n-1}(3\times 2^{i})[\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-6))\times(6\times 2^{n-i}-7)\}^{2}+$$
$$\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-5))\times(6\times 2^{n-i}-6)\}^{2}]\times$$
$$\prod_{i=0}^{n-1}(6\times 2^{i})[\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-5))\times(6\times 2^{n-i}-6)\}^{2}+0^{2}]$$
$$=108\prod_{i=0}^{n-1}2^{3i}[(108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+42)^{2}$$
$$\{(108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+42)^{2}+$$
$$(108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+42)^{2}\}$$
$$(108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times\times 2^{n}+42)^{2}]$$

Theorem 2.8.5: Let NS[n] be a Nanostar Dendrimer. Then,

$$RF_C\prod(NS[n]) = 108\prod_{i=0}^{n-1} 2^{3i}\ [\{(108\times 2^{2n-i} – 36\times 2^{2n-2i} + 6\times 2^{n-i} – 126\times 2^{n} + 41)^2 + 1\}$$
$$\{\{108\times 2^{2n-i} – 36\times 2^{2n-2i} + 6\times 2^{n-i} – 126\times 2^{n} + 41\}^2 +$$
$$\{108\times 2^{2n-i} – 36\times 2^{2n-2i} – 6\times 2^{n-i} – 108\times 2^{n} + 41\}^2\}$$
$$\{(108\times 2^{2n-i} – 36\times 2^{2n-2i} – 6\times 2^{n-i} – 108\times 2^{n} + 41)^2 + 1\}\}$$

Proof: $RF_C\prod(NS[n]) = \prod_{e=uv\in E(NS[n])}[(c(u) – 1)^2 + (c(v) – 1)^2]$

$$= 6\prod_{i=0}^{n-1} 2^{i}\ [\{\{(((18\times 2^{n}) – 12) – (6\times 2^{n-i} – 6))\times(6\times 2^{n-i} – 7)\} – 1\}^2 + \{0 – 1\}^2]\times$$
$$3\prod_{i=0}^{n-1} 2^{i}\ [\{\{(((18\times 2^{n}) – 12) – (6\times 2^{n-i} – 6))\times(6\times 2^{n-i} – 7)\} – 1\}^2 +$$
$$\{\{(((18\times 2^{n}) – 12) – (6\times 2^{n-i} – 5))\times(6\times 2^{n-i} – 6)\} – 1\}^2\ \times$$
$$6\prod_{i=0}^{n-1} 2^{i}\ [\{\{(((18\times 2^{n}) – 12) – (6\times 2^{n-i} – 5) – 1\}^2 + \{0 – 1\}^2]$$
$$= 108\prod_{i=0}^{n-1} 2^{3i}\ [\{(108\times 2^{2n-i} – 36\times 2^{2n-2i} + 6\times 2^{n-i} – 126\times 2^{n} + 41)^2 + 1\}$$
$$\{(108\times 2^{2n-i} – 36\times 2^{2n-2i} + 6\times 2^{n-i} – 126\times 2^{n} + 41)^2 +$$
$$(108\times 2^{2n-i} – 36\times 2^{2n-2i} – 6\times 2^{n-i} – 108\times 2^{n} + 41)^2\}$$
$$\{(108\times 2^{2n-i} – 36\times 2^{2n-2i} – 6\times 2^{n-i} – 108\times 2^{n} + 41)^2 + 1\}]$$

Theorem 2.8.6: Let NS[n] be a Nanostar Dendrimer. Then,

$$RM_{2c}\prod(NS[n]) = 108\prod_{i=0}^{n-1} 2^{3i}\ [\{108\times 2^{2n-i} – 36\times 2^{2n-2i} + 6\times 2^{n-i} – 126\times 2^{n} + 41\}$$
$$\{108\times 2^{2n-i} – 36\times 2^{2n-2i} + 6\times 2^{n-i} – 126\times 2^{n} + 41\}$$
$$\{108\times 2^{2n-i} – 36\times 2^{2n-2i} – 6\times 2^{n-i} – 108\times 2^{n} + 41\}$$
$$\{108\times 2^{2n-i} – 36\times 2^{2n-2i} – 6\times 2^{n-i} – 108\times 2^{n} + 41\}]$$

Proof: $RM_{2C}\prod(NS[(n]) = \prod_{e=uv\in E(NS[n])} [(c(u)-1)\,(c(v)-1)]$

$$= \prod_{i=0}^{n-1} (6\times 2^{i})\,[\{\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-6))\times (6\times 2^{n-i}-7)\}-1\}\{0-1\}]\times$$

$$\prod_{i=0}^{n-1} (3\times 2^{i})\,[\{\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-6))\times (6\times 2^{n-i}-7)\}-1\}$$
$$\{\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-5))\times (6\times 2^{n-i}-6)\}-1\}]\times$$

$$\prod_{i=0}^{n-1} (6\times 2^{i})\,[\{\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-5))\times (6\times 2^{n-i}-6)\}-1\}\{0-1\}]$$

$$= 108\prod_{i=0}^{n-1} 2^{3i}\,[\{108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+41\}$$
$$\{108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+41\}$$
$$\{108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+41\}$$
$$\{108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+41\}]$$

Theorem 2.8.7: Let NS[n] be a Nanostar Dendrimer. Then,

$$RHM_{2C}\prod(NS[n]) = 108\prod_{i=0}^{n-1} 2^{3i}\,[\{108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+41\}^{2}$$
$$\{(108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+41)$$
$$(108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+41)\}^{2}$$
$$\{108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+41\}^{2}]$$

Proof: $RHM_{2C}\prod(NS[n]) = \prod_{e=uv\in E(NS[n])} [(c(u)-1)\,(c(v)-1)]^{2}$

$$= \prod_{i=0}^{n-1} (6\times 2^{i})\,[\{\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-6))\times (6\times 2^{n-i}-7)\}-1\}\{0-1\}]^{2}\times$$

$$\prod_{i=0}^{n-1} (3\times 2^{i})\,[\{\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-6))\times (6\times 2^{n-i}-7)\}-1\}$$
$$\{\{(((18\times 2^{n})-12)-(6\times 2^{n-i}-5))\times (6\times 2^{n-i}-6)\}-1\}]^{2}\times$$

$$\prod_{i=0}^{n-1} (6\times 2^{i})\,[\{\{[((18\times 2^{n})-12)-(6\times 2^{n-i}-5)]\times (6\times 2^{n-i}-6)\}-1\}]\{0-1\}]^{2}$$

$$=108 \prod_{i=0}^{n-1} 2^{3i} \ [\{108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^{n} + 41\}^{2}$$

$$\{(108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^{n} + 41)$$

$$(108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^{n} + 41)\}^{2}$$

$$\{108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^{n} + 41\}^{2}]$$

2.9 Multiplicative of General reduced forgotten, General reduced second Zagreb, General Reduced hyper second Zagreb cutting number topological indices of Nanostar Dendrimer NS[n]

Now, we compute multiplicative of General reduced forgotten, General Reduced second Zagreb and General Reduced hyper second Zagreb cutting number topological indices for NS[n] in the following theorems.

Theorem 2.9.1: Let NS[n] be a Nanostar Dendrimer. Then,

$$RF_C{}^{a} \prod(NS[n]) = 108 \prod_{i=0}^{n-1} 2^{3i} [(108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^{n} + 41)^{2} + 1]^{a}$$

$$[\{108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^{n} + 41\}^{2}$$

$$\{108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^{n} + 41\}^{2}]^{a}$$

$$[(108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^{n} + 41)^{2} + 1]^{a}$$

Proof: $RF_C{}^{a} \prod(NS[n]) = \prod_{e=uv \in E(NS[n])} [(c(u) - 1)^{2} + (c(v) - 1)^{2}]^{a}$

$$= \prod_{i=0}^{n-1}(6 \times 2^{i}) [\{\{(((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 6)) \times (6 \times 2^{n-i} - 7)\} - 1\}^{2} + (0 - 1)^{2}]^{a} +$$

$$\prod_{i=0}^{n-1}(3 \times 2^{i}) [\{(((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 6)) \times (6 \times 2^{n-i} - 7)\} - 1]^{2} +$$

$$[\{(((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 5)) \times (6 \times 2^{n-i} - 6)\} - 1\}^{2}]^{a} +$$

$$\prod_{i=0}^{n-1}(6 \times 2^{i}) [\{\{(((18 \times 2^{n}) - 12) - (6 \times 2^{n-i} - 5)) \times (6 \times 2^{n-i} - 6)\} - 1\}^{2} + (0 - 1)^{2}]^{a}$$

$$= 108 \prod_{i=0}^{n-1} 2^{3i} [(108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^{n} + 41)^{2} + 1]^{a}$$

$$[\{108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^{n} + 41\}^{2}$$

$$\{108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^n + 41\}^2]^a$$

$$[(108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^n + 41)^2 + 1]^a$$

Theorem 2.9.2: Let NS[n] be a Nanostar Dendrimer. Then,

$RM_{2C}{}^a \prod (NS[n]) =$

$$108 \prod_{i=0}^{n-1} 2^{3i} [\{108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^n + 41\}^a$$
$$\{(108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^n + 41)$$
$$(108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^n + 41)\}^a$$
$$\{108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^n + 41\}^a]$$

Proof: $RM_{2c}\prod^a (NS[n]) = \prod_{e=uv\in E(NS[n])}[(c(u) - 1)\ (c(v) - 1)]^a$

$$= \prod_{i=0}^{n-1}(6 \times 2^i)[\{\{(((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 6)) \times$$
$$(6\times 2^{n-i} - 7)\} - 1\}\{0 - 1\}]^a \times$$
$$\prod_{i=0}^{n-1} (3 \times 2^i) [\{\{(((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 6)) \times$$
$$(6 \times 2^{n-i} - 7)\} - 1\}$$
$$\{\{(((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 5)) \times$$
$$(6 \times 2^{n-i} - 6)\} - 1\}]^a \times$$
$$\prod_{i=0}^{n-1} (6 \times 2^i) [\{\{(((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 5)) \times$$
$$(6 \times 2^{n-i} - 6)\} - 1\}\{0 - 1\}]^a$$
$$= 108 \prod_{i=0}^{n-1} 2^{3i} [\{108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^n + 41\}^a$$
$$\{(108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^n + 41)$$
$$(108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^n + 41)\}^a$$
$$\{108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^n + 41\}^a]$$

Theorem 2.9.3: Let NS[n] be a Nanostar Dendrimer. Then,

$RHM_{2C}{}^a \prod(NS[n]) =$

$$108 \prod_{i=0}^{n-1} 2^i [108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^n + 41]^{2a}$$
$$[\{108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126\times 2^n + 41\}$$
$$\{108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^n + 41\}]^{2a}$$
$$[108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^n + 41]^{2a}$$

Proof: $RHM_{2C}{}^{a} \prod (NS[n]) = \prod_{e=uv\in E(NS[n])}[(c(u)-1)(c(v)-1)]^{2a}$

$$= \prod_{i=0}^{n-1} (6\times 2^{i}) [\{\{\{((18\times 2^{n})-12)-(6\times 2^{n-i}-6)\} (6\times 2^{n-i}-7)\}-1\}\{0-1\}]^{2a}$$

$$\prod_{i=0}^{n-1} (3\times 2^{i}) [\{\{\{((18\times 2^{n})-12)-(6\times 2^{n-i}-6)\}\times (6\times 2^{n-i}-7)\}-1\} \{\{\{((18\times 2^{n})-12)-(6\times 2^{n-i}-5)\}\times (6\times 2^{n-i}-6)\}-1\}]^{2a} +$$

$$\prod_{i=0}^{n-1} (6\times 2^{i}) [\{\{\{((18\times 2^{n})-12)-(6\times 2^{n-i}-5)\}\times (6\times 2^{n-i}-6)\}-1\}\{0-1\}]^{2a}$$

$$= 108\prod_{i=0}^{n-1} 2^{i} [108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+41]^{2a} \times [\{108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+41\} \{108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+41\}]^{2a} \times [108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+41]^{2a}$$

Example: 1

For the Nanostar Dendrimer D_n for n=1, we shall compute the indices.(Refer Fig 1)

Number of edges	cutting number of end vertices (c(u), c(v))
3×2^{0}	$\{(3(19\times 2^{n-1}-13)^{2}, ((57\times 2^{n-1}-38)-(19\times 2^{n-1}-13))\times (19\times 2^{n-1}-13-1)\}$
3×2^{1}	$\{((57\times 2^{n-1}-38)-(19\times 2^{n-1}-13))\times (19\times 2^{n-1}-13-1), 0\}$
3×2^{1}	(0, 0)
3×2^{1}	$\{((57\times 2^{n-1}-38)-(19\times 2^{n-1}-13)+5)\times (19\times 2^{n-1}-13-6), 0\}$

$M_{1C}[D_1] = \sum_{uv \in E(\mathrm{Dn})} (c(u) + c(v))$

$= (3\times 2^0)\ [(3(19\times 2^{n-1} - 13)^2 + ((57\times 2^{n-1} - 38) - (19\times 2^{n-1} - 13))\times$

$(19\times 2^{n-1} - 13 - 1)] +$

$(3\times 2^1)\ [((57\times 2^{n-1} - 38) - (19\times 2^{n-1} - 13))\times$

$(19\times 2^{n-1} - 13 - 1) + 0] +$

$(3\times 2^1)\ [((57\times 2^{n-1} - 38) - (19\times 2^{n-1} - 13) + 5)\times$

$(19\times 2^{n-1} - 13 - 6) + 0]$

$= (3\times 2^0)[173] + (3\times 2^1)[65] + (3\times 2^1)[0] = 909.$

CHAPTER 3

VERTEX BASED CUTTING NUMBER TOPOLOGICAL INDICES OF NANOSTAR DENDRIMER NS[n]

In this chapter, we calculate First Zagreb vertex cutting number index, F-vertex cutting number index, Y-index vertex cutting number index, Inverse Degree vertex cutting number index, Modified Zagreb vertex cutting number index, Zeroth-order general Randic vertex cutting number index, Reduced first Zagreb vertex cutting number index, Reduced F-index and Reduced modified first Zagreb vertex cutting number indices of G. Also we define multiplicative and polynomial indices of a graph G. Here we introduced new indices on cutting number namely

(i) First Zagreb, F-index and Y-index vertex cutting number indices of G, as

$$M_{1VC}(G) = \sum_{u \in V(G)} [c(u)]^2 \tag{3.1.1}$$

$$F_{VC}(G) = \sum_{u \in V(G)} [c(u)]^3 \tag{3.1.2}$$

$$Y_{VC}(G) = \sum_{u \in V(G)} [c(u)]^4 \tag{3.1.3}$$

(ii) Inverse Degree index, Modified Zagreb and Zeroth-order general Randic vertex cutting number indices of G are defined as

$$ID_{VC}(G) = \sum_{u \in V(G)} \left[\frac{1}{c(u)}\right] \tag{3.1.4}$$

$$^{m}M_{1VC}(G) = \sum_{u \in V(G)} \left[\frac{1}{c(u)^2}\right] \tag{3.1.5}$$

$$^{O}R_{\alpha VC}(G) = \sum_{u \in V(G)} [c(u)]^{\alpha} \tag{3.1.6}$$

(iii) Reduced first Zagreb, Reduced F-index and Reduced modified first Zagreb vertex cutting number indices of G are defined as

$$RM_{1VC}(G) = \sum_{u \in V(G)} (c(u)-1)^2 \quad (3.1.7)$$

$$RF_{VC}(G) = \sum_{u \in V(G)} (c(u)-1)^3 \quad (3.1.8)$$

$$^{m}RM_{1VC}(G) = \sum_{u \in V(G)} \left[\frac{1}{(c(u)-1)^2}\right] \quad (3.1.9)$$

Evaluate cutting topological indices of this Nanostar, we required number of vertices and cutting number of their end vertices. This is given in the Table (2)

The Nanostar Dendrimer NS[n]

Number of vertices	**Cutting number**
$2^{n-1} \times 3 \times 5$ $2^{n-1} \times 3$	0 $[(18 \times 2^n) - 12) - 6] \times 5$
$2^{n-2} \times 3^2$ $2^{n-1} \times 3$ $2^{n-2} \times 3$	0 $[(18 \times 2^n) - 12) - 7] \times 6$ $[(18 \times 2^n) - 12) - 18] \times 17$
$2^{n-3} \times 3^2$ $2^{n-2} \times 3$ $2^{n-3} \times 3$	0 $[(18 \times 2^n) - 12) - 19] \times 18$ $[(18 \times 2^n) - 12) - 42] \times 41$
. . .	. . .
$2^i \times 3^2$ $2^i \times 3$ $2^i \times 3$	0 $[(18 \times 2^n) - (6 \times 2^{n-i} - 5)] \times (6 \times 2^{n-i} - 6)$ $[(18 \times 2^n) - (6 \times 2^{n-i} - 6)] \times (6 \times 2^{n-i} - 7)$
$2^0 \times 3^2\, 2^0 \times 3$ $2^0 \times 3$	0 $[(18 \times 2^n) - (6 \times 2^{n-i} - 5)] \times (6 \times 2^{n-i} - 6)$ $[(18 \times 2^n) - (6 \times 2^{n-i} - 6)] \times (6 \times 2^{n-i} - 7)$
3 3	0 $[(18 \times 2^n) - (6 \times 2^{n-i} - 5)] \times (6 \times 2^{n-i} - 6)$
Total number of vertices	$18 \times 2^n - 12$

Table (2)

3.1 First Zagreb, F-index and Y-index vertex based cutting number topological indices of Nanostar Dendrimer NS[n]

In this section, we introduced new indices of First Zagreb, F-index and Y-index vertex based cutting number topological indices.

Theorem 3.1.1: Let NS[n] be a Nanostar Dendrimer. Then

$$M_{1VC}(NS[n]) = 3\sum_{i=0}^{n-1} 2^{i}\,[108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n} + 42]^{2} +$$
$$3\sum_{i=0}^{n-1} 2^{i}\,[108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n} + 42]^{2}$$

Proof: $M_{1VC}(NS[n]) = \sum_{u\in V(NS[n])} [c(u)]^{2}$

$$= \sum_{i=0}^{n-1}(2^{i}\times 3)[\{((18\times 2^{n}) - 12) - ((6\times 2^{n-i}) - 5)\} \times ((6\times 2^{n-i}) - 6)]^{2} +$$
$$\sum_{i=0}^{n-1}(2^{i}\times 3)[\{((18\times 2^{n}) - 12) - ((6\times 2^{n-i}) - 6)\} \times ((6\times 2^{n-i}) - 7)]^{2}$$
$$M_{1VC}(NS[n]) = 3\sum_{i=0}^{n-1} 2^{i}\,[108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n} + 42]^{2} +$$
$$3\sum_{i=0}^{n-1} 2^{i}\,[108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n} + 42]^{2}$$

Theorem 3.1.2: Let NS[n] be a Nanostar Dendrimer. Then,

$$F_{VC}(NS[n]) = 3\sum_{i=0}^{n-1} 2^{i}\,[108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n} + 42]^{3} +$$
$$3\sum_{i=0}^{n-1} 2^{i}\,[108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n} + 42]^{3}$$

Proof: $F_{VC}(NS[n]) = \sum_{u\in V(NS[n])} [c(u)]^{3}$

$$= \sum_{i=0}^{n-1} (2^{i}\times 3)\,[\{((18\times 2^{n}) - 12) - ((6\times 2^{n-i}) - 5)\} \times ((6\times 2^{n-i}) - 6)]^{3} +$$
$$\sum_{i=0}^{n-1} (2^{i}\times 3)[\{((18\times 2^{n}) - 12) - ((6\times 2^{n-i}) - 6)\} \times ((6\times 2^{n-i}) - 7)]^{3}$$
$$F_{VC}(NS[n]) = 3\sum_{i=0}^{n-1} 2^{i}\,[108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n} + 42]^{3} +$$
$$3\sum_{i=0}^{n-1} 2^{i}\,[108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n} + 42]^{3}$$

Theorem 3.1.3: Let NS[n] be a Nanostar Dendrimer. Then,

$$Y_{VC}(NS[n]) = 3\sum_{i=0}^{n-1} 2^{i}\,[108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n} + 42]^{4} +$$
$$3\sum_{i=0}^{n-1} 2^{i}\,[108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n} + 42]^{4}$$

Proof: $Y_{VC}(NS[n]) = \sum_{u\in V(NS[n])} [c(u)]^4$

$$=\sum_{i=0}^{n-1} (2^i \times 3)[\{((18\times 2^n)-12)-((6\times 2^{n-i})-5)\} \times ((6\times 2^{n-i})-6)]^4 +$$

$$\sum_{i=0}^{n-1} (2^i \times 3)[\{((18\times 2^n)-12)-((6\times 2^{n-i})-6)\} \times ((6\times 2^{n-i})-7)]^4$$

$$Y_{VC}(NS[n]) = 3\sum_{i=0}^{n-1} 2^i [108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^n + 42]^4 +$$

$$3\sum_{i=0}^{n-1} 2^i [108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^n + 42]^4$$

3.2 Inverse Degree index, Modified Zagreb and Zeroth-order general Randic vertex based cutting number topological indices of Nanostar Dendrimer NS[n]

In this section, we calculated new indices of Inverse Degree index, Modified Zagreb and Zeroth-order general Randic vertex based cutting number topological indices.

Theorem 3.2.1: Let NS[n] be a Nanostar Dendrimer. Then,

$$ID_{VC}(NS[n]) = 3\sum_{i=0}^{n-1} 2^i \left[\frac{1}{108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^n + 42}\right] +$$

$$3\sum_{i=0}^{n-1} 2^i \left[\frac{1}{108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^n + 42}\right]$$

Proof: $ID_{VC}(NS[n]) = \sum_{u\in V(NS[n])} \left[\frac{1}{c(u)}\right]$

$$= \sum_{i=0}^{n-1} (2^i \times 3) \left[\frac{1}{[\{((18\times 2^n)-12)-((6\times 2^{n-i})-5)\} \times ((6\times 2^{n-i})-6)]}\right] +$$

$$\sum_{i=0}^{n-1} (2^i \times 3) \left[\frac{1}{[\{((18\times 2^n)-12)-((6\times 2^{n-i})-6)\} \times ((6\times 2^{n-i})-7)]}\right]$$

$$ID_{VC}(NS[n]) = 3\sum_{i=0}^{n-1} 2^i \left[\frac{1}{108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^n + 42}\right] +$$

$$3\sum_{i=0}^{n-1} 2^i \left[\frac{1}{108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^n + 42}\right]$$

Theorem 3.2.2: Let NS[n] be a Nanostar Dendrimer. Then,

$${}^{m}M_{1VC}(NS[n]) = 3\sum_{i=0}^{n-1} 2^i \left[\frac{1}{[108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^n + 42]^2}\right] +$$

$$3\sum_{i=0}^{n-1} 2^i \left[\frac{1}{[108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^n + 42]^2}\right]$$

Proof: $^{m}M_{1VC}(NS[n]) = \sum_{u\in V(NS[n])} \left[\frac{1}{(C(V))^{2}}\right]$

$$= \sum_{i=0}^{n-1} (2^{i} \times 3)\left[\frac{1}{[\{((18\times 2^{n})-12)-((6\times 2^{n-i})-5)\}\times((6\times 2^{n-i})-6)]^{2}}\right] +$$

$$\sum_{i=0}^{n-1} (2^{i} \times 3)\left[\frac{1}{[\{((18\times 2^{n})-12)-((6\times 2^{n-i})-6)\}\times((6\times 2^{n-i})-7)]^{2}}\right]$$

$$^{m}M_{1VC}(NS[n]) = 3\sum_{i=0}^{n-1} 2^{i}\left[\frac{1}{[108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+42]^{2}}\right] +$$

$$3\sum_{i=0}^{n-1} 2^{i}\left[\frac{1}{[108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+42]^{2}}\right]$$

Theorem 3.2.3: Let NS[n] be a Nanostar Dendrimer. Then,

$$^{O}R_{\alpha VC}(NS[n]) = 3\sum_{i=0}^{n-1} 2^{i}[108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+42]^{\alpha} +$$

$$3\sum_{i=0}^{n-1} 2^{i}[108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+42]^{\alpha}$$

Proof: $^{O}R_{\alpha VC}(NS[n]) = \sum_{u\in V(NS[n])} [c(u)]^{\alpha}$

$$= \sum_{i=0}^{n-1}(2^{i}\times 3)[\{((18\times 2^{n})-12)-((6\times 2^{n-i})-5)\}\times((6\times 2^{n-i})-6)]^{\alpha} +$$

$$\sum_{i=0}^{n-1}(2^{i}\times 3)[\{((18\times 2^{n})-12)-((6\times 2^{n-i})-6)\}\times((6\times 2^{n-i})-7)]^{\alpha}$$

$$^{O}R_{\alpha V}(NS[n]) = 3\sum_{i=0}^{n-1} 2^{i}[108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+42]^{\alpha} +$$

$$3\sum_{i=0}^{n-1} 2^{i}[108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+42]^{\alpha}$$

3.3 Reduced first Zagreb, Reduced F-index and Reduced modified first Zagreb vertex based cutting number topological indices of Nanostar Dendrimer NS[n]

In this section, we calculated new indices of Reduced first Zagreb, Reduced F-index and Reduced modified first Zagreb vertex based cutting number topological indices.

Theorem 3.3.1: Let NS[n] be a Nanostar Dendrimer. Then,

$$RM_{1VC}(NS[n]) = 3\sum_{i=0}^{n-1} 2^{i}[108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n} + 41]^{2} + 3\sum_{i=0}^{n-1} 2^{i}[108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n} + 41]^{2}$$

Proof: $RM_{1VC}(NS[n]) = \sum_{u\in V(NS[n])} [c(u) - 1]^{2}$

$$= \sum_{i=0}^{n-1} (2^{i}\times 3)[\{\{((18\times 2^{n}) - 12) - ((6\times 2^{n-i}) - 5)\}\times((6\times 2^{n-i}) - 6)\} - 1]^{2} + \sum_{i=0}^{n-1} (2^{i}\times 3)[\{\{((18\times 2^{n}) - 12) - ((6\times 2^{n-i}) - 6)\}\times((6\times 2^{n-i}) - 7)\} - 1]^{2}$$

$$RM_{1VC}(NS[n]) = 3\sum_{i=0}^{n-1} 2^{i}[108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n} + 41]^{2} + 3\sum_{i=0}^{n-1} 2^{i}[108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n} + 41]^{2}$$

Theorem 3.3.2: Let NS[n] be a Nanostar Dendrimer. Then,

$$RF_{VC}(NS[n]) = 3\sum_{i=0}^{n-1} 2^{i}[108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n} + 41]^{3} + 3\sum_{i=0}^{n-1} 2^{i}[108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n} + 41]^{3}$$

Proof: $RF_{VC}(NS[n]) = \sum_{u\in V(NS[n])} [c(u) - 1]^{3}$

$$= \sum_{i=0}^{n-1} (2^{i}\times 3)[\{\{((18\times 2^{n}) - 12) - ((6\times 2^{n-i}) - 5)\}\times((6\times 2^{n-i}) - 6)\} - 1]^{3} + \sum_{i=0}^{n-1} (2^{i}\times 3)[\{\{((18\times 2^{n}) - 12) - ((6\times 2^{n-i}) - 6)\}\times((6\times 2^{n-i}) - 7)\} - 1]^{3}$$

$$RF_{VC}(NS[n]) = 3\sum_{i=0}^{n-1} 2^{i}[108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n} + 41]^{3} + 3\sum_{i=0}^{n-1} 2^{i}[108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n} + 41]^{3}$$

Theorem 3.3.3: Let NS[n] be a Nanostar Dendrimer. Then,

$$^{m}RM_{1VC}(NS[n]) = 3\sum_{i=0}^{n-1} 2^{i}\left[\frac{1}{[108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n} + 41]^{2}}\right] + 3\sum_{i=0}^{n-1} 2^{i}\left[\frac{1}{[108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n} + 41]^{2}}\right]$$

Proof: $^{m}RM_{1VC}(NS[n]) = \sum_{u\in V(NS[n])}\left[\frac{1}{(c(u)-1)^{2}}\right]$

$$= \sum_{i=0}^{n-1}(2^{i}\times 3)\left[\frac{1}{[\{\{((18\times 2^{n})-12)-((6\times 2^{n-i})-5)\}\times((6\times 2^{n-i})-6)\}-1]^{2}}\right]+$$

$$\sum_{i=0}^{n-1}(2^{i}\times 3)\left[\frac{1}{[\{\{((18\times 2^{n})-12)-((6\times 2^{n-i})-6)\}\times((6\times 2^{n-i})-7)\}-1]^{2}}\right]$$

$$^{m}RM_{1VC}(NS[n]) = 3\sum_{i=0}^{n-1}2^{i}\left[\frac{1}{[108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+41]^{2}}\right]+$$

$$3\sum_{i=0}^{n-1}2^{i}\left[\frac{1}{[108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+41]^{2}}\right]$$

3.4 Multiplicative of first Zagreb, F-index and Y-index vertex based cutting number topological indices of Nanostar Dendrimer NS[n]

In this section, we compute Multiplicative of First Zagreb, F-index and Y-index vertex based cutting number topological indices.

Theorem 3.4.1: Let NS[n] be a Nanostar Dendrimer. Then,

$$M_{1VC}\prod(NS[n])=9\prod_{i=0}^{n-1}2^{2i}[(108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+42)^{2}$$
$$(108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+42)^{2}]$$

Proof: $M_{1VC}\prod(NS[n]) = \prod_{u\in V(NS[n])}[c(u)]^{2}$

$$= \prod_{i=0}^{n-1}(2^{i}\times 3)[\{((18\times 2^{n})-12)-((6\times 2^{n-i})-5)\}\times((6\times 2^{n-i})-6)]^{2}\times$$

$$\prod_{i=0}^{n-1}(2^{i}\times 3)[\{((18\times 2^{n})-12)-((6\times 2^{n-i})-6)\}\times((6\times 2^{n-i})-7)]^{2}$$

$$M_{1VC}\prod(NS[n])=9\prod_{i=0}^{n-1}2^{2i}[(108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+42)^{2}$$
$$(108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+42)^{2}]$$

Theorem 3.4.2: Let NS[n] be a Nanostar Dendrimer. Then,

$$F_{VC}\prod(NS[n]) = 9\prod_{i=0}^{n-1}2^{2i}[(108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+42)^{3}$$
$$(108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+42)^{3}]$$

Proof: $F_{VC}\prod(NS[n]) = \prod_{u\in V(NS[n])} [c(u)]^{3}$

$$= \prod_{i=0}^{n-1}(2^{i} \times 3)[\{((18\times 2^{n})-12)-((6 \times 2^{n-i})-5)\} \times ((6 \times 2^{n-i})-6)]^{3} \times$$

$$\prod_{i=0}^{n-1}(2^{i} \times 3)\ [\{((18\times 2^{n})-12)-((6 \times 2^{n-i})-6)\} \times ((6 \times 2^{n-i})-7)]^{3}$$

$$F_{VC}\prod(NS[n]) = 9\prod_{i=0}^{n-1} 2^{2i}\,[(108\times 2^{2n-i}-36\times 2^{2n-2i}-6 \times 2^{n-i}-108\times 2^{n}+42)^{3}$$

$$(108\times 2^{2n-i}-36\times 2^{2n-2i}+6 \times 2^{n-i}-126\times 2^{n}+42)^{3}]$$

Theorem 3.4.3: Let NS[n] be a Nanostar Dendrimer. Then,

$$Y_{VC}\prod(NS[n]) = 9\prod_{i=0}^{n-1} 2^{2i}\,[(108\times 2^{2n-i}-36\times 2^{2n-2i}-6 \times 2^{n-i}-108\times 2^{n}+42)^{4}$$

$$(108\times 2^{2n-i}-36\times 2^{2n-2i}+6 \times 2^{n-i}-126\times 2^{n}+42)^{4}]$$

Proof: $Y_{VC}\prod(NS[n]) = \prod_{u\in V(NS[n])} [c(u)]^{4}$

$$= \prod_{i=0}^{n-1}(2^{i} \times 3)[\{((18\times 2^{n})-12)-((6 \times 2^{n-i})-5)\} \times ((6 \times 2^{n-i})-6)]^{4} \times$$

$$\prod_{i=0}^{n-1}(2^{i} \times 3)[\{((18\times 2^{n})-12)-((6 \times 2^{n-i})-6)\} \times ((6 \times 2^{n-i})-7)]^{4}$$

$$= 9\prod_{i=0}^{n-1} 2^{2i}\,[(108\times 2^{2n-i}-36\times 2^{2n-2i}-6 \times 2^{n-i}-108\times 2^{n}+42)^{4}$$

$$(108\times 2^{2n-i}-36\times 2^{2n-2i}+6 \times 2^{n-i}-126\times 2^{n}+42)^{4}]$$

3.5 Multiplicative of Inverse Degree index, Modified Zagreb and Zeroth-order general Randic vertex based cutting number topological indices of Nanostar Dendrimer NS[n]

In this section, we introduced new indices namely, multiplicative of Inverse Degree Index, Modified Zagreb, Zeroth-order general Randic vertex based cutting number topological indices.

Theorem 3.5.1: Let NS[n] be a Nanostar Dendrimer. Then,

$$ID_{VC}\prod(NS[n])=9\prod_{i=0}^{n-1} 2^{2i} \left[\begin{matrix}\left(\frac{1}{108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+42}\right)\\ \left(\frac{1}{108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+42}\right)\end{matrix}\right]$$

Proof: $ID_{VC}\prod(NS[n]) = \prod_{u\in V(NS[n])} \left[\frac{1}{C(u)}\right]$

$$= \prod_{i=0}^{n-1}(2^{i} \times 3)\ \left[\frac{1}{\{(((18\times 2^{n})-12)-((6\times 2^{n-i})-5))\times((6\times 2^{n-i})-6)\}}\right] \times$$

$$\prod_{i=0}^{n-1} (2^{i} \times 3) \left[\frac{1}{\{(((18\times 2^{n}) - 12) - ((6 \times 2^{n-i}) - 6)) \times ((6 \times 2^{n-i}) - 7)\}}\right]$$

$$ID_{VC}\prod(NS[n]) = 9 \prod_{i=0}^{n-1} 2^{2i} \left[\begin{matrix}\left(\frac{1}{108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6 \times 2^{n-i} - 108\times 2^{n}+42}\right) \\ \left(\frac{1}{108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6 \times 2^{n-i} - 126\times 2^{n}+42}\right)\end{matrix}\right]$$

Theorem 3.5.2: Let NS[n] be a Nanostar Dendrimer. Then,

$${}^{m}M_{1VC}\prod(NS[n]) = 9 \prod_{i=0}^{n-1} 2^{2i} \left[\begin{matrix}\left(\frac{1}{(108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6 \times 2^{n-i} - 108\times 2^{n}+42)^{2}}\right) \\ \left(\frac{1}{(108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6 \times 2^{n-i} - 126\times 2^{n}+42)^{2}}\right)\end{matrix}\right]$$

Proof: ${}^{m}M_{1VC}\prod(NS[n]) = \prod_{u\in V(NS[n])} \left[\frac{1}{(C(u))^{2}}\right]$

$$= \prod_{i=0}^{n-1} (2^{i} \times 3) \left[\frac{1}{\{(((18\times 2^{n}) - 12) - ((6 \times 2^{n-i}) - 5)) \times ((6 \times 2^{n-i}) - 6)\}^{2}}\right] \times$$

$$\prod_{i=0}^{n-1} (2^{i} \times 3) \left[\frac{1}{\{(((18\times 2^{n}) - 12) - ((6 \times 2^{n-i}) - 6)) \times ((6 \times 2^{n-i}) - 7)\}^{2}}\right]$$

$${}^{m}M_{1VC}\prod(NS[n]) = 9 \prod_{i=0}^{n-1} 2^{2i} \left[\begin{matrix}\left(\frac{1}{(108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6 \times 2^{n-i} - 108\times 2^{n}+42)^{2}}\right) \\ \left(\frac{1}{(108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6 \times 2^{n-i} - 126\times 2^{n}+42)^{2}}\right)\end{matrix}\right]$$

Theorem 3.5.3: Let NS[n] be a Nanostar Dendrimer. Then,

$${}^{O}R_{\alpha VC}\prod(NS[n]) = 9 \prod_{i=0}^{n-1} 2^{2i} \, [(108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^{n} + 42)^{\alpha}$$
$$(108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^{n} + 42)^{\alpha}]$$

Proof: ${}^{O}R_{\alpha vc}\prod(NS[n]) = \prod_{u\in V(NS[n])} [c(u)]^{\alpha}$

$$= \prod_{i=0}^{n-1}(2^{i} \times 3)\{(((18 \times 2^{n}) - 12) - ((6 \times 2^{n-i}) - 5)) \times ((6 \times 2^{n-i}) - 6)\}^{\alpha} \times$$

$$\prod_{i=0}^{n-1}(2^{i} \times 3)\{(((18 \times 2^{n}) - 12) - ((6 \times 2^{n-i}) - 6)) \times ((6 \times 2^{n-i}) - 7)\}^{\alpha}$$

$$= 9 \prod_{i=0}^{n-1} 2^{2i} \, [(108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^{n} + 42)^{\alpha}$$
$$(108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^{n} + 42)^{\alpha}]$$

3.6 Multiplicative of Reduced first Zagreb, Reduced F- index and Reduced modified first Zagreb vertex based cutting number topological indices of Nanostar Dendrimer NS[n]

In this section, we introduced Multiplicative of new indices namely, Reduced first Zagreb, Reduced F-index and Reduced modified first Zagreb vertex based cutting number topological indices.

Theorem 3.6.1: Let NS[n] be a Nanostar Dendrimer. Then,

$$RM_{1VC}\prod(NS[n]) = 9\prod_{i=0}^{n-1} 2^{2i}\ [(108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n} + 41)^{2} + (108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n} + 41)^{2}]$$

Proof: $RM_{1VC}\prod(NS[n]) = \prod_{u\in V(NS[n])} (c(u)-1)^{2}$

$$= \prod_{i=0}^{n-1} (2^{i}\times 3)\ [\{(((18\times 2^{n}) - 12) - ((6\times 2^{n-i}) - 5)) \times ((6\times 2^{n-i}) - 6)\} - 1]^{2} \times \prod_{i=0}^{n-1} (2^{i}\times 3)\ [\{(((18\times 2^{n}) - 12) - ((6\times 2^{n-i}) - 6)) \times ((6\times 2^{n-i}) - 7)\} - 1]^{2}$$

$$RM_{1VC}\prod(NS[n]) = 9\prod_{i=0}^{n-1} 2^{2i}\ [(108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n} + 41)^{2} + (108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n} + 41)^{2}]$$

Theorem 3.6.2: Let NS[n] be a Nanostar Dendrimer. Then,

$$RF_{VC}\prod(NS[n]) = 9\prod_{i=0}^{n-1} 2^{2i}\ [(108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n} + 41)^{3} \times (108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n} + 41)^{3}]$$

Proof: $RF_{VC}\prod(NS[n]) = \prod_{u\in V(NS[n])} (c(u)-1)^{3}$

$$= \prod_{i=0}^{n-1} (2^{i}\times 3)\ [\{(((18\times 2^{n}) - 12) - ((6\times 2^{n-i}) - 5)) \times ((6\times 2^{n-i}) - 6)\} - 1]^{3} \times \prod_{i=0}^{n-1} (2^{i}\times 3)\ [\{(((18\times 2^{n}) - 12) - ((6\times 2^{n-i}) - 6)) \times ((6\times 2^{n-i}) - 7)\} - 1]^{3}$$

$$RF_{VC}\prod(NS[n]) = 9\prod_{i=0}^{n-1} 2^{2i}\ [(108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n} + 41)^{3} \times (108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n} + 41)^{3}]$$

Theorem 3.6.3: Let NS[n] be a Nanostar Dendrimer. Then,

$$m_{RM_{1VC}}\prod(NS[n]) = 9\prod_{i=0}^{n-1} 2^{2i}\left[\begin{array}{c}\left(\frac{1}{(108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n}+41)^{2}}\right)\\ \left(\frac{1}{(108\times 2^{2n-i} - 36\times 2^{2n-2i}+6\times 2^{n-i} - 126\times 2^{n}+41)^{2}}\right)\end{array}\right]$$

Proof: $m_{RM_{1VC}}\prod(NS[n]) = \prod_{u\in V(NS[n])}\left[\frac{1}{(c(u)-1)^{2}}\right]$

$$= \prod_{i=0}^{n-1}(2^{i}\times 3)\left[\frac{1}{[\{(((18\times 2^{n}) - 12) - ((6\times 2^{n-i}) - 5))\times((6\times 2^{n-i}) - 6)\} - 1]^{2}}\right]\times$$

$$\prod_{i=0}^{n-1}(2^{i}\times 3)\left[\frac{1}{[\{(((18\times 2^{n}) - 12) - ((6\times 2^{n-i}) - 6))\times((6\times 2^{n-i}) - 7)\} - 1]^{2}}\right]$$

$$RM_{1VC}\prod(NS[n]) = 9\prod_{i=0}^{n-1} 2^{2i}\left[\begin{array}{c}\left(\frac{1}{(108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n}+41)^{2}}\right)\\ \left(\frac{1}{(108\times 2^{2n-i} - 36\times 2^{2n-2i}+6\times 2^{n-i} - 126\times 2^{n}+41)^{2}}\right)\end{array}\right]$$

3.7 Polynomial of First Zagreb, F-index and Y-index vertex based cutting number topological indices of Nanostar Dendrimer NS[n]

In this section, we introduced polynomial of new indices namely, First Zagreb, F-index and Y-index vertex based cutting number topological indices.

Theorem 3.7.1: Let NS[n] be a Nanostar Dendrimer. Then,

$$M_{1VC}(NS[n], x) = 3\sum_{i=0}^{n-1} 2^{i}\, x^{(108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n} + 42)^{2}} + 3\sum_{i=0}^{n-1} 2^{i}\, x^{(108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n} + 42)^{2}}$$

Proof: $M_{1VC}(NS[n], x) = \sum_{u\in V(NS[n])}[c(u)]^{2}$

$$= \sum_{i=1}^{n}(2^{i}\times 3)\, x^{[\{((18\times 2^{n}) - 12) - ((6\times 2^{n-i}) - 5)\}\times((6\times 2^{n-i}) - 6)]^{2}} +$$

$$\sum_{i=1}^{n}(2^{i}\times 3)\, x^{[\{((18\times 2^{n}) - 12) - ((6\times 2^{n-i}) - 6)\}\times((6\times 2^{n-i}) - 7)]^{2}}$$

$$M_{1VC}(NS[n], x) = 3\sum_{i=0}^{n-1} 2^{i}\, x^{(108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n} + 42)^{2}} +$$

$$3\sum_{i=0}^{n-1} 2^{i}\, x^{(108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n} + 42)^{2}}$$

Theorem 3.7.2: Let NS[n] be a Nanostar Dendrimer. Then,

$$F_{VC}(NS[n], x) = 3\sum_{i=0}^{n-1} 2^{i}\ x^{[(108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^{n} + 42)]^{3}} +$$

$$3\sum_{i=0}^{n-1} 2^{i}\ x^{[(108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^{n} + 42)]^{3}}$$

Proof: $F_{VC}(NS[n], x) = \sum_{u \in V(NS[n])} [c(u)]^{3}$

$$= \sum_{i=0}^{n-1}(2^{i} \times 3)\ x^{[\{((18 \times 2^{n}) - 12) - ((6 \times 2^{n-i}) - 5)\} \times ((6 \times 2^{n-i}) - 6)]^{3}} +$$

$$\sum_{i=0}^{n-1}(2^{i} \times 3)\ x^{[\{((18 \times 2^{n}) - 12) - ((6 \times 2^{n-i}) - 6)\} \times ((6 \times 2^{n-i}) - 7)]^{3}}$$

$$F_{VC}(NS[n], x) = 3\sum_{i=0}^{n-1} 2^{i}\ x^{[(108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^{n} + 42)]^{3}} +$$

$$3\sum_{i=0}^{n-1} 2^{i}\ x^{[(108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^{n} + 42)]^{3}}$$

Theorem 3.7.3: Let NS[n] be a Nanostar Dendrimer. Then,

$$Y_{VC}(NS[n], x) = 3\sum_{i=0}^{n-1} 2^{i}\ x^{[(108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^{n} + 42)]^{4}} +$$

$$3\sum_{i=0}^{n-1} 2^{i}\ x^{[(108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^{n} + 42)]^{4}}$$

Proof: $Y_{VC}(NS[n], x) = \sum_{u \in V(NS[n])} [c(u)]^{4}$

$$= \sum_{i=0}^{n-1}(2^{i} \times 3)\ x^{[\{((18 \times 2^{n}) - 12) - ((6 \times 2^{n-i}) - 5)\} \times ((6 \times 2^{n-i}) - 6)]^{4}} +$$

$$\sum_{i=0}^{n-1}(2^{i} \times 3)\ x^{[\{((18 \times 2^{n}) - 12) - ((6 \times 2^{n-i}) - 6)\} \times ((6 \times 2^{n-i}) - 7)]^{4}}$$

$$Y_{VC}(NS[n], x) = 3\sum_{i=0}^{n-1} 2^{i}\ x^{[(108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^{n} + 42)]^{4}} +$$

$$3\sum_{i=0}^{n-1} 2^{i}\ x^{[(108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^{n} + 42)]^{4}}$$

3.8 Polynomial of Inverse Degree index, Modified Zagreb and Zeroth-order general Randic vertex based cutting number topological indices of Nanostar Dendrimer NS[n]

In this section, we introduced polynomial of new indices, Inverse Degree index, Modified Zagreb, Zeroth-order general Randic vertex based cutting number topological indices.

Theorem 3.8.1: Let NS[n] be a Nanostar Dendrimer. Then,

$$\mathrm{ID_{VC}}(\mathrm{NS}[n], x)=3\sum_{i=0}^{n-1} 2^{i}\left[\frac{1}{(108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+42)}\right]+$$

$$3\sum_{i=0}^{n-1} 2^{i}\left[\frac{1}{(108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+42)}\right]$$

Proof: $\mathrm{ID_{VC}}(\mathrm{NS}[n], x) = \sum_{u\in V(\mathrm{NS}[n])} x^{\left[\frac{1}{C(u)}\right]}$

$$=\sum_{i=0}^{n-1}(2^{i}\times 3)\; x^{\left[\frac{1}{[\{((18\times 2^{n})-12)-((6\times 2^{n-i})-5)\}\times((6\times 2^{n-i})-6)]}\right]}+$$

$$\sum_{i=0}^{n-1}(2^{i}\times 3)\; x^{\left[\frac{1}{[\{((18\times 2^{n})-12)-((6\times 2^{n-i})-6)\}\times((6\times 2^{n-i})-7)]}\right]}$$

$$\mathrm{ID_{VC}}(\mathrm{NS}[n], x)=3\sum_{i=0}^{n-1} 2^{i}\left[\frac{1}{(108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+42)}\right]+$$

$$3\sum_{i=0}^{n-1} 2^{i}\left[\frac{1}{(108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+42)}\right]$$

Theorem 3.8.2: Let NS[n] be a Nanostar Dendrimer. Then,

$$\mathrm{M_{1VC}}^{m}(\mathrm{NS}[n], x)=3\sum_{i=0}^{n-1} 2^{i}\left[\frac{1}{(108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+42)^{2}}\right]+$$

$$3\sum_{i=0}^{n-1} 2^{i}\left[\frac{1}{(108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+42)^{2}}\right]$$

Proof: $\mathrm{M_{1VC}}^{m}(\mathrm{NS}[n], x) = \sum_{u\in V(\mathrm{NS}[n])} x^{\left[\frac{1}{(C(u))^2}\right]}$

$$=\sum_{i=0}^{n-1}(2^{i}\times 3)\; x^{\left[\frac{1}{[\{((18\times 2^{n})-12)-((6\times 2^{n-i})-5)\}\times((6\times 2^{n-i})-6)]^{2}}\right]}+$$

$$\sum_{i=0}^{n-1}(2^{i}\times 3)\; x^{\left[\frac{1}{[\{((18\times 2^{n})-12)-((6\times 2^{n-i})-6)\}\times((6\times 2^{n-i})-7)]^{2}}\right]}$$

$$\mathrm{M_{1VC}}^{m}(\mathrm{NS}[n], x)=3\sum_{i=0}^{n-1} 2^{i}\left[\frac{1}{(108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+42)^{2}}\right]+$$

$$3\sum_{i=0}^{n-1} 2^{i}\left[\frac{1}{(108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+42)^{2}}\right]$$

Theorem 3.8.3: Let NS[n] be a Nanostar Dendrimer. Then,

$$^{O}R_{\alpha VC}(NS[n], x) = 3\sum_{i=1}^{n} 2^{i}\ x^{\left(108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n} + 42\right)^{\alpha}} +$$
$$3\sum_{i=1}^{n} 2^{i}\ x^{\left(108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n} + 42\right)^{\alpha}}$$

Proof: $^{O}R_{\alpha VC}(NS[n], x) = \sum_{u\in V(NS[n])} x^{[c(u)]^{\alpha}}$

$$= \sum_{i=1}^{n}(2^{i} \times 3)\ x^{[\{((18\times 2^{n}) - 12) - ((6\times 2^{n-i}) - 5)\} \times ((6\times 2^{n-i}) - 6)]^{\alpha}} +$$
$$\sum_{i=1}^{n}(2^{i} \times 3)\ x^{[\{((18\times 2^{n}) - 12) - ((6\times 2^{n-i}) - 6)\} \times ((6\times 2^{n-i}) - 7)]^{\alpha}}$$

$$^{O}R_{\alpha VC}(NS[n], x) = 3\sum_{i=1}^{n} 2^{i}\ x^{\left(108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n} + 42\right)^{\alpha}} +$$
$$3\sum_{i=1}^{n} 2^{i}\ x^{\left(108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n} + 42\right)^{\alpha}}$$

3.9 Polynomial of Reduced first Zagreb, Reduced F-index and the Reduced modified first Zagreb vertex based cutting number topological indices of Nanostar Dendrimer NS[n]

In this section, we introduced polynomial of new indices namely, Reduced first Zagreb, Reduced F-index and Reduced modified first Zagreb vertex based cutting number topological indices.

Theorem 3.9.1: Let NS[n] be a Nanostar Dendrimer. Then,

$$RM_{1VC}(NS[n], x) = 3\sum_{i=0}^{n-1} 2^{i}\ x^{\left(108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n} + 41\right)^{2}} +$$
$$3\sum_{i=0}^{n-1} 2^{i}\ x^{\left(108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n} + 41\right)^{2}}$$

Proof: $RM_{1VC}(NS[n], x) = \sum_{u\in V(NS[n])} x^{[c(u) - 1]^{2}}$

$$= \sum_{i=0}^{n-1}(2^{i} \times 3)\ x^{[[\{((18\times 2^{n}) - 12) - ((6\times 2^{n-i}) - 5)\} \times ((6\times 2^{n-i}) - 6)] - 1]^{2}} +$$
$$\sum_{i=0}^{n-1}(2^{i} \times 3)\ x^{[[\{((18\times 2^{n}) - 12) - ((6\times 2^{n-i}) - 6)\} \times ((6\times 2^{n-i}) - 7)] - 1]^{2}}$$

$$RM_{1VC}(NS[n], x) = 3\sum_{i=0}^{n-1} 2^{i}\ x^{\left(108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n} + 41\right)^{2}} +$$
$$3\sum_{i=0}^{n-1} 2^{i}\ x^{\left(108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n} + 41\right)^{2}}$$

Theorem 3.9.2: Let NS[n] be a Nanostar Dendrimer. Then,

$$RF_{VC}(NS[n], x) = 3\sum_{i=0}^{n-1} 2^{i}\, x^{[108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n} + 41]^{3}} +$$
$$3\sum_{i=0}^{n-1} 2^{i}\, x^{[108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n} + 41]^{3}}$$

Proof: $RF_{VC}(NS[n], x) = \sum_{u\in V(NS[n])} x^{[c(u)-1]^{3}}$

$$= \sum_{i=0}^{n-1} (2^{i} \times 3) x^{[\{\{((18\times 2^{n}) - 12) - ((6\times 2^{n-i}) - 5)\}\times((6\times 2^{n-i}) - 6)\} - 1]^{3}} +$$
$$\sum_{i=0}^{n-1} (2^{i} \times 3) x^{[\{\{((18\times 2^{n}) - 12) - ((6\times 2^{n-i}) - 6)\}\times((6\times 2^{n-i}) - 7)\} - 1]^{3}}$$

$$RF_{VC}(NS[n], x) = 3\sum_{i=0}^{n-1} 2^{i}\, x^{[108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n} + 41]^{3}} +$$
$$3\sum_{i=0}^{n-1} 2^{i}\, x^{[108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n} + 41]^{3}}$$

Theorem 3.9.3: Let NS[n] be a Nanostar Dendrimer. Then,

$$^{m}RM_{1VC}(NS[n], x) = 3\sum_{i=0}^{n-1} 2^{i}\, x^{\left[\frac{1}{[108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n} + 41]^{2}}\right]} +$$
$$3\sum_{i=0}^{n-1} 2^{i}\, x^{\left[\frac{1}{[108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n} + 41]^{2}}\right]}$$

Proof: $^{m}RM_{1VC}(NS[n], x) = \sum_{u\in V(NS[n])} x^{\left[\frac{1}{(c(u)-1)^{2}}\right]}$

$$= \sum_{i=0}^{n-1} (2^{i} \times 3)\, x^{\left[\frac{1}{[\{\{((18\times 2^{n}) - 12) - ((6\times 2^{n-i}) - 5)\}\times((6\times 2^{n-i}) - 6)\} - 1]^{2}}\right]} +$$
$$\sum_{i=0}^{n-1} (2^{i} \times 3) x^{\left[\frac{1}{[\{\{((18\times 2^{n}) - 12) - ((6\times 2^{n-i}) - 6)\}\times((6\times 2^{n-i}) - 7)\} - 1]^{2}}\right]}$$

$$^{m}RM_{1VC}(NS[n], x) = 3\sum_{i=0}^{n-1} 2^{i}\, x^{\left[\frac{1}{(108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n} + 41)^{2}}\right]} +$$
$$3\sum_{i=0}^{n-1} 2^{i}\, x^{\left[\frac{1}{(108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n} + 41)^{2}}\right]}$$

CHAPTER 4

EDGE BASED CUTTING NUMBER TOPOLOGICAL INDICES OF NANOSTAR DENDRIMER D_n

In this chapter, we define First, Second, Third, Fourth and Fifth cutting number Indices, SK, SK_1 and SK_2 cutting number Indices and Nano-Zagreb, sum Nano-Zagreb cutting number indices of a graph G. Here we introduced new indices on cutting number namely

(i) First, Second, Third, Fourth and Fifth cutting number indices as,

$$M_{1C}(G) = \sum_{uv\in E(G)} [c(u) + c(v)]$$

$$M_{2C}(G) = \sum_{uv\in E(G)} [c(u)c(v)]$$

$$M_{3C}(G) = \sum_{uv\in E(G)} |c(u) - c(v)|$$

$$M_{4C}(G) = \sum_{uv\in E(G)} c(u)[(c(u) + c(v)]$$

$$M_{5C}(G) = \sum_{uv\in E(G)} c(v)[c(u) + c(v)]$$

(ii) SK, SK_1 and SK_2 cutting number indices are defined as,

$$SK_C(G) = \sum_{uv\in E(G)} [\frac{c(u)+c(v)}{2}]$$

$$SK_{1C}(G) = \sum_{uv\in E(G)} [\frac{c(u)c(v)}{2}]$$

$$SK_{2C}(G) = \sum_{uv\in E(G)} [\frac{c(u)+c(v)}{2}]^2$$

(iii) Nano-Zagreb and sum Nano-Zagreb cutting number indices are defined as,

$$\mathcal{NZ}_C(G) = \sum_{uv\in E(G)} [c^2(u) - c^2(v)]$$

$$\chi_\alpha\mathcal{NZ}_C(G) = \sum_{uv\in E(G)} [c^2(u) - c^2(v)]^\alpha$$

Consider the Nanostar Dendrimer D_n, where n is the defining parameter as illustrated in Fig. 4.1 & 4.2. The number of vertices in Nanostar Dendrimer D_n is equal to $| V(D_n) | = 57 \times 2^{n-1} - 12$ and the number of edges is $| E(D_n) | = 33 \times 2^n - 45$.

To Evaluate cutting topological indices of this Nanostar Dendrimer D_n, we required number of edge and cutting number of their end vertices. This is given in the table (3).

Fig. 4.1 Nanostar Dendrimer D_n for n = 1.

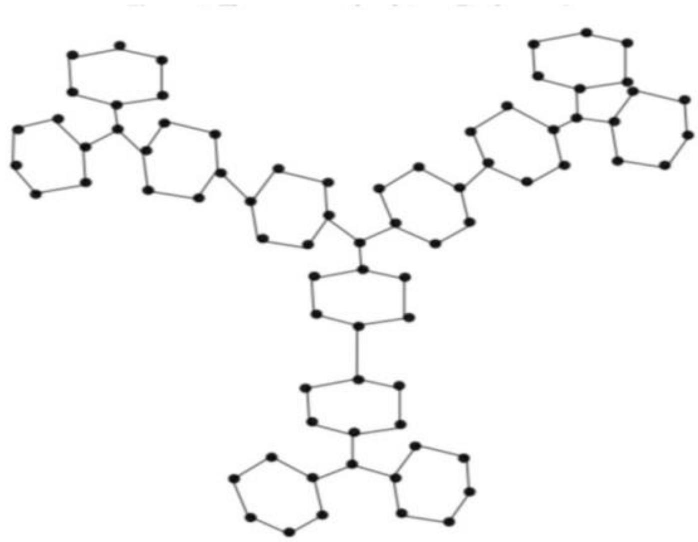


Fig. 4.2 Nanostar Dendrimer D_n for n = 2.

The Nanostar Dendrimer D_n

Number of edges e = uv	cutting number of end vertices (c(u), c(v))
3×2^{0} 3×2^{1} 3×2^{1} 3×2^{1}	$[(3(19 \times 2^{n-1} - 13)^{2}, ((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \times (19 \times 2^{n-1} - 13 - 1)]$ $[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \times (19 \times 2^{n-1} - 13 - 1), 0]$ $(0, 0)$ $[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13) + 5) \times (19 \times 2^{n-1} - 13 - 6), 0]$
. . .	. . .
$3 \times 2^{i-2}$ $3 \times 2^{i-1}$ $3 \times 2^{i-1}$ $3 \times 2^{i-1}$ $3 \times 2^{i-2}$ $3 \times 2^{i-1}$ 3×2^{i} 3×2^{i} 3×2^{i}	$[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 5) \times (19 \times 2^{n-(i-1)} - 13 - 6),$ $((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 6) \times (19 \times 2^{n-(i-1)} - 13 - 7)]$ $[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 6) \times (19 \times 2^{n-(i-1)} - 13 - 7), 0]$ $(0, 0)$ $[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 11) \times (2\,(19 \times 2^{n-i} - 13) + 1), 0]$ $[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 11) \times (2\,(19 \times 2^{n-i} - 13) + 1),$ $((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 12) \times (2(19 \times 2^{n-i} - 13) + (19 \times 2^{n-i} - 13)^{2}]$ $[(57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 12) \times (2(19 \times 2^{n-i} - 13) + (19 \times 2^{n-i} - 13)^{2}, ((57 \times 2^{n-1} - 38) - (19 \times 2^{n-i} - 13)) \times (19 \times 2^{n-i} - 13 - 1)]$ $[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-i} - 13)) \times (19 \times 2^{n-i} - 13 - 1), 0]$ $(0, 0)$ $[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-i} - 13) + 5) \times (19 \times 2^{n-i} - 13 - 6), 0]$
. . .	. . .
$3 \times 2^{n-2}$ $3 \times 2^{n-1}$	$[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(n-1)} - 13) + 5) \times (19 \times 2^{n-(n-1)} - 13 - 6),$ $((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(n-1)} - 13) + 6) \times (19 \times 2^{n-(n-1)} - 13 - 7)]$

$3 \times 2^{n-1}$	$[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(n-1)} - 13) + 6) \times (19 \times 2^{n-(n-1)} - 13 - 7), 0]$
$3 \times 2^{n-1}$	$(0, 0)$
$3 \times 2^{n-2}$	$[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(n-1)} - 13) + 11) \times (2\,(19 \times 2^{n-n} - 13) + 1), 0]$
$3 \times 2^{n-1}$	$[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(n-1)} - 13) + 11) \times (2\,(19 \times 2^{n-n} - 13) + 1), ((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(n-1)} - 13) + 12) \times (2\,(19 \times 2^{n-n} - 13) + (19 \times 2^{n-n} - 13)^2]$ $[(57 \times 2^{n-1} - 38) - (19 \times 2^{n-(n-1)} - 13) + 12) \times (2\,(19 \times 2^{n-n} - 13) + (19 \times 2^{n-n} - 13)^2, ((57 \times 2^{n-1} - 38) - (19 \times 2^{n-n} - 13)) \times (19 \times 2^{n-n} - 13 - 1)]\backslash$
3×2^{n}	$[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-n} - 13)) \times (19 \times 2^{n-n} - 13 - 1), 0]$
3×2^{n}	$(0, 0)$
3×2^{n}	$[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-n} - 13) + 5) \times (19 \times 2^{n-n} - 13 - 6), 0]$
Total Number of Edges	$33 \times 2^{n} - 45.$

Table (3)

4.1 First, Second, Third, Fourth and Fifth Zagreb cutting number topological indices of Nanostar Dendrimer D_n

In this section, we compute First, Second, Third, Fourth and Fifth Zagreb cutting number topological indices of Nanostar Dendrimer D_n in the following theorems.

Theorem 4.1.1: Let D_n be the Nanostar Dendrimer. Then,

$$M_{1C}[D_n] = [\tfrac{48735}{4}] \times n \times 2^{2n} - \left[\tfrac{254277}{8}\right] \times 2^{2n} + [\tfrac{190989}{4}] \times 2^{n} - 16182.$$

Proof: From table (3), we derive the first Zagreb index of the Nanostar Dendrimer D_n as follows:

$$M_{1C}[D_n] = \sum_{uv \in E(D_n)} (c(u) + c(v))$$

$$= (3 \times 2^{0})\,[\{(3(19 \times 2^{n-1} - 13)^{2}\} + \{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \times (19 \times 2^{n-1} - 13 - 1)\}] +$$

$$(3 \times 2^{1})\,[\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \times (19 \times 2^{n-1} - 13 - 1)\} + 0] +$$

$$(3 \times 2^1)\ [\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13) + 5) \times$$
$$(19 \times 2^{n-1} - 13 - 6)\} + 0] +$$
$$3\sum_{i=2}^{n} 2^{i-2}\ [\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(\mathrm{i}-1)} - 13)+5) \times$$
$$(19 \times 2^{n-(\mathrm{i}-1)} - 13 - 6)\}+\{((57\times 2^{n-1} - 38) -$$
$$(19 \times 2^{n-(\mathrm{i}-1)} - 13)+6) \times (19 \times 2^{n-(\mathrm{i}-1)} - 13 - 7)\}] +$$
$$3\sum_{i=2}^{n} 2^{i-1}\ [\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(\mathrm{i}-1)} - 13) + 6) \times$$
$$(19 \times 2^{n-(\mathrm{i}-1)} - 13 - 7)\} + 0] +$$
$$3\sum_{i=2}^{n} 2^{i-1}\ [\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(\mathrm{i}-1)} - 13) + 11) \times$$
$$(2(19 \times 2^{n-(\mathrm{i}-1)} - 13) + 1)\} + 0] +$$
$$3\sum_{i=2}^{n} 2^{i-2}\ [\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(\mathrm{i}-1)} - 13) + 11) \times$$
$$(2(19 \times 2^{n-\mathrm{i}} - 13) +1)\}+\{((57 \times 2^{n-1} - 38) -$$
$$(19 \times 2^{n-(\mathrm{i}-1)} - 13) + 12) \times (2(19 \times 2^{n-\mathrm{i}} - 13)) +$$
$$(19 \times 2^{n-\mathrm{i}} - 13)^2\}]+$$
$$3\sum_{i=2}^{n} 2^{i-1}\ [\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(\mathrm{i}-1)} - 13) + 12) \times$$
$$2(19 \times 2^{n-\mathrm{i}} - 13) + (19 \times 2^{n-\mathrm{i}} - 13)^2\} +$$
$$\{((57\times 2^{n-1} - 38) - (19 \times 2^{n-\mathrm{i}} - 13) \times$$
$$(19 \times 2^{n-\mathrm{i}} - 13 - 1)\}] +$$
$$3\sum_{i=2}^{n} 2^{i}\ [\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-\mathrm{i}} - 13)) \times$$
$$(19 \times 2^{n-\mathrm{i}} - 13 - 1)\} + 0] +$$
$$3\sum_{i=2}^{n} 2^{i}\ [\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-\mathrm{i}} - 13) + 5) \times$$
$$(19 \times 2^{n-\mathrm{i}} - 13 - 6)\} + 0]$$

After simplification, we get

$$M_{1C}[D_n] = [\tfrac{48735}{4}] \times n \times 2^{2n} - \left[\tfrac{254277}{8}\right] \times 2^{2n} + [\tfrac{190989}{4}] \times 2^{n} - 16182.$$

Theorem 4.1.2: Let D_n be the Nanostar Dendrimer. Then,

$$M_{2C}[D_n] = \left[\frac{116116011}{112}\right] \times 2^{4n} + \left[\frac{61493823}{16}\right] \times 2^{3n} - \left[\frac{86236725}{8}\right] \times 2^{2n} + \left[\frac{103542435}{14}\right] \times 2^{n} + \left[\frac{9178425}{4}\right] \times n \times 2^{2n} - 3333474 \times n \times 2^{3n} - 1497900.$$

Proof: From table (3), we derive the second Zagreb index of the Nanostar Dendrimer D_n as follows:

$$M_{2C}[D_n] = \sum_{uv\in E(\mathrm{Dn})} (c(u)\, c(v))$$

$$= (3 \times 2^0) [\{(3(19\times 2^{n-1} - 13)^2\} \times ((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \times (19 \times 2^{n-1} - 13 - 1)] +$$

$$(3 \times 2^1) [((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \times (19 \times 2^{n-1} - 13 - 1) \times 0] +$$

$$(3 \times 2^1) [((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13) + 5) \times (19 \times 2^{n-1} - 13 - 6) \times 0] +$$

$$3 \sum_{i=2}^{n} 2^{i-2} [\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(\mathrm{i}-1)} - 13) + 5) \times (19 \times 2^{n-(\mathrm{i}-1)} - 13 - 6)\} \times \{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(\mathrm{i}-1)} - 13) + 6) \times (19 \times 2^{n-(\mathrm{i}-1)} - 13 - 7)\}] +$$

$$3 \sum_{i=2}^{n} 2^{i-1} [((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(\mathrm{i}-1)} - 13) + 6) \times (19 \times 2^{n-(\mathrm{i}-1)} - 13 - 7) \times 0] +$$

$$3 \sum_{i=2}^{n} 2^{i-1} [((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(\mathrm{i}-1)} - 13) + 11) \times (2(19 \times 2^{n-\mathrm{i}} - 13) + 1) \times 0] +$$

$$3 \sum_{i=2}^{n} 2^{i-2} [\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(\mathrm{i}-1)} - 13) + 11) \times (2(19 \times 2^{n-\mathrm{i}} - 13) +1\} \times \{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(\mathrm{i}-1)} - 13) + 12) \times (2(19 \times 2^{n-\mathrm{i}} - 13) + (19 \times 2^{n-\mathrm{i}} - 13)^2\}] +$$

$$3 \sum_{i=2}^{n} 2^{i-1} [\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(\mathrm{i}-1)} - 13) + 12) \times 2(19 \times 2^{n-\mathrm{i}} - 13) + (19 \times 2^{n-\mathrm{i}} - 13)^2\} \times \{((57\times 2^{n-1} - 38) - (19 \times 2^{n-\mathrm{i}} - 13) \times (19 \times 2^{n-\mathrm{i}} - 13 - 1)\}] +$$

$$3 \sum_{i=2}^{n} 2^{i} [((57 \times 2^{n-1} - 38) - (19 \times 2^{n-\mathrm{i}} - 13)) \times (19 \times 2^{n-\mathrm{i}} - 13 - 1) \times 0] +$$

$$3 \sum_{i=2}^{n} 2^{i} [((57\times 2^{n-1} - 38) - (19 \times 2^{n-\mathrm{i}} - 13) + 5) \times (19 \times 2^{n-\mathrm{i}} - 13 - 6) \times 0]$$

After simplification, we obtain

$$\mathrm{M}_{2C}[\mathrm{D}_n] = \left[\frac{116116011}{112}\right] \times 2^{4n} + \left[\frac{61493823}{16}\right] \times 2^{3n} - \left[\frac{86236725}{8}\right] \times 2^{2n} + \left[\frac{103542435}{14}\right] \times 2^{n} + \left[\frac{9178425}{4}\right] \times n \times 2^{2n} - 3333474 \times n \times 2^{3n} - 1497900.$$

Theorem 4.1.3: Let D_n be the Nanostar Dendrimer. Then,

$$\mathrm{M}_{3C}[\mathrm{D}_n] = \left[\frac{29241}{4}\right] \times n \times 2^{2n} + 570 \times n \times 2^{n} - \left[\frac{153843}{8}\right] \times 2^{2n} + \left[\frac{111639}{4}\right] \times 2^{n} - 8760.$$

Proof: From table (3), we derive the third Zagreb index of the Nanostar Dendrimer D_n as follows:

$$\mathrm{M}_{3C}[\mathrm{D}_n] = \sum_{uv\in E(\mathrm{Dn})} |\, c(u) - c(v) \,|$$

$$\begin{aligned}
&= (3 \times 2^{0})\ |\ [(3(19 \times 2^{n-1} - 13)^{2} - ((57 \times 2^{n-1} - 38) - \\
&\qquad (19 \times 2^{n-1} - 13)) \times (19 \times 2^{n-1} - 13 - 1)]\ | + \\
&(3 \times 2^{1})\ |\ [((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \times \\
&\qquad (19 \times 2^{n-1} - 13 - 1) - 0]\ | + \\
&(3 \times 2^{1})\ |\ [((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13) + 5) \times \\
&\qquad (19 \times 2^{n-1} - 13 - 6) - 0]\ | + \\
&3 \sum_{i=2}^{n} 2^{i-2}\ |\ [\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(\mathrm{i}-1)} - 13) + 5) \times \\
&\qquad (19 \times 2^{n-(\mathrm{i}-1)} - 13 - 6)\} - \{((57 \times 2^{n-1} - 38) - \\
&\qquad (19 \times 2^{n-(\mathrm{i}-1)} - 13) + 6) \times (19 \times 2^{n-(\mathrm{i}-1)} - 13 - 7)\}]| + \\
&3 \sum_{i=2}^{n} 2^{i-1}\ |\ [((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(\mathrm{i}-1)} - 13) + 6) \times \\
&\qquad (19 \times 2^{n-(\mathrm{i}-1)} - 13 - 7) - 0]\ | + \\
&3 \sum_{i=2}^{n} 2^{i-1}\ |\ [((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(\mathrm{i}-1)} - 13) + 11) \times \\
&\qquad (2(19 \times 2^{n-\mathrm{i}} - 13) + 1) - 0]\ | + \\
&3 \sum_{i=2}^{n} 2^{i-2}\ |\ [\{((57 \times 2^{n-1} - 38) - \left(19 \times 2^{n-(\mathrm{i}-1)} - 13\right) + 11) \times \\
&\qquad (2(19 \times 2^{n-\mathrm{i}} - 13) + 1\} - \{((57 \times 2^{n-1} - 38) - \\
&\qquad (19 \times 2^{n-(\mathrm{i}-1)} - 13) + 12) \times (2(19 \times 2^{n-\mathrm{i}} - 13) + \\
&\qquad (19 \times 2^{n-\mathrm{i}} - 13)^{2}\}]| + \\
&3 \sum_{i=2}^{n} 2^{i-1}\ |\ [\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(\mathrm{i}-1)} - 13) + 12) \times \\
&\qquad 2(19 \times 2^{n-\mathrm{i}} - 13) + \left(19 \times 2^{n-\mathrm{i}} - 13\right)^{2}\} -
\end{aligned}$$

$$\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-\mathrm{i}} - 13)$$
$$(19 \times 2^{n-\mathrm{i}} - 13 - 1)\}] \mid +$$
$$3\sum_{i=2}^{n} 2^{i} \mid [((57\times 2^{n-1} - 38) - (19 \times 2^{n-\mathrm{i}} - 13)) \times$$
$$(19 \times 2^{n-\mathrm{i}} - 13 - 1) - 0] \mid +$$
$$3\sum_{i=2}^{n} 2^{i} \mid [((57 \times 2^{n-1} - 38) - (19 \times 2^{n-\mathrm{i}} - 13) + 5) \times$$
$$(19 \times 2^{n-\mathrm{i}} - 13 - 6) - 0] \mid$$

After some calculation, we get

$$M_{3C}[D_n] = \left[\frac{29241}{4}\right] \times n \times 2^{2n} + 570 \times n \times 2^{n} - \left[\frac{153843}{8}\right] \times 2^{2n} + \left[\frac{111639}{4}\right] \times 2^{n} - 8760.$$

Theorem 4.1.4: Let D_n be the Nanostar Dendrimer. Then,

$$M_{4C}[D_n] = \left[\frac{4119186168}{896}\right] \times 2^{4n} - \left[\frac{189906577}{16}\right] \times 2^{3n} - \left[\frac{238905981}{8}\right] \times 2^{2n} + \left[\frac{446809616}{14}\right] \times 2^{n} - 6272547.$$

Proof: From table (3), we derive the fourth Zagreb index of the Nanostar Dendrimer D_n as follows:

$$M_{4C}[D_n] = \sum_{uv\in E(\mathrm{Dn})} c(u)[(c(u) + c(v)]$$
$$= (3 \times 2^{0})\ [(3(19 \times 2^{n-1} - 13)^{2}\ \{3(19 \times 2^{n-1} - 13)^{2} +$$
$$((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \times$$
$$(19 \times 2^{n-1} - 13 - 1)\}] +$$
$$(3 \times 2^{1})\ [\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \times$$
$$(19 \times 2^{n-1} - 13 - 1)\}$$
$$\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \times$$
$$(19 \times 2^{n-1} - 13 - 1) + 0\}]$$
$$(3 \times 2^{1})\ [\{((57\times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13) + 5) \times$$
$$((19 \times 2^{n-1} - 13) - 6)\}$$
$$\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13) + 5) \times$$
$$((19 \times 2^{n-1} - 13) - 6) + 0\}] +$$
$$3\sum_{i=2}^{n} 2^{i-2}\ [\{((57\times 2^{n-1} - 38) - (19 \times 2^{n-(\mathrm{i}-1)} - 13) + 5) \times$$
$$(19 \times 2^{n-(\mathrm{i}-1)} - 13) - 6)\}\{((57\times 2^{n-1} - 38) -$$

$$(19\times 2^{n-(\mathrm{i}-1)}-13)+5)\times(19\times 2^{n-(\mathrm{i}-1)}-13-6)+$$
$$((57\times 2^{n-1}-38)-(19\times 2^{n-(\mathrm{i}-1)}-13)+6)\times$$
$$(19\times 2^{n-(\mathrm{i}-1)}-13-7)\}]$$
$$3\sum_{i=2}^{n}2^{i-1}\ [\{((57\times 2^{n-1}-38)-(19\times 2^{n-(\mathrm{i}-1)}-13)+6)\times$$
$$(19\times 2^{n-(\mathrm{i}-1)}-13\ -7)\}\{((57\times 2^{n-1}-38)-$$
$$(19\times 2^{n-(\mathrm{i}-1)}-13)\ +6)\times(19\times 2^{n-(\mathrm{i}-1)}-13\ -7)+0\}]$$
$$3\sum_{i=2}^{n}2^{i-1}\,[((57\times 2^{n-1}-38)-(19\times 2^{n-(\mathrm{i}-1)}-13)+11)\times$$
$$(2(19\times 2^{n-\mathrm{i}}-13)+1)\{((57\times 2^{n-1}-38)-$$
$$(19\times 2^{n-(\mathrm{i}-1)}-13)+11)\times(2(19\times 2^{n-\mathrm{i}}-13))+1)+0\}]+$$
$$3\sum_{i=2}^{n}2^{i-2}\,[\{((57\times 2^{n-1}-38)-(19\times 2^{n-(\mathrm{i}-1)}-13)+11)\times$$
$$(2(19\times 2^{n-\mathrm{i}}-13)+1)\}\{\{((57\times 2^{n-1}-38)-$$
$$(19\times 2^{n-(\mathrm{i}-1)}-13)+11)\times(2(19\times 2^{n-\mathrm{i}}-13)+1)\}+$$
$$\{((57\times 2^{n-1}-38)-(19\times 2^{n-(\mathrm{i}-1)}-13)+12)\times$$
$$2(19\times 2^{n-\mathrm{i}}-13)+(19\times 2^{n-\mathrm{i}}-13)^2\}\}]+$$
$$3\sum_{i=2}^{n}2^{i-1}\,[\{((57\times 2^{n-1}-38)-(19\times 2^{n-(\mathrm{i}-1)}-13)\ +12)\times$$
$$2(19\times 2^{n-\mathrm{i}}-13)+(19\times 2^{n-\mathrm{i}}-13)^2\}$$
$$\{\{((57\times 2^{n-1}-38)-(19\times 2^{n-\mathrm{i}}-13)+12)\times$$
$$2(19\times 2^{n-\mathrm{i}}-13)+(19\times 2^{n-\mathrm{i}}-13)^2\}\ +$$
$$\{((57\times 2^{n-1}-38)-(19\times 2^{n-\mathrm{i}}-13)\times$$
$$(19\times 2^{n-\mathrm{i}}-13-1)\}\}]+$$
$$3\sum_{i=2}^{n}2^{i}\ [\{((57\times 2^{n-1}-38)-(19\times 2^{n-\mathrm{i}}-13))\times(19\times 2^{n-\mathrm{i}}-13-1)\}$$
$$(57\times 2^{n-1}-38)-(19\times 2^{n-\mathrm{i}}-13))\times$$
$$(19\times 2^{n-\mathrm{i}}-13-1)+0\}]+$$
$$3\sum_{i=2}^{n}2^{i}[\{((57\times 2^{n-1}-38)-(19\times 2^{n-\mathrm{i}}-13)+5)\times$$
$$(19\times 2^{n-\mathrm{i}}-13-6)\}\{((57\times 2^{n-1}-38)-$$
$$(19\times 2^{n-\mathrm{i}}-13)+5)\times(19\times 2^{n-\mathrm{i}}-13-6)+0\}]$$

After a bit calculation, we get

$$M_{4C}[D_n]=\left[\frac{4\,119186168}{896}\right]\times 2^{4n}-\left[\frac{189906577}{16}\right]\times 2^{3n}-\left[\frac{238905981}{8}\right]\times 2^{2n}+\left[\frac{446809616}{14}\right]\times 2^{n}-\ 6272547.$$

Theorem 4.1.5: Let D_n be the Nanostar Dendrimer. Then,

$$M_{5C}[D_n]= \left[\frac{423412929}{224}\right] \times 2^{4n} + \left[\frac{111579685}{16}\right] \times 2^{3n} - \left[\frac{160540215}{8}\right] \times 2^{2n} + \left[\frac{399550415}{28}\right] \times 2^{n} + \left[\frac{18087183}{4}\right] \times n \times 2^{2n} - 197790 \times n \times 2^{n} - 6111369 \times n \times 2^{3n} - 3069747.$$

Proof: From table (3), we compute the fifth Zagreb index of the Nanostar Dendrimer D_n as follows:

$$M_{5C}[D_n] = \sum_{uv\in E(Dn)} c(v)[c(u) + c(v)]$$

$$= (3 \times 2^{0}) [\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \times (19 \times 2^{n-1} - 13 - 1)\} \{3(19 \times 2^{n-1} - 13)^{2} + ((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \times (19 \times 2^{n-1} - 13 - 1)\}] +$$

$$= 3 \sum_{i=2}^{n} 2^{i-2} [\{((57\times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 6) \times (19 \times 2^{n-(i-1)} - 13 - 7)\} \{(((57\times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 5) \times (19 \times 2^{n-(i-1)} - 13 - 6)) + (((57\times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 6) \times (19 \times 2^{n-(i-1)} - 13 - 7))\}]$$

$$3 \sum_{i=2}^{n} 2^{i-2} [\{((57\times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 12) \times (2(19 \times 2^{n-i} - 13) + (19 \times 2^{n-i} - 13)^{2}\} \{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 11) \times (2(19 \times 2^{n-i} - 13) + 1\} + \{((57\times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 12) \times (2(19 \times 2^{n-i} - 13) + (19 \times 2^{n-i} - 13)^{2}\}] +$$

$$3 \sum_{i=2}^{n} 2^{i-1}[\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-i} - 13) \times (19 \times 2^{n-i} - 13 - 1)\} \{(((57\times 2^{n-1} - 38 - (19 \times 2^{n-(i-1)} - 13) + 12) \times 2(19 \times 2^{n-i} - 13) + (19 \times 2^{n-i} - 13)^{2}) + \{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-i} - 13) \times (19 \times 2^{n-i} - 3 - 1)\}$$

After simplification, we get

$$M_{5C}[D_n]= \left[\frac{423412929}{224}\right] \times 2^{4n} + \left[\frac{111579685}{16}\right] \times 2^{3n} - \left[\frac{160540215}{8}\right] \times 2^{2n} + \left[\frac{399550415}{28}\right] \times 2^{n} + \left[\frac{18087183}{4}\right] \times n \times 2^{2n} - 197790 \times n \times 2^{n} - 6111369 \times n \times 2^{3n} - 3069747.$$

4.2 SK, SK₁ and SK₂ cutting number topological indices of Nanostar Dendrimer D_n

In this section, we calculate SK, SK_1 and SK_2 cutting number topological indices of Nanostar Dendrimer D_n in the following theorems.

Theorem 4.2.1: Let D_n be the Nanostar Dendrimer. Then,

$$SK_C[D_n] = [\frac{48735}{8}] \times n \times 2^{2n} - \left[\frac{254277}{16}\right] \times 2^{2n} + [\frac{190989}{8}] \times 2^{n} - 8091.$$

Proof: From table (3), the SK index of the Nanostar Dendrimer D_n , can be written as

$$SK_C[D_n] = \sum_{uv\in E[\mathrm{Dn}]} \left[\frac{c(u)+c(v)}{2}\right]$$

$$= (3 \times 2^0)\left[\frac{[(3(19 \times 2^{n-1} - 13)^2 + ((57 \times 2^{n-1} - 38) - (19\times 2^{n-1} - 13)) \times (19 \times 2^{n-1} - 13 - 1)]}{2}\right] +$$

$$(3 \times 2^1) \left[\frac{[((57 \times 2^{n-1} - 38) - (19\times 2^{n-1} - 13)) \times (19 \times 2^{n-1} - 13 - 1) + 0]}{2}\right] +$$

$$(3 \times 2^1) \left[\frac{[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13) + 5) \times (19 \times 2^{n-1} - 13 - 6) + 0]}{2}\right] +$$

$$3\sum_{i=2}^{n} 2^{i-2}\left[\frac{[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 5) \times (19 \times 2^{n-(i-1)} - 13 - 6) + ((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 6) \times (19 \times 2^{n-(i-1)} - 13 - 7)]}{2}\right] +$$

$$3\sum_{i=2}^{n} 2^{i-1}\left[\frac{[((57 \times 2^{n-1} - 38) - (19\times 2^{n-(i-1)} - 13) + 6) \times (19 \times 2^{n-(i-1)} - 13 - 7) + 0]}{2}\right] +$$

$$3\sum_{i=2}^{n} 2^{i-1}\left[\frac{[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 11) \times (2(19 \times 2^{n-(i-1)} - 13) + 1)) + 0]}{2}\right] +$$

$$3\sum_{i=2}^{n} 2^{i-2}\left[\frac{[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 11) \times (2(19 \times 2^{n-(i-1)} - 13) + 1)) + ((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 12) \times 2(19\times 2^{n-i} - 13) + (19 \times 2^{n-i} - 13)^2]}{2}\right] +$$

$$3\sum_{i=2}^{n} 2^{i-1}\left[\frac{[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 12) \times 2(19\times 2^{n-i} - 13) + (19 \times 2^{n-i} - 13)^2 + ((57 \times 2^{n-1} - 38) - (19 \times 2^{n-i} - 13)) \times (19 \times 2^{n-i} - 13 - 1)]}{2}\right] +$$

$$3\sum_{i=2}^{n} 2^{i}\left[\frac{[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-i} - 13)) \times (19 \times 2^{n-i} - 13 - 1) + 0]}{2}\right] +$$

$$3\sum_{i=2}^{n} 2^{i}\left[\frac{[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13) + 5) \times (19 \times 2^{n-i} - 13 - 6) + 0]}{2}\right]$$

After simplification, we obtain

$$SK_C[D_n] = [\frac{48735}{8}] \times n \times 2^{2n} - \left[\frac{254277}{16}\right] \times 2^{2n} + [\frac{190989}{8}] \times 2^n - 8091.$$

Theorem 4.2.2: Let D_n be the Nanostar Dendrimer. Then,

$$SK_{1C}[D_n] = \left[\frac{116116011}{224}\right] \times 2^{4n} + \left[\frac{61493823}{32}\right] \times 2^{3n} - \left[\frac{86236725}{16}\right] \times 2^{2n} + \left[\frac{103542435}{28}\right] \times 2^n + \left[\frac{9178425}{8}\right] \times n \times 2^{2n} - 1666737 \times n \times 2^{3n} - 748950.$$

Proof: From table (3), SK_1 index of the Nanostar Dendrimer D_n , can be written as

$$SK_{1C}[D_n] = \sum_{uv \in E[Dn]} [\frac{C(u)C(v)}{2}]$$

$$= (3 \times 2^0) \left[\frac{[(3(19 \times 2^{n-1} - 13)^2 \times ((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \times (19 \times 2^{n-1} - 13 - 1)]}{2}\right] +$$

$$3 \sum_{i=2}^{n} 2^{i-2} \left[\frac{[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13 + 5) \times (19 \times 2^{n-(i-1)} - 13 - 6) \times ((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13 + 6) \times (19 \times 2^{n-(i-1)} - 13 - 7)]}{2}\right] +$$

$$3 \sum_{i=2}^{n} 2^{i-2} \left[\frac{[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 11) \times (2(19 \times 2^{n-(i-1)} - 13) + 1)) \times ((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 12) \times 2(19 \times 2^{n-i} - 13) + (19 \times 2^{n-i} - 13)^2]}{2}\right] +$$

$$3 \sum_{i=2}^{n} 2^{i-1} \left[\frac{[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13 + 12) \times 2\left(19 \times 2^{n-i} - 13\right) + \left(19 \times 2^{n-i} - 13\right)^2 \times ((57 \times 2^{n-1} - 38) - (19 \times 2^{n-i} - 13)) \times (19 \times 2^{n-i} - 13 - 1)]}{2}\right]$$

After some calculation, we get

$$SK_{1C}[D_n] = \left[\frac{116116011}{224}\right] \times 2^{4n} + \left[\frac{61493823}{32}\right] \times 2^{3n} - \left[\frac{86236725}{16}\right] \times 2^{2n} + \left[\frac{103542435}{28}\right] \times 2^n + \left[\frac{9178425}{8}\right] \times n \times 2^{2n} - 1666737 \times n \times 2^{3n} - 748950.$$

Theorem 4.2.3: Let D_n be the Nanostar Dendrimer. Then,

$$SK_{2C}[D_n] = \left[\frac{2375100225}{64}\right] \times n^2 \times 2^{4n} - \left[\frac{24784379190}{128}\right] \times n \times 2^{4n} + \left[\frac{18615697830}{64}\right] \times n \times 2^{3n} - \left[\frac{788629770}{8}\right] \times n \times 2^{2n} + \left[\frac{64656792729}{256}\right] \times 2^{4n} - \left[\frac{97128219906}{128}\right] \times 2^{3n} + \left[\frac{44706218949}{64}\right] \times 2^{2n} - \left[\frac{3090583998}{8}\right] \times 2^n + 65464281.$$

Proof: From table (3), SK_2 index of the Nanostar Dendrimer D_n , can be written as

$$SK_{2C}[D_n] = \sum_{uv\in E[Dn]} \left[\frac{c(u)+c(v)}{2}\right]^2$$

$$= (3\times 2^0)\left[\frac{[(3(19\times 2^{n-1}-13)^2+((57\times 2^{n-1}-38)-(19\times 2^{n-1}-13))\times(19\times 2^{n-1}-13-1)]}{2}\right]^2+$$

$$(3\times 2^1)\left[\frac{[((57\times 2^{n-1}-38)-(19\times 2^{n-1}-13))\times(19\times 2^{n-1}-13-1)+0]}{2}\right]^2+$$

$$(3\times 2^1)\left[\frac{[((57\times 2^{n-1}-38)-(19\times 2^{n-1}-13)+5)\times(19\times 2^{n-1}-13-6)+0]}{2}\right]^2+$$

$$3\sum_{i=2}^{n} 2^{i-2}\left[\frac{\begin{array}{c}[((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+5)\times(19\times 2^{n-(i-1)}-13-6)+\\ ((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+6)\times(19\times 2^{n-(i-1)}-13-7)]\end{array}}{2}\right]^2+$$

$$3\sum_{i=2}^{n} 2^{i-1}\left[\frac{[((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+6)\times(19\times 2^{n-(i-1)}-13-7)+0]}{2}\right]^2+$$

$$3\sum_{i=2}^{n} 2^{i-1}\left[\frac{\begin{array}{c}[((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+11)\times(2(19\times 2^{n-i}-13)+1))\\ +0]\end{array}}{2}\right]^2+$$

$$3\sum_{i=2}^{n} 2^{i-2}\left[\frac{\begin{array}{c}[((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+11)\times(2(19\times 2^{n-i}-13)+1))+\\ ((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+12)\times 2(19\times 2^{n-i}-13)+(19\times 2^{n-i}-13)^2]\end{array}}{2}\right]^2+$$

$$3\sum_{i=2}^{n} 2^{i-1}\left[\frac{\begin{array}{c}[((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+12)\times 2(19\times 2^{n-i}-13)+(19\times 2^{n-i}-13)^2+\\ ((57\times 2^{n-1}-38)-(19\times 2^{n-i}-13))\times(19\times 2^{n-i}-13-1)]\end{array}}{2}\right]^2+$$

$$3\sum_{i=2}^{n} 2^{i}\left[\frac{[((57\times 2^{n-1}-38)-(19\times 2^{n-i}-13))\times(19\times 2^{n-i}-13-1)+0]}{2}\right]^2+$$

$$3\sum_{i=2}^{n} 2^{i}\left[\frac{[((57\times 2^{n-1}-38)-(19\times 2^{n-1}-13)+5)\times(19\times 2^{n-i}-13-6)+0]}{2}\right]^2$$

After a bit calculation, we get

$$SK_{2C}[D_n] = \left[\frac{2375100225}{64}\right]\times n^2\times 2^{4n} - \left[\frac{24784379190}{128}\right]\times n\times 2^{4n} + \left[\frac{18615697830}{64}\right]\times n\times 2^{3n} - \left[\frac{788629770}{8}\right]\times n\times 2^{2n} + \left[\frac{64656792729}{256}\right]\times 2^{4n} - \left[\frac{97128219906}{128}\right]\times 2^{3n} + \left[\frac{44706218949}{64}\right]\times 2^{2n} - \left[\frac{3090583998}{8}\right]\times 2^{n} + 65464281$$

4.3 Nano-Zagreb and sum Nano-Zagreb cutting number topological indices of Nanostar Dendrimer D_n

Theorem 4.3.1: Let D_n be the Nanostar Dendrimer. Then,

$$\mathcal{NZ}_C(D_n) = \left[\frac{606383613}{224}\right] \times 2^{4n} + \left[\frac{178533994}{16}\right] \times 2^{3n} - \left[\frac{220107354}{8}\right] \times 2^{2n} + \left[\frac{415733777}{28}\right] \times 2^{n} - 8889264 \times n \times 2^{3n} + 395580 \times n \times 2^{n} + \left[\frac{9074457}{2}\right] \times n \times 2^{2n}.$$

Proof: From table (3), we compute the Nano-Zagreb index of the Nanostar Dendrimer D_n as follows:

$$\mathcal{NZ}_C(D_n) = \sum_{uv\in E(D_n)} (c^2(u) - c^2(v))$$

$$= (3 \times 2^0)\{\{(3(19\times 2^{n-1} - 13)^2\}^2 - \{((57\times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \times (19 \times 2^{n-1} - 13 - 1)\}^2\} +$$

$$(3 \times 2^1)\{\{((57\times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \times (19 \times 2^{n-1} - 13 - 1)\}^2 - 0^2\} +$$

$$(3 \times 2^1)\{\{((57\times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13) + 5) \times (19 \times 2^{n-1} - 13 - 6)\}^2 - 0^2\} +$$

$$3 \sum_{i=2}^{n} 2^{i-2} \{\{((57\times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 5) \times (19 \times 2^{n-(i-1)} - 13 - 6)\}^2 - \{((57\times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 6) \times (19 \times 2^{n-(i-1)} - 13 - 7)\}^2\} +$$

$$3 \sum_{i=2}^{n} 2^{i-1}\{\{((57\times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 6) \times (19 \times 2^{n-(i-1)} - 13 - 7)\}^2 - 0^2\} +$$

$$3 \sum_{i=2}^{n} 2^{i-1}\{\{((57\times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 11) \times (2(19 \times 2^{n-(i-1)} - 13) + 1)\}^2 - 0^2\} +$$

$$3 \sum_{i=2}^{n} 2^{i-2} \{\{((57\times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 11) \times (2(19 \times 2^{n-i} - 13) + 1)\}^2 - \{((57\times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13)+12) \times (2(19 \times 2^{n-i} - 13) + (19 \times 2^{n-i} - 13)^2\}^2\} +$$

$$3\sum_{i=2}^{n} 2^{i-1}\{\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+12)\times$$
$$2(19\times 2^{n-i}-13)\}+(19\times 2^{n-i}-13)^2\}^2 -$$
$$\{((57\times 2^{n-1}-38)-(19\times 2^{n-i}-13))\times$$
$$(19\times 2^{n-i}-13-1)\}^2\}+$$
$$3\sum_{i=2}^{n} 2^{i}\ \{\{((57\times 2^{n-1}-38)-(19\times 2^{n-i}-13))\times$$
$$(19\times 2^{n-i}-13-1)\}^2-0^2\}+$$
$$3\sum_{i=2}^{n} 2^{i}\ \{\{((57\times 2^{n-1}-38)-(19\times 2^{n-i}-13)+5)\times$$
$$(19\times 2^{n-i}-13-6)\}^2-0^2\}$$

After simplification, we get

$$\mathcal{N}Z_C(D_n) = \left[\frac{606383613}{224}\right]\times 2^{4n}+\left[\frac{178533994}{16}\right]\times 2^{3n}-\left[\frac{220107354}{8}\right]\times 2^{2n}+\left[\frac{415733777}{28}\right]\times$$
$$2^{n}-8889264\times n\times 2^{3n}+395580\times n\times 2^{n}+\left[\frac{9074457}{2}\right]\times n\times 2^{2n}.$$

Theorem 4.3.2: Let D_n be the Nanostar Dendrimer. Then,

$$\chi_{\alpha}NZ_C(G)=3[651605\times 2^{4n-4}-1755904\times 2^{3n-3}+1775037\times 2^{2n-2}-$$
$$797848\times 2^{n-1}+134549]^{\alpha}$$
$$6[521284\times 2^{4n-4}-1454108\times 2^{3n-3}+1519449\times 2^{2n-2}-$$
$$704900\times 2^{n-1}+122500]^{\alpha}+$$
$$6[521284\times 2^{4n-4}-1591288\times 2^{3n-3}+1763124\times 2^{2n-2}-837520\times$$
$$2^{n-1}+144400]^{\alpha}$$
$$3\sum_{i=2}^{n} 2^{i-2}[912247\times 2^{4n-4i}-781926\times 2^{4n-3i}-987696\times$$
$$2^{3n-3i}+1111158\times 2^{3n-2i}+123462\times 2^{3n-i}+262086\times$$
$$2^{2n-2i}-851238\times 2^{2n-i}+234840\times 2^{n-i}+3203514\times$$
$$2^{2n-2}-873981\times 2^{2n}+274284\times 2^{n}-134549]^{\alpha}$$
$$3\sum_{i=2}^{n} 2^{i-1}[5212840\times 2^{4n-4i}-8210223\times 2^{4n-3i}+\left[\frac{12901779}{4}\right]\times$$
$$2^{4n-2i}-1467826\times 2^{3n-3i}+6234831\times 2^{3n-2i}-$$
$$3950784\times 2^{3n-i}-2821215\times 2^{2n-2i}+1494540\times$$
$$2^{2n-i}+467780\times 2^{n-i}+2030625\times 2^{2n-2}+714780\times$$
$$2^{2n}-997500\times 2^{n-1}-905274\times 2^{n}+401449]^{\alpha}$$

$3\sum_{i=2}^{n} 2^{i}[2345778 \times 2^{4n-4i-2} + 260642 \times 2^{4n-4i} - 781926 \times$
$2^{4n-3i} + 164616 \times 2^{3n-3i} + 432117 \times 2^{3n-2i} - 432117 \times$
$2^{3n-i} - 2345778 \times 2^{3n-i-2} - 483018 \times 2^{2n-2i} +$
$977949 \times 2^{2n-i} - 160740 \times 2^{n-i} + 1172889 \times 2^{2n-2} +$
$159201 \times 2^{2n} - 690840 \times 2^{n} + 266900]^{\alpha}$

Proof: From table (3), we compute the sum Nano-Zagreb index of the Nanostar Dendrimer D_n as follows:

$$\chi_{\alpha}\mathcal{N}Z_C(G) = \sum_{uv \in E(G)} (c^2(u) - c^2(v))^{\alpha}$$
$$= (3 \times 2^0)[\{(3(19 \times 2^{n-1} - 13)^2\}^2 - \{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \times (19 \times 2^{n-1} - 13 - 1)\}^2]^{\alpha} +$$
$$(3 \times 2^1)[\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \times (19 \times 2^{n-1} - 13 - 1)\}^2 - 0^2]^{\alpha} +$$
$$(3 \times 2^1)[\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13) + 5) \times (19 \times 2^{n-1} - 13 - 6)\}^2 - 0^2]^{\alpha} +$$
$$3\sum_{i=2}^{n} 2^{i-2}[\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 5) \times (19 \times 2^{n-(i-1)} - 13 - 6)\}^2 - \{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 6) \times (19 \times 2^{n-(i-1)} - 13 - 7)\}^2]^{\alpha} +$$
$$3\sum_{i=2}^{n} 2^{i-1}[\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 6) \times (19 \times 2^{n-(i-1)} - 13 - 7)\}^2 - 0^2]^{\alpha} +$$
$$3\sum_{i=2}^{n} 2^{i-1}[\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 11) \times (2(19 \times 2^{n-(i-1)} - 13) + 1)\}^2 - 0^2]^{\alpha} +$$
$$3\sum_{i=2}^{n} 2^{i-2}[\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 11) \times (2(19 \times 2^{n-i} - 13) + 1)\}^2 - \{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 12) \times (2(19 \times 2^{n-i} - 13) + (19 \times 2^{n-i} - 13)^2\}^2]^{\alpha} +$$
$$3\sum_{i=2}^{n} 2^{i-1}[\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 12) \times 2(19 \times 2^{n-i} - 13)\} + (19 \times 2^{n-i} - 13)^2\}^2 - \{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-i} - 13)) \times (19 \times 2^{n-i} - 13 - 1)\}^2]^{\alpha} +$$

$$3\sum_{i=2}^{n} 2^{i}\,[\{((57\times 2^{n-1}-38)-(19\times 2^{n-\mathrm{i}}-13))\times (19\times 2^{n-\mathrm{i}}-13-1)\}^{2}-0^{2}]^{\alpha}+$$

$$3\sum_{i=2}^{n} 2^{i}[\{((57\times 2^{n-1}-38)-(19\times 2^{n-\mathrm{i}}-13)+5)\times (19\times 2^{n-\mathrm{i}}-13-6)\}^{2}-0^{2}]^{\alpha}$$

After simplification, we obtain

$$= 3[651605\times 2^{4n-4}-1755904\times 2^{3n-3}+1775037\times 2^{2n-2}-797848\times 2^{n-1}+134549]^{\alpha}$$

$$6[521284\times 2^{4n-4}-1454108\times 2^{3n-3}+1519449\times 2^{2n-2}-704900\times 2^{n-1}+122500]^{\alpha}+$$

$$6[521284\times 2^{4n-4}-1591288\times 2^{3n-3}+1763124\times 2^{2n-2}-837520\times 2^{n-1}+144400]^{\alpha}$$

$$3\sum_{i=2}^{n} 2^{i-2}[912247\times 2^{4n-4i}-781926\times 2^{4n-3i}-987696\times 2^{3n-3i}+1111158\times 2^{3n-2i}+123462\times 2^{3n-i}+262086\times 2^{2n-2i}-851238\times 2^{2n-i}+234840\times 2^{n-i}+3203514\times 2^{2n-2}-873981\times 2^{2n}+274284\times 2^{n}-134549]^{\alpha}$$

$$3\sum_{i=2}^{n} 2^{i-1}[5212840\times 2^{4n-4i}-8210223\times 2^{4n-3i}+[\tfrac{12901779}{4}]\times 2^{4n-2i}-1467826\times 2^{3n-3i}+6234831\times 2^{3n-2i}-3950784\times 2^{3n-i}-2821215\times 2^{2n-2i}+1494540\times 2^{2n-i}+467780\times 2^{n-i}+2030625\times 2^{2n-2}+714780\times 2^{2n}-997500\times 2^{n-1}-905274\times 2^{n}+401449]^{\alpha}$$

$$3\sum_{i=2}^{n} 2^{i}[2345778\times 2^{4n-4i-2}+260642\times 2^{4n-4i}-781926\times 2^{4n-3i}+164616\times 2^{3n-3i}+432117\times 2^{3n-2i}-432117\times 2^{3n-i}-2345778\times 2^{3n-i-2}-483018\times 2^{2n-2i}+977949\times 2^{2n-i}-160740\times 2^{n-i}+1172889\times 2^{2n-2}+159201\times 2^{2n}-690840\times 2^{n}+266900]^{\alpha}$$

CHAPTER 5

EDGE BASED POLYNOMIAL CUTTING NUMBER TOPOLOGICAL INDICES OF NANOSTAR DENDRIMER D_n

In this chapter, we define polynomial of First, Second, Third, Fourth and Fifth cutting number indices, polynomial of SK, SK_1 and SK_2 cutting number indices and polynomial of Nano-Zagreb and sum Nano-Zagreb cutting number indices of a graph G. Here we introduced new indices polynomial of cutting number namely

(i) polynomial of First, Second, Third, Fourth and Fifth Zagreb cutting number indices as,

$$M_{1C}(G, x) = \sum_{uv\in E(G)} x^{(c(u)+c(v))}$$

$$M_{2C}(G, x) = \sum_{uv\in E(G)} x^{(c(u)\,c(v))}$$

$$M_{3C}(G, x) = \sum_{uv\in E(G)} x^{|c(u)-c(v)|}$$

$$M_{4C}(G, x) = \sum_{uv\in E(G)} x^{c(u)[(c(u)+c(v)]}$$

$$M_{5C}(G, x) = \sum_{uv\in E(G)} x^{c(v)[c(u)+c(v)]}$$

(ii) polynomial of SK, SK_1 and SK_2 cutting number indices are defined as,

$$SK_C(G, x) = \sum_{uv\in E(G)} x^{[\frac{C(u)+C(v)}{2}]}$$

$$SK_{1C}(G, x) = \sum_{uv\in E(G)} x^{[\frac{C(u)\times C(v)}{2}]}$$

$$SK_{2C}(G, x) = \sum_{uv\in E(G)} x^{[\frac{C(u)+C(v)}{2}]^2}$$

(iii) polynomial of Nano-Zagreb and sum Nano-Zagreb cutting number indices are defined as,

$$\mathcal{NZ}_C(G, x) = \sum_{uv\in E(G)} x^{(c^2(u)-c^2(v))}$$

$$\chi_\alpha\mathcal{NZ}_C(G, x) = \sum_{uv\in E(G)} x^{(c^2(u)-c^2(v))^\alpha}$$

5.1 Polynomial of First, Second, Third, Fourth and Fifth Zagreb cutting number based topological indices of Nanostar Dendrimer D_n

We compute polynomial of First, Second, Third, Fourth and Fifth Zagreb cutting number topological indices of Nanostar Dendrimer D_n in the following theorems.

Theorem 5.1.1: Let D_n be the Nanostar Dendrimer. Then,

$$
\begin{aligned}
M_{1C}[D_n, x] = {} & 3x^{[1805 \times 2^{2n-2} - 2489 \times 2^{n-1} + 857]} + \\
& 6x^{[722 \times 2^{2n-2} - 1007 \times 2^{n-1} + 350]} + \\
& 6x^{[722 \times 2^{2n-2} - 1102 \times 2^{n-1} + 380]} + \\
& 3\sum_{i=2}^{n} 2^{i-2} x^{[4332 \times 2^{2n-i} - 5415 \times 2^{2n-2i} + 418 \times 2^{n-i} - 5130 \times 2^{n-1} + 1617]} + \\
& 3\sum_{i=2}^{n} 2^{i-1} x^{\substack{[2166 \times 2^{2n-i} - 4332 \times 2^{2n-2i} + 247 \times 2^{n-i} - 4845 \times 2^{n-1} + \\ 3249 \times 2^{2n-i-1} + 1587]}} + \\
& 3\sum_{i=2}^{n} 2^{i} x^{[2166\times 2^{2n-i-1} - 722\times 2^{2n-2i} - 228 \times 2^{n-i} - 1881 \times 2^{n-1} + 730]}
\end{aligned}
$$

Proof: using table (3), we can write the polynomial of first Zagreb D_n as follows,

$$
\begin{aligned}
M_{1C}[D_n, x] = {} & \sum_{uv \in E(Dn)} x^{(c(u)+c(v))} \\
= {} & (3 \times 2^0)\, x^{\substack{[\{(3(19 \times 2^{n-1} - 13)^2\} + \{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \times \\ (19 \times 2^{n-1} - 13 - 1)\}]}} + \\
& (3 \times 2^1)\, x^{[\{((57 \times 2^{n-1} - 38) - (19\times 2^{n-1} - 13)) \times (19\times 2^{n-1} - 13 - 1)\} + 0]} + \\
& (3 \times 2^1)\, x^{[\{((57 \times 2^{n-1} - 38) - (19\times 2^{n-1} - 13) + 5) \times (19\times 2^{n-1} - 13 - 6)\} + 0]} + \\
& 3\sum_{i=2}^{n} 2^{i-2} x^{\substack{[\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 5) \times (19 \times 2^{n-(i-1)} - 13 - 6)\} + \\ \{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 6) \times (19 \times 2^{n-(i-1)} - 13 - 7)\}]}} + \\
& 3\sum_{i=2}^{n} 2^{i-1} x^{[\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 6) \times (19 \times 2^{n-(i-1)} - 13 - 7)\} + 0]} + \\
& 3\sum_{i=2}^{n} 2^{i-1} x^{[\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 11) \times (2(19 \times 2^{n-i} - 13) + 1)\} + 0]} + \\
& 3\sum_{i=2}^{n} 2^{i-2} x^{\substack{[\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 11) \times (2(19 \times 2^{n-i} - 13) + 1)\} + \\ \{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 12) \times (2(19 \times 2^{n-i} - 13)) + \\ (19 \times 2^{n-i} - 13)^2\}]}} + \\
& 3\sum_{i=2}^{n} 2^{i-1} x^{\substack{[\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 12) \times 2(19 \times 2^{n-i} - 13) + \\ (19 \times 2^{n-i} - 13)^2\} + \{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-i} - 13)) \times \\ (19 \times 2^{n-i} - 13 - 1)\}]}} + \\
& 3\sum_{i=2}^{n} 2^{i} x^{[\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-i} - 13)) \times (19 \times 2^{n-i} - 13 - 1)\} + 0]} + \\
& 3\sum_{i=2}^{n} 2^{i} x^{[\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-i} - 13) + 5) \times (19 \times 2^{n-i} - 13 - 6)\} + 0]}
\end{aligned}
$$

After simplification, we obtain

$$= 3x^{[1805 \times 2^{2n-2} - 2489 \times 2^{n-1} + 857]} +$$
$$6x^{[722 \times 2^{2n-2} - 1007 \times 2^{n-1} + 350]} +$$
$$6x^{[722 \times 2^{2n-2} - 1102 \times 2^{n-1} + 380]} +$$
$$3\sum_{i=2}^{n} 2^{i-2}\, x^{[4332 \times 2^{2n-i} - 5415 \times 2^{2n-2i} + 418 \times 2^{n-i} - 5130 \times 2^{n-1} + 1617]} +$$
$$3\sum_{i=2}^{n} 2^{i-1}\, x^{\substack{[2166 \times 2^{2n-i} - 4332 \times 2^{2n-2i} + 247 \times 2^{n-i} - 4845 \times 2^{n-1} + \\ 3249 \times 2^{2n-i-1} + 1587]}} +$$
$$3\sum_{i=2}^{n} 2^{i}\, x^{[2166 \times 2^{2n-i-1} - 722 \times 2^{2n-2i} - 228 \times 2^{n-i} - 1881 \times 2^{n-1} + 730]}$$

Theorem: 5.1.2 Let D_n be the Nanostar Dendrimer. Then,

$$M_{2C}[D_n, x] = 3x^{[781926 \times 2^{4n-4} - 2160585 \times 2^{3n-3} + 2237478 \times 2^{2n-2} - 1029249 \times 2^{n-1} + 177450]} +$$
$$3\sum_{i=2}^{n} 2^{i-2}\, x^{\substack{[2345778 \times 2^{4n-2i} - 5864445 \times 2^{4n-3i} + 5802714 \times 2^{3n-2i} - 2777895 \times 2^{3n-i} + \\ 3648988 \times 2^{4n-4i} - 2210042 \times 2^{2n-2i} + 1442556 \times 2^{2n-i} - 422370 \times 2^{n} + \\ 308655 \times 2^{2n} - 3806745 \times 2^{3n-2i-1} - 452694 \times 2^{3n-3i} + 211926 \times 2^{n-i} + \\ 1055925 \times 2^{2n-1} - 1241175 \times 2^{n-1} + 321850]}} +$$
$$3\sum_{i=2}^{n} 2^{i-1}\, x^{\substack{[1172889 \times 2^{4n-2i-1} - 781926 \times 2^{4n-3i} + 473271 \times 2^{3n-2i} - 1666737 \times 2^{3n-i-1} - \\ 390963 \times 2^{4n-3i-1} + 2405343 \times 2^{2n-i-1} + 390963 \times 2^{4n-4i} + 226347 \times 2^{3n-3i} - \\ 105963 \times 2^{n-i} + 295659 \times 2^{2n} - 394212 \times 2^{2n-i} - 461643 \times 2^{n} + 177450]}}$$

Proof: using table (3), we can write the polynomial of second Zagreb D_n as follows,

$$M_{2C}[D_n, x] = \sum_{uv \in E(\mathrm{Dn})} x^{(c(u)\, c(v))}$$

$$= (3 \times 2^0)\, x^{[\{(3(19 \times 2^{n-1} - 13)^2\} \times ((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \times (19 \times 2^{n-1} - 13 - 1)]} +$$
$$(3 \times 2^1)\, x^{[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \times (19 \times 2^{n-1} - 13 - 1) \times 0]} +$$
$$(3 \times 2^1)\, x^{[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13) + 5) \times (19 \times 2^{n-1} - 13 - 6) \times 0]} +$$
$$3\sum_{i=2}^{n} 2^{i-2}\, x^{\substack{[\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 5) \times (19 \times 2^{n-(i-1)} - 13 - 6)\} \times \\ \{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 6) \times (19 \times 2^{n-(i-1)} - 13 - 7)\}]}} +$$
$$3\sum_{i=2}^{n} 2^{i-1}\, x^{[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 6) \times (19 \times 2^{n-(i-1)} - 13 - 7) \times 0]} +$$
$$3\sum_{i=2}^{n} 2^{i-1}\, x^{[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 11) \times (2(19 \times 2^{n-i} - 13) + 1) \times 0]} +$$

$$3\sum_{i=2}^{n} 2^{i-2}\, x^{\begin{array}{c}[\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+11)\times(2(19\times 2^{n-i}-13)+1\}\times \\ \{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+12)\times \\ (2(19\times 2^{n-i}-13)+(19\times 2^{n-i}-13)^2\}]\end{array}} +$$

$$3\sum_{i=2}^{n} 2^{i-1}\, x^{\begin{array}{c}[\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+12)\times 2(19\times 2^{n-i}-13)+ \\ (19\times 2^{n-i}-13)^{2}\}\times\{((57\times 2^{n-1}-38)-(19\times 2^{n-i}-13))\times \\ (19\times 2^{n-i}-13-1)\}]\end{array}} +$$

$$3\sum_{i=2}^{n} 2^{i} x^{[((57\times 2^{n-1}-38)-(19\times 2^{n-i}-13))\times(19\times 2^{n-i}-13-1)\times 0]} +$$

$$3\sum_{i=2}^{n} 2^{i}\, x^{[((57\times 2^{n-1}-38)-(19\times 2^{n-i}-13)+5)\times(19\times 2^{n-i}-13-6)\times 0]}$$

After simplification, we get

$$= 3x^{[781926\times 2^{4n-4}-2160585\times 2^{3n-3}+2237478\times 2^{2n-2}-1029249\times 2^{n-1}+177450]} +$$

$$3\sum_{i=2}^{n} 2^{i-2}\, x^{\begin{array}{c}[2345778\times 2^{4n-2i}-5864445\times 2^{4n-3i}+5802714\times 2^{3n-2i}-2777895\times 2^{3n-i}+ \\ 3648988\times 2^{4n-4i}-2210042\times 2^{2n-2i}+1442556\times 2^{2n-i}-422370\times 2^{n}+ \\ 308655\times 2^{2n}-3806745\times 2^{3n-2i-1}-452694\times 2^{3n-3i}+211926\times 2^{n-i}+ \\ 1055925\times 2^{2n-1}-1241175\times 2^{n-1}+321850]\end{array}} +$$

$$3\sum_{i=2}^{n} 2^{i-1}\, x^{\begin{array}{c}[1172889\times 2^{4n-2i-1}-781926\times 2^{4n-3i}+473271\times 2^{3n-2i}-1666737\times 2^{3n-i-1}- \\ 390963\times 2^{4n-3i-1}+2405343\times 2^{2n-i-1}+390963\times 2^{4n-4i}+226347\times 2^{3n-3i}- \\ 562077\times 2^{2n-2i}-105963\times 2^{n-i}+295659\times 2^{2n}-394212\times \\ 2^{2n-i}-461643\times 2^{n}+177450]\end{array}}$$

Theorem 5.1.3: Let D_n be the Nanostar Dendrimer. Then,

$$M_{3C}[D_n,x] = (3\times 2^{0})\, x^{(361\times 2^{2n-2}-475\times 2^{n-1}+157)} +$$
$$(3\times 2^{1})\ x^{(722\times 2^{2n-2}-1007\times 2^{n-1}+350)} +$$
$$(3\times 2^{1})\ x^{(722\times 2^{2n-2}-1102\times 2^{n-1}+380)} +$$
$$3\ \sum_{i=2}^{n} 2^{i-2}\, x^{[342\times 2^{n-i}-361\times 2^{2n-2i}+114\times 2^{n-1}-157]} +$$
$$3\ \sum_{i=2}^{n} 2^{i-1}\, x^{\begin{array}{c}[2166\times 2^{2n-i}-3610\times 2^{2n-2i}+665\times 2^{n-i}-3249\times 2^{n-1}+ \\ 1083\times 2^{2n-i-1}+887]\end{array}} +$$
$$3\ \sum_{i=2}^{n} 2^{i}\, x^{[2166\times 2^{2n-i-1}-722\times 2^{2n-2i}-228\times 2^{n-i}-1881\times 2^{n-1}+730]}$$

Proof: using table (3), we can write the polynomial of third Zagreb D_n as follows,

$$M_{3C}[D_n,\,] = \sum_{uv\in E(\mathrm{Dn})} x^{|c(u)-c(v)|}$$

$$= (3\times 2^{0})\, x^{|[(3(19\times 2^{n-1}-13)^{2}-((57\times 2^{n-1}-38)-(19\times 2^{n-1}-13))\times(19\times 2^{n-1}-13-1)]|} +$$

$$(3\times 2^{1})\, x^{|[((57\times 2^{n-1}-38)-(19\times 2^{n-1}-13))\times(19\times 2^{n-1}-13-1)-0]|} +$$

$$(3\times 2^{1})\, x^{|[((57\times 2^{n-1}-38)-(19\times 2^{n-1}-13)+5)\times(19\times 2^{n-1}-13-6)-0]|} +$$

$$3\sum_{i=2}^{n} 2^{i-2}\ x^{\left|\begin{array}{c}[\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+5)\times(19\times 2^{n-(i-1)}-13-6)\}-\\ \{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+6)\times(19\times 2^{n-(i-1)}-13-7)\}]\end{array}\right|}\ +$$

$$3\sum_{i=2}^{n} 2^{i-1}\ x^{|[((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+6)\times(19\times 2^{n-(i-1)}-13-7)-0]|}\ +$$

$$3\sum_{i=2}^{n} 2^{i-1}\ x^{|[((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+11)\times(2(19\times 2^{n-(i-1)}-13)+1)-0]|}\ +$$

$$3\sum_{i=2}^{n} 2^{i-2}\ x^{\left|\begin{array}{c}[\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+11)\times(2(19\times 2^{n-i}-13)+1\}-\\ \{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+12)\times\\ (2(19\times 2^{n-i}-13)+(19\times 2^{n-i}-13)^2\}]\end{array}\right|}\ +$$

$$3\sum_{i=2}^{n} 2^{i-1}\ x^{\left|\begin{array}{c}[\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+12)\times 2(19\times 2^{n-i}-13)+\\ (19\times 2^{n-i}-13)^2\}-\{((57\times 2^{n-1}-38)-(19\times 2^{n-i}-13))\times(19\times 2^{n-i}-13-1)\}]\end{array}\right|}\ +$$

$$3\sum_{i=2}^{n} 2^{i}\ x^{|[((57\times 2^{n-1}-38)-(19\times 2^{n-i}-13))\times(19\times 2^{n-i}-13-1)-0]|}\ +$$

$$3\sum_{i=2}^{n} 2^{i}\ x^{|[((57\times 2^{n-1}-38)-(19\times 2^{n-i}-13)+5)\times(19\times 2^{n-i}-13-6)-0]|}$$

After some calculation, we obtain

$$\begin{aligned}M_{3C}[D_n, x] = {} & (3\times 2^0)\ x^{(361\times 2^{2n-2}-475\times 2^{n-1}+157)} + \\ & (3\times 2^1)\ x^{(722\times 2^{2n-2}-1007\times 2^{n-1}+350)} + \\ & (3\times 2^1)\ x^{(722\times 2^{2n-2}-1102\times 2^{n-1}+380)} + \\ & 3\sum_{i=2}^{n} 2^{i-2}\, x^{[342\times 2^{n-i}-361\times 2^{2n-2i}+114\times 2^{n-1}-157]} + \\ & 3\sum_{i=2}^{n} 2^{i-1}\, x^{\begin{array}{c}[2166\times 2^{2n-i}-3610\times 2^{2n-2i}+665\times 2^{n-i}-3249\times 2^{n-1}+\\ 1083\times 2^{2n-i-1}+887]\end{array}} + \\ & 3\sum_{i=2}^{n} 2^{i}\, x^{[2166\times 2^{2n-i-1}-722\times 2^{2n-2i}-228\times 2^{n-i}-1881\times 2^{n-1}+730]}\end{aligned}$$

Theorem 5.1.4: Let D_n be the Nanostar Dendrimer. Then,

$$\begin{aligned}M_{4C}[D_n, x] = {} & 3x^{[1954815\times 2^{4n-4}-5370597\times 2^{3n-3}+5531964\times 2^{2n-2}-2531997\times 2^{n-1}+434499]} + \\ & 6x^{[521284\times 2^{4n-4}-1591288\times 2^{3n-3}+1763124\times 2^{2n-2}-837520\times 2^{n-1}+144400]} + \\ & 6x^{[521284\times 2^{4n-4}-1454108\times 2^{3n-3}+1519449\times 2^{2n-2}-704900\times 2^{n-1}+122500]} + \\ & 3\sum_{i=2}^{n} 2^{i-2}\, x^{\begin{array}{c}[4691556\times 2^{4n-2i}-12119853\times 2^{4n-3i}+7819260\times 2^{4n-4i}-6111369\times 2^{3n-2i-1}-\\ 5494059\times 2^{3n-i}+11399658\times 2^{3n-2i}-1550134\times 2^{3n-3i}-4142114\times 2^{2n-2i}+\\ 2469240\times 2^{2n-i}+1637496\times 2^{2n}-3906495\times 2^{n-1}+475646\times 2^{n-i}+588750]\end{array}} + \\ & 3\sum_{i=2}^{n} 2^{i-1}\, x^{\begin{array}{c}[2345778\times 2^{4n-2i}+3518667\times 2^{4n-2i-1}-10165038\times 2^{4n-3i}+6769833\times 2^{3n-2i}-\\ 5494059\times 2^{3n-i}+11399658\times 2^{3n-2i}-1550134\times 2^{3n-3i}-4142114\times 2^{2n-2i}+\\ 2469240\times 2^{2n-i}+1637496\times 2^{2n}-3906495\times 2^{n-1}+475646\times 2^{n-i}+588750]\end{array}} + \\ & 3\sum_{i=2}^{n} 2^{i}\, x^{[2166\times 2^{2n-i-1}-722\times 2^{2n-2i}-228\times 2^{n-i}-1881\times 2^{n-1}+730]}\end{aligned}$$

Proof: using table (3), we can write the polynomial of Fourth Zagreb D_n as follows,

$$M_{4C}[D_n, x] = \sum_{uv\in E(D_n)} c(u)[(c(u)+c(v)]$$

$$= (3\times 2^0)\, x^{[(3(19\times 2^{n-1}-13)^2\{3(19\times 2^{n-1}-13)^2+((57\times 2^{n-1}-38)-(19\times 2^{n-1}-13))\times(19\times 2^{n-1}-13-1)\}]} +$$

$$(3\times 2^1)\, x^{[\{((57\times 2^{n-1}-38)-(19\times 2^{n-1}-13))\times(19\times 2^{n-1}-13-1)\}\{((57\times 2^{n-1}-38)-(19\times 2^{n-1}-13))\times(19\times 2^{n-1}-13-1)+0\}]} +$$

$$(3\times 2^1)\, x^{[\{((57\times 2^{n-1}-38)-(19\times 2^{n-1}-13)+5)\times(19\times 2^{n-1}-13-6)\}\{((57\times 2^{n-1}-38)-(19\times 2^{n-1}-13)+5)\times(19\times 2^{n-1}-13-6)+0\}]}$$

$$3\sum_{i=2}^{n} 2^{i-2}\, x^{[\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+5)\times(19\times 2^{n-(i-1)}-13-6)\}\times\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+5)(19\times 2^{n-(i-1)}-13-6)+((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+6)\times(19\times 2^{n-(i-1)}-13-7)\}]}$$

$$3\sum_{i=2}^{n} 2^{i-1}\, x^{[\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+6)\times(19\times 2^{n-(i-1)}-13-7)\}\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+6)\times(19\times 2^{n-(i-1)}-13-7)+0]}$$

$$3\sum_{i=2}^{n} 2^{i-1}\, x^{[\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+11)\times(2(19\times 2^{n-i}-13)+1)\}\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+11)\times(2(19\times 2^{n-i}-13)+1)+0\}]} +$$

$$3\sum_{i=2}^{n} 2^{i-2}\, x^{[\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+11)\times(2(19\times 2^{n-i}-13)+1)\}+\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+11)\times(2(19\times 2^{n-i}-13)+1)+((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+12)\times 2(19\times 2^{n-i}-13))+(19\times 2^{n-i}-13)^2\}]} +$$

$$3\sum_{i=2}^{n} 2^{i-1}\, x^{[\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+12)\times 2(19\times 2^{n-i}-13)+(19\times 2^{n-i}-13)^2\}\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+12)\times 2(19\times 2^{n-i}-13)+(19\times 2^{n-i}-13)^2\}+\{((57\times 2^{n-1}-38)-(19\times 2^{n-i}-13)\times(19\times 2^{n-i}-13-1)\}]} +$$

$$3\sum_{i=2}^{n} 2^{i}\, x^{[((57\times 2^{n-1}-38)-(19\times 2^{n-i}-13))\times(19\times 2^{n-i}-13-1)(57\times 2^{n-1}-38)-(19\times 2^{n-i}-13))\times(19\times 2^{n-i}-13-1+0\}]} +$$

$$3\sum_{i=2}^{n} 2^{i}\, x^{[((57\times 2^{n-1}-38)-(19\times 2^{n-i}-13)+5)\times(19\times 2^{n-i}-13-6)\{((57\times 2^{n-1}-38)-(19\times 2^{n-i}-13)+5)\times(19\times 2^{n-i}-13-6)+0\}]}$$

After an easy simplification, we get

$$= 3x^{[1954815\times 2^{4n-4}-5370597\times 2^{3n-3}+5531964\times 2^{2n-2}-2531997\times 2^{n-1}+434499]} +$$

$$6x^{[521284\times 2^{4n-4}-1591288\times 2^{3n-3}+1763124\times 2^{2n-2}-837520\times 2^{n-1}+144400]} +$$

$$6x^{[521284\times 2^{4n-4}-1454108\times 2^{3n-3}+1519449\times 2^{2n-2}-704900\times 2^{n-1}+122500]} +$$

$$3\sum_{i=2}^{n} 2^{i-2}\, x^{[4691556\times 2^{4n-2i}-12119853\times 2^{4n-3i}+7819260\times 2^{4n-4i}-6111369\times 2^{3n-2i-1}-5494059\times 2^{3n-i}+11399658\times 2^{3n-2i}-1550134\times 2^{3n-3i}-4142114\times 2^{2n-2i}+2469240\times 2^{2n-i}+1637496\times 2^{2n}-3906495\times 2^{n-1}+475646\times 2^{n-i}+588750]} +$$

$$3\sum_{i=2}^{n} 2^{i-1}\, x^{[2345778\times 2^{4n-2i}+3518667\times 2^{4n-2i-1}-10165038\times 2^{4n-3i}+6769833\times 2^{3n-2i}-5494059\times 2^{3n-i}+11399658\times 2^{3n-2i}-1550134\times 2^{3n-3i}-4142114\times 2^{2n-2i}+2469240\times 2^{2n-i}+1637496\times 2^{2n}-3906495\times 2^{n-1}+475646\times 2^{n-i}+588750]} +$$

$$3\sum_{i=2}^{n} 2^{i}\, x^{[2166\times 2^{2n-i-1}-722\times 2^{2n-2i}-228\times 2^{n-i}-1881\times 2^{n-1}+730]}$$

Theorem 5.1.5 Let D_n be the Nanostar Dendrimer. Then,

$$M_{5C}[D_n, x] = 3x^{[1303210 \times 2^{4n-4} - 3614693 \times 2^{3n-3} + 3756927 \times 2^{2n-2} - 1734149 \times 2^{n-1} + 299950]} +$$

$$3\sum_{i=2}^{n} 2^{i-2}\, x^{\begin{array}{c}[4691556 \times 2^{4n-2i} - 11337927 \times 2^{4n-3i} + 6907013 \times 2^{4n-4i} - \\ 9156765 \times 2^{3n-2i-1} - 5617521 \times 2^{3n-i} + 11811198 \times 2^{3n-2i} - \\ 562438 \times 2^{3n-3i} - 4404200 \times 2^{2n-2i} + 3320478 \times 2^{2n-i} + \\ 633555 \times 2^{2n} - 855570 \times 2^{n} + 4308174 \times 2^{2n-2} - \\ 2743923 \times 2^{n-1} + 240806 \times 2^{n-i} + 72399]\end{array}} +$$

$$3\sum_{i=2}^{n} 2^{i-1}\, x^{\begin{array}{c}[3518667 \times 2^{4n-2i-2} - 781926 \times 2^{4n-3i} + 535002 \times 2^{3n-2i} - \\ 617310 \times 2^{3n-i} + 928131 \times 2^{2n-i-1} - 1172889 \times 2^{4n-3i-1} + \\ 377245 \times 2^{3n-3i} + 521284 \times 2^{4n-4i} - 771096 \times 2^{2n-2i} - 252263 \times \\ 2^{n-i} + 890226 \times 2^{2n-i} + 454860 \times 2^{2n} - 740943 \times 2^{n} + 299950]\end{array}} +$$

$$12 + 36\,(2^{n-1} - 1).$$

Proof: using table (3), we can write the polynomial of Fifth Zagreb D_n as follows,

$$M_{5C}[D_n, x] = \sum_{uv \in E(D_n)} c(v)[c(u) + c(v)]$$

$$= (3 \times 2^0)x^{\begin{array}{c}[\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \times (19 \times 2^{n-1} - 13 - 1)\}\{3(19 \times 2^{n-1} - 13)^2 + \\ ((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \times (19 \times 2^{n-1} - 13 - 1)\}]\end{array}} +$$

$$(3 \times 2^1)\, x^0 + (3 \times 2^1)\, x^0 +$$

$$3\sum_{i=2}^{n} 2^{i-2}\, x^{\begin{array}{c}[\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 6) \times (19 \times 2^{n-(i-1)} - 13 - 7)\} \\ \{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 5) \times (19 \times 2^{n-(i-1)} - 13 - 6) + \\ ((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 6) \times (19 \times 2^{n-(i-1)} - 13 - 7)\}]\end{array}} +$$

$$3\sum_{i=2}^{n} 2^{i-1}\, x^0 + 3\sum_{i=2}^{n} 2^{i-1}\, x^0 +$$

$$3\sum_{i=2}^{n} 2^{i-2}\, x^{\begin{array}{c}[\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 12) \times (2(19 \times 2^{n-i} - 13) + (19 \times 2^{n-i} - 13)^2\} \\ \{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 11) \times (2(19 \times 2^{n-i} - 13) + 1\} + \\ \{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 12) \times \\ (2(19 \times 2^{n-i} - 13) + (19 \times 2^{n-i} - 13)^2\}]\end{array}} +$$

$$3\sum_{i=2}^{n} 2^{i-1}\, x^{\begin{array}{c}[\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-i} - 13) \times (19 \times 2^{n-i} - 13 - 1)\}\{((57 \times 2^{n-1} - 38) - \\ (19 \times 2^{n-(i-1)} - 13) + 12) \times 2(19 \times 2^{n-i} - 13) + \left(19 \times 2^{n-i} - 13\right)^2 + \\ \{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-i} - 13) \times (19 \times 2^{n-i} - 3 - 1)\}]\end{array}} +$$

$$3\sum_{i=2}^{n} 2^{i}\, x^0 + 3\sum_{i=2}^{n} 2^{i}\, x^0$$

After simplification, we obtain

$$M_{5C}[D_n, x] = 3x^{[1303210\times 2^{4n-4} - 3614693\times 2^{3n-3} + 3756927\times 2^{2n-2} - 1734149\times 2^{n-1} + 299950]} +$$

$$3\sum_{i=2}^{n} 2^{i-2}\, x^{\left[\begin{array}{c}4691556\times 2^{4n-2i} - 11337927\times 2^{4n-3i} + 6907013\times 2^{4n-4i} - \\ 9156765\times 2^{3n-2i-1} - 5617521\times 2^{3n-i} + 11811198\times 2^{3n-2i} - \\ 562438\times 2^{3n-3i} - 4404200\times 2^{2n-2i} + 3320478\times 2^{2n-i} + \\ 633555\times 2^{2n} - 855570\times 2^{n} + 4308174\times 2^{2n-2} - \\ 2743923\times 2^{n-1} + 240806\times 2^{n-i} + 72399\end{array}\right]} +$$

$$3\sum_{i=2}^{n} 2^{i-1}\, x^{\left[\begin{array}{c}3518667\times 2^{4n-2i-2} - 781926\times 2^{4n-3i} + 535002\times 2^{3n-2i} - \\ 617310\times 2^{3n-i} + 928131\times 2^{2n-i-1} - 1172889\times 2^{4n-3i-1} + \\ 377245\times 2^{3n-3i} + 521284\times 2^{4n-4i} - 771096\times 2^{2n-2i} - 252263\times \\ 2^{n-i} + 890226\times 2^{2n-i} + 454860\times 2^{2n} - 740943\times 2^{n} + 299950\end{array}\right]} +$$

$$12 + 36(2^{n-1} - 1).$$

5.2 Polynomial of SK, SK1 and SK2 cutting number topological indices of Nanostar Dendrimer D_n

Theorem 5.2.1: Let D_n be the Nanostar Dendrimer. Then,

$$SK_C[D_n, x] = 3x^{\left[\left(\frac{4693}{8}\right)\times 2^{2n} - \left(\frac{6707}{4}\right)\times 2^{n} + \left(\frac{2317}{2}\right)\right]} +$$

$$3\sum_{i=2}^{n} 2^{i-2}\, x^{\left[2166\times 2^{2n-i} - \left(\frac{5415}{2}\right)\times 2^{2n-2i} - 2565\times 2^{n-1} + 209\times 2^{n-i} + \left(\frac{1617}{2}\right)\right]} +$$

$$3\sum_{i=2}^{n} 2^{i-1}\, x^{\left[\left(\frac{7581}{4}\right)\times 2^{2n-i} - 2166\times 2^{2n-2i} + \left(\frac{247}{2}\right)\times 2^{n-i} - \left(\frac{4845}{2}\right)\times 2^{n-1} + \left(\frac{1587}{2}\right)\right]} +$$

$$3\sum_{i=2}^{n} 2^{i}\, x^{\left[\left(\frac{1083}{4}\right)\times 2^{2n-i} - 361\times 2^{2n-2i} - 114\times 2^{n-i} - \left(\frac{1881}{2}\right)\times 2^{n-1} + 365\right]}$$

Proof: From table (3), we compute polynomial of SK index of the Nanostar Dendrimer

D_n as follows:

$$SK_C[D_n, x] = \sum_{uv\in E(G)} x^{\left[\frac{c(u)+c(v)}{2}\right]}$$

$$= (3\times 2^{0})\, x^{\left[\frac{[(3(19\times 2^{n-1} - 13)^{2} + ((57\times 2^{n-1} - 38) - (19\times 2^{n-1} - 13))\times(19\times 2^{n-1} - 13 - 1)]}{2}\right]} +$$

$$(3\times 2^{1})\, x^{\left[\frac{[((57\times 2^{n-1} - 38) - (19\times 2^{n-1} - 13))\times(19\times 2^{n-1} - 13 - 1) + 0]}{2}\right]} +$$

$$(3\times 2^{1})\, x^{\left[\frac{[((57\times 2^{n-1} - 38) - (19\times 2^{n-1} - 13) + 5)\times((19\times 2^{n-1} - 13) - 6) + 0]}{2}\right]} +$$

$$3\sum_{i=2}^{n} 2^{i-2}\, x^{\left[\frac{\begin{array}{c}[((57\times 2^{n-1} - 38) - (19\times 2^{n-1} - 13) + 5)\times(19\times 2^{n-(i-1)} - 13 - 6) + \\ ((57\times 2^{n-1} - 38) - (19\times 2^{n-1} - 13) + 6)\times(19\times 2^{n-(i-1)} - 13 - 7)]\end{array}}{2}\right]} +$$

$$3\sum_{i=2}^{n} 2^{i-1}\ x^{\left[\frac{[((57\times 2^{n-1}\ -\ 38)-(19\times 2^{n-1}-13)+6)\times(19\times 2^{n-(i-1)}-13-7)+0]}{2}\right]}+$$

$$3\sum_{i=2}^{n} 2^{i-1}\ x^{\left[\frac{[((57\times 2^{n-1}\ -\ 38)-(19\times 2^{n-(i-1)}-13)+11)\times(2(19\times 2^{n-(i-1)}-13)+1))+0\,]}{2}\right]}+$$

$$3\sum_{i=2}^{n} 2^{i-2}\, x^{\left[\frac{[((57\times 2^{n-1}\ -\ 38)-(19\times 2^{n-(i-1)}-13)+11)\times(2(19\times 2^{n-(i-1)}-13)+1))+((57\times 2^{n-1}\ -\ 38)-(19\times 2^{n-(i-1)}-13)+11)\ \times 2\left(19\times 2^{n-i}-13\right)+\left(19\times 2^{n-i}-13\right)^{2}]}{2}\right]}+$$

$$3\sum_{i=2}^{n} 2^{i-1}\, x^{\left[\frac{[((57\times 2^{n-1}\ -\ 38)-(19\times 2^{n-(i-1)}-13)+11)\times 2\left(19\times 2^{n-i}-13\right)+\left(19\times 2^{n-i}-13\right)^{2}+((57\times 2^{n-1}\ -\ 38)-(19\times 2^{n-(i-1)}-13))\times(19\times 2^{n-i}-13-1)]}{2}\right]}+$$

$$3\sum_{i=2}^{n} 2^{i}\quad x^{\left[\frac{[((57\times 2^{n-1}\ -\ 38)-(19\times 2^{n-1}-13))\times(19\times 2^{n-i}-13-1)+0]}{2}\right]}+$$

$$3\sum_{i=2}^{n} 2^{i}\ x^{\left[\frac{[((57\times 2^{n-1}\ -\ 38)-(19\times 2^{n-1}-13)+5)\times(19\times 2^{n-i}-13-6)+0\,]}{2}\right]}+$$

$$SK_C[D_n, x] = 3x^{\left[\left(\frac{4693}{8}\right)\times 2^{2n}-\left(\frac{6707}{4}\right)\times 2^{n}+\left(\frac{2317}{2}\right)\right]}+$$

$$3\sum_{i=2}^{n} 2^{i-2}\, x^{\left[2166\times 2^{2n-i}-\left(\frac{5415}{2}\right)\times 2^{2n-2i}\ -\ 2565\times 2^{n-1}+209\times 2^{n-i}+\left(\frac{1617}{2}\right)\right]}+$$

$$3\sum_{i=2}^{n} 2^{i-1}\ x^{\left[\left(\frac{7581}{4}\right)\times 2^{2n-i}-2166\times 2^{2n-2i}+\left(\frac{247}{2}\right)\times 2^{n-i}-\left(\frac{4845}{2}\right)\times 2^{n-1}+\left(\frac{1587}{2}\right)\right]}+$$

$$3\sum_{i=2}^{n} 2^{i}\ x^{\left[\left(\frac{1083}{4}\right)\times 2^{2n-i}-361\times 2^{2n-2i}-\ 114\times 2^{n-i}-\left(\frac{1881}{2}\right)\times 2^{n-1}+365\right]}$$

Theorem 5.2.2: Let D_n be the Nanostar Dendrimer. Then,

$$SK_{1C}[D_n, x] = 3x^{[390963\times 2^{4n-4}\ -\ 2160585\times 2^{3n-4}\ +1118739\times 2^{2n-2}-1029249\times 2^{n-2}\ +88725]}+$$

$$3\sum_{i=2}^{n} 2^{i-2}\, x^{\left[\begin{array}{c}1824494\times 2^{4n-4i}-\left(\frac{5864445}{2}\right)\times 2^{4n-3i}\ +1172889\times 2^{4n-2i}\ -\ 226347\times 2^{3n-3i}\ + \\ \left(\frac{7798683}{4}\right)\times 2^{3n-2i}\ -\left(\frac{5555790}{4}\right)\times 2^{3n-i}\ -\ 1105021\times 2^{2n-2i}\ +721278\times 2^{2n-i}\ + \\ 105963\times 2^{n-i}+\left(\frac{1673235}{4}\right)\times 2^{2n}\ -\left(\frac{2085915}{4}\right)\times 2^{n}\ +160925.\end{array}\right]}+$$

$$3\sum_{i=2}^{n} 2^{i-1}\ x^{\left[\begin{array}{c}\left(\frac{1172889}{4}\right)\times 2^{4n-2i}-\left(\frac{1954815}{4}\right)\times 2^{4n-3i}+\left(\frac{390963}{2}\right)\times 2^{4n-4i}\ +\left(\frac{226347}{2}\right)\times 2^{3n-3i}\ + \\ \left(\frac{473271}{2}\right)\times 2^{3n-2i}-\left(\frac{1666737}{4}\right)\times 2^{3n-i}-\left(\frac{562077}{2}\right)\times 2^{2n-2i}\ +\left(\frac{1616919}{4}\right)\times 2^{2n-i}\ + \\ \left(\frac{295659}{2}\right)\times 2^{2n}-\left(\frac{105963}{2}\right)\times 2^{n-i}\ -\left(\frac{461643}{2}\right)\times 2^{n}+88725.\end{array}\right]}+$$

Proof: From table (3), we compute polynomial of SK_1 index of the Nanostar Dendrimer D_n as follows:

$$SK_{1C}[D_n,x] = \sum_{uv\in E(G)} x^{\left[\frac{c(u)c(v)}{2}\right]}$$

$$= 3x^{\left[\frac{[3(19\times 2^{n-1}-13)^2\,((57\times 2^{n-1}-38)-(19\times 2^{n-1}-13))\times(19\times 2^{n-1}-13-1)]}{2}\right]} +$$

$$3\sum_{i=2}^{n} 2^{i-2}\, x^{\left[\frac{[((57\times 2^{n-1}-38)-(19\times 2^{n-1}-13)+5)\times(19\times 2^{n-(i-1)}-13-6)\times ((57\times 2^{n-1}-38)-(19\times 2^{n-1}-13)+6)\times(19\times 2^{n-(i-1)}-13-7)]}{2}\right]} +$$

$$3\sum_{i=2}^{n} 2^{i-2}\, x^{\left[\frac{[((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+11)\times(2(19\times 2^{n-i}-13)+1)\times [((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+12)\times 2(19\times 2^{n-i}-13)+(19\times 2^{n-i}-13)^2]}{2}\right]} +$$

$$3\sum_{i=2}^{n} 2^{i-1}\, x^{\left[\frac{[((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+12)\times + 2(19\times 2^{n-i}-13)+(19\times 2^{n-i}-13)^2\times ((57\times 2^{n-1}-38)-(19\times 2^{n-i}-13)\times(19\times 2^{n-i}-13-1)]}{2}\right]}$$

After some calculation, we get

$$SK_{1C}[D_n, x] = 3x^{[390963\times 2^{4n-4} - 2160585\times 2^{3n-4} + 1118739\times 2^{2n-2} - 1029249\times 2^{n-2} + 88725]} +$$

$$3\sum_{i=2}^{n} 2^{i-2}\, x^{\left[1824494\times 2^{4n-4i} - \left(\frac{5864445}{2}\right)\times 2^{4n-3i} + 1172889\times 2^{4n-2i} - 226347\times 2^{3n-3i} + \left(\frac{7798683}{4}\right)\times 2^{3n-2i} - \left(\frac{5555790}{4}\right)\times 2^{3n-i} - 1105021\times 2^{2n-2i} + 721278\times 2^{2n-i} + 105963\times 2^{n-i} + \left(\frac{1673235}{4}\right)\times 2^{2n} - \left(\frac{2085915}{4}\right)\times 2^{n} + 160925.\right]} +$$

$$3\sum_{i=2}^{n} 2^{i-1}\, x^{\left[\left(\frac{1172889}{4}\right)\times 2^{4n-2i} - \left(\frac{1954815}{4}\right)\times 2^{4n-3i} + \left(\frac{390963}{2}\right)\times 2^{4n-4i} + \left(\frac{226347}{2}\right)\times 2^{3n-3i} + \left(\frac{473271}{2}\right)\times 2^{3n-2i} - \left(\frac{1666737}{4}\right)\times 2^{3n-i} - \left(\frac{562077}{2}\right)\times 2^{2n-2i} + \left(\frac{1616919}{4}\right)\times 2^{2n-i} + \left(\frac{295659}{2}\right)\times 2^{2n} - \left(\frac{105963}{2}\right)\times 2^{n-i} - \left(\frac{461643}{2}\right)\times 2^{n} + 88725.\right]} +$$

Theorem: 5.2.3 Let D_n be the Nanostar Dendrimer. Then,

$$SK_{2C}[D_n, x]= [\tfrac{3}{4}]x^{[5343161\times 2^{4n-4} - 15076082\times 2^{3n-3} + 15854037\times 2^{2n-2} - 7350986\times 2^{n-1} + 1268249]}+$$

$$[\tfrac{3}{4}]\sum_{i=2}^{n} 2^{i-2}\, x^{\begin{bmatrix} 9383112\times 2^{4n-2i} - 23457780\times 2^{4n-3i} + 14726273\times 2^{4n-4i} - 2112572\times 2^{3n-3i} + \\ 15576789\times 2^{3n-2i} - 11111580\times 2^{3n-i} + 5789718\times 2^{2n-i} - 8546314\times 2^{2n-2i} + \\ 716452\times 2^{n-i} + 13392378\times 2^{2n-2} - 4180779\times 2^{n} + 1312049 \end{bmatrix}}+$$

$$[\tfrac{3}{4}]\sum_{i=2}^{n} 2^{i-1}\, x^{\begin{bmatrix} \left(\frac{19939113}{4}\right)\times 2^{4n-2i} - 10946964\times 2^{4n-3i} + 6255408\times 2^{4n-4i} - 713336\times 2^{3n-3i} + \\ 7304835\times 2^{3n-2i} - 6481755\times 2^{3n-i} - 4363407\times 2^{2n-2i} + 4203123\times 2^{2n-i} + \\ \left(\frac{8528625}{4}\right)\times 2^{2n} - 2885910\times 2^{n} - 36746\times 2^{n-i} + 1001349 \end{bmatrix}}+$$

$$[\tfrac{3}{4}]\sum_{i=2}^{n} 2^{i}\, x^{\begin{bmatrix} 2345778\times 2^{4n-2i-2} - 781926\times 2^{4n-3i} + 144039\times 2^{3n-2i} - \left(\frac{2037123}{2}\right)\times 2^{3n-i} + \\ 957372\times 2^{2n-i} + 260642\times 2^{4n-4i} + 164616\times 2^{3n-3i} + 288078\times 2^{3n-2i} - \\ 483018\times 2^{2n-2i} + 20577\times 2^{2n-i} - 160740\times 2^{n-i} + \left(\frac{1809693}{2}\right)\times 2^{2n} - \\ 690840\times 2^{n} + 266900 \end{bmatrix}}$$

Proof: From table (3), we compute polynomial of SK_2 index of the Nanostar Dendrimer D_n as follows:

$$SK_{2C}[D_n,x] = \sum_{uv\in E(G)} x^{[\frac{c(u)+c(v)}{2}]^2}$$

$$= (3\times 2^0)\, x^{\left[\frac{[(3(19\times 2^{n-1} - 13)^2 + ((57\times 2^{n-1} - 38) - (19\times 2^{n-1} - 13))\times(19\times 2^{n-1} - 13 - 1)]}{2}\right]^2}+$$

$$(3\times 2^1)\, x^{\left[\frac{[((57\times 2^{n-1} - 38) - (19\times 2^{n-1} - 13))\times(19\times 2^{n-1} - 13 - 1) + 0]}{2}\right]^2}+$$

$$(3\times 2^1)\, x^{\left[\frac{[((57\times 2^{n-1} - 38) - (19\times 2^{n-1} - 13) + 5)\times((19\times 2^{n-1} - 13) - 6) + 0]}{2}\right]^2}+$$

$$3\sum_{i=2}^{n} 2^{i-2}\, x^{\left[\frac{\begin{matrix}[((57\times 2^{n-1} - 38) - (19\times 2^{n-1} - 13) + 5)\times(19\times 2^{n-(i-1)} - 13 - 6) + \\ ((57\times 2^{n-1} - 38) - (19\times 2^{n-1} - 13) + 6)\times(19\times 2^{n-(i-1)} - 13 - 7)]\end{matrix}}{2}\right]^2}+$$

$$3\sum_{i=2}^{n} 2^{i-1}\, x^{\left[\frac{[((57\times 2^{n-1} - 38) - (19\times 2^{n-1} - 13) + 6)\times(19\times 2^{n-(i-1)} - 13 - 7) + 0]}{2}\right]^2}+$$

$$3\sum_{i=2}^{n} 2^{i-1}\, x^{\left[\frac{[((57\times 2^{n-1} - 38) - (19\times 2^{n-(i-1)} - 13) + 11)\times(2(19\times 2^{n-(i-1)} - 13) + 1)) + 0]}{2}\right]^2}+$$

$$3\sum_{i=2}^{n} 2^{i-2}\, x^{\left[\frac{[((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+11)\times(2(19\times 2^{n-(i-1)}-13)+1))+((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+11)\times 2(19\times 2^{n-i}-13)+(19\times 2^{n-i}-13)^{2}]}{2}\right]^{2}} +$$

$$3\sum_{i=2}^{n} 2^{i-1}\, x^{\left[\frac{[((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+11)\times 2(19\times 2^{n-i}-13)+(19\times 2^{n-i}-13)^{2}+((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13))\times(19\times 2^{n-i}-13-1)]}{2}\right]^{2}} +$$

$$3\sum_{i=2}^{n} 2^{i}\, x^{\left[\frac{[((57\times 2^{n-1}-38)-(19\times 2^{n-1}-13))\times(19\times 2^{n-i}-13-1)+0]}{2}\right]^{2}} +$$

$$3\sum_{i=2}^{n} 2^{i}\, x^{\left[\frac{[((57\times 2^{n-1}-38)-(19\times 2^{n-1}-13)+5)\times(19\times 2^{n-i}-13-6)+0]}{2}\right]^{2}} +$$

$$SK_{2C}[D_n,x]=\left[\frac{3}{4}\right]x^{[5343161\times 2^{4n-4}-15076082\times 2^{3n-3}+15854037\times 2^{2n-2}-7350986\times 2^{n-1}+1268249]}+$$

$$\left[\frac{3}{4}\right]\sum_{i=2}^{n} 2^{i-2}\, x^{\left[\begin{array}{c}9383112\times 2^{4n-2i}-23457780\times 2^{4n-3i}+14726273\times 2^{4n-4i}-2112572\times 2^{3n-3i}+\\ 15576789\times 2^{3n-2i}-11111580\times 2^{3n-i}+5789718\times 2^{2n-i}-8546314\times 2^{2n-2i}+\\ 716452\times 2^{n-i}+13392378\times 2^{2n-2}-4180779\times 2^{n}+1312049\end{array}\right]}+$$

$$\left[\frac{3}{4}\right]\sum_{i=2}^{n} 2^{i-1}\, x^{\left[\begin{array}{c}\left(\frac{19939113}{4}\right)\times 2^{4n-2i}-10946964\times 2^{4n-3i}+6255408\times 2^{4n-4i}-713336\times 2^{3n-3i}+\\ 7304835\times 2^{3n-2i}-6481755\times 2^{3n-i}-4363407\times 2^{2n-2i}+4203123\times 2^{2n-i}+\\ \left(\frac{8528625}{4}\right)\times 2^{2n}-2885910\times 2^{n}-36746\times 2^{n-i}+1001349\end{array}\right]}+$$

$$\left[\frac{3}{4}\right]\sum_{i=2}^{n} 2^{i}\, x^{\left[\begin{array}{c}2345778\times 2^{4n-2i-2}-781926\times 2^{4n-3i}+144039\times 2^{3n-2i}-\left(\frac{2037123}{2}\right)\times 2^{3n-i}+\\ 957372\times 2^{2n-i}+260642\times 2^{4n-4i}+164616\times 2^{3n-3i}+288078\times 2^{3n-2i}-\\ 483018\times 2^{2n-2i}+20577\times 2^{2n-i}-160740\times 2^{n-i}+\left(\frac{1809693}{2}\right)\times 2^{2n}-\\ 690840\times 2^{n}+266900\end{array}\right]}$$

5.3 Polynomial of Nano-Zagreb and sum Nano-Zagreb cutting number topological indices of Nanostar Dendrimer D_n

Theorem: 5.3.1 Let D_n be the Nanostar Dendrimer. Then,

$$\mathcal{NZ}_C[D_n,x] = 3\, x^{(651605\times 2^{4n-4}-175594\times 2^{3n-3}+1775037\times 2^{2n-2}-797848\times 2^{n-1}+134549)} +$$

$$6\, x^{(521284\times 2^{4n-4}-1454108\times 2^{3n-3}+1519449\times 2^{2n-2}-704900\times 2^{n-1}+1225000)} +$$

$$6\, x^{(521284\times 2^{4n-4}-1591288\times 2^{3n-3}+1763124\times 2^{2n-2}-837520\times 2^{n-1}+144400)} +$$

$$3\sum_{i=2}^{n} 2^{i-2}\, x^{\left(\begin{array}{c}912247\times 2^{4n-4i}-781926\times 2^{4n-3i}-987696\times 2^{3n-3i}+1111158\times 2^{3n-2i}+\\ 123462\times 2^{3n-i}+262086\times 2^{2n-2i}-851238\times 2^{2n-i}+234840\times 2^{n-i}+\\ 3203514\times 2^{2n-2}-873981\times 2^{2n}+274284\times 2^{n}-134549\end{array}\right)} +$$

$$3\sum_{i=2}^{n} 2^{i-1}\, x^{(5212840 \times 2^{4n-4i} - 821223 \times 2^{4n-3i} + [\frac{1291779}{4}] \times 2^{4n-2i} - 1467826 \times 2^{3n-3i} + 6234831 \times 2^{3n-2i} - 3950784 \times 2^{3n-i} + 1494540 \times 2^{2n-i} - 2821215 \times 2^{2n-2i} + 467780 \times 2^{n-i} + 714780 \times 2^{2n} + 2030625 \times 2^{2n-2} - 1404024 \times 2^{n} + 401449)} +$$

$$3\sum_{i=2}^{n} 2^{i}\, x^{(260642 \times 2^{4n-4i} - 781926 \times 2^{4n-3i} + 2345778 \times 2^{4n-2i-2} + 164616 \times 2^{3n-3i} + 432117 \times 2^{3n-2i} - 432117 \times 2^{3n-i} - 2345778 \times 2^{3n-i-2} - 483018 \times 2^{2n-2i} + 977949 \times 2^{2n-i} - 160740 \times 2^{n-i} + 159201 \times 2^{2n} + 1172889 \times 2^{2n-2} - 690840 \times 2^{2n} + 266900)}$$

Proof: From table (3), we derive polynomial of Nano-Zagreb index of the Nanostar Dendrimer D_n as follows:

$$\mathcal{NZ}c[D_n,x] = \sum_{uv\in E(D_n)} x^{(c^2(u) - c^2(v))}$$

$$= 3x^{\{\{(3(19 \times 2^{n-1} - 13)^2\}^2 - \{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \times (19 \times 2^{n-1} - 13 - 1)\}^2\}} +$$

$$6x^{\{\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \times (19 \times 2^{n-1} - 13 - 1)\}^2 - 0^2\}} +$$

$$6x^{\{\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13) + 5) \times (19 \times 2^{n-1} - 13 - 6)\}^2 - 0^2\}} +$$

$$3\sum_{i=2}^{n} 2^{i-2}\, x^{\{\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 5) \times (19 \times 2^{n-(i-1)} - 13 - 6)\}^2 - \{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 6) \times (19 \times 2^{n-(i-1)} - 13 - 7)\}^2\}} +$$

$$3\sum_{i=2}^{n} 2^{i-1}\, x^{\{\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 6) \times (19 \times 2^{n-(i-1)} - 13 - 7)\}^2 - 0^2\}} +$$

$$3\sum_{i=2}^{n} 2^{i-1}\, x^{\{\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 11) \times (2(19 \times 2^{n-i} - 13) + 1)\}^2 - 0^2\}} +$$

$$3\sum_{i=2}^{n} 2^{i-2}\, x^{\{\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 11) \times (2(19 \times 2^{n-i} - 13) + 1)\}^2 - \{((57 \times 2^{n-1} - 38) - (19\times 2^{n-(i-1)} - 13) + 12) \times (2(19\times 2^{n-i} - 13) + (19\times 2^{n-i} - 13)^2\}^2\}} +$$

$$3\sum_{i=2}^{n} 2^{i-1}\, x^{\{((57\times 2^{n-1} - 38) - (19\times 2^{n-(i-1)} - 13) + 12) \times (2(19\times 2^{n-i} - 13) + (19\times 2^{n-i} - 13)^2\}^2 - \{\{((57\times 2^{n-1} - 38) - (19\times 2^{n-i} - 13)) \times (19\times 2^{n-i} - 14)\}^2\}} +$$

$$3\sum_{i=2}^{n} 2^{i}\, x^{\{\{((57 \times 2^{n-1} - 38) - (19\times 2^{n-i} - 13)) \times (19 \times 2^{n-i} - 14)\}^2 - 0^2\}} +$$

$$3\sum_{i=2}^{n} 2^{i}\, x^{\{\{((57 \times 2^{n-1} - 38) - (19\times 2^{n-i} - 13) + 5) \times \{\{(19 \times 2^{n-i} - 13 - 6)\}^2 - 0^2\}}$$

After simplification, we get

$$\mathcal{NZ}c[D_n,x] = 3x^{(651605 \times 2^{4n-4} - 175594 \times 2^{3n-3} + 1775037 \times 2^{2n-2} - 797848 \times 2^{n-1} + 134549)} +$$

$$6x^{(521284 \times 2^{4n-4} - 1454108 \times 2^{3n-3} + 1519449 \times 2^{2n-2} - 704900 \times 2^{n-1} + 1225000)} +$$

$$6x^{(521284 \times 2^{4n-4} - 1591288 \times 2^{3n-3} + 1763124 \times 2^{2n-2} - 837520 \times 2^{n-1} + 144400)} +$$

$$3\sum_{i=2}^{n} 2^{i-2}\, x^{\begin{array}{c}(912247 \times 2^{4n-4i} - 781926 \times 2^{4n-3i} - 987696 \times 2^{3n-3i} + 1111158 \times 2^{3n-2i} + \\ 123462 \times 2^{3n-i} + 262086 \times 2^{2n-2i} - 851238 \times 2^{2n-i} + 234840 \times 2^{n-i} + \\ 3203514 \times 2^{2n-2} - 873981 \times 2^{2n} + 274284 \times 2^{n} - 134549)\end{array}} +$$

$$3\sum_{i=2}^{n} 2^{i-1}\, x^{\begin{array}{c}(5212840 \times 2^{4n-4i} - 821223 \times 2^{4n-3i} + [\frac{1291779}{4}] \times 2^{4n-2i} - 1467826 \times 2^{3n-3i} + \\ 6234831 \times 2^{3n-2i} - 3950784 \times 2^{3n-i} + 1494540 \times 2^{2n-i} - 2821215 \times 2^{2n-2i} + \\ 467780 \times 2^{n-i} + 714780 \times 2^{2n} + 2030625 \times 2^{2n-2} - 1404024 \times 2^{n} + 401449)\end{array}} +$$

$$3\sum_{i=2}^{n} 2^{i}\, x^{\begin{array}{c}(260642 \times 2^{4n-4i} - 781926 \times 2^{4n-3i} + 2345778 \times 2^{4n-2i-2} + 164616 \times 2^{3n-3i} + \\ 432117 \times 2^{3n-2i} - 432117 \times 2^{3n-i} - 2345778 \times 2^{3n-i-2} - 483018 \times 2^{2n-2i} + \\ 977949 \times 2^{2n-i} - 160740 \times 2^{n-i} + 159201 \times 2^{2n} + 1172889 \times 2^{2n-2} - \\ 690840 \times 2^{2n} + 266900)\end{array}}$$

Theorem: 5.3.2 Let D_n be the Nanostar Dendrimer. Then,

$\chi_\alpha NZ_C[D_n, x]$

$$= 3x^{[651605 \times 2^{4n-4} - 1755904 \times 2^{3n-3} + 1775037 \times 2^{2n-2} - 797848 \times 2^{n-1} + 134549]^{\alpha}} +$$

$$6x^{[521284 \times 2^{4n-4} - 1454108 \times 2^{3n-3} + 1519449 \times 2^{2n-2} - 704900 \times 2^{n-1} + 122500]^{\alpha}} +$$

$$6x^{[521284 \times 2^{4n-4} - 1591288 \times 2^{3n-3} + 1763124 \times 2^{2n-2} - 837520 \times 2^{n-1} + 144400]^{\alpha}} +$$

$$3\sum_{i=2}^{n} 2^{i-2}\, x^{\begin{array}{c}[912247 \times 2^{4n-4i} - 781926 \times 2^{4n-3i} - 987696 \times 2^{3n-3i} + 1111158 \times \\ 2^{3n-2i} + 123462 \times 2^{3n-i} + 262086 \times 2^{2n-2i} - 851238 \times 2^{2n-i} + 234840 \times \\ 2^{n-i} + 3203514 \times 2^{2n-2} - 873981 \times 2^{2n} + 274284 \times 2^{n} - 134549]^{\alpha}\end{array}} +$$

$$3\sum_{i=2}^{n} 2^{i-1} x^{\begin{array}{c}[5212840 \times 2^{4n-4i} - 8210223 \times 2^{4n-3i} + [\frac{12901779}{4}] \times 2^{4n-2i} - \\ 67826 \times 2^{3n-3i} + 146234831 \times 2^{3n-2i} - 3950784 \times 2^{3n-i} - 2821215 \times 2^{2n-2i} + \\ 1494540 \times 2^{2n-i} + 467780 \times 2^{n-i} + 2030625 \times 2^{2n-2} + 714780 \times 2^{2n} - 997500 \times \\ 2^{n-1} - 905274 \times 2^{n} + 401449]^{\alpha}\end{array}}$$

$$+3\sum_{i=2}^{n} 2^{i}\, x^{\begin{array}{c}[2345778 \times 2^{4n-4i-2} + 260642 \times 2^{4n-4i} - 781926 \times 2^{4n-3i} + 164616 \times 2^{3n-3i} + \\ 432117 \times 2^{3n-2i} - 432117 \times 2^{3n-i} - 2345778 \times 2^{3n-i-2} - 483018 \times 2^{2n-2i} + \\ 977949 \times 2^{2n-i} - 160740 \times 2^{n-i} + 1172889 \times 2^{2n-2} + 159201 \times 2^{2n} - \\ 690840 \times 2^{n} + 266900]^{\alpha}\end{array}}$$

Proof: From table (3), we derive polynomial of sum Nano-Zagreb index of the Nanostar Dendrimer D_n as follows:

$$\chi_\alpha \mathcal{N}Z_C[D_n, x] = \sum_{uv \in E(G)} x^{(c^2(u) - c^2(v))^{\alpha}}$$

$$= (3 \times 2^0)\, x^{\begin{array}{c}\{\{(3(19 \times 2^{n-1} - 13)^2\}^2 - \{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \times \\ (19 \times 2^{n-1} - 13 - 1)\}^2\}^{\alpha}\end{array}} +$$

$$(3 \times 2^1)\, x^{\{\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \times (19 \times 2^{n-1} - 13 - 1)\}^2 - 0^2\}^{\alpha}} +$$

$$(3 \times 2^1)\, x^{\{\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13) + 5) \times (19 \times 2^{n-1} - 13 - 6)\}^2 - 0^2\}^{\alpha}} +$$

$$\{\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 5) \times (19 \times 2^{n-(i-1)} - 13 - 6)\}^2 -$$

$$3\sum_{i=2}^{n} 2^{i-2}\, x^{\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 6) \times (19 \times 2^{n-(i-1)} - 13 - 7)\}^2\,\}^{\alpha}} +$$

$$3\sum_{i=2}^{n} 2^{i-1}\, x^{\{\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 6) \times (19 \times 2^{n-(i-1)} - 13 - 7)\}^2 - 0^2\,\}^{\alpha}} +$$

$$3\sum_{i=2}^{n} 2^{i-1}\, x^{\{\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 11) \times (2(19 \times 2^{n-(i-1)} - 13) + 1)\}^2 - 0^2\,\}^{\alpha}} +$$

$$3\sum_{i=2}^{n} 2^{i-2}\, x^{\begin{array}{c}\{\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 11) \times (2(19 \times 2^{n-i} - 13) + 1)\}^2 - \\ \{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 12) \times (19 \times 2^{n-i} - 13)^2\}^2\,\}^{\alpha}\end{array}} +$$

$$3\sum_{i=2}^{n} 2^{i-1}\, x^{\begin{array}{c}\{\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 12) \times 2(19 \times 2^{n-i} - 13)\} + \\ (19 \times 2^{n-i} - 13)^2\}^2 - \{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-i} - 13)) \times (19 \times 2^{n-i} - 13 - 1)\}^2\,\}^{\alpha}\end{array}} +$$

$$3\sum_{i=2}^{n} 2^{i}\, x^{\{\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-i} - 13)) \times (19 \times 2^{n-i} - 13 - 1)\}^2 - 0^2\,\}^{\alpha}} +$$

$$3\sum_{i=2}^{n} 2^{i} x^{\{\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-i} - 13) + 5) \times (19 \times 2^{n-i} - 13 - 6)\}^2 - 0^2\,\}^{\alpha}}$$

After simplification, we obtain

$$\chi_{\alpha} NZ_{C}[D_n, x] = 3x^{[651605 \times 2^{4n-4} - 1755904 \times 2^{3n-3} + 1775037 \times 2^{2n-2} - 797848 \times 2^{n-1} + 134549]^{\alpha}} +$$

$$6x^{[521284 \times 2^{4n-4} - 1454108 \times 2^{3n-3} + 1519449 \times 2^{2n-2} - 704900 \times 2^{n-1} + 122500]^{\alpha}} +$$

$$6x^{[521284 \times 2^{4n-4} - 1591288 \times 2^{3n-3} + 1763124 \times 2^{2n-2} - 837520 \times 2^{n-1} + 144400]^{\alpha}} +$$

$$3\sum_{i=2}^{n} 2^{i-2}\, x^{\begin{array}{c}[912247 \times 2^{4n-4i} - 781926 \times 2^{4n-3i} - 987696 \times 2^{3n-3i} + 1111158 \times \\ 2^{3n-2i} + 123462 \times 2^{3n-i} + 262086 \times 2^{2n-2i} - 851238 \times 2^{2n-i} + 234840 \times \\ 2^{n-i} + 3203514 \times 2^{2n-2} - 873981 \times 2^{2n} + 274284 \times 2^{n} - 134549]^{\alpha}\end{array}} +$$

$$3\sum_{i=2}^{n} 2^{i-1} x^{\begin{array}{c}[5212840 \times 2^{4n-4i} - 8210223 \times 2^{4n-3i} + [\frac{12901779}{4}] \times 2^{4n-2i} - 67826 \times 2^{3n-3i} + \\ 146234831 \times 2^{3n-2i} - 3950784 \times 2^{3n-i} - 2821215 \times 2^{2n-2i} + 1494540 \times 2^{2n-i} + \\ 467780 \times 2^{n-i} + 2030625 \times 2^{2n-2} + 714780 \times 2^{2n} - 997500 \times 2^{n-1} - \\ 905274 \times 2^{n} + 401449]^{\alpha}\end{array}} +$$

$$3\sum_{i=2}^{n} 2^{i}\, x^{\begin{array}{c}[2345778 \times 2^{4n-4i-2} + 260642 \times 2^{4n-4i} - 781926 \times 2^{4n-3i} + 164616 \times 2^{3n-3i} + \\ 432117 \times 2^{3n-2i} - 432117 \times 2^{3n-i} - 2345778 \times 2^{3n-i-2} - 483018 \times 2^{2n-2i} + \\ 977949 \times 2^{2n-i} - 160740 \times 2^{n-i} + 1172889 \times 2^{2n-2} + 159201 \times 2^{2n} - \\ 690840 \times 2^{n} + 266900]^{\alpha}\end{array}}$$

CHAPTER 6

EDGE BASED MULTIPLICATIVE CUTTING NUMBER TOPOLOGICAL INDICES OF NANOSTAR DENDRIMER D_n

In this chapter, we define Multiplicative of First, Second, Third, Fourth and Fifth cutting number indices, Multiplicative of SK, SK_1 and SK_2 cutting number indices, Multiplicative of Nano-Zagreb and sum Nano-Zagreb cutting number indices of a graph G. Here we introduced new indices on cutting number namely

(i) Multiplicative of First, Second, Third, Fourth and Fifth cutting number indices as,

$$M_{1C}\prod(G) = \prod_{uv\in E(G)}(c(u)+c(v))$$

$$M_{2C}\prod(G) = \prod_{uv\in E(G)}(c(u)c(v))$$

$$M_{3C}\prod(G) = \prod_{uv\in E(G)} |\, c(u) - c(v) \,|$$

$$M_{4C}\prod(G) = \prod_{uv\in E(G)} c(u)[(c(u)+c(v)]$$

$$M_{5C}\prod(G) = \prod_{uv\in E(G)} c(v)[(c(u)+c(v)]$$

(ii) Multiplicative of SK, SK_1 and SK_2 cutting number indices are defined as,

$$SK_C\prod(G) = \prod_{uv\in E(G)}[\frac{c(u)+c(v)}{2}]$$

$$SK_{1C}\prod(G) = \prod_{uv\in E(G)}[\frac{c(u)c(v)}{2}]$$

$$SK_{2C}\prod(G) = \prod_{uv\in E(G)}[\frac{c(u)+c(v)}{2}]^2$$

(iii) Multiplicative of Nano-Zagreb and sum Nano-Zagreb cutting number indices are defined as,

$$\mathcal{NZ}c\prod(G) = \prod_{uv\in E(G)} (c^2(u) - c^2(v))$$

$$\chi_\alpha\mathcal{NZ}c\prod(G) = \prod_{uv\in E(G)} (c^2(u) - c^2(v))^\alpha$$

6.1 Multiplicative of First, Second, Third, Fourth and Fifth Zagreb cutting number topological indices of Nanostar Dendrimer D_n

Theorem 6.1.1: Multiplicative of first Zagreb index of Nanostar Dendrimer D_n, is given by

$$M_{1C}\prod [D_n] = 108[(1805 \times 2^{2n-2} - 2489 \times 2^{n-1} + 857)$$
$$(722 \times 2^{2n-2} - 1007 \times 2^{n-1} + 350)$$
$$(722 \times 2^{2n-2} - 1102 \times 2^{n-1} + 380)] \times$$
$$3\prod_{i=2}^{n} 2^{i-2} [(2166 \times 2^{2n-i} - 2888 \times 2^{2n-2i} - 2223\times 2^{n-1} + 760)$$
$$(2166 \times 2^{2n-i} - 2527\times 2^{2n-2i} + 418 \times 2^{n-i} - 2907\times 2^{n-1} + 857)] \times$$
$$3\prod_{i=2}^{n} 2^{i-1}[(1083 \times 2^{2n-i} - 1444\times 2^{2n-2i} + 38\times 2^{n-i} - 1140 \times 2^{n-1} + 380)$$
$$(1083 \times 2^{2n-i} - 1444 \times 2^{2n-2i} + 418\times 2^{n-i} - 1425 \times 2^{n-1} + 350)$$
$$(3249 \times 2^{2n-i-1} - 1444\times 2^{2n-2i} - 209 \times 2^{n-i} - 2280 \times 2^{n-1} + 857)] \times$$
$$3\prod_{i=2}^{n} 2^{i}[(1083 \times 2^{2n-i-1} - 361\times 2^{2n-2i} - 209 \times 2^{n-i} - 798 \times 2^{n-1} + 350)$$
$$(1083 \times 2^{2n-i-1} - 361 \times 2^{2n-2i} - 19 \times 2^{n-i} - 1083 \times 2^{n-1} + 380)]$$

Proof: using the data given in table (3), multiplicative of first Zagreb index of Nanostar Dendrimer D_n, can be written as

$$M_{1C}\prod[D_n] = \prod_{uv\in E(D_n)} \left(c(u) + c(v)\right)$$
$$= (3 \times 2^0)\ [\{(3(19\times 2^{n-1} - 13)^2\} + \{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \times (19 \times 2^{n-1} - 13 - 1)\}]$$
$$(3 \times 2^1)\ [\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \times (19 \times 2^{n-1} - 13 - 1)\} + 0] \times$$
$$(3 \times 2^1)\ [\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13) + 5) \times (19 \times 2^{n-1} - 13 - 6)\} + 0] \times$$
$$3\prod_{i=2}^{n} 2^{i-2}\ [\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13)+5) \times (19 \times 2^{n-(i-1)} - 13 - 6)\}+\{((57\times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13)+6) \times (19 \times 2^{n-(i-1)} - 13 - 7)\}] \times$$
$$3 \prod_{i=2}^{n} 2^{i-1}\ [\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 6) \times (19 \times 2^{n-(i-1)} - 13 - 7)\} + 0] \times$$
$$3 \prod_{i=2}^{n} 2^{i-1}\ [\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 11) \times (2(19 \times 2^{n-(i-1)} - 13) + 1)\} + 0] \times$$

$$3\prod_{i=2}^{n}2^{i-2}\ [\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+11)\times$$
$$(2(19\times 2^{n-i}-13)+1\}+\{((57\times 2^{n-1}-38)-$$
$$\left(19\times 2^{n-(i-1)}-13\right)+12)\times\ (2\left(19\times 2^{n-i}-13\right)+$$
$$(19\times 2^{n-i}-13)^{2}\}]\times$$
$$3\prod_{i=2}^{n}2^{i-1}[\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+12)\times$$
$$2(19\times 2^{n-i}-13)+(19\times 2^{n-i}-13)^{2}\}+\{((57\times 2^{n-1}-38)-$$
$$(19\times 2^{n-i}-13)\times(19\times 2^{n-i}-13-1)\}]\times$$
$$3\prod_{i=2}^{n}2^{i}\ [\{((57\times 2^{n-1}-38)-(19\times 2^{n-i}-13))\times$$
$$(19\times 2^{n-i}-13-1)\}+0]\ \times$$
$$3\prod_{i=2}^{n}2^{i}[\{((57\times 2^{n-1}-38)-(19\times 2^{n-i}-13)+5)\times$$
$$(19\times 2^{n-i}-13-6)\}+0]$$

After simplification, we obtain

$$M_{1C}\Pi[D_n]=108[(1805\times 2^{2n-2}-2489\times 2^{n-1}+857)$$
$$(722\times 2^{2n-2}-1007\times 2^{n-1}+350)$$
$$(722\times 2^{2n-2}-1102\times 2^{n-1}+380)]\times$$
$$3\prod_{i=2}^{n}2^{i-2}\ [(2166\times 2^{2n-i}-2888\times 2^{2n-2i}-2223\times 2^{n-1}+760)$$
$$(2166\times 2^{2n-i}-2527\times 2^{2n-2i}+418\times 2^{n-i}-2907\times 2^{n-1}+857)]\times$$
$$3\prod_{i=2}^{n}2^{i-1}\ [(1083\times 2^{2n-i}-1444\times 2^{2n-2i}+38\times 2^{n-i}-1140\times 2^{n-1}+380)$$
$$(1083\times 2^{2n-i}-1444\times 2^{2n-2i}+418\times 2^{n-i}-1425\times 2^{n-1}+350)$$
$$(3249\times 2^{2n-i-1}-1444\times 2^{2n-2i}-209\times 2^{n-i}-2280\times 2^{n-1}+857)]\times$$
$$3\prod_{i=2}^{n}2^{i}[(1083\times 2^{2n-i-1}-361\times 2^{2n-2i}-209\times 2^{n-i}-798\times 2^{n-1}+350)$$
$$(1083\times 2^{2n-i-1}-361\times 2^{2n-2i}-19\times 2^{n-i}-1083\times 2^{n-1}+380)]$$

Theorem: 6.1.2 Multiplicative of second Zagreb index of Nanostar Dendrimer D_n, is given by

$$M_{2C}\Pi[D_n]=3[(1083\times 2^{2n-2}-1482\times 2^{n-1}+507)$$
$$(722\times 2^{2n-2}-1007\times 2^{n-1}+350)]\times$$
$$3\prod_{i=2}^{n}2^{i-2}\ [(1083\times 2^{2n-i}-1444\times 2^{2n-2i}-38\times 2^{n-i}-1083\times 2^{n-1}+380)$$
$$(1083\times 2^{2n-i}-1444\times 2^{2n-2i}+38\times 2^{n-i}-1140\times 2^{n-1}+380)$$
$$(1083\times 2^{2n-i}-1444\times 2^{2n-2i}+418\times 2^{n-i}-1425\times 2^{n-1}+350)$$
$$(1083\times 2^{2n-i}-1083\times 2^{2n-2i}-1482\times 2^{n-1}+507)]$$

$$3\prod_{i=2}^{n}2^{i-1}\,[(1083\times 2^{2n-\mathrm{i}} - 1083\times 2^{2n-2\mathrm{i}} - 1482\times 2^{n-1} + 507)$$

$$(1083\times 2^{2n-\mathrm{i}-1} - 361\times 2^{2n-2\mathrm{i}} - 209\times 2^{n-\mathrm{i}} - 798\times 2^{n-1} + 350)]$$

Proof: $M_{2C}\Pi[D_n] = \prod_{uv\in E(\mathrm{Dn})}\big(c(u)c(v)\big)$

$$= (3\times 2^0)\ [\{(3(19\times 2^{n-1} - 13)^2\} \times ((57\times 2^{n-1} - 38) -$$
$$(19\times 2^{n-1} - 13)) \times (19\times 2^{n-1} - 13 - 1)] \times$$
$$(3\times 2^1)\ [((57\times 2^{n-1} - 38) - (19\times 2^{n-1} - 13)) \times$$
$$(19\times 2^{n-1} - 13 - 1) \times 0] \times$$
$$(3\times 2^1)\ [((57\times 2^{n-1} - 38) - (19\times 2^{n-1} - 13) + 5) \times$$
$$(19\times 2^{n-1} - 13) - 6) \times 0] \times$$
$$3\prod_{i=2}^{n}2^{i-2}\ [\{((57\times 2^{n-1} - 38) - (19\times 2^{n-(\mathrm{i}-1)} - 13) + 5) \times$$
$$(19\times 2^{n-(\mathrm{i}-1)} - 13 - 6)\} \times \{((57\times 2^{n-1} - 38) -$$
$$(19\times 2^{n-(\mathrm{i}-1)} - 13) + 6) \times (19\times 2^{n-(\mathrm{i}-1)} - 13 - 7)\}] \times$$
$$3\prod_{i=2}^{n}2^{i-1}\,[((57\times 2^{n-1} - 38) - (19\times 2^{n-(\mathrm{i}-1)} - 13) + 6) \times$$
$$(19\times 2^{n-(\mathrm{i}-1)} - 13 - 7) \times 0] \times$$
$$3\prod_{i=2}^{n}2^{i-1}\,[((57\times 2^{n-1} - 38) - (19\times 2^{n-(\mathrm{i}-1)} - 13) + 11) \times$$
$$(2(19\times 2^{n-\mathrm{i}} - 13) + 1) \times 0] \times$$
$$3\prod_{i=2}^{n}2^{i-2}\,[\{((57\times 2^{n-1} - 38) - (19\times 2^{n-(\mathrm{i}-1)} - 13) + 11) \times$$
$$(2(19\times 2^{n-\mathrm{i}} - 13) + 1\} \times \{((57\times 2^{n-1} - 38) -$$
$$\big(19\times 2^{n-(\mathrm{i}-1)} - 13\big) + 12) \times (2(19\times 2^{n-\mathrm{i}} - 13) +$$
$$\big(19\times 2^{n-\mathrm{i}} - 13\big)^2\}] \times$$
$$3\prod_{i=2}^{n}2^{i-1}\,[\{((57\times 2^{n-1} - 38) - (19\times 2^{n-(\mathrm{i}-1)} - 13) + 12) \times$$
$$2(19\times 2^{n-\mathrm{i}} - 13) + (19\times 2^{n-\mathrm{i}} - 13)^2\} \times \{((57\times 2^{n-1} - 38) -$$
$$(19\times 2^{n-\mathrm{i}} - 13) \times (19\times 2^{n-\mathrm{i}} - 13 - 1)\}] \times$$
$$3\prod_{i=2}^{n}2^{i}\quad [((57\times 2^{n-1} - 38) - (19\times 2^{n-\mathrm{i}} - 13)) \times$$
$$(19\times 2^{n-\mathrm{i}} - 13 - 1) \times 0] \times$$
$$3\prod_{i=2}^{n}2^{i}\quad [((57\times 2^{n-1} - 38) - (19\times 2^{n-\mathrm{i}} - 13) + 5) \times$$
$$(19\times 2^{n-\mathrm{i}} - 13 - 6) \times 0]$$

After simplification, we get

$$M_{2C}\Pi[D_n] = 3[(1083\times 2^{2n-2} - 1482\times 2^{n-1} + 507)$$
$$(722\times 2^{2n-2} - 1007\times 2^{n-1} + 350)] \times$$
$$3\prod_{i=2}^{n}2^{i-2}\,[(1083\times 2^{2n-\mathrm{i}} - 1444\times 2^{2n-2\mathrm{i}} - 38\times 2^{n-\mathrm{i}} - 1083\times 2^{n-1} + 380)$$

$$(1083 \times 2^{2n-\mathrm{i}} - 1444 \times 2^{2n-2\mathrm{i}} + 38 \times 2^{n-\mathrm{i}} - 1140 \times 2^{n-1} + 380)$$
$$(1083 \times 2^{2n-\mathrm{i}} - 1444 \times 2^{2n-2\mathrm{i}} + 418 \times 2^{n-\mathrm{i}} - 1425 \times 2^{n-1} + 350)$$
$$(1083 \times 2^{2n-\mathrm{i}} - 1083 \times 2^{2n-2\mathrm{i}} - 1482 \times 2^{n-1} + 507)]$$
$$3 \prod_{i=2}^{n} 2^{i-1}[(1083 \times 2^{2n-\mathrm{i}} - 1083 \times 2^{2n-2\mathrm{i}} - 1482 \times 2^{n-1} + 507)$$
$$(1083 \times 2^{2n-\mathrm{i}-1} - 361 \times 2^{2n-2\mathrm{i}} - 209 \times 2^{n-\mathrm{i}} - 798 \times 2^{n-1} + 350)]$$

Theorem 6.1.3 Multiplicative of third Zagreb index Nanostar Dendrimer D_n, is given by $M_{3C}\Pi[D_n] = 108[(361 \times 2^{2n-2} - 475 \times 2^{n-1} + 157)$

$$(722 \times 2^{2n-2} - 1007 \times 2^{n-1} + 350)$$
$$(722 \times 2^{2n-2} - 1102 \times 2^{n-1} + 380)] \times$$
$$3 \prod_{i=2}^{n} 2^{i-2} [(-76 \times 2^{n-i} + 57 \times 2^{n-1})$$
$$(418 \times 2^{n-\mathrm{i}} + 57 \times 2^{n-1} - 361 \times 2^{2n-2\mathrm{i}} - 157)] \times$$
$$3 \prod_{i=2}^{n} 2^{i-1} [(1083 \times 2^{2n-\mathrm{i}} - 1444 \times 2^{2n-2\mathrm{i}} + 38 \times 2^{n-\mathrm{i}} - 1140 \times 2^{n-1} + 380)$$
$$(1083 \times 2^{2n-\mathrm{i}} - 1444 \times 2^{2n-2\mathrm{i}} - 1425 \times 2^{n-1} + 418 \times 2^{n-\mathrm{i}} + 350)$$
$$(1083 \times 2^{2n-\mathrm{i}-1} - 722 \times 2^{2n-2\mathrm{i}} - 684 \times 2^{n-1} + 209 \times 2^{n-\mathrm{i}} + 157)]$$
$$3 \prod_{i=2}^{n} 2^{i} [(1083 \times 2^{2n-\mathrm{i}} - 361 \times 2^{2n-2\mathrm{i}} - 209 \times 2^{n-\mathrm{i}} - 798 \times 2^{n-1} + 350)$$
$$(1083 \times 2^{2n-\mathrm{i}-1} - 361 \times 2^{2n-2\mathrm{i}} - 19 \times 2^{n-\mathrm{i}} - 1083 \times 2^{n-1} + 380)]$$

Proof: $M_{3C}\Pi[D_n] = \prod_{uv \in E(\mathrm{Dn})} |c(u) - c(v)|$

$$= (3 \times 2^0) \mid [(3(19 \times 2^{n-1} - 13)^2 - ((57 \times 2^{n-1} - 38) -$$
$$(19 \times 2^{n-1} - 13)) \times (19 \times 2^{n-1} - 13 - 1)] \mid \times$$
$$(3 \times 2^1) \mid [((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \times$$
$$(19 \times 2^{n-1} - 13 - 1) - 0] \mid \times$$
$$(3 \times 2^1) \mid [((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13) + 5) \times$$
$$(19 \times 2^{n-1} - 13 - 6) - 0] \mid \times$$
$$3 \prod_{i=2}^{n} 2^{i-2} \mid [\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(\mathrm{i}-1)} - 13) + 5) \times$$
$$(19 \times 2^{n-(\mathrm{i}-1)} - 13 - 6)\} - \{((57 \times 2^{n-1} - 38) -$$
$$(19 \times 2^{n-(\mathrm{i}-1)} - 13) + 6) \times (19 \times 2^{n-(\mathrm{i}-1)} - 13 - 7)\}] \mid \times$$
$$3 \prod_{i=2}^{n} 2^{i-1} \mid [((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(\mathrm{i}-1)} - 13) + 6) \times$$
$$(19 \times 2^{n-(\mathrm{i}-1)} - 13 - 7) - 0] \mid \times$$
$$3 \prod_{i=2}^{n} 2^{i-1} \mid [((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(\mathrm{i}-1)} - 13) + 11) \times$$
$$(2(19 \times 2^{n-\mathrm{i}} - 13) + 1) - 0] \mid \times$$

$$3\prod_{i=2}^{n} 2^{i-2} \mid [\{((57\times 2^{n-1} - 38) - (19\times 2^{n-(i-1)} - 13) + 11)\times$$
$$(2(19\times 2^{n-i} - 13) + 1\} - \{((57\times 2^{n-1} - 38) -$$
$$(19\times 2^{n-(i-1)} - 13) + 12) \times (2(19\times 2^{n-i} - 13) +$$
$$(19\times 2^{n-i} - 13)^{2}\}]| \times$$
$$3\prod_{i=2}^{n} 2^{i-1} \mid [\{((57\times 2^{n-1} - 38) - (19\times 2^{n-(i-1)} - 13) + 12)\times$$
$$2(19\times 2^{n-i} - 13) + (19\times 2^{n-i} - 13)^{2}\} - \{((57\times 2^{n-1} - 38) -$$
$$(19\times 2^{n-i} - 13)(19\times 2^{n-i} - 13 - 1)\}]| \times$$
$$3\prod_{i=2}^{n} 2^{i} \mid [((57\times 2^{n-1} - 38) - (19\times 2^{n-i} - 13))\times$$
$$(19\times 2^{n-i} - 13 - 1) - 0] \mid \times$$
$$3\prod_{i=2}^{n} 2^{i} \mid [((57\times 2^{n-1} - 38) - (19\times 2^{n-i} - 13) + 5)\times$$
$$(19\times 2^{n-i} - 13 - 6) - 0] \mid$$

After simplification, we get

$$M_{3C}\Pi[D_n] = 108[(361\times 2^{2n-2} - 475\times 2^{n-1} + 157)$$
$$(722\times 2^{2n-2} - 1007\times 2^{n-1} + 350)$$
$$(722\times 2^{2n-2} - 1102\times 2^{n-1} + 380)]\times$$
$$3\prod_{i=2}^{n} 2^{i-2} [(-76\times 2^{n-i} + 57\times 2^{n-1})$$
$$(418\times 2^{n-i} + 57\times 2^{n-1} - 361\times 2^{2n-2i} - 157)]\times$$
$$3\prod_{i=2}^{n} 2^{i-1} [(1083\times 2^{2n-i} - 1444\times 2^{2n-2i} + 38\times 2^{n-i} - 1140\times 2^{n-1} + 380)$$
$$(1083\times 2^{2n-i} - 1444\times 2^{2n-2i} - 1425\times 2^{n-1} + 418\times 2^{n-i} + 350)$$
$$(1083\times 2^{2n-i-1} - 722\times 2^{2n-2i} - 684\times 2^{n-1} + 209\times 2^{n-i} + 157)]\times$$
$$3\prod_{i=2}^{n} 2^{i} [(1083\times 2^{2n-i} - 361\times 2^{2n-2i} - 209\times 2^{n-i} - 798\times 2^{n-1} + 350)$$
$$(1083\times 2^{2n-i-1} - 361\times 2^{2n-2i} - 19\times 2^{n-i} - 1083\times 2^{n-1} + 380)]$$

Theorem 6.1.4: Multiplicative of fourth Zagreb index of Nanostar Dendrimer D_n, is given by $M_{4C}\Pi[D_n] = 108[(1083\times 2^{2n-2} - 1482\times 2^{n-1} + 507)$

$$(1805\times 2^{2n-2} - 2489\times 2^{n-1} + 857)$$
$$(722\times 2^{2n-2} - 1007\times 2^{n-1} + 350)^{2}$$
$$(722\times 2^{2n-2} - 1102\times 2^{n-1} + 380)^{2}]\times$$
$$3\prod_{i=2}^{n} 2^{i-2} [(1083\times 2^{2n-i} - 1444\times 2^{2n-2i} - 38\times 2^{n-i} - 1083\times 2^{n-1} + 380)$$
$$(2166\times 2^{2n-i} - 2888\times 2^{2n-2i} - 2223\times 2^{n-1} + 760)$$

$(1083 \times 2^{2n-i} - 1444 \times 2^{2n-2i} + 418 \times 2^{n-i} - 1425 \times 2^{n-1} + 350)$

$(2166 \times 2^{2n-i} - 2527 \times 2^{2n-2i} - 2907 \times 2^{n-1} + 418 \times 2^{n-i} + 857)] \times$

$3 \prod_{i=2}^{n} 2^{i-1} [(1083 \times 2^{2n-i} - 1444 \times 2^{2n-2i} + 38 \times 2^{n-i} - 1140 \times 2^{n-1} + 380)^2$

$(1083 \times 2^{2n-i} - 1444 \times 2^{2n-2i} - 1425 \times 2^{n-1} + 418 \times 2^{n-i} + 350)^2$

$(1083 \times 2^{2n-i} - 1083 \times 2^{2n-2i} - 1482 \times 2^{n-1} + 507)^2$

$(1083 \times 2^{2n-i} + 1083 \times 2^{2n-i-1} - 1444 \times 2^{2n-2i} - 209 \times$

$2^{n-i} - 2280 \times 2^{n-1} + 857)^2] \times$

$3 \prod_{i=2}^{n} 2^{i} [(1083 \times 2^{2n-i-1} - 361 \times 2^{2n-2i} - 209 \times 2^{n-i} - 798 \times 2^{n-1} + 350)^2$

$(1083 \times 2^{2n-i-1} - 361 \times 2^{2n-2i} - 19 \times 2^{n-i} - 1083 \times 2^{n-1} + 380)^2]$

Proof: $M_{4C}\Pi[D_n] = \prod_{uv \in E(D_n)} c(u)[(c(u) + c(v)]$

$= (3 \times 2^0) [(3(19 \times 2^{n-1} - 13)^2 \{3(19 \times 2^{n-1} - 13)^2 +$

$((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \times (19 \times 2^{n-1} - 13 - 1)\}] \times$

$(3 \times 2^1)[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \times$

$(19 \times 2^{n-1} - 13 - 1) \{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \times$

$(19 \times 2^{n-1} - 13 - 1) + 0\}] \times$

$(3 \times 2^1) [((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13) + 5) \times$

$((19 \times 2^{n-1} - 13) - 6) \times \{((57 \times 2^{n-1} - 38) -$

$(19 \times 2^{n-1} - 13) + 5) \times ((19 \times 2^{n-1} - 13) - 6) + 0\}] \times$

$3 \prod_{i=2}^{n} 2^{i-2} [\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 5) \times$

$(19 \times 2^{n-(i-1)} - 13) - 6)\} \{((57 \times 2^{n-1} - 38) -$

$(19 \times 2^{n-(i-1)} - 13) + 5) (19 \times 2^{n-(i-1)} - 13 - 6) +$

$((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 6) \times$

$(19 \times 2^{n-(i-1)} - 13 - 7)\}] \times$

$3 \prod_{i=2}^{n} 2^{i-1} [((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 6) \times$

$(19 \times 2^{n-(i-1)} - 13 - 7)\{((57 \times 2^{n-1} - 38) -$

$(19 \times 2^{n-(i-1)} - 13) + 6) \times (19 \times 2^{n-(i-1)} - 13 - 7) + 0\}] \times$

$3 \prod_{i=2}^{n} 2^{i-1} [((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 11) \times$

$(2(19 \times 2^{n-i} - 13) + 1)\{((57 \times 2^{n-1} - 38) -$

$(19 \times 2^{n-(i-1)} - 13) + 11) \times (2(19 \times 2^{n-i} - 13) + 1) + 0\}] \times$

$$3\prod_{i=2}^{n} 2^{i-2}\ [\{((57\times 2^{n-1}-38)-(19\times 2^{n-(\mathrm{i}-1)}-13)+11)\times$$

$$(2(19\times 2^{n-\mathrm{i}}-13)+1)\}\{\{((57\times 2^{n-1}-38)-$$

$$(19\times 2^{n-(\mathrm{i}-1)}-13)+11)\times(2(19\times 2^{n-\mathrm{i}}-13)+1)\}+$$

$$\{((57\times 2^{n-1}-38)-(19\times 2^{n-(\mathrm{i}-1)}-13)+12)\times$$

$$2(19\times 2^{n-\mathrm{i}}-13)+(19\times 2^{n-\mathrm{i}}-13)^2\}\}]\times$$

$$3\prod_{i=2}^{n} 2^{i-1}\ [\{((57\times 2^{n-1}-38)-(19\times 2^{n-(\mathrm{i}-1)}-13)+12)\times$$

$$2(19\times 2^{n-\mathrm{i}}-13)+(19\times 2^{n-\mathrm{i}}-13)^2\}\{\{((57\times 2^{n-1}-38)-$$

$$(19\times 2^{n-\mathrm{i}}-13)+12)\times 2(19\times 2^{n-\mathrm{i}}-13)+$$

$$(19\times 2^{n-\mathrm{i}}-13)^2\}+\{((57\times 2^{n-1}-38)-\left(19\times 2^{n-\mathrm{i}}-13\right)\times$$

$$(19\times 2^{n-\mathrm{i}}-13-1)\}\}]\times$$

$$3\prod_{i=2}^{n} 2^{i}\ [((57\times 2^{n-1}-38)-(19\times 2^{n-\mathrm{i}}-13))\times(19\times 2^{n-\mathrm{i}}-13-1)$$

$$(57\times 2^{n-1}-38)-(19\times 2^{n-\mathrm{i}}-13))\times(19\times 2^{n-\mathrm{i}}-13-1)+0\}]\times$$

$$3\prod_{i=2}^{n} 2^{i}\ [((57\times 2^{n-1}-38)-(19\times 2^{n-\mathrm{i}}-13)+5)\times(19\times 2^{n-\mathrm{i}}-13-6)$$

$$\{((57\times 2^{n-1}-38)-(19\times 2^{n-\mathrm{i}}-13)+5)\times(19\times 2^{n-\mathrm{i}}-13-6)+0\}]$$

After simplification, we get

$$M_{4C}\prod[D_n] = 108[(1083\times 2^{2n-2}-1482\times 2^{n-1}+507)$$

$$(1805\times 2^{2n-2}-2489\times 2^{n-1}+857)$$

$$(722\times 2^{2n-2}-1007\times 2^{n-1}+350)^2$$

$$(722\times 2^{2n-2}-1102\times 2^{n-1}+380)^2\ ]\times$$

$$3\prod_{i=2}^{n} 2^{i-2}\ [(1083\times 2^{2n-\mathrm{i}}-1444\times 2^{2n-2\mathrm{i}}-38\times 2^{n-\mathrm{i}}-1083\times 2^{n-1}+380)$$

$$(2166\times 2^{2n-\mathrm{i}}-2888\times 2^{2n-2\mathrm{i}}-2223\times 2^{n-1}+760)$$

$$(1083\times 2^{2n-\mathrm{i}}-1444\times 2^{2n-2\mathrm{i}}+418\times 2^{n-\mathrm{i}}-1425\times 2^{n-1}+350)$$

$$(2166\times 2^{2n-\mathrm{i}}-2527\times 2^{2n-2\mathrm{i}}-2907\times 2^{n-1}+418\times 2^{n-\mathrm{i}}\ \ 857)]\times$$

$$3\prod_{i=2}^{n} 2^{i-1}\ [(1083\times 2^{2n-\mathrm{i}}-1444\times 2^{2n-2\mathrm{i}}+38\times 2^{n-\mathrm{i}}-1140\times 2^{n-1}+380)^2$$

$$(1083\times 2^{2n-\mathrm{i}}-1444\times 2^{2n-2\mathrm{i}}-1425\times 2^{n-1}+418\times 2^{n-\mathrm{i}}+350)^2$$

$$(1083\times 2^{2n-\mathrm{i}}-1083\times 2^{2n-2\mathrm{i}}-1482\times 2^{n-1}+507)$$

$$(1083\times 2^{2n-\mathrm{i}}+1083\times 2^{2n-\mathrm{i}-1}-1444\times 2^{2n-2\mathrm{i}}-209\times 2^{n-\mathrm{i}}-$$

$$2280\times 2^{n-1}+857)]\times$$

$$3\prod_{i=2}^{n}2^{i}\,[(1083\times 2^{2n-i-1}-361\times 2^{2n-2i}-209\times 2^{n-i}-798\times 2^{n-1}+350)^{2}$$
$$(1083\times 2^{2n-i-1}-361\times 2^{2n-2i}-19\times 2^{n-i}-1083\times 2^{n-1}+380)^{2}]$$

Theorem 6.1.5: Multiplicative of fifth Zagreb index of Nanostar Dendrimer D_n, is given by $M_{5C}\Pi[D_n] = 3[(722\times 2^{2n-2}-1007\times 2^{n-1}+350)$

$$(1805\times 2^{2n-2}-2489\times 2^{n-1}+857)]\times$$
$$3\prod_{i=2}^{n}2^{i-2}\,[(1083\times 2^{2n-i}-1444\times 2^{2n-2i}+38\times 2^{n-i}-1140\times 2^{n-1}+380)$$
$$(2166\times 2^{2n-i}-2888\times 2^{2n-2i}-2223\times 2^{n-1}+760)$$
$$(1083\times 2^{2n-i}-1083\times 2^{2n-2i}-1482\times 2^{n-1}+507)$$
$$(2166\times 2^{2n-i}-2527\times 2^{2n-2i}-2907\times 2^{n-1}+418\times 2^{n-i}+857)]\times$$
$$3\prod_{i=2}^{n}2^{i-1}\,[(1083\times 2^{2n-i-1}-361\times 2^{2n-2i}-209\times 2^{n-i}-798\times 2^{n-1}+350)$$
$$(1083\times 2^{2n-i}+1083\times 2^{2n-i-1}-1444\times 2^{2n-2i}-209\times 2^{n-i}-$$
$$2280\times 2^{n-1}+857)]$$

Proof: $M_{5C}\Pi[D_n] = \prod_{uv\in E(\mathrm{Dn})} c(v)[(c(u)+c(v)]$

$$=(3\times 2^{0})\,[\{((57\times 2^{n-1}-38)-(19\times 2^{n-1}-13))\times(19\times 2^{n-1}-13-1)\}$$
$$\{3(19\times 2^{n-1}-13)^{2}+((57\times 2^{n-1}-38)-(19\times 2^{n-1}-13))\times$$
$$(19\times 2^{n-1}-13-1)\}]\times$$
$$3\prod_{i=2}^{n}2^{i-2}\,[\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+6)\times$$
$$(19\times 2^{n-(i-1)}-13-7)\}\times$$
$$\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+5)\times$$
$$(19\times 2^{n-(i-1)}-13-6)+$$
$$((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+6)\times$$
$$(19\times 2^{n-(i-1)}-13-7)\}]\times$$
$$3\prod_{i=2}^{n}2^{i-2}\,[\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+12)\times$$
$$(2(19\times 2^{n-i}-13)\times(19\times 2^{n-i}-13)^{2}\}$$
$$\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+11)\times$$
$$(2(19\times 2^{n-i}-13)+1\}+\{((57\times 2^{n-1}-38)-$$
$$(19\times 2^{n-(i-1)}-13)+12)\times(2\big(19\times 2^{n-i}-13\big)+$$
$$\big(19\times 2^{n-i}-13\big)^{2}\}]\times$$

$$3\prod_{i=2}^{n}2^{i-2}\ [\{((57\times 2^{n-1}-38)-(19\times 2^{n-i}-13)\times(19\times 2^{n-i}-13-1)\}$$

$$\{((57\times 2^{n-1}-38-(19\times 2^{n-(i-1)}-13)+12)\times 2(19\times 2^{n-i}-13)+$$

$$\left(19\times 2^{n-i}-13\right)^{2}+\{((57\times 2^{n-1}-38)-(19\times 2^{n-i}-13)\times$$

$$(19\times 2^{n-i}-3-1)\}]$$

After an easy simplification, we get

$$M_{5C}\Pi[D_n] = 3[(722\times 2^{2n-2}-1007\times 2^{n-1}+350)$$

$$(1805\times 2^{2n-2}-2489\times 2^{n-1}+857)]\times$$

$$3\prod_{i=2}^{n}2^{i-2}[(1083\times 2^{2n-i}-1444\times 2^{2n-2i}+38\times 2^{n-i}-1140\times 2^{n-1}+380)$$

$$(2166\times 2^{2n-i}-2888\times 2^{2n-2i}-2223\times 2^{n-1}+760)$$

$$(1083\times 2^{2n-i}-1083\times 2^{2n-2i}-1482\times 2^{n-1}+507)$$

$$(2166\times 2^{2n-i}-2527\times 2^{2n-2i}-2907\times 2^{n-1}+418\times 2^{n-i}+857)]\times$$

$$3\prod_{i=2}^{n}2^{i-1}[(1083\times 2^{2n-i-1}-361\times 2^{2n-2i}-209\times 2^{n-i}-798\times 2^{n-1}+350)$$

$$(1083\times 2^{2n-i}+1083\times 2^{2n-i-1}-1444\times 2^{2n-2i}-209\times 2^{n-i}-2280\times 2^{n-1}+857)]$$

6.2 Multiplicative of SK, SK_1 and SK_2 cutting number topological indices of Nanostar Dendrimer D_n

Theorem 6.2.1: Multiplicative of SK index of Nanostar Dendrimer D_n, is given by

$$SK_C\Pi[D_n] = [\tfrac{108}{8}][(1805\times 2^{2n-2}-2489\times 2^{n-1}+857)$$

$$(722\times 2^{2n-2}-1007\times 2^{n-1}+350)$$

$$(722\times 2^{2n-2}-1102\times 2^{n-1}+380)]\times$$

$$[\tfrac{3}{2}]\prod_{i=2}^{n}2^{i-2}\ [(2166\times 2^{2n-i}-2888\times 2^{2n-2i}-2223\times 2^{n-1}+760)$$

$$(2166\times 2^{2n-i}-2527\times 2^{2n-2i}+418\times 2^{n-i}-2907\times$$

$$2^{n-1}+857)]\times$$

$$[\tfrac{3}{2}]\prod_{i=2}^{n}2^{i-1}\ [(1083\times 2^{2n-i}-1444\times 2^{2n-2i}+38\times 2^{n-i}-1140\times 2^{n-1}+380)$$

$$(1083\times 2^{2n-i}-1444\times 2^{2n-2i}+418\times 2^{n-i}-1425\times 2^{n-1}+350)$$

$$(3249\times 2^{2n-i-1}-1444\times 2^{2n-2i}-209\times 2^{n-i}-2280\times 2^{n-1}+857)]\times$$

$$[\tfrac{3}{2}]\prod_{i=2}^{n} 2^{i}[(1083 \times 2^{2n-\mathrm{i}-1} - 361\times 2^{2n-2\mathrm{i}} - 209 \times 2^{n-\mathrm{i}} - 798 \times 2^{n-1} + 350)$$

$$(1083 \times 2^{2n-\mathrm{i}-1} - 361 \times 2^{2n-2\mathrm{i}} - 19 \times 2^{n-\mathrm{i}} - 1083 \times 2^{n-1} + 380)]$$

Proof: $SK_C\Pi[D_n] = \prod_{uv\in E(\mathrm{Dn})}[\frac{c(u)+c(v)}{2}]$

$$= [\tfrac{3}{2}]\ [\{(3(19\times 2^{n-1} - 13)^{2}\} + \{((57 \times 2^{n-1} - 38) -$$

$$(19 \times 2^{n-1} - 13)) \times (19 \times 2^{n-1} - 13 - 1)\}]$$

$$[\tfrac{6}{2}]\ [\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \times$$

$$(19 \times 2^{n-1} - 13 - 1)\} + 0] \times$$

$$[\tfrac{6}{2}]\ [\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13) + 5) \times$$

$$(19 \times 2^{n-1} - 13 - 6)\} + 0] \times$$

$$[\tfrac{3}{2}]\prod_{i=2}^{n} 2^{i-2}\ [\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(\mathrm{i}-1)} - 13)+5) \times$$

$$(19 \times 2^{n-(\mathrm{i}-1)} - 13 - 6)\}+\{((57\times 2^{n-1} - 38) -$$

$$(19 \times 2^{n-(\mathrm{i}-1)} - 13)+6) \times (19 \times 2^{n-(\mathrm{i}-1)} - 13 - 7)\}] \times$$

$$[\tfrac{3}{2}]\prod_{i=2}^{n} 2^{i-1}\ [\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(\mathrm{i}-1)} - 13) + 6) \times$$

$$(19 \times 2^{n-(\mathrm{i}-1)} - 13 - 7)\} + 0] \times$$

$$[\tfrac{3}{2}]\prod_{i=2}^{n} 2^{i-1}\ [\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(\mathrm{i}-1)} - 13) + 11) \times$$

$$(2(19 \times 2^{n-(\mathrm{i}-1)} - 13) + 1)\} + 0] \times$$

$$[\tfrac{3}{2}]\prod_{i=2}^{n} 2^{i-2}\ [\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(\mathrm{i}-1)} - 13) + 11) \times$$

$$(2(19 \times 2^{n-\mathrm{i}} - 13) + 1\} + \{((57 \times 2^{n-1} - 38) -$$

$$(19 \times 2^{n-(\mathrm{i}-1)} - 13) + 12) \times (2(19 \times 2^{n-(\mathrm{i}-1)} - 13) +$$

$$(19 \times 2^{n-\mathrm{i}} - 13)^{2}\}] \times$$

$$[\tfrac{3}{2}]\prod_{i=2}^{n} 2^{i-1}\ [\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(\mathrm{i}-1)} - 13) + 12) \times$$

$$2(19 \times 2^{n-\mathrm{i}} - 13)+(19 \times 2^{n-\mathrm{i}} - 13)^{2}\} + \{((57\times 2^{n-1} - 38) -$$

$$(19 \times 2^{n-\mathrm{i}} - 13) \times (19 \times 2^{n-\mathrm{i}} - 13 - 1)\}] \times$$

$$[\tfrac{3}{2}]\prod_{i=2}^{n} 2^{i}\ [\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-\mathrm{i}} - 13)) \times$$

$$(19 \times 2^{n-\mathrm{i}} - 13 - 1)\} + 0] \times$$

$$[\tfrac{3}{2}]\prod_{i=2}^{n} 2^{i}[\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-\mathrm{i}} - 13) + 5) \times$$

$$(19 \times 2^{n-\mathrm{i}} - 13 - 6)\} + 0]$$

After a bit calculation, we get

$$SK_C\Pi[D_n] = [\tfrac{108}{8}][(1805 \times 2^{2n-2} - 2489 \times 2^{n-1} + 857)$$

$$(722 \times 2^{2n-2} - 1007 \times 2^{n-1} + 350)$$

$$(722 \times 2^{2n-2} - 1102 \times 2^{n-1} + 380)] \times$$

$$[\tfrac{3}{2}]\prod_{i=2}^{n} 2^{i-2} [(2166 \times 2^{2n-i} - 2888 \times 2^{2n-2i} - 2223 \times 2^{n-1} + 760)$$

$$(2166 \times 2^{2n-i} - 2527 \times 2^{2n-2i} + 418 \times 2^{n-i} - 2907 \times 2^{n-1} + 857)] \times$$

$$[\tfrac{3}{2}]\prod_{i=2}^{n} 2^{i-1} [(1083 \times 2^{2n-i} - 1444 \times 2^{2n-2i} + 38 \times 2^{n-i} - 1140 \times 2^{n-1} + 380)$$

$$(1083 \times 2^{2n-i} - 1444 \times 2^{2n-2i} + 418 \times 2^{n-i} - 1425 \times 2^{n-1} + 350)$$

$$(3249 \times 2^{2n-i-1} - 1444 \times 2^{2n-2i} - 209 \times 2^{n-i} - 2280 \times 2^{n-1} + 857)] \times$$

$$[\tfrac{3}{2}]\prod_{i=2}^{n} 2^{i}[(1083 \times 2^{2n-i-1} - 361 \times 2^{2n-2i} - 209 \times 2^{n-i} - 798 \times 2^{n-1} + 350)$$

$$(1083 \times 2^{2n-i-1} - 361 \times 2^{2n-2i} - 19 \times 2^{n-i} - 1083 \times 2^{n-1} + 380)]$$

Theorem: 6.2.2 Multiplicative of SK_1 index of Nanostar Dendrimer D_n, is given by

$$SK_{1C}\Pi[D_n] = [\tfrac{3}{2}][(1083 \times 2^{2n-2} - 1482 \times 2^{n-1} + 507)$$

$$(722 \times 2^{2n-2} - 1007 \times 2^{n-1} + 350)] \times$$

$$[\tfrac{3}{2}] \prod_{i=2}^{n} 2^{i-2} [(1083 \times 2^{2n-i} - 1444 \times 2^{2n-2i} - 38 \times 2^{n-i} - 1083 \times 2^{n-1} + 380)$$

$$(1083 \times 2^{2n-i} - 1444 \times 2^{2n-2i} + 38 \times 2^{n-i} - 1140 \times 2^{n-1} + 380)$$

$$(1083 \times 2^{2n-i} - 1444 \times 2^{2n-2i} + 418 \times 2^{n-i} - 1425 \times 2^{n-1} + 350)$$

$$(1083 \times 2^{2n-i} - 1083 \times 2^{2n-2i} - 1482 \times 2^{n-1} + 507)]$$

$$[\tfrac{3}{2}] \prod_{i=2}^{n} 2^{i-1} [(1083 \times 2^{2n-i} - 1083 \times 2^{2n-2i} - 1482 \times 2^{n-1} + 507)$$

$$(1083 \times 2^{2n-i-1} - 361 \times 2^{2n-2i} - 209 \times 2^{n-i} - 798 \times 2^{n-1} + 350)]$$

Proof: $SK_{1C}\Pi[D_n] = \prod_{uv\in E(Dn)}[\frac{c(u)c(v)}{2}]$

$$= [\frac{3}{2}]\ [\{(3(19\times 2^{n-1} - 13)^{2}\} \times ((57 \times 2^{n-1} - 38) -$$

$$(19 \times 2^{n-1} - 13)) \times (19 \times 2^{n-1} - 13 - 1)] \times$$

$$[\frac{3}{2}](3 \times 2^1)\ [((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \times$$

$$(19 \times 2^{n-1} - 13 - 1) \times 0] \times$$

$$[\frac{3}{2}](3 \times 2^1)\ [((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13) + 5) \times$$

$$(19 \times 2^{n-1} - 13) - 6) \times 0] \times$$

$$[\frac{9}{2}]\ \prod_{i=2}^{n} 2^{i-2}\ [\{((57 \times 2^{n-1} - 38) -(19 \times 2^{n-(i-1)} - 13) + 5) \times$$

$$(19 \times 2^{n-(i-1)} - 13 - 6)\} \times \{((57 \times 2^{n-1} - 38) -$$

$$(19 \times 2^{n-(i-1)} - 13) + 6) \times (19 \times 2^{n-(i-1)} - 13 - 7)\}] \times$$

$$[\frac{9}{2}]\ \prod_{i=2}^{n} 2^{i-1}\ [((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 6) \times$$

$$(19 \times 2^{n-(i-1)} - 13 - 7) \times 0] \times$$

$$[\frac{9}{2}]\ \prod_{i=2}^{n} 2^{i-1}\ [((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 11) \times$$

$$(2(19 \times 2^{n-i} - 13) + 1) \times 0] \times$$

$$[\frac{9}{2}]\ \prod_{i=2}^{n} 2^{i-2}\ [\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 11) \times$$

$$(2(19 \times 2^{n-i} - 13) + 1\} \times \{((57 \times 2^{n-1} - 38) -$$

$$\left(19 \times 2^{n-(i-1)} - 13\right) + 12) \times (2(19 \times 2^{n-i} - 13) +$$

$$\left(19 \times 2^{n-i} - 13\right)^2\}] \times$$

$$[\frac{3}{2}]\ \prod_{i=2}^{n} 2^{i-1}\ [\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 12) \times$$

$$2(19 \times 2^{n-i} - 13) + (19 \times 2^{n-i} - 13)^{2}\} \times \{((57 \times 2^{n-1} - 38) -$$

$$(19 \times 2^{n-i} - 13) \times (19 \times 2^{n-i} - 13 - 1)\}] \times$$

$$[\frac{3}{2}]\ \prod_{i=2}^{n} 2^{i}\ [((57 \times 2^{n-1} - 38) - (19 \times 2^{n-i} - 13)) \times$$

$$(19 \times 2^{n-i} - 13 - 1) \times 0] \times$$

$$[\frac{3}{2}]\ \prod_{i=2}^{n} 2^{i}\ [((57 \times 2^{n-1} - 38) - (19 \times 2^{n-i} - 13) + 5) \times$$

$$(19 \times 2^{n-i} - 13 - 6) \times 0]$$

After simplification, we get

$$SK_{1C}\Pi[D_n] = [\frac{3}{2}]\ [(1083 \times 2^{2n-2} - 1482 \times 2^{n-1} + 507)$$

$(722 \times 2^{2n-2} - 1007 \times 2^{n-1} + 350)] \times$

$[\frac{3}{2}] \prod_{i=2}^{n} 2^{i-2}$ $[(1083 \times 2^{2n-i} - 1444 \times 2^{2n-2i} - 38 \times 2^{n-i} - 1083 \times 2^{n-1} + 380)$

$(1083 \times 2^{2n-i} - 1444 \times 2^{2n-2i} + 38 \times 2^{n-i} - 1140 \times 2^{n-1} + 380)$

$(1083 \times 2^{2n-i} - 1444 \times 2^{2n-2i} + 418 \times 2^{n-i} - 1425 \times 2^{n-1} + 350)$

$(1083 \times 2^{2n-i} - 1083 \times 2^{2n-2i} - 1482 \times 2^{n-1} + 507)]$

$[\frac{3}{2}] \prod_{i=2}^{n} 2^{i-1}[(1083 \times 2^{2n-i} - 1083 \times 2^{2n-2i} - 1482 \times 2^{n-1} + 507)$

$(1083 \times 2^{2n-i-1} - 361 \times 2^{2n-2i} - 209 \times 2^{n-i} - 798 \times 2^{n-1} + 350)]$

Theorem 6.2.3: Multiplicative of SK_2 index of Nanostar Dendrimer D_n, is given by

$SK_{2C}\Pi[D_n] = [\frac{108}{64}]$ $[(1805 \times 2^{2n-2} - 2489 \times 2^{n-1} + 857)^2$

$(722 \times 2^{2n-2} - 1007 \times 2^{n-1} + 350)^2$

$(722 \times 2^{2n-2} - 1102 \times 2^{n-1} + 380)^2] \times$

$[\frac{3}{4}]\prod_{i=2}^{n} 2^{i-2}$ $[(2166 \times 2^{2n-i} - 2888 \times 2^{2n-2i} - 2223 \times 2^{n-1} + 760)^2$

$(2166 \times 2^{2n-i} - 2527 \times 2^{2n-2i} + 418 \times 2^{n-i} - 2907 \times 2^{n-1} + 857)^2] \times$

$[\frac{3}{4}]\prod_{i=2}^{n} 2^{i-1}$ $[(1083 \times 2^{2n-i} - 1444 \times 2^{2n-2i} + 38 \times 2^{n-i} - 1140 \times 2^{n-1} + 380)^2$

$(1083 \times 2^{2n-i} - 1444 \times 2^{2n-2i} + 418 \times 2^{n-i} - 1425 \times 2^{n-1} + 350)^2$

$(3249 \times 2^{2n-i-1} - 1444 \times 2^{2n-2i} - 209 \times 2^{n-i} - 2280 \times 2^{n-1} + 857)^2] \times$

$[\frac{3}{4}]\prod_{i=2}^{n} 2^{i}$ $[(1083 \times 2^{2n-i-1} - 361 \times 2^{2n-2i} - 209 \times 2^{n-i} - 798 \times 2^{n-1} + 350)^2$

$(1083 \times 2^{2n-i-1} - 361 \times 2^{2n-2i} - 19 \times 2^{n-i} - 1083 \times 2^{n-1} + 380)^2]$

Proof: $SK_{2C}[D_n] = \prod_{uv \in E(Dn)} [\frac{c(u)+c(v)}{2}]^2$

$= [\frac{3}{4}]$ $[\{(3(192^{n-1} - 13)^2\} + \{((57 \times 2^{n-1} - 38) -$

$(19 \times 2^{n-1} - 13)) \times (19 \times 2^{n-1} - 13 - 1)\}]^2$

$[\frac{6}{4}]$ $[\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \times$

$(19 \times 2^{n-1} - 13 - 1)\} + 0]^2 \times$

$[\frac{6}{4}]$ $[\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13) + 5) \times$

$(19 \times 2^{n-1} - 13 - 6)\} + 0]^2 \times$

$[\frac{3}{4}] \prod_{i=2}^{n} 2^{i-2}$ $[\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 5) \times$

$(19 \times 2^{n-(i-1)} - 13 - 6)\} + \{((57 \times 2^{n-1} - 38) -$

$$(19\times\ 2^{n-(\mathrm{i}-1)}-13)+6)\times(19\times 2^{n-(\mathrm{i}-1)}-13-7)\}]^{2}\ \times$$

$$[\tfrac{3}{4}]\prod_{i=2}^{n}2^{i-1}\ [\{((57\times 2^{n-1}\ -38)-(19\times 2^{n-(\mathrm{i}-1)}-13)+6)\times$$

$$(19\times 2^{n-(\mathrm{i}-1)}-13\ -7)\}\ +0]^{2}\ \times$$

$$[\tfrac{3}{4}]\prod_{i=2}^{n}2^{i-1}\ [\{((57\times 2^{n-1}\ -38)-(19\times 2^{n-(\mathrm{i}-1)}-13)+11)\times$$

$$(2(19\times 2^{n-(\mathrm{i}-1)}-13)+1)\}+0]^{2}\ \times$$

$$[\tfrac{3}{4}]\ \prod_{i=2}^{n}2^{i-2}\ \ [\{((57\times 2^{n-1}\ -38)\ -(19\times 2^{n-(\mathrm{i}-1)}-13)+11)\times$$

$$(2(19\times 2^{n-\mathrm{i}}-13)\ +1\}+\{((57\times 2^{n-1}\ -38)-$$

$$\left(19\times 2^{n-(\mathrm{i}-1)}-13\right)+12)\ \times\ (2\left(19\times 2^{n-\mathrm{i}}-13\right)+$$

$$(19\times 2^{n-\mathrm{i}}-13)^{2}\ \}]^{2}\ \times$$

$$[\tfrac{3}{4}]\ \prod_{i=2}^{n}2^{i-1}\ [\{((57\times 2^{n-1}\ -38)\ -(19\times 2^{n-(\mathrm{i}-1)}-13)+12)\times$$

$$2(19\times 2^{n-\mathrm{i}}-13)+(19\times 2^{n-\mathrm{i}}-13)^{2}\}+\{((57\times 2^{n-1}\ -38)-$$

$$(19\times\ 2^{n-\mathrm{i}}-13)\times(19\times 2^{n-\mathrm{i}}-13-1)\}]^{2}\ \times$$

$$[\tfrac{3}{4}]\ \prod_{i=2}^{n}2^{i}\ [\{((57\times 2^{n-1}\ -38)-(19\times 2^{n-\mathrm{i}}-13))\times$$

$$(19\times 2^{n-\mathrm{i}}-13-1)\}+0]^{2}\ \ \times$$

$$[\tfrac{3}{4}]\prod_{i=2}^{n}2^{i}[\{((57\times 2^{n-1}\ -38)-(19\times 2^{n-\mathrm{i}}-13)+5)\times$$

$$(19\times 2^{n-\mathrm{i}}-13-6)\}+0]^{2}$$

After simplification, we get

$$SK_{2C}\Pi[D_n]=[\tfrac{108}{8}][(1805\times 2^{2n-2}-2489\times 2^{n-1}+857)^{2}$$

$$(722\times 2^{2n-2}-1007\times 2^{n-1}+350)^{2}$$

$$(722\times 2^{2n-2}-1102\times 2^{n-1}+380)^{2}\ ]\times$$

$$[\tfrac{3}{4}]\prod_{i=2}^{n}2^{i-2}\ [(2166\times 2^{2n-\mathrm{i}}\ -2888\times 2^{2n-2\mathrm{i}}\ \ -2223\times 2^{n-1}\ +760)^{2}$$

$$(2166\times 2^{2n-\mathrm{i}}-2527\times 2^{2n-2\mathrm{i}}\ +418\times 2^{n-\mathrm{i}}-2907\times 2^{n-1}+857)^{2}\ ]\times$$

$$[\tfrac{3}{4}]\prod_{i=2}^{n}2^{i-1}\ [(1083\times 2^{2n-\mathrm{i}}\ -1444\times 2^{2n-2\mathrm{i}}\ +38\times 2^{n-\mathrm{i}}-1140\times 2^{n-1}+380)^{2}$$

$$(1083\times 2^{2n-i} - 1444 \times 2^{2n-2i} + 418\times 2^{n-i} - 1425\times 2^{n-1} + 350)^2$$

$$(3249\times 2^{2n-i-1} - 1444\times 2^{2n-2i} - 209\times 2^{n-i} - 2280\times 2^{n-1} + 857)^2\,] \times$$

$$[\tfrac{3}{4}]\prod_{i=2}^{n} 2^{i}\,[(1083\times 2^{2n-i-1} - 361\times 2^{2n-2i} - 209\times 2^{n-i} - 798\times 2^{n-1} + 350)^2$$

$$(1083\times 2^{2n-i-1} - 361\times 2^{2n-2i} - 19\times 2^{n-i} - 1083\times 2^{n-1} + 380)^2\,]$$

6.3 Multiplicative of Nano-Zagreb and sum Nano-Zagreb cutting number topological indices of Nanostar Dendrimer D_n

Theorem 6.3.1: Multiplicative of Nano-Zagreb index of Nanostar Dendrimer D_n, is given by $\mathcal{NZ}_c\prod(D_n)=108(651605\times 2^{4n-4} - 1755904\times 2^{3n-3} + 1775037\times 2^{2n-2} -$

$$797848\times 2^{n-1} + 134549)$$

$$(521284\times 2^{4n-4} - 1454108\times 2^{3n-3} + 1519449\times 2^{2n-2} -$$

$$704900\times 2^{n-1} + 122500)$$

$$(521284\times 2^{4n-4} - 1591288\times 2^{3n-3} + 1763124\times 2^{2n-2} -$$

$$837520\times 2^{n-1} + 144400)] \times$$

$$3\prod_{i=2}^{n} 2^{i-2}\,[(912247\times 2^{4n-4i} - 781926\times 2^{4n-3i} - 987696\times$$

$$2^{3n-3i} + 1111158\times 2^{3n-2i} + 123462\times 2^{3n-i} +$$

$$262086\times 2^{2n-2i} - 851238\times 2^{2n-i} + 234840\times$$

$$2^{n-i} + 3203514\times 2^{2n-2} - 873981\times 2^{2n} + 274284\times$$

$$2^{n} - 134549)] \times$$

$$3\prod_{i=2}^{n} 2^{i-1}\,[(5212840\times 2^{4n-4i} - 8210223\times 2^{4n-3i} + \left[\frac{12901779}{4}\right]\times$$

$$2^{4n-2i} - 1467826\times 2^{3n-3i} + 6234831\times 2^{3n-2i} -$$

$$3950784\times 2^{3n-i} + 1494540\times 2^{2n-i} - 2821215\times$$

$$2^{2n-2i} + 467780\times 2^{n-i} + 714780\times 2^{2n} + 203062\times$$

$$2^{2n-2} - 1404024\times 2^{n} + 401449]$$

$$3\prod_{i=2}^{n} 2^{i}\,[260642\times 2^{4n-4i} - 781926\times 2^{4n-3i} + 2345778\times 2^{4n-2i-2} +$$

$$164616\times 2^{3n-3i} + 432117\times 2^{3n-2i} - 432117\times 2^{3n-i} - 2345778$$

$$\times 2^{3n-i-2} - 483018\times 2^{2n-2i} + 977949\times 2^{2n-i} - 160740\times$$

$$2^{n-i} + 159201\times 2^{2n} + 1172889\times 2^{2n-2} - 690840\times 2^{2n} +$$

$$266900]$$

Proof: $\mathcal{NZ}c\prod(D_n) = \prod_{e=uv\in E(Dn)} (c^2(u) - c^2(v))$

$$= (3\times 2^0)[\{(3(19\times 2^{n-1} - 13)^2\}^2 - \{((57\times 2^{n-1} - 38) - (19\times 2^{n-1} - 13))\times(19\times 2^{n-1} - 14)\}^2] \times$$

$$(3\times 2^1)[\{((57\times 2^{n-1} - 38) - (19\times 2^{n-1} - 13))\times (19\times 2^{n-1} - 14)\}^2 - 0^2] \times$$

$$(3\times 2^1)[\{((57\times 2^{n-1} - 38) - (19\times 2^{n-1} - 13) + 5)\times (19\times 2^{n-1} - 13 - 6)\}^2 - 0^2] \times$$

$$3\prod_{i=2}^{n} 2^{i-2}[\{((57\times 2^{n-1} - 38) - \left(19\times 2^{n-(i-1)} - 13\right) + 5)\times (19\times 2^{n-(i-1)} - 13 - 6)\}^2 - \{((57\times 2^{n-1} - 38) - \left(19\times 2^{n-(i-1)} - 13\right) + 6)\times(19\times 2^{n-(i-1)} - 13 - 7)\}^2]$$

$$3\prod_{i=2}^{n} 2^{i-1}[\{((57\times 2^{n-1} - 38) - (19\times 2^{n-(i-1)} - 13)+6)\times (19\times 2^{n-(i-1)} - 13 - 7)\}^2 - 0^2]$$

$$3\prod_{i=2}^{n} 2^{i-1}[\{((57\times 2^{n-1} - 38) - (19\times 2^{n-(i-1)} - 13)+11)\times (2(19\times 2^{n-i} - 13)+1)\}^2 - 0^2]\times$$

$$3\prod_{i=2}^{n} 2^{i-2}[\{((57\times 2^{n-1} - 38) - (19\times 2^{n-(i-1)} - 13)+11)\times (2(19\times 2^{n-i} - 13)+1)\}^2 - \{((57\times 2^{n-1} - 38) - (19\times 2^{n-(i-1)} - 13)+12)\times (2(19\times 2^{n-i} - 13)+ \left(19\times 2^{n-i} - 13\right)^2\}^2]\times$$

$$3\prod_{i=2}^{n} 2^{i-1}[\{((57\times 2^{n-1} - 38) - (19\times 2^{n-(i-1)} - 13)+12)\times 2(19\times 2^{n-i} - 13)\}+ \left(19\times 2^{n-i} - 13\right)^2]^2 - \{((57\times 2^{n-1} - 38) - (19\times 2^{n-i} - 13))\times (19\times 2^{n-i} - 14)\}^2] \times$$

$$3\prod_{i=2}^{n} 2^{i}[\{((57\times 2^{n-1} - 38) - (19\times 2^{n-i} - 13))\times (19\times 2^{n-i} - 14)\}^2 - 0^2] \times$$

$$3\prod_{i=2}^{n} 2^{i}[((57\times 2^{n-1} - 38) - (19\times 2^{n-i} - 13)+5)\times (19\times 2^{n-i} - 13 - 6)\}^2 - 0^2]$$

After simplification we get

$\mathcal{NZ}c\prod(D_n) = 108\prod(651605\times 2^{4n-4} - 1755904\times 2^{3n-3} + 1775037\times 2^{2n-2} -$

$797848 \times 2^{n-1}$ +134549)

($521284 \times 2^{4n-4} - 1454108 \times 2^{3n-3} + 1519449 \times 2^{2n-2} -$

$704900 \times 2^{n-1} + 122500$)

($521284 \times 2^{4n-4} - 1591288 \times 2^{3n-3} + 1763124 \times 2^{2n-2} -$

$837520 \times 2^{n-1} + 144400$)] $\times$

$3\prod_{i=2}^{n} 2^{i-2}$ [($912247 \times 2^{4n-4i} - 781926 \times 2^{4n-3i} - 987696 \times$

$2^{3n-3i} + 1111158 \times 2^{3n-2i} + 123462 \times 2^{3n-i} +$

$262086 \times 2^{2n-2i} - 851238 \times 2^{2n-i} + 234840 \times$

$2^{n-i} + 3203514 \times 2^{2n-2} - 873981 \times 2^{2n} + 274284 \times$

$2^{n} - 134549$)] $\times$

$3\prod_{i=2}^{n} 2^{i-1}$ [($5212840 \times 2^{4n-4i} - 8210223 \times 2^{4n-3i} + \left[\frac{12901779}{4}\right] \times$

$2^{4n-2i} - 1467826 \times 2^{3n-3i} + 6234831 \times 2^{3n-2i} -$

$3950784 \times 2^{3n-i} + 1494540 \times 2^{2n-i} - 2821215 \times$

$2^{2n-2i} + 467780 \times 2^{n-i} + 714780 \times 2^{2n} + 2030625 \times$

$2^{2n-2} - 1404024 \times 2^{n} + 401449$]

$3\prod_{i=2}^{n} 2^{i}$ [$260642 \times 2^{4n-4i} - 781926 \times 2^{4n-3i} + 2345778 \times 2^{4n-2i-2} +$

$164616 \times 2^{3n-3i} + 432117 \times 2^{3n-2i} - 432117 \times 2^{3n-i} - 2345778$

$\times 2^{3n-i-2} - 483018 \times 2^{2n-2i} + 977949 \times 2^{2n-i} - 160740 \times$

$2^{n-i} + 159201 \times 2^{2n} + 1172889 \times 2^{2n-2} - 690840 \times 2^{2n} +$

266900]

Theorem 6.3.2: Multiplicative of sum Nano-Zagreb index of Nanostar Dendrimer D_n, is given by $\mathcal{NZ}_c\prod(D_n) = 108\prod(651605 \times 2^{4n-4} - 1755904 \times 2^{3n-3} + 1775037 \times 2^{2n-2} -$

$797848 \times 2^{n-1}$ +134549)

($521284 \times 2^{4n-4} - 1454108 \times 2^{3n-3} + 1519449 \times 2^{2n-2} -$

$704900 \times 2^{n-1} + 122500$)

($521284 \times 2^{4n-4} - 1591288 \times 2^{3n-3} + 1763124 \times 2^{2n-2} -$

$837520 \times 2^{n-1} + 144400$)] $\times$

$3\prod_{i=2}^{n} 2^{i-2}$ [($912247 \times 2^{4n-4i} - 781926 \times 2^{4n-3i} - 987696 \times$

$2^{3n-3i} + 1111158 \times 2^{3n-2i} + 123462 \times 2^{3n-i} +$

$262086 \times 2^{2n-2i} - 851238 \times 2^{2n-i} + 234840 \times$

$2^{n-\mathrm{i}}$ +3203514 × 2^{2n-2} – 873981× 2^{2n} +274284×

2^{n} –134549)] ×

$3\prod_{i=2}^{n} 2^{i-1}$ [(5212840 × $2^{4n-4\mathrm{i}}$ – 8210223 × $2^{4n-3\mathrm{i}}$ + $\left[\frac{12901779}{4}\right]$ ×

$2^{4n-2\mathrm{i}}$ –1467826 × $2^{3n-3\mathrm{i}}$ + 6234831 × $2^{3n-2\mathrm{i}}$ –

3950784 × $2^{3n-\mathrm{i}}$ + 1494540 × $2^{2n-\mathrm{i}}$ – 2821215 ×

$2^{2n-2\mathrm{i}}$ +467780× $2^{n-\mathrm{i}}$ +714780× 2^{2n} +203062×

2^{2n-2} – 1404024 × 2^{n} + 401449]

$3\prod_{i=2}^{n} 2^{i}$ [260642 × $2^{4n-4\mathrm{i}}$ – 781926 × $2^{4n-3\mathrm{i}}$ + 2345778 × $2^{4n-2\mathrm{i}-2}$ +

164616 × $2^{3n-3\mathrm{i}}$ +432117× $2^{3n-2\mathrm{i}}$ – 432117 × $2^{3n-\mathrm{i}}$ –2345778

× $2^{3n-\mathrm{i}-2}$ – 483018 × $2^{2n-2\mathrm{i}}$ + 977949 × $2^{2n-\mathrm{i}}$ – 160740 ×

$2^{n-\mathrm{i}}$ +159201× 2^{2n} +1172889 × 2^{2n-2} – 690840 × 2^{2n} +

266900]

Proof: $\mathcal{NZ}_C\prod(\mathrm{D_n}) = \prod_{e=uv\in E(\mathrm{Dn})} (c^2(u) - c^2(v))$

$= (3\times 2^0)[\{(3(19\times 2^{n-1} - 13)^2\}^2 - \{((57\times 2^{n-1} - 38) -$

$(19\times 2^{n-1} - 13)) \times (19\times 2^{n-1} - 14)\}^2] \times$

$(3\times 2^1)[\{((57\times 2^{n-1} - 38) - (19\times 2^{n-1} - 13)) \times$

$(19\times 2^{n-1} - 14)\}^2 - 0^2] \times$

$(3\times 2^1)\ [\{((57\times 2^{n-1} - 38) - (19\times 2^{n-1} - 13) + 5) \times$

$(19\times 2^{n-1} - 13 - 6)\}^2 - 0^2] \times$

$3\prod_{i=2}^{n} 2^{i-2} [\{((57\times 2^{n-1} - 38) - (19\times 2^{n-(\mathrm{i}-1)} - 13) + 5) \times$

$(19\times 2^{n-(\mathrm{i}-1)} - 13 - 6)\}^2 - \{((57\times 2^{n-1} - 38) -$

$(19\times 2^{n-(\mathrm{i}-1)} - 13) + 6) \times (19\times 2^{n-(\mathrm{i}-1)} - 13 - 7)\}^2]$

$3\prod_{i=2}^{n} 2^{i-1} [\{((57\times 2^{n-1} - 38) - (19\times 2^{n-(\mathrm{i}-1)} - 13) + 6) \times$

$(19\times -13 - 7)\}^2 - 0^2]$

$3\prod_{i=2}^{n} 2^{i-1} [\{((57\times 2^{n-1} - 38) - (19\times 2^{n-(\mathrm{i}-1)} - 13) + 11) \times$

$(2(19\times 2^{n-(\mathrm{i}-1)} - 13) + 1)\}^2 - 0^2] \times$

$3\prod_{i=2}^{n} 2^{i-2} [\{((57\times 2^{n-1} - 38) - (19\times 2^{n-(\mathrm{i}-1)} - 13) + 11) \times$

$(2(19\times 2^{n-\mathrm{i}} - 13) + 1)\}^2 - \{((57\times 2^{n-1} - 38) -$

$(19\times 2^{n-(\mathrm{i}-1)} - 13) + 12) \times (2(19\times 2^{n-\mathrm{i}} - 13) +$

$(19\times 2^{n-\mathrm{i}} - 13)^2\}^2]\times$

$$3\prod_{i=2}^{n} 2^{i-1}[\{((57\times 2^{n-1} - 38) - (19\times 2^{n-(i-1)} - 13)+12)\times 2(19\times 2^{n-i} - 13)\} + \left(19\times 2^{n-i} - 13\right)^2]^2 - \{((57\times 2^{n-1} - 38) - (19\times 2^{n-i} - 13))\times (19\times 2^{n-i} - 14)\}^2] \times$$

$$3\prod_{i=2}^{n} 2^{i}\,[\{((57\times 2^{n-1} - 38) - (19\times 2^{n-i} - 13))\times (19\times 2^{n-i} - 14)\}^2 - 0^2] \times$$

$$3\prod_{i=2}^{n} 2^{i}\,[((57\times 2^{n-1} - 38) - (19\times 2^{n-i} - 13) + 5)\times (19\times 2^{n-i} - 13 - 6)\}^2 - 0^2]$$

After simplification we get

$$\mathcal{NZ}_c\Pi(D_n) = 108\prod (651605\times 2^{4n-4} - 1755904\times 2^{3n-3} + 1775037\times 2^{2n-2} - 797848\times 2^{n-1} + 134549)$$

$$(521284\times 2^{4n-4} - 1454108\times 2^{3n-3} + 1519449\times 2^{2n-2} - 704900\times 2^{n-1} + 122500)$$

$$(521284\times 2^{4n-4} - 1591288\times 2^{3n-3} + 1763124\times 2^{2n-2} - 837520\times 2^{n-1} + 144400)] \times$$

$$3\prod_{i=2}^{n} 2^{i-2}\,[(912247\times 2^{4n-4i} - 781926\times 2^{4n-3i} - 987696\times 2^{3n-3i} + 1111158\times 2^{3n-2i} + 123462\times 2^{3n-i} + 262086\times 2^{2n-2i} - 851238\times 2^{2n-i} + 234840\times 2^{n-i} + 3203514\times 2^{2n-2} - 873981\times 2^{2n} + 274284\times 2^{n} - 134549)] \times$$

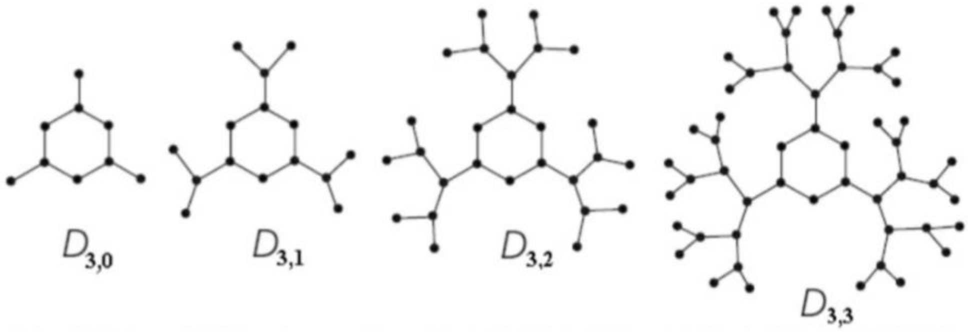
$$3\prod_{i=2}^{n} 2^{i-1}\,[(5212840\times 2^{4n-4i} - 8210223\times 2^{4n-3i} + \left[\frac{12901779}{4}\right]\times 2^{4n-2i} - 1467826\times 2^{3n-3i} + 6234831\times 2^{3n-2i} - 3950784\times 2^{3n-i} + 1494540\times 2^{2n-i} - 2821215\times 2^{2n-2i} + 467780\times 2^{n-i} + 714780\times 2^{2n} + 2030625\times 2^{2n-2} - 1404024\times 2^{n} + 401449]$$

$$3\prod_{i=2}^{n} 2^{i}\,[260642\times 2^{4n-4i} - 781926\times 2^{4n-3i} + 2345778\times 2^{4n-2i-2} + 164616\times 2^{3n-3i} + 432117\times 2^{3n-2i} - 432117\times 2^{3n-i} - 2345778\times 2^{3n-i-2} - 483018\times 2^{2n-2i} + 977949\times 2^{2n-i} - 160740\times 2^{n-i} + 159201\times 2^{2n} + 1172889\times 2^{2n-2} - 690840\times 2^{2n} + 266900]$$

CHAPTER 7

ECCENTRIC BASED TOPOLOGICAL INDICES OF MULTIPLICATIVE AND THEIR POLYNOMIAL OF HEXOGONAL CORE DENDRIMER $D_{3,n-1}$.

In this chapter, we calculate eccentricity based Redefined First, Second, and Third Zagreb indices, Modified First, Second, Third, Fourth and Fifth Zagreb indices, Randic, Reciprocal Randic, Reduced Reciprocal Randic and Generalized Randic indices. Also we define multiplicative and polynomial indices of Hexoconal core Dendrimer $D_{3,n-1}$.



Edge Set	**No. of Edges E = uv**	**Eccentricity of end vertices (e(u), e(v))**
E1	6	(k+1, k+2)
E2	$3x2^0 = 3$	(k+2, k+3)
E3	$3x2^1 = 6$	(k+3, k+4)
E4	$3x2^2 =12$	(k+4, k+5)
E5	$3x2^3 = 24$	(k+5, k+6)
E6	$3x2^4 = 48$	(k+6, k+7)
E7	$3x2^5 = 96$	(k+7, k+8)
.	.	.
.	.	.
.	.	.
E_{n-1}	$3x2^{n-1}$	(K+n+1, k+n+2)

Table (4)

7.1 Redefined First, Second, and Third Zagreb indices, Modified First, Second, Third, Fourth and Fifth Zagreb indices, Randic, Reciprocal Randic, Reduced Reciprocal Randic and Generalized Randic indices of eccentricity based topological indices of Hexoconal core Dendrimer $D_{3,n-1}$.

In this section, we calculated eccentricity based Redefined First, Second and Third Zagreb indices, Modified First, Second, Third, Fourth and Fifth Zagreb indices, Randic, Reciprocal Randic, Reduced Reciprocal Randic and Generalized Randic topological indices of Hexoconal core Dendrimer $D_{3,n-1}$.

(i) Redefined First, Second and Third Zagreb indices of G, as

$$ReZG_1(G) = \sum_{uv\in E(G)} \left(\frac{e(u)+e(v)}{e(u)\, e(v)}\right) \quad (7.1.1.)$$

$$ReZG_2(G) = \sum_{uv\in E(G)} \left(\frac{e(u)e(v)}{e(u)+e(v)}\right) \quad (7.1.2)$$

$$ReZG_3(G) = \sum_{uv\in E(G)} [e(u)e(v)\,(e(u)+e(v))] \quad (7.1.3)$$

(ii) Modified First, Second, Third, Fourth and Fifth Zagreb indices of G, are defined

$$m_{M_1}(G) = \sum_{uv\in E(G)}[\frac{1}{e(u)+e(v)}] \quad (7.1.4)$$

$$m_{M_2}(G) = \sum_{uv\in E(G)} \left[\frac{1}{e(u)e(v)}\right] \quad (7.1.5)$$

$$m_{M_3}(G) = \sum_{uv\in E(G)}[\frac{1}{|e(u)-e(v)|}] \quad (7.1.6)$$

$$m_{M_4}(G) = \sum_{uv\in E(G)} \left[\frac{1}{e(u)(e(u)+e(v))}\right] \quad (7.1.7)$$

$$m_{M_5}(G) = \sum_{uv\in E(G)}[\frac{1}{e(v)(e(u)+e(v))}] \quad (7.1.8)$$

(iii) Randic, Reciprocal Randic, Reduced Reciprocal Randic and Generalized Randic indices of G, are defined

$$R(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{e(u)e(v)}} \quad (7.1.9)$$

$$RR(G) = \sum_{uv\in E(G)} \sqrt{e(u)e(v)} \quad (7.1.10)$$

$$RRR(G) = \sum_{uv\in E(G)} \sqrt{(e(u)-1)(e(v)-1)} \quad (7.1.11)$$

$$R_a(G) = \sum_{uv\in E(G)}[e(u)e(v)]^a \quad (7.1.12)$$

In the following theorem we compute these topological indices of Hexoconal core Dendrimer $D_{3,n-1}$.

Theorem 7.1.1: The Redefined first, second and third Zagreb indices of Hexagonal core Dendrimer $D_{3,n-1}$, n =0,1,2,3…… are given by

(i) $ReZG_1(D_{3,n-1}) = 6\left[\frac{2k+3}{(k+1)\,(k+2)}\right] + 3\sum_{i=1}^{n} 2^{i-1}\left[\frac{(k+i+1)+(k+i+2)}{(k+i+1)\,(k+i+2)}\right]$

(ii) $ReZG_2(D_{3,n-1}) = 6\left[\frac{(k+1)\,(k+2)}{2k+3}\right] + 3\sum_{i=1}^{n} 2^{i-1}\left[\frac{(k+i+1)+(k+i+2)}{2k+2i+3}\right]$

(iii) $ReZG_3(D_{3,n-1}) = 6(k+1)(k+2)(2k+3) + 3\sum_{i=1}^{n} 2^{i-1}\left[(k+i+1)\,(k+i+2)\right.$

$\left.(2k+2i+3)\right]$

Proof: By using table (4) and from equation (7.1.1 to 7.1.3), we deduce derive

(i) $ReZG_1(G) = \sum_{uv\in E(G)}\left(\frac{e(u)+e(v)}{e(u)\,e(v)}\right)$

$ReZG_1(D_{3,n-1}) = (6\times2^0)\left[\frac{(k+1)+(k+2)}{(k+1)\,(k+2)}\right] + (3\times2^0)\left[\frac{(k+2)+(k+3)}{(k+2)\,(k+3)}\right] +$

$(3\times2^1)\left[\frac{(k+3)+(k+4)}{(k+3)\,(k+4)}\right] + \ldots\ldots + (3\times2^{n-1})\left[\frac{(k+n+1)+(k+n+2)}{(K+n+1)\,(K+n+2)}\right]$

$ReZG_1(D_{3,n-1}) = 6\left[\frac{2k+3}{(k+1)\,(k+2)}\right] + 3\sum_{i=1}^{n} 2^{i-1}\left[\frac{(k+i+1)+(k+i+2)}{(k+i+1)\,(k+i+2)}\right]$

(ii) $ReZG_2(G) = \sum_{uv\in E(G)}\left(\frac{e(u)e(v)}{e(u)+e(v)}\right)$

$ReZG_2(D_{3,n-1}) = (6\times2^0)\left[\frac{(k+1)\,(k+2)}{(k+1)+(k+2)}\right] + (3\times2^0\,)\left[\frac{(k+2)(k+3)}{(k+2)+(k+3)}\right] +$

$(3\times2^1)\left[\frac{(k+3)(k+4)}{(k+3)+(k+4)}\right] + \ldots\ldots + (3\times2^{n-1})\left[\frac{(k+n+1)\,(k+n+2)}{(k+n+1)+(k+n+2)}\right]$

$ReZG_2(D_{3,n-1}) = 6\left[\frac{(k+1)\,(k+2)}{2k+3}\right] + 3\sum_{i=1}^{n} 2^{i-1}\left[\frac{(k+i+1)+(k+i+2)}{2k+2i+3}\right]$

(iii) $ReZG_3(G) = \sum_{uv\in E(G)}\left[e(u)e(v)\,(e(u)+e(v))\right]$

$ReZG_3(D_{3,n-1}) = (6\times2^0)\left[(k+1)+(k+2)+(2k+3)\right] +$

$(3\times2^0)\left[(k+2)+(k+3)+(2k+5)\right] +$

$(3\times 2^1)\ [(k+3)+(k+4)+(2K+7)\ ]\ + \ldots\ldots +$

$(3\times 2^{n-1})\ [(k+n+1)+(k+n+2)+(2k+2n+3)\ ]$

$$ReZG_3(D_{3,n-1}) = 6\ (k+1)\ (k+2)\ (2k+3) + 3\sum_{i=1}^{n} 2^{i-1}\ [(k+i+1) + (k+i+2)(2k+2i+3)]$$

Theorem 7.1.2: Modified first, second, third, fourth and fifth Zagreb indices of Hexogonal core Dendrimer $D_{3,n-1}$, $n = 0,1,2,3\ldots.$ are given by

(i) $m_{M_1}(G) = \frac{6}{(k+1)+(k+2)} + 3\sum_{i=1}^{n} 2^{i-1}\left[\frac{1}{(k+i+1)+(k+i+2)}\right]$

(ii) $m_{M_2}(G) = \frac{6}{(k+1)(k+2)} + 3\sum_{i=1}^{n} 2^{i-1}\left[\frac{1}{(k+i+1)+(k+i+2)}\right]$

(iii) $m_{M_3}(G) = \frac{6}{|(k+1)-(k+2)|} + 3\sum_{i=1}^{n} 2^{i-1}\left[\frac{1}{|(k+i+1)-(k+i+2)|}\right]$

(iv) $m_{M_4}(G) = \frac{6}{(k+1)(2k+3)} + 3\sum_{i=1}^{n} 2^{i-1}\left[\frac{1}{(k+i+1)+(2k+2i+3)}\right]$

(v) $m_{M_5}(G) = \frac{6}{(k+2)+(2k+3)} + 3\sum_{i=1}^{n} 2^{i-1}\left[\frac{1}{(k+i+1)+(2k+2i+3)}\right]$

Proof: By using table (4) and from equation (7.1.4 to 7.1.8), we deduce derive

(i) $m_{M_1}(G) = \sum_{uv\in E(G)}[\frac{1}{e(u)+e(v)}]$

$m_{M_1}(D_{3,n-1}) = (6\times 2^0)\left[\frac{1}{(k+1)+(k+2)}\right] + (3\times 2^0)\left[\frac{1}{(k+2)+(k+3)}\right] +$

$(3\times 2^1)\left[\frac{1}{(k+3)+(k+4)}\right] + \ldots\ldots\ldots + (3\times 2^{n-1})\left[\frac{1}{(k+n+1)+(k+n+2)}\right]$

$m_{M_1}(D_{3,n-1}) = \frac{6}{(k+1)+(k+2)} + 3\sum_{i=1}^{n} 2^{i-1}\left[\frac{1}{(k+i+1)+(k+i+2)}\right]$

(ii) $m_{M_2}(G) = \sum_{uv\in E(G)}[\frac{1}{e(u)e(v)}]$

$m_{M_2}(D_{3,n-1}) = (6\times 2^0)\left[\frac{1}{(k+1)(k+2)}\right] + (3\times 2^0)\left[\frac{1}{(k+2)(k+3)}\right] +$

$(3\times 2^1)\left[\frac{1}{(k+3)(k+4)}\right] + \ldots\ldots\ldots + (3\times 2^{n-1})\left[\frac{1}{(k+n+1)(k+n+2)}\right]$

$m_{M_2}(D_{3,n-1}) = \frac{6}{(k+1)(k+2)} + 3\sum_{i=1}^{n} 2^{i-1}\left[\frac{1}{(k+i+1)+(k+i+2)}\right]$

(iii) $m_{M_3}(G) = \sum_{uv\in E(G)}[\frac{1}{|e(u) - e(v)|}]$

$$m_{M_3}(D_{3,n-1}) = (6\times 2^0)\left[\frac{1}{|(k+1)-(k+2)|}\right] + (3\times 2^0)\left[\frac{1}{|(k+2)-(k+3)|}\right] +$$

$$(3\times 2^1)\left[\frac{1}{|(k+3)-(k+4)|}\right] + \dots\dots\dots + (3\times 2^{n-1})\left[\frac{1}{|(k+n+1)-(k+n+2)|}\right]$$

$$m_{M_3}(D_{3,n-1}) = \frac{6}{|(k+1)-(k+2)|} + 3\sum_{i=1}^{n} 2^{i-1}\left[\frac{1}{|(k+i+1)-(k+i+2)|}\right]$$

(iv) $m_{M_4}(G) = \sum_{uv\in E(G)}[\frac{1}{e(u)(e(u)+e(v))}]$

$$m_{M_4}(D_{3,n-1}) = (6\times 2^0)\left[\frac{1}{(k+1)(2k+3)}\right] + (3\times 2^0)\left[\frac{1}{(k+2)(2k+5)}\right] +$$

$$(3\times 2^1)\left[\frac{1}{(k+3)(2k+7)}\right] + \dots\dots\dots + (3\times 2^{n-1})\left[\frac{1}{(k+n+1)+(2k+2n+3)}\right]$$

$$m_{M_4}(D_{3,n-1}) = \frac{6}{(k+1)(2k+3)} + 3\sum_{i=1}^{n} 2^{i-1}\left[\frac{1}{(k+i+1)+(2k+2i+3)}\right]$$

(v) $m_{M_5}(G) = \sum_{uv\in E(G)}[\frac{1}{e(v)(e(u)+e(v))}]$

$$m_{M_5}(D_{3,n-1}) = (6\times 2^0)\left[\frac{1}{(k+2)+(2k+3)}\right] + (3\times 2^0)\left[\frac{1}{(k+3)+(2k+5)}\right] +$$

$$(3\times 2^1)\left[\frac{1}{(k+4)+(2k+7)}\right] + \dots\dots\dots + (3\times 2^{n-1})\left[\frac{1}{(k+n+2)+(2k+2n+3)}\right]$$

$$m_{M_5}(D_{3,n-1}) = \frac{6}{(k+2)+(2k+3)} + 3\sum_{i=1}^{n} 2^{i-1}\left[\frac{1}{(k+i+1)+(2k+2i+3)}\right]$$

Theorem 7.1.3: Randic, Reciprocal Randic, Reduced Reciprocal Randic and Generalized Randic indices of Hexogonal core Dendrimer $D_{3,n-1}$, $n = 0,1,2,3$ ... are given by

(i) $R(G) = \frac{6}{\sqrt{(k+1)(k+2)}} + 3\sum_{i=1}^{n} 2^{i-1}\left[\frac{1}{\sqrt{(k+i+1)(k+i+2)}}\right]$

(ii) $RR(G) = 6\sqrt{(k+1)(k+2)} + 3\sum_{i=1}^{n} 2^{i-1}\sqrt{(k+i+1)(k+i+2)}$

(iii) $RRR(G) = 6\sqrt{k(k+1)} + 3\sum_{i=1}^{n} 2^{i-1}\sqrt{(k+i)(k+i+1)}$

(iv) $R_\alpha(G) = 6[(k+1)(k+2)]^\alpha + 3\sum_{i=1}^{n} 2^{i-1}[(k+i+1)(k+i+2)]^\alpha$

Proof: By using table (4) and from equation (7.1.9 to 7.1.12), we deduce derive

(i) $$R(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{e(u)e(v)}}$$

$$= (6\times2^0)\frac{1}{\sqrt{(k+1)(k+2)}} + (3\times2^0)\frac{1}{\sqrt{(k+2)(k+3)}} +$$

$$(3\times2^1)\frac{1}{\sqrt{(k+3)(k+4)}} + \dots\dots\dots + \quad (3\times2^{n-1})\frac{1}{\sqrt{(k+n+1)(k+n+2)}}$$

$$R(G) = \frac{6}{\sqrt{(k+1)(k+2)}} + 3\sum_{i=1}^{n} 2^{i-1}\left[\frac{1}{\sqrt{(k+i+1)(k+i+2)}}\right]$$

(ii) $$RR(G) = \sum_{uv\in E(G)} \sqrt{e(u)e(v)}$$

$$= (6\times2^0)\sqrt{(k+1)(k+2)} + (3\times2^0)\sqrt{(k+2)(k+3)} +$$

$$(3\times2^1)\sqrt{(k+3)(k+4)} + \dots\dots\dots +$$

$$(3\times2^{n-1})\sqrt{(k+n+1)(k+n+2)}$$

$$RR(G) = 6\sqrt{(k+1)(k+2)} + 3\sum_{i=1}^{n} 2^{i-1}\sqrt{(k+i+1)(k+i+2)}$$

(iii) $$RRR(G) = \sum_{uv\in E(G)} \sqrt{(e(u)-1)(e(v)-1)}$$

$$= (6\times2^0)\sqrt{k(k+1)} + (3\times2^0)\sqrt{(k+1)(k+2)} +$$

$$(3\times2^1)\sqrt{(k+2)(k+3)} + \dots\dots\dots +$$

$$(3\times2^{n-1})\sqrt{(k+n)(k+n+1)}$$

$$RRR(G) = 6\sqrt{k(k+1)} + 3\sum_{i=1}^{n} 2^{i-1}\sqrt{(k+i)(k+i+1)}$$

(iv) $$R_\alpha(G) = \sum_{uv\in E(G)} [e(u)e(v)]^\alpha$$

$$= (6\times2^0)\,[(k+1)(k+2)]^\alpha + (3\times2^0)\,[(k+2)(k+3)]^\alpha +$$

$$(3\times2^1)\ \ [(k+3)(k+4)]^\alpha + \dots\dots\dots +$$

$$(3\times2^{n-1})\,[(k+n+1)(k+n+2)]^\alpha$$

$$= 6[(k+1)(k+2)]^\alpha + 3\sum_{i=1}^{n} 2^{i-1}\,[(k+i+1)(k+i+2)]^\alpha$$

7.2 Multiplicative of Redefined first, second, and third Zagreb indices, Modified first, second, third, fourth and fifth Zagreb indices, Randic, Reciprocal Randic, Reduced Reciprocal Randic and Generalized Randic indices of eccentricity based topological indices of Hexoconal core Dendrimer $D_{3,n-1}$.

In this section, we calculate eccentricity based Multiplicative of Redefined first, second and third Zagreb indices, Modified first, second, Ttird, fourth and fifth Zagreb indices, Randic, Reciprocal Randic, Reduced Reciprocal Randic and Generalized Randic topological indices of Hexoconal core Dendrimer $D_{3,n-1}$.

(i) Multiplicative of Redefined first, second and third Zagreb indices of G, as

$$ReZG_1\prod(G) = \prod_{uv\in E(G)}\left(\frac{e(u)+e(v)}{e(u)\,e(v)}\right) \tag{7.2.1}$$

$$ReZG_2\prod(G) = \prod_{uv\in E(G)}\left(\frac{e(u)e(v)}{e(u)+e(v)}\right) \tag{7.2.2}$$

$$ReZG_3(G) = \prod_{uv\in E(G)}[e(u)e(v)(e(u)+e(v)] \tag{7.2.3}$$

(ii) Multiplicative of modified first, second, third, fourth and fifth Zagreb indices of G, are defined

$$m_{M_1}\prod(G) = \prod_{uv\in E(G)}\left[\frac{1}{e(u)+e(v)}\right] \tag{7.2.4}$$

$$m_{M_2}\prod(G) = \prod_{uv\in E(G)}\left[\frac{1}{e(u)e(v)}\right] \tag{7.2.5}$$

$$m_{M_3}\prod(G) = \prod_{uv\in E(G)}\left[\frac{1}{|e(u)-e(v)|}\right] \tag{7.2.6}$$

$$m_{M_4}\prod(G) = \prod_{uv\in E(G)}\left[\frac{1}{e(u)(e(u)+e(v))}\right] \tag{7.2.7}$$

$$m_{M_5}\prod(G) = \prod_{uv\in E(G)}\left[\frac{1}{e(v)(e(u)+e(v))}\right] \tag{7.2.8}$$

(iii) Multiplicative of Randic, Reciprocal Randic, Reduced Reciprocal Randic and Generalized Randic indices of G, are defined

$$R\prod(G) = \prod_{uv\in E(G)}\frac{1}{\sqrt{e(u)e(v)}} \tag{7.2.9}$$

$$RR\prod(G) = \prod_{uv\in E(G)}\sqrt{e(u)e(v)} \tag{7.2.10}$$

$$\mathrm{RRR}\prod(\mathrm{G}) = \prod_{uv\in E(\mathrm{G})}\sqrt{(e(u)-1)(e(v)-1)} \qquad (7.2.11)$$

$$\mathrm{R}_{\alpha}\prod(\mathrm{G}) = \prod_{uv\in E(\mathrm{G})}[e(u)e(v)]^{\alpha} \qquad (7.2.12)$$

Theorem7.2.1: Multiplicative of redefined first, second and third Zagreb indices of Hexogonal core Dendrimer $D_{3,n-1}$, $n = 0,1,2,3$ ... are given by

(i) $\mathrm{ReZG_1}\prod(\mathrm{G}) = 6\left[\frac{2k+3}{(k+1)\,(k+2)}\right] \times 3\prod_{i=1}^{n} 2^{i-1}\left[\frac{(k+i+1)+(k+i+2)}{(k+i+1)\,(k+i+2)}\right]$

(ii) $\mathrm{ReZG_2}\prod(\mathrm{G}) = 6\left[\frac{(k+1)\,(k+2)}{2k+3}\right] \times 3\prod_{i=1}^{n} 2^{i-1}\left[\frac{(k+i+1)+(k+i+2)}{2k+2i+3}\right]$

(iv) $\mathrm{ReZG_3}(\mathrm{G}) = 6(k+1)(k+2)(2k+3) \times 3\prod_{i=1}^{n} 2^{i-1}[(k+i+1)\,(k+i+2)$

$(2k+2i+3)]$

Proof: By using table (4) and from equation (7.2.1 to 7.2.3), we deduce derive

(i) $\mathrm{ReZG_1}\prod(\mathrm{G}) = \prod_{uv\in E(\mathrm{G})}\left(\frac{e(u)+e(v)}{e(u)\,e(v)}\right)$

$$= (6\times2^0)\left[\frac{(k+1)+(k+2)}{(k+1)\,(k+2)}\right] \times (3\times2^0)\left[\frac{(k+2)+(k+3)}{(k+2)\,(k+3)}\right] \times$$

$$(3\times2^1)\left[\frac{(k+3)+(k+4)}{(k+3)\,(k+4)}\right] \times \ldots\ldots\times (3\times2^{n-1})\left[\frac{(k+n+1)+(k+n+2)}{(K+n+1)\,(K+n+2)}\right]$$

$$= 6\left[\frac{2k+3}{(k+1)\,(k+2)}\right] \times 3\prod_{i=1}^{n} 2^{i-1}\left[\frac{(k+i+1)+(k+i+2)}{(k+i+1)\,(k+i+2)}\right]$$

(ii) $\mathrm{ReZG_2}\prod(\mathrm{G}) = \prod_{uv\in E(\mathrm{G})}\left(\frac{e(u)e(v)}{e(u)+e(v)}\right)$

$$= (6\times2^0)\left[\frac{(k+1)\,(k+2)}{(k+1)+(k+2)}\right] \times (3\times2^0)\left[\frac{(k+2)(k+3)}{(k+2)+(k+3)}\right] \times$$

$$(3\times2^1)\left[\frac{(k+3)(k+4)}{(k+3)+(k+4)}\right] \times \ldots\ldots\ldots\times (3\times2^{n-1})\left[\frac{(k+n+1)\,(k+n+2)}{(k+n+1)+(k+n+2)}\right]$$

$$= 6\left[\frac{(k+1)\,(k+2)}{2k+3}\right] \times 3\prod_{i=1}^{n} 2^{i-1}\left[\frac{(k+i+1)+(k+i+2)}{2k+2i+3}\right]$$

(iii) $\mathrm{ReZG_3}\prod(\mathrm{G}) = \prod_{uv\in E(\mathrm{G})}[e(u)e(v)(e(u)+e(v)]$

$$= (6\times2^0)\,[(k+1)(k+2)(2k+3)] \times$$

$$(3\times2^0)\,[(k+2)(k+3)(2k+5)] \times$$

$$(3\times2^1)\ [(k+3)+(k+4)+(2K+7)\,]\times \ldots\ldots\times$$

$$(3\times2^{n-1})\,[(k+n+1)+(k+n+2)+(2k+2n+3)\,]$$

$$= 6\,(k+1)\,(k+2)\,(2k+3)\times 3\prod_{i=1}^{n} 2^{i-1}\,[(k+i+1)+$$

$$(k+i+2)(2k+2i+3)]$$

Theorem 7.2.2: Multiplicative of modified first, second, third, fourth and fifth Zagreb indices of Hexogonal core Dendrimer $D_{3,n-1}$, $n = 0,1,2,3 \ldots$ are given by

(i) $m_{M_1}\prod(G) = \frac{6}{(k+1)+(k+2)}\times 3\prod_{i=1}^{n} 2^{i-1}\left[\frac{1}{(k+i+1)+(k+i+2)}\right]$

(ii) $m_{M_2}\prod(G) = \frac{6}{(k+1)(k+2)}\times 3\prod_{i=1}^{n} 2^{i-1}\left[\frac{1}{(k+i+1)+(k+i+2)}\right]$

(iii) $m_{M_3}\prod(G) = \frac{6}{|(k+1)-(k+2)|}\times 3\prod_{i=1}^{n} 2^{i-1}\left[\frac{1}{|(k+i+1)-(k+i+2)|}\right]$

(iv) $m_{M_4}\prod(G) = \frac{6}{(k+1)(2k+3)}\times 3\prod_{i=1}^{n} 2^{i-1}\left[\frac{1}{(k+i+1)+(2k+2i+3)}\right]$

(v) $m_{M_5}\prod(G) = \frac{6}{(k+2)+(2k+3)}\times 3\prod_{i=1}^{n} 2^{i-1}\left[\frac{1}{(k+i+1)+(2k+2i+3)}\right]$

Proof: By using table (4) and from equation (7.2.4 to 7.2.8), we deduce derive

(i) $m_{M_1}\prod(G) = \prod_{uv\in E(G)}[\frac{1}{e(u)+e(v)}]$

$$= (6\times2^0)\left[\frac{1}{(k+1)+(k+2)}\right]\times(3\times2^0)\left[\frac{1}{(k+2)+(k+3)}\right]\times$$

$$(3\times2^1)\left[\frac{1}{(k+3)+(k+4)}\right]\times\ldots\ldots\ldots\times(3\times2^{n-1})\left[\frac{1}{(k+n+1)+(k+n+2)}\right]$$

$$m_{M_1}\prod(G) = \frac{6}{(k+1)+(k+2)}\times 3\prod_{i=1}^{n} 2^{i-1}\left[\frac{1}{(k+i+1)+(k+i+2)}\right]$$

(ii) $m_{M_2}\prod(G) = \prod_{uv\in E(G)}[\frac{1}{e(u)e(v)}]$

$$= (6\times2^0)\left[\frac{1}{(k+1)(k+2)}\right]\times(3\times2^0)\left[\frac{1}{(k+2)(k+3)}\right]\times$$

$$(3\times2^1)\left[\frac{1}{(k+3)(k+4)}\right]\times\ldots\ldots\ldots\times(3\times2^{n-1})\left[\frac{1}{(k+n+1)(k+n+2)}\right]$$

$$m_{M_2}\prod(G) = \frac{6}{(k+1)(k+2)}\times 3\prod_{i=1}^{n} 2^{i-1}\left[\frac{1}{(k+i+1)+(k+i+2)}\right]$$

(iii) $m_{M_3}\prod(G) = \prod_{uv\in E(G)}[\frac{1}{|e(u)-e(v)|}]$

$$= (6\times2^0)\left[\frac{1}{|(k+1)-(k+2)|}\right]\times(3\times2^0)\left[\frac{1}{|(k+2)-(k+3)|}\right]\times$$

$$(3\times2^1)\left[\frac{1}{|(k+3)-(k+4)|}\right]\times\ldots\ldots\ldots\times\ (3\times2^{n-1})\left[\frac{1}{|(k+n+1)-(k+n+2)|}\right]$$

$$m_{M_3}\prod(G) = \frac{6}{|(k+1)-(k+2)|}\times 3\prod_{i=1}^{n}2^{i-1}\left[\frac{1}{|(k+i+1)-(k+i+2)|}\right]$$

(iv) $$m_{M_4}\prod(G) = \prod_{uv\in E(G)}\left[\frac{1}{e(u)(e(u)+e(v))}\right]$$

$$= (6\times2^0)\left[\frac{1}{(k+1)(2k+3)}\right]\times(3\times2^0)\left[\frac{1}{(k+2)(2k+5)}\right]\times$$

$$(3\times2^1)\left[\frac{1}{(k+3)(2k+7)}\right]\times\ldots\ldots\ldots\times(3\times2^{n-1})\left[\frac{1}{(k+n+1)+(2k+2n+3)}\right]$$

$$m_{M_4}\prod(G) = \frac{6}{(k+1)(2k+3)}\times 3\prod_{i=1}^{n}2^{i-1}\left[\frac{1}{(k+i+1)+(2k+2i+3)}\right]$$

(v) $$m_{M_5}\prod(G) = \prod_{uv\in E(G)}\left[\frac{1}{e(v)(e(u)+e(v))}\right]$$

$$= (6\times2^0)\left[\frac{1}{(k+2)+(2k+3)}\right]\times(3\times2^0)\left[\frac{1}{(k+3)+(2k+5)}\right]\times$$

$$(3\times2^1)\left[\frac{1}{(k+4)+(2k+7)}\right]\times\ldots\ldots\ldots\times(3\times2^{n-1})\left[\frac{1}{(k+n+2)+(2k+2n+3)}\right]$$

$$m_{M_5}\prod(G) = \frac{6}{(k+2)+(2k+3)}\times 3\prod_{i=1}^{n}2^{i-1}\left[\frac{1}{(k+i+1)+(2k+2i+3)}\right]$$

Theorem 7.2.3: Multiplicative of Randic, Reciprocal Randic, Reduced Reciprocal Randic and Generalized Randic indices of Hexogonal core Dendrimer $D_{3,n-1}$, $n = 0,1,2,3\ldots$ are given by

(i) $R\prod(G) = \frac{6}{\sqrt{(k+1)(k+2)}}\times 3\prod_{i=1}^{n}2^{i-1}\left[\frac{1}{\sqrt{(k+i+1)(k+i+2)}}\right]$

(ii) $RR\prod(G) = 6\sqrt{(k+1)(k+2)}\times 3\prod_{i=1}^{n}2^{i-1}\sqrt{(k+i+1)(k+i+2)}$

(iii) $RRR\prod(G) = 6\sqrt{k(k+1)}\times 3\prod_{i=1}^{n}2^{i-1}\sqrt{(k+i)(k+i+1)}$

(iv) $R_\alpha\prod(G) = 6[(k+1)(k+2)]^{\alpha}\times 3\prod_{i=1}^{n}2^{i-1}[(k+i+1)(k+i+2)]^{\alpha}$

Proof: By using table (4) and from equation (7.2.9 to 7.2.12), we deduce derive

(i) $R\prod(G) = \prod_{uv\in E(G)} \frac{1}{\sqrt{e(u)e(v)}}$

$$(3\times 2^1)\frac{1}{\sqrt{(k+3)(k+4)}} \times \ldots\ldots\ldots \times (3\times 2^{n-1})\frac{1}{\sqrt{(k+n+1)(k+n+2)}}$$

$$R\prod(G) = \frac{6}{\sqrt{(k+1)(k+2)}} \times 3\prod_{i=1}^{n} 2^{i-1}\left[\frac{1}{\sqrt{(k+i+1)(k+i+2)}}\right]$$

(ii) $RR\prod(G) = \prod_{uv\in E(G)} \sqrt{e(u)e(v)}$

$$= (6\times 2^0)\sqrt{(k+1)(k+2)} \times (3\times 2^0)\sqrt{(k+2)(k+3)} \times$$

$$(3\times 2^1)\sqrt{(k+3)(k+4)} \times \ldots\ldots\ldots \times$$

$$(3\times 2^{n-1})\sqrt{(k+n+1)(k+n+2)}$$

$$RR\prod(G) = 6\sqrt{(k+1)(k+2)} \times 3\prod_{i=1}^{n} 2^{i-1}\sqrt{(k+i+1)(k+i+2)}$$

(iii) $RRR\prod(G) = \prod_{uv\in E(G)} \sqrt{(e(u)-1)(e(v)-1)}$

$$= (6\times 2^0)\sqrt{k(k+1)} \times (3\times 2^0)\sqrt{(k+1)(k+2)} \times (3\times 2^1)$$

$$\sqrt{(k+2)(k+3)} \times \ldots\ldots\ldots \times (3\times 2^{n-1})\sqrt{(k+n)(k+n+1)}$$

$$RRR\prod(G) = 6\sqrt{k(k+1)} \times 3\prod_{i=1}^{n} 2^{i-1}\sqrt{(k+i)(k+i+1)}$$

(iv) $R_\alpha\prod(G) = \prod_{uv\in E(G)}[e(u)e(v)]^\alpha$

$$= (6\times 2^0)[(k+1)(k+2)]^\alpha \times (3\times 2^0)[(k+2)(k+3)]^\alpha \times (3\times 2^1)$$

$$[(k+3)(k+4)]^\alpha \times \ldots\ldots\ldots \times (3\times 2^{n-1})[(k+n+1)(k+n+2)]^\alpha$$

$$R_\alpha\prod(G) = 6[(k+1)(k+2)]^\alpha \times 3\prod_{i=1}^{n} 2^{i-1}[(k+i+1)(k+i+2)]^\alpha$$

7.3 Polynomial of redefined first, second, and third Zagreb indices, modified first, second, third, fourth and fifth Zagreb indices, Randic, Reciprocal Randic, Reduced Reciprocal Randic and Generalized Randic indices of eccentricity based topological indices of Hexoconal core Dendrimer $D_{3,n-1}$.

In this section, we derive eccentricity based polynomial of redefined first, second and third Zagreb indices, Modified First, Second, Third, Fourth and Fifth Zagreb indices, Randic, Reciprocal Randic, Reduced Reciprocal Randic and Generalized Randic topological indices of Hexoconal core Dendrimer $D_{3,n-1}$.

(i) polynomial Redefined First, Second and Third Zagreb indices of G, as

$$ReZG_1(G,x)=\sum_{uv\in E(G)} x^{\left(\frac{e(u)+e(v)}{e(u)\,e(v)}\right)} \tag{7.3.1}$$

$$ReZG_2(G,x)=\sum_{uv\in E(G)} x^{\left(\frac{e(u)e(v)}{e(u)+e(v)}\right)} \tag{7.3.2}$$

$$ReZG_3(G,x)=\sum_{uv\in E(G)} x^{e(u)e(v)[e(u)+e(v)]} \tag{7.3.3}$$

(ii) polynomial Modified First, Second, Third, Fourth and Fifth Zagreb indices of G, are defined

$$m_{M_1}(G,x)=\sum_{uv\in E(G)} x^{[\frac{1}{e(u)+e(v)}]} \tag{7.3.4}$$

$$m_{M_2}(G,x)=\sum_{uv\in E(G)} x^{[\frac{1}{e(u)e(v)}]} \tag{7.3.5}$$

$$m_{M_3}(G,x)=\sum_{uv\in E(G)} x^{[\frac{1}{|e(u)-e(v)|}]} \tag{7.3.6}$$

$$m_{M_4}(G,x)=\sum_{uv\in E(G)} x^{[\frac{1}{e(u)(e(u)+e(v))}]} \tag{7.3.7}$$

$$m_{M_5}(G,x)=\sum_{uv\in E(G)} x^{[\frac{1}{e(v)(e(u)+e(v))}]} \tag{7.3.8}$$

(iii) polynomial Randic, Reciprocal Randic, Reduced Reciprocal Randic and Generalized Randic indices of G, are defined

$$R(G,x)=\sum_{uv\in E(G)} x^{[\frac{1}{\sqrt{e(u)e(v)}}]} \tag{7.3.9}$$

$$RR(G,x)=\sum_{uv\in E(G)} x^{\sqrt{e(u)e(v)}} \tag{7.3.10}$$

$$RRR(G, x) = \sum_{uv\in E(G)} x^{\sqrt{(e(u)-1)(e(v)-1)}} \quad (7.3.11)$$

$$R_\alpha(G, x) = \sum_{uv\in E(G)} x^{[e(u)e(v)]^\alpha} \quad (7.3.12)$$

In the following theorem, we compute these topological indices of Hexagonal core Dendrimer $D_{3,n-1}$.

Theorem7.3.1: polynomial Redefined First, Second and Third Zagreb indices of Hexogonal core Dendrimer $D_{3,n-1}$, n = 0,1,2,3 ... are given by

(i) $ReZG_1(G, x) = 6\, x^{\left[\frac{2k+3}{(k+1)\,(k+2)}\right]} + 3\sum_{i=1}^{n} 2^{i-1}\, x^{\left[\frac{(k+i+1)+\,(k+i+2)}{(k+i+1)\,(k+i+2)}\right]}$

(ii) $ReZG_2(G, x) = 6\, x^{\left[\frac{(k+1)\,(k+2)}{2k+3}\right]} + 3\sum_{i=1}^{n} 2^{i-1}\, x^{\left[\frac{(k+i+1)\,+\,(k+i+2)}{2k+2i+3}\right]}$

(iii) $ReZG_3(G, x) = 6\, x^{(k+1)(k+2)(2k+3)} + 3\sum_{i=1}^{n} 2^{i-1}\, x^{[(k+i+1)\,(k+i+2)(2k+2i+3)]}$

Proof: Let G = $D_{3,n-1}$, n = 0,1,2,3 ... be the graph of Hexagonal core Dendrimer. By using equation (7.3.1 to 7.3.3) and Table , we obtain

(i) $ReZG_1(G, x) = \sum_{uv\in E(G)} x^{\left(\frac{e(u)+e(v)}{e(u)\,e(v)}\right)}$

$$= (6\times 2^0)\, x^{\left[\frac{(k+1)+\,(k+2)}{(k+1)\,(k+2)}\right]} + (3\times 2^0)\, x^{\left[\frac{(k+2)+\,(k+3)}{(k+2)\,(k+3)}\right]} +$$

$$(3\times 2^1)\, x^{\left[\frac{(k+3)+\,(k+4)}{(k+3)\,(k+4)}\right]} + \ldots\ldots\ldots + (3\times 2^{n-1})\, x^{\left[\frac{(k+n+1)+\,(k+n+2)}{(K+n+1)\,(K+n+2)}\right]}$$

$$ReZG_1(G, x) = 6\, x^{\left[\frac{2k+3}{(k+1)\,(k+2)}\right]} + 3\sum_{i=1}^{n} 2^{i-1}\, x^{\left[\frac{(k+i+1)+\,(k+i+2)}{(k+i+1)\,(k+i+2)}\right]}$$

(ii) $ReZG_2(G, x) = \sum_{uv\in E(G)} x^{\left(\frac{e(u)e(v)}{e(u)+e(v)}\right)}$

$$= (6\times 2^0)\, x^{\left[\frac{(k+1)\,(k+2)}{(k+1)+(k+2)}\right]} + (3\times 2^0\,)\, x^{\left[\frac{(k+2)(k+3)}{(k+2)+\,(k+3)}\right]} +$$

$$(3\times 2^1) x^{\left[\frac{(k+3)(k+4)}{(k+3)+(k+4)}\right]} + \ldots\ldots\ldots + (3\times 2^{n-1}) x^{\left[\frac{(k+n+1)\,(k+n+2)}{(k+n+1)+(k+n+2)}\right]}$$

$$ReZG_2(G, x) = 6\, x^{\left[\frac{(k+1)\,(k+2)}{2k+3}\right]} + 3\sum_{i=1}^{n} 2^{i-1}\, x^{\left[\frac{(k+i+1)+\,(k+i+2)}{2k+2i+3}\right]}$$

(iii) $\quad ReZG_3(G, x) = \sum_{uv\in E(G)} x^{e(u)e(v)[e(u)+e(v)]}$

$$= (6\times 2^0)\, x^{[(k+1)(k+2)(2k+3)]} + (3\times 2^0)\, x^{[(k+2)(k+3)(2k+5)]} +$$

$$(3\times 2^1)\, x^{[(k+3)+(k+4)+(2k+7)\,]} + \ldots..+$$

$$(3\times 2^{n-1})\, x^{[(k+n+1)+(k+n+2)+(2k+2n+3)\,]}$$

$$ReZG_3(G, x) = 6\, x^{(k+1)\,(k+2)\,(2k+3)} + 3\sum_{i=1}^{n} 2^{i-1}\, x^{[(k+i+1)+(k+i+2)(2k+2i+3)]}$$

Theorem 7.3.2: Modified First, Second, Third, Fourth and Fifth Zagreb polynomial indices of Hexogonal core Dendrimer $D_{3,n-1}$, $n = 0,1,2,3$ ... are given by

(i) $\quad m_{M_1}(G,x) = 6\, x^{\left[\frac{1}{(k+1)+(k+2)}\right]} + 3\sum_{i=1}^{n} 2^{i-1}\, x^{\left[\frac{1}{(k+i+1)+(k+i+2)}\right]}$

(ii) $\quad m_{M_2}(G,x) = 6\, x^{\left[\frac{1}{(k+1)(k+2)}\right]} + 3\sum_{i=1}^{n} 2^{i-1}\, x^{\left[\frac{1}{(k+i+1)+(k+i+2)}\right]}$

(iii) $\quad m_{M_3}(G,x) = 6\, x^{\left[\frac{1}{|(k+1)\,-(k+2)|}\right]} + 3\sum_{i=1}^{n} 2^{i-1} x^{\left[\frac{1}{|(k+i+1)\,-(k+i+2)|}\right]}$

(iv) $\quad m_{M_4}(G,x) = 6\, x^{\left[\frac{1}{(k+1)(2k+3)}\right]} + 3\sum_{i=1}^{n} 2^{i-1}\, x^{\left[\frac{1}{(k+i+1)+(2k+2i+3)}\right]}$

(v) $\quad m_{M_5}(G,x) = 6\, x^{\left[\frac{1}{(k+2)+(2k+3)}\right]} + 3\sum_{i=1}^{n} 2^{i-1} x^{\left[\frac{1}{(k+i+1)+(2k+2i+3)}\right]}$

Proof: Let $G = D_{3,n-1}$, $n = 0,1,2,3$ ... be the graph of Hexagonal core Dendrimer. By using equation (7.3.4 to 7.3.8) and Table(4) , we derive

(i) $\quad m_{M_1}(G,x) = \sum_{uv\in E(G)} x^{\left[\frac{1}{e(u)+e(v)}\right]}$

$$= (6\times 2^0)\, x^{\left[\frac{1}{(k+1)+(k+2)}\right]} + (3\times 2^0)\, x^{\left[\frac{1}{(k+2)+(k+3)}\right]} +$$

$$(3\times 2^1)\, x^{\left[\frac{1}{(k+3)+(k+4)}\right]} + \ldots\ldots..+ (3\times 2^{n-1})\, x^{\left[\frac{1}{(k+n+1)+(k+n+2)}\right]}$$

$$m_{M_1}(G,x) = 6\, x^{\left[\frac{1}{(k+1)+(k+2)}\right]} + 3\sum_{i=1}^{n} 2^{i-1}\, x^{\left[\frac{1}{(k+i+1)+(k+i+2)}\right]}$$

(ii) $\quad m_{M_2}(G,x) = \sum_{uv\in E(G)} x^{\left[\frac{1}{e(u)e(v)}\right]}$

$$= (6\times 2^0)\, x^{\left[\frac{1}{(k+1)(k+2)}\right]} + (3\times 2^0)\, x^{\left[\frac{1}{(k+2)(k+3)}\right]} +$$

$$(3\times2^{1})\, x^{\left[\frac{1}{(k+3)(k+4)}\right]} + \ldots\ldots\ldots + (3\times2^{n-1})\, x^{\left[\frac{1}{(k+n+1)(k+n+2)}\right]}$$

$$m_{M_2}(G,x) = 6\, x^{\left[\frac{1}{(k+1)(k+2)}\right]} + 3\sum_{i=1}^{n} 2^{i-1}\, x^{\left[\frac{1}{(k+i+1)+(k+i+2)}\right]}$$

(iii) $$m_{M_3}(G,x) = \sum_{uv\in E(G)} x^{\left[\frac{1}{|e(u)\,-\,e(v)|}\right]}$$

$$= (6\times2^{0})\, x^{\left[\frac{1}{|(k+1)\,-(k+2)|}\right]} + (3\times2^{0})\, x^{\left[\frac{1}{|(k+2)\,-(k+3)|}\right]} +$$

$$(3\times2^{1})\, x^{\left[\frac{1}{|(k+3)\,-(k+4)|}\right]} + \ldots\ldots\ldots + (3\times2^{n-1})\, x^{\left[\frac{1}{|(k+n+1)\,-(k+n+2)|}\right]}$$

$$m_{M_3}(G,x) = 6\, x^{\left[\frac{1}{|(k+1)\,-(k+2)|}\right]} + 3\sum_{i=1}^{n} 2^{i-1} x^{\left[\frac{1}{|(k+i+1)\,-(k+i+2)|}\right]}$$

(iv) $$m_{M_4}(G,x) = \sum_{uv\in E(G)} x^{\left[\frac{1}{e(u)(e(u)+e(v))}\right]}$$

$$= (6\times2^{0})\, x^{\left[\frac{1}{(k+1)(2k+3)}\right]} + (3\times2^{0})\, x^{\left[\frac{1}{(k+2)(2k+5)}\right]} +$$

$$(3\times2^{1})\, x^{\left[\frac{1}{(k+3)(2k+7)}\right]} + \ldots\ldots\ldots + (3\times2^{n-1})\, x^{\left[\frac{1}{(k+n+1)+(2k+2n+3)}\right]}$$

$$m_{M_4}(G,x) = 6\, x^{\left[\frac{1}{(k+1)(2k+3)}\right]} + 3\sum_{i=1}^{n} 2^{i-1}\, x^{\left[\frac{1}{(k+i+1)+(2k+2i+3)}\right]}$$

(v) $$m_{M_5}(G,x) = \sum_{uv\in E(G)} x^{\left[\frac{1}{e(v)(e(u)+e(v))}\right]}$$

$$= (6\times2^{0})\, x^{\left[\frac{1}{(k+2)+(2k+3)}\right]} + (3\times2^{0})\, x^{\left[\frac{1}{(k+3)+(2k+5)}\right]} +$$

$$(3\times2^{1})\, x^{\left[\frac{1}{(k+4)+(2k+7)}\right]} + \ldots\ldots\ldots + (3\times2^{n-1})\, x^{\left[\frac{1}{(k+n+2)+(2k+2n+3)}\right]}$$

$$m_{M_5}(G,x) = 6\, x^{\left[\frac{1}{(k+2)+(2k+3)}\right]} + 3\sum_{i=1}^{n} 2^{i-1} x^{\left[\frac{1}{(k+i+1)+(2k+2i+3)}\right]}$$

Theorem 7.3.3: Randic, Reciprocal Randic, Reduced Reciprocal Randic and Generalized Randic polynomial indices of Hexogonal core Dendrimer $D_{3,n-1}$, $n = 0,1,2,3$ ... are given by

(i) $$R\,(G,x) = 6x^{\left[\frac{1}{\sqrt{(k+1)(k+2)}}\right]} + 3\sum_{i=1}^{n} 2^{i-1}\, x^{\left[\frac{1}{\sqrt{(k+i+1)(k+i+2)}}\right]}$$

(ii) $$RR\,(G,x) = 6\, x^{\sqrt{(k+1)(k+2)}} + 3\sum_{i=1}^{n} 2^{i-1} x^{\sqrt{(k+i+1)(k+i+2)}}$$

(iii) RRR (G, x) $= 6\, x^{\sqrt{k(k+1)}} + 3\sum_{i=1}^{n} 2^{i-1}\, x^{\sqrt{(k+i)(k+i+1)}}$

(iv) R_α (G, x) $= 6x^{[(k+1)(k+2)]^{\alpha}} + 3\sum_{i=1}^{n} 2^{i-1}\, x^{[(k+i+1)(k+i+2)]^{\alpha}}$

Proof: Let G = $D_{3,n-1}$, n = 0,1,2,3 ... be the graph of Hexagonal core Dendrimer. By using equation (7.3.9 to 7.3.12) and Table (4) , we have

(i) $R(G, x) = \sum_{uv\in E(G)} x^{\left[\frac{1}{\sqrt{e(u)e(v)}}\right]}$

$$= (6\times2^0)\, x^{\left[\frac{1}{\sqrt{(k+1)(k+2)}}\right]} + (3\times2^0)\, x^{\left[\frac{1}{\sqrt{(k+2)(k+3)}}\right]} + (3\times2^1)\, x^{\left[\frac{1}{\sqrt{(k+3)(k+4)}}\right]} + \dots\dots\dots + (3\times2^{n-1})\, x^{\left[\frac{1}{\sqrt{(k+n+1)(k+n+2)}}\right]}$$

$$R(G, x) = 6x^{\left[\frac{1}{\sqrt{(k+1)(k+2)}}\right]} + 3\sum_{i=1}^{n} 2^{i-1}\, x^{\left[\frac{1}{\sqrt{(k+i+1)(k+i+2)}}\right]}$$

(ii) $RR(G, x) = \sum_{uv\in E(G)} x^{\sqrt{e(u)e(v)}}$

$$= (6\times2^0)\, x^{\sqrt{(k+1)(k+2)}} + (3\times2^0)\, x^{\sqrt{(k+2)(k+3)}} + (3\times2^1)\, x^{\sqrt{(k+3)(k+4)}} + \dots\dots\dots + (3\times2^{n-1})\, x^{\sqrt{(k+n+1)(k+n+2)}}$$

$$RR(G, x) = 6\, x^{\sqrt{(k+1)(k+2)}} + 3\sum_{i=1}^{n} 2^{i-1} x^{\sqrt{(k+i+1)(k+i+2)}}$$

(iii) $RRR(G, x) = \sum_{uv\in E(G)} x^{\sqrt{(e(u)-1)(e(v)-1)}}$

$$= (6\times2^0)\, x^{\sqrt{k(k+1)}} + (3\times2^0)\, x^{\sqrt{(k+1)(k+2)}} + (3\times2^1)\, x^{\sqrt{(k+2)(k+3)}} + \dots\dots\dots + (3\times2^{n-1})\, x^{\sqrt{(k+n)(k+n+1)}}$$

$$RRR(G, x) = 6\, x^{\sqrt{k(k+1)}} + 3\sum_{i=1}^{n} 2^{i-1}\, x^{\sqrt{(k+i)(k+i+1)}}$$

(iv) $R_\alpha(G, x) = \sum_{uv\in E(G)} x^{[e(u)e(v)]^{\alpha}}$

$$= (6\times2^0)\, x^{[(k+1)(k+2)]^{\alpha}} + (3\times2^0)\, x^{[(k+2)(k+3)]^{\alpha}} + (3\times2^1)\, x^{[(k+3)(k+4)]^{\alpha}} + \dots\dots\dots + (3\times2^{n-1})\, x^{[(k+n+1)(k+n+2)]^{\alpha}}$$

$$R_\alpha(G, x) = 6x^{[(k+1)(k+2)]^{\alpha}} + 3\sum_{i=1}^{n} 2^{i-1}\, x^{[(k+i+1)(k+i+2)]^{\alpha}}$$

Table 5: Edges of the type uv∈E(G) and eccentricity for Dendrimers $D_{3,1}$ and $D_{3,2}$.

Dendrimer	Edges of the type uv∈E(G) with eccentricity	Frequency
$D_{3,1}$	(4,5)	3
	(5,6)	6
$D_{3,2}$	(3,4)	6
	(4,5)	6
	(5,6)	3

This method to compute eccentricity based topological indices is as given below. The eccentricity based Redefined first Zagreb index,

$$ReZG_1(G) = \sum_{uv\in E(G)} \left(\frac{e(u)+e(v)}{e(u)\,e(v)}\right)$$

$$ReZG_1(D_{3,1}) = 3\left(\frac{4+5}{4\times5}\right) + 6\left(\frac{5+6}{5\times6}\right)$$

$$= 3.55$$

Redefined second Zagreb index for $D_{3,1}$ (Table 5) is computed as

The values of $ReZG_1(D_{3,1})$ and $ReZG_2(D_{3,1})$ are given in table (6).

Table 6: The values of $ReZG_1(D_{3,1})$ and $ReZG_2(D_{3,1})$ of Dendrimers $D_{3,1}$ and $D_{3,2}$.

Dendrimer	Redefined first Zagreb index($ReZG_1(D_{3,1})$)	Redefined second Zagreb index($ReZG_1(D_{3,1})$)
$D_{3,1}$	3.55	23.03
$D_{3,2}$	7.3	31.79

Here eccentricity based Redefined second Zagreb version has greater value then Redefined second Zagreb Index.

CHAPTER 8

ECCENTRIC BASED TOPOLOGICAL INDICES OF MULTIPLICATIVE AND THEIR POLYNOMIAL OF DENDRIMER D(n)

In this chapter, we obtained eccentricity based Hyper first, second, third, fourth and fifth Zagreb indices, Square reduced sum, Square reduced product, Square reduced Arithmetic, Square reduced Geometric and Square reduced Harmonic indices, First F, Second F, The Minus F and Square F indices. Also we define multiplicative and polynomial indices of Nanostar Dendrimer D(n).

In this chapter, we focus on the special infinite class of Nanostar Dendrimers D(n) (here n is the step of growth) which is descripted as follows:

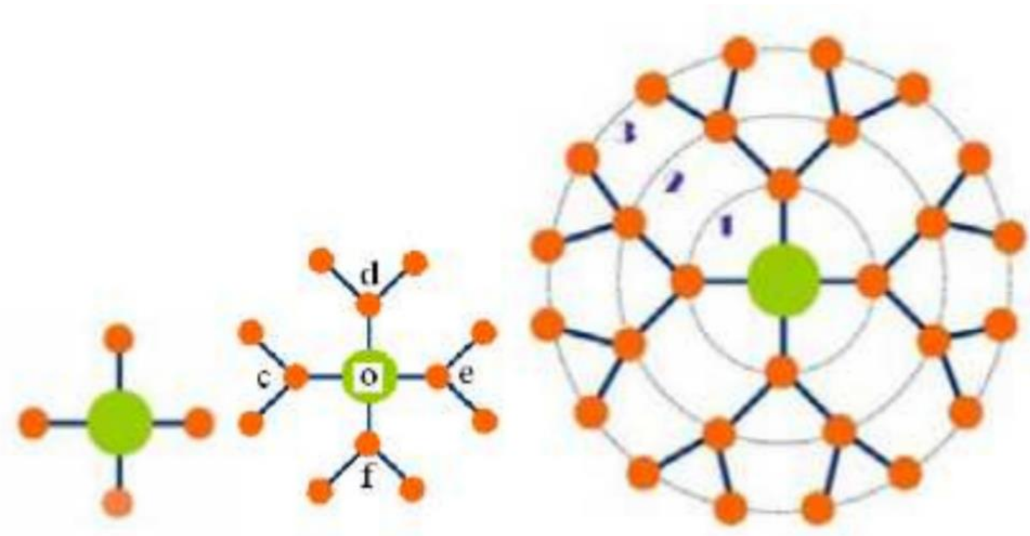


Figure 8.1. The structures of D(1), D(2) and D(3)

By the analysis of its molecular structure, we see that the edge set of D(n) can be divided into n parts:

E_1: ec(u) = n and ec(v) = n+1;

E_2: ec(u) = n+1 and ec(v) = n+2;

……………….

E_i: ec(u) = n+$i + 1$ and ec(v) = n+i;

……………….

E_n: ec(u) = 2n−1 and ec(v) = 2n.

Furthermore, we have | E_1 | = 4, | E_2 | = 8, ……., | E_i | = 2^{i+1}…….. and | E_n | = 2^{n+1} .

8.1 Sombor, Modified Sombor, Reduced Sombor, Reduced Modified Sombor indices, The first and second (a,b)-KA indices, The first and second Reduced (a,b)-KA indices, square reduced sum, product, Arithmetic, Geometric and Harmonic indices, First and Second F, Minus F, Square F indices of eccentricity based topological indices of Nanostar Dendrimer D(n).

In this section, we obtain exact values for Sombor, Modified Sombor, Reduced Sombor, Reduced Modified Sombor indices, The first and second (a,b)–KA indices, The first and second Reduced (a,b)–KA indices, Square reduced sum, Square reduced product, Square reduced Arithmetic, Square reduced Geometric and Square reduced Harmonic indices, First F, Second F, The Minus F, Square F indices of eccentricity based topological indices of Nanostar Dendrimer D(n).

(i) Sombor, Modified Sombor, Reduced Sombor and Reduced modified Sombor indices of G, as

$$SO(G) = \sum_{uv\in E(G)} \sqrt{e(u)^2 + e(v)^2}$$

$$m_{SO}(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{e(u)^2+e(v)^2}}$$

$$RSO(G) = \sum_{uv\in E(G)} \sqrt{(e(u) - 1)^{\,2} + (e(v) - 1)^2}$$

$$m_{RSO}(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{(e(u)-1)^{\,2}+(e(v)-1)^2}}$$

(ii) The First and Second (a, b) - KA indices and the First and Second Reduced (a, b) - KA indices of G are defined

$$KA^1_{a,b}(G) = \sum_{uv\in E(G)} [e(u)^a + e(v)^a]^b$$

$$KA^2_{a,b}(G) = \sum_{uv\in E(G)} [e(u)^a e(v)^a]^b$$

$$RKA^1_{a,b}(G) = \sum_{uv\in E(G)} [(e(u)-1)^a + (e(v)-1)^a]^b$$

$$RKA^2_{a,b}(G) = \sum_{uv\in E(G)} [(e(u)-1)^a (e(v)-1)^a]^b$$

(iii) Square Reduced sum, Square Reduced product, Square Reduced Arithmetic, Square Reduced Geometric and Square Reduced Harmonic indices of G, are defined

$$SRS(G) = \sum_{uv\in E(G)} [(e(u)-1)^2 + (e(v)-1)^2]$$

$$SRP(G) = \sum_{uv\in E(G)} [(e(u)-1)^2 ((e(v)-1)^2]$$

$$SRA(G) = \sum_{uv\in E(G)} [\frac{[(e(u)-1)^2 + (e(v)-1)^2]}{2}]$$

$$SRH(G) = \sum_{uv\in E(G)} [\frac{2}{[(e(u)-1)^2 + (e(v)-1)^2]}]$$

$$SRG(G) = \sum_{uv\in E(G)} \sqrt{[(e(u)-1)^2 \ (e(v)-1)^2]}$$

(iv) First F, Second F, Minus F, Square F indices of G, are defined

$$F_1(G) = \sum_{uv\in E(G)} [e(u)^2 + e(v)^2]$$

$$F_2(G) = \sum_{uv\in E(G)} [e(u)^2 \ e(v)^2]$$

$$MF(G) = \sum_{uv\in E(G)} |e(u)^2 - e(v)^2|$$

$$QF(G) = \sum_{uv\in E(G)} |e(u)^2 - e(v)^2|^2$$

Theorem 8.1.1: Let G be D(n). Then we have

(i) $SO(D(n)) = 4\sum_{i=1}^{n} 2^{i-1} \sqrt{(n+i-1)^2 + (n+i)^2}$

(ii) $m_{SO}(D(n)) = 4\sum_{i=1}^{n} 2^{i-1} \frac{1}{\sqrt{(n+i-1)^2+(n+i)^2}}$

(iii) $RSO(D(n)) = 4\sum_{i=1}^{n} 2^{i-1} \sqrt{(n+i-2)^2 + (n+i-1)^2}$

(iv) $m_{RSO}(D(n)) = 4\sum_{i=1}^{n} 2^{i-1} \frac{1}{\sqrt{(n+i-2)^2+(n+i-1)^2}}$

Proof: (i) SO(G) = $\sum_{uv\in E(\mathrm{G})} \sqrt{e(u)^2+e(v)^2}$

$$\mathrm{SO(D(n))} = (4\times 2^0)\sqrt{n^2+(n+1)^2} +$$

$$(4\times 2^1)\sqrt{(n+1)^2+(n+2)^2} +\ldots\ldots\ldots+$$

$$(4\times 2^{n-2})\sqrt{(2n-2)^2+(2n-1)^2} +$$

$$(4\times 2^{n-1})\sqrt{(2n-1)^2+(2n)^2}$$

$$\mathrm{SO(D(n))} = 4\sum_{i=1}^{n} 2^{i-1}\sqrt{(n+i-1)^2+(n+i)^2}$$

(ii) m_{SO} (G) = $\sum_{uv\in E(\mathrm{G})} \frac{1}{\sqrt{e(u)^2+e(v)^2}}$

$$m_{\mathrm{SO}}(\mathrm{D(n)}) = (4\times 2^0)\frac{1}{\sqrt{n^2+(n+1)^2}} + (4\times 2^1)\frac{1}{\sqrt{(n+1)^2+(n+2)^2}} +\ldots\ldots\ldots+$$

$$(4\times 2^{n-2})\frac{1}{\sqrt{(2n-2)^2+(2n-1)^2}} + (4\times 2^{n-1})\frac{1}{\sqrt{(2n-1)^2+(2n)^2}}$$

$$m_{\mathrm{SO}}(\mathrm{D(n)}) = 4\sum_{i=1}^{n} 2^{i-1}\frac{1}{\sqrt{(n+i-1)^2+(n+i)^2}}$$

(iii) RSO(G) = $\sum_{uv\in E(\mathrm{G})} \sqrt{(e(u)-1)^{\,2}+(e(v)-1)^2}$

$$\mathrm{RSO(D(n))} = (4\times 2^0)\sqrt{(n-1)^2+n^2} + (4\times 2^1)\sqrt{n^2+(n+1)^2} +\ldots\ldots\ldots+$$

$$(4\times 2^{n-2})\sqrt{(2n-3)^2+(2n-2)^2} +$$

$$(4\times 2^{n-1})\sqrt{(2n)^2+(2n-1)^2}$$

$$\mathrm{RSO(D(n))} = 4\sum_{i=1}^{n} 2^{i-1}\sqrt{(n+i-2)^2+(n+i-1)^2}$$

(iv) $m_{\mathrm{RSO}}(\mathrm{G}) = \sum_{uv\in E(\mathrm{G})} \frac{1}{\sqrt{(e(u)-1)^{\,2}+(e(v)-1)^2}}$

$$m_{\mathrm{RSO}}(\mathrm{D(n)}) = (4\times 2^0)\frac{1}{\sqrt{(n-1)^2+n^2}} + (4\times 2^1)\frac{1}{\sqrt{n^2+(n+1)^2}} +\ldots\ldots\ldots+$$

$$(4\times 2^{n-2})\frac{1}{\sqrt{(2n-3)^2+(2n-2)^2}} + (4\times 2^{n-1})\frac{1}{\sqrt{(2n)^2+(2n-1)^2}}$$

$$m_{\mathrm{RSO}}(\mathrm{D(n)}) = 4\sum_{i=1}^{n} 2^{i-1}\frac{1}{\sqrt{(n+i-2)^2+(n+i-1)^2}}$$

Theorem 8.1.2: Let G be D(n). Then we have

(i) $KA^1_{a,b}(D(n)) = 4\sum_{i=1}^{n} 2^{i-1} [(n+i-1)^a + (n+i)^a]^b$

(ii) $KA^2_{a,b}(D(n)) = 4\sum_{i=1}^{n} 2^{i-1} [(n+i-1)^a(n+i)^a]^b$

(iii) $RKA^1_{a,b}(D(n)) = \sum_{uv\in E(G)} [(e(u)-1)^a + (e(v)-1)^a]^b$

(iv) $RKA^2_{a,b}(D(n)) = \sum_{uv\in E(G)} [(e(u)-1)^a(e(v)-1)^a]^b$

Proof: (i) $KA^1_{a,b}(G) = \sum_{uv\in E(G)} [e(u)^a + e(v)^a]^b$

$KA^1_{a,b}(D(n)) = (4\times 2^0)[n^a + (n+1)^a]^b +$

$(4\times 2^1)[(n+1)^a + (n+2)^a]^b + \ldots\ldots\ldots +$

$(4\times 2^{n-2})[(2n-2)^a + (2n-1)^a]^b +$

$(4\times 2^{n-1})[(2n-1)^a + (2n)^a]^b$

$KA^1_{a,b}(D(n)) = 4\sum_{i=1}^{n} 2^{i-1} [(n+i-1)^a + (n+i)^a]^b$

(ii) $KA^2_{a,b}(G) = \sum_{uv\in E(G)} [e(u)^a e(v)^a]^b$

$KA^2_{a,b}(D(n)) = (4\times 2^0)[n^a(n+1)^a]^b +$

$(4\times 2^1)[(n+1)^a(n+2)^a]^b + \ldots\ldots\ldots +$

$(4\times 2^{n-2})[(2n-2)^a(2n-1)^a]^b +$

$(4\times 2^{n-1})[(2n-1)^a(2n)^a]^b$

$KA^2_{a,b}(D(n)) = 4\sum_{i=1}^{n} 2^{i-1} [(n+i-1)^a(n+i)^a]^b$

(iii) $RKA^1_{a,b}(G) = \sum_{uv\in E(G)} [(e(u)-1)^a + (e(v)-1)^a]^b$

$RKA^1_{a,b}(D(n)) = (4\times 2^0)[(n-1)^a + n^a]^b + (4\times 2^1)[n^a + (n+1)^a]^b + \ldots\ldots\ldots +$

$(4\times 2^{n-2})[(2n-3)^a + (2n-2)^a]^b +$

$(4\times 2^{n-1})[(2n)^a + (2n-1)^a]^b$

$RKA^1_{a,b}(D(n)) = 4\sum_{i=1}^{n} 2^{i-1} [(n+i-2)^a + (n+i-1)^a]^b$

(iv) $RKA^2_{a,b}(G) = \sum_{uv\in E(G)} [(e(u)-1)^a(e(v)-1)^a]^b$

$RKA^2_{a,b}(D(n)) = (4\times 2^0)[(n-1)^a n^a]^b + (4\times 2^1)[n^a(n+1)^a]^b + \ldots\ldots\ldots +$

$(4\times 2^{n-2})[(2n-3)^a(2n-2)^a]^b +$

$(4\times 2^{n-1})[(2n)^a(2n-1)^a]^b$

$RKA^2_{a,b}(D(n)) = 4\sum_{i=1}^{n} 2^{i-1}[(n+i-2)^a(n+i-1)^a]^b$

Theorem 8.1.3: Let G be D(n). Then we have

(i) $SRS(D(n)) = 4\sum_{i=1}^{n} 2^{i-1}[(n+i-2)^2+(n+i-1)^2]$

(ii) $SRP(D(n)) = 4\sum_{i=1}^{n} 2^{i-1}[(n+i-2)^2(n+i-1)^2]$

(iii) $SRA(D(n)) = 2\sum_{i=1}^{n} 2^{i-1}[(n+i-2)^2(n+i-1)^2]$

(iv) $SRH(D(n)) = 8\sum_{i=1}^{n} 2^{i-1}[\frac{1}{[(n+i-2)^2(n+i-1)^2}]$

(v) $SRG(D(n)) = 4\sum_{i=1}^{n} 2^{i-1}[(n+i-2)(n+i-1)]$

Proof: (i) $SRS(G) = \sum_{uv\in E(G)} [(e(u)-1)^2+(e(v)-1)^2]$

$SRS(D(n)) = (4\times 2^0)[(n-1)^2+n^2] + (4\times 2^1)[n^2+(n+1)^2] +$

$(4\times 2^2)[n^2+(n+1)^2] + \ldots\ldots\ldots +$

$(4\times 2^{n-1})[(2n-2)^2+(2n-1)^2]$

$SRS(D(n)) = 4\sum_{i=1}^{n} 2^{i-1}[(n+i-2)^2+(n+i-1)^2]$

(ii) $SRP(G) = \sum_{uv\in E(G)} [(e(u)-1)^2((e(v)-1)^2]$

$SRP(D(n)) = (4\times 2^0)[(n-1)^2n^2] + (4\times 2^1)[n^2(n+1)^2] +$

$(4\times 2^2)[n^2(n+1)^2] + \ldots\ldots\ldots +$

$(4\times 2^{n-1})[(2n-2)^2(2n-1)^2]$

$SRP(D(n)) = 4\sum_{i=1}^{n} 2^{i-1}[(n+i-2)^2(n+i-1)^2]$

(iii) $\quad SRA(G) = \sum_{uv\in E(G)} [\frac{[(e(u)-1)^2 + (e(v)-1)^2]}{2}]$

$$SRA(D(n)) = [\tfrac{1}{2}]\{(4\times 2^0)\,((n-1)^2 + n^2) + (4\times 2^1)\,(n^2 + (n+1)^2) +$$

$$(4\times 2^2)\,(n^2 + (n+1)^2) + \ldots\ldots\ldots\ldots +$$

$$(4\times 2^{n-1})\,((2n-2)^2 + (2n-1)^2]\}$$

$$SRA(D(n)) = 2\sum_{i=1}^{n} 2^{i-1}\,[(n+i-2)^2(n+i-1)^2]$$

(iv) $\quad SRH(G) = \sum_{uv\in E(G)} [\frac{2}{[(e(u)-1)^2 + (e(v)-1)^2]}]$

$$SRH(D(n)) = 2\{(4\times 2^0)\frac{1}{[(n-1)^2 + n^2]} + (4\times 2^1)\frac{1}{[n^2 + (n+1)^2]} +$$

$$(4\times 2^2)\frac{1}{[n^2 + (n+1)^2]} + \ldots\ldots\ldots\ldots +$$

$$(4\times 2^{n-1})\frac{1}{[(2n-2)^2 + (2n-1)^2]}\}$$

$$SRH(D(n)) = 8\sum_{i=1}^{n} 2^{i-1}[\frac{1}{[(n+i-2)^2(n+i-1)^2]}]$$

(v) $\quad SRG(G) = \sum_{uv\in E(G)} \sqrt{[(e(u)-1)^2\ (e(v)-1)^2]}$

$$SRG(D(n)) = (4\times 2^0)\sqrt{(n-1)^2\ n^2} + (4\times 2^1)\sqrt{n^2\ (n+1)^2} +$$

$$(4\times 2^2)\sqrt{n^2\ (n+1)^2} + \ldots\ldots\ldots\ldots +$$

$$(4\times 2^{n-1})\sqrt{(2n-2)^2\ (2n-1)^2}$$

$$SRG(D(n)) = 4\sum_{i=1}^{n} 2^{i-1}\,[(n+i-2)(n+i-1)]$$

Theorem 8.1.4: Let G be D(n). Then we have

(i) $\quad F_1(D(n)) = 4\sum_{i=1}^{n} 2^{i-1}\,[(n+i-1)^2 + (n+i)^2]$

(ii) $\quad F_2(D(n)) = 4\sum_{i=1}^{n} 2^{i-1}\,[(n+i-1)^2(n+i)^2]$

(iii) $\quad MF(D(n)) = 4\sum_{i=1}^{n} 2^{i-1}\,|(n+i-1)^2 - (n+i)^2|$

(iv) $\quad QF(D(n)) = 4\sum_{i=1}^{n} 2^{i-1}\,|(n+i-1)^2 - (n+i)^2|^2$

Proof: (i) $F_1(G) = \sum_{i=1}^{n}[e(u)^2 + e(v)^2]$

$F_1(D(n)) = (4\times 2^0)\,[n^2+(n+1)^2] + (4\times 2^1)\,[(n+1)^2+(n+2)^2] +$

$(4\times 2^2)\,[(n+2)^2+(n+3)^2] + \ldots\ldots\ldots. +$

$(4\times 2^{n-2})\,[(2n-2)^2 + (2n-1)^2 +$

$(4\times 2^{n-1})\,[(2n-1)^2 + (2n)^2]$

$F_1(D(n)) = 4\sum_{i=1}^{n} 2^{i-1}\,[(n+i-1)^2 + (n+i)^2]$

(ii) $F_2(G) = \sum_{i=1}^{n}[e(u)^2\, e(v)^2]$

$F_2(D(n)) = (4\times 2^0)\,[n^2(n+1)^2] + (4\times 2^1)\,[(n+1)^2(n+2)^2] +$

$(4\times 2^2)\,[(n+2)^2(n+3)^2] + \ldots\ldots\ldots. +$

$(4\times 2^{n-2})\,[(2n-2)^2(2n-1)^2] +$

$(4\times 2^{n-1})\,[(2n-1)^2(2n)^2]$

$F_2(D(n)) = 4\sum_{i=1}^{n} 2^{i-1}\,[(n+i-1)^2(n+i)^2]$

(iii) $MF(G) = \sum_{i=1}^{n} |e(u)^2 - e(v)^2|$

$MF(D(n)) = (4\times 2^0)\,|n^2 - (n+1)^2| + (4\times 2^1)\,|(n+1)^2 - (n+2)^2| +$

$(4\times 2^2)\,|(n+2)^2 - (n+3)^2| + \ldots\ldots\ldots. +$

$(4\times 2^{n-2})\,|(2n-2)^2 - (2n-1)^2| +$

$(4\times 2^{n-1})\,|(2n-1)^2 - (2n)^2|$

$MF(D(n)) = 4\sum_{i=1}^{n} 2^{i-1}\,|(n+i-1)^2 - (n+i)^2|$

(iv) $QF(G) = \sum_{i=1}^{n} |e(u)^2 - e(v)^2|^2$

$QF(D(n)) = (4\times 2^0)\,|n^2 - (n+1)^2|^2 + (4\times 2^1)\,|(n+1)^2 - (n+2)^2|^2$

$(4\times 2^2)\,|(n+2)^2 - (n+3)^2|^2 + \ldots\ldots\ldots. +$

$(4\times 2^{n-2})\,|(2n-2)^2 - (2n-1)^2|^2 +$

$(4\times 2^{n-1})|(2n-1)^2 - (2n)^2|^2$

$QF(D(n)) = 4\sum_{i=1}^{n} 2^{i-1}\,|(n+i-1)^2 - (n+i)^2|^2$

8.2 Sombor, Modified Sombor, Reduced Sombor, Reduced Modified Sombor indices, The first and second (a,b)–KA indices, The first and second Reduced (a,b)–KA indices, Square Reduced sum, product, Arithmetic, Geometric and Harmonic indices, First and Second F, Minus F, Square F indices of eccentricity based polynomial topological indices of Nanostar Dendrimer D[n].

(i) Sombor, Modified Sombar, Reduced Sombar, Reduced Modified Sombar polynomials indices of G, as

$$SO(G, x) = \sum_{uv\in E(G)} x^{\sqrt{e(u)^2+e(v)^2}}$$

$$m_{SO}(G, x) = \sum_{uv\in E(G)} x^{\frac{1}{\sqrt{e(u)^2+e(v)^2}}}$$

$$RSO(G, x) = \sum_{uv\in E(G)} x^{\sqrt{(e(u)-1)^2+(e(v)-1)^2}}$$

$$m_{RSO}(G, x) = \sum_{uv\in E(G)} x^{\frac{1}{\sqrt{(e(u)-1)^2+(e(v)-1)^2}}}$$

(ii) The first and second (a, b) - KA indices, The first and second Reduced (a, b) - KA polynomial indices of G, are defined

$$KA^1{}_{a,b}(G, x) = \sum_{uv\in E(G)} x^{[e(u)^a+e(v)^a]^b}$$

$$KA^2{}_{a,b}(G, x) = \sum_{uv\in E(G)} x^{[e(u)^a e(v)^a]^b}$$

$$RKA^1{}_{a,b}(G, x) = \sum_{uv\in E(G)} x^{[(e(u)-1)^a+(e(v)-1)^a]^b}$$

$$RKA^2{}_{a,b}(G, x) = \sum_{uv\in E(G)} x^{[(e(u)-1)^a(e(v)-1)^a]^b}$$

(iii) Square Reduced sum, Square Reduced product, Square Reduced Arithmetic, Square Reduced Geometric and Square Reduced Harmonic polynomial indices of G, are defined

$$SRS(G, x) = \sum_{uv\in E(G)} x^{[(e(u)-1)^2 + (e(v)-1)^2]}$$

$$SRP(G, x) = \sum_{uv\in E(G)} x^{[(e(u)-1)^2 ((e(v)-1)^2]}$$

$$SRA(G, x) = \sum_{uv\in E(G)} x^{[\frac{[(e(u)-1)^2 + (e(v)-1)^2]}{2}]}$$

$$SRH(G, x) = \sum_{uv\in E(G)} x^{[\frac{2}{[(e(u)-1)^2 + (e(v)-1)^2]}]}$$

$$SRG(G, x) = \sum_{uv\in E(G)} x^{\sqrt{[(e(u)-1)^2\ (e(v)-1)^2]}}$$

(iv) First F, Second F, Minus F, Square F polynomial indices of G, are defined

$F_1(G, x) = \sum_{uv\in E(G)} x^{[e(u)^2 + e(v)^2]}$

$F_2(G, x) = \sum_{uv\in E(G)} x^{[e(u)^2\, e(v)^2]}$

$MF(G, x) = \sum_{uv\in E(G)} x^{|e(u)^2 - e(v)^2|}$

$QF(G, x) = \sum_{uv\in E(G)} x^{|e(u)^2 - e(v)^2|^2}$

Theorem 8.2.1: Let G be D(n) Nanostar Dendrimer. Then we have

(i) $SO(D(n), x) = 4\sum_{i=1}^{n} 2^{i-1}\, x^{\sqrt{(n+i-1)^2+(n+i)^2}}$

(ii) $m_{SO}(D(n), x) = 4\sum_{i=1}^{n} 2^{i-1}\, x^{\frac{1}{\sqrt{(n+i-1)^2+(n+i)^2}}}$

(iii) $RSO(D(n), x) = 4\sum_{i=1}^{n} 2^{i-1}\, x^{\sqrt{(n+i-2)^2+(n+i-1)^2}}$

(iv) $m_{RSO}(D(n), x) = 4\sum_{i=1}^{n} 2^{i-1}\, x^{\frac{1}{\sqrt{(n+i-2)^2+(n+i-1)^2}}}$

Proof: (i) $SO(G, x) = \sum_{uv\in E(G)} x^{\sqrt{e(u)^2+e(v)^2}}$

$$SO(D(n), x) = (4\times 2^0)\, x^{\sqrt{n^2+(n+1)^2}} + (4\times 2^1)\, x^{\sqrt{(n+1)^2+(n+2)^2}} + \ldots\ldots\ldots + (4\times 2^{n-2})\, x^{\sqrt{(2n-2)^2+(2n-1)^2}} + (4\times 2^{n-1})\, x^{\sqrt{(2n-1)^2+(2n)^2}}$$

$$SO(D(n), x) = 4\sum_{i=1}^{n} 2^{i-1}\, x^{\sqrt{(n+i-1)^2+(n+i)^2}}$$

(ii) $m_{SO}(G, x) = \sum_{uv\in E(G)} x^{\frac{1}{\sqrt{e(u)^2+e(v)^2}}}$

$$m_{SO}(D(n), x) = (4\times 2^0)\, x^{\frac{1}{\sqrt{n^2+(n+1)^2}}} + (4\times 2^1)\, x^{\frac{1}{\sqrt{(n+1)^2+(n+2)^2}}} + \ldots\ldots\ldots + (4\times 2^{n-2})\, x^{\frac{1}{\sqrt{(2n-2)^2+(2n-1)^2}}} + (4\times 2^{n-1})\, x^{\frac{1}{\sqrt{(2n-1)^2+(2n)^2}}}$$

$$m_{SO}(D(n), x) = 4\sum_{i=1}^{n} 2^{i-1}\, x^{\frac{1}{\sqrt{(n+i-1)^2+(n+i)^2}}}$$

(iii) $\mathrm{RSO}(G, x) = \sum_{uv\in E(G)} x^{\sqrt{(e(u)-1)^2+(e(v)-1)^2}}$

$$\mathrm{RSO}(D(n), x) = (4\times 2^0)\, x^{\sqrt{(n-1)^2+n^2}} + (4\times 2^1) x^{\sqrt{n^2+(n+1)^2}} + \ldots\ldots\ldots +$$

$$(4\times 2^{n-2})\, x^{\sqrt{(2n-3)^2+(2n-2)^2}} + (4\times 2^{n-1}) x^{\sqrt{(2n)^2+(2n-1)^2}}$$

$$\mathrm{RSO}(D(n), x) = 4\sum_{i=1}^{n} 2^{i-1}\, x^{\sqrt{(n+i-2)^2+(n+i-1)^2}}$$

(iv) $m_{\mathrm{RSO}}(G, x) = \sum_{uv\in E(G)} x^{\frac{1}{\sqrt{(e(u)-1)^2+(e(v)-1)^2}}}$

$$m_{\mathrm{RSO}}(D(n), x) = (4\times 2^0)\, x^{\frac{1}{\sqrt{(n-1)^2+n^2}}} + (4\times 2^1)\, x^{\frac{1}{\sqrt{n^2+(n+1)^2}}} + \ldots\ldots\ldots +$$

$$(4\times 2^{n-2})\, x^{\frac{1}{\sqrt{(2n-3)^2+(2n-2)^2}}} + (4\times 2^{n-1})\, x^{\frac{1}{\sqrt{(2n)^2+(2n-1)^2}}}$$

$$m_{\mathrm{RSO}}(D(n), x) = 4\sum_{i=1}^{n} 2^{i-1}\, x^{\frac{1}{\sqrt{(n+i-2)^2+(n+i-1)^2}}}$$

Theorem 8.2.2: Let G be D(n) Nanostar Dendrimer. Then we have

(i) $KA^1_{a,b}(G, x) = \sum_{uv\in E(G)} x^{[e(u)^a+e(v)^a]^b}$

(ii) $KA^2_{a,b}(G, x) = \sum_{uv\in E(G)} x^{[e(u)^a e(v)^a]^b}$

(iii) $RKA^1_{a,b}(G, x) = \sum_{uv\in E(G)} x^{[(e(u)-1)^a+(e(v)-1)^a]^b}$

(iv) $RKA^2_{a,b}(G, x) = \sum_{uv\in E(G)} x^{[(e(u)-1)^a(e(v)-1)^a]^b}$

Proof: (i) $KA^1_{a,b}(G, x) = \sum_{uv\in E(G)} x^{[e(u)^a+e(v)^a]^b}$

$$KA^1_{a,b}(D(n), x) = (4\times 2^0)\, x^{[n^a+(n+1)^a]^b} + (4\times 2^1)\, x^{[(n+1)^a+(n+2)^a]^b} + \ldots\ldots\ldots +$$

$$(4\times 2^{n-2})\, x^{[(2n-2)^a+(2n-1)^a]^b} + (4\times 2^{n-1})\, x^{[(2n-1)^a+(2n)^a]^b}$$

$$KA^1_{a,b}(D(n), x) = 4\sum_{i=1}^{n} 2^{i-1}\, x^{[(n+i-1)^a+(n+i)^a]^b}$$

(ii) $KA^2_{a,b}(G, x) = \sum_{uv\in E(G)} x^{[e(u)^a e(v)^a]^b}$

$KA^2_{a,b}(D(n), x) = (4\times 2^0)\, x^{[n^a(n+1)^a]^b} + (4\times 2^1)\, x^{[(n+1)^a(n+2)^a]^b} + \ldots\ldots\ldots +$

$(4\times 2^{n-2})\, x^{[(2n-2)^a(2n-1)^a]^b} + (4\times 2^{n-1}) x^{[(2n-1)^a(2n)^a]^b}$

$KA^2_{a,b}(D(n), x) = 4\sum_{i=1}^{n} 2^{i-1}\, x^{[(n+i-1)^a(n+i)^a]^b}$

(iii) $RKA^1_{a,b}(G, x) = \sum_{uv\in E(G)} x^{[(e(u)-1)^a+(e(v)-1)^a]^b}$

$RKA^1_{a,b}(D(n), x) = (4\times 2^0)\, x^{[(n-1)^a+n^a]^b} + (4\times 2^1)\, x^{[n^a+(n+1)^a]^b} + \ldots\ldots\ldots +$

$(4\times 2^{n-2})\, x^{[(2n-3)^a+(2n-2)^a]^b} + (4\times 2^{n-1})\, x^{[(2n)^a+(2n-1)^a]^b}$

$RKA^1_{a,b}(D(n), x) = 4\sum_{i=1}^{n} 2^{i-1}\, x^{[(n+i-2)^a+(n+i-1)^a]^b}$

(iv) $RKA^2_{a,b}(G, x) = \sum_{uv\in E(G)} x^{[(e(u)-1)^a(e(v)-1)^a]^b}$

$RKA^2_{a,b}(D(n), x) = (4\times 2^0)\, x^{[(n-1)^a n^a]^b} + (4\times 2^1)\, x^{[n^a(n+1)^a]^b} + \ldots\ldots\ldots +$

$(4\times 2^{n-2})\, x^{[(2n-3)^a(2n-2)^a]^b} + (4\times 2^{n-1})\, x^{[(2n)^a(2n-1)^a]^b}$

$RKA^2_{a,b}(D(n), x) = 4\sum_{i=1}^{n} 2^{i-1}\, x^{[(n+i-2)^a(n+i-1)^a]^b}$

Theorem 8.2.3: Let G be D(n) Nanostar Dendrimer. Then we have

(i) $SRS(D(n), x) = 4\sum_{i=1}^{n} 2^{i-1}\, x^{[(n+i-2)^2+(n+i-1)^2]}$

(ii) $SRP(D(n), x) = 4\sum_{i=1}^{n} 2^{i-1}\, x^{[(n+i-2)^2(n+i-1)^2]}$

(iii) $SRA(D(n), x) = 2\sum_{i=1}^{n} 2^{i-1}\, x^{[(n+i-2)^2+(n+i-1)^2]}$

(iv) $SRH(D(n), x) = 8\sum_{i=1}^{n} 2^{i-1}\{\frac{1}{x^{[(n+i-2)^2+(n+i-1)^2]}}\}$

(v) $SRG(D(n), x) = 4\sum_{i=1}^{n} 2^{i-1}\, x^{[(n+i-2)(n+i-1)]}$

Proof:

(i) $SRS(G, x) = \sum_{uv\in E(G)} x^{[(e(u)-1)^2 + (e(v)-1)^2]}$

$SRS(D(n), x) = (4\times 2^0)\, x^{[(n-1)^2 + n^2]} + (4\times 2^1)\, x^{[n^2 + (n+1)^2]} +$

$$(4 \times 2^2)\, x^{[n^2 + (n+1)^2]} + \ldots\ldots\ldots + (4 \times 2^{n-1}) x^{[(2n-2)^2 + (2n-1)^2]}$$

$$SRS(D(n), x) = 4\sum_{i=1}^{n} 2^{i-1}\, x^{[(n+i-2)^2 + (n+i-1)^2]}$$

(ii) $SRP(G, x) = \sum_{uv \in E(G)} x^{[(e(u)-1)^2\ ((e(v)-1)^2]}$

$$SRP(D(n), x) = (4 \times 2^0)\, x^{[(n-1)^2\ n^2]} + (4 \times 2^1)\, x^{[n^2\ (n+1)^2]} +$$

$$(4 \times 2^2)\, x^{[n^2\ (n+1)^2]} + \ldots\ldots\ldots +$$

$$(4 \times 2^{n-1}) x^{[(2n-2)^2 (2n-1)^2]}$$

$$SRP(D(n), x) = 4\sum_{i=1}^{n} 2^{i-1}\, x^{[(n+i-2)^2 (n+i-1)^2]}$$

(iii) $SRA(G, x) = \sum_{uv \in E(G)} x^{[\frac{[(e(u)-1)^2 + (e(v)-1)^2]}{2}]}$

$$SRA(D(n), x) = [\tfrac{1}{2}]\ \{(4 \times 2^0)\, x^{[(n-1)^2 + n^2]} + (4 \times 2^1)\, x^{[n^2 + (n+1)^2]} +$$

$$(4 \times 2^2)\, x^{[n^2 + (n+1)^2]} + \ldots\ldots\ldots +$$

$$(4 \times 2^{n-1}) x^{[(2n-2)^2 + (2n-1)^2]}\ \}$$

$$SRA(D(n), x) = 2\sum_{i=1}^{n} 2^{i-1}\, x^{[(n+i-2)^2 + (n+i-1)^2]}$$

(iv) $SRH(G, x) = \sum_{uv \in E(G)} x^{[\frac{2}{[(e(u)-1)^2 + (e(v)-1)^2]}]}$

$$SRH(D(n), x) = 2\{(4 \times 2^0)\, x^{[\frac{1}{[(n-1)^2 + n^2]}]} + (4 \times 2^1)\, x^{[\frac{1}{[n^2 + (n+1)^2]}]} +$$

$$(4 \times 2^2)\, x^{\left[\frac{1}{[n^2 + (n+1)^2]}\right]} + \cdots \ldots \ldots .. +$$

$$(4 \times 2^{n-1})\, x^{[\frac{1}{[(2n-2)^2 + (2n-1)^2]}]}\ \}$$

$$SRH(D(n), x) = 8\sum_{i=1}^{n} 2^{i-1}\, x^{[\frac{1}{[(n+i-2)^2 + (n+i-1)^2]}]}$$

(v) $SRG(G, x) = \sum_{uv \in E(G)} x^{\sqrt{[(e(u)-1)^2\ (e(v)-1)^2]}}$

$$SRG(D(n), x) = (4 \times 2^0)\, x^{\sqrt{(n-1)^2\ n^2}} + (4 \times 2^1) x^{\sqrt{n^2\ (n+1)^2}} +$$

$$(4 \times 2^2)\, x^{\sqrt{n^2\ (n+1)^2}} + \ldots\ldots\ldots + (4 \times 2^{n-1})\, x^{\sqrt{(2n-2)^2\ (2n-1)^2}}$$

$$SRG(D(n), x) = 4\sum_{i=1}^{n} 2^{i-1}\, x^{[(n+i-2)(n+i-1)]}$$

Theorem 8.2.4: Let G be D(n) Nanostar Dendrimer. Then we have

(i) $F_1(D(n), x) = 4\sum_{i=1}^{n} 2^{i-1}\ x^{[(n+i-1)^2+(n+i)^2]}$

(ii) $F_2(D(n), x) = 4\sum_{i=1}^{n} 2^{i-1}\ x^{[(n+i-1)^2(n+i)^2]}$

(iii) $MF(D(n), x) = 4\sum_{i=1}^{n} 2^{i-1}\ x^{|(n+i-1)^2-(n+i)^2|}$

(iv) $QF(D(n), x) = 4\sum_{i=1}^{n} 2^{i-1} x^{|(n+i-1)^2-(n+i)^2|^2}$

Proof: (i) $F_1(G, x) = \sum_{i=1}^{n} x^{[e(u)^2 + e(v)^2]}$

$$F_1(D(n), x) = (4 \times 2^0)\, x^{[n^2+(n+1)^2]} + (4 \times 2^1)\, x^{[(n+1)^2+(n+2)^2]} + (4 \times 2^2)\, x^{[(n+2)^2+(n+3)^2]} + \ldots\ldots\ldots.. + (4 \times 2^{n-2})\, x^{[(2n-2)^2+(2n-1)^2]} + (4 \times 2^{n-1})\, x^{[(2n-1)^2+(2n)^2]}$$

$$F_1(D(n), x) = 4\sum_{i=1}^{n} 2^{i-1}\ x^{[(n+i-1)^2+(n+i)^2]}$$

(ii) $F_2(G, x) = \sum_{i=1}^{n} x^{[e(u)^2\ e(v)^2]}$

$$F_2(D(n), x) = (4 \times 2^0)\, x^{[n^2(n+1)^2]} + (4 \times 2^1)\, x^{[(n+1)^2(n+2)^2]} + (4 \times 2^2)\, x^{[(n+2)^2(n+3)^2]} + \ldots\ldots\ldots.. + (4 \times 2^{n-2})\, x^{[(2n-2)^2(2n-1)^2]} + (4 \times 2^{n-1})\, x^{[(2n-1)^2(2n)^2]}$$

$$F_2(D(n), x) = 4\sum_{i=1}^{n} 2^{i-1}\ x^{[(n+i-1)^2(n+i)^2]}$$

(iii) $MF(G, x) = \sum_{i=1}^{n} x^{|e(u)^2 - e(v)^2|}$

$$MF(D(n), x) = (4 \times 2^0)\, x^{|n^2-(n+1)^2|} + (4 \times 2^1)\, x^{|(n+1)^2-(n+2)^2|} + (4 \times 2^2)\, x^{|(n+2)^2-(n+3)^2|} + \ldots\ldots\ldots.. + (4 \times 2^{n-2})\, x^{|(2n-2)^2-(2n-1)^2|} + (4 \times 2^{n-1})\, x^{|(2n-1)^2-(2n)^2|}$$

$$MF(D(n), x) = 4\sum_{i=1}^{n} 2^{i-1}\ x^{|(n+i-1)^2-(n+i)^2|}$$

(iv) $QF(G, x) = \sum_{i=1}^{n} x^{|e(u)^2 - e(v)^2|^2}$

$$QF(D(n), x) = (4 \times 2^0)\, x^{|n^2-(n+1)^2|^2} + (4 \times 2^1)\, x^{|(n+1)^2-(n+2)^2|^2}$$

$(4 \times 2^2)\, x^{|(n+2)^2-(n+3)^2|^2} + \ldots\ldots\ldots\ldots +$

$(4 \times 2^{n-2})\, x^{|(2n-2)^2-(2n-1)^2|^2} + (4 \times 2^{n-1})\, x^{|(2n-1)^2-(2n)^2|^2}$

$$QF(D(n), x) = 4\sum_{i=1}^{n} 2^{i-1}\, x^{|(n+i-1)^2-(n+i)^2|^2}$$

8.3 Sombor, Modified Sombor, Reduced Sombor, Reduced Modified Sombor indices, The first and second (a,b)–KA indices, The first and second Reduced (a,b)–KA indices, square Reduced sum, product, Arithmetic,Geometric and Harmonic indices, First and Second F, Minus F, square F indices of eccentricity based Multiplicative topological indices of Nanostar Dendrimer D[n].

(i) Sombor, Modified Sombar, Reduced Sombar, Reduced Modified Sombar Multiplicative indices of G, as

$$SO\Pi(G) = \prod_{uv\in E(G)} \sqrt{e(u)^2 + e(v)^2}$$

$$m_{SO}\Pi(G) = \prod_{uv\in E(G)} \frac{1}{\sqrt{e(u)^2+e(v)^2}}$$

$$RSO\Pi(G) = \prod_{uv\in E(G)} \sqrt{(e(u)-1)^2 + (e(v)-1)^2}$$

$$m_{RSO}\Pi(G) = \prod_{uv\in E(G)} \frac{1}{\sqrt{(e(u)-1)^2+(e(v)-1)^2}}$$

(ii) The first and second (a, b) - KA indices, The first and second Reduced (a, b) – KA Multiplicative indices of G, are defined

$$KA^1{}_{a,b}\Pi(G) = \prod_{uv\in E(G)} [e(u)^a + e(v)^a]^b$$

$$KA^2{}_{a,b}\Pi(G) = \prod_{uv\in E(G)} [e(u)^a e(v)^a]^b$$

$$RKA^1{}_{a,b}\Pi(G) = \prod_{uv\in E(G)} [(e(u)-1)^a + (e(v)-1)^a]^b$$

$$RKA^2{}_{a,b}\Pi(G) = \prod_{uv\in E(G)} [(e(u)-1)^a (e(v)-1)^a]^b$$

(iii) square Reduced sum, square Reduced product, square Reduced Arithmetic, square Reduced Geometric and square Reduced Harmonic Multiplicative indices of G, are defined

$$SRS\Pi(G) = \prod_{uv\in E(G)} [(e(u)-1)^2 + (e(v)-1)^2]$$

$$SRP\Pi(G) = \prod_{uv\in E(G)} [(e(u)-1)^2\, ((e(v)-1)^2]$$

$$SRA\Pi(G) = \prod_{uv\in E(G)}\left[\frac{[(e(u)-1)^2+(e(v)-1)^2]}{2}\right]$$

$$SRH\Pi(G) = \prod_{uv\in E(G)}\left[\frac{2}{[(e(u)-1)^2+(e(v)-1)^2]}\right]$$

$$SRG\Pi(G) = \prod_{uv\in E(G)}\sqrt{[(e(u)-1)^2\ (e(v)-1)^2]}$$

(iv) First F, Second F, Minus F, Square F polynomial indices of G, are defined

$$F_1\Pi(G) = \prod_{uv\in E(G)}[e(u)^2 + e(v)^2]$$

$$F_2\Pi(G) = \prod_{uv\in E(G)}[e(u)^2\, e(v)^2]$$

$$MF\Pi(G) = \prod_{uv\in E(G)}|e(u)^2 - e(v)^2|$$

$$QF\Pi(G) = \prod_{uv\in E(G)}|e(u)^2 - e(v)^2|^2$$

Theorem 8.3.1: Let D(n) be the Nanostar Dendrimer. Then

(i) $SO\Pi(D(n)) = 4\prod_{i=1}^{n} 2^{i-1}\sqrt{(n+i-1)^2+(n+i)^2}$

(ii) $m_{SO}\Pi(D(n)) = 4\prod_{i=1}^{n} 2^{i-1}\frac{1}{\sqrt{(n+i-1)^2+(n+i)^2}}$

(iii) $RSO\Pi(D(n)) = 4\prod_{i=1}^{n} 2^{i-1}\sqrt{(n+i-2)^2+(n+i-1)^2}$

(iv) $m_{RSO}\Pi(D(n)) = 4\prod_{i=1}^{n} 2^{i-1}\frac{1}{\sqrt{(n+i-2)^2+(n+i-1)^2}}$

Proof: (i) $SO\Pi(G) = \prod_{uv\in E(G)}\sqrt{e(u)^2+e(v)^2}$

$$\begin{aligned} SO\Pi(D(n)) = {} & (4\times 2^0)\sqrt{n^2+(n+1)^2}\times \\ & (4\times 2^1)\sqrt{(n+1)^2+(n+2)^2}\times\ldots\ldots\ldots\times \\ & (4\times 2^{n-2})\sqrt{(2n-2)^2+(2n-1)^2}\times \\ & (4\times 2^{n-1})\sqrt{(2n-1)^2+(2n)^2} \end{aligned}$$

$$SO\Pi(D(n)) = 4\prod_{i=1}^{n} 2^{i-1}\sqrt{(n+i-1)^2+(n+i)^2}$$

(ii) $m_{\mathrm{SO}}\Pi(\mathrm{G}) = \prod_{uv\in E(\mathrm{G})} \frac{1}{\sqrt{e(u)^2+e(v)^2}}$

$$m_{\mathrm{SO}}\Pi(\mathrm{D(n)}) = (4\times 2^0)\frac{1}{\sqrt{n^2+(n+1)^2}} \times (4\times 2^1)\frac{1}{\sqrt{(n+1)^2+(n+2)^2}} \times \ldots\ldots\ldots\times$$

$$(4\times 2^{n-2})\frac{1}{\sqrt{(2n-2)^2+(2n-1)^2}} \times (4\times 2^{n-1})\frac{1}{\sqrt{(2n-1)^2+(2n)^2}}$$

$$m_{\mathrm{SO}}\Pi(\mathrm{D(n)}) = 4\prod_{i=1}^{n} 2^{i-1}\frac{1}{\sqrt{(n+i-1)^2+(n+i)^2}}$$

(iii) $\mathrm{RSO}\Pi(\mathrm{G}) = \prod_{uv\in E(\mathrm{G})} \sqrt{(e(u)-1)^{\,2}+(e(v)-1)^2}$

$$\mathrm{RSO}\Pi(\mathrm{D(n)}) = (4\times 2^0)\sqrt{(n-1)^2+n^2} \times (4\times 2^1)\sqrt{n^2+(n+1)^2} \times \ldots\ldots\ldots\times$$

$$(4\times 2^{n-2})\sqrt{(2n-3)^2+(2n-2)^2}\times$$

$$(4\times 2^{n-1})\sqrt{(2n)^2+(2n-1)^2}$$

$$\mathrm{RSO}\Pi(\mathrm{D(n)}) = 4\prod_{i=1}^{n} 2^{i-1}\sqrt{(n+i-2)^2+(n+i-1)^2}$$

(iv) $m_{\mathrm{RSO}}\Pi(\mathrm{G}) = \prod_{uv\in E(\mathrm{G})} \frac{1}{\sqrt{(e(u)-1)^{\,2}+(e(v)-1)^2}}$

$$m_{\mathrm{RSO}}\Pi(\mathrm{D(n)}) = (4\times 2^0)\frac{1}{\sqrt{(n-1)^2+n^2}} \times (4\times 2^1)\frac{1}{\sqrt{n^2+(n+1)^2}} \times \ldots\ldots\ldots\times$$

$$(4\times 2^{n-2})\frac{1}{\sqrt{(2n-3)^2+(2n-2)^2}} \times (4\times 2^{n-1})\frac{1}{\sqrt{(2n)^2+(2n-1)^2}}$$

$$m_{\mathrm{RSO}}\Pi(\mathrm{D(n)}) = 4\prod_{i=1}^{n} 2^{i-1}\frac{1}{\sqrt{(n+i-2)^2+(n+i-1)^2}}$$

Theorem 8.3.2: Let D(n) be the Nanostar Dendrimer. Then

(i) $KA^1_{a,b}\,\Pi(\mathrm{D(n)}) = 4\prod_{i=1}^{n} 2^{i-1}\,[(n+i-1)^a+(n+i)^a]^b$

(ii) $KA^2_{a,b}\,\Pi(\mathrm{D(n)}) = 4\prod_{i=1}^{n} 2^{i-1}\,[(n+i-1)^a(n+i)^a]^b$

(iii) $RKA^1_{a,b}\Pi(\mathrm{D(n)}) = 4\prod_{i=1}^{n} 2^{i-1}\,[(n+i-2)^a+(n+i-1)^a]^b$

(iv) $RKA^2_{a,b}\,\Pi(\mathrm{D(n)}) = 4\prod_{i=1}^{n} 2^{i-1}\,[(n+i-2)^a(n+i-1)^a]^b$

Proof: (i) $KA^1_{a,b}\Pi(G) = \prod_{uv\in E(G)}[e(u)^a + e(v)^a]^b$

$KA^1_{a,b}\Pi(D(n)) = (4\times 2^0)\ [n^a + (n+1)^a]^b \times$

$(4\times 2^1)[(n+1)^a + (n+2)^a]^b\ \times\ldots\ldots\ldots\times$

$(4\times 2^{n-2})[(2n-2)^a + (2n-1)^a]^b \times$

$(4\times 2^{n-1})[(2n-1)^a + (2n)^a]^b$

$KA^1_{a,b}\Pi(D(n)) = 4\prod_{i=1}^{n} 2^{i-1}\ [(n+i-1)^a + (n+i)^a]^b$

(ii) $KA^2_{a,b}\Pi(D(n)) = \prod_{uv\in E(G)}[e(u)^a e(v)^a]^b$

$KA^2_{a,b}\Pi(D(n)) = (4\times 2^0)\ [n^a(n+1)^a]^b \times$

$(4\times 2^1)[(n+1)^a(n+2)^a]^b\ \times\ldots\ldots\ldots\times$

$(4\times 2^{n-2})[(2n-2)^a(2n-1)^a]^b \times$

$(4\times 2^{n-1})[(2n-1)^a(2n)^a]^b$

$KA^2_{a,b}\Pi(D(n)) = 4\prod_{i=1}^{n} 2^{i-1}\ [(n+i-1)^a(n+i)^a]^b$

(iii) $RKA^1_{a,b}\Pi(D(n)) = \prod_{uv\in E(G)}[(e(u)-1)^a + (e(v)-1)^a]^b$

$RKA^1_{a,b}\Pi(D(n)) = (4\times 2^0)[(n-1)^a + n^a]^b \times$

$(4\times 2^1)[n^a + (n+1)^a]^b\ \times\ldots\ldots\ldots\times$

$(4\times 2^{n-2})\ [(2n-3)^a + (2n-2)^a]^b \times$

$(4\times 2^{n-1})[(2n)^a + (2n-1)^a]^b$

$RKA^1_{a,b}\Pi(D(n)) = 4\prod_{i=1}^{n} 2^{i-1}\ [(n+i-2)^a + (n+i-1)^a]^b$

(iv) $RKA^2_{a,b}\Pi(G) = \prod_{uv\in E(G)}[(e(u)-1)^a(e(v)-1)^a]^b$

$RKA^2_{a,b}\Pi(D(n)) = (4\times 2^0)[(n-1)^a n^a]^b \times$

$(4\times 2^1)[n^a(n+1)^a]^b\ \times\ldots\ldots\ldots\times$

$(4\times 2^{n-2})\ [(2n-3)^a(2n-2)^a]^b \times$

$(4\times 2^{n-1})[(2n)^a(2n-1)^a]^b$

$RKA^2_{a,b}\Pi(D(n)) = 4\prod_{i=1}^{n} 2^{i-1}\ [(n+i-2)^a(n+i-1)^a]^b$

Theorem 8.3.3: Let D(n) be the Nanostar Dendrimer. Then

(i) $SRS\Pi(D(n)) = 4\prod_{i=1}^{n} 2^{i-1}\,[(n+i-2)^2 + (n+i-1)^2]$

(ii) $SRP\Pi(D(n)) = 4\prod_{i=1}^{n} 2^{i-1}\,[(n+i-2)^2(n+i-1)^2]$

(iii) $SRA\Pi(D(n)) = 2\prod_{i=1}^{n} 2^{i-1}\,[(n+i-2)^2(n+i-1)^2]$

(iv) $SRH\Pi(D(n)) = 8\prod_{i=1}^{n} 2^{i-1}\{\frac{1}{[(n+i-2)^2+(n+i-1)^2}\}$

(v) $SRG\Pi(D(n)) = 4\prod_{i=1}^{n} 2^{i-1}\,[(n+i-2)(n+i-1)]$

Proof: (i) $SRS\Pi(G) = \prod_{uv\in E(G)}[(e(u)-1)^2 + (e(v)-1)^2]$

$SRS\Pi(D(n)) = (4\times 2^0)\,[(n-1)^2 + n^2] \times (4\times 2^1)\,[n^2 + (n+1)^2] \times$

$(4\times 2^2)\,[n^2 + (n+1)^2] \times \ldots\ldots\ldots\ldots \times$

$(4\times 2^{n-1})\,[(2n-2)^2 + (2n-1)^2]$

$SRS\Pi(D(n)) = 4\prod_{i=1}^{n} 2^{i-1}\,[(n+i-2)^2 + (n+i-1)^2]$

(ii) $SRP\Pi(G) = \prod_{uv\in E(G)}[(e(u)-1)^2\,((e(v)-1)^2]$

$SRP\Pi(D(n)) = (4\times 2^0)\,[(n-1)^2\ n^2] \times (4\times 2^1)\,[n^2\ (n+1)^2] \times$

$(4\times 2^2)\,[n^2\ (n+1)^2] \times \ldots\ldots\ldots\ldots \times$

$(4\times 2^{n-1})\,[(2n-2)^2\ (2n-1)^2]$

$SRP\Pi(D(n)) = 4\prod_{i=1}^{n} 2^{i-1}\,[(n+i-2)^2(n+i-1)^2]$

(iii) $SRA\Pi(G) = \prod_{uv\in E(G)}[\frac{[(e(u)-1)^2 + (e(v)-1)^2]}{2}]$

$SRA\Pi(D(n)) = [\frac{1}{2}]\ \{(4\times 2^0)\,[(n-1)^2 + n^2] \times (4\times 2^1)\,[n^2 + (n+1)^2] \times$

$(4\times 2^2)\,[n^2 + (n+1)^2] \times \ldots\ldots\ldots\ldots \times$

$(4\times 2^{n-1})\,[(2n-2)^2 + (2n-1)^2]\}$

$SRA\Pi(D(n)) = 2\prod_{i=1}^{n} 2^{i-1}\,[(n+i-2)^2 + (n+i-1)^2]$

(iv) SRH∏(G) $= \prod_{uv\in E(G)}[\frac{2}{[(e(u)-1)^2 + (e(v)-1)^2]}]$

$$SRH\prod(D(n)) = (4\times 2^0)\frac{1}{[(n-1)^2+n^2]}\times(4\times 2^1)\frac{1}{[n^2+(n+1)^2]}\times$$

$$(4\times 2^2)\frac{1}{[n^2+(n+1)^2]}\times \;\ldots\ldots\ldots..\times(4\times 2^{n-1})\frac{1}{[(2n-2)^2+(2n-1)^2]}\}$$

$$SRH\prod(D(n)) = 8\prod_{i=1}^{n}2^{i-1}\{\frac{1}{[(n+i-2)^2+(n+i-1)^2}\}$$

(v) SRG∏(G) $= \prod_{uv\in E(G)}\sqrt{[(e(u)\;-1)^2\;\;(e(v)\;-1)^2]}$

$$= (4\times 2^0)\sqrt{(n-1)^2\;\;n^2}\times(4\times 2^1)\sqrt{n^2\;(n+1)^2}\;\times$$

$$(4\times 2^2)\sqrt{n^2\;(n+1)^2}\;\times\ldots\ldots\ldots..\times$$

$$(4\times 2^{n-1})\sqrt{(2n-2)^2\;\;(2n-1)^2}$$

$$SRG\prod(D(n)) = 4\prod_{i=1}^{n}2^{i-1}\;[(n+i-2)(n+i-1)$$

Theorem 8.3.4: Let D(n) be the Nanostar Dendrimer. Then

(i) $F_1\prod[D(n)] = 4\prod_{i=1}^{n}2^{i-1}\;[(n+i-1)^2+(n+i)^2]$

(ii) $F_2\prod[D(n)] = 4\prod_{i=1}^{n}2^{i-1}\;[(n+i-1)^2(n+i)^2]$

(iii) $MF\prod[D(n)] = 4\prod_{i=1}^{n}2^{i-1}\;|(n+i-1)^2-(n+i)^2|$

(iv) $QF\prod[D(n)] = 4\prod_{i=1}^{n}2^{i-1}\;|(n+i-1)^2-(n+i)^2|^2$

Proof: (i) $F_1\prod(G) = \prod_{i=1}^{n}[e(u)^2\;+\;e(v)^2]$

$$F_1\prod[D(n)] = (4\times 2^0)\;[n^2+(n+1)^2]\times(4\times 2^1)\;[(n+1)^2+(n+2)^2]\times$$

$$(4\times 2^2)\;[(n+2)^2+(n+3)^2]\;\times\ldots\ldots\ldots..\times$$

$$(4\times 2^{n-2})\;[(2n-2)^2+(2n-1)^2]\;\times$$

$$(4\times 2^{n-1})\;[(2n-1)^2+(2n)^2]$$

$$F_1\prod[D(n)] = 4\prod_{i=1}^{n}2^{i-1}\;\;[(n+i-1)^2+(n+i)^2]$$

(ii) $F_2\prod(G) = \prod_{i=1}^{n}[e(u)^2\ e(v)^2]$

$F_2\prod[D(n)] = (4\times 2^0)\ [n^2(n+1)^2] \times (4\times 2^1)\ [(n+1)^2(n+2)^2] \times$

$(4\times 2^2)\ [(n+2)^2(n+3)^2] \times \ldots\ldots\ldots\ldots\times$

$(4\times 2^{n-2})[(2n-2)^2(2n-1)^2] \times$

$(4\times 2^{n-1})\ [(2n-1)^2(2n)^2]$

$F_2\prod[D(n)] = 4\prod_{i=1}^{n} 2^{i-1}\ [(n+i-1)^2(n+i)^2]$

(iii) $MF\prod(G) = \prod_{i=1}^{n}|e(u)^2 - e(v)^2|$

$MF\prod[D(n)] = (4\times 2^0)\ |n^2-(n+1)^2| \times (4\times 2^1)\ |(n+1)^2-(n+2)^2| \times$

$(4\times 2^2)\ |(n+2)^2-(n+3)^2| \times \ldots\ldots\ldots\ldots\times$

$(4\times 2^{n-2})\ |(2n-2)^2-(2n-1)^2| \times$

$(4\times 2^{n-1})\ |(2n-1)^2-(2n)^2|$

$MF\prod[D(n)] = 4\prod_{i=1}^{n} 2^{i-1}\ |(n+i-1)^2-(n+i)^2|$

(iv) $QF\prod(G) = \prod_{i=1}^{n}|e(u)^2 - e(v)^2|^2$

$QF\prod[D(n)] = (4\times 2^0)\ |n^2-(n+1)^2|^2 \times (4\times 2^1)\ |(n+1)^2-(n+2)^2|^2 \times$

$(4\times 2^2)\ |(n+2)^2-(n+3)^2|^2 + \ldots\ldots\ldots\ldots+$

$(4\times 2^{n-2})\ |(2n-2)^2-(2n-1)^2|^2 +$

$(4\times 2^{n-1})|(2n-1)^2-(2n)^2|^2$

$QF\prod[D(n)] = 4\prod_{i=1}^{n} 2^{i-1}\ |(n+i-1)^2-(n+i)^2|^2$

CHAPTER 9

STATUS BASED TOPOLOGICAL INDICES OF NANOSTAR DENDRIMER D(n)

In this chapter, we calculate status based General first and second status index, General first and second Gourava status index, General modified first and second Gourava status index. Also we define polynomial index Nanostar Dendrimer D(n).

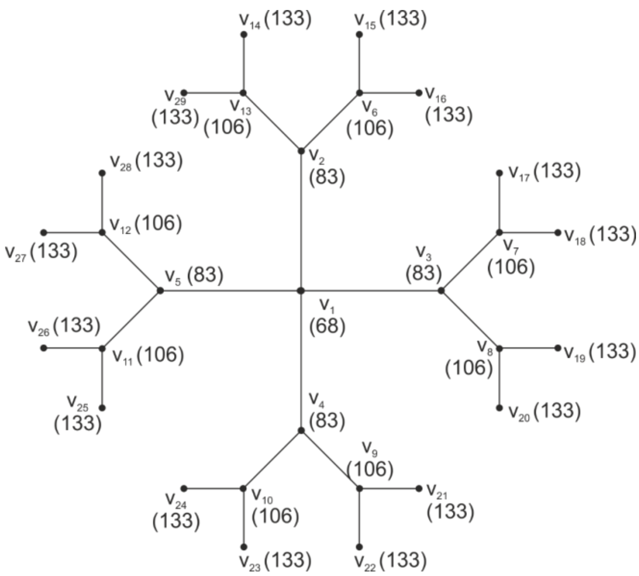


Figure 9.1. The molecular graph of dendrimer D(3)

To derive the value of $\sigma(u_i)$:

$$\sigma(u_1) = n.2^{n+2} - 1.\ 2^{n+1} + 3$$

$$\sigma(u_2) = (n+1)\ 2^{n+2} - 3.\ 2^{n} + 2$$

$$\sigma(u_3) = (n+2)\ 2^{n+2} - 7.\ 2^{n-1} + 1$$

$$\sigma(u_4) = (n+3)\ 2^{n+2} - 15.\ 2^{n-2} + 0$$

$$\sigma(u_5) = (n+4)\ 2^{n+2} - 31.\ 2^{n-2} - 1$$

……………………

$$\sigma(u_{i-1}) = (n+(i-2))2^{n+2} - (2^{i-1} - 1).2^{n-(i-3)} + (4-i+1)$$

$$\sigma(u_i) = (n+(i-1))2^{n+2} - (2^{i} - 1).2^{n-(i-2)} + (4-i)$$

We can find the value of status as follows:

D(1): $\sigma(u_0) = 4$ and $\sigma(u_1) = 7$(4 times)

D(2): $\sigma(u_0) = 20$, $\sigma(u_1) = 27$(4 times) and $\sigma(u_2) = 38$ (8 times)

D(3): $\sigma(u_0) = 68$, $\sigma(u_1) = 83$(4 times), $\sigma(u_2) = 106$ (8 times) and $\sigma(u_3) = 133$ (16 times)

D(4): $\sigma(u_0) = 196$, $\sigma(u_1) = 227$(4 times), $\sigma(u_2) = 274$ (8 times), $\sigma(u_3) = 329$ (16 times)

and $\sigma(u_4) = 388$ (32 times) and so on.

9.1 General first and second status index, General first and second Gourava status index and General modified first and second Gourava status topological indices of Nanostar Dendrimer D(n).

In this section, we introduced new indices namely General first and second status indices, General first and second Gourava status indices and General modified first and second Gourava status indices. General first and second status indices, General first and second Gourava status indices and General modified first and second Gourava status indices of a graph G and are defined as

$$S_1^{\,a}(G) = \sum_{uv\in E(G)} [\sigma(u) + \sigma(v)]^a \tag{9.1.1}$$

$$S_2^{\,a}(G) = \sum_{uv\in E(G)} [\sigma(u)\,\sigma(v)]^a \tag{9.1.2}$$

$$SG_1^{\,a}(G) = \sum_{uv\in E(G)} [\sigma(u) + \sigma(v) + \sigma(u)\sigma(v)]^a \tag{9.1.3}$$

$$SG_2^{\,a}(G) = \sum_{uv\in E(G)} [(\sigma(u) + \sigma(v))(\sigma(u)\,\sigma(v))]^a \tag{9.1.4}$$

$$m_{SG_1{}^a(G)} = \sum_{uv\in E(G)} \left[\frac{1}{(\sigma(u)+\sigma(v)+\sigma(u)\sigma(v))^a}\right] \tag{9.1.5}$$

$$m_{SG_2{}^a(G)} = \sum_{uv\in E(G)} \left[\frac{1}{(\sigma(u)+\sigma(v))(\sigma(u)\sigma(v))^a}\right] \tag{9.1.6}$$

Now, we shall calculate General first and second status indices, General first and second Gourava status indices and General modified first and second Gourava status indices of Nanostar Dendrimer D(n).

Theorem 9.1.1: General first status index of a Nanostar Dendrimer D(n) is

$$S_1^{\,a}(D(n)) = \sum_{i=1}^{n} 2^{i+1}\ [(8n+8i-20)\,2^{n} + 12\times 2^{n-i} + 9 - 2i]^{a} \text{ ------(1).}$$

Proof: General first status index of a Nanostar Dendrimer D(n), we have

$$S_1^{\,a}(G) = \sum_{uv\in E(G)} [\sigma(u)+\sigma(v)]^{a}$$

$$S_1^{\,a}(D(n)) = \sum_{i=1}^{n} 2^{i+1}[\sigma(u_{i-1})+\sigma(u_i)]^{a}$$

$$= \sum_{i=1}^{n} 2^{i+1}[(n+i-2)\,2^{n+2} - (2^{i-1}-1)\,2^{n-i+3} + (5-i) +$$

$$(n+i-1)2^{n+2} - (2^{i}-1)\,2^{n-i+2} + (4-i)]^{a}$$

$$S_1^{\,a}(D(n)) = \sum_{i=1}^{n} 2^{i+1}\ [(8n+8i-20)\,2^{n} + 12\times 2^{n-i} + 9 - 2i]^{a}$$

We obtain the following results by using theorem 9.1.1.

Corollary 9.1.1: First status index of D(n) is given by

$$S_1(D(n)) = \sum_{i=1}^{n} 2^{i+1}\,[(8n+8i-20)\,2^{n} + 12\times 2^{n-i} + 9 - 2i].$$

Proof: put a = 1 in equation (1), we get the desired result.

Corollary 9.1.2: First hyper status index of D(n) is given by

$$HS_1(D(n)) = \sum_{i=1}^{n} 2^{i+1}\,[(8n+8i-20)\,2^{n} + 12\times 2^{n-i} + 9 - 2i]^{2}$$

Proof: put a = 2 in equation (1), we get the desired result.

Corollary 9.1.3: Sum connectivity status index of D(n) is given by

$$SS(D(n)) = \sum_{i=1}^{n} 2^{i+1}\frac{1}{\sqrt{(8n+8i-20)\,2^{n} + 12\times 2^{n-i} + 9 - 2i}}$$

Proof: Put a = -1/2 in equation (1), we get the desired result.

Corollary 9.1.4: Reciprocal sum connectivity status index of D(n) is given by

$$RSS(D(n)) = \sum_{i=1}^{n} 2^{i+1}\sqrt{(8n+8i-20)\,2^{n} + 12\times 2^{n-i} + 9 - 2i}$$

Proof: put a = 1/2 in equation (1), we get the desired result.

Theorem 9.1.2: General second status index of a Nanostar Dendrimer D(n) is

$$S_2^{\,a}(D(n)) = \sum_{i=1}^{n} 2^{i+1}\ [16\{(n+i)^2 - 5(n+i) + 6\}\ 2^{2n} +$$

$$\{48(n+i) - 112\}2^{2n-i} + 32 \times 2^{2n-2i} + 20 - 9i + i^2$$

$$4\{21n + 32i - 5ni - 5i^2 - 46)\ 2^n + (52 - 12i)\ 2^{n-i}\ ]^{\,a} \text{ -------(2).}$$

Proof: General second status index of a Nanostar Dendrimer D(n), we have

$$S_2^{\,a}(G) = \sum_{uv \in E(G)} [\sigma(u)\ \sigma(v)]^{\,a}$$

$$S_2^{\,a}(D(n)) = \sum_{i=1}^{n} 2^{i+1}[\sigma(u_{i-1})\ \sigma(u_i)]^{\,a}$$

$$= \sum_{i=1}^{n} 2^{i+1}[(n+i-2)2^{n+2} - (2^{i-1}-1)2^{n-i+3} + (5-i) \times$$

$$(n+i-1)2^{n+2} - (2^{i}-1)2^{n-i+2} + (4-i)]^{\,a}$$

$$S_2^{\,a}(D(n)) = \sum_{i=1}^{n} 2^{i+1}\ [16\{(n+i)^2 - 5(n+i) + 6\}\ 2^{2n} +$$

$$(48(n+i) - 112)\ 2^{2n-i} + 32 \times 2^{2n-2i} + 20 - 9i + i^2 +$$

$$4(21n + 32i - 5ni - 5i^2 - 46)\ 2^n + (52 - 12i)\ 2^{n-i}\ ]^{\,a}$$

From theorem 9.1.2, we obtain the following results.

Corollary 9.1.5: second status index of D(n) is given by

$$S_2(D(n)) = \sum_{i=1}^{n} 2^{i+1}\ [16((n+i)^2 - 5(n+i) + 6)\ 2^{2n} +$$

$$(48(n+i) - 112)\ 2^{2n-i} + 32 \times 2^{2n-2i} + 20 - 9i + i^2 +$$

$$4(21n + 32i - 5ni - 5i^2 - 46)\ 2^n + (52 - 12i)\ 2^{n-i}\ ]$$

Proof: put a = 1 in equation (2), we get the desired result.

Corollary 9.1.6: Second hyper status index of D(n) is given by

$$HS_2(D(n)) = \sum_{i=1}^{n} 2^{i+1}\ [16\{(n+i)^2 - 5(n+i) + 6\}\, 2^{2n} + \{48(n+i) - 112\}\, 2^{2n-i} + 32 \times 2^{2n-2i} + 20 - 9i + i^2 + 4\{21n + 32i - 5ni - 5i^2 - 46\}\, 2^{n} + (52 - 12i)\, 2^{n-i}\ ]^2$$

Proof: put a = 2 in equation (2), we get the desired result.

Corollary 9.1.7: Product connectivity status index of D(n) is given by

$$PS(D(n)) = \sum_{i=1}^{n} 2^{i+1} \frac{1}{\sqrt{\begin{matrix} 16\{(n+i)^2 - 5(n+i) + 6\}\, 2^{2n} \\ \{48(n+i) - 112\}\, 2^{2n-i} + 32 \times 2^{2n-2i} + 20 - 9i + i^2 + \\ 4\{21n + 32i - 5ni - 5i^2 - 46\}\, 2^{n} + (52 - 12i)\, 2^{n-i} \end{matrix}}}$$

Proof: put a = -1/2 in equation (2), we get the desired result.

Corollary 9.1.8: Reciprocal product connectivity status index of D(n) is given by

$$RPS(D(n)) = \sum_{i=1}^{n} 2^{i+1} \sqrt{\begin{matrix} 16\{(n+i)^2 - 5(n+i) + 6\}\, 2^{2n} \\ \{48(n+i) - 112\}\, 2^{2n-i} + 32 \times 2^{2n-2i} + \\ 20 - 9i + i^2 + (52 - 12i)\, 2^{n-i} + \\ 4\{21n + 32i - 5ni - 5i^2 - 46\}\, 2^{n} \end{matrix}}$$

Proof: put a = 1/2 in equation (2), we get the desired result.

Theorem 9.1.3: General first Gourava status index of a Nanostar Dendrimer D(n) is

$$SG_1^{\,a}(D(n)) = \sum_{i=1}^{n} 2^{i+1}[\{92n + 136i - 20ni - 20i^2 - 204\}\, 2^{n} + \{64 - 12i\}2^{n-i} + 16\{(n+i)^2 - 5(n+i) + 6\}2^{2n} + \{48(n+i) - 112\}2^{2n-i} + 32 \times 2^{2n-2i} + 29 - 11i + i^2]^{a} \quad \text{-----(3)}$$

Proof: General first Gourava status index of a Nanostar Dendrimer D(n), we have

$SG_1{}^a(G) = \sum_{uv\in E(G)} [\sigma(u) + \sigma(v) + \sigma(u)\sigma(v)]^a$

$$SG_1{}^a\big(D(n)\big) = \sum_{uv\in E(G)} [\sigma(u_{i-1}) + \sigma(u_i) + \sigma(u_{i-1})\sigma(u_i)]^a$$

$$= \sum_{i=1}^{n} 2^{i+1}[(n+i-2)2^{n+2} - \big(2^{i-1}-1\big)2^{n-i+3} + (5-i) +$$

$$(n+i-1)2^{n+2} - \big(2^{i}-1\big)2^{n-i+2} + (4-i) +$$

$$((n+i-2)2^{n+2} - \big(2^{i-1}-1\big)2^{n-i+3} + (5-i) \times$$

$$(n+i-1)2^{n+2} - \big(2^{i}-1\big)2^{n-i+2} + (4-i))]^a$$

$$SG_1{}^a\big(D(n)\big) = \sum_{i=1}^{n} 2^{i+1}[\{92n + 136i - 20ni - 20i^2 - 204\}\, 2^n +$$

$$\{64 - 12i\}2^{n-i} + 16\{(n+i)^2 - 5(n+i) + 6\}2^{2n} +$$

$$\{48(n+i) - 112\}2^{2n-i} + 32 \times 2^{2n-2i} + 29 - 11i + i^2]^a$$

The following results are obtained by using theorem 9.1.3.

Corollary 9.1.9: First Gourava status index of D(n) is given by

$$SG_1(D(n)) = \sum_{i=1}^{n} 2^{i+1}[\{92n + 136i - 20ni - 20i^2 - 204\}\, 2^n +$$

$$\{64 - 12i\}2^{n-i} + 16\{(n+i)^2 - 5(n+i) + 6\}2^{2n} +$$

$$\{48(n+i) - 112\}2^{2n-i} + 32 \times 2^{2n-2i} + 29 - 11i + i^2]$$

Proof: put a = 1 in equation (3), we get the desired result.

Corollary 9.1.10: First hyper Gourava status index of D(n) is given by

$$HSG_1(D(n)) = \sum_{i=1}^{n} 2^{i+1}\,[\{92n + 136i - 20ni - 20i^2 - 204\}2^n +$$

$$\{64 - 12i\}2^{n-i} + 16\{(n+i)^2 - 5(n+i) + 6\}2^{2n} +$$

$$\{48(n+i) - 112\}\, 2^{2n-i} + 32 \times 2^{2n-2i} + 29 - 11i + i^2]^2$$

Proof: put a = 2 in equation (3), we get the desired result.

Corollary 9.1.11: Sum connectivity Gourava status index of D(n) is given by

$$SSG_1(D(n)) = \sum_{i=1}^{n} 2^{i+1} \frac{1}{\sqrt{\begin{array}{c}\{92n+136i-20ni-20i^2-204\}2^n + \\ \{64-12i\}2^{n-i} + 16\{(n+i)^2-5(n+i)+6\}2^{2n} + \\ \{48(n+i)-112\}2^{2n-i} + 32\times 2^{2n-2i} + 29-11i+i^2\end{array}}}$$

Proof: put a = -1/2 in equation (3), we get the desired result.

Corollary 9.1.12: Reciprocal sum connectivity Gourava status index of D(n) is given by

$$RSSG_1(D(n)) = \sum_{i=1}^{n} 2^{i+1} \sqrt{\begin{array}{c}\{92n+136i-20ni-20i^2-204\}2^n + \\ \{64-12i\}2^{n-i} + 16\{(n+i)^2-5(n+i)+6\}2^{2n} + \\ \{48(n+i)-112\}2^{2n-i} + 32\times 2^{2n-2i} + 29-11i+i^2\end{array}}$$

Proof: put a = 1/2 in equation (3), we get the desired result.

Theorem 9.1.4: General second Gourava status index of a Nanostar Dendrimer D(n) is

$$\begin{aligned} SG_2{}^{a}(D(n)) = \sum_{i=1}^{n} 2^{i+1} [\{(8n+8i-20)\,2^n + 12\times 2^{n-i} + 9-2i\} \times \\ \{16\{(n+i)^2-5(n+i)+6\}2^{2n} + \\ \{48(n+i)-112\}2^{2n-i} + 32\times 2^{2n-2i} + 20-9i+i^2 + \\ 4\{21n+32i-5ni-5i^2-46\}2^n + \{52-12i\}\,2^{n-i}\}]^{a} \end{aligned}$$ ---------(4).

Proof: General second Gourava status index of Nanostar Dendrimer D(n), we have

$$SG_2{}^{a}(G) = \sum_{uv\in E(G)} [(\sigma(u)+\sigma(v))(\sigma(u)\,\sigma(v))]^{a}$$

$$\begin{aligned} SG_2{}^{a}(D(n)) &= \sum_{uv\in E(G)} [\sigma(u_{i-1})+\sigma(u_i))(\sigma(u_{i-1})\sigma(u_i))]^{a} \\ &= \sum_{i=1}^{n} 2^{i+1}[(n+i-2)2^{n+2} - (2^{i-1}-1)2^{n-i+3} + (5-i) + \\ &\quad (n+i-1)2^{n+2} - (2^{i}-1)2^{n-i+2} + (4-i) + \\ &\quad \{((n+i-2)2^{n+2} - (2^{i-1}-1)2^{n-i+3} + (5-i)) \\ &\quad ((n+i-1)2^{n+2} - (2^{i}-1)2^{n-i+2} + (4-i))\}]^{a} \end{aligned}$$

$$SG_2{}^a(D(n))=\sum_{i=1}^{n} 2^{i+1}\,[\{(8n+8i-20)\,2^n+12\times 2^{n-i}+9-2i\}\times$$
$$\{16\{(n+i)^2-5(n+i)+6\}2^{2n}+$$
$$\{48(n+i)-112\}2^{2n-i}+32\times 2^{2n-2i}+20-9i+i^2+$$
$$4\{21n+32i-5ni-5i^2-46\}2^n+\{52-12i\}\,2^{n-i}\}]^a$$

We establish the following results by theorem 9.1.4.

Corollary 9.1.13: Second status Gourava index of D(n) is given by

$$SG_2(D(n))=\sum_{i=1}^{n} 2^{i+1}\,[\{(8n+8i-20)\,2^n+12\times 2^{n-i}+9-2i\}\times$$
$$\{16\{(n+i)^2-5(n+i)+6\}2^{2n}+$$
$$\{48(n+i)-112\}2^{2n-i}+32\times 2^{2n-2i}+20-9i+i^2+$$
$$4\{21n+32i-5ni-5i^2-46\}2^n+\{52-12i\}\,2^{n-i}\}]$$

Proof: put a = 1 in equation (4), we get the desired result.

Corollary 9.1.14: Second hyper status index of D(n) is given by

$$HSG_2(D(n))=\sum_{i=1}^{n} 2^{i+1}\,[\{(8n+8i-20)\,2^n+12\times 2^{n-i}+9-2i\}\times$$
$$16\{(n+i)^2-5(n+i)+6\}2^{2n}+$$
$$\{48(n+i)-112\}2^{2n-i}+32\times 2^{2n-2i}+20-9i+i^2)$$
$$4\{21n+32i-5ni-5i^2-46\}2^n+\{52-12i\}\,2^{n-i}\,]^2$$

Proof: put a = 2 in equation (4), we get the desired result.

Corollary 9.1.15: Product connectivity Gourava status index of D(n) is given by

$$PSG_2(D(n))=\sum_{i=1}^{n} 2^{i+1}\,\frac{1}{\sqrt{\begin{array}{c}16\{(n+i)^2-5(n+i)+6\}\,2^{2n}\\ \{48(n+i)-112\}\,2^{2n-i}+32\times 2^{2n-2i}+20-9i+i^2+\\ 4\{21n+32i-5ni-5i^2-46\}\,2^n+(52-12i)\,2^{n-i}\end{array}}}$$

Proof: put a = -1/2 in equation (4), we get the desired result.

Corollary 9.1.16: Reciprocal product connectivity status index of D(n) is given by

$$RPSG_2(D(n)) = \sum_{i=1}^{n} 2^{i+1} \sqrt{\begin{matrix} 16\{(n+i)^2 - 5(n+i) + 6\}\, 2^{2n} \\ \{48(n+i) - 112\}\, 2^{2n-i} + 32 \times 2^{2n-2i} + \\ 20 - 9i + i^2 + (52 - 12i)\, 2^{n-i} + \\ 4\{21n + 32i - 5ni - 5i^2 - 46\}\, 2^n \end{matrix}}$$

Proof: put a = 1/2 in equation (4), we get the desired result.

Theorem 9.1.5: General modified first Gourava status index of a Nanostar Dendrimer

D(n) is $m_{SG_1^a D(n)} = \sum_{i=1}^{n} 2^{i+1} \left[\frac{1}{\begin{matrix}[\{92n+136i - 20ni - 20i^2 - 204\}2^n \\ \{64 - 12i\}2^{n-i} + 16\{(n+i)^2 - 5(n+i)+6\}2^{2n} + \\ \{48(n+i) - 112\}2^{2n-i} + 32 \times 2^{2n-2i} + 29 - 11i + i^2]^a\end{matrix}} \right]$ -----(5).

Proof: General modified first Gourava status index of a Nanostar Dendrimer D(n), we have

$$m_{SG_1^a}(G) = \sum_{uv\in E(G)} \left[\frac{1}{(\sigma(u)+\sigma(v)+\sigma(u)\sigma(v))^a}\right]$$

$$m_{SG_1^a}(D(n)) = \sum_{uv\in E(G)} \left[\frac{1}{(\sigma(u_{i-1})+\sigma(u_i)+\sigma(u_{i-1})\sigma(u_i))^a}\right]$$

$$= \sum_{i=1}^{n} 2^{i+1} \left[\frac{1}{\begin{matrix}(n+i-2)2^{n+2} - (2^{i-1}-1)2^{n-i+3} + (5-i) + (n+i-1)2^{n+2} - (2^i - 1)2^{n-i+2} + (4-i) + \\ ((n+i-2)2^{n+2} - (2^{i-1}-1)2^{n-i+3} + (5-i) \times (n+i-1)2^{n+2} - (2^i-1)2^{n-i+2} + (4-i))^a\end{matrix}} \right]$$

$$= \sum_{i=1}^{n} 2^{i+1} \left[\frac{1}{\begin{matrix}[\{92n+136i - 20ni - 20i^2 - 204\}2^n \\ \{64 - 12i\}2^{n-i} + 16\{(n+i)^2 - 5(n+i)+6\}2^{2n} + \\ \{48(n+i) - 112\}2^{2n-i} + 32 \times 2^{2n-2i} + 29 - 11i + i^2]^a\end{matrix}} \right]$$

From theorem 9.1.5, we establish the following results.

Corollary 9.1.17: Modified first Gourava status index of a D(n) is given by

$$m_{SG_1(D(n))} = \sum_{i=1}^{n} 2^{i+1}\left[\frac{1}{\begin{array}{c}[\{92n+136i-20ni-20i^2-204\}2^n\\ \{64-12i\}2^{n-i}+16\{(n+i)^2-5(n+i)+6\}2^{2n}+\\ \{48(n+i)-112\}2^{2n-i}+32\times 2^{2n-2i}+29-11i+i^2\end{array}}\right]$$

Proof: put a = 1 in equation (5), we get the desired result.

Corollary 9.1.18: Modified first hyper Gourava status index of a D(n) is given by

$$m_{HSG_1(D(n))} = \sum_{i=1}^{n} 2^{i+1}\left[\frac{1}{\begin{array}{c}[\{92n+136i-20ni-20i^2-204\}2^n\\ \{64-12i\}2^{n-i}+16\{(n+i)^2-5(n+i)+6\}2^{2n}+\\ \{48(n+i)-112\}\,2^{2n-i}+32\times 2^{2n-2i}+29-11i+i^2]^2\end{array}}\right]$$

Proof: put a = 2 in equation (5), we get the desired result.

Theorem 9.1.6: General modified second Gourava status index of a Nanostar Dendrimer D(n) is

$$m_{SG_2{}^a(D(n))} = \sum_{i=1}^{n} 2^{i+1}\left[\frac{1}{\begin{array}{c}[\{(8n+8i-20)\,2^n+12\times 2^{n-i}+9-2i\}\times\\ \{16\{(n+i)^2-5(n+i)+6\}2^{2n}+\\ \{48(n+i)-112\}2^{2n-i}+32\times 2^{2n-2i}+29-11i+i^2\\ 4\{21n+32i-5ni-5i^2-46\}2^n+\{52-12i\}\,2^{n-i}\}\,]^a\end{array}}\right] \text{-----------(6).}$$

Proof: General modified second Gourava status index of a Nanostar Dendrimer D(n),we have

$$m_{SG_2{}^a(G)} = \sum_{uv\in E(G)}\left[\frac{1}{(\sigma(u)+\sigma(v))(\sigma(u)\sigma(v))^a}\right]$$

$$m_{SG_2{}^a(D(n))} = \sum_{uv\in E(G)}\left[\frac{1}{(\sigma(u_{i-1})+\sigma(u_i))(\sigma(u_{i-1})\sigma(u_i))^a}\right]$$

$$= \sum_{i=1}^{n} 2^{i+1} \left[\frac{1}{\begin{array}{c}(n+i-2)2^{n+2} - (2^{i-1}-1)2^{n-i+3}+(5-i)+(n+i-1)2^{n+2} - (2^{i}-1)2^{n-i+2}+(4-i)+ \\ \{((n+i-2)2^{n+2} - (2^{i-1}-1)2^{n-i+3}+(5-i))\,((n+i-1)2^{n+2} - (2^{i}-1)2^{n-i+2}+(4-i))\}]\end{array}} \right]^{a}$$

$$= \sum_{i=1}^{n} 2^{i+1} \left[\frac{1}{\begin{array}{c}[\{(8n+8i-20)\, 2^{n} +12 \times 2^{n-i} + 9 -2i\} \times \\ \{16\{(n+i)^2 - 5(n+i)+6\}2^{2n}+ \\ \{48(n+i)- 112\}2^{2n-i}+32 \times 2^{2n-2i}+29 -11i+i^2 \\ 4\{21n + 32i -5ni -5i^2- 46\}2^{n} + \{52 -12i\}\, 2^{n-i}\}]\end{array}} \right]^{a}$$

We establish the following results from theorem 9.1.6.

Corollary 9.1.19: Modified second Gourava status index of a D(n) is given by

$$m_{SG_2}{}^{a}(D(n)) = \sum_{i=1}^{n} 2^{i+1} \left[\frac{1}{\begin{array}{c}[\{(8n+8i-20)\, 2^{n} +12 \times 2^{n-i} + 9 -2i\} \times \\ 16\{(n+i)^2 - 5(n+i)+6\}2^{2n}+ \\ \{48(n+i)- 112\}2^{2n-i} + 32 \times 2^{2n-2i}+29 -11i+i^2 \\ 4\{21n + 32i -5ni -5i^2- 46\}2^{n} + \{52 -12i\}\, 2^{n-i}\}]\end{array}} \right]$$

Proof: put a = 1 in Equation (6), we get the desired result.

Corollary 9.1.20: Modified second hyper Gourava status index of a D(n) is given by

$$m_{HSG_2}{}^{a}(D(n)) = \sum_{i=1}^{n} 2^{i+1} \left[\frac{1}{\begin{array}{c}[\{(8n+8i-20)\, 2^{n} +12 \times 2^{n-i} + 9 -2i\} \times \\ 16\{(n+i)^2 - 5(n+i)+6\}2^{2n}+ \\ \{48(n+i)- 112\}2^{2n-i} + 32 \times 2^{2n-2i}+29 -11i+i^2 \\ 4\{21n + 32i -5ni -5i^2- 46\}2^{n} + \{52 -12i\}\, 2^{n-i}\}]\end{array}} \right]^{2}$$

Proof: put a = 2 in equation (6), we get the desired result.

9.2 General first and second status polynomial index, General first and second Gourava status polynomial index and General modified first and second Gourava status polynomial topological indices of Nanostar Dendrimer D(n).

In this section, we introduced new indices namely General first and second status polynomial indices, General first and second Gourava status polynomial indices and General modified first and second Gourava status polynomial indices. The General first and second status polynomial indices, General first and second Gourava status polynomial indices and General modified first and second Gourava status polynomial indices of a graph G and are defined as

$$S_1{}^{a}(G,x) = \sum_{uv\in E(G)} x^{[\sigma(u)+\sigma(v)]^{a}} \tag{9.2.1}$$

$$S_2{}^{a}(G,x) = \sum_{uv\in E(G)} x^{[\sigma(u)\times\sigma(v)]^{a}} \tag{9.2.2}$$

$$SG_1{}^{a}(G,x) = \sum_{uv\in E(G)} x^{[\sigma(u)+\sigma(v)+\sigma(u)\sigma(v)]^{a}} \tag{9.2.3}$$

$$SG_2{}^{a}(G,x) = \sum_{uv\in E(G)} x^{[(\sigma(u)+\sigma(v))(\sigma(u)\sigma(v))]^{a}} \tag{9.2.4}$$

$$m_{SG_1{}^{a}(G,x)} = \sum_{uv\in E(G)} x^{\left[\frac{1}{(\sigma(u)+\sigma(v)+\sigma(u)\sigma(v))^{a}}\right]} \tag{9.2.5}$$

$$m_{SG_2{}^{a}(G,x)} = \sum_{uv\in E(G)} x^{\left[\frac{1}{((\sigma(u)+\sigma(v))(\sigma(u)\sigma(v)))^{a}}\right]} \tag{9.2.6}$$

Now, we shall compute General first and second status polynomial indices, General first and second Gourava status polynomial indices and General modified first and second Gourava status polynomial indices of Nanostar Dendrimer D(n).

Theorem 9.2.1: General first status polynomial index of a Nanostar Dendrimer D(n) is

$$S_1{}^{a}(D(n),x) = \sum_{i=1}^{n} 2^{i+1}\ x^{[(8n+8i-20)\,2^{n}+12\times 2^{n-i}+9-2i]^{a}} \text{ ---------(1).}$$

Proof: $S_1^{\,a}(G, x) = \sum_{uv \in E(G)} x^{[\sigma(u)+\sigma(v)]^a}$

$$S_1^{\,a}(D(n), x) = \sum_{i=1}^{n} 2^{i+1} x^{[\sigma(u_{i-1})+\sigma(u_i)]^a}$$

$$= \sum_{i=1}^{n} 2^{i+1} x^{[(n+i-2)\,2^{n+2} - (2^{i-1}-1)\,2^{n-i+3} + (5-i) + (n+i-1)2^{n+2} - (2^{i}-1)\,2^{n-i+2} + (4-i)]^a}$$

$$S_1^{\,a}(D(n), x) = \sum_{i=1}^{n} 2^{i+1} \; x^{[(8n+8i-20)\,2^{n} + 12\times 2^{n-i} + 9 - 2i]^a}$$

Corollary 9.2.1: First status index of D(n) is given by

$$S_1(D(n)) = \sum_{i=1}^{n} 2^{i+1}\,[(8n + 8i - 20)\,2^{n} + 12\times 2^{n-i} + 9 - 2i].$$

Proof: put a = 1 in equation (1), we get the desired result.

Corollary 9.2.2: First hyper status index of D(n) is given by

$$HS_1\big(D(n)\big) = \sum_{i=1}^{n} 2^{i+1}\,[(8n + 8i - 20)\,2^{n} + 12 \times 2^{n-i} + 9 - 2i]^2$$

Corollary 9.2.3: If D(n) is a Nanostar Dendrimer then

(i) $S_1(D(n), x) = \sum_{i=1}^{n} 2^{i+1} x^{[(8n+8i-20)\,2^{n} + 12\times 2^{n-i} + 9 - 2i]}$

(ii) $HS_1(D(n), x) = \sum_{i=1}^{n} 2^{i+1} x^{[(8n+8i-20)\,2^{n}+12\times 2^{n-i}+9-2i]^{2}}$

(iii) $SS(D(n), x) = \sum_{i=1}^{n} 2^{i+1} x^{\left[\frac{1}{\sqrt{(8n+8i-20)\,2^{n} + 12\times 2^{n-i} + 9 - 2i}}\right]}$

(iv) $RSS(D(n), x) = \sum_{i=1}^{n} 2^{i+1} x^{\sqrt{(8n+8i-20)\,2^{n}+12\times 2^{n-i}+9-2i}}$

Proof: put a = 1, 2, -1/2, 1/2 in equation (1), we get the desired results.

Theorem 9.2.2: General second status index of a Nanostar Dendrimer D(n) is

$$S_2{}^a(D(n),x) = \sum_{i=1}^{n} 2^{i+1}\, x^{[16\{(n+i)^2-5(n+i)+6\}\,2^{2n} + \{48(n+i)-112\}2^{2n-i} + 32\times 2^{2n-2i} + 20-9i+i^2 + 4\{21n+32i-5ni-5i^2-46\}\,2^n + \{52-12i\}\,2^{n-i}]^a} \text{ -------(2).}$$

Proof: $S_2{}^a(G,x) = \sum_{uv\in E(G)} x^{[\sigma(u)\times\sigma(v)]^a}$

$$S_2{}^a(D(n),x) = \sum_{i=1}^{n} 2^{i+1} x^{[\sigma(u_{i-1})\times\sigma(u_i)]^a}$$

$$= \sum_{i=1}^{n} 2^{i+1} x^{[(n+i-2)2^{n+2} - (2^{i-1}-1)2^{n-i+3} + (5-i)\times (n+i-1)2^{n+2} - (2^{i}-1)2^{n-i+2} + (4-i)]^a}$$

$$S_2{}^a(D(n),x) = \sum_{i=1}^{n} 2^{i+1}\, x^{[16\{(n+i)^2-5(n+i)+6\}\,2^{2n} + \{48(n+i)-112\}\,2^{2n-i} + 32\times 2^{2n-2i} + 20-9i+i^2 + 4\{21n+32i-5ni-5i^2-46\}\,2^n + \{52-12i\}\,2^{n-i}]^a}$$

Corollary 9.2.4: If D(n) is a Nanostar Dendrimer then

(i) $S_2(D(n),x) = \sum_{i=1}^{n} 2^{i+1}\, x^{[16\{(n+i)^2-5(n+i)+6\}\,2^{2n} + \{48(n+i)-112\}\,2^{2n-i} + 32\times 2^{2n-2i} + 20-9i+i^2 + 4\{21n+32i-5ni-5i^2-46\}\,2^n + \{52-12i\}\,2^{n-i}]}$

(ii) $HS_2(D(n),x) = \sum_{i=1}^{n} 2^{i+1}\, x^{[16\{(n+i)^2-5(n+i)+6\}\,2^{2n} + \{48(n+i)-112\}\,2^{2n-i} + 32\times 2^{2n-2i} + 20-9i+i^2 + 4\{21n+32i-5ni-5i^2-46\}\,2^n + \{52-12i\}\,2^{n-i}]^2}$

(iii) $PS(D(n),x) = \sum_{i=1}^{n} 2^{i+1}\, x^{\left[\frac{1}{\sqrt{16\{(n+i)^2-5(n+i)+6\}\,2^{2n}\ \{48(n+i)-112\}\,2^{2n-i} + 32\times 2^{2n-2i} + 20-9i+i^2 + 4\{21n+32i-5ni-5i^2-46\}\,2^n + \{52-12i\}\,2^{n-i}}}\right]}$

(iv) $RPS(D(n),x) = \sum_{i=1}^{n} 2^{i+1}\, x^{\sqrt{16\{(n+i)^2-5(n+i)+6\}\,2^{2n}\ \{48(n+i)-112\}\,2^{2n-i} + 32\times 2^{2n-2i} + 20-9i+i^2+(52-12i)\,2^{n-i} + 4\{21n+32i-5ni-5i^2-46\}\,2^n}}$

Proof: put a = 1, 2, -1/2, 1/2 in equation (2), we get the desired results.

Theorem 9.2.5: General first Gourava status index of a Nanostar Dendrimer D(n) is

$$SG_1^{\,a}(D(n),x) = \sum_{i=1}^{n} 2^{i+1} x^{\left[\begin{array}{c}\{92n+136i-20ni-20i^2-204\}\,2^{n}+\\ \{64-12i\}2^{n-i}+16\{(n+i)^2-5(n+i)+6\}2^{2n}+\\ \{48(n+i)-112\}2^{2n-i}+32\times 2^{2n-2i}+29-11i+i^2\end{array}\right]^{a}} \qquad \text{------ (3)}$$

Proof: $SG_1^{\,a}(G,x) = \sum_{uv\in E(G)} x^{[\sigma(u)+\sigma(v)+\sigma(u)\sigma(v)]^{a}}$

$$SG_1^{\,a}(D(n),x) = \sum_{uv\in E(G)} x^{[\sigma(u_{i-1})+\sigma(u_i)+\sigma(u_{i-1})\sigma(u_i)]^{a}}$$

$$= \sum_{i=1}^{n} 2^{i+1} x^{\left[\begin{array}{c}(n+i-2)2^{n+2}-(2^{i-1}-1)2^{n-i+3}+(5-i)+\\ (n+i-1)2^{n+2}-(2^{i}-1)2^{n-i+2}+(4-i)+\\ ((n+i-2)2^{n+2}-(2^{i-1}-1)2^{n-i+3}+(5-i)\times\\ (n+i-1)2^{n+2}-(2^{i}-1)2^{n-i+2}+(4-i))\end{array}\right]^{a}}$$

$$SG_1^{\,a}(D(n),x) = \sum_{i=1}^{n} 2^{i+1} x^{\left[\begin{array}{c}\{92n+136i-20ni-20i^2-204\}\,2^{n}+\\ \{64-12i\}2^{n-i}+16\{(n+i)^2-5(n+i)+6\}2^{2n}+\\ \{48(n+i)-112\}2^{2n-i}+32\times 2^{2n-2i}+29-11i+i^2\end{array}\right]^{a}}$$

Corollary 9.2.6: If D(n) is a Nanostar Dendrimer then

(i) $$SG_1(D(n),x) = \sum_{i=1}^{n} 2^{i+1} x^{\left[\begin{array}{c}\{92n+136i-20ni-20i^2-204\}\,2^{n}+\\ \{64-12i\}2^{n-i}+16\{(n+i)^2-5(n+i)+6\}2^{2n}+\\ \{48(n+i)-112\}2^{2n-i}+32\times 2^{2n-2i}+29-11i+i^2\end{array}\right]}$$

(ii) $$HSG_1(D(n),x) = \sum_{i=1}^{n} 2^{i+1} x^{\left[\begin{array}{c}\{92n+136i-20ni-20i^2-204\}2^{n}+\\ \{64-12i\}2^{n-i}+16\{(n+i)^2-5(n+i)+6\}2^{2n}+\\ \{48(n+i)-112\}\,2^{2n-i}+32\times 2^{2n-2i}+29-11i+i^2\end{array}\right]^{2}}$$

(iii) $$SSG_1(D(n),x) = \sum_{i=1}^{n} 2^{i+1} x^{\frac{1}{\sqrt{\begin{array}{c}\{92n+136i-20ni-20i^2-204\}2^{n}+\\ \{64-12i\}2^{n-i}+16\{(n+i)^2-5(n+i)+6\}2^{2n}+\\ \{48(n+i)-112\}\,2^{2n-i}+32\times 2^{2n-2i}+29-11i+i^2\end{array}}}}$$

(iv) $$RSSG_1(D(n),x) = \sum_{i=1}^{n} 2^{i+1} x^{\sqrt{\begin{array}{c}\{92n+136i-20ni-20i^2-204\}2^{n}+\\ \{64-12i\}2^{n-i}+16\{(n+i)^2-5(n+i)+6\}2^{2n}+\\ \{48(n+i)-112\}\,2^{2n-i}+32\times 2^{2n-2i}+29-11i+i^2\end{array}}}$$

Proof: put a = 1, 2, -1/2, 1/2 in equation (3), we get the desired results.

Theorem 9.2.4: General second Gourava status index of a Nanostar Dendrimer D(n) is

$$SG_2{}^{a}(D(n),x)=\sum_{i=1}^{n} 2^{i+1}\, x^{\left[\{(8n+8i-20)\,2^{n}+12\times 2^{n-i}+9-2i\}\times \{16\{(n+i)^2-5(n+i)+6\}2^{2n}+\{48(n+i)-112\}2^{2n-i}+32\times 2^{2n-2i}+20-9i+i^2+4\{21n+32i-5ni-5i^2-46\}2^{n}+\{52-12i\}\,2^{n-i}\}\right]^{a}} \quad ------ (4).$$

Proof: $SG_2{}^{a}(G,x)=\sum_{uv\in E(G)} x^{[(\sigma(u)+\sigma(v))(\sigma(u)\,\sigma(v))]^{a}}$

$$SG_2{}^{a}(D(n),x)=\sum_{uv\in E(G)} x^{[\sigma(u_{i-1})+\sigma(u_i))(\sigma(u_{i-1})\sigma(u_i))]^{a}}$$

$$=\sum_{i=1}^{n} 2^{i+1} x^{\left[(n+i-2)2^{n+2}-(2^{i-1}-1)\,2^{n-i+3}+(5-i)+(n+i-1)2^{n+2}-\left(2^{i}-1\right)2^{n-i+2}+(4-i)+\{((n+i-2)2^{n+2}-(2^{i-1}-1)2^{n-i+3}+(5-i))((n+i-1)2^{n+2}-\left(2^{i}-1\right)2^{n-i+2}+(4-i))\}\right]^{a}}$$

$$SG_2{}^{a}(D(n),x)=\sum_{i=1}^{n} 2^{i+1}\, x^{\left[\{(8n+8i-20)\,2^{n}+12\times 2^{n-i}+9-2i\}\times \{16\{(n+i)^2-5(n+i)+6\}2^{2n}+\{48(n+i)-112\}2^{2n-i}+32\times 2^{2n-2i}+20-9i+i^2+4\{21n+32i-5ni-5i^2-46\}2^{n}+\{52-12i\}\,2^{n-i}\}\right]^{a}}$$

Corollary 9.2.7: If D(n) is a Nanostar Dendrimer then

(i) $$SG_2(D(n),x)=\sum_{i=1}^{n} 2^{i+1}\, x^{\left[\{(8n+8i-20)\,2^{n}+12\times 2^{n-i}+9-2i\}\times \{16\{(n+i)^2-5(n+i)+6\}2^{2n}+\{48(n+i)-112\}2^{2n-i}+32\times 2^{2n-2i}+20-9i+i^2+4\{21n+32i-5ni-5i^2-46\}2^{n}+\{52-12i\}\,2^{n-i}\}\right]}$$

(ii) $$SG_2(D(n),x)=\sum_{i=1}^{n} 2^{i+1}\, x^{\left[\{(8n+8i-20)\,2^{n}+12\times 2^{n-i}+9-2i\}\times 16\left\{(n+i)^2-5(n+i)+6\right\}2^{2n}+\{48(n+i)-112\}2^{2n-i}+32\times 2^{2n-2i}+20-9i+i^2)+4\{21n+32i-5ni-5i^2-46\}2^{n}+\{52-12i\}\,2^{n-i}\right]^{2}}$$

(iii) $$PSG_2(D(n),x)=\sum_{i=1}^{n} 2^{i+1} x^{\frac{1}{\sqrt{16\left\{(n+i)^2-5(n+i)+6\right\}2^{2n}\{48(n+i)-112\}\,2^{2n-i}+32\times 2^{2n-2i}+20-9i+i^2+4\{21n+32i-5ni-5i^2-46\}\,2^{n}+(52-12i)\,2^{n-i}}}}$$

(iv) $$RPSG_2(D(n),x)=\sum_{i=1}^{n} 2^{i+1}\, x^{\sqrt{16\{(n+i)^2-5(n+i)+6\}\,2^{2n}\{48(n+i)-112\}\,2^{2n-i}+32\times 2^{2n-2i}+20-9i+i^2+(52-12i)\,2^{n-i}+4\{21n+32i-5ni-5i^2-46\}\,2^{n}}}$$

Proof: put a = 1, 2, -1/2, 1/2 in equation (4), we get the desired results.

Theorem 9.2.5: General modified first Gourava status index of a Nanostar Dendrimer

D(n) is $m_{SG_1{}^a D(n,x)} = \sum_{i=1}^{n} 2^{i+1} x^{\left[\frac{1}{[\{92n+136i-20ni-20i^2-204\}2^n \{64-12i\}2^{n-i}+16\{(n+i)^2-5(n+i)+6\}2^{2n}+\{48(n+i)-112\}2^{2n-i}+32\times 2^{2n-2i}+29-11i+i^2]^a}\right]}$ --------------(5).

Proof: $m_{SG_1{}^a(G,x)} = \sum_{uv\in E(G)} x^{\left[\frac{1}{(\sigma(u)+\sigma(v)+\sigma(u)\sigma(v))^a}\right]}$

$$m_{SG_1{}^a(D(n),x)} = \sum_{uv\in E(G)} x^{\left[\frac{1}{(\sigma(u_{i-1})+\sigma(u_i)+\sigma(u_{i-1})\sigma(u_i))^a}\right]}$$

$$= \sum_{i=1}^{n} 2^{i+1} x^{\left[\frac{1}{(n+i-2)2^{n+2}-(2^{i-1}-1)2^{n-i+3}+(5-i)+(n+i-1)2^{n+2}-(2^i-1)2^{n-i+2}+(4-i)+((n+i-2)2^{n+2}-(2^{i-1}-1)2^{n-i+3}+(5-i)\times(n+i-1)2^{n+2}-(2^i-1)2^{n-i+2}+(4-i))]^a}\right]}$$

$$= \sum_{i=1}^{n} 2^{i+1} x^{\left[\frac{1}{[\{92n+136i-20ni-20i^2-204\}2^n \{64-12i\}2^{n-i}+16\{(n+i)^2-5(n+i)+6\}2^{2n}+\{48(n+i)-112\}2^{2n-i}+32\times 2^{2n-2i}+29-11i+i^2]^a}\right]}$$

Corollary 9.2.8: If D(n) is a Nanostar Dendrimer then

(i) $m_{SG_1(D(n),x)} = \sum_{i=1}^{n} 2^{i+1} x^{\left[\frac{1}{[\{92n+136i-20ni-20i^2-204\}2^n \{64-12i\}2^{n-i}+16\{(n+i)^2-5(n+i)+6\}2^{2n}+\{48(n+i)-112\}2^{2n-i}+32\times 2^{2n-2i}+29-11i+i^2}\right]}$

(ii) $$m_{HSG_1(D(n),x)} = \sum_{i=1}^{n} 2^{i+1} x^{\left[\frac{1}{[\{92n+136i-20ni-20i^2-204\}2^n \{64-12i\}2^{n-i}+16\{(n+i)^2-5(n+i)+6\}2^{2n}+\{48(n+i)-112\}2^{2n-i}+32\times 2^{2n-2i}+29-11i+i^2]}\right]^2}$$

Proof: put a = 1,2, in equation (5), we get the desired results.

Theorem 9.2.6: General modified second Gourava status index of a Nanostar Dendrimer D(n) is

$$m_{SG_2}{}^{a}(D(n),x) = \sum_{i=1}^{n} 2^{i+1} x^{\left[\frac{1}{[\{(8n+8i-20)\,2^n+12\times 2^{n-i}+9-2i\}\times \{16\{(n+i)^2-5(n+i)+6\}2^{2n}+\{48(n+i)-112\}2^{2n-i}+32\times 2^{2n-2i}+29-11i+i^2 \, 4\{21n+32i-5ni-5i^2-46\}2^n+\{52-12i\}\,2^{n-i}\}]}\right]^a} \text{--------------(6).}$$

Proof: $m_{SG_2}{}^{a}(G,x) = \sum_{uv\in E(G)} x^{\left[\frac{1}{(\sigma(u)+\sigma(v))(\sigma(u)\sigma(v))}\right]^a}$

$$m_{SG_2}{}^{a}(D(n),x) = \sum_{uv\in E(G)} x^{\left[\frac{1}{(\sigma(u_{i-1})+\sigma(u_i))(\sigma(u_{i-1})\sigma(u_i))}\right]^a}$$

$$= \sum_{i=1}^{n} 2^{i+1} x^{\left[\frac{1}{(n+i-2)2^{n+2}-(2^{i-1}-1)2^{n-i+3}+(5-i)+(n+i-1)2^{n+2}-(2^{i}-1)2^{n-i+2}+(4-i)+\{((n+i-2)2^{n+2}-(2^{i-1}-1)2^{n-i+3}+(5-i))((n+i-1)2^{n+2}-(2^{i}-1)2^{n-i+2}+(4-i))\}]}\right]^a}$$

$$= \sum_{i=1}^{n} 2^{i+1} x^{\left[\frac{1}{[\{(8n+8i-20)\,2^n+12\times 2^{n-i}+9-2i\}\times \{16\{(n+i)^2-5(n+i)+6\}2^{2n}+\{48(n+i)-112\}2^{2n-i}+32\times 2^{2n-2i}+29-11i+i^2 \, 4\{21n+32i-5ni-5i^2-46\}2^n+\{52-12i\}\,2^{n-i}\}]}\right]^a}$$

Corollary 9.2.9: If D(n) is a Nanostar Dendrimer then

(i) $$m_{SG_2{}^a}(D(n),x) = \sum_{i=1}^{n} 2^{i+1} x^{\left[\frac{1}{\begin{gathered}[\{(8n+8i-20)\,2^{n}+12\times 2^{n-i}+9-2i\}\times \\ 16\{(n+i)^2-5(n+i)+6\}2^{2n}+ \\ \{48(n+i)-112\}2^{2n-i}+32\times 2^{2n-2i}+29-11i+i^2 \\ 4\{21n+32i-5ni-5i^2-46\}2^{n}+\{52-12i\}\,2^{n-i}\}]\end{gathered}}\right]}$$

(ii) $$m_{HSG_2{}^a}(D(n),x) = \sum_{i=1}^{n} 2^{i+1} x^{\left[\frac{1}{\begin{gathered}[\{(8n+8i-20)\,2^{n}+12\times 2^{n-i}+9-2i\}\times \\ 16\{(n+i)^2-5(n+i)+6\}2^{2n}+ \\ \{48(n+i)-112\}2^{2n-i}+32\times 2^{2n-2i}+29-11i+i^2 \\ 4\{21n+32i-5ni-5i^2-46\}2^{n}+\{52-12i\}\,2^{n-i}\}]^{2}\end{gathered}}\right]}$$

Proof: put a = 1,2, in equation (6), we get the desired results.

APPLICATIONS

During the last few years, numerous graph-theoretic methods have been developed for the analysis and prediction of physiochemical, environmental, and bio medicinal properties of molecules.

Use of topological indices in structure activity relationship studies seemsto play an important role in situations where biological activity is determined predominantly by topological architecture of molecular structure. One of the main medical and social problems nowadays is HIV and we are in need of Anti-HIV therapy which is in need of new drugs with less toxic, active against the drug resistant mutants. It was analyzed Weiner index, Zagreb index playeda vital role in solving this problem.

It has been considered as the main source of medicines and during the past two decades thousands of compounds and their metabolites with several different type of biological activity such as anti microbial, anti inflammatory, anti-malarial, antioxidant, anti-HIV, and anti Cancer activity. Study of topological indices helps in acquiring that medicine with less toxic.

The constitutional formula of a molecule is in essence, a planar graph where vertices represent the atoms and edges are the covalent bonds. Sincesuch a graph adequately depicts the topology of the molecule, it is not surprising that the graph- theoretic approaches in explaining the physical and biological properties of diverse groups of chemicals.

In this work we have introduced many topological indices with multiplicative version, edge version, and eccentricity. These indices give the nearer values depicting the physical and biological properties of molecules

CONCLUSION

In this work we have considered some familes of Denderimers namely, (i) Nanostar Dendrimer NS[n] (ii) Nanostar Dendrimer D_n (iii)Nanostar Dendrimer D(n) (iv) Hexogonal core Dendrimer $D_{3,n-1.}$ We have introduced 213 new indices namely Arithmetic, Harmonic, Geometric Arithmetic, Arithmetic Geometric, sum and product connectivity, ABC, Inverse sum, Augumented Zagreb indices etc. and also we define multiplicative and polynomial indices of a graph G. These indices may surely give hands to the chemist in finding in more and more appropriate or nearer values in studying about molecules. In the future, we are interested to study and compute topological indices of multiplicative and polynomial of various families of Dendrimers or nanostructures.

LIST OF PUBLICATIONS JOURNALS

1. P. Gladyis and G. Srividhya, **ON SOME VERTEX CUTTING NUMBER TOPOLOGICAL INDICES OF NANOSTAR DENDRIMER NS[n],** Journal of Sambodhi, Vol-43, No.-03(VIII) 2020, 2249-6661.

2. M. Bhanumathi, P. Gladyis and G. Srividhya, **On some cutting number topological indices of nanostar Dendrimer NS[n]**, Malaya Journal of Matematik, Vol. 8, No. 4, 2177–2185, 2020.**(UGC – Care list – Group I).**

3. P. Gladyis and G. Srividhya, **Certain Topological indices and their polynomials of some cutting number nanostar Dendrimer NS[n],** Malaya Journal of Matematik, Vol. 9, No. 1, 425–430, 2021.**(UGC – Care list – Group I).**

4. P. Gladyis, G. Srividhya and R. Rohini, **The Eccentricity Version of Some Square Reduced Indices of Nanostar Dendrimer D[N],** International Journal of Scientific Engineering and Applied Science – Vol.-7, Issue-5, May 2021, 2395–3470.

5. R. Rohini, P. Gladyis and G. Srividhya, **Eccentricity Based on Topological Indices of Boron Nanotubes,** International Journal of Scientific Engineering and Applied Science – Vol.-7, Issue-5, May 2021, 2395–3470.

6. R. Rohini, P. Gladyis and G. Srividhya, **Some status indices of binary tree graphs,** Advances and Applications in Discrete Mathematics – Vol.-28, Issue-1, Pages115-132(September 2021), 0974–1658.**(UGC – Care list – Group II)(Web of Science).**

7. Gladyis .P, Srividhya .G, and Rohini .R **"ON ZAGREB CUTTING NUMBER TOPOLOGICAL INDICES OF NANOSTAR DENDRIMER D_n"**, in Advances and Applications in Discrete Mathematics, ISSN 0974-1658, Vol.28, No.2, 2021, pages 235-252. **(UGC – Care list – Group II)(Web of Science).**

8. Gladyis .P, Srividhya .G, and Rohini .R **"ON ZAGREB POLYNOMIAL CUTTING NUMBER TOPOLOGICAL INDICES OF NANOSTAR DENDRIMER D_n"**, in Advances and Applications in Discrete Mathematics, P-ISSN 0974-1658, Vol.29, No.2, 2022, pages 169-185. **(UGC – Care list – Group II)(Web of Science).**

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ON ZAGREB POLYNOMIAL CUTTING NUMBER TOPOLOGICAL INDICES OF NANOSTAR DENDRIMER D_n

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Abstract

In this paper, we propose generalized version of the first, second, third, fourth and fifth Zagreb polynomial indices of nanostar dendrimer D_n, which are based on cutting number of the vertices.

1. Introduction

A molecular graph is a graph such that its vertices correspond to the atoms and the edges to the bonds. Chemical graph theory is a branch of Mathematical Chemistry which has an important effect on the development of the chemical sciences. A topological index is a numerical parameter mathematically derived from the graph structure.

A *cutting number* $c(v)$ of a vertex $v \in V(G)$ in a connected graph G is the number of pairs of vertices $\{v, w\}$ such that v and w are in different components of $G - v$ [4].

In this paper, we introduce cutting number based topological indices of graphs. For two-connected graphs, cutting number of each vertex is zero. So, we define these indices, for graphs with cut vertices only.

Aouchiche et al. [1, 2, 5] defined first and the second Zagreb polynomials as

$$M_1(G, x) = \sum_{uv \in E(G)} x^{d_u + d_v},$$

$$M_2(G, x) = \sum_{uv \in E(G)} x^{d_u d_v}.$$

Fath-Tabar [5] defined the third Zagreb polynomial index:

$$M_3(G, x) = \sum_{uv \in E(G)} x^{| d_u - d_v |}.$$

In the year 2016, following Zagreb type polynomials were defined [10]:

$$M_4(G, x) = \sum_{uv \in E(G)} x^{d_u (d_u + d_v)},$$

$$M_5(G, x) = \sum_{uv \in E(G)} x^{d_v(d_u + d_v)}.$$

2. Computing the Zagreb Polynomial Topological Indices of Nanostar Dendrimer D_n Using Cutting Number

2.1. First Zagreb polynomial cutting number index

$$M_{1C}(G, x) = \sum_{uv \in E(G)} x^{c(u)+c(v)},$$

where $c(u)$ and $c(v)$ are the cutting numbers of u and v in G.

2.2. Second Zagreb polynomial cutting number index

$$M_{2C}(G, x) = \sum_{uv \in E(G)} x^{c(u)c(v)},$$

where $c(u)$ and $c(v)$ are the product of the cutting numbers of u and v in G.

2.3. Third Zagreb polynomial cutting number index

$$M_{3C}(G, x) = \sum_{uv \in E(G)} x^{|c(u)-c(v)|},$$

where $c(u)$ and $c(v)$ are the difference of the cutting numbers of u and v in G.

2.4. Fourth Zagreb polynomial cutting number index

$$M_{4C}(G, x) = \sum_{uv \in E(G)} x^{c(u)(c(u)+c(v))},$$

where $c(u)$ and $c(v)$ are the cutting numbers of u and v in G.

2.5. Fifth Zagreb polynomial cutting number index

$$M_{5C}(G, x) = \sum_{uv \in E(G)} x^{c(v)(c(u)+c(v))},$$

where $c(u)$ and $c(v)$ are the cutting numbers of u and v in G.

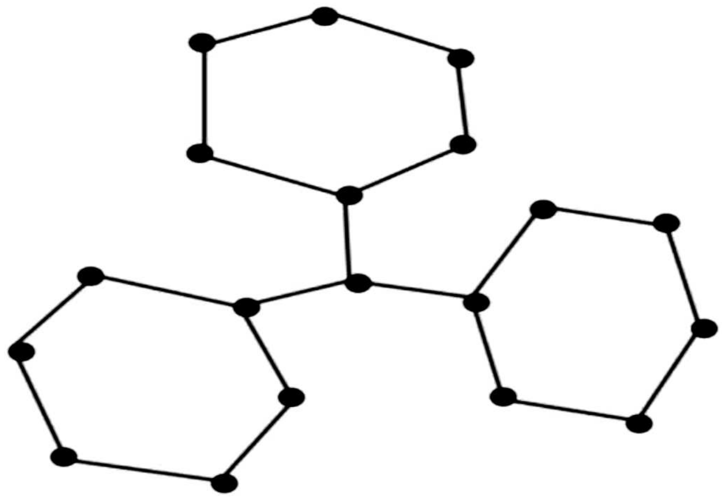

Figure 1. Nanostar dendrimer D_n for $n = 1$.

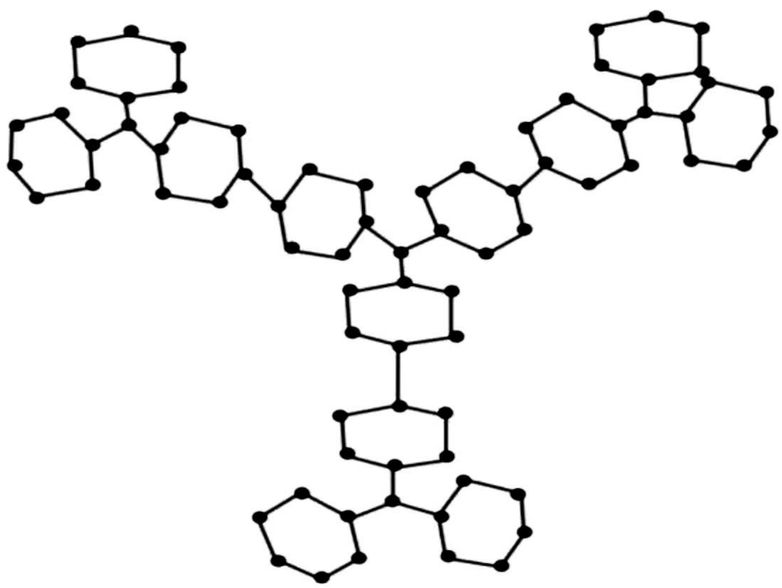

Figure 2. Nanostar dendrimer D_n for $n = 2$.

Table 1

Number of edges $e = uv$	**Cutting number of end vertices** $(c(u),\ c(v))$
3×2^0	$\{(3(19 \times 2^{n-1} - 13)^2, ((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \times (19 \times 2^{n-1} - 13 - 1)\}$
3×2^1	$\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \times (19 \times 2^{n-1} - 13 - 1),\ 0\}$
3×2^1	$(0,\ 0)$
3×2^1	$\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13) + 5) \times (19 \times 2^{n-1} - 13 - 6),\ 0\}$
$\vdots$	$\vdots$
$3 \times 2^{i-2}$	$\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 5) \times (19 \times 2^{n-(i-1)} - 13 - 6),$ $((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 6) \times (19 \times 2^{n-(i-1)} - 13 - 7)\}$
$3 \times 2^{i-1}$	$\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 6) \times (19 \times 2^{n-(i-1)} - 13 - 7),\ 0\}$
$3 \times 2^{i-1}$	$(0,\ 0)$
$3 \times 2^{i-1}$	$\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 11) \times (2(19 \times 2^{n-i} - 13) + 1),\ 0\}$
$3 \times 2^{i-2}$	$\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 11) \times (2(19 \times 2^{n-i} - 13) + 1),$ $((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 12) \times (2(19 \times 2^{n-i} - 13)$ $+ (19 \times 2^{n-i} - 13)^2)\}$
$3 \times 2^{i-1}$	$\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 12) \times (2(19 \times 2^{n-i} - 13)$ $+ (19 \times 2^{n-i} - 13)^2),\ ((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13))$ $\times (19 \times 2^{n-(i-1)} - 13 - 1)\}$
3×2^{i}	$\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-i} - 13)) \times (19 \times 2^{n-i} - 13 - 1),\ 0\}$
3×2^{i}	$(0,\ 0)$
3×2^{i}	$\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-i} - 13) + 5) \times (19 \times 2^{n-i} - 13 - 6),\ 0\}$
$\vdots$	$\vdots$
$3 \times 2^{n-2}$	$\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(n-1)} - 13) + 5) \times (19 \times 2^{n-(n-1)} - 13 - 6),$ $((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(n-1)} - 13) + 6) \times (19 \times 2^{n-(n-1)} - 13 - 7)\}$
$3 \times 2^{n-1}$	$\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(n-1)} - 13) + 6) \times (19 \times 2^{n-(n-1)} - 13 - 7),\ 0\}$
$3 \times 2^{n-1}$	$(0,\ 0)$

$3\times 2^{n-1}$	$\{((57\times 2^{n-1}-38)-(19\times 2^{n-(n-1)}-13)+11)\times(2(19\times 2^{n-n}-13)+1), 0\}$
$3\times 2^{n-2}$	$\{((57\times 2^{n-1}-38)-(19\times 2^{n-(n-1)}-13)+11)\times(2(19\times 2^{n-n}-13)+1),$ $((57\times 2^{n-1}-38)-(19\times 2^{n-(n-1)}-13)+12)\times(2(19\times 2^{n-n}-13)$ $+(19\times 2^{n-n}-13)^2)\}$
$3\times 2^{n-1}$	$\{((57\times 2^{n-1}-38)-(19\times 2^{n-(n-1)}-13)+12)\times(2(19\times 2^{n-n}-13)$ $+(19\times 2^{n-n}-13)^2), ((57\times 2^{n-1}-38)-(19\times 2^{n-n}-13))$ $\times(19\times 2^{n-n}-13-1)\}$
3×2^{n}	$\{((57\times 2^{n-1}-38)-(19\times 2^{n-n}-13))\times(19\times 2^{n-n}-13-1), 0\}$
3×2^{n}	$(0, 0)$
3×2^{n}	$\{((57\times 2^{n-1}-38)-(19\times 2^{n-n}-13)+5)\times(19\times 2^{n-n}-13-6), 0\}$
Total number of edges	$33\times 2^{n}-45$

3. Main Results

3.1. Nanostar dendrimer D_n

Consider the nanostar dendrimer D_n, where n is the defining parameter as illustrated in Figures 1 and 2. The number of vertices in nanostar dendrimer D_n is equal to $|V(D_n)| = 57\times 2^{n-1}-12$ and the number of edges is $|E(D_n)| = 33\times 2^{n}-45.$

3.2. Polynomial version of first, second, third, fourth and fifth Zagreb cutting number topological indices of nanostar dendrimer D_n

We compute polynomial version of first, second, third, fourth and fifth Zagreb polynomial cutting number topological indices of nanostar dendrimer D_n in the following theorems.

Theorem 3.1. *Let* D_n *be the nanostar dendrimer. Then*

$$M_{1C}[D_n, x]$$

$$= 3x^{[1805\times 2^{2n-2}-2489\times 2^{n-1}+857]}$$

$$+ 6x^{[1444\times 2^{2n-2}-2109\times 2^{n-1}+730]}$$

$$+ 3\sum_{i=2}^{n} 2^{i-2} x^{[4332\times 2^{2n-i}-5415\times 2^{2n-2i}+418\times 2^{n-i}-5130\times 2^{n-1}+1617]}$$

$$+ 3\sum_{i=2}^{n} 2^{i-1} x^{\substack{[2166\times 2^{2n-i}-4332\times 2^{2n-2i}+247\times 2^{n-i} \\ -4845\times 2^{n-1}+3249\times 2^{2n-i-1}+1587]}}$$

$$+ 3\sum_{i=2}^{n} 2^{i} x^{[2166\times 2^{2n-i-1}-722\times 2^{2n-2i}-228\times 2^{n-i}-1881\times 2^{n-1}+730]}.$$

Proof. Using Table 1, we can write the first Zagreb polynomial of D_n as follows:

$$M_{1C}[D_n, x]$$

$$= \sum_{uv\in E(D_n)} x^{(c(u)+c(v))}$$

$$= (3\times 2^0)x^{[\{(3(19\times 2^{n-1}-13)^2)\}+\{((57\times 2^{n-1}-38)-(19\times 2^{n-1}-13))\times(19\times 2^{n-1}-13-1)\}]}$$

$$+ (3\times 2^1)x^{[\{((57\times 2^{n-1}-38)-(19\times 2^{n-1}-13))\times(19\times 2^{n-1}-13-1)\}+0]}$$

$$+ (3\times 2^1)x^{[\{((57\times 2^{n-1}-38)-(19\times 2^{n-1}-13)+5)\times(19\times 2^{n-1}-13-6)\}+0]}$$

$$+ 3\sum_{i=2}^{n} 2^{i-2} x^{\substack{[\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+5)\times(19\times 2^{n-(i-1)}-13-6)\} \\ +\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+6)\times(19\times 2^{n-(i-1)}-13-7)\}]}}$$

$$+ 3\sum_{i=2}^{n} 2^{i-1} x^{[\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+6)\times(19\times 2^{n-(i-1)}-13-7)\}+0]}$$

$$+ 3\sum_{i=2}^{n} 2^{i-1} x^{[\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+11)\times(2(19\times 2^{n-i}-13)+1)\}+0]}$$

$$+ 3\sum_{i=2}^{n} 2^{i-2} x^{[\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+11)\times(2(19\times 2^{n-i}-13)+1)\} +\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+12)\times(2(19\times 2^{n-i}-13)) +(19\times 2^{n-i}-13)^2\}]}$$

$$+ 3\sum_{i=2}^{n} 2^{i-1} x^{[\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+12)\times 2(19\times 2^{n-i}-13) +(19\times 2^{n-i}-13)^2\}+\{((57\times 2^{n-1}-38)-(19\times 2^{n-i}-13)) \times(19\times 2^{n-i}-13-1)\}]}$$

$$+ 3\sum_{i=2}^{n} 2^{i} x^{[\{((57\times 2^{n-1}-38)-(19\times 2^{n-i}-13))\times(19\times 2^{n-i}-13-1)\}+0]}$$

$$+ 3\sum_{i=2}^{n} 2^{i} x^{[\{((57\times 2^{n-1}-38)-(19\times 2^{n-i}-13)+5)\times(19\times 2^{n-i}-13-6)\}+0]}.$$

After simplification, we obtain

$$= 3x^{[1805\times 2^{2n-2}-2489\times 2^{n-1}+857]}$$

$$+ 6x^{[1444\times 2^{2n-2}-2109\times 2^{n-1}+730]}$$

$$+ 3\sum_{i=2}^{n} 2^{i-2} x^{[4332\times 2^{2n-1}-5415\times 2^{2n-2i}+418\times 2^{n-i}-5130\times 2^{n-1}+1617]}$$

$$+ 3\sum_{i=2}^{n} 2^{i-1} x^{[2166\times 2^{2n-i}-4332\times 2^{2n-2i}+247\times 2^{n-i}-4845\times 2^{n-1} +3249\times 2^{2n-i-1}+1587]}$$

$$+ 3\sum_{i=2}^{n} 2^{i} x^{[2166\times 2^{2n-i-1}-722\times 2^{2n-2i}-288\times 2^{n-i}-1881\times 2^{n-1}+730]}.$$

Theorem 3.2. *Let* D_n *be the nanostar dendrimer. Then*

$M_{2C}[D_n, x]$

$$= 3x^{[781926\times 2^{4n-4}-2160585\times 2^{3n-3}+2237478\times 2^{2n-2}-1029249\times 2^{n-1}+177450]}$$

$$+\,3\sum_{i=2}^{n} 2^{i-2} x^{\begin{array}{l}[2345778\times 2^{4n-2i}-5864445\times 2^{4n-3i}+5802714\times 2^{3n-2i}-2777895\\ \times 2^{3n-i}+3648988\times 2^{4n-4i}-2210042\times 2^{2n-2i}+1442556\times 2^{2n-i}\\ -422370\times 2^{n}+308655\times 2^{2n}-3806745\times 2^{3n-2i-1}-452694\times 2^{3n-3i}\\ +211926\times 2^{n-i}+1055925\times 2^{2n-1}-1241175\times 2^{n-1}+321850]\end{array}}$$

$$+\,3\sum_{i=2}^{n} 2^{i-1} x^{\begin{array}{l}[1172889\times 2^{4n-2i-1}-781926\times 2^{4n-3i}+473271\times 2^{3n-2i}\\ -1666737\times 2^{3n-i-1}-390963\times 2^{4n-3i-1}+2405343\times 2^{2n-i-1}\\ +390963\times 2^{4n-4i}+226347\times 2^{3n-3i}-105963\times 2^{n-i}\\ +295659\times 2^{2n}-394212\times 2^{2n-i}-461643\times 2^{n}+177450]\end{array}}.$$

Proof. Using Table 1, we can write the second Zagreb polynomial of D_n as follows:

$$M_{2C}[D_n,\ x]$$

$$=\sum_{uv\in E(D_n)} x^{(c(u)c(v))}$$

$$=(3\times 2^{0})\,x^{[\{(3(19\times 2^{n-1}-13)^{2})\}+((57\times 2^{n-1}-38)-(19\times 2^{n-1}-13))\times(19\times 2^{n-1}-13-1)]}$$

$$+\,(3\times 2^{1})\,x^{[((57\times 2^{n-1}-38)-(19\times 2^{n-1}-13))\times(19\times 2^{n-1}-13-1)\times 0]}$$

$$+\,(3\times 2^{1})\,x^{[((57\times 2^{n-1}-38)-(19\times 2^{n-1}-13)+5)\times(19\times 2^{n-1}-13-6)\times 0]}$$

$$+\,3\sum_{i=2}^{n} 2^{i-2} x^{\begin{array}{l}[\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+5)\times(19\times 2^{n-(i-1)}-13-6)\}\\ \times\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+6)\times(19\times 2^{n-(i-1)}-13-7)\}]\end{array}}$$

$$+\,3\sum_{i=2}^{n} 2^{i-1} x^{[((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+6)\times(19\times 2^{n-(i-1)}-13-7)\times 0]}$$

$$+\,3\sum_{i=2}^{n} 2^{i-1} x^{[((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+11)\times(2(19\times 2^{n-i}-13)+1)\times 0]}$$

$$+\,3\sum_{i=2}^{n} 2^{i-2} x^{\begin{array}{l}[\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+11)\times(2(19\times 2^{n-i}-13)+1)\}\\ \times\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+12)\times(2(19\times 2^{n-i}-13))\\ +(19\times 2^{n-i}-13)^{2}\}]\end{array}}$$

$$+ 3\sum_{i=2}^{n} 2^{i-1} x^{[\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+12)\times 2(19\times 2^{n-i}-13) + (19\times 2^{n-i}-13)^2\}\times\{((57\times 2^{n-1}-38)-(19\times 2^{n-i}-13))\times(19\times 2^{n-i}-13-1)\}]}$$

$$+ 3\sum_{i=2}^{n} 2^{i} x^{[((57\times 2^{n-1}-38)-(19\times 2^{n-i}-13))\times(19\times 2^{n-i}-13-1)\times 0]}$$

$$+ 3\sum_{i=2}^{n} 2^{i} x^{[((57\times 2^{n-1}-38)-(19\times 2^{n-i}-13)+5)\times(19\times 2^{n-i}-13-6)\times 0]}.$$

After simplification, we get

$$M_{2C}[D_n, x]$$

$$= 3x^{[781926\times 2^{4n-4}-2160585\times 2^{3n-3}+2237478\times 2^{2n-2}-1029249\times 2^{n-1}+177450]}$$

$$+ 3\sum_{i=2}^{n} 2^{i-2} x^{[2345778\times 2^{4n-2i}-5864445\times 2^{4n-3i}+5802714\times 2^{3n-2i}-2777895\times 2^{3n-i}+3648988\times 2^{4n-4i}-2210042\times 2^{2n-2i}+1442556\times 2^{2n-i}-422370\times 2^{n}+308655\times 2^{2n}-3806745\times 2^{3n-2i-1}-452694\times 2^{3n-3i}+211926\times 2^{n-i}+1055925\times 2^{2n-1}-1241175\times 2^{n-1}+321850]}$$

$$+ 3\sum_{i=2}^{n} 2^{i-1} x^{[1172889\times 2^{4n-2i-1}-781926\times 2^{4n-3i}+473271\times 2^{3n-2i}-1666737\times 2^{3n-i-1}-390963\times 2^{4n-3i-1}+2405343\times 2^{2n-i-1}+390963\times 2^{4n-4i}+226347\times 2^{3n-3i}-105963\times 2^{n-i}+295659\times 2^{2n}-394212\times 2^{2n-i}-461643\times 2^{n}+177450]}.$$

Theorem 3.3. *Let* D_n *be the nanostar dendrimer. Then*

$$M_{3C}[D_n, x]$$

$$= 3x^{(361\times 2^{2n-2}-475\times 2^{n-1}+157)}$$

$$+ 6x^{(1444\times 2^{2n-2}-2109\times 2^{n-1}+730)}$$

$$+ \sum_{i=2}^{n} 2^{i-2} x^{[342\times 2^{n-i}-361\times 2^{2n-2i}+114\times 2^{n-1}-157]}$$

$$+\,3\sum_{i=2}^{n} 2^{i-1} x^{\substack{[2166\times 2^{2n-i}-3610\times 2^{2n-2i}+665\times 2^{n-i} \\ -3249\times 2^{n-1}+1083\times 2^{2n-i-1}+887]}}$$

$$+\,3\sum_{i=2}^{n} 2^{i} x^{[2166\times 2^{2n-i-1}-722\times 2^{2n-2i}-228\times 2^{n-i}-1881\times 2^{n-1}+730]}.$$

Proof. Using Table 1, we can write the third Zagreb polynomial of D_n as follows:

$$M_{3C}[D_n, x]$$

$$= \sum_{uv\in E(D_n)} x^{|\,c(u)-c(v)\,|}$$

$$= (3\times 2^{0})\, x^{|\,[(3(19\times 2^{n-1}-13)^{2})-((57\times 2^{n-1}-38)-(19\times 2^{n-1}-13))\times(19\times 2^{n-1}-13-1)]\,|}$$

$$+\,(3\times 2^{1})\, x^{|\,[((57\times 2^{n-1}-38)-(19\times 2^{n-1}-13))\times(19\times 2^{n-1}-13-1)\times 0]\,|}$$

$$+\,(3\times 2^{1})\, x^{|\,[((57\times 2^{n-1}-38)-(19\times 2^{n-1}-13)+5)\times(19\times 2^{n-1}-13-6)-0]\,|}$$

$$+\,3\sum_{i=2}^{n} 2^{i-2} x^{\substack{|\,[\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+5)\times(19\times 2^{n-(i-1)}-13-6)\} \\ -\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+6)\times(19\times 2^{n-(i-1)}-13-7)\}]\,|}}$$

$$+\,3\sum_{i=2}^{n} 2^{i-1} x^{|\,[((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+6)\times(19\times 2^{n-(i-1)}-13-7)-0]\,|}$$

$$+\,3\sum_{i=2}^{n} 2^{i-1} x^{\substack{|\,[((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+11) \\ \times(2(19\times 2^{n-(i-1)}-13)+1)-0]\,|}}$$

$$+\,3\sum_{i=2}^{n} 2^{i-2} x^{\substack{|\,[\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+11)\times(2(19\times 2^{n-i}-13)+1)\} \\ -\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+12)\times(2(19\times 2^{n-i}-13)) \\ +(19\times 2^{n-i}-13)^{2}\}]\,|}}$$

$$+3\sum_{i=2}^{n} 2^{i-1} x^{\begin{array}{l}|[\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+12)\times 2(19\times 2^{n-i}-13)\\ +(19\times 2^{n-i}-13)^2\}-\{((57\times 2^{n-1}-38)-(19\times 2^{n-i}-13))\\ \times(19\times 2^{n-i}-13-1)\}]|\end{array}}$$

$$+3\sum_{i=2}^{n} 2^{i} x^{|[((57\times 2^{n-1}-38)-(19\times 2^{n-i}-13))\times(19\times 2^{n-i}-13-1)-0]|}$$

$$+3\sum_{i=2}^{n} 2^{i} x^{|[((57\times 2^{n-1}-38)-(19\times 2^{n-i}-13)+5)\times(19\times 2^{n-i}-13-6)-0]|}.$$

After some calculation, we obtain

$$M_{3C}[D_n,\ x]$$

$$= 3x^{(361\times 2^{2n-2}-475\times 2^{n-1}+157)}$$

$$+ 6x^{(1444\times 2^{2n-2}-2109\times 2^{n-1}+730)}$$

$$+ \sum_{i=2}^{n} 2^{i-2} x^{[342\times 2^{n-i}-361\times 2^{2n-2i}+114\times 2^{n-1}-157]}$$

$$+ 3\sum_{i=2}^{n} 2^{i-1} x^{\begin{array}{l}[2166\times 2^{2n-i}-3610\times 2^{2n-2i}+665\times 2^{n-i}\\ -3249\times 2^{n-1}+1083\times 2^{2n-i-1}+887]\end{array}}$$

$$+ 3\sum_{i=2}^{n} 2^{i} x^{[2166\times 2^{2n-i-1}-722\times 2^{2n-2i}-228\times 2^{n-i}-1881\times 2^{n-1}+730]}.$$

Theorem 3.4. *Let* D_n *be the nanostar dendrimer. Then*

$$M_{4C}[D_n,\ x]$$

$$= 3x^{[1954815\times 2^{4n-4}-5370597\times 2^{3n-3}+5531964\times 2^{2n-2}-2531997\times 2^{n-1}+434499]}$$

$$+ 6x^{[1042568\times 2^{4n-4}-3045396\times 2^{3n-3}+3282573\times 2^{2n-2}-1542420\times 2^{n-1}+266900]}$$

$$+ 3\sum_{i=2}^{n} 2^{i-2} x^{\begin{array}{l}[46911556\times 2^{4n-2i}-12119853\times 2^{4n-3i}+7819260\times 2^{4n-4i}\\ -6111369\times 2^{3n-2i-1}-5494059\times 2^{3n-i}+11399658\times 2^{3n-2i}\\ -1550134\times 2^{3n-3i}-4142114\times 2^{2n-2i}+2469240\times 2^{2n-i}\\ +1637496\times 2^{2n}-3906495\times 2^{n-1}+475646\times 2^{n-i}+588750]\end{array}}$$

$$+3\sum_{i=2}^{n} 2^{i-1} x^{\substack{[2345778\times 2^{4n-2i}+3518667\times 2^{4n-2i-1}-10165038\times 2^{4n-3i} \\ +6769833\times 2^{3n-2i}-5494059\times 2^{3n-i}+11399658\times 2^{3n-2i} \\ -1550134\times 2^{3n-3i}-4142114\times 2^{2n-2i}+2469240\times 2^{2n-i} \\ +1637496\times 2^{2n}-3906495\times 2^{n-1}+475646\times 2^{n-i}+588750]}}$$

$$+3\sum_{i=2}^{n} 2^{i} x^{[2166\times 2^{2n-i-1}-722\times 2^{2n-2i}-288\times 2^{n-i}-1881\times 2^{n-1}+730]}.$$

Proof. Using Table 1, we can write the fourth Zagreb polynomial of D_n as follows:

$$M_{4C}[D_n,\ x]$$

$$= \sum_{uv\in E(D_n)} c(u)[c(u)+c(v)]$$

$$= (3\times 2^{0})\, x^{\substack{[(3(19\times 2^{n-1}-13)^{2})\{3(19\times 2^{n-1}-13)^{2}+((57\times 2^{n-1}-38) \\ -(19\times 2^{n-1}-13))\times(19\times 2^{n-1}-13-1)\}]}}$$

$$+ (3\times 2^{1})\, x^{\substack{[\{((57\times 2^{n-1}-38)-(19\times 2^{n-1}-13))\times(19\times 2^{n-1}-13-1)\} \\ \{((57\times 2^{n-1}-38)-(19\times 2^{n-1}-13))\times(19\times 2^{n-1}-13-1)+0\}]}}$$

$$+ (3\times 2^{1})\, x^{\substack{[\{((57\times 2^{n-1}-38)-(19\times 2^{n-1}-13)+5)\times(19\times 2^{n-1}-13-6)\} \\ \{((57\times 2^{n-1}-38)-(19\times 2^{n-1}-13)+5)\times(19\times 2^{n-1}-13-6)+0\}]}}$$

$$+3\sum_{i=2}^{n} 2^{i-2} x^{\substack{[\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+5)\times(19\times 2^{n-(i-1)}-13-6)\} \\ \times\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+5)(19\times 2^{n-(i-1)}-13-6) \\ +((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+6)\times(19\times 2^{n-(i-1)}-13-7)\}]}}$$

$$+3\sum_{i=2}^{n} 2^{i-1} x^{\substack{[\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+6)\times(19\times 2^{n-(i-1)}-13-7)\} \\ \{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+6)\times(19\times 2^{n-(i-1)}-13-7)+0\}]}}$$

$$+3\sum_{i=2}^{n} 2^{i-1} x^{\substack{[\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+11)\times(2(19\times 2^{n-i}-13)+1)\} \\ \{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+11)\times(2(19\times 2^{n-i}-13)+1)+0\}]}}$$

$$+\,3\sum_{i=2}^{n} 2^{i-2}\, x^{\begin{array}{l}[\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+11)\times(2(19\times 2^{n-i}-13)+1)\}\\ +\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+11)\times(2(19\times 2^{n-i}-13)+1)\\ +((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+12)\times 2(19\times 2^{n-i}-13))\\ +(19\times 2^{n-i}-13)^{2}\}]\end{array}}$$

$$+\,3\sum_{i=2}^{n} 2^{i-1}\, x^{\begin{array}{l}[\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+12)\times 2(19\times 2^{n-i}-13)\\ +(19\times 2^{n-i}-13)^{2}\}\times\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+12)\\ \times 2(19\times 2^{n-i}-13)^{2}\}+\{((57\times 2^{n-1}-38)-(19\times 2^{n-i}-13))\\ \times(19\times 2^{n-i}-13-1)\}]\end{array}}$$

$$+\,3\sum_{i=2}^{n} 2^{i}\, x^{\begin{array}{l}[((57\times 2^{n-1}-38)-(19\times 2^{n-i}-13))\times(19\times 2^{n-i}-13-1)\\ ((57\times 2^{n-1}-38)-(19\times 2^{n-i}-13))\times(19\times 2^{n-i}-13-1+0)]\end{array}}$$

$$+\,3\sum_{i=2}^{n} 2^{i}\, x^{\begin{array}{l}[((57\times 2^{n-1}-38)-(19\times 2^{n-i}-13)+5)\times(19\times 2^{n-i}-13-6)\\ ((57\times 2^{n-1}-38)-(19\times 2^{n-i}-13)+5)\times(19\times 2^{n-i}-13-6)+0]\end{array}}.$$

After an easy simplification, we get

$$= 3x^{[1954815\times 2^{4n-4}-5370597\times 2^{3n-3}+5531964\times 2^{2n-2}-2531997\times 2^{n-1}+434499]}$$

$$+\,6x^{[1042568\times 2^{4n-4}-3045396\times 2^{3n-3}+3282573\times 2^{2n-2}-1542420\times 2^{n-1}+266900]}$$

$$+\,3\sum_{i=2}^{n} 2^{i-2}\, x^{\begin{array}{l}[46911556\times 2^{4n-2i}-12119853\times 2^{4n-3i}+7819260\times 2^{4n-4i}\\ -6111369\times 2^{3n-2i-1}-5494059\times 2^{3n-i}+11399658\times 2^{3n-2i}\\ -1550134\times 2^{3n-3i}-4142114\times 2^{2n-2i}+2469240\times 2^{2n-i}\\ +1637496\times 2^{2n}-3906495\times 2^{n-1}+475646\times 2^{n-i}+588750]\end{array}}$$

$$+\,3\sum_{i=2}^{n} 2^{i-1}\, x^{\begin{array}{l}[2345778\times 2^{4n-2i}+3518667\times 2^{4n-2i-1}-10165038\times 2^{4n-3i}\\ +6769833\times 2^{3n-2i}-5494059\times 2^{3n-i}+11399658\times 2^{3n-2i}\\ -1550134\times 2^{3n-3i}-4142114\times 2^{2n-2i}+2469240\times 2^{2n-i}\\ +1637496\times 2^{2n}-3906495\times 2^{n-1}+475646\times 2^{n-i}+588750]\end{array}}$$

$$+\,3\sum_{i=2}^{n} 2^{i}\, x^{[2166\times 2^{2n-i-1}-722\times 2^{2n-2i}-288\times 2^{n-i}-1881\times 2^{n-1}+730]}.$$

Theorem 3.5. *Let* D_n *be the nanostar dendrimer*. *Then*

$$M_{5C}[D_n,\, x]$$

$$= 3x^{[1303210\times 2^{4n-4}-3614693\times 2^{3n-3}+3756927\times 2^{2n-2}-1734149\times 2^{n-1}+299950]}$$

$$+ 3\sum_{i=2}^{n} 2^{i-2} x^{\begin{array}{l}[4691556\times 2^{4n-2i}-11337927\times 2^{4n-3i}+6907013\times 2^{4n-4i}\\ -9156765\times 2^{3n-2i-1}-5617521\times 2^{3n-i}+11811198\times 2^{3n-2i}\\ -562438\times 2^{3n-3i}-4404200\times 2^{2n-2i}+3320478\times 2^{2n-i}\\ +633555\times 2^{2n}-855570\times 2^{n}+4308174\times 2^{2n-2}\\ -2743923\times 2^{n-1}+240806\times 2^{n-i}+72399]\end{array}}$$

$$+ 3\sum_{i=2}^{n} 2^{i-1} x^{\begin{array}{l}[3518667\times 2^{4n-2i-2}-781926\times 2^{4n-3i}+535002\times 2^{3n-2i}\\ -617310\times 2^{3n-i}+928131\times 2^{2n-i-1}-1172889\times 2^{4n-3i-1}\\ +377245\times 2^{3n-3i}+521284\times 2^{4n-4i}-771096\times 2^{2n-2i}\\ -252263\times 2^{n-i}+890226\times 2^{2n-i}+454860\times 2^{2n}\\ -740943\times 2^{n}+299950]\end{array}}$$

$$+ 12 + 36(2^{n-1} - 1).$$

Proof. Using Table 1, we can write the fifth Zagreb polynomial of D_n as follows:

$$M_{5C}[D_n,\, x]$$

$$= \sum_{uv\in E(D_n)} c(v)[c(u) + c(v)]$$

$$= (3\times 2^0)\, x^{\begin{array}{l}[\{((57\times 2^{n-1}-38)-(19\times 2^{n-1}-13))\times(19\times 2^{n-1}-13-1)\}\\ \{3(19\times 2^{n-1}-13)^2+((57\times 2^{n-1}-38)-(19\times 2^{n-1}-13))\times(19\times 2^{n-1}-13-1)\}]\end{array}}$$

$$+ (3\times 2^1)\, x^0 + (3\times 2^1)\, x^0$$

$$+ 3\sum_{i=2}^{n} 2^{i-2} x^{\begin{array}{l}[\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+6)\times(19\times 2^{n-(i-1)}-13-7)\}\\ \times\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+5)\times(19\times 2^{n-(i-1)}-13-6)\\ +((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+6)\times(19\times 2^{n-(i-1)}-13-7)\}]\end{array}}$$

$$+ \sum_{i=2}^{n} 2^{i-1} x^0 + 3\sum_{i=2}^{n} 2^{i-1} x^0$$

$$+\,3\sum_{i=2}^{n} 2^{i-2}\,x^{\substack{[\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+12)\times(2(19\times 2^{n-i}-13)) \\ +(19\times 2^{n-i}-13)^2\}\times\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+11) \\ \times(2(19\times 2^{n-i}-13)+1)\}+\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+12) \\ \times(2(19\times 2^{n-i}-13))+(19\times 2^{n-i}-13)^2\}]}}$$

$$+\,3\sum_{i=2}^{n} 2^{i-1}\,x^{\substack{[\{((57\times 2^{n-1}-38)-(19\times 2^{n-i}-13))\times(19\times 2^{n-i}-13-1)\} \\ \times\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+12)\times 2(19\times 2^{n-i}-13) \\ +(19\times 2^{n-i}-13)^2+\{((57\times 2^{n-1}-38)-(19\times 2^{n-i}-13)) \\ \times(19\times 2^{n-i}-13-1)\}]}}$$

$$+\sum_{i=2}^{n} 2^{i}x^{0}+3\sum_{i=2}^{n} 2^{i}x^{0}.$$

After simplification, we obtain

$$M_{5C}[D_n,\ x]$$

$$=3x^{[1303210\times 2^{4n-4}-3614693\times 2^{3n-3}+3756927\times 2^{2n-2}-1734149\times 2^{n-1}+299950]}$$

$$+\,3\sum_{i=2}^{n} 2^{i-2}\,x^{\substack{[4691556\times 2^{4n-2i}-11337927\times 2^{4n-3i}+6907013\times 2^{4n-4i} \\ -9156765\times 2^{3n-2i-1}-5617521\times 2^{3n-i}+11811198\times 2^{3n-2i} \\ -562438\times 2^{3n-3i}-4404200\times 2^{2n-2i}+3320478\times 2^{2n-i} \\ +633555\times 2^{2n}-855570\times 2^{n}+4308174\times 2^{2n-2} \\ -2743923\times 2^{n-1}+240806\times 2^{n-i}+72399]}}$$

$$+\,3\sum_{i=2}^{n} 2^{i-1}\,x^{\substack{[3518667\times 2^{4n-2i-2}-781926\times 2^{4n-3i}+535002\times 2^{3n-2i} \\ -617310\times 2^{3n-i}+928131\times 2^{2n-i-1}-1172889\times 2^{4n-3i-1} \\ +377245\times 2^{3n-3i}+521284\times 2^{4n-4i}-771096\times 2^{2n-2i} \\ -252263\times 2^{n-i}+890226\times 2^{2n-i}+454860\times 2^{2n} \\ -740943\times 2^{n}+299950]}}$$

$$+\,12+36(2^{n-1}-1).$$

4. Conclusion

We computed first, second, third, fourth and fifth Zagreb polynomial topological indices of nanostar dendrimer D_n.

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ON ZAGREB CUTTING NUMBER TOPOLOGICAL INDICES OF NANOSTAR DENDRIMER D_n

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Abstract

In this manuscript, we have computed first, second, third, fourth and fifth cutting number Zagreb indices of nanostar dendrimer D_n.

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1. Introduction

In this paper, we consider only simple and connected graph G with vertex set $V(G)$ and edge set $E(G)$. The degree of a vertex $v \in V(G)$ is the number of lines incident to v and denoted by d_v. The concept of "topological index" was first proposed by Hosoya for the characterizing topological nature of a graph. Topological indices are the mathematical measures which correspond to the structures of any simple finite graph. The topological indices correlate certain physicochemical properties such as boiling point, stability of chemical compounds. They are invariant under the graph isomorphism. The significance of topological indices is usually associated with quantitative structures property relationship (QSPR) and quantitative structure activity relationship (QSAR).

A molecular graph is a representation of the structural formula of a chemical compound in terms of graph theory, whose vertices correspond to the atoms of the compound and edges correspond to chemical bonds. In the chemical literature, several dozens of vertex-degree-based topological indices have been and are currently considered and applied in QSPR/QSAR studies.

A cutting number [3], $c(v)$ of a vertex $v \in V(G)$ in a connected graph G is the number of pairs of points $\{v, w\}$ such that v and w are in different components of $G - v$. In this paper, we introduce cutting number based topological indices of graphs. For two-connected graphs, cutting number of each node is zero. So, we define these indices, for graphs with cut vertices only.

In our previous research papers [2] and [5], we introduced first, second and third cutting number Zagreb indices. On connection with those Zagreb indices, we introduce fourth and fifth cutting number Zagreb indices in the present research paper.

One of the oldest and well known topological indices are the first and second Zagreb indices, first introduced by Gutman and Trinajstić in [6], and it is defined as

$$M_1(G) = \sum_{v \in V} d_G(v)^2 = \sum_{uv \in E(G)} (d_u + d_v),$$

$$M_2(G) = \sum_{uv \in E(G)} (d_u d_v).$$

In [6], Gutman and Trinajstić introduced the second Zagreb index:

$$M_2(G) = \sum_{uv \in E(G)} (d_u d_v).$$

Astaneh-Asl and Fath-Tabar [1] defined the third Zagreb index

$$M_3(G) = \sum_{uv \in E(G)} | d_u - d_v |.$$

In [6], Gutman and Trinajstić introduced Zagreb type polynomials defined as

$$M_4(G, x) = \sum_{uv \in E(G)} x^{d_u(d_u + d_v)},$$

$$M_5(G, x) = \sum_{uv \in E(G)} x^{d_v(d_u + d_v)}.$$

2. Cutting Number Topological Indices of Nanostar Dendrimer D_n

In this section, we define fourth Zagreb cutting number index and fifth Zagreb cutting number index.

2.1. Fourth Zagreb (M_{4C}) cutting number index

$$M_{4C}(G) = \sum_{uv \in E(G)} c(u)(c(u) + c(v)),$$

where $c(u)$ and $c(v)$ are the cutting numbers of *u* and *v* in *G*.

2.2. Fifth Zagreb (M_{5C}) cutting number index

$$M_{5C}(G) = \sum_{uv \in E(G)} c(v)(c(u) + c(v)),$$

where $c(u)$ and $c(v)$ are the cutting numbers of *u* and *v* in *G*.

2.3. Nanostar dendrimer D_n

Dendrimers are recognized as one of the major commercially available nano scale building blocks, large and complex molecules with very well defined chemical structure. From polymer chemistry point of view, dendrimers are nearly perfect mono disperse macromolecules which are regular and highly branched three dimensional architecture. They consist of three major architectural components: core, branches and end groups. The nanostar dendrimer is a part of a new group of macro particles that appear to be photon funnels just like artificial antennas. These macro molecules and more precisely those containing phosphorus are used in the formation of nano tubes, micro and macro capsules, nanolatex, colored glasses, chemical sensors, modified electrodes and so on.

Here, we consider nanostar dendrimer D_n, where n is the defining parameter as illustrated in Figures 1 and 2. The number of nodes in nanostar dendrimer D_n is equal to $|V(D_n)| = 57 \times 2^{n-1} - 12$ and the number of links is $|E(D_n)| = 33 \times 2^n - 45$.

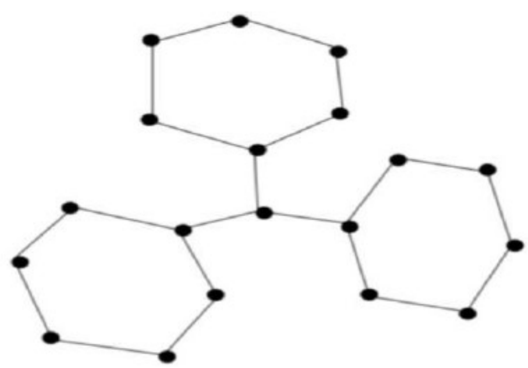

Figure 1. Nanostar dendrimer D_n for $n = 1$.

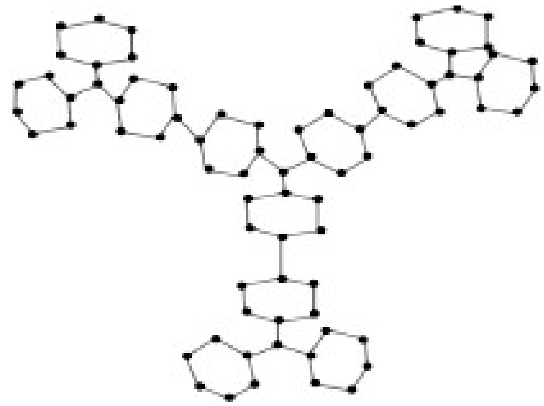

Figure 2. Nanostar dendrimer D_n for $n = 2$.

Table 1. The number of edges of D_n with pairs of cutting numbers of end-vertices

Number of edges $e = uv$	Cutting number of end vertices $(c(u), c(v))$
3×2^0	$\{3(19 \times 2^{n-1} - 13)^2, ((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \times (19 \times 2^{n-1} - 13 - 1)\}$
3×2^1	$\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \times (19 \times 2^{n-1} - 13 - 1), 0\}$
3×2^1	$(0, 0)$
3×2^1	$\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13) + 5) \times (19 \times 2^{n-1} - 13 - 6), 0\}$
$\vdots$	$\vdots$
$3 \times 2^{i-2}$	$\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 5) \times (19 \times 2^{n-(i-1)} - 13 - 6), ((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 6) \times (19 \times 2^{n-(i-1)} - 13 - 7)\}$
$3 \times 2^{i-1}$	$\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 6) \times (19 \times 2^{n-(i-1)} - 13 - 7), 0\}$
$3 \times 2^{i-1}$	$(0, 0)$
$3 \times 2^{i-1}$	$\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 11) \times (2(19 \times 2^{n-i} - 13) + 1), 0\}$
$3 \times 2^{i-2}$	$\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 11) \times (2(19 \times 2^{n-i} - 13) + 1), ((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 12) \times 2(19 \times 2^{n-i} - 13) + (19 \times 2^{n-i} - 13)^2\}$

$3\times 2^{i-1}$	$\{((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13)+12)\times 2(19\times 2^{n-i}-13)$ $+(19\times 2^{n-i}-13)^2,\ ((57\times 2^{n-1}-38)-(19\times 2^{n-(i-1)}-13))$ $\times(19\times 2^{n-(i-1)}-13-1)\}$
3×2^{i}	$\{((57\times 2^{n-1}-38)-(19\times 2^{n-i}-13))\times(19\times 2^{n-i}-13-1),\ 0\}$
3×2^{i}	$(0,\ 0)$
3×2^{i}	$\{((57\times 2^{n-1}-38)-(19\times 2^{n-i}-13)+5)\times(19\times 2^{n-i}-13-6),\ 0\}$
$\vdots$	$\vdots$
$3\times 2^{n-2}$	$\{((57\times 2^{n-1}-38)-(19\times 2^{n-(n-1)}-13)+5)\times(19\times 2^{n-(n-1)}-13-6),$ $((57\times 2^{n-1}-38)-(19\times 2^{n-(n-1)}-13)+6)\times(19\times 2^{n-(n-1)}-13-7)\}$
$3\times 2^{n-1}$	$\{((57\times 2^{n-1}-38)-(19\times 2^{n-(n-1)}-13)+6)$ $\times(19\times 2^{n-(n-1)}-13-7),\ 0\}$
$3\times 2^{n-1}$	$(0,\ 0)$
$3\times 2^{n-1}$	$\{((57\times 2^{n-1}-38)-(19\times 2^{n-(n-1)}-13)+11)$ $\times(2(19\times 2^{n-n}-13)+1),\ 0\}$
$3\times 2^{n-2}$	$\{((57\times 2^{n-1}-38)-(19\times 2^{n-(n-1)}-13)+11)\times(2(19\times 2^{n-n}-13)+1),$ $((57\times 2^{n-1}-38)-(19\times 2^{n-(n-1)}-13)+12)\times 2(19\times 2^{n-n}-13)$ $+(19\times 2^{n-n}-13)^2\}$
$3\times 2^{n-1}$	$\{((57\times 2^{n-1}-38)-(19\times 2^{n-(n-1)}-13)+12)\times 2(19\times 2^{n-n}-13)$ $+(19\times 2^{n-n}-13)^2,\ ((57\times 2^{n-1}-38)-(19\times 2^{n-n}-13))$ $\times(19\times 2^{n-n}-13-1)\}$
3×2^{n}	$\{((57\times 2^{n-1}-38)-(19\times 2^{n-n}-13))\times(19\times 2^{n-n}-13-1),\ 0\}$
3×2^{n}	$(0,\ 0)$
3×2^{n}	$\{((57\times 2^{n-1}-38)-(19\times 2^{n-n}-13)+5)\times(19\times 2^{n-n}-13-6),\ 0\}$

3. Main Results

In this section, we obtain the exact values for first, second, third, fourth and fifth Zagreb cutting indices for nanostar dendrimer D_n.

Theorem 3.1. *Let* D_n *be the nanostar dendrimer. Then*

$$M_{1C}[D_n] = \left[\frac{48735}{4}\right] \times n \times 2^{2n} - \left[\frac{254277}{8}\right] \times 2^{2n}$$

$$+ \left[\frac{190989}{4}\right] \times 2^{n} - 16182.$$

Proof. From Table 1, we compute the first Zagreb cutting index of the nanostar dendrimer D_n as follows:

$$\begin{aligned}
M_{1C}[D_n] &= \sum\nolimits_{uv \in E(D_n)} (c(u) + c(v)) \\
&= (3 \times 2^0)[(3(19 \times 2^{n-1} - 13)^2 + ((57 \times 2^{n-1} - 38) \\
&\quad - (19 \times 2^{n-1} - 13)) \times (19 \times 2^{n-1} - 13 - 1)] \\
&\quad + (3 \times 2^1)[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \\
&\quad \times (19 \times 2^{n-1} - 13 - 1) + 0] \\
&\quad + (3 \times 2^1)[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13) + 5) \\
&\quad \times (19 \times 2^{n-1} - 13 - 6) + 0] \\
&\quad + 3\sum\nolimits_{i=2}^{n} 2^{i-2}[\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 5) \\
&\quad \times (19 \times 2^{n-(i-1)} - 13 - 6)\} + \{((57 \times 2^{n-1} - 38) \\
&\quad - (19 \times 2^{n-(i-1)} - 13) + 6) \times (19 \times 2^{n-(i-1)} - 13 - 7)\}] \\
&\quad + 3\sum\nolimits_{i=2}^{n} 2^{i-1}[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 6) \\
&\quad \times (19 \times 2^{n-(i-1)} - 13 - 7) + 0] \\
&\quad + 3\sum\nolimits_{i=2}^{n} 2^{i-1}[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 11) \\
&\quad \times (2(19 \times 2^{n-(i-1)} - 13) + 1) + 0]
\end{aligned}$$

$$+ 3\sum_{i=2}^{n} 2^{i-2}[\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 11)$$

$$\times (2(19 \times 2^{n-i} - 13) + 1)\} + \{((57 \times 2^{n-1} - 38)$$

$$- (19 \times 2^{n-(i-1)} - 13) + 12)$$

$$\times 2(19 \times 2^{n-i} - 13) + (19 \times 2^{n-(i-1)} - 13)^2\}]$$

$$+ 3\sum_{i=2}^{n} 2^{i-1}[\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 12)$$

$$\times 2(19 \times 2^{n-i} - 13) + (19 \times 2^{n-i} - 13)^2\}$$

$$+ \{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-i} - 13)) \times (19 \times 2^{n-i} - 13 - 1)\}]$$

$$+ 3\sum_{i=2}^{n} 2^{i}[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-i} - 13))$$

$$\times (19 \times 2^{n-i} - 13 - 1) + 0]$$

$$+ 3\sum_{i=2}^{n} 2^{i}[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-i} - 13) + 5)$$

$$\times (19 \times 2^{n-i} - 13 - 6) + 0].$$

After simplification, we get

$$M_{1C}[D_n] = \left[\frac{48735}{4}\right] \times n \times 2^{2n} - \left[\frac{254277}{8}\right] \times 2^{2n}$$

$$+ \left[\frac{190989}{4}\right] \times 2^{n} - 16182.$$

Theorem 3.2. *Let* D_n *be the nanostar dendrimer. Then*

$$M_{2C}[D_n] = \left[\frac{116116011}{112}\right] \times 2^{4n} + \left[\frac{61493823}{16}\right] \times 2^{3n} - \left[\frac{86236725}{8}\right] \times 2^{2n}$$

$$+ \left[\frac{103542435}{14}\right] \times 2^{n} + \left[\frac{9178435}{4}\right] \times n \times 2^{2n}$$

$$- 3333474 \times n \times 2^{3n} - 1497900.$$

Proof. From Table 1, we derive the second Zagreb cutting index of the nanostar dendrimer D_n as follows:

$$
\begin{aligned}
M_{2C}[D_n] = {} & \sum_{uv \in E(D_n)} (c(u)c(v)) \\
= {} & (3 \times 2^0)[(3(19 \times 2^{n-1} - 13)^2 \times ((57 \times 2^{n-1} - 38) \\
& - (19 \times 2^{n-1} - 13)) \times (19 \times 2^{n-1} - 13 - 1)] \\
& + (3 \times 2^1)[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \\
& \times (19 \times 2^{n-1} - 13 - 1) \times 0] \\
& + (3 \times 2^1)[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13) + 5) \\
& \times (19 \times 2^{n-1} - 13 - 6) \times 0] \\
& + 3\sum_{i=2}^{n} 2^{i-2}[\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 5) \\
& \times (19 \times 2^{n-(i-1)} - 13 - 6)\} \times \{((57 \times 2^{n-1} - 38) \\
& - (19 \times 2^{n-(i-1)} - 13) + 6) \times (19 \times 2^{n-(i-1)} - 13 - 7)\}] \\
& + 3\sum_{i=2}^{n} 2^{i-1}[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 6) \\
& \times (19 \times 2^{n-(i-1)} - 13 - 7) \times 0] \\
& + 3\sum_{i=2}^{n} 2^{i-1}[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 11) \\
& \times (2(19 \times 2^{n-i} - 13) + 1) \times 0] \\
& + 3\sum_{i=2}^{n} 2^{i-2}[\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 11) \\
& \times (2(19 \times 2^{n-i} - 13) + 1)\} \times \{((57 \times 2^{n-1} - 38)
\end{aligned}
$$

$$- (19 \times 2^{n-(i-1)} - 13) + 12) \times 2(19 \times 2^{n-i} - 13)$$

$$+ (19 \times 2^{n-i} - 13)^2\}]$$

$$+ 3\sum_{i=2}^{n} 2^{i-1}[\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 12)$$

$$\times 2(19 \times 2^{n-i} - 13) + (19 \times 2^{n-i} - 13)^2\}$$

$$\times \{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-i} - 13)(19 \times 2^{n-i} - 13 - 1)\}]$$

$$+ 3\sum_{i=2}^{n} 2^{i}[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-i} - 13))$$

$$\times (19 \times 2^{n-i} - 13 - 1) \times 0]$$

$$+ 3\sum_{i=2}^{n} 2^{i}[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-i} - 13) + 5)$$

$$\times (19 \times 2^{n-i} - 13 - 6) \times 0].$$

After simplification, we obtain

$$M_{2C}[D_n]$$

$$= \left[\frac{116116011}{112}\right] \times 2^{4n} + \left[\frac{61493823}{16}\right] \times 2^{3n} - \left[\frac{86236725}{8}\right] \times 2^{2n}$$

$$+ \left[\frac{103542435}{14}\right] \times 2^{n} + \left[\frac{9178425}{4}\right] \times n \times 2^{2n}$$

$$- 3333474 \times n \times 2^{3n} - 1497900.$$

Theorem 3.3. *Let* D_n *be the nanostar dendrimer. Then*

$$M_{3C}[D_n] = \left[\frac{29241}{4}\right] \times n \times 2^{2n} + 570 \times n + 2^{n} - \left[\frac{153843}{8}\right] \times 2^{2n}$$

$$+ \left[\frac{111639}{4}\right] \times 2^{n} - 8760.$$

Proof. From Table 1, we calculate the third Zagreb cutting index of the nanostar dendrimer D_n as follows:

$$
\begin{aligned}
M_{3C}[D_n] &= \sum_{uv \in E(D_n)} |c(u) - c(v)| \\
&= (3 \times 2^0)|[3(19 \times 2^{n-1} - 13)^2 - ((57 \times 2^{n-1} - 38) \\
&\quad - (19 \times 2^{n-1} - 13)) \times (19 \times 2^{n-1} - 13 - 1)]| \\
&\quad + (3 \times 2^1)|[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \\
&\quad \times (19 \times 2^{n-1} - 13 - 1) - 0]| \\
&\quad + (3 \times 2^1)|[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13) + 5) \\
&\quad \times (19 \times 2^{n-1} - 13 - 6) - 0]| \\
&\quad + 3\sum_{i=2}^{n} 2^{i-2}|[\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 5) \\
&\quad \times (19 \times 2^{n-(i-1)} - 13 - 6)\} \\
&\quad - \{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 6) \\
&\quad \times (19 \times 2^{n-(i-1)} - 13 - 7)\}]| \\
&\quad + 3\sum_{i=2}^{n} 2^{i-1}|[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 6) \\
&\quad \times (19 \times 2^{n-(i-1)} - 13 - 7) - 0]| \\
&\quad + 3\sum_{i=2}^{n} 2^{i-1}|[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 11) \\
&\quad \times (2(19 \times 2^{n-(i-1)} - 13) + 1) - 0]| \\
&\quad + 3\sum_{i=2}^{n} 2^{i-2}|[\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 11) \\
&\quad \times 2(19 \times 2^{n-i} - 13) + 1\}
\end{aligned}
$$

$$- \{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 12)$$

$$\times 2(19 \times 2^{n-i} - 13) + (19 \times 2^{n-i} - 13)^2\}]|$$

$$+ 3\sum_{i=2}^{n} 2^{i-1}|[\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 12)$$

$$\times 2(19 \times 2^{n-i} - 13) + (19 \times 2^{n-i} - 13)^2\}$$

$$- \{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-i} - 13)) \times (19 \times 2^{n-i} - 13 - 1)\}]|$$

$$+ 3\sum_{i=2}^{n} 2^{i}|[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-i} - 13))$$

$$\times (19 \times 2^{n-i} - 13 - 1) - 0]|$$

$$+ 3\sum_{i=2}^{n} 2^{i}|[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-i} - 13) + 5)$$

$$\times (19 \times 2^{n-i} - 13 - 6) - 0]|.$$

After simplification, we get

$$M_{3C}[D_n] = \left[\frac{29241}{6}\right] \times n \times 2^{2n} + 570 \times n \times 2^n - \left[\frac{153843}{8}\right] \times 2^{2n}$$

$$+ \left[\frac{111639}{4}\right] \times 2^n - 8760.$$

Theorem 3.4. *Let* D_n *be the nanostar dendrimer. Then*

$$M_{4C}(D_n) = \left[\frac{4119186168}{896}\right] \times 2^{4n} - \left[\frac{189906577}{16}\right] \times 2^{3n} - \left[\frac{238905981}{8}\right]$$

$$\times 2^{2n} + \left[\frac{446809616}{14}\right] \times 2^n - 6272547.$$

Proof. Using Table 1, we can write the fourth Zagreb cutting index of D_n as follows:

$$
\begin{aligned}
M_{4C}[D_n] &= \sum_{uv \in E(D_n)} c(u)[c(u) + c(v)] \\
&= (3 \times 2^0)[3(19 \times 2^{n-1} - 13)^2\{3(19 \times 2^{n-1} - 13)^2 \\
&\quad + ((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \times (19 \times 2^{n-1} - 13 - 1)\}] \\
&\quad + (3 \times 2^1)[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \\
&\quad \times (19 \times 2^{n-1} - 13 - 1)\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \\
&\quad \times (19 \times 2^{n-1} - 13 - 1) + 0\}] \\
&\quad + (3 \times 2^1)[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13) + 5) \\
&\quad \times (19 \times 2^{n-1} - 13 - 6)\{((57 \times 2^{n-1} - 38) \\
&\quad - (19 \times 2^{n-1} - 13) + 5) \times (19 \times 2^{n-1} - 13 - 6) + 0\}] \\
&\quad + 3\sum_{i=2}^{n} 2^{i-2}[\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13) + 5) \\
&\quad \times (19 \times 2^{n-(i-1)} - 13 - 6)\} + \{((57 \times 2^{n-1} - 38) \\
&\quad - (19 \times 2^{n-(i-1)} - 13) + 5) \times (19 \times 2^{n-(i-1)} - 13 - 6) \\
&\quad + ((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 6) \\
&\quad \times (19 \times 2^{n-(i-1)} - 13 - 7)\}] \\
&\quad + 3\sum_{i=2}^{n} 2^{i-1}[\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 6) \\
&\quad \times (19 \times 2^{n-(i-1)} - 13 - 7)\}\{((57 \times 2^{n-1} - 38) \\
&\quad - (19 \times 2^{n-(i-1)} - 13) + 6) \\
&\quad \times (19 \times 2^{n-(i-1)} - 13 - 7) + 0\}]
\end{aligned}
$$

$$+ 3\sum_{i=2}^{n} 2^{i-1}[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 11)$$

$$\times (2(19 \times 2^{n-i} - 13) + 1)\{((57 \times 2^{n-1} - 38)$$

$$- (19 \times 2^{n-(i-1)} - 13) + 11) \times (2(19 \times 2^{n-i} - 13) + 1) + 0\}]$$

$$+ 3\sum_{i=2}^{n} 2^{i-2}\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 11)$$

$$\times (2(19 \times 2^{n-i} - 13) + 1)\}[\{((57 \times 2^{n-1} - 38)$$

$$- (19 \times 2^{n-(i-1)} - 13) + 11) \times (2(19 \times 2^{n-i} - 13) + 1)\}$$

$$+ \{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 12)$$

$$\times 2(19 \times 2^{n-i} - 13) + (19 \times 2^{n-i} - 13)^2\}]$$

$$+ 3\sum_{i=2}^{n} 2^{i-1}\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 12)$$

$$\times 2(19 \times 2^{n-i} - 13) + (19 \times 2^{n-i} - 13)^2\}[\{((57 \times 2^{n-1} - 38)$$

$$- (19 \times 2^{n-i} - 13) + 12) \times 2(19 \times 2^{n-i} - 13)$$

$$+ (19 \times 2^{n-i} - 13)^2\} + \{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-i} - 13))$$

$$\times (19 \times 2^{n-i} - 13 - 1)\}]$$

$$+ 3\sum_{i=2}^{n} 2^{i}[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-i} - 13))$$

$$\times (19 \times 2^{n-i} - 13 - 1)\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-i} - 13))$$

$$\times (19 \times 2^{n-i} - 13 - 1) + 0\}]$$

$$+ 3\sum_{i=2}^{n} 2^{i}[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-i} - 13) + 5)$$

$$\times (19 \times 2^{n-i} - 13 - 6)\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-i} - 13) + 5)$$

$$\times (19 \times 2^{n-i} - 13 - 6) + 0\}].$$

By doing some calculation, we get

$$M_{4C}[D_n] = \left[\frac{4119186168}{896}\right] \times 2^{4n} - \left[\frac{189906577}{16}\right] \times 2^{3n}$$

$$- \left[\frac{238905981}{8}\right] \times 2^{2n} + \left[\frac{446809616}{14}\right] \times 2^{n} - 6272547.$$

Theorem 3.5. *Let* D_n *be the nanostar dendrimer. Then*

$$M_{5C}[D_n] = \left[\frac{423412929}{224}\right] \times 2^{4n} + \left[\frac{111579685}{16}\right] \times 2^{3n} - \left[\frac{160540215}{8}\right]$$

$$\times 2^{2n} + \left[\frac{399550415}{28}\right] \times 2^{n} + \left[\frac{18087183}{4}\right] \times n \times 2^{2n}$$

$$- 197790 \times n \times 2^{n} - 6111369 \times n \times 2^{3n} - 3069747.$$

Proof. Using the data given in Table 1, the fifth Zagreb cutting index of D_n can be written as

$$M_{5C}[D_n] = \sum_{uv \in E(D_n)} c(v)[c(u) + c(v)]$$

$$= (3 \times 2^{0})[\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13))$$

$$\times (19 \times 2^{n-1} - 13 - 1)\}\{3(19 \times 2^{n-1} - 13)^2 + ((57 \times 2^{n-1} - 38)$$

$$- (19 \times 2^{n-1} - 13))(19 \times 2^{n-1} - 13 - 1)\}]$$

$$+ 3\sum_{i=2}^{n} 2^{i-2}[\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 6)$$

$$\times (19 \times 2^{n-(i-1)} - 13 - 7)\}\{((57 \times 2^{n-1} - 38)$$

$$- (19 \times 2^{n-(i-1)} - 13) + 5)$$

$$\times (19 \times 2^{n-(i-1)} - 13 - 6)((57 \times 2^{n-1} - 38)$$

$$- (19 \times 2^{n-(i-1)} - 13) + 6) \times (19 \times 2^{n-(i-1)} - 13 - 7)\}]$$

$$+ 3\sum_{i=2}^{n} 2^{i-2}[\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13) + 12)$$

$$\times 2(19 \times 2^{n-i} - 13) + (19 \times 2^{n-i} - 13)^2\}\{((57 \times 2^{n-1} - 38)$$

$$- (19 \times 2^{n-(i-1)} - 13) + 11)$$

$$\times (2(19 \times 2^{n-i} - 13) + 1)\} + \{((57 \times 2^{n-1} - 38)$$

$$- (19 \times 2^{n-(i-1)} - 13) + 12)$$

$$\times 2(19 \times 2^{n-i} - 13) + (19 \times 2^{n-i} - 13)^2\}]$$

$$+ 3\sum_{i=2}^{n} 2^{i-1}[\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-i} - 13))$$

$$\times (19 \times 2^{n-i} - 13 - 1)\}\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-(i-1)} - 13)$$

$$+ 12) \times 2(19 \times 2^{n-i} - 13) + (19 \times 2^{n-i} - 13)^2\}$$

$$+ \{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-i} - 13)) \times (19 \times 2^{n-i} - 13 - 1)\}].$$

After some calculation, we get

$$M_{5C}[D_n] = \left[\frac{423412929}{224}\right] \times 2^{4n} + \left[\frac{111579685}{16}\right] \times 2^{3n} - \left[\frac{160540215}{8}\right]$$

$$\times 2^{2n} + \left[\frac{399550415}{28}\right] \times 2^{n} + \left[\frac{18087183}{4}\right] \times n \times 2^{2n}$$

$$- 197790 \times n \times 2^{n} - 6111369 \times n \times 2^{3n} - 3069747.$$

Example 1. For the nanostar dendrimer D_n for $n = 1$, we shall compute the indices (refer Figure 1)

Number of edges $e = uv$	Cutting number of end vertices $(c(u), c(v))$
3×2^0	$\{3(19 \times 2^{n-1} - 13)^2, ((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \times (19 \times 2^{n-1} - 13 - 1)\}$
3×2^1	$\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13)) \times (19 \times 2^{n-1} - 13 - 1), 0\}$
3×2^1	$(0, 0)$
3×2^1	$\{((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13) + 5) \times (19 \times 2^{n-1} - 13 - 6), 0\}$

$$M_{1C}[D_n] = \sum\nolimits_{uv \in E(D_n)} (c(u) + c(v))$$

$$= (3 \times 2^0)[3(19 \times 2^{n-1} - 13)^2 + ((57 \times 2^{n-1} - 38)$$

$$- (19 \times 2^{n-1} - 13)) \times (19 \times 2^{n-1} - 13 - 1)]$$

$$+ (3 \times 2^1)[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13))$$

$$\times (19 \times 2^{n-1} - 13 - 1) + 0]$$

$$+ (3 \times 2^1)[((57 \times 2^{n-1} - 38) - (19 \times 2^{n-1} - 13) + 5)$$

$$\times (19 \times 2^{n-1} - 13 - 6) + 0]$$

$$= (3 \times 2^0)[173] + (3 \times 2^1)[65] + (3 \times 2^1)[0] = 909.$$

4. Conclusion

In this article, we have computed first, second, third, fourth and fifth Zagreb cutting indices. It would be interesting to investigate other topological indices of nanostar dendrimer D_n in future.

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