A STUDY ON FUZZY SEMIGRAPHS AND THEIR PROPERTIES

Thesis Submitted to the

Bharathidasan University for the award of the Degree of

DOCTOR OF PHILOSOPHY IN MATHEMATICS

by

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(Ref. No. 16706/Ph.D.K1/Mathematics/P-T (3-5)/January 2019/Date 05.01.2019)

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CERTIFICATE

This thesis entitled " **A STUDY ON FUZZY SEMIGRAPHS AND THEIR PROPERTIES"** Submitted by **P.RENGANATHAN** (Ref.No.16706 /Ph.D.K1 / Mathematics/P-T(3-5)/January 2019/Date 05.01.2019) for the fulfilment of the degree of Doctor of Philosophy in Mathematics to Bharathidasan University, Tiruchirappalli is a record of original work done by him under my guidance. It has not previously formed the basis for the award of any Degree, Diploma, Associateship, Fellowship or any such similar title of any University or Institution.

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DECLARATION

I, **P.RENGANATHAN**, hereby declare that the thesis entitled **"A STUDY ON FUZZY SEMIGRAPHS AND THEIR PROPERTIES"** submitted to the Bharathidasan University, in partial fulfilment of the requirements for the award of the degree Doctor of Philosophy in Mathematics is a record of this work has been originally carried out by me under the guidance and supervision of **Dr. K. RADHA, Associate Professor, P.G & Research Department of Mathematics, Thanthai Periyar Government Arts and Science College (Autonomous), Tiruchirappalli-620 023** and that this work has not been submitted elsewhere for any other degree.

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ACKNOWLEDGEMENT

Praise and thanks to **GOD ALMIGHTY**, for having showered His Grace and Blessings on me on the completion of my thesis.

I sincerely express my gratitude to my research supervisor **Dr. K. RADHA**, P.G. & Research Department of Mathematics, Thanthai Periyar Government Arts and Science College (Autonomous), Tiruchirappalli for accepting me as a research scholar to do research in the field of Fuzzy Graph Theory. I am deeply indebted to her constant encouragement and sincere effort at drawing out the best in order to mould me into a more innovative and committed person in the field of research work.

I am extremely grateful to **Dr. J. SUGANTHI,** Principal, Thanthai Periyar Government Arts and Science College (Autonomous), Tiruchirappalli, for having given the permission to pursue my Ph.D. in this esteemed institution and providing me with necessary facilities.

I convey my wholehearted acknowledgement to the doctoral committee member **Dr. R. JAHIR HUSAIN,** P.G. & Research Department of Mathematics, Jamal Mohamed College (Autonomous), Tiruchirappalli, for his valuable suggestions during the development of my thesis. I express my heartfelt thanks to the other doctoral committee member **Dr. M. MARAGATHAM,** P.G. & Research Department of Mathematics, Thanthai Periyar Government Arts and Science College (Autonomous), Tiruchirappalli, for her sustaining support.

I thank **Dr. S. SETHURAMAN,** Head and all the staff members of P.G. & Research Department of Mathematics, Thanthai Periyar Government Arts and Science College, Tiruchirappalli.

I express my sincere gratitude to **Dr. A. NAGOOR GANI,** P.G. & Research Department of Mathematics, Jamal Mohamed College (Autonomous), Tiruchirappalli, for his valuable guidance and constant encouragement.

I express my sincere thanks to **Dr. R. RAVICHANDRAN,** Principal, Doctor Kalaignar Government Arts College, Kulithalai.

I dedicate my thesis to my parents **P.PICHAIMUTHU** and **P. PICHAIAMMAL** to whom I owe all that I am today.

My deepest thanks to my wife **K. JEEVA** for extending her constant and consistent support in all my efforts throughout our journey together.

Special thanks to my daughters **R.SRIVARDHINI** and **K.R.SREEDHARSHINI** for their moral support and their driving energy.

Finally, I am thankful to my friends, relatives, and well wishers for their immense love and affection which always helps me sail smoothly in all my efforts. There are many others whom I might have inadvertently left out and I sincerely thank them for their help.

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Chapter 1

Introduction

CHAPTER -1

INTRODUCTION

In this chapter, introduction on graph theory, semigraph theory, fuzzy graph theory and some basic definitions that are required for the research work and organization of the thesis work are given.

1.1. INTRODUCTION

The study of graph theory came into existence with the Konigsberg bridge problem in 1735[3]. Graph theory [13, 14, 15] finds its wide range of applications in the field of chemistry, physics, social sciences, communication engineering, etc., Because of this diversity of applications it is useful to develop and study the object in abstract terms and to interpret its results in terms of the objects of any specific case in which one may wish. Graph theory plays an important role in several areas of Computer science such as Artificial Intelligence, Formal Languages, Computer Graphics, Operating Systems, etc.,

Fuzzy sets are introduced by Lofti A. Zadeh [63] in 1965. The study of Fuzzy sets and system play a vital role in pattern recognition, image processing, robotics, artificial intelligence, decision making, data analysis, data mining, etc. Fuzzy mathematical Theories contribute considerably to Economics, Finance, Management, Industries, Electronics and communications.

Fuzzy sets and Fuzzy relations gave birth to fuzzy graphs [11, 12, 57]. Fuzzy relations play a crucial role in the areas of clustering analysis, neural networks, computer networks, pattern recognition, etc… In each of the above areas the basic mathematical structure is that of fuzzy graphs.

The notion of fuzzy graphs was introduced by Rosenfeld [58] in the year 1975. In recent years, fuzzy graph theory has emerged as one of the vast areas of research. Applications of fuzzy graphs include data mining, image segmentation, clustering, image capturing, networking, communication, planning, scheduling, etc..,

Crisp graphs and fuzzy graphs are structurally similar, but when there is uncertainty on vertices and edges then fuzzy graphs have separate importance.

1.2. REVIEW OF LITERATURE

The notion of fuzzy graph was introduced by Rosenfeld. A [58] in the year 1975.Theconcept of fuzziness in crisp graph theory, like connectedness, completeness, isomorphism, degree sequence, etc were studied afterwards.

Eccentricity concepts and centre of fuzzy graphs were developed by Bhattacharya [6] . The notions of strong arcs, fuzzy end nodes and geodesics in fuzzy graphs were contributed by Bhutani and Rosenfeld [8, 9, 10]. Bhutani. K.R, Mordeson and Rosenfeld[12] studied the Degrees of End nodes and Cut nodes in Fuzzy graphs. Dewdney. A[18] has contributed some useful results on Degree sequences in complexes and hypergraphs. C. Berg. [5] studied some properties of hypergraphs.

Boonyasombet. V [16] contributed some useful results to Degree sequences of connected hypergraphs and hypertrees. Choudam. S .A [17] studied Graphic and 3-hypergraphic sequence.

Bhave. N. S., Bam. B. Y and and Deshpande .C.M [7] studied the Characterization of Degree Sequences of Linear Hypergraphs. Mordeson. J.N and Nair. P.S [27, 28, 29] made a comparative study on Fuzzy Graphs with Fuzzy Hypergraphs and also studied Fuzziness and soft computing.

Sampathkumar. E [60] introduced the concept of graphoids. Acharya. B. D and Sampathkumar. E [1] introduced and contributed considerably to Graphoidal covers and graphoidal covering number of a graph. B. D. Acharya coined the term semigraph for graphoids which was introduced and developed by E. Sampathkumar.

Bam. B. Y and Bhave. N. S [4] contributed considerably to degree, degree sequences, complete semigraphs ,connectivity in semigraphs.

Sampathkumar. E and Pushpalatha. L [59] introduced the concept of bipartite semigraphs and Dendroids. They also contributed towards isomorphism of semigraphs matrix representation of Semigraphs

K. Kayathri and S. Pethanachi Selvam [24,25] studied the Edge Completeness in semigraphs.

Operations of union, join, cartesian product and composition of fuzzy graphs were introduced and some of their properties were studied by Mordeson. J.N and Peng. C.S [30] .

Nagoor Gani. A. and Bhsheer Ahamed. M [39] defined order, size and effective edges in fuzzy graphs. Nagoorgani. A and Jahir Hussain. R [44] defined and studied the properties of effective fuzzy Euler graph and fuzzy Hamiltonian graph. Sunitha. M.S and Vijayakumar. A [62] gave a definition for the complement of a fuzzy graph. They studied some properties of fuzzy bridges and fuzzy cut vertices and gave a characterization of fuzzy tree using them. They also studied some metric aspects [62] of fuzzy graphs

Nagoor Gani. A and Radha. K [47, 48] introduced regular and totally regular fuzzy graphs and obtained some useful results on regular fuzzy graphs..

Nagoor Gani. A and Malarvizhi. J [45,46] discussed the concept of isometry in fuzzy graphs and studied its properties and also studied some properties of isomorphism on fuzzy graphs[45]. Nagoor Gani. A and Fathima Kani. B [43] introduced alpha, beta and gamma product of fuzzy graphs.

Radha. K and Kumaravel. N [56] introduced the concept of edge regular fuzzy graphs. They also studied the degree of an edge in union and join of two fuzzy graphs. Radha. K and Arumugam. S [54] made a study on direct sum of two fuzzy graphs. Radha. K and Rosemine. A [57] introduced the concept of degree sequence of a fuzzy graph.

1.3. SCOPE AND ORGANIZATION OF THE THESIS

Chapter I In this chapter, introduction on fuzzy graph theory, semigraph theory, fuzzy semigraphs and some basic definitions that are required for the research work and organization of the thesis work are studied.

Chapter II In this chapter, the concept of Fuzzy semigraphs is introduced. The fuzzy sub semigraphs, spanning fuzzy sub semigraphs and induced fuzzy subsemigraphs are defined. The effective fuzzy semigraph is introduced and some results on effective fuzzy semigraphs have been derived. The concept of bipartite fuzzy semigraphs is introduced and some of their properties are studied.

Chapter III The end vertex fuzzy graph, fuzzy adjacency graph, fuzzy consecutive adjacency graph are studied. It is proved that the adjacency fuzzy graph is connected if and only if the given fuzzy semigraph is connected. Many properties of these fuzzy graphs are derived.

Chapter IV In this chapter, isomorphism, weak isomorphism and co-weak isomorphism of fuzzy semigraphs are introduced and some of their properties are studied. End vertex isomorphism (ev-isomorphism), edge isomorphism (e-isomorphism) and adjacency isomorphism (a-isomorphism) of fuzzy semigraphs are defined. Properties of effective edges and effective fuzzy semigraphs under isomorphism are studied. Also, it is proved that isomorphism is an equivalence relation and week isomorphism is a partial order relation.

Chapter V In this chapter, various degrees of a vertex in a fuzzy semigraph are defined. Degree, edge degree, adjacent degree and consecutive adjacent degree of a vertex in a fuzzy semigraph are introduced. Their properties under various isomorphisms are discussed. The degree of an edge is also defined.

Chapter VI Various regular properties of fuzzy semigraphs are studied. The regular, edge degree regular, adjacency regular and consecutive adjacency regular fuzzy semigraphs are introduced and some of their properties are studied. Also biregular fuzzy semigraphs and totally regular fuzzy semigraphs are defined.

1.4. BASIC CONCEPTS

Definition 1.4.1[58]

Let V be a non-empty finite set and $E \subseteq V \times V$. A fuzzy graph $G\colon(\sigma,\mu)$ is a pair of functions $\sigma\colon V \to [0,1]$ and $\mu\colon E \to [0,1]$ such that $\mu(x,y) \leq \sigma(x) \wedge \wedge\, \sigma(y)$ for all $x, y \in V$. Underlying crisp graph of $G\colon(\sigma,\mu)$ is denoted by $G^*\colon(V,E)$.

Definition 1.4.2[58]

A fuzzy graph $H:(\tau,\ \rho)$ is called fuzzy subgraph of $G:(\sigma,\mu)$if $\tau(u) \leq \sigma(u), \forall\, u \in V$ and $\rho(uv) \leq \mu(uv), \forall\, u, v \in$. If $\tau(u) = \sigma(u), \forall u \in V$ and $\rho(uv) \leq \mu(uv), \forall\, u, v \in V$, then H is called a spanning fuzzy subgraph of G. $H:(\tau,\ \rho)$ is called induced fuzzy subgraph of $G:(\sigma,\mu)$induced by τ if $\tau(u) = \sigma(u), \forall\, u \in V(H)$ and $\rho(uv) = \mu(uv), \forall u, v \in V(H)$.

Definition 1.4.3[44]

G is an effective fuzzy graph if $\mu(xy) = \sigma(x) \wedge \sigma(y)$for all $x, y \in$ E. G is a complete fuzzy graph if $\mu(xy) = \sigma(x) \wedge \sigma(y)$for all $x, y \in$ V.

Definition 1.4.4[39]

The order and size of a fuzzy graph $G:(\sigma,\mu)$ are defined by $O(G) = \sum_{x \in V} \sigma(x)andS(G) = \sum_{xy \in E} \mu(xy)$.

Definition 1.4.5[40]

If $\mu(xy) > 0$ then x and y are called neighbours, x and y are said to lie on the edge $e = xy$. A path ρ in a fuzzy graph $G:(\sigma,\mu)$ is a sequence of distinct nodes $v_0, v_1, v_2, \dots, v_n$ such that $\mu(v_i, v_{i-1}) > 0, 1 \leq i \leq n$. Here '$n$' is called the length of the path. The consecutive pairs (v_i, v_{i-1}) are called arcs of the path.

Definition 1.4.6[54]

Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*:(V,E)$.The degree of vertex x is $d_G(x) = \sum_{x\neq y} \mu(xy)$. If each vertex in G has same degree k, then G is said to be a regular fuzzy graph or k -regular fuzzy graph.

Definition 1.4.7 [34]

Let G: (σ,μ) be a fuzzy graph. Then the total degree of the vertex u is td(u) = $\sum_{u\neq v} \mu(uv) + \sigma(u) = d(u) + \sigma(u)$. If the total degree of all the vertices is r, then G is said to be a totally regular fuzzy graph of degree r or a r-totally regular fuzzy graph.

Definition 1.4.8[27]

If u, v are nodes in $G:(\sigma,\mu)$ and if they are connected by means of a path then the strength of that path is defined as $\sum_{i=1}^{n} \mu\,(v_{i-1}, v_i)$. ie., it is the strength of the weakest arc. If u, v are connected by means of paths of length ' k ' then $\mu^k(u,v) = sup\{\mu(u,v_1)\wedge\mu(v_1,v_2)\wedge\mu(v_2,v_3)\wedge \dots \wedge\mu(v_{k-1},v)/$ $u, v_1, v_2, v_3, \dots, v_{k-1}v \in V\}$

If $u, v \in V$, the strength of connectedness between u and v is,

$$\mu^\infty(u,v) = sup\{\mu^k(u,v)/\ k = 1,2,3,\dots\}$$

Definition 1.4.9[41]

A fuzzy graph $G:(\sigma,\mu)$ is connected if $\mu^{\infty}(u,v) > 0$ for all $u,v \in V$. An edge xy is said to be a strong edge if $\mu(x,y) > \mu^{\infty}(u,v)$. A vertex x is said to be an isolated vertex if $\mu(xy) = 0,\ \forall\, y \neq x$.

Definition 1.4.10[36]

A homomorphism of fuzzy graphs $f: G \to G'$ is a map $f: V \to V'$which satisfies $\sigma(u) \leq \sigma'\big(f(u)\big)\, for\ all$ u ∈ V , $\mu(uv) \leq \mu'(f(u)f(v))$ for all $u,v \in V$.

Definition 1.4.11[36]

A isomorphism of fuzzy graphs $f: G \to G'$ is a bijective map $f: V \to V'$ which satisfies $\sigma(u) = \sigma'\big(f(u)\big)\, for\ all$ u ∈ V and $\mu(uv) = \mu'(f(u)f(v))$ for all $u,v \in V$.

Definition 1.4.12[36]

A weak isomorphism of fuzzy graphs $f: G \to G'$ is a map $f: V \to V'\ which$ is bijective and satisfies $\sigma(u) = \sigma'\big(f(u)\big)\, for\ all$ u ∈ V and $\mu(uv) \leq \mu'(f(u)f(v))$ for all $u,v \in V$.

Definition 1.4.13[36]

A co-weak isomorphism of fuzzy graphs $f: G \to G'$ is a map$f: V \to V'$ which is bijective and satisfies $\sigma(u) \leq \sigma'\big(f(u)\big)$ for all u ∈ Vand $\mu(uv) = \mu'(f(u)f(v))$ for all $u,v \in V$.

Definition 1.4.14[59]

G is e-bipartite if its vertex set V can be partitioned into sets$\{V_1, V_2\}$ such that $V_1, V_2$are e- independent.

Definition 1.4.15[58]

G is strongly bipartite if its vertex set V can be partitioned into sets$\{V_1, V_2\}$ such that V_1, V_2 are strongly independent.

Definition 1.4.16[59]

The edge clique number of G is the maximum cardinality of an edge clique in G.

Definition 1.4.17[41]

Fuzzy independent set: Two vertices are said to be Fuzzy independent if there is no strong arc between them. A subset S of V is fuzzy independent of G if any two vertices of S are fuzzy independent.

Definition 1.4.18 [35]

A fuzzy graph G is fuzzy bipartite then its vertex set V can be partitioned into sets $\{V_1, V_2\}$ such that V_1 and V_2 are fuzzy independent sets. These $V_1, V_2$are called fuzzy bipartitions of V.

Definition 1.4.19 [35]

The size of a fuzzy bipartite graph is defined to be the sum of the membership values of all strong arcs of it.

Definition 1.4.20 [35]

A fuzzy bipartite graph G with fuzzy bipartition $\{V_1, V_2\}$is said to be complete fuzzy bipartite if between every vertex of V_1 and each vertex of V_2 there is a strong arc.

Definition 1.4.21[60]

A semigraph is a pair (V, X), where V is a non-empty set of elements called vertices and X is a set of n-tuples called edges of distinct vertices for various $n \geq 2$ satisfying the following conditions:

1. Any two edges have at most one vertex in common
2. Two edges $E_1 = (u_{1,} u_2, \dots, u_n), E_2 = \left(v_{1,} v_2, \dots, v_m\right)$ are considered to be equal if and only if

   a) $m = n$ b) either $u_i = v_i$ for i = 1 to n or $u_i = v_{n-i+1}$ for I = 1to n.

In the edge $E = \left(u_{1,} u_2, \dots, u_n\right)$, $u_1$and u_n are called the end vertices and all vertices in between them are called middle vertices (m- vertices). If a middle vertex is an end vertex of some other edge, it is called middle end vertex (me-vertices).

Definition 1.4.22[60]

A subedge (fs-edge) of an edge $E = (v_{1,} v_2, \dots, v_n)$ is a $k -$tuple $E = (v_{i_1}, v_{i_2}, \dots . v_{i_k})$ where $1 \leq i_1 < i_2 < \cdots < i_k \leq n$ or $1 \leq i_k < i_{k-1} < \cdots < i_1 \leq n$.

Definition 1.4.23[60]

A partial edge (fp-edge) of an edge $E = (v_{1,}v_2, \ldots, v_n)$ is a $(j - i + n)$-tuple $E = (v_{i,}v_{i-1}, \ldots, v_j)$ where $1 \leq i \leq n$.

Definition 1.4.24[60]

Let $G = (V, X)$be a semigraph. There are three different types of graphs associated with G.

The end vertex graph G_e: Two vertices in G_e are adjacent if and only if they are end vertices of an edge in G.

The Adjacency graph G_a: Two vertices in G_a are adjacent if and only if they are adjacent in G.

The Consecutive Adjacency graph G_{ca}: Two vertices in G_{ca} are adjacent if and only if they are consecutively adjacent vertices in G.

Definition 1.4.25[59]

Let $G_1: (V_1, X_1)$ and $G_2: (V_2, X_2)$ be two semigraphs and f be a bijection from V_1 to V_2. Let $x = (v_1, v_2, \ldots, v_n)$ be an edge in G_1. f is an isomorphism if $(f(v_1), f(v_2), \ldots, f(v_n))$is an edge in G_2.f is an end vertex isomorphism (ev-isomorphism) if the set $\{f(v_1), f(v_2), \ldots, f(v_n)\}$ forms an edge in G_2 with end vertices $f(v_1)$ and $f(v_n)$. f is an edge isomorphism (e –isomorphism) if the set $\{f(v_1), f(v_2), \ldots, f(v_n)\}$ forms an edge in G_2 . f is an adjacency isomorphism (a-isomorphism) if the adjacent vertices in G_1 are mapped onto adjacent vertices in G_2.

Definition 1.4.26[58]

A set S of vertices in a semigraph $G = (V, X)$ is independent if no edge is a subset of S.

S is e-independent if no two end vertices of an edge belong to S.

S is strongly independent if no two adjacent vertices belong to S.

Definition 1.4.27[60]

A semigraph G is bipartite if its vertex set V can be partitioned into sets $\{V_1, V_2\}$ such that V_1, V_2 are independent.

Definition 1.4.28[59]

A semigraph G is edge complete if any two edges in G are adjacent.

Definition 1.4.28[59]

A set S of edges in a semigraph G is said to form an edge clique if any two edges in S are adjacent.

Definition 1.4.29[58]

A semigraph G is r-uniform if all the edges of G contains r vertices.

Definition 1.4.30 [55]

A vertex u in a fuzzy graph G is simplicial in G if $N_G[u]$ is a complete fuzzy graph. In other words, u is simplicial in G if u is simplicial in G$*$ and each edge of $N_G[u]$ is an effective edge.

Chapter 2

Fuzzy Semigraphs

CHAPTER -2

FUZZY SEMIGRAPHS

In this chapter, the concept of fuzzy semigraphs is introduced. The fuzzy sub semigraphs, spanning fuzzy sub semigraphs and induced fuzzy subsemigraphs are defined. The effective fuzzy semigraph is introduced and some results on effective fuzzy semigraphs have been derived. The concept of bipartite fuzzy semigraphs is introduced and some of their properties are studied.

2. 1. FUZZY GRAPHS ASSOCIATED WITH SEMI GRAPHS

In this section, fuzzy semigraph is introduced.

Definition 2.1.1

Consider a semigraph$G^*: (V, £, X)$. A fuzzy semigraph on $G^*: (V, £, X)$ is defined as $G: (\sigma, \mu, \eta)$ where $\sigma: V \to [0,1]$, $\mu: V \times V \to [0,1]$,:$\eta: X \to [0,1]$ are such that

(i) $\mu(u, v) \leq \sigma(u) \wedge \sigma(v) \;\; \forall (u, v) \in V \times V$

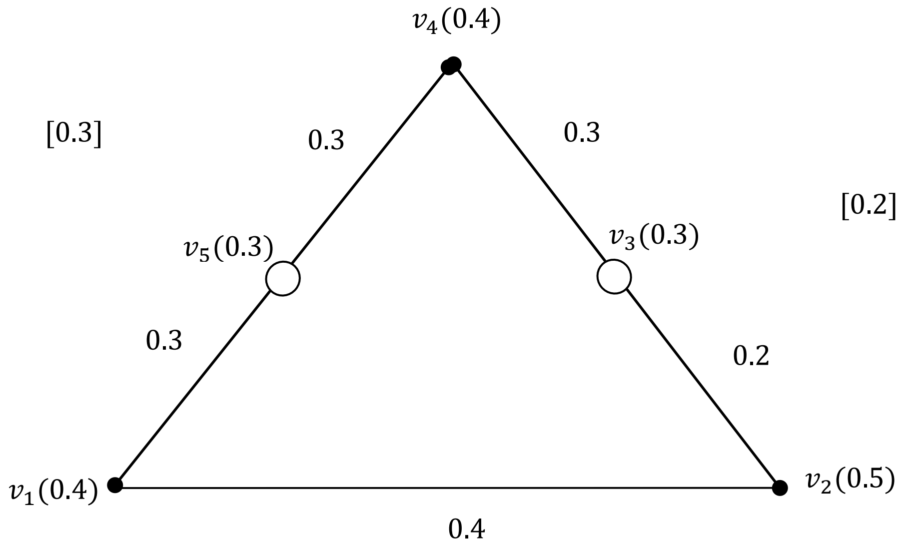
(ii) $\eta(E) = \mu(u_1, u_2) \wedge \mu(u_2, u_3) \wedge \dots \wedge \mu(u_{n-1}, u_n) \leq \sigma(u_1) \wedge \sigma(u_n)$,

$E = (u_{1,} u_2, \dots, u_n)$, $n \geq 2$ is an edge in G.

Here (σ, μ) is a fuzzy graph on $(V, £)$.

Example 2.1.2

To illustrate the above definition, we consider the fuzzy semigraph $G:(\sigma,\mu,\eta)$ in Figure 2.1.



$\boldsymbol{Fig\ 2.1}$: $\boldsymbol{G}$: $(\boldsymbol{\sigma},\boldsymbol{\mu},\boldsymbol{\eta})$

Here V = $\{v_{1,}v_2, v_3, v_4, v_5\}$

£ = $\{v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_1\}$

X = $\{E_1 = (v_1, v_2), E_2 = (v_2, v_3, v_4), E_3 = (v_4, v_5, v_1)\}$

$\eta(E_1)$ = 0.4, $\eta(E_2)$ = 0.2, $\eta(E_3)$ = 0.3.

$\mu(v_1v_2) = 0.4, \mu(v_2v_3) = 0.2, \mu(v_3v_4) = 0.3, \mu(v_4v_5) = 0.3, \mu(v_5v_1) = 0.3$

(σ,μ,η) satisfy the condition of the fuzzy semigraph.

Definition 2.1.3

A fuzzy semigraph G:(σ, μ, η) on G*(V, £, X) is a fuzzy semigraph of stage p if σ(u) = p, for all u∈V, μ(uv) = p, for all uv∈ £ and η(E) = p, for all E∈X where $0 < \text{p} \leq 1$.

Definition 2.1.4

Consider a fuzzy semigraph G:(σ, μ, η) on G*(V, £, X).

The order of G is O(G) = $\sum_{u \in V} \sigma(u)$.

The size of G is S(G) = $\sum_{E \in X} \eta(E)$.

The total size of G is TS(G) = $\sum_{e \in £} \mu(e)$.

Example 2.1.5

For the fuzzy semigraph in Fig.2.1,

O(G) = 1.3, S(G) = 0.9, TS(G) = 1.5

Definition 2.1.6

Consider a fuzzy semigraph G:(σ, μ, η) on G*(V, £, X). A path in G is a sequence of vertices $v_1 v_2 \ldots, v_n$ such that $v_i v_{i+1} \in$ £ and $\mu(v_i v_{i+1}) > 0$ for all i.

Definition 2.1.7

Consider a fuzzy semigraph G:(σ, μ, η) on G*(V, £, X). G is connected if there is a path between any two vertices of G.

Theorem 2.1.8

If G:(σ, μ, η) is a r-uniform fuzzy semigraph of stage p on G^*(V, £, X) with m edges, then size of G is S(G) = mp and the total size of G , TS(G)= m(r-1)p

Proof:

Since G is a fuzzy semigraph of stage p,

σ(u) = p, ∀ u∈V, μ(uv) = p, for all uv∈ £

and η(E) =p, for all E∈X. where 0< p≤ 1.

Therefore S(G) $= \sum_{E \in X} \eta(E)$

$=\sum_{E \in X} p$

= mp.

Total size is TS(G) = $\sum_{e \in £} \mu(e)$

$=\sum_{e \in £} p$

= m(r-1)p.

Theorem 2.1.9

If G;(σ, μ, η) is a fuzzy semigraph of stage p on G^*:(V, £, X), then the order of G is, O(G) = *np*, where n is the number of vertices of G.

Proof: Since G is a fuzzy semigraph of stage p, σ(u) = p, ∀ u∈V, μ(uv) = p, for all uv∈ £ and η(E) = p, for all E∈X. where 0<p≤ 1.

Now order of G, O(G) = $\sum_{u \in V} \sigma(u)$

Therefore O(G) = $\sum_{u \in V} p$

= $n.p$, which proves the theorem.

Theorem 2.1.10

Let G;(σ, μ, η) be a fuzzy semigraph on G^*:(V, £, X). Then G is connected if and only if there is a path between any two end vertices of G.

Proof:

If G is connected, then by the definition 2.1.7, there is a path between any two end vertices of G.

Conversely, suppose there is a path between any two end vertices of G.

Let u and v be any two middle vertices of G.

Let E_1 and E_2 be the edges containing u and v respectively.

Let u_i and v_i be the end vertices of E_i, i = 1, 2.

By hypothesis, there is a path P between u_1 and u_2.

Then the partial edge (u, ⋯, u_1) of E_1, the path P and the partial edge (u_1, ⋯, v) of E_2 constitute a path between u and v. Hence G is connected.

2.2. EFFECTIVE FUZZY SEMIGRAPHS

Definition 2.2.1

An edge $E = (u_{1,} u_2, \dots, u_n)$ in X of a fuzzy semigraph is called an effective edge if

$$\eta(E) = \eta(u_{1,} u_2, \dots, u_n) = \sigma(u_1) \wedge \sigma(u_n)$$

and $\mu(u_i u_j) = \sigma(u_i) \wedge \sigma(u_j)$ for all $i \neq j$.

E = $(u_{1,}u_2, \dots, u_n)$ in X of a fuzzy semigraph is called an e-effective edge if

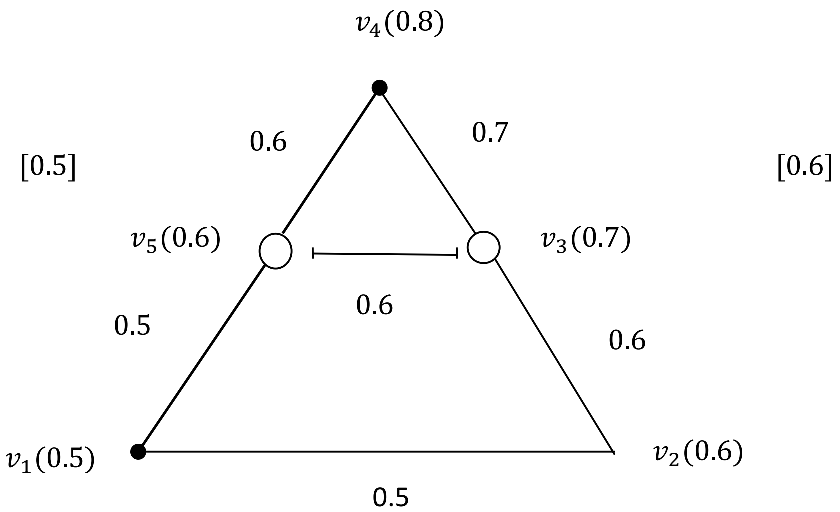
$$\eta(E) = \eta(u_{1,}u_2, \dots, u_n) = \sigma(u_1)\wedge\sigma(u_n).$$

Definition 2.2.2

A fuzzy semigraph G: (σ, μ, η) $on\ G^*$: (V, £, X) is said to be an effective fuzzy semigraph if all the edges of G are effective edges.

A fuzzy semigraph G: (σ, μ, η) $on\ G^*$: (V, £, X) is said to be an e-effective fuzzy semigraph if all the edges of G are e-effective edges.

An effective fuzzy semigraph is shown in Fig (2.2)



$\boldsymbol{Fig\ 2.2: Effective\ Semigraph}$

Definition 2.2.3

A fuzzy subsemigraph H of a fuzzy semigraph G: (σ, μ, η) is said to be a fuzzy effective sub semigraph if all its edges are effective edges.

Example 2.2.4

Consider the fuzzy semigraph in Fig. 2.3, The fuzzy subsemigraph of 2.3 is given in the Fig. 2.4.

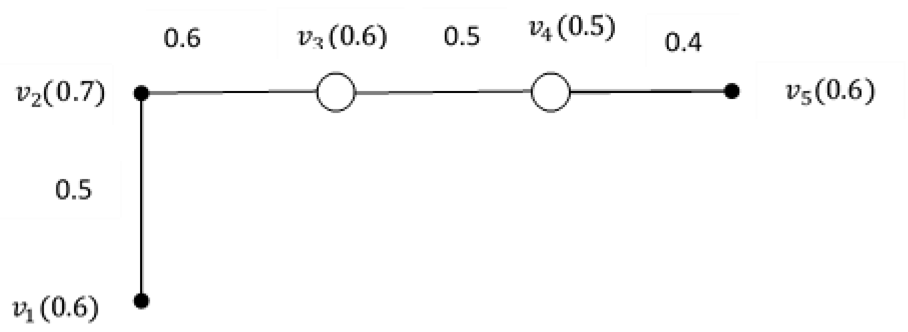


Fig. 2.3 Fuzzy Semigraph

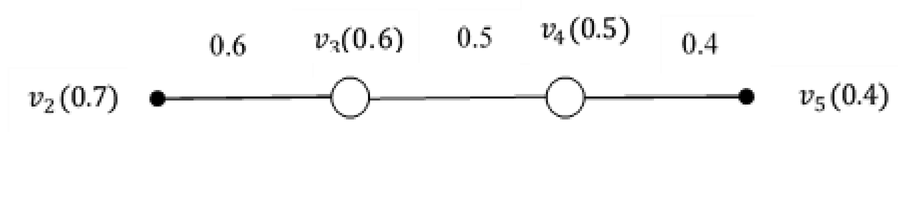


Fig 2.4 – Fuzzy Effective subsemigraph of the fuzzy semigraph in Fig 3.2

Remark 2.2.5

Fuzzy sub semigraphs and spanning fuzzy sub semigraphs of an effective fuzzy semigraph need not be effective.

Theorem 2.2.6

Induced sub semigraphs of an effective fuzzy semigraph are effective.

Proof:

Since membership values are preserved in induced fuzzy sub semigraphs, induced sub semigraphs of an effective fuzzy semigraph are effective.

Theorem 2.2.7

Any fuzzy semigraph G: (σ, μ, η) *on* G^*: (V, £, X) of stage p is an edge effective fuzzy semigraph.

Proof:

Since G is a fuzzy semigraph of stage p,

$\sigma(u) = p$, $\forall$ u$\in$V, $\mu(uv) = p$, for all $uv \in$ £,

and η(E) = p, for all $E \in$ X where $0 < p \leq 1$.

Then $\mu(uv) = p = \sigma(u) \wedge \sigma(v), \forall uv \in$ £, and

If E = $(u_1, u_2, \cdots, u_n)$, then

$$\eta(E) = \mu(u_1u_2) \wedge \mu(u_2u_3) \wedge \mu(u_3u_4) \wedge \ldots\ldots \wedge \mu(u_{n-1}u_n)$$

$$= p \wedge p \wedge \cdots \wedge p$$

$$= p$$

Therefore $\eta(E) = p = \sigma(u_1) \wedge \sigma(u_2), \forall E \in X$

Hence G:(σ, μ, η) in an edge effective fuzzy semigraph.

Remark 2.2.8

The converse of the above theorem 2.2.7 need not be true.

Definition 2.2.9

A fuzzy semigraph $G:(\sigma,\mu,\eta)\ on\ G^*:(V,X)$ is a complete fuzzy semigraph if

1.$G^*$is a complete semigraph.(ie) any two vertices lie on the same edge

2. G is an effective fuzzy semigraph.

Example 2.2.10

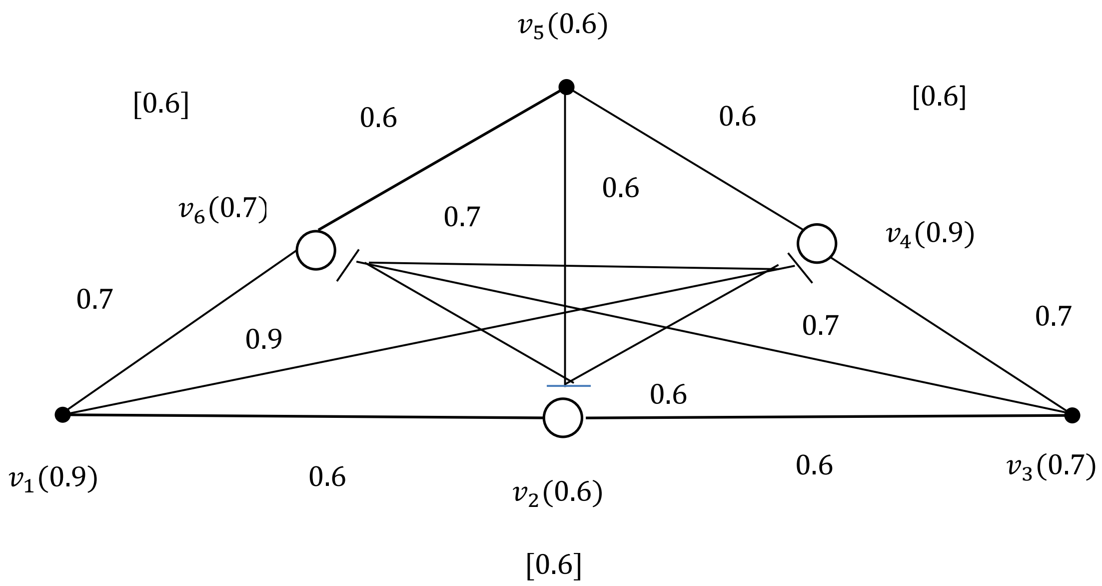


Fig 2.5: $\boldsymbol{Complete\ Fuzzy\ Semigraph}$

Remark 2.2.11

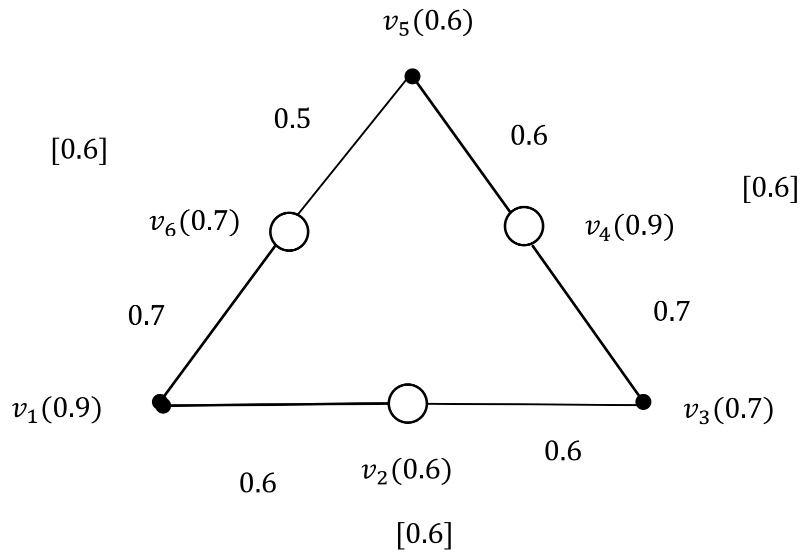
A complete fuzzy semigraph on *n* vertices is not unique.

Definition 2.2.12

A complete fuzzy semigraph containing a given fuzzy semigraph as an induced sub semigraph is denoted by *K*(G).

Example 2.2.13

The fuzzy semigraph in Fig.2.5 is a complete fuzzy semigraph K(G) containing G in Fig.2.6.



$\boldsymbol{Fig\ 2.6: G: (\sigma, \mu, \eta) Fuzzy\ Semigraph\ G}$

2.3. BIPARTITE FUZZY SEMIGRAPHS

Definition 2.3.1

Bipartite Fuzzy Semigraph: Let $G: (\sigma, \mu, \eta)$be a fuzzy semigraph.Then $G$is said to be bipartite if its vertex set V can be partitioned into $\{V_1, V_2\}$sets such that V_1 and V_2 are independent. Here $G^*: (V, £, X)$ is the underlying semigraph and (σ, μ) is a fuzzy graph.

Definition2.3.2

e -Bipartite Fuzzy Semigraph: Let $G:(\sigma,\mu,\eta)$be a fuzzy semigraph on $G^*:(V,£,X)$ Then G is said to be e-bipartite if its vertex set V can be partitioned into $\{V_1,V_2\}$such that $V_1$and V_2 are e- independent.

Definition: 2.3.3

Strongly Bipartite Fuzzy Semigraph: Let $G:(\sigma,\mu,\eta)$ be a fuzzy semigraph on $G^*:(V,£,X)$ Then G is said to be strongly bipartite if its vertex set V can be partitioned into sets $\{V_1,V_2\}$such that $V_1$and V_2 are strongly independent.

Definition 2.3.4

The size of a bipartite fuzzy semigraph $G:(\sigma,\mu,\eta)$is defined to be the sum of the membership values of all edges of it.

Size of $G=\sum_{E\in X}\eta(E)$

Theorem 2.3.5

A fuzzy semigraph $G:(\sigma,\mu,\eta)$is e-bipartite if and only if, its e-graph $G_e$is bipartite.

Proof:

Suppose $G:(\sigma,\mu,\eta)$is e- bipartite, This implies that V can be partitioned into sets $\{V_1, V_2\}$ such that V_1 and V_2 are e- independent.

Therefore no two end vertices of an edge of G belong to V_1

Similarly, no two end vertices of an edge of G belong to V_2

This implies that no two end vertices of an edge of G_e belong to V_1 and no two end vertices of an edge of G_e belong to V_2

Since all the vertices of V_1 and V_2 are e- independent,

V_1 and V_2 are e- independent in G_e.

Therefore G_e is bipartite.

Conversely, suppose G_e is bipartite.

Then the vertex set V where all the vertices are the end vertices of G can be partitioned into sets $\{V_1, V_2\}$ such that V_1 and V_2 are e-independent.

ie . G is e-bipartite.

Theorem 2.3.6

If a fuzzy semigraph $G: (\sigma, \mu, \eta)$ is e-bipartite then it is bipartite.

Proof:

Suppose $G: (\sigma, \mu, \eta)$ is e- bipartite.

This implies that V can be partitioned into sets $\{V_1, V_2\}$ such that V_1 and V_2 are e- independent.

Here $G^*: (V, X)$ is the underlying semigraph and (σ, μ) is a fuzzy graph.

ie. no two end vertices of an edge of Gbelong to V_1.

Similarly, no two end vertices of an edge of Gbelong to V_2.

ie . no edge of G belong to V_1

Similarly, no edge of G belong to V_2.

This implies that $V_1$and V_2 are independent in G.

Hence G is bipartite.

Chapter 3

Fuzzy Graphs associated with given Fuzzy Semigraph

CHAPTER – 3

FUZZY GRAPHS ASSOCIATED WITH GIVEN FUZZY SEMIGRAPH

In this chapter, the end vertex fuzzy graph, adjacency fuzzy graph, consecutive adjacency fuzzy graph are defined and their properties are studied. It is proved that the adjacency fuzzy graph is connected if and only if the given fuzzy semigraph is connected. Many properties of these fuzzy graphs are derived.

3.1 END VERTEX FUZZY GRAPH

Definition 3.1.1

End Vertex Fuzzy Graph (e-Fuzzy Graph) G_e:

Let $G: (\sigma, \mu, \eta)$ be a fuzzy semigraph $on\ G^*: (V, £, X)$.

Define $G_e: (\sigma_e, \eta_e)$on $(V_e, £_e)$ where $V_e = V$

and $£_e = \{uv\ /\ u$ and v are end vertices of an edge E in $G\}$ as

$\sigma_e(u) = \sigma(u)$ for every u in V

and $\eta_e(uv) = \eta(E)$ for every $uv \in £_e$, where E is an edge in G with end vertices u and v in G.

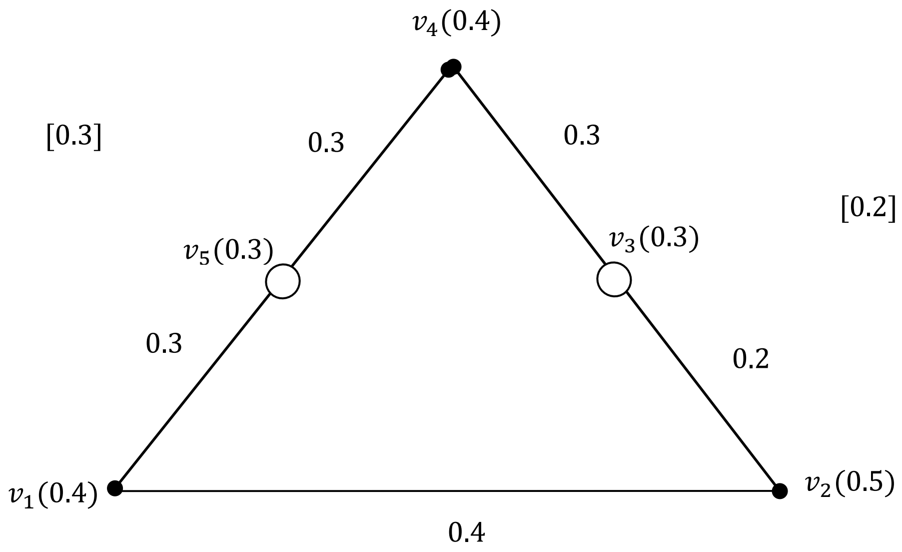
$$\begin{aligned} \text{Now } \eta_e(uv) &= \eta(uv) \\ &\leq \sigma(u) \wedge \sigma(v) \\ &= \sigma_e(u) \wedge \sigma_e(v) \end{aligned}$$

Hence (σ_e, η_e) satisfy the condition of fuzzy graph.

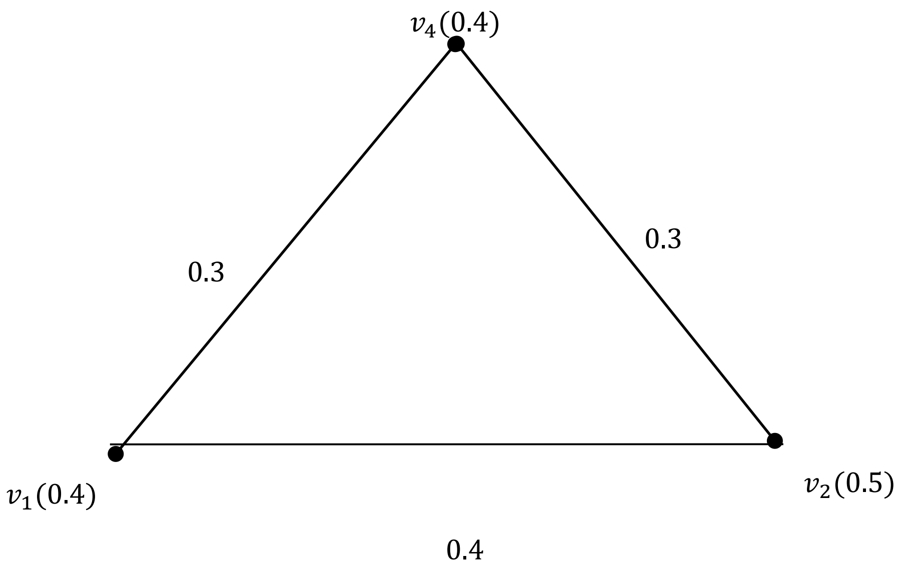
Therefore $G_e: (\sigma_e, \eta_e)$ is a fuzzy graph on (V_e, E_e). This is called the end vertex fuzzy graph of G.

Example 3.1.2

The end vertex fuzzy graph (e-fuzzy graph) G_e of G is given in Figure 2.2.



$\boldsymbol{Fig\ 3.1}: \boldsymbol{G}: (\boldsymbol{\sigma}, \boldsymbol{\mu}, \boldsymbol{\eta})$



$\boldsymbol{Fig\ 3.2}: \boldsymbol{G_e} -$ **End vertex Fuzzy Graph of the Graph**

Theorem 3.1.3:

Let $G: (\sigma, \mu, \eta)$be a connected fuzzy semigraph on $G^*: (V, £, X)$. The end vertex fuzzy graph has no isolated vertices if and only if G has no middle vertices and no middle- end vertices.

Proof:

Since only the end vertices of G are adjacent in G_e, the middle vertices and middle end vertices of G are not adjacent to any vertex of G_e. Hence the theorem follows.

Theorem 3.1.4

Let $G: (\sigma, \mu, \eta)$be a connected fuzzy semigraph. The endvertex fuzzy graph is connected if and only if G has no middle vertices and no middle- end vertices.

Proof:

Since only the end vertices of G are adjacent in G_e, the middle vertices and middle end vertices of G are not adjacent to any vertex of G_e. Hence if G_e is connected, it has no isolated vertices. Therefore G has no middle and middle-end vertex.

Conversely if G has no middle vertices and no middle- end vertices, there is no isolated vertices in G_e. The remaining vertices of G_e correspond to end vertices of G. Also G is connected. Hence G_e is connected.

Theorem 3.1.5

Let G:(σ, μ, η) be a fuzzy semigraph $on\ G^*: (V, £, X)$.Then the size of G and the size of its end vertex fuzzy graph are equal.

Proof: Let G_e be the end vertex fuzzy graph of G.

Let E = ($u_1, u_2, u_3, \ldots, u_n$) be an edge in G.

Then u_1u_n is an edge in G_e and

$\mu_e(u_1, u_n) = \mu(u_1u_2) \wedge \mu(u_2u_3) \wedge \ldots \wedge \mu(u_{n-1}u_n) = \eta(E)$

Therefore $S(G_e) = \sum_{uv \in £_e} \eta_e(uv)$

$= \sum_{(u, \cdots, v) \in E} \eta(u, \cdots, v)$

$= S(G)$.

Theorem 3.1.6

The end vertex fuzzy graph of an effective fuzzy semigraph is an effective fuzzy graph.

Proof:

Let $G: (\sigma, \mu, \eta)$ be an effective fuzzy semigraph on $G^*: (V, £, X)$.

Let uv be any edge in G_e.

Then for any edge E with end vertices u and v,

$$\eta(E) = \sigma(u) \wedge \sigma(v)$$

Let uv be any edge in G_e.

Then u and v are end vertices of G.

Therefore $\eta_e(uv) = \eta(E)$, where E is an edge with end vertices u and v

$$= \sigma(u) \wedge \sigma(v)$$

$= \sigma_e(u) \wedge \sigma_e(v)$, Hence G_e is an effective fuzzy graph.

Theorem 3.1.7

The end vertex fuzzy graph of an e-effective fuzzy semigraph is an effective fuzzy graph.

Proof:

If $G: (\sigma, \mu, \eta)$ is an e-effective fuzzy semigraph on $G^*: (V, £, X)$, then for every pair of end vertices u and v in G,

$$\eta_e(uv) = \eta(u, \cdots, v) = \sigma(u) \wedge \sigma(v) = \sigma_e(u) \wedge \sigma_e(v)$$

Hence the theorem follows.

3.2. ADJACENCY FUZZY GRAPH

Definition 3.2.1

Adjacency Fuzzy Graph (a-Fuzzy Graph) G_a:

Let $G: (\sigma, \mu, \eta)$ be a fuzzy semigraph on $G^*: (V, £, X)$.

Define $G_a: (\sigma_a, \eta_a)$on $(V_a, £_a)$ where $V_a = V$

and $£_a = \{uv \,/ u$ and v are adjacent in $G\}$ as

$\sigma_a(u) = \sigma(u)$ for every u in V and

$\mu_a(uv) = \mu(uv_i) \wedge \wedge \mu(v_i v_{i+1}) \wedge \wedge \ldots \wedge \wedge \mu(v_j v)$ for every $uv \in £_a$,

where $(u, v_i, v_{i+1}, \ldots, v_j, v)$ is an edge or a partial edge of G.

$$\text{Here } \mu_a(uv) \leq \sigma(u) \wedge \sigma(v_i) \wedge \dots \wedge \sigma(v_j) \wedge \sigma(v)$$

$$\leq \sigma(u) \wedge \sigma(v)$$

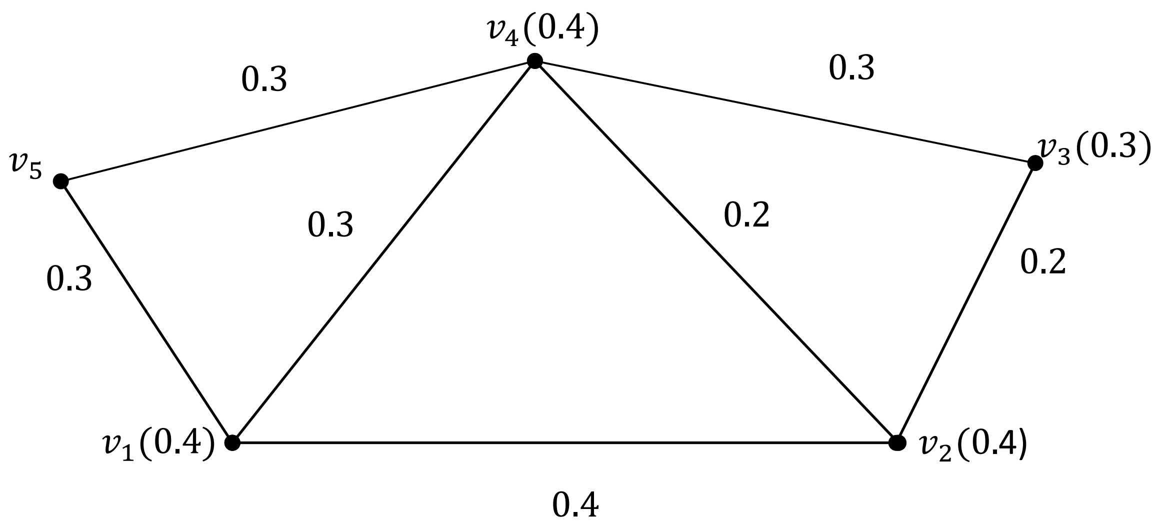
$$= \sigma_a(u) \wedge \sigma_a(v)$$

Hence (σ_a, η_a) satisfy the condition of fuzzy graph.

$G_a: (\sigma_a, \eta_a)$ is called the adjacency fuzzy graph of the semigraph G.

Example 3.2.2

The adjacency fuzzy graph (a-fuzzy graph) $G_a$of G in Fig3.1 is given in Fig.3.3.



$\boldsymbol{Fig3.3}$: $\boldsymbol{G_a}$ – **Adjacency Fuzzy Graph**

Theorem 3.2.3: Let $G:(\sigma,\mu,\eta)$ be a connected fuzzy semigraph on $G^*:(V,£,X)$.The adjacency fuzzy graph is connected if and only if G is connected.

Proof:

Since adjacent vertices in G are all adjacent in G_a, G_a is connected if and only if G is connected.

Theorem 3.2.4

Let $G:(\sigma,\mu,\eta)$be a connected fuzzy semigraph. The adjacency fuzzy graph is connected if and only if G is connected.

Proof:

Since consecutively adjacent vertices in G are all adjacent in G_{ca}, G_{ca} is connected if and only if G is connected.

Remark 3.2.5

Adjacency Fuzzy Graph (Ga-Fuzzy Graph) of an e-effective fuzzy semigraph need not be effective.

Remark 3.2.6

Adjacency fuzzy graph of an effective fuzzy semigraph need not be effective.

Theorem 3.2.7

If G:(σ, μ, η) is both effective and e-effective then its consecutive adjacency fuzzy graph is effective.

Proof:

In the consecutive adjacency fuzzy graph associated with the given fuzzy semigraph G with vertex set V, two vertices are adjacent if and only if they are adjacent in G.

Since G is effective, μ(xy)= $\sigma(x) \wedge \sigma(y) = c, \forall xy \in$ £ and $\eta(E) = \eta\left(u_{1,} u_2, \dots, u_n\right) = \sigma(u_1) \wedge \sigma(u_n)$, where $E = \left(u_{1,} u_2, \dots, u_n\right)$ is an edge in G

Also σ_{ca} (u) = $\sigma(u)$, $\forall$ u$\in$ V_{ca}.

Now μ_{ca} (xy) = μ(xy), $\forall$ xy ϵ E_{ca}

= $\sigma(x) \wedge \sigma(y)$, $\forall$ xy ϵ $£_{ca}$

= $\sigma_{ca}(x) \wedge \sigma_{ca}(y)$, $\forall$ xy ϵ $£_{ca}$

which implies that G_{ca} is effective.

Theorem 3.2.8

Let G^*: (V, E, X) be a semigraph. The subgraph of adjacency graph G_a* induced by the vertices of any edge in G^* is complete.

Proof:

Let E=($u_{1,} u_2, \cdots, u_n$) be any edge in G^*.

Then any two vertices u_i and u_j, i $\neq$ j, are adjacent in G^*.

Hence they are adjacent in G_a*.

Therefore the subgraph induced by the vertices $u_1, u_2, \cdots, u_n$ is complete.

Corollary 3.2.9

Let $G:(\sigma,\mu,\eta)$ be an effective fuzzy semigraph on a semigraph$G^*:(V,E,X)$. Then the fuzzy subgraph of the adjacency fuzzy graph G_a induced by the vertices of any edge in G is complete.

Proof:

Since the adjacency fuzzy graph of an effective fuzzy semigraph is effective, by theorem 3.2.8, the fuzzy subgraph of the adjacency fuzzy graph G_a induced by the vertices of any edge in G is complete.

Theorem 3.2.10

Let $G^*:(V,£,X)$ be a semigraph. Let E=$(u_1, u_2, \cdots, u_n)$ be an edge of G* which has no middle-end vertex. Then the vertices u_2, u_3, $\cdots, u_{n-1}$ are simplicial in the adjacency graph G_a* of G^*.

Proof:

By theorem 3.2.8, the subgraphs induced by u_2, u_3, $\cdots, u_{n-1}$ are all complete. Also E has no middle-end vertex. Therefore the neighbours of u_i for i $\neq 1$, i $\neq n$, are $u_1, u_2, \cdots, u_{i-1}, u_{i+1}, \cdots$, u_{n-1}, u_n in G_a. Hence the subgraph induced by $N[u_2]$, $N[u_3]$, $\cdots, N[u_{n-1}]$ are all complete. Therefore u_2, u_3, $\cdots, u_{n-1}$ are simplicial in G^*.

Corollary 3.2.11

Let $G:(\sigma,\mu,\eta)$ be an effective fuzzy semigraph on a semigraph $G^*:(V,£,X)$ with more than one edge. Let E=(u_1,u_2 , ⋯,u_n) be a edge of G which has no middle-end vertex. Then the vertices u_2 , u_3 ,⋯,u_{n-1} are simplicial in the adjacency graph G_a* of G*.

Proof:

Since the adjacency fuzzy graph of an effective fuzzy semigraph is effective, the result follows from theorem 3.2.10.

Remark 3.2.12

If the semigraph G^* has more than one edge, then the end vertices u_1 and u_n will be adjacent to other end vertices. Hence they need not be adjacent in G_a.

Theorem 3.2.13

If the semigraph G^* has only one edge, then all the vertices of the adjacency graph G_a* are simplicial vertices.

Proof:

By theorem 3.2.8, the subgraph induced by the edge is complete. Therefore the adjacency graph G_a* is complete. Therefore for any vertex u, the subgraph induced by N[u] is complete. Hence all the vertices of the adjacency graph G_a* are simplicial vertices.

Theorem 3.2.14

Let $G:(\sigma,\mu,\eta)$ be an effective fuzzy semigraph with only one edge. Then all the vertices of the adjacency fuzzy graph G_a are simplicial vertices.

Proof:

By Corollary 3.2.13, all the vertices of the adjacency graph G_a* are simplicial vertices. Also since G is effective, the adjacency fuzzy graph G_a is also effective. Hence all the vertices of the adjacency fuzzy graph G_a* are simplicial vertices.

3.3 CONSECUTIVE ADJACENCY FUZZY GRAPH

Definition 3.3.1

Let $G:(\sigma,\mu,\eta)$ be a fuzzy semigraph on $G^*:(V,£,X)$.

Define $G_{ca}:(\sigma_{ca},\eta_{ca})$ on (V_{ca}, $£_{ca}$) where $V_{ca}=V$ and

$£_{ca}=\{uv\ /u \text{ and } v \text{ are consecutively adjacent in } G\}$ as,

$\sigma_{ca}(u)=\sigma(u)$ for every u in V

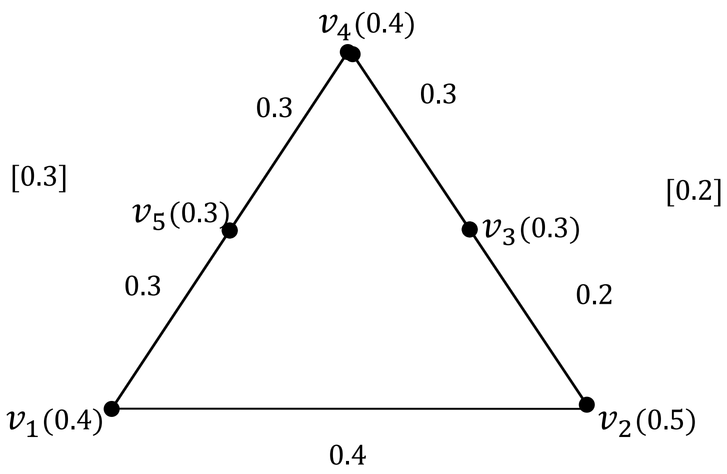
and $\mu_{ca}(uv)=\mu(uv)$ for every $uv\in £_{ca}$,

$$\begin{aligned}\text{Then }\ \mu_{ca}(uv) &= \mu(uv)\\ &\le \sigma(u)\wedge\sigma(v)\\ &= \sigma_{ca}(u)\wedge\sigma_{ca}(v)\end{aligned}$$

Hence (σ_{ca},η_{ca}) satisfy the condition of fuzzy graph. This is called the consecutive adjacency fuzzy graph o rca-fuzzy graph G_{ca}.

Example 3.3.2:

The consecutive adjacency fuzzy graph (*ca*-fuzzy graph) G_{ca} of G in Fig.3.1 is given in Figure 3.4.



$\boldsymbol{Fig3.4}$: $\boldsymbol{G_{ca}}$ – **Consecutive adjacency Fuzzy Graph**

Remark 3.3.3

Consecutive Adjacency fuzzy graph of an *e*-effective fuzzy semigraph need not be effective.

Theorem 3.3.4

G is an effective fuzzy semigraph if and only if the consecutive adjacency fuzzy graph of G is an effective fuzzy graph.

Proof:

In the consecutive adjacency vertex fuzzy graph G_{ca}: (σ_{ca},μ_{ca}) with vertex set V, two vertices are adjacent if and only if they are consecutively adjacent in G. Also $\sigma_{ca}(u) = \sigma(u)$ for every u in V and $\mu_{ca}(u) = \mu(uv)$ for every pair of consecutive adjacent vertices u and v in G.

Hence G is effective if and only if $\mu(uv) = \sigma(u) \wedge \sigma(v)$

if and only if $\mu_{ca}(uv) = \sigma_{ca}(u) \wedge \sigma_{ca}(v)$

if and only if G_{ca} is effective.

Chapter 4

Various Isomorphisms on Fuzzy Semigraphs

CHAPTER – 4

VARIOUS ISOMORPHISMS OF FUZZYSEMIGRAPHS

In this chapter, isomorphism, weak isomorphism and co-weak isomorphism of fuzzy semigraphs are introduced and some of their properties are studied. End vertex isomorphism (ev-isomorphism), edge isomorphism (e-isomorphism) and adjacency isomorphism (a-isomorphism) of fuzzy semigraphs are defined. Properties of effective edges and effective fuzzy semigraphs under isomorphism are studied. Also, it is proved that isomorphism is an equivalence relation and week isomorphism is a partial order relation.

4.1. VARIOUS ISOMORPHISMS

In this section four types of isomorphisms, namely, isomorphism, end vertex isomorphism, an edge isomorphism (e-isomorphism), an adjacency isomorphism (a-isomorphism) are defined.

Definition 4.1.1:

Let $G:(\sigma,\mu,\eta)$ and $G':(\sigma',\mu',\eta')$ be two fuzzy semigraphs with underlying semigraphs $G^*:(V,£,X)$ and $G'^*:(V'£',X')$. An isomorphism of fuzzy semigraphs $f:G\to G'$ is a bijective map denoted by $f:V\to V'$ which satisfies

1. If $E=(v_1,v_2,\ldots,v_n)$ is an edge in G then $\{f(v_1),f(v_2),\ldots,f(v_n)\}$forms an edge in G'

2. $\sigma(u) = \sigma'(f(u))$ for all $u \in$ V,

3. $\mu(uv) = \mu'(f(u)f(v))$ for all $uv \in$ £ and

4. $\eta(E) = \eta'(f(E))$, for all $E \in$ X.

Definition 4.1.2

An **end vertex isomorphism (ev-isomorphism)** of fuzzy semigraphs $f: G \to G'$ is a bijective $f: V \to V'$ which satisfies

1. If $E = (v_1, v_2, \dots, v_n)$ is an edge in G then $\{f(v_1), f(v_2), \dots, f(v_n)\}$ forms an edge in G' with end vertices $f(v_1)$ and $f(v_n)$.
2. $\sigma(u) = \sigma'(f(u))$ for all $u \in$ V,
3. $\mu(uv) = \mu'(f(u)f(v))$ for all $uv \in$ £ and
4. $\eta(E) = \eta'(f(E))$, for all $E \in$ X.

Definition 4.1.3:

An **edge isomorphism (e-isomorphism)** of fuzzy semigraphs $f: G \to G'$ is a bijective map $f: V \to V'$ which satisfies

1. If $E = (v_1, v_2, \dots, v_n)$ is an edge in G, then $\{f(v_1), f(v_2), \dots, f(v_n)\}$ forms an edge in G'
2. $\sigma(u) = \sigma'(f(u))$ for all $u \in$ V,
3. $\mu(uv) = \mu'(f(u)f(v))$ for all $uv \in$ £ and
4. $\eta(E) = \eta'(f(E))$, for all $E \in$ X.

Definition 4.1.4

An **adjacency isomorphism (a-isomorphism)** of fuzzy semigraphs $f: G \rightarrow G'$ is a bijective $f: V \rightarrow V'$ which satisfies

1. the adjacent vertices in G are mapped onto adjacent vertices in G',
2. $\sigma(u) = \sigma'(f(u))$ for all $u \in$ V,
3. $\mu(uv) = \mu'(f(u)f(v))$ for all $uv \in$ £ and
4. $\eta(E) = \eta'(f(E))$, for all $E \in$ X.

Theorem 4.1.5

An edge isomorphism of a fuzzy semigraph is an adjacency isomorphism but the converse need not be true.

Proof:

Since any two vertices in an edge of a fuzzy semigraph are adjacent, the theorem follows.

Remark 4.1.6

The converse of theorem 4.15 need not be true.

Theorem 4.1.7

An end vertex isomorphism of a fuzzy semigraph is an edge isomorphism.

Proof:

For en end vertex isomorphism, if $E = (v_1, v_2, \dots, v_n)$ is an edge in G, then $(f(v_1), f(v_2), \dots, f(v_n))$forms an edge in G' with end vertices $f(v_1)$ and $f(v_n)$. Hence f is an edge isomorphism.

Remark 4.1.8

The converse of theorem 4.1.7 need not be true.

Theorem 4.1.9

An isomorphism of a fuzzy semigraph is both an edge isomorphism and the end vertex isomorphism.

Proof:

For an isomorphism, if $E = (v_1, v_2, \dots, v_n)$ is an edge in G then $(f(v_1), f(v_2), \dots, f(v_n))$ forms an edge in G'.

It follows that $\{f(v_1), f(v_2), \dots, f(v_n)\}$ forms an edge in G' with end vertices $f(v_1)$ and $f(v_n)$. Hence it is edge isomorphism as well as end vertex isomorphism.

Theorem 4.1.10

Isomorphism between fuzzy semigraphs is an equivalence relation.

Proof:

Let $G: (\sigma, \mu, \eta)$,$G': (\sigma', \mu', \eta')$and $G'': (\sigma'', \mu'', \eta'')$ be fuzzy semigraphs with vertex sets V, $V'$and V''a respectively.

Let $f: V \to V'$ be such that $f(v) = v, \forall\, v \in V$. This mapping f is a bijection.

Also, $\sigma(u) = \sigma\big(f(u)\big)$ for all $u \in V$,

$\mu(uv) = \mu(f(u)f(v))$ for all $uv \in$ £,

$\eta(E) = \eta(f(E))$ for all $E \in X$.

Thus f is an isomorphism from G to itself.

Hence isomorphism is a reflexive relation.

Let $f: G \to G'$ be an isomorphism between the fuzzy semigraphs G and G' then the mapping $f: V \to V'$ is

$\sigma(u) = \sigma'\big(f(u)\big)$ for all $u \in V$ and ...... (1)

$\mu(uv) = \mu'(f(u)f(v)\,)$ for all $uv \in$ £ ...... (2)

η(E)= $\eta'(f(E))$, for all $E \in X$ ...... (3)

Since f is bijective, for u' in V', there exists u in V such that $f^{-1}(u') = u$. Hence by (1) $\sigma(f^{-1}(u')) = \sigma'\big(\text{f(u)}\big) = \sigma'(u')$ for all $u' \in V'$.

Similarly,

$\mu(f^{-1}(u')f^{-1}(v')) = \big(f(u)f(v)\big) = (\mu'(u'v'))$ for all $u'v' \in$ £'... (4)

Hence we get a 1-1, onto map $f^{-1}: V \to V'$ whichis an isomorphism.

Thus G is isomorphic to G' implies G' is isomorphic to G.

Hence isomorphism satisfies the symmetric relation.

Let $f: V \to V'$ and $g: V' \to V''$ be isomorphisms from fuzzy semigraphs G to G' and G' to G'' respectively.

Then $g\ \circ$ f is 1-1 and onto map from $V \to V''$ where

$(g\ \circ \text{f}\,)(u) = g(f(u)$, for all $u \in V$

Since $f: G \rightarrow G'$is an isomorphism between the fuzzy semigraphs G and G'then the mapping $f: V \rightarrow V'$ given by

$f(v) = v'$for all v in V is bijective and satisfies

$$\sigma(u) = \sigma'(f(u)) \text{ for all } u \in V \text{ and} \qquad \text{...... (5)}$$

$$\mu(uv) = \mu'(f(u)f(v)) \text{ for all } uv \in \text{£} \qquad \text{...... (6)}$$

Since $g: G' \rightarrow G''$is an isomorphism between the semigraphs G' and G'' then the mapping $g: V' \rightarrow V''$ given by

$$g(v') = v'' \text{for all } v' \text{in } V' \qquad \text{...... (7)}$$

is bijective and satisfies

$$\mu'(u'v') = \mu''(g(u')g(v')) \text{ for all } u'v' \in \text{£}' \qquad \text{...... (8)}$$

Using (5) and (7) and using $f(v) = v' f$or all v in V

$\sigma(u) = \sigma'(u') = \sigma''(u'') = \sigma''(g(u')) \ \ = \sigma''(g\big(f(u)\big))$for all u in V

From (6) and (8), we have

$$\begin{aligned} \mu(uv) &= \mu'(f(u)f(v)) \\ &= \mu'(u'v') \\ &= \mu''(u''v'') \\ &= \mu''((g(u')g(v')) \\ &= \mu''((g(f(u)g(f(v)), \forall uv \in \text{£} \end{aligned}$$

Therefore $g \circ f$is an isomorphism between G and G'.

Hence isomorphism between fuzzy semigraphs is transitive and hence it is an equivalence relation.

Theorem 4.1.11

Let $G:(\sigma,\mu,\eta)$ and $G':(\sigma',\mu',\eta')$ be two isomorphic fuzzy semigraphs, then an edge in G is an effective edge if and only if the corresponding image edge in $G'$is effective.

Proof:

Let $f: G \rightarrow G'$ be an isomorphism between the fuzzy semigraphs G and $G'$with underlying sets V and V'.

Let E be an effective edge in Gthen

$\eta(E) = \mu(u_1u_2) \wedge \mu(u_2u_3) \wedge \dots \wedge \mu(u_{n-1}u_n) = \sigma(u_1) \wedge \wedge\, \sigma(u_n)$ ...... (9)

Where $E = (u_1, u_2, \dots, u_n)$

Since f is an isomorphism, $\mu(uv) = \mu'\big(f(u)f(v)\big)$ for all $uv \in £$ and

Since $x$is an effective edge.We have, $\mu(uv) = \sigma(u) \wedge \sigma(v)$, $\forall uv \in £$

$\eta'(f(x)) = \mu'((f(u_1)f(u_2)) \wedge \mu'((f(u_2)f(u_3)) \wedge \dots \wedge \mu'((f(u_{n-1})f(u_n))$

$= \mu(u_1u_2) \wedge \mu(u_2u_3) \wedge \dots \wedge \mu(u_{n-1}u_n)$ (Since f is an isomorphism)

$= \sigma(u_1) \wedge \sigma(u_n)$, (Using (9))

$= \sigma'(f(u_1)) \wedge \sigma'(f(u_n))$ (Since f is an isomorphism)

Hence $f(E)$ is an effective edge in G'.

Conversely, suppose $f(E)$ is an effective edge in G'.

Since f is a bijective isomorphism, the pre- image of the edge $f(E)$in $G'$is also effective in G.

Theorem 4.1.12

If G and G' are isomorphic fuzzy semigraphs then G is an effective fuzzy semigraph if and only if $G'$is also effective.

Proof:

Since G is isomorphic to G', there is an isomorphism $f: G \to G'$which is a bijection and satisfies

1. $E = (v_1, v_2, \dots, v_n)$ is an edge in G, then $(f(v_1), f(v_2), \dots, f(v_n))$ forms an edge in G'.
2. $\sigma(u) = \sigma'(f(u))$ for all $u \in V$.
3. $\mu(uv) = \mu'(f(u)f(v))$ for all $uv \in £$ and
4. $\eta(E) = \eta'(f(E))$, for all $E \in X$.

Since G is effective

$\eta(E) = \mu(u_1u_2) \wedge \mu(u_2u_3) \wedge \dots \wedge \mu(u_{n-1}u_n) = \sigma(u_1) \wedge \sigma(u_n)$, for all $E \in X$.

and $\mu(u_iu_{i+1}) = \sigma(u_i) \wedge \sigma(u_{i+1})$ for all i.

Therefore

$$\begin{aligned}\mu'(f(u_i)f(u_{i+1})) &= \mu(u_iu_{i+1})\\ &= \sigma(u_i) \wedge \sigma(u_{i+1})\\ &= \sigma'(f(u_i)) \wedge \sigma'(f(u_{i+1})), \text{ for all i.}\end{aligned}$$

Also $\eta(E) = \sigma(u_1) \wedge \sigma(u_n)$ gives

$\eta'(f(E)) = \sigma'(f(u_1)) \wedge \sigma'((u_n))$, for all $E \in X$.

Hence G' is effective.

Theorem 4.1.13

Isomorphism of fuzzy semigraphs preserves order and size.

Proof:

Let G:(σ, μ, η) and G′:(σ', μ', η') be two fuzzy semigraphs on G*:(V, £, X) and G′*:(V′, £ ′, X′) respectively.

Let f: G→G' be an isomorphism between the fuzzy semigraphs G and G′.

Then the mapping f: V→ $V'$is such that,

σ(u)= σ'(f(u)) for all u∈V and

μ(uv)= μ'(f(u)f(v)) for all uv ∈ £

η(E)= $\eta'(f(E))$, for all E∈ X.

Therefore

Order of G = $\sum_{u \in V} \sigma(u)$

= $\sum_{u \in V} \sigma'(f(u))$

= Order of G′

Size of G = S(G) = $\sum_{E \in X} \eta\ (E)$ = $\sum_{E \in X} \eta'(f(E))$ = Size of G′

Similarly T S(G) = T S(G′).

Remark 4.1.14

Converse of the above theorem 4.1.13 need not be true.

4.2. WEAK ISOMORPHISMS

Definition 4.2.1

Let $G:(\sigma,\mu,\eta)$ and $G':(\sigma',\mu',\eta')$ be two fuzzy semigraphs with underlying semigraphs $G^*(V,£\,,X)$ and $G'^*(V',£',X')$.

A **weak isomorphism of fuzzy semigraphs** $f:G\to G'$ is a bijective map $f:V\to V'$ which satisfies

1. If $E=(v_1,v_2,\dots,v_n$) is an edge in G then$\{f(v_1),f(v_2),\dots,f(v_n)\}$forms an edge in G'.
2. $\sigma(u)=\sigma'(f(u))$ for all $u\in V$.
3. $\mu(uv)\le\mu'(f(u)f(v))$ for all $uv\in £$.

A **weak-end vertex isomorphism (weak-ev isomorphism)** of fuzzy semigraphs $f:G\to G'$ is a bijective map $f:V\to V'$which satisfies,

1. If $E=(v_1,v_2,\dots,v_n)$ is an edge in G then $\{f(v_1),f(v_2),\dots,f(v_n)\}$forms an edge in $G'$with end vertices $f(v_1)$ and $f(v_n)$.
2. $\sigma(u)=\sigma'(f(u))$ for all $u\in V$.
3. $\mu(uv)\le\mu'(f(u)f(v))$ for all $uv\in £$.

A **weak-edge isomorphism (weak-e isomorphism)** of fuzzy semigraphs

$f:G\to G'$ is a bijective map $f:V\to V'$which satisfies,

1. If $E = (v_1, v_2, \dots, v_n)$ is an edge in G then $\{f(v_1), f(v_2), \dots, f(v_n)\}$forms an edge in $G'$with end vertices $f(v_1)$ and $f(v_n)$.
2. $\sigma(u) = \sigma'(f(u))$ for all $u \in V$.
3. $\mu(uv) \leq \mu'(f(u)f(v))$ for all $uv \in$ £.

A **weak-adjacency isomorphism (weak-a isomorphism)** of fuzzy semigraphs $f: G \rightarrow G'$ is a bijective map $f: V \rightarrow V'$which satisfies,

1. If the adjacent vertices in G are mapped onto adjacent vertices in G′,
2. $\sigma(u) = \sigma'(f(u))$ for all $u \in V$.
3. $\mu(uv) \leq \mu'(f(u)f(v))$ for all $uv \in$ £.

Theorem 4.2.2

Weak is isomorphism between fuzzy semigraphs is a partial order relation.

Proof:

Let $G: (\sigma, \mu, \eta)$,$G': (\sigma', \mu', \eta')$ and$G'': (\sigma'', \mu'', \eta'')$be fuzzy semigraphs with vertex sets V, V' and V''respectively.

Let $f: V \rightarrow V'$such that $f(v) = v, \forall v \in V$.

This mapping f is a bijection.

Hence $\sigma(u) = \sigma(f(u))$ for all $u \in V$,

$\mu(uv) = \mu(f(u)f(v))$ for all $uv \in$ £

Thus $f$is a weak isomorphism to itself and hence it satisfies reflexive relation.

Thus G is weak isomorphic to itself.

Let $f: V \to V'$and $g: V' \to V$be weak isomorphisms on fuzzy semigraphs G to G' and G' to G respectively

Then $f: V \to V'$ is a bijective map $(u) = u'$, $\forall u \in V$ satisfying

$\sigma(u) = \sigma'(f(u)), \forall u \in V$ and

$\mu(uv) \leq \mu'\big(f(u)f(v)\big), \forall uv \in £.$ .... (10)

Similarly $g: V' \to V$is a bijective map $(u') = u$, $\forall u' \in V'$ satisfying

$\sigma'(u') = \sigma\big(g(u')\big) = \sigma(g\big(f(u)\big)), \forall u' \in V'$ and

$\mu'(u'v') \leq \mu\big(g(f(u)gf(v)\big), \forall u'v' \in £'.$ .... (11)

Inequalities (10) and (11) hold good on the underlying vertex sets V and V' only when G and G' have the same number of edges and the corresponding membership values of the edges are equal. Hence G and G' are identical.

Thus Weak isomorphism between fuzzy semigraphs is antisymmetric.

Let $f: V \to V'$ and $g: V' \to V''$ be weak isomorphisms on fuzzy semigraphs G to G' and G' to G''respectively.

Then $g \circ f$is 1-1 and onto map from $V \to V''$where

$(g \circ f)\ (u) = g(f(u))$, for all $u \in V$

Since $f: G \rightarrow G'$is a weak isomorphism between the fuzzy semigraphs G and G' then the mapping $f: V \rightarrow V'$is bijective,

$f(v) = v'$for all $v$in V such that

$\sigma(u) = \sigma'(f(u))$ for all $u \in V$ and .... (12)

$\mu(uv) \leq \mu'((f(u)f(v))$ for all $uv \in £$ .... (13)

Since $g: G' \rightarrow G''$is an isomorphism between the fuzzy semigraphs G' and G''then the mapping $g: V' \rightarrow V''$is bijective.

$g(v') = v''$ for all v inV' such that

$\sigma'(u') = \sigma''(g(u'))$ for all $u' \in V'$ .... (14)

and $\mu'(u'v') \leq \mu''(g(u')g(v'))$ for all $u'v' \in V'$ .... (15)

Using (12), (14) and using $f(v) = v'$for all v in$V$

$\sigma(u) = \sigma'(u') = \sigma''(u'') = \sigma''(g(u'))$

$= \sigma''(g(f(u)))$, for all u in V

From (13) and (15), we have

$\mu(uv) \leq \mu'((f(u)f(v)))$, for all $u, v \in V$

$= \mu'(u'v')$

$= \mu''(u''v'')$

$= \mu''(g(u')g(v'))$,

$\leq \mu''(g(f(u))g(f(v)))$,$\forall u, v \in V$

Therefore $g \circ f$ is a weak isomorphism between G and G''

Hence isomorphism between fuzzy semigraphs is transitive and hence the weak isomorphism between fuzzy semigraphs is a partial order relation.

Theorem 4.2.3

If $f: G \to G'$ is a weak isomorphism on fuzzy semigraphs G and G' and if G is an effective fuzzy semigraph, then G' is also an effective fuzzy semigraph.

Proof:

Since $f: G \to G'$ is a weak isomorphism, it is bijective and satisfies if $E = (v_1, v_2, \dots, v_n)$ is an edge in G, then $= (f(v_1), f(v_2), \dots, f(v_n))$is an edge in G',

$\sigma(u) = \sigma'(f(u))$, for all $u \in V$

$\mu(uv) \leq \mu'(f(u)f(v))$ for all $uv \in$ £

Since G is an effective fuzzy semigraph,

$\mu(uv) = \sigma(u) \wedge \sigma(v)$

Now $\mu'(f(u)f(v)) = \mu(uv)$

$= \sigma(u) \wedge \sigma(v)$

$\geq \sigma'\big(f(u)\big) \wedge \sigma'(f(v))$

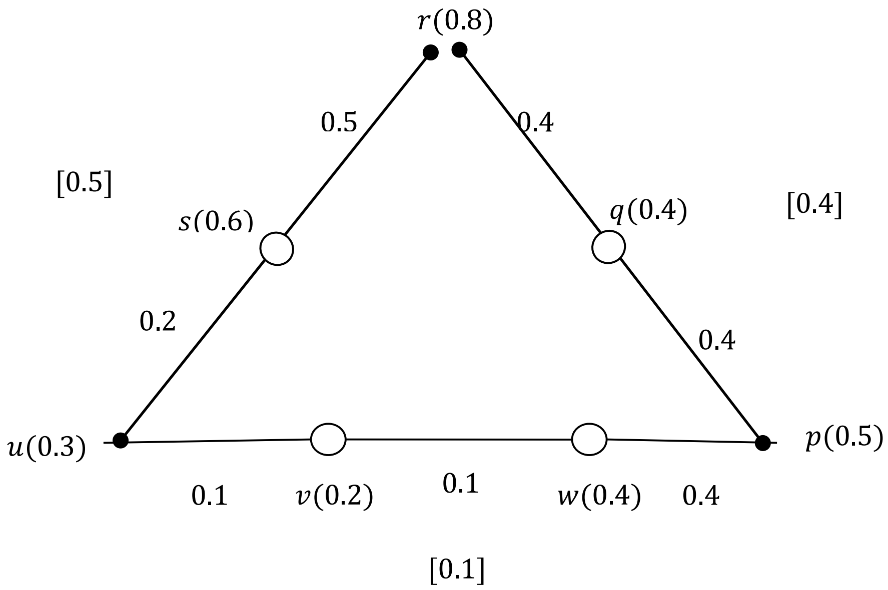
But $\mu'(f(u)f(v)) \leq \sigma'\big(f(u)\big) \wedge \sigma'(f(v))$

Hence $\mu'\big(f(u)f(v)\big) = \sigma'\big(f(u)\big) \wedge \sigma'(f(v))$

Thus G is an effective fuzzy semigraph.

Remark 4.2.4

If G is weak isomorphic to G′, then the effectiveness of G′ need not imply the effectiveness of G. The fuzzy semigraph G in Fig.4.1 is weak isomorphic to the fuzzy semigraph G' in Fig.4.2. Here G' is effective but G is not effective.



$\boldsymbol{Fig\ 4.1}$: $\boldsymbol{G}$: $(\boldsymbol{\sigma}, \boldsymbol{\mu}, \boldsymbol{\eta})$

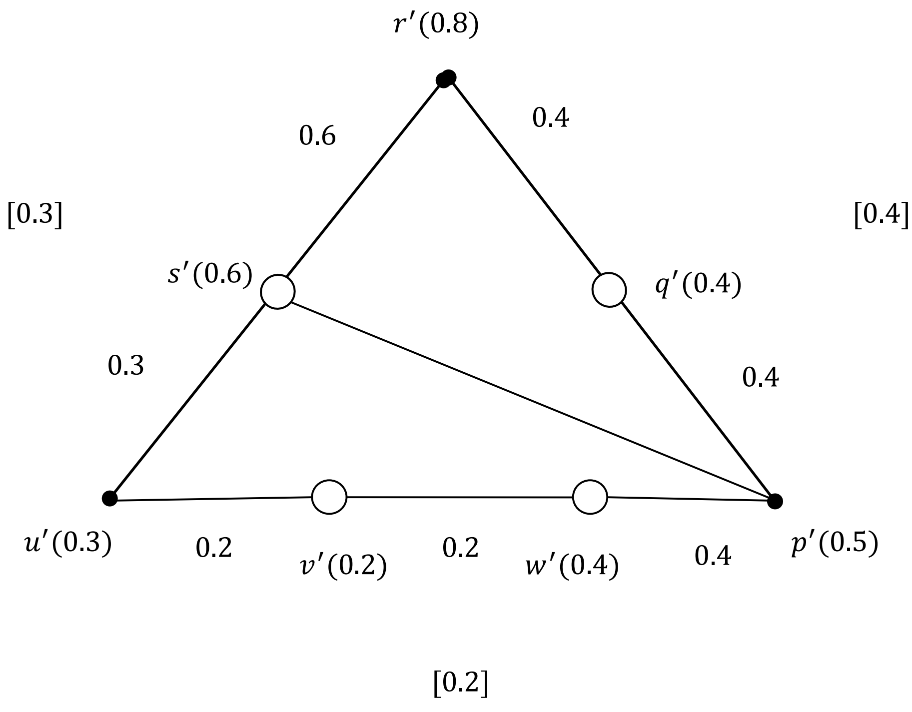


Fig 4.2: $\boldsymbol{G}'$: $(\boldsymbol{\sigma}', \boldsymbol{\mu}', \boldsymbol{\eta}')$

Remark 4.2.5

It is observed from Fig 4.1 and Fig.4.2 that order of fuzzy semigraphs are preserved under weak isomorphisms.

Theorem 4.2.6

The weak isomorphism of fuzzy semigraphs preserves order.

Proof:

Let G:(σ, μ, η) and G′:(σ', μ', η') be two fuzzy semigraphs on G*:(V, £, X) and G′*:(V′, £′, X′) respectively.

Let f: G→G' be an isomorphism between the fuzzy semigraphs G and G′.

Then the mapping f: V→ V′is such that,

$\sigma(u) = \sigma'(f(u))$ for all $u \in V$ and

$\mu(uv) \leq \mu'(f(u)f(v))$ for all $uv \in$ £

Therefore Order of G is $= \sum_{u \in V} \sigma(u)$

$= \sum_{u \in V} \sigma'(f(u))$

= Order of G′

Remark 4.2.7

Converse of the above theorem 4.2.5 need not be true.

Remark 4.2.8

The week isomorphism need not preserve effective property.

4.3. CO -WEAK ISOMORPHISMS

Definition 4.3.1

A co-weak isomorphism of fuzzy semigraphs $f: G \to G'$ is a bijective map $f: V \to V'$ which satisfies

1. If $E = (v_1, v_2, \dots, v_n)$ is an edge in G then $\{f(v_1), f(v_2), \dots, f(v_n)\}$ forms an edge in G'.
2. $\sigma(u) \leq \sigma'(f(u))$, for all $u \in V$.
3. $\mu(uv) = \mu'(f(u)f(v))$,for all $uv \in$ £.
4. $\eta(E) = \eta'(f(E))$, for all $E \in X$.

Definition 4.3.2

Let $G: (\sigma, \mu, \eta)$ and $G': (\sigma', \mu', \eta')$ be two fuzzy semigraphs with underlying semigraphs $G^*(V, £, X)$ and $G'^*(V', £', X')$ respectively. A co-weak end vertex isomorphism (co-weak ev-isomorphism) of fuzzy semigraphs $f: G \to G'$is a bijective map denoted by $f: V \to V'$and which satisfies,

1. If $E = (v_1, v_2, \dots, v_n)$ is an edge in G then $\{f(v_1), f(v_2), \dots, f(v_n)\}$ forms an edge in $G'$with end vertices $f(v_1)$ and $f(v_n)$,
2. $\sigma(u) \leq \sigma'(f(u))$ for all $u \in V$.
3. $\mu(uv) = \mu'(f(u)f(v))$ for all $uv \in$ £.
4. $\eta(E) = \eta'(f(E))$, for all $E \in X$.

Definition 4.3.3

A co-weak edge isomorphism (co-weak edge isomorphism) of fuzzy semigraphs $f: G \to G'$ is a bijective map denoted by $f: V \to V'$and which satisfies

1. If $E = (v_1, v_2, \dots, v_n)$ is an edge in G then $\{f(v_1), f(v_2), \dots, f(v_n)\}$ forms an edge in G'
2. $\sigma(u) \leq \sigma'(f(u))$ for all $u \in V$.
3. $\mu(uv) = \mu'(f(u)f(v))$ for all $uv \in$ £.
4. $\eta(E) = \eta'(f(E))$, for all $E \in X$.

Definition 4.3.4

A co-weak adjacency isomorphism (co-weak a isomorphism) of fuzzy semigraphs $f: G \to G'$ is a bijective map $f: V \to V'$which satisfies,

1. If the adjacent vertices in G are mapped onto adjacent vertices in G'
2. $\sigma(u) \leq \sigma'\big(f(u)\big)\ for\ all\ u \in V,$
3. $\mu(uv) = \mu'(f(u)f(v))$ for all $uv \in$ £ and
4. $\eta(E) = \eta'(f(E))$, for all $E \in X$.

Theorem 4.3.5

If $f: G \to G'$ is a co-weak isomorphism on fuzzy semigraphs G and G' and if G' is an effective fuzzy semigraph then G is also an effective fuzzy semigraph.

Proof:

Since $f: G \rightarrow G'$ is a co-weak isomorphism, it is bijective and satisfies if $E = (v_1, v_2, \dots, v_n)$ be an edge in G then $= (f(v_1), f(v_2), \dots, f(v_n))$is an edge in G'.

$\sigma(u) \leq \sigma'(f(u))$, for all $u \in V$ .... (16)

$\mu(uv) = \mu'(f(u)f(v))$ for all $uv \in$ £

and $\eta(E) = \eta'(f(E))$, for all $E \in X$.

Since G' is an effective fuzzy semigraph,

$\mu'((f(u)f(v)) = \sigma'(f(u)) \wedge \sigma'(f(v)), \forall\, f(u)f(v) \in$ £′ .... (17)

Now $\mu(uv) = \mu'(f(u)f(v))$

$= \sigma'(f(u)) \wedge \sigma'(f(v))$ (using (17)

$\geq \sigma(u) \wedge \sigma(v)$) .... (18)

But $\mu(uv) \leq \sigma(u) \wedge \sigma(v)$ .... (19)

Using (18) and (19),

$\mu(uv) = \sigma(u) \wedge \sigma(v)$, for all $uv \in$ £ .... (20)

Thus G is an effective fuzzy semigraph.

Remark 4.3.6

The effectiveness of G need not imply the effectiveness of G' when G is co-week isomorphic to G'.

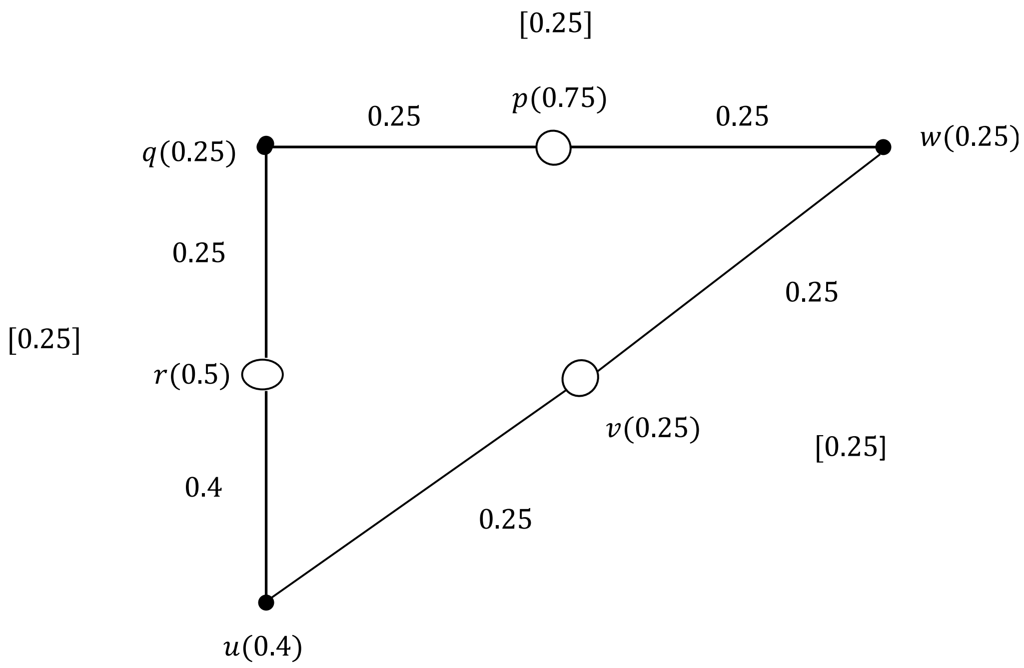


Fig 4.3: $\boldsymbol{G}: (\boldsymbol{\sigma}, \boldsymbol{\mu}, \boldsymbol{\eta})$

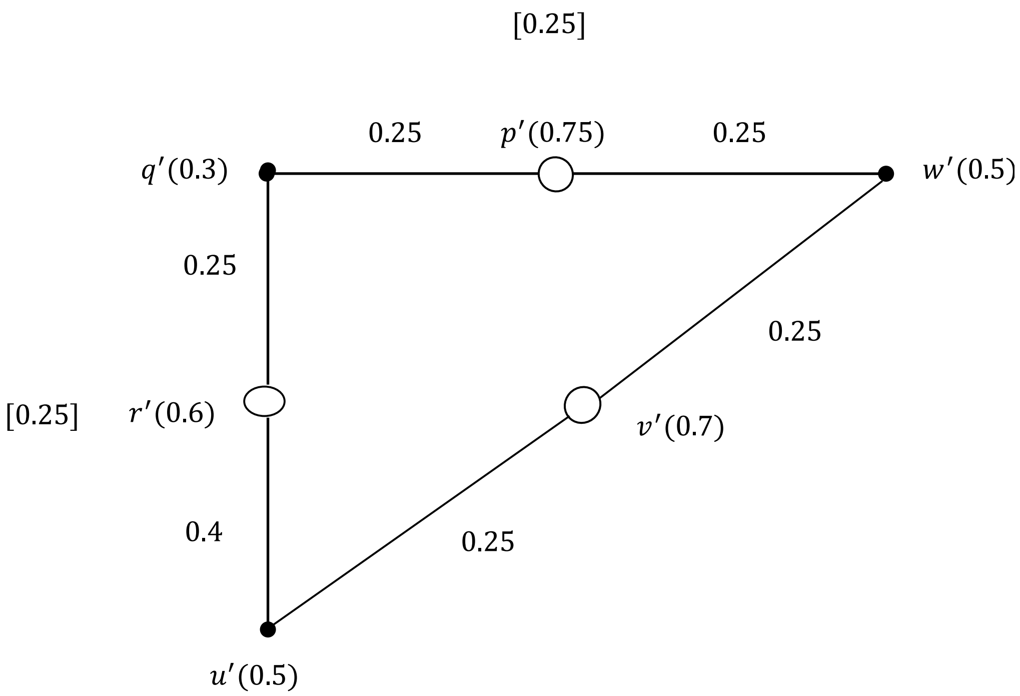


Fig 4.4: $\boldsymbol{G}': (\boldsymbol{\sigma}', \boldsymbol{\mu}', \boldsymbol{\eta}')$

Here G is co-weak isomorphic to G' . G is an effective fuzzy semigraph but G is not effective.

Remark 4.3.7

It is observed from Fig 4.3 and 4.4 that co-weak isomorphisms of fuzzy semigraphs preserve the size of fuzzy semigraphs.

Theorem 4.3.8

The co-weak Isomorphism of fuzzy semigraphs preserves the size of fuzzy semigraphs.

Proof:

Let G:(σ, μ, η) and G$'$:(σ', μ', η') be two fuzzy semigraphs on G*:(V, £, X) and G$'$*:(V$'$, £$'$, X$'$) respectively.

Let f: G→G$'$ be an isomorphism between the fuzzy semigraphs G and G$'$.

Then the mapping f: V→ $V'$is such that,

σ(u)≤ σ'(f(u)) for all u∈V and

μ(uv)= μ'(f(u)f(v)) for all uv ∈ £

η(E)= $\eta'(f(E))$, for all E∈ X.

Therefore

Size of G = S(G) = $\sum_{E \in X} \eta\ (E)$ = $\sum_{E \in X} \eta'(f(E))$ = Size of G$'$.

Similarly T S(G) = T S(G$'$).

Remark 4.3.9

Converse of the above theorem 4.3.8 need not be true.

That is, there are fuzzy semigraphs with same size which are not co- weak isomorphic.

For example consider the fuzzy semigraphs G in Fig.4.5 and G′ in Fig.4.6.

Size of G = S(G) = 0.75 = Size of G′.

Total size of G = 1.65 = Total size of G′.

But the fuzzy semigraph *G* is not co-weak isomorphic to G′.

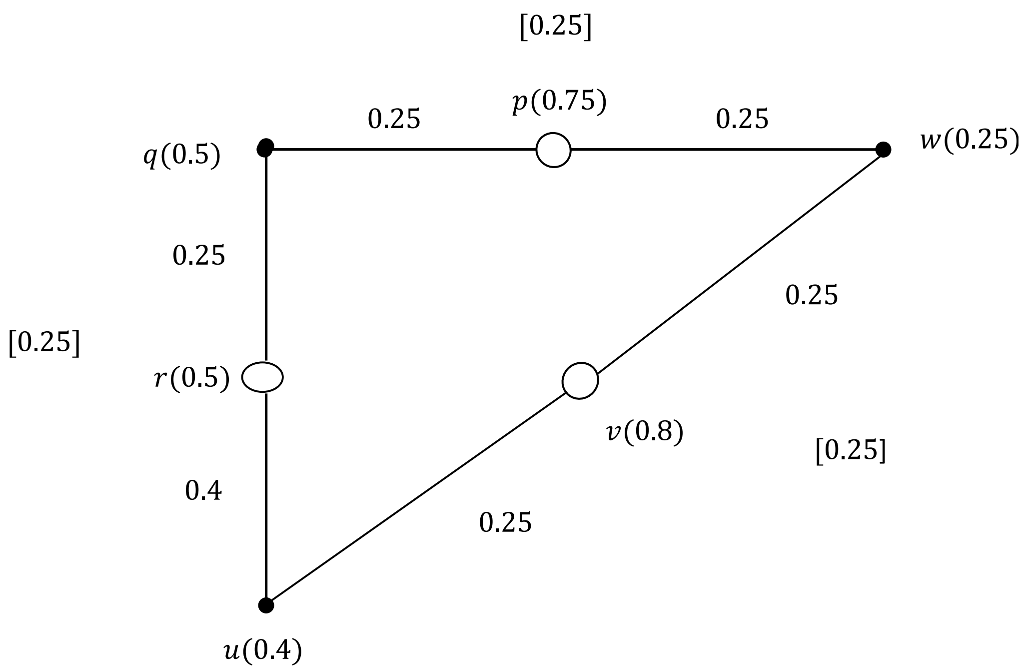


Fig 4.5: $\boldsymbol{G}: (\boldsymbol{\sigma}, \boldsymbol{\mu}, \boldsymbol{\eta})$

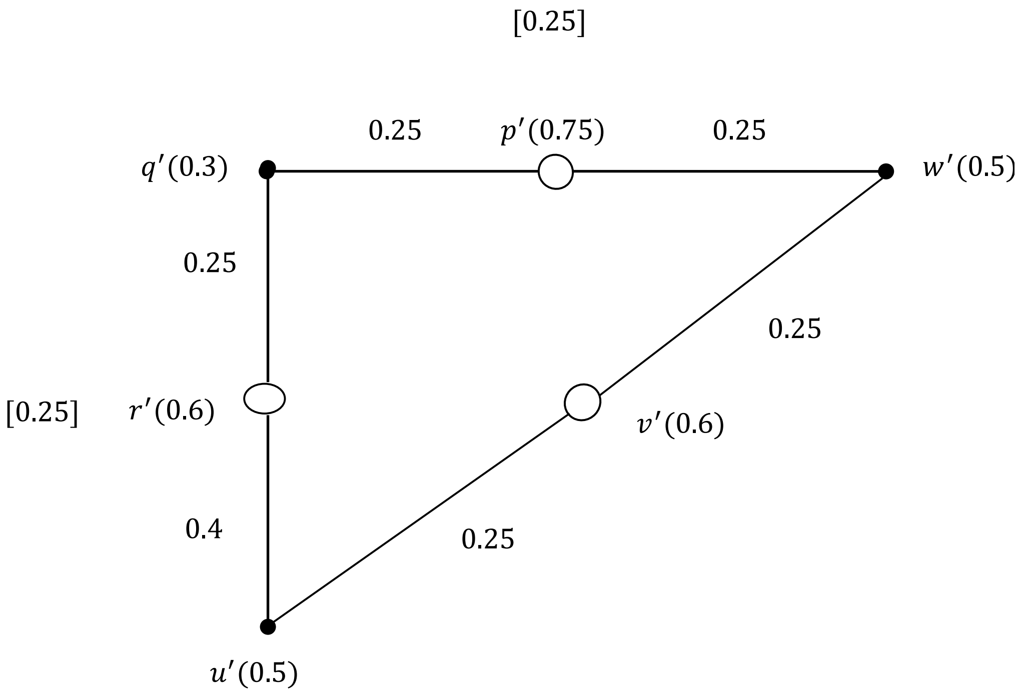


Fig 4.6: $\boldsymbol{G'}:(\boldsymbol{\sigma'},\boldsymbol{\mu'},\boldsymbol{\eta'})$

Chapter 5

Degrees of Vertices and Edges in Fuzzy Semigraphs

CHAPTER – 5

DEGREES OF VERTICES AND EDGES IN FUZZY SEMIGRAPHS

In this chapter, various degrees of a vertex in a fuzzy semigraph are defined. Degree, edge degree, adjacent degree and consecutive adjacent degree of a vertex in a fuzzy semigraph are introduced. Their properties under various isomorphisms are discussed. The degree of an edge is also defined.

5.1. DEGREE OF A VERTEX IN FUZZY SEMIGRAPH

Definition 5.1.1

Let $G:(\sigma,\mu,\eta)$ be a fuzzy semigraph on $G^*:(V,£,X)$. Let u be any vertex in G, then the degree of u denoted by $d(u)$ is defined by $d(u)=\sum\eta(E)$ where the summation runs over all edges E having u as an end vertex.

The total degree of u denoted by $td(u)$ is defined by $td(u)=\sum\eta(E)$ where the summation runs over all edges E having u as an end vertex.

Example 5.1.2

Consider the following fuzzy semigraph G: (σ,μ,η) in fig. 5.1.

The degree of the vertices are

d(v_1) = 0.8, d(v_2) = 0, d(v_3) = 1.0, d(v_4) = 1.0, d(v_5) = 0.4, d(v_6) = 0.4.

The total degree of the vertices are

$td(v_1)$ = 1.4, $td(v_2)$ = 0.5, $td(v_3)$ = 1.7, td (v_4) = 1.8, $td(v_5)$ = 0.8, $td(v_6)$ = 0.9.

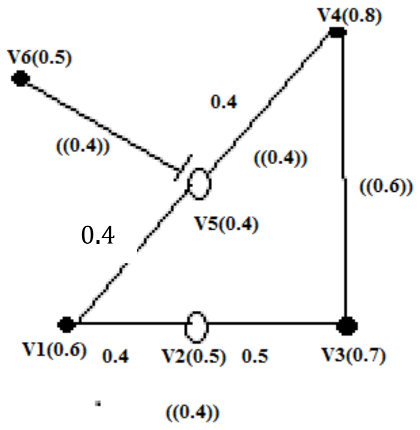


Fig.5.1 G: (σ, μ, η)

Theorem 5.1.3

Let $G(\sigma, \mu, \eta)$ be a fuzzy semigraph on $G^*(V,£,X)$. The degree of a middle vertex of an edge is zero.

Proof:

Since the middle vertex is not the end vertex of any edge, its degree is zero.

Theorem 5.1.4

Let $G(\sigma, \mu, \eta)$ be a fuzzy semigraph on $G^*(V,£,X)$. The degree of a vertex v in the end vertex fuzzy graph G_e is the degree of u in G, if v is the end vertex or middle end vertex of G. The degree of v in G_e is 0 if v is a middle vertex of G.

Proof:

Let $u \in V$ be an end vertex of G.

Then $d_{Ge}(u) = \sum_{uv \in £(G_e)} \eta_e(uv)$

$= \sum \quad \eta(E)$, the summation runs over all edges with one end u

$= d_G(u)$

Let u∈v be a middle vertex of G.

Then u is not adjacent to any other vertex of G_e.

Therefore $d_{G_e}(u) = 0$.

Theorem 5.1.5

Let $G: (\sigma, \mu, \eta)$ be a fuzzy semigraph on $G^*: (V, £, X)$ where $V = \{v_1, v_2, \ldots, v_n\}$ and $X = \{E_1, E_2, \ldots, E_m\}$, then $\sum_{i=1}^{n} deg(v_i) = 2S(G) \leq 2m$.

Proof :

$\sum_{i=1}^{n} deg(v_i) = \sum_{i=1}^{n} \sum \eta(E)$ where E runs over all the edges E having v as an end vertex.

In $\sum_{i=1}^{n} \sum \eta(E)$, each $\eta(E)$ appears twice.

Therefore $\sum_{i=1}^{n} deg(v_i) = 2 \sum_{i=1}^{m} \eta(E_i) = 2S(G)$

Also $2S(G) = \sum_{i=1}^{m} \eta(E_i) \leq 2 \sum_{i=1}^{m} 1 = 2m$

Therefore $\sum_{i=1}^{n} deg(v_i) = 2S(G) \leq 2m$

Theorem 5.1.6

Let $G: (\sigma, \mu, \eta)$ be a complete fuzzy semigraph on $G^*: (V, X, E)$. Let $n - 1$ vertices be strictly end vertices and one vertex be middle vertex of k semiedges $(v_i, u, v_{k+i}), i = 1, 2, \ldots, k$. Then

i) $d(u) \leq n - 1 - 2k$

ii) $d(v_i) \leq n - 2,\ i = 1,2, \ldots, 2k$

iii) $d(v) \le n-1, v \in V - \{u, v_1, v_2, \dots, v_{2k}\}$

iv) $S(G) \le \frac{n^2-n-2k}{2}$

Proof:

Since any two edges in x have atmost one vertex in common, the end vertices of 1- semiedges $v_1, v_2, \dots, v_{2k}$are all distinct.

There fore$2k \le n-1$, this implies k $k \le \frac{(n-1)}{2}$

Hence the maximum possible value of K is $[\frac{n-1}{2}]$

$V - \{u, v_1, v_2, \dots, v_{2k}\}$ has $n-1-2k$ elements.

Let $V - \{u, v_1, v_2, \dots, v_{2k}\} = \{u_1, u_2, \dots, u_{n-1-2k}\}$

Let $E_i = (v_i, u, v_{k+i}), i = 1,2, \dots, k$

u is the only middle vertex and all the other vertices are strictly end vertices.

Since G is complete, any two vertices must lie on the same edge.

$$X = \{E_1, E_2, \dots, E_k\} \cup \{(u, u_i)/i = 1,2, \dots, n-1-2k \cup \{(u_i, v_j)/\ i$$

$$= 1, 2, \dots, n-1-2k, j = 1,2, \dots ,2k\} \cup \{u_i, u_j)/i < j,$$

$$i, j = 1,2, \dots, n-1-2k\} \cup \{(v_i, v_j)/j = 1,2, \dots ,2k, j$$

$$\neq k + i \text{ if } i = 1,2, \dots, k, j \neq i - k\ if\ i = k+1, k+2, \dots ,2k\}$$

(i) $d(u) = \sum \eta(E)$over all E with u as an end vertex.

Therefore $d(u) = \sum_{i=1}^{n-1-2k} \eta(uu_i)$

$$\le \sum_{i=1}^{n-1-2k} 1$$

$$= n-1-2k$$

ii) For $i = 1,2,\dots,k$

$d(v_i) = \sum \eta(E)$ over all E with v_i as an end vertex.

$$= \sum_{\substack{j=1 \\ i\neq j, j\neq k+i}}^{2k} \eta(v_i, v_j) + \eta(v_i, u, v_k + i) + + \sum_{j=1}^{n-1-2k} \eta(u_j, v_i)$$

$$\leq \sum_{\substack{j=1 \\ i\neq j, j\neq k+i}}^{2k} 1 + 1 + \sum_{j=1}^{n-1-2k} 1$$

$$= 2k - 2 + 1 + n - 1 - 2k$$

$$= n - 2$$

For $i = k+1, k+2, \dots, 2k$

$$d(v_i) = \sum_{\substack{j=1 \\ i\neq j, j\neq i-k}}^{2k} \eta(v_i, v_j) + \eta(v_i - k, u, v_k + i, v_i) + \sum_{j=1}^{n-1-2k} \eta(u_j, v_i)$$

$$\leq 2k - 2 + 1 + n - 1 - 2k$$

$$= n - 2$$

Hence $d(v_i) \leq n - 2, i = 1,2,\dots,2k$.

(iii) For $i = 1,2,\dots,n-1-2k$,

$$d(v) = \sum_{\substack{j=1 \\ \cdots}}^{2k} 1 + 1 + \sum_{j=1}^{2k} \eta(u_i, v_j) + \sum_{\substack{j=1 \\ j\neq i}}^{n-1-2k} \eta(u_j, v_i)$$

$$\leq 1 + 2k + n - 1 - 2k - 1$$

$$= n - 1$$

(iv) By theorem 5.1.4,

$$2S(G) = \sum_{v\in V} d(v)$$

$$= d(u) + \sum_{i=1}^{2k} d(v_i) + \sum_{i=1}^{n-1-2k} d(u_i)$$

$$\leq n - 1 - 2k + \sum_{i=1}^{2k} n - 2 + \sum_{i=1}^{n-1-2k} n - 1$$

$= n - 1 - 2k + 2k(n-2) + (n-1-2k)(n-1)$

$= n^2 - n - 2k$

Therefore $S(G) \leq \frac{n^2-n-2k}{2}$

Theorem 5.1.7

Let $G:(\sigma,\mu,\eta)$ be a complete fuzzy semigraph on $G^*:(V,£,X)$ such that μ is a constant function of constant value c .Let $n-1$vertices be strictly end vertices and one vertex be middle vertex of k_1-semiedges (v_i,u,v_{k+i}), $i = 1,2,\dots,k$. Then

i) $d(u) = (n-1-2k)c$

ii) $d(v_i) = (n-2)c, i = 1,2,\dots,2k$

iii) $d(v) = (n-1)c, v \in V - \{u, v_1, v_2, \dots, v_{2k}\}$

iv) $2S(G) = n^2 - n - 2k$

Proof:

Consider V and X as in theorem 5.1.3,

Since $\mu(e) = c, \forall\ e \in E$,

$\eta(E) = c, \forall\ E \in X$,

(i) $d(u) = \sum_{i=1}^{n-1-2k} \eta(u,u_i)$

$= \sum_{i=1}^{n-1-2k} c$

$= (n-1-2k)c$

(ii) $d(v_i) = \sum^{2k}_{\substack{j=1 \\ i\neq j, j\neq k+i}} \eta(v_i, v_j) + \eta(v_i, u, v_k + i) + \sum_{j=1}^{n-1-2k} \eta(u_j, v_i)$

$= \sum^{2k}_{\substack{j=1 \\ i\neq j, j\neq k+i}} c + c + \sum_{j=1}^{n-1-2k} c$

$= (2k-2)c + c + (n-1-2k)c$

$= (n-2)c$

For $i = k+1, k+2, \dots, 2k$,

$d(v_i) = \sum^{2k}_{\substack{j=1 \\ i\neq j, j\neq i-k}} \eta\left(v_i, v_j\right) + \eta(v_i - k, u, v_k + i, v_i) + \sum_{j=1}^{n-1-2k} \eta((u_j, v_i)$

$= \sum^{2k}_{\substack{j=1 \\ i\neq j, j\neq i-k}} c + c + \sum_{j=1}^{n-1-2k} c$

$= (2k-2)c + c + (n-1-2k)c$

$= (n-2)c$

(iii) $i = 1,2, \dots, n-1-2k,$

$d(v) = \eta(uu_i) + \sum_{j=1}^{2k} \eta(u_i v_j) + \sum_{\substack{j=1 \\ j\neq i}}^{n-1-2k} \eta(u_i v_j)$

$= c + 2kc + (n-1-2k)c$

$= (n-1)c$

(iv) $2S(G) = \sum_{v\in V} d(v)$, by theorem 5.1.5

$= d(u) + \sum_{i=1}^{2k} d(v_i) + \sum_{i=1}^{n-1-2k} d(u_i)$

$= (n-1-2k)c + \sum_{i=1}^{2k} (n-2)c + \sum_{i=1}^{n-1-2k} (n-1)c$

$= (n-1-2k)c + 2k(n-2)c + (n-1-2k)(n-1)c$

$= (n^2 - n - 2k)c$

Theorem 5.1.8

The degree of a vertex is preserved under an isomorphism.

Proof:

Under an isomorphism $f: G \rightarrow G'$, $\eta(E) = \eta'(f(E))$, for all $E \in$ X.

Therefore $d_G(u) = \sum \eta(E)$, over all E with u as an end vertex.

$= \sum \eta'(f(E))$, over all $f(E)$ with $f(u)$ as an end vertex.

$= d_{G'}(f(u))$

The following theorems can be proved in a similar manner.

Theorem 5.1.9

The degree of a vertex is preserved under an end vertex isomorphism.

Theorem 5.1.10

The degree of a vertex is preserved under an edge isomorphism.

Theorem 5.1.11

The degree of a vertex is preserved under an adjacency isomorphism.

Theorem 5.1.12

The degree of a vertex is preserved under a co-weak isomorphism.

Theorem 5.1.13

The degree of a vertex is preserved under a co-weak end vertex isomorphism.

Theorem 5.1.14

The degree of a vertex is preserved under a co-weak edge isomorphism.

Theorem 5.1.15

The degree of a vertex is preserved under a co-weak adjacency isomorphism.

Remark 5.1.16

The degree of a vertex need not be preserved under weak isomorphism, weak end vertex isomorphism, weak edge isomorphism and weak adjacency isomorphism since $\eta(E) \leq \eta'(f(E))$, for all $E \in \mathrm{X}$ under any of these isomorphisms.

Theorem 5.1.17

The total degree of a vertex is preserved under an isomorphism, end vertex isomorphism, edge isomorphism and adjacency isomorphism.

Remark 5.1.18

The total degree of a vertex need not be preserved under any weak isomorphism and under any co-weak isomorphism.

5.2. EDGE DEGREE OF A VERTEX IN FUZZY SEMIGRAPH

Definition 5.2.1:

Let $G:(\sigma,\mu,\eta)$ be a fuzzy semigraph on $G^*:(V,£,X)$. Let u be any vertex in G, then the edge degree of u denoted by $d^e(u)$ is defined by $d^e(u) = \sum \eta(E)$ where the summation runs over all edges E containing u.

The edge total degree of u denoted by $td^e(u)$ is defined by $td^e(u) = \sum \eta(E) + \sigma(u)$ where the summation runs over all edges E containing u.

Example 5.2.2

Consider the fuzzy semigraph in fig. 5.1. The edge degree of the vertices are $d^e(v_1) = 0.8$, $d^e(v_2) = 0.4$, $d^e(v_3) = 1.0$, $d^e(v_4) = 1.0$, $d^e(v_5) = 0.8$, $d^e(v_6) = 0.4$

The edge degree of the vertices are

$td^e(v_1) = 1.4$, $td^e(v_2) = 0.9$, $td^e(v_3) = 1.7$, $td^e(v_4) = 1.8$, $td^e(v_5) = 1.2$, $td^e(v_6) = 0.9$

Theorem 5.2.3

Let G:(σ,μ,η) be a fuzzy semigraph on a semigraph G*:(V,£,X). If u is an end vertex in G, then $d_G(u) = d_G^e(u)$.

Proof:

If u is the end vertex, then the edges containing u are precisely the edges with end vertex u.

Therefore

$d_G(u)$ = ∑η(E), where the summation runs over all edges E with u

as an end vertex.

= ∑η(E), where the summation runs over all edges E containing u.

= $d_G^e(u)$.

Theorem 5.2.4

Let G*:(V,£, X) be a semigraph. If u is an end vertex of an edge in G*, then $d_{G^*}(u) = d_{G^*}^e(u)$.

Proof:

If u is the end vertex, then the edges containing u are precisely the edges with end vertex u.

Therefore $d_{G^*}(u) = d_{G^*}^e(u)$.

Theorem 5.2.5

Let G:(σ,µ,η) be a fuzzy semigraph such that µ is a constant function. If u is an end vertex, then $d(u) = d^e(u)$.

Proof:

Let µ(e) = c for every e in€ where c is a constant.

Then η(E) = c for every edge E. Therefore

$d_G(u)$ = ∑η(E), where the summation runs over all edges E with u

as an end vertex.

= c × the number of edges E with u as an end vertex.

= $cd_{G^*}(u)$

$d^{e}_{G}(u) = \sum\eta(E)$, the summation runs over all edges E containing u.

$= c \times$ the number of edges E containing the vertex u

$= c d^{e}_{G^*}(u)$

From theorem 5.2.4, if u is an end vertex,

$d_{G^*}(u) = d^{e}_{G^*}(u)$.

Hence $d_G(u) = d^{e}_{G}(u)$.

Theorem 5.2.6

The edge degree of a vertex is preserved under an isomorphism.

Proof:

Under an isomorphism $f: G \to G'$, $\eta(E) = \eta'(f(E))$, for all $E \in X$.

Therefore $d^{e}_{G}(u) = \sum\eta(E)$, over all E containing u.

$= \sum \eta'(f(E))$, over all $f(E)$ containin $f(u)$.

$= d^{e}_{G'}(f(u))$

The following results follow in a similar manner.

Theorem 5.2.7

The edge degree of a vertex is preserved under an end vertex isomorphism.

Theorem 5.2.8

The edge degree of a vertex is preserved under an edge isomorphism.

Theorem 5.2.9

The edge degree of a vertex is preserved under an adjacency isomorphism.

Theorem 5.2.10

The edge degree of a vertex is preserved under a co-weak isomorphism.

Theorem 5.2.11

The edge degree of a vertex is preserved under a co-weak end vertex isomorphism.

Theorem 5.2.12

The edge degree of a vertex is preserved under a co-weak edge isomorphism.

Theorem 5.2.13

The edge degree of a vertex is preserved under a co-weak adjacency isomorphism.

Remark 5.2.14

The edge degree of a vertex need not be preserved under weak isomorphism, weak end vertex isomorphism, weak edge isomorphism and weak adjacency isomorphism since $\eta(E) \leq \eta'(f(E))$, for all $E \in X$ under any of these isomorphisms.

Theorem 5.2.15

The total edge degree of a vertex is preserved under an isomorphism, end vertex isomorphism, edge isomorphism and adjacency isomorphism.

Remark 5.2.16

The total edge degree of a vertex need not be preserved under any weak isomorphism and under any co-weak isomorphism.

5.3. ADJACENT DEGREE OF A VERTEX IN FUZZY SEMIGRAPH

Definition 5.3.1

Let $G:(\sigma,\mu,\eta)$ be a fuzzy semigraph on $G^*:(V,£,X)$. Let u be any vertex in G. Then the adjacent degree of u denoted by $d^a(u)$ is defined by $d^a(u) = \sum\sum_{e\in E}\mu(e)$ where the outer summation runs over all edges E containing u.

The adjacent total degree of u denoted by $td^a(u)$ is defined by $td^a(u) = \sum\sum_{e\in E}\mu(e) + \sigma(u)$ where the outer summation runs over all edges E containing u.

Example 5.3.2

Consider the fuzzy semigraph in fig. 5.1.

The adjacent degrees of the vertices are

$d^a(v_1) = 1.6$, $d^a(v_2) = 0.9$, $d^a(v_3) = 1.5$, $d^a(v_4) = 1.3$, $d^a(v_5) = 0.8$, $d^a(v_6) = 0.4$

The adjacent total degrees of the vertices are

$td^a(v_1) = 2.2$, $td^a(v_2) = 1.4$, $td^a(v_3) = 2.2$, $td^a(v_4) = 2.1$, $td^a(v_5) = 1.2$, $td^a(v_6) = 0.9$

Theorem 5.3.3

Let G*: (V,£, X) be a k-uniform semigraph. If u is an end vertex in G*, then $d^a_{G^*}(u)$ = (k - 1) $d_{G^*}(u)$ = (k - 1) $d^e_{G^*}(u)$.

Proof:

Since G^* is k-uniform and u is an end vertex, the number of vertices adjacent to u from a single edge is k-1.

The number of edges adjacent to u is $d_{G^*}(u)$.

Therefore $d^a_{G^*}(u)$ = (k-1)$d_{a^*}(u)$.

If u is the end vertex, then the edges containing u are precisely the edges with end vertex u.

Therefore $d_{G^*}(u) = d^e_{G^*}(u)$.

Hence $d^a_{G^*}(u)$ = (k-1)$d^e_{G^*}(u)$.

Theorem 5.3.4

Let G:(σ,μ,η) be a k-uniform fuzzy semigraph on G*:(V,£, X) such that μ is a constant function of constant value c. If u is an end vertex in G, then

$$d^a_G(u) = (k - 1).c.d_{G^*}(u)$$

$$d^a_G(u)=(k-1).c.d^e_{G^*}(u).$$

Proof:

The adjacent degree of u is

$d^a_G(u) = \sum_E \sum_{e\in £} \mu(e)$, outer summation runs over all edges with end vertex u

$= \sum_E \sum_{e\in £} c$, outer summation runs over all edges with end vertex u

$= \sum_E (k-1)c$, the summation runs over all edges with end vertex u

$= (k - 1)c d_{G^*}(u)$

Also since $d_{G^*}(u) = d^e_{G^*}(u)$, $d^a_G(u) = (k - 1)c d^e_{G^*}(u)$.

Theorem 5.3.5

Let $G^*(V, £, X)$ be a k-uniform semigraph. If u is a middle vertex in G^*, then $d^a_{G^*}(u) = (k - 1)$.

Proof:

Since G^* is k-uniform, the number of vertices adjacent to a middle vertex u is $k - 1$. Also u is a middle vertex of exactly one edge.

Hence $d^a_{G^*}(u) = (k - 1)$.

Theorem 5.3.6

Let $G:(\sigma,\mu,\eta)$ be a k-uniform fuzzy semigraph on $G^*:(V, £, X)$ such that μ is a constant function of constant value c. If u is a middle vertex in G^*, then $d^a_{G^*}(u) = (k - 1)c$.

Proof:

Since G^* is k-uniform, the number of vertices adjacent to a middle vertex u is $k - 1$. Also u is a middle vertex of exactly one edge.

Therefore the adjacent degree of u is

$$d^a_G(u) = \sum_{e \in £} \mu(e) = \sum_{e \in £} c = (k - 1)c.$$

Theorem 5.3.7

Let G*(V,£, X) be a k-uniform semigraph. If u is a middle-end vertex in G*, then $d^a_{G^*}(u) = (k - 1)[\, d_{G^*}(u) + 1]$.

Proof:

$d_{G^*}(u)$ edges have u as end vertex.

The $(k - 1)d_{G^*}(u)$ vertices in them are all adjacent to u.

Also the k - 1 vertices in the edge containing u as middle vertex are also adjacent to the vertex u.

Hence $d^a_{G^*}(u) = (k - 1)d_{G^*}(u) + (k - 1)$

$= (k - 1)[\, d_{G^*}(u) + 1]$.

Theorem 5.3.8

Let G:(σ,μ,η) be a k-uniform fuzzy semigraph on G*:(V,£, X) such that μ is a constant function of constant value c. If u is a middle-end vertex in G*, then $d^a_{G^*}(u) = (k - 1)c[\, d_{G^*}(u) + 1]$.

Proof:

$d_{G^*}(u)$ is the number of edges E with u as end vertex. Only one edge has u as a middle vertex.

Therefor the adjacent degree of u is

$d^a_G(u) = \sum_E \sum_{e \in £} \mu(e)$, outer summation runs over all edges E containing u

$= \sum_E \sum_{e \in £} c$, outer summation runs over all edges E containing u

$= \sum_E (k-1)c$, the summation runs over all edges E containing u

$= \sum_{E}(k-1)c$, the summation runs over all edges E with u as end vertex

$+ \sum_{E}(k-1)c$, the summation runs over all edges E with u as a middle vertex

$= (k - 1)c d_{G^*}(u) + (k - 1)c.$

$= (k - 1)c[\, d_{G^*}(u) + 1].$

Theorem 5.3.9

The adjacent degree of a vertex is preserved under an isomorphism.

Proof:

Under an isomorphism $f: G \to G'$, $\mu(uv) = \mu'(f(u)f(v))$ for all $uv \in £$

Therefore

$d_G^a(u) = \sum_{E} \sum_{e \in £} \mu(e)$, the outer summation runs over all edges containing u

$= \sum_{f(E)} \sum_{e \in £} \mu'(f(e))$, the outer summation runs over all edges $f(E)$ containing $f(u)$

$= d_{G'}^a(f(u))$

Hence the theorem.

The same result holds for other isomorphisms and co-weak isomorphisms under the same reasoning.

Theorem 5.3.10

The adjacent degree of a vertex is preserved under an end vertex isomorphism.

Theorem 5.3.11

The adjacent degree of a vertex is preserved under an edge isomorphism.

Theorem 5.3.12

The adjacent degree of a vertex is preserved under an adjacency isomorphism.

Theorem 5.3.13

The adjacent degree of a vertex is preserved under a co-weak isomorphism.

Theorem 5.3.14

The adjacent degree of a vertex is preserved under a co-weak end vertex isomorphism.

Theorem 5.3.15

The adjacent degree of a vertex is preserved under a co-weak edge isomorphism.

Theorem 5.3.16

The adjacent degree of a vertex is preserved under a co-weak adjacency isomorphism.

Remark 5.3.17

The adjacent degree of a vertex need not be preserved under weak isomorphism, weak end vertex isomorphism, weak edge isomorphism and weak adjacency isomorphism since $\mu(uv) \leq \mu'(f(u)f(v))$ for all $uv \in$ £ under any of these weak isomorphisms.

Theorem 5.3.18

The total adjacent degree of a vertex is preserved under an isomorphism, end vertex isomorphism, edge isomorphism and adjacency isomorphism.

Remark 5.3.19

The total adjacent degree of a vertex need not be preserved under any weak isomorphism and under any co-weak isomorphism.

5.4. CONSECUTIVE ADJACENT DEGREE OF A VERTEX IN FUZZY SEMIGRAPH

Definition 5.4.1

The consecutive adjacent degree of u denoted by $d^{ca}(u)$ is defined by $d^{ca}(u) = \sum \mu(uv)$ where the summation runs over all vertices v which are consecutively adjacent to u.

The consecutive adjacent total degree of u denoted by $td^{ca}(u)$ is defined by $td^{ca}(u) = \sum \mu(uv) + \sigma(u)$ where the summation runs over all vertices v which are consecutively adjacent to u.

Example 5.4.2

Consider the fuzzy semigraph in fig. 5.1.

The Consecutive adjacent degrees of the vertices are

$d^{ca}(v_1) = 0.8$, $d^{ca}(v_2) = 0.9$, $d^{ca}(v_3) = 1.1$, $d^{ca}(v_4) = 1.0$, $d^{ca}(v_5) = 1.2$, $d^{ca}(v_6) = 0.4$

The Consecutive adjacent total degrees of the vertices are

$td^{ca}(v_1) = 1.4$, $td^{ca}(v_2) = 1.4$, $td^{ca}(v_3) = 1.8$, $td^{ca}(v_4) = 1.8$, $td^{ca}(v_5) = 1.6$, $td^{ca}(v_6) = 0.9$

Theorem 5.4.3

Let G*:(V,£, X) be a semigraph. If u is an end vertex of an edge in G*, then $d_{G^*}(u) = d^e_{G^*}(u) = d^{ca}_{G^*}(u)$.

Proof:

If u is the end vertex, then the edges containing u are precisely the edges with end vertex u.

Therefore $d_{G^*}(u) = d^e_{G^*}(u)$.

Also if u is the end vertex, then the number of edges with end vertex u is the same as the number of vertices consecutively adjacent to u.

Therefore $d_{G^*}(u) = d^{ca}_{G^*}(u)$.

Theorem 5.4.4

Let G:(σ,μ,η) be a fuzzy semigraph such that μ is a constant function. If u is an end vertex, then $d(u) = d^e(u) = d^{ca}(u)$.

Proof:

Let $\mu(e) = c$ for every $e \in$ £ where c is a constant.

Then $\eta(E)$ = c for every edge E.

Therefore

$d_G(u) = \eta(E)$, the summation runs over all edges E with u as an end vertex.

= c × the number of edges E with u as an end vertex.

= $c.d._{G^*}(u)$

$d^{e}_{G}(u) = \sum\eta(E)$, the summation runs over all edges E containing u.

$= c \times$ the number of edges E containing the vertex u

$= cd^{e}_{G^*}(u)$

$d^{ca}_{G}(u) = \sum_{uv\in E} \mu(uv)$

$= c\sum_{uv\in E} 1$

$= cd^{ca}_{G^*}(u)$

From theorem 5.2.4, if u is an end vertex, $d_{G^*}(u) = d^{e}_{G^*}(u)$ and therefore

$d_{G^*}(u) = d^{e}_{G^*}(u) = d^{ca}_{G^*}(u)$.

Hence $d_G(u) = d^{e}_{G}(u) = d^{ca}_{G}(u)$.

Theorem 5.4.5

Let G*: (V,£, X) be a *k*-uniform semigraph. If *u* is an end vertex in *G**, then $d^{a}_{G*}(u) = (k - 1)d^{ca}_{G^*}(u)$.

[or $d^{a}_{G^*}(u) = (k - 1)\, d^{e}_{G^*}(u)$ or $d^{a}_{G*}(u) = (k - 1)(k - 1)\, d_{G^*}(u)$.]

Proof:

Since G^* is *k*-uniform and *u* is an end vertex, the number of vertices adjacent to *u* from a single edge is *k*-1.

The number of edges adjacent to *u* is $d_{G^*}(u)$.

Therefore $d^{a}_{G^*}(u) = (k - 1)d_{G^*}(u)$.

From theorem 5.2.4, $d_{G^*}(u) = d^{e}_{G^*}(u) = d_{G^*}(u)$.

Hence $d^{a}_{G*}(u) = (k - 1)d^{ca}_{G^*}(u)$.

The other expressions follow in a similar manner.

Theorem 5.4.6

Let G:(σ,μ,η) be a k-uniform fuzzy semigraph on G*:(V,£, X) such that μ is a constant function of constant value c. If u is an end vertex in G*, then $d^{a}_{G*}(u)$= (k - 1)$cd^{ca}_{G^*}(u)$.

[or $d^{a}_{G^*}(u)$= (k - 1)c $d^{e}_{G^*}(u)$ or $d^{a}_{G*}(u)$= (k - 1)$cd_{G^*}(u)$.]

Proof: Let u ∊V be any vertex.

Then $d_{G_a}(u) = \sum_{uv\in E(G_{ca})} \mu_{ca}(uv)$

$= \sum_{uv\in E} \mu(uv)$

$= \sum_{uv\in E} c$

$= (k - 1)cd_{G^*}(u)$.

If u is an end vertex, then $d_{G^*}(u) = d^{e}_{G^*}(u) = d^{ca}_{G^*}(u)$.

Hence the theorem follows.

Theorem 5.4.7

Let G(σ, μ, η) be a fuzzy semigraph on G*(V,£,X). Then the degree of a vertex u in the consecutive adjacency fuzzy graph G_{ca} is $d^{ca}_{G}(u)$.

Proof: Let u ∊V be any vertex.

Then $d_{G_a}(u) = \sum_{uv\in E(G_{ca})} \mu_{ca}(uv)$

$= \sum_{uv\in E} \mu(uv)$

$= d^{ca}_{G}(u)$.

Theorem 5.4.8

Let G:(σ,μ,η) be a fuzzy semigraph on G*:(V,£,X) such that μis a constant function. Then for any vertex u,

i. $d_G(u) = cd_{G^*}(u)$

ii. $d_G^e(u) = cd_{G^*}^e(u)$

iii. $d_G^{ca}(u) = cd_{G^*}^{ca}(u)$

iv. $d_G^a\ (u) = cd_{G^*}^a(u)$

Proof:

Let μ(e) = c for every e ∈ £.

Then η(E) = c for every E ∈ X.

i. $d_G(u) = \sum\eta(E)$, the summation runs over all edges E with u as an end,

$= \sum c$, the summation runs over all vertex edges with u as an end vertex.

$= cd_{G^*}(u)$

ii. $d_G^e(u) = \sum\eta(E)$, the summation runs over all edges containing u

$= \sum c$, the summation runs over all edges containing u

$= cd_{G^*}^e(u)$

iii. $d_G^{ca}(u) = \sum_{uv\in E}\mu(uv)$

$= \sum_{uv\in E} c$

$= cd_{G^*}^{ca}(u)$

iv. $d_G^a(u) = \sum_E \sum_{e\in E}\mu(e)$, the outer summation runs over all E containing u

$= \sum_{E} \sum_{e \in E} c$

$= c d^{a}_{G^*}(u)$

The following theorem gives the relation between the four degrees of a fuzzy semigraph.

Theorem 5.4.9

For any vertex u in a fuzzy semigraph $G: (\sigma, \mu, \eta)$ on a semigraph $G^*: (V, X)$, $d(u) \leq d^{e}(u) \leq d^{ca}(u) \leq d^{a}(u)$

Proof :

In $d(u) = \sum \eta(E)$, E runs over all the edges having u as an end vertex.

But in $d^{e}(u) = \sum \eta(E)$, edges having u as middle vertex are also included.

Therefore $d(u) \leq d^{e}(u)$

For any edge E, $\eta(E) \leq \mu(e)$ for every $e \in E$

Which implies $\sum \eta(E) \leq \mu(uv)$ where the summation on L. H. S runs over all the edges containing u and the summation on R. H. S runs over all the vertices v consecutively adjacent to u.

Hence $d^{e}(u) \leq d^{ca}(u)$.

If $uv \in E$, then $\mu(uv) \leq \sum_{e \in E} \mu(e)$. Hence $d^{ca}(u) \leq d^{a}(u)$

Therefore $d(u) \leq de(u) \leq d^{ca}(u) \leq d^{a}(u)$

Remark 5.4.10

In the fuzzy semigraph in Fig 5.1,

$$d(v_1) = 0.8,\ d^e(v_1) = 0.8,\ d^a(v_1) = 1.6,\ d^{ca}(v_1) = 0.8$$

Therefore $d(v_1) \leq d^e(v_1) \leq\ d^{ca}(v_1)\ \leq d^a(v_1)$.

Remark 5.4.11

If u is an end vertex in a semigraph G:(σ,μ,η), then $d_G(u)$ or $d_G^e(u)$ need not be equal to $d_G^{ca}(u)$. For example, consider the semigraph in Fig.5.3

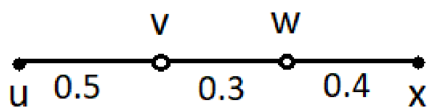


Fig 5.2 G:(σ,μ,η)

Here u is the end vertex of the edge E = (u, v, x, w).

d(u) = η(E) = 0.3

d^e(u) = η(E) = 0.3

But d^{ca}(u) = μ(uv) = 0.5

Therefore d(u) ≠ d^{ca}(u)

and d^e(u) ≠ d^{ca}(u).

Theorem 5.4.12

The consecutive adjacent degree of a vertex is preserved under an isomorphism.

Proof:

Under an isomorphism $f: G \to G'$, $\mu(uv) = \mu'(f(u)f(v))$ for all $uv \in £$

Therefore

$$d_G^{ca}(\mathrm{u}) = \sum_{\mathrm{uv}\in\mathrm{E}} \mu(\mathrm{uv})$$

$$= \sum_{\mathrm{uv}\in\mathrm{E}} \mu'(f(u)f(v))$$

$$= d_{G'}^{ca}(f(u))$$

The following theorems hold similarly under the same reasoning.

Theorem 5.4.13

The consecutive adjacent degree of a vertex is preserved under an end vertex isomorphism.

Theorem 5.4.14

The consecutive adjacent degree of a vertex is preserved under an edge isomorphism.

Theorem 5.4.15

The consecutive adjacent degree of a vertex is preserved under an adjacency isomorphism.

Theorem 5.4.16

The consecutive adjacent degree of a vertex is preserved under a co-weak isomorphism.

Theorem 5.4.17

The consecutive adjacent degree of a vertex is preserved under a co-weak end vertex isomorphism.

Theorem 5.4.18

The consecutive adjacent degree of a vertex is preserved under a co-weak edge isomorphism.

Theorem 5.4.19

The consecutive adjacent degree of a vertex is preserved under a co-weak adjacency isomorphism.

Remark 5.4.20

The consecutive adjacent degree of a vertex need not be preserved under weak isomorphism, weak end vertex isomorphism, weak edge isomorphism and weak adjacency isomorphism since $\mu(uv) \leq \mu'(f(u)f(v))$ for all $uv \in £$ under any of these weak isomorphisms.

Theorem 5.4.21

The total consecutive adjacent degree of a vertex is preserved under an isomorphism, end vertex isomorphism, edge isomorphism and adjacency isomorphism.

Remark 5.4.22

The total consecutive adjacent degree of a vertex need not be preserved under any weak isomorphism and under any co-weak isomorphism.

5.5 THE DEGREE OF AN EDGE IN FUZZY SEMIGRAPH

Definition 5.5.1

Let G:(σ,μ,η) be a fuzzy semigraph on G^*:(V, £,X). The degree of an edge E in X of G is $d_G(E) = \sum_{E_i \neq E} \eta(E_i)$ where the summation runs over all edges E_i adjacent to E.

Definition 5.5.2

The degree of the partial edge e = uv ϵ £ is

$$d_G(e) = \sum_{\substack{uw\varepsilon£,\\ w\neq u}} \mu(uw) + \sum_{\substack{vw\varepsilon£,\\ w\neq u}} \mu(vw).$$

Example 5.5.3

Consider the fuzzy semigraph in Fig.5.1.

Here V = $\{v_1, v_2, v_3, v_4, v_5, v_6\}$

£ = $\{v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_1, v_5v_6\}$

X = $\{E_1 = (v_1, v_2, v_3), E_2 = (v_3, v_4), E_3 = (v_4, v_5, v_1), E_4 = (v_5, v_6)\}$

$\eta(E_1)$ = 0.4, $\eta(E_2)$ = 0.6, $\eta(E_3)$ = 0.4, $\eta(E_4)$ = 0.4.

$\mu(v_1v_2) = 0.4$, $\mu(v_2v_3) = 0.5$, $\mu(v_3v_4) = 0.6$,

$\mu(v_4v_5) = 0.4$, $\mu(v_5v_1) = 0.4$, $\mu(v_5v_6) = 0.4$

Therefore the degree of the edges in *X* are

$d(E_1)$ = 0.4 + 0.6 = 1, $d(E_2)$ = 0.4 + 0.4 = 0.8

$d(E_3)$ = 0.4 + 0.6 + 0.4 = 1.4, $d(E_4)$ = 0.4

The degree of the edges in £ are

$$d(v_1v_2) = 0.4 + 0.5 = 0.9, \qquad d(v_2v_3) = 0.4 + 0.6 = 1,$$

$$d(v_3v_4) = 0.5 + 0.4 = 0.9\,, \qquad d(v_4v_5) = 0.6 + 0.4 = 1,$$

$$d(v_5v_1) = 0.4 + 0.4 + 0.4 = 1.2, \quad d(v_5v_6) = 0.4 + 0.4 = 0.8.$$

Theorem 5.5.4

Let G:(σ, μ, η) be a fuzzy semigraph on G*:(V, £, X). The degree of an edge E in G is $d_G(e)=\sum_{v\varepsilon E}[d^e\,(v) - \eta(E)]$.

Proof:

The edges adjacent to E are the edges incident on the end vertices and on the middle-end vertices of E.

Also for any middle vertex u of E, the edge degree of u is

$d^e(u) = \eta(E) \Rightarrow d^e(u) - \eta(E) = 0$.

Now $d_G(u) = \sum_{\substack{E_i \neq E \\ E_i \text{ adjacent to } E}} \eta(E_i)$

$$= \sum_{\substack{E_i \neq E, E_i \text{ has e-vertex of E,} \\ E_i \text{ adjacent to } E}} \eta(E_i) + \sum_{\substack{E_i \neq E, E_i \text{ has me-vertex of E,} \\ E_i \text{ adjacent to } E}} \eta(E_i)$$

$$= \sum_{v \text{is e-vertex of E}}[d^e\,(v) - \eta(E)] + \sum_{v \text{ is me-vertex of E}}[d^e\,(V) - \eta(E)]$$

$$= \sum_{V\text{is e-vertex of E}}[d^e\,(V) - \eta(E)] + \sum_{v \text{ is me-vertex of E}}[d^e\,(V) - \eta(E)]$$
$$+ \sum_{v\text{is m-vertex of E}}[d^e\,(V) - \eta(E)]$$

$$= \sum_{v\in E}[d_G^e(V) - \eta(E)]$$

Corollary 5.5.5

Let G:(σ, μ, η) be a k-uniform fuzzy semigraph on G^*:(V, £, X). The degree of an edge E∈ X is $d_G(E)=\sum_{v\in E}[d_G^e(v) - k\eta(E)]$.

Proof:

Since G is k-uniform, the number of vertices is k.

Therefore $d_G(E) = \sum_{V\in E}[d^e(v) - \eta(E)$

$= \sum_{v\in E}[d^e(v) - \sum_{v\in E}\eta(E)$

$= \sum_{v\in E}[d_G^e(v) - k\eta(E)]$

Corollary 5.5.6

Let G:(σ,μ,η) be a k-uniform fuzzy semigraph on G^*:(V, £,X) such that μ is a constant function of constant value c. Then the degree of an edge E∈X is $d_G(E) = c\sum_{v\in E} d_{G^*}^e(v) - kc$.

Proof:

Since μ(e) = c for every e∈ £,

η(E) = c for every E∈X.

Also $d^e(v) = cd_{G^*}^e(v)$ for every v ∈ V.

Therefore by corollary 5.5.5,

$d_G(E) = \sum_{v\in E}[d^e(v) - k\eta(E)]$

$= c\sum_{v\in E} d_{G^*}^e(v) - kc$

Corollary 5.5.7

Let $G(\sigma,\mu,\eta)$ be a fuzzy semigraph on $G^*(V, £, X)$ such that μ is a constant function. Then the degree of an edge E is $d_G(E) = c\sum_{v\in F}[d^e_{G*}(v) - 1]$.

Proof :

Here $d^e_G(v) = c d^e_{G*}(v)$ and $\eta(E) = c$,

Therefore by theorem 5.5.4,

$$d_G(E) = \sum_{v\in E}[d^e(v) - \eta(E)]$$

$$= \sum_{v\in E}[c d^e_{G*}(v) - c]$$

$$= c\sum_{v\in E}[d^e_{G*}(v) - 1]$$

Theorem 5.5.8

Let $G:(\sigma,\mu,\eta)$ be a fuzzy semigraph on $G^*:(V, £, X)$. Let $E \in X$ be an edge with end vertices u and v and with no middle-end vertex. Then the degree of E is $d_G(E) = d_G(u) + d_G(v) - 2\eta(E)$.

Proof:

Since E has no middle-end vertex, the edges adjacent to E are incident on the end vertices of E.

Therefore

$d_G(E) = \sum_{E_i\neq E}\eta(E_i)$, the summation runs over all E_i adjacent to E

$$= \sum_{\substack{E_i \text{ is adjacent to } E, E_i\neq E \\ \text{one end of Ei is u}}}\eta(E_i) + \sum_{\substack{E_i \text{ is adjacent to } E, E_i\neq E \\ \text{one end of Ei is v}}}\eta(E_i)$$

$$= d_G(u) - \eta(E) + d_G(v) - \eta(E)$$

Hence $d_G(E) = d_G(u) + d_G(v) - 2\eta(E)$.

Theorem 5.5.9

Let G:(σ, μ, η) be a fuzzy semigraph on G*:(V, £, X). The degree of the partial edge e = uv in £ is $d_G^{ca}(u) + d_G^{ca}(v) - 2\mu(uv)$.

Proof:

The degree of the partial edge e = uv is

$$d_G(e) = \sum_{\substack{uw\varepsilon£,\\ w\neq v}} \mu(uw)+\sum_{\substack{vw\varepsilon£,\\ w\neq u}} \mu(vw).$$

$$= \sum_{\substack{uw\varepsilon£,\\ w\neq v}} \mu(uw) + \mu(uv) + \sum_{\substack{vw\varepsilon£,\\ w\neq u}} \mu(vw) + \mu(uv) - 2\mu(uv)$$

$$= \sum_{uw\varepsilon£} \mu(uw) + \sum_{vw\varepsilon£} \mu(vw) - 2\mu(uv)$$

$$= d_G^{ca}(u) + d_G^{ca}(v) - 2\mu(uv)$$

Theorem 5.5.10

Let G(σ, μ, η)be a fuzzy semigraph on G^*(V,£,X) such that μ is a constant function .Then the degree of the edge E∊X is $d_G(E) = cd_{G^*}(E)$.

Proof:

Since $\mu(e) = c$, for every e∊£,

$\eta(E) = c$, for every E∊ X.

Therefore, $d_G(E) = \sum_{E_i \text{ adjacent to } E} \eta(E_i)$

= c× the number of edges adjacent to E

$= c.d_{G^*}(E)$

Theorem 5.5.11

Let $G(\sigma, \mu, \eta)$ be a fuzzy semigraph on $G^*(V,£,X)$ such that μ is a constant function.Then the degree of the partial edge $e = uv \in £$ is $d_G(e) = cd_{G^*}(e)$.

Proof:

The degree of e = uv is

$$d_G(e) = \sum_{\substack{uw\in £,\\ w\neq u}} \mu(uw) + \sum_{\substack{vw\in £,\\ w\neq v}} \mu(vw)$$

$$= \sum_{\substack{uw\in £,\\ w\neq u}} c + \sum_{\substack{vw\in £,\\ w\neq v}} c$$

$$= cd_{G^*}(e)$$

Theorem 5.5.12

Let $G(\sigma, \mu, \eta)$ be a fuzzy semigraph on $G^*(V,£,X)$. Let E be an edge in G with end vertices u and v. If E has no middle – end vertex, then the degree of the edge uv in e- fuzzy graph G_e is the degree of the edge E in the fuzzy semigraph G. That is, $d_{G_e}(uv) = d_G(E)$.

Proof:

Since E has no middle – end vertex, the edges adjacent to E or the edges incident on the end vertices u and v.

Now $d_{G_e}(uv) = \sum_{\substack{w\neq u,\\ uw\in E(G_e)}} \mu_e(uw) + \sum_{\substack{w\neq v,\\ vw\in E(G_e)}} \mu_e(vw)$

$$= \sum_{\substack{E_i \text{ incident on } u\\ E_i\neq E}} \eta(E_i) + \sum_{\substack{E_i \text{ incident on } v\\ E_i\neq E}} \eta(E_i)$$

$$= \sum_{\substack{E_i \text{ adjacent to } E \\ E_i \neq E}} \eta(E_i)$$

$$= d_G(E).$$

Theorem 5.5.13

Let $G(\sigma, \mu, \eta)$ be a fuzzy semigraph on $G^{**}(V, £, X)$. Let E be an edge in G with end vertices *u* and *v*. If E has no middle – end vertices, then the degree of the edge *uv* in the end vertex graph is $d_{G_e}(uv) = d_G(E) - \sum \eta(E_i)$ where the summation in the second term runs over all edges E_i which are incident on the middle-end vertices of E.

Proof:

The edges adjacent to E in G are the edges incident on the end vertices u and v and on the middle-end vertices of E.

Therefore

$$d_{G_e}(u) = \sum_{\substack{w \neq u \\ ue \in E(G_e)}} \mu(uw) + \sum_{\substack{w \neq v \\ ve \in E(G_e)}} \mu(vw)$$

$$= \sum_{\substack{E_i \neq E \\ E_i \text{ is incident on } u}} \eta(E_i) + \sum_{\substack{E_i \neq E \\ E_i \text{ is incident on } v}} \eta(E_i)$$

$$= \sum_{\substack{E_i \neq E \\ E_i \text{ is adjacent to } E}} \eta(E_i) - \sum_{\substack{E_i \neq E \\ E_i \text{ is incident on me-vertex of } E}} \eta(E_i)$$

$$= d_G(E) - \sum_{\substack{E_i \text{ is incident on} \\ \text{me-vertexof } E}} \eta(E_i)$$

Theorem 5.5.14

The degree of an edge is preserved under an isomorphism, end vertex isomorphism, edge isomorphism and adjacency isomorphism.

Theorem 5.5.15

The degree of an edge is preserved under a co-weak isomorphism, co-weak end vertex isomorphism, co-weak edge isomorphism and co-weak adjacency isomorphism.

Remark 5.5.16

The degree of an edge need not be preserved under any weak isomorphism.

Chapter 6

Various Regular Properties of Fuzzy Semigraphs

CHAPTER – 6

VARIOUS REGULAR PROPERTIES OF FUZZY SEMIGRAPHS

Various regular properties of fuzzy semigraphs are studied. The regular, edge degree regular, adjacency regular and consecutive adjacency regular fuzzy semigraphs are introduced and some of their properties are studied. Also biregular fuzzy semigraphs and totally regular fuzzy semigraphs are defined and their properties are studied.

6.1. REGULAR FUZZY SEMIGRAPHS

Definition 6.1.1

Let $G: (\sigma, \mu, \eta)$ be a fuzzy semigraph on semigraph $G^*: (V, £, X)$. G is a regular fuzzy semigraph of degree *r* or *r*-regular fuzzy semigraph if the degree of each vertex is *r*.

Definition 6.1.2

Let $G: (\sigma, \mu, \eta)$ be a fuzzy semigraph on semigraph $G^*: (V, £, X)$. G is a v-edge degree regular fuzzy semigraph of vertex edge degree *r* or *r*- v-edge degree regular fuzzy semigraph if the edge degree of each vertex is *r*.

Definition 6.1.3

Let $G\colon(\sigma,\mu,\eta)$ be a fuzzy semigraph on semigraph $G^*\colon(V,£,X)$. G is an adjacent regular fuzzy semigraph of adjacent degree r or r- adjacent regular fuzzy semigraph if the adjacent degree of each vertex is r.

Definition 6.1.4

Let $G\colon(\sigma,\mu,\eta)$ be a fuzzy semigraph on semigraph $G^*\colon(V,£,X)$. G is a consecutive adjacent regular fuzzy semigraph of consecutive adjacent degree r or r- consecutive adjacent regular fuzzy semigraph if the consecutive adjacent degree of each vertex is r.

Theorem 6.1.5

Let $G\colon(\sigma,\mu,\eta)$ be a fuzzy semigraph on a r-regular semigraph $G^*\colon(V,£,X)$ such that μ is a constant function. Then the end vertex fuzzy graph G_e is regular.

Proof :

Let $\mu(e)=c$ for every .

Then $\eta(E)=c$ for every $E\epsilon X$

Let E be an edge in G with end vertices u and v.

Then $\mu_e(uv)=\eta(E)=c$

This is true for every edge uv in the end vertex fuzzy graph G_e .

Therefore μ_e is also a constant function of constant value c.

Also, $d_{{G_e}^*}(u)=d_{G^*}=r$

Now $\quad d_{G_e}(u) = \sum_{uv \epsilon E(G_e)} \mu(uv)$

$= \sum_{uv \epsilon E(G_e)} c$

$= c d_{{G_e}^*}(u)$

$= cr$

This is true for every vertex u in G_e.

Hence G_e is cr-regular.

Corollary 6.1.6

If G$(\sigma, \mu, \eta)$is a fuzzy semigraph of stage c on an r-regular semigraph, then its end vertex fuzzy graph is regular.

Proof:

Since G is a fuzzy semigraph of stage c, μ is a constant function of constant value *c*. Therefore the result follows from theorem 6.1.5.

Theorem 6.1.7

Let $G: (\sigma, \mu, \eta)$ be an effective fuzzy semigraph on a r-regular semigraph$G^*: (V, £, X)$ such that σ is a constant function. Then the end vertex fuzzy graph G_e is regular.

Proof :

Let $\sigma(u) = c$ for every uϵV .

Let E be any edge of G with end vertices u and v.

Since G is an effective fuzzy semigraph, $\eta(E) = \sigma(u) \wedge \sigma(v) = c$

Therefore $\mu_e(uv) = \eta(E) = c$

This is true for every edge uv in the end vertex fuzzy graph G_e .

Then proceeding as in theorem 6.1.5, $G_e$is cr-regular.

Theorem 6.1.8

Let $G: (\sigma, \mu, \eta)$ be a fuzzy semigraph on a semigraph $G^*: (V, £, X)$. Then G is a r-regular fuzzy semigraph if and only if G_e is a r-regular fuzzy graph.

Proof:

The degree of an end vertex u of G is

$d_G(u) = \sum \eta(E)$, the summation runs over all edges E with u as an end vertex.

$$= \sum_{(u,\ldots,v)\in X} \eta(u, \ldots, v)$$

$$= \sum_{uv\in E(G_e)} \mu(uv)$$

$$= d_{G_e}(u)$$

This is true for every end verex u of G.

Hence the fuzzy semigraph G is r-regular if and only if its end vertex fuzzy graph is r-regular.

Theorem 6.1.9

Let $G: (\sigma, \mu, \eta)$ be a fuzzy semigraph on a semigraph $G^*: (V, £, X)$. Then G is a r-consecutive adjacent regular fuzzy semigraph if and only if its consecutive adjacency fuzzy graph G_{ca} is r-regular.

Proof:

The consecutive adjacent degree of a vertex u in G is

$$d_G^{ca}(u) = \sum_{uv \in E(G_{ca})} \mu(uv)$$

$$= d_{G_{ca}}(u)$$

$$= \text{the degree of u in } G_{ca} \text{ .}$$

Hence the fuzzy semigraph G is r- consecutive adjacent regular if and only if its consecutive adjacency fuzzy graph G_{ca} is r-regular.

Theorem 6.1.10

Let $G: (\sigma, \mu, \eta)$ be a fuzzy semigraph on a semigraph $G^*: (V, £, X)$ such that μ is a constant function and its end vertex graph $G_e{*}$ is a cycle. Then G is a regular fuzzy semigraph. The end vertex fuzzy graph G_e is also regular.

Proof:

Let E_1, $E_2, \cdots, E_m$ be the edges of G such that its end vertex graph $G_e{*}$ is a cycle.

Since μ is a constant function, the degree of each edge E_i in G is $2c$. Therefore G is $2c$ – regular. Similarly the end vertex fuzzy graph G_e is $2c$-regular.

6.2 BIREGULAR FUZZY SEMIGRAPHS

Definition 6.2.1

Let $G: (\sigma, \mu, \eta)$ be a fuzzy semigraph on semigraph $G^*: (V, £, X)$. G is a biregular fuzzy semigraph of degree (k, r) or (k, r)-biregular fuzzy semigraph if the degree of each vertex is either k or r.

Definition 6.2.2

Let $G: (\sigma, \mu, \eta)$ be a fuzzy semigraph on semigraph $G^*: (V, £, X)$. G is a v-edge degree biregular fuzzy semigraph of vertex edge degree (k, r) or (k, r)-v-edge degree biregular fuzzy semigraph if the edge degree of each vertex is either k or r.

Definition 6.2.3

Let $G: (\sigma, \mu, \eta)$ be a fuzzy semigraph on semigraph $G^*: (V, £, X)$. G is an adjacent biregular fuzzy semigraph of adjacent degree (k, r) or (k, r)- adjacent biregular fuzzy semigraph if the adjacent degree of each vertex is either k or r.

Definition 6.2.4

Let $G: (\sigma, \mu, \eta)$ be a fuzzy semigraph on semigraph $G^*: (V, £, X)$. G is a consecutive adjacent biregular fuzzy semigraph of consecutive adjacent degree (k, r) or (k, r)- consecutive adjacent biregular fuzzy semigraph if the consecutive adjacent degree of each vertex is either k or r.

Theorem 6.2.5

Let $G:(\sigma,\mu,\eta)$ be a k-uniform fuzzy semigraph on a semigraph $G^*:(V,£,X)$ such that μ is a constant function, with no middle-end vertex and its end vertex graph G_e* is a star.

i. The semigraph G is a v-edge degree biregular and adjacent biregular

ii. G is a consecutive adjacent biregular fuzzy semigraph if and only if the number of edges is 2.

Proof:

Let E_1, $E_2, \cdots, E_m$ be the edges of G with a common end vertex u such that its end vertex graph G_e* is a star with u as an apex vertex.

Let μ be a constant function of constant value c. G is k-uniform.

i. The edge degree of the vertex u in G is mc. The edge degree of all the other vertices is c. Hence G is a (mc, c)- v-edge degree biregular fuzzy semigraph.

The adjacent degree of the vertex u in G is $mc(k-1)$. The adjacent degree of all the other vertices is $c(k-1)$. Hence G is a ($mc(k-1)$, $c(k-1)$)- adjacent biregular fuzzy semigraph.

ii. The consecutive adjacent degree of the vertex u in G is mc. The consecutive adjacent degree of the end vertices is c. The consecutive adjacent degree of the middle vertices vertices is $2c$.

Hence G is a biregular fuzzy semigraph if and only if $mc = 2c$

if and only if $m = 2$

Theorem 6.2.6

Let $G:(\sigma,\mu,\eta)$ be a k-uniform fuzzy semigraph on a semigraph $G^*:(V,£,X)$ such that μ is a constant function and its end vertex graph G_e* is a cycle. Then the adjacency fuzzy graph G_a of G is a biregular fuzzy graph.

Proof:

Let $u_1, u_2, \cdots, u_n$ be the end vertices of G such that its end vertex graph G_e* is a cycle $u_1u_2\cdots u_nu_1$.

Since G is k-uniform, the fuzzy subgraph induced by the k vertices of each edge $(u_i,\cdots, u_{i+1})$ is complete in the adjacency graph G_a*.

The adjacency graph G_a* has the cycle $u_1u_2\cdots u_nu_1$ together with the n complete graphs formed by the vertices of the edge $(u_i,\cdots, u_{i+1})$, $1\leq i\leq n$, where $u_{n+1}=u_1$.

Therefore $d_{G_a^*}(v_i) = 2(k-1), i = 1,2,\cdots,n$

If u is any middle vertex, $d_{G_a^*}(u) = k - 1$.

Let $\mu(e) = c$ for every $e\in E$. Then $\mu_a(e) = \mu(e)=c$ for every $e\in E_a$.

Now

$$d_{G_a}(u) = \sum_{uv\epsilon E_a} \mu_a(uv)$$

$$= c\ d_{G_a^*}(u)$$

Therefore $d_{G_a}(u) = 2c(k-1)$, if u is an end vertex of an edge in G

and $d_{G_a}(u) = c\,(k\text{-}1)$, if u is a middle vertex of an edge in G.

Therefore G_a is $(2c(k\text{-}1),c(k\text{-}1))$- biregular.

6.3 TOTALLY REGULAR FUZZY SEMIGRAPHS

Definition 6.3.1

Let $G: (\sigma, \mu, \eta)$ be a fuzzy semigraph on semigraph $G^*: (V, £, X)$. G is a regular fuzzy semigraph of total degree r or r-totally regular fuzzy semigraph if the total degree of each vertex is r.

Definition 6.3.2

Let $G: (\sigma, \mu, \eta)$ be a fuzzy semigraph on semigraph $G^*: (V, £, X)$. G is a v-edge degree totally regular fuzzy semigraph of total vertex edge degree r or r- v- total edge degree regular fuzzy semigraph if the total edge degree of each vertex is r.

Definition 6.3.3

Let $G: (\sigma, \mu, \eta)$ be a fuzzy semigraph on semigraph $G^*: (V, £, X)$. G is an adjacent totally regular fuzzy semigraph of adjacent total degree r or r-adjacent totally regular fuzzy semigraph if the adjacent total degree of each vertex is r.

Definition 6.3.4

Let $G: (\sigma, \mu, \eta)$ be a fuzzy semigraph on semigraph $G^*: (V, £, X)$. G is a consecutive adjacent totally regular fuzzy semigraph of consecutive adjacent total degree r or r- consecutive adjacent totally regular fuzzy semigraph if the consecutive adjacent total degree of each vertex is r.

Theorem 6.3.5

Let $G(\sigma, \mu, \eta)$ is a fuzzy semigraph on $G^*(V, £, X)$. Then σ is a constant function if and only if the following are equivalent.

i). G is a regular fuzzy semigraph.

ii). G is a totally regular fuzzy semigraph.

Proof

Suppose σ is a constant function of constant value c.

Assume that G is a k-regular fuzzy semigraph.

Then $d_e(u) = k$, for all $u \in V$

$td_e(u) = d_e(u) + \sigma(u)$

$= k + p$, for all $u \in V$

Therefore G is a $(k + p)$-totally regular fuzzy semigraph.

Hence (i) implies (ii).

Suppose that G_e is a q- totally regular fuzzy semigraph.

Therefore $td_e(u) = q, \forall\ u \in V$

$d_e(u) + \sigma(u) = q, \forall\ u \in V$

$d_e(u) = q - \sigma(u), \forall\ u \in V$

$= q - p$

Hence G is a $(q - p)$-regular fuzzy semigraph.

Hence (ii) implies (i).

Therefore (i) and (ii) are equivalent.

Conversely assume that (i) and (ii) are equivalent.

That is, G is a regular fuzzy semigraph if and only if it is a totally regular fuzzy semigraph.

Assume that σ is not a constant function.

Therefore $\sigma(u) \neq \sigma(v)$ for atleast one pair of vertices $u, v \in$V

$$td_e(u) = d_e(u) + \sigma(u)$$

$$td_e(v) = d_e(v) + \sigma(v)$$

If G is a regular fuzzy semigraph, the above two equation imply that $td_e(u) \neq td_e(v)$

Hence G is not a totally regular fuzzy semigraph which is a contradiction.

Hence σ is a constant function.

The following theorems hold similarly under the same reasoning

Theorem 6.3.6

Let G(σ, μ, η) is a fuzzy semigraph on G*(V, £, X). Then σ is a constant function if and only if the following are equivalent.

i). G is a v-edge degree regular fuzzy semigraph.

ii). G is a v-edge degree totally regular fuzzy semigraph.

Theorem 6.3.7

Let G(σ, μ, η) is a fuzzy semigraph on G*(V, £, X). Then σ is a constant function if and only if the following are equivalent.

i). G is an adjacent regular fuzzy semigraph.

ii). G is an adjacent totally regular fuzzy semigraph.

Theorem 6.3.8

Let G(σ, μ, η) is a fuzzy semigraph on G^*(V, £, X). Then σ is a constant function if and only if the following are equivalent.

i). G is a consecutive adjacent regular fuzzy semigraph.

ii). G is a consecutive adjacent totally regular fuzzy semigraph.

Theorem 6.3.9

Let G(σ, μ, η) is a fuzzy semigraph on G^*(V,E,X). Then σ is a constant function if and only if the following are equivalent.

i. The end vertex fuzzy graph G_e of G is a regular fuzzy graph.

ii. G_e is a totally regular fuzzy graph.

Theorem 6.3.10

Let G(σ, μ, η) is a fuzzy semigraph on G^*(V,E,X). Then σ is a constant function if and only if the following are equivalent.

i. The adjacency fuzzy graph G_a of G is a regular fuzzy graph.

ii. G_a is a totally regular fuzzy graph.

Theorem 6.3.11

Let G(σ, μ, η) is a fuzzy semigraph on G^*(V,E,X). Then σ is a constant function if and only if the following are equivalent.

i. The consecutive adjacency fuzzy graph G_{ca} of G is a regular fuzzy graph.

ii. G_{ca} is a totally regular fuzzy graph.

Theorem 6.3.12

If $G(\sigma, \mu, \eta)$ is a fuzzy semigraph of stage c on r-regular fuzzy semigraph on $G^*(V,E,X)$, then its end vertex fuzzy graph G_e is totally regular.

Proof:

Let G_e be the end vertex fuzzy graph of G. Then

$$td_{G_e}(u) = d_{G_e}(u) + \sigma(u)$$

$$= \sum_{u \neq v} \mu(uv) + \sigma(u)$$

$$= cr + c$$

$$= c(r + 1)$$

Therefore G_e is $c(r+1)$-totally regular.

Conclusion

CONCLUSION

The concept of fuzzy semigraph have been introduced. The definitions of fuzzy sub semigraphs spanning fuzzy subsemigraph have been provided and some definitions have also been illustrated with examples.

The concept of effective fuzzy semigraph is explained and properties of effectiveness of the three associated fuzzy graphs of an effective fuzzy semigraphs are also discussed which may be used for future studies and research. Various degrees of a vertex of a fuzzy semigraph are defined and some results on degrees of vertices in a fuzzy semigraph are studied. Different types of isomorphisms on fuzzy semigraphs are studied. Complete fuzzy semigraph is defined. Transport networks and telecommunication networks can be modeled as fuzzy semigraphs. Hence our findings may be useful for future studies and research.

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Publications

PUBLICATIONS

1. K.Radha. and P,Renganathan " On Fuzzy Semigraphs ", Our Heritage, Vol. 68, Issue 4, Jan 2020, 397-404. (UGC CARE LISTJOURNAL)

2. K.Radha. and P,Renganathan " Effective Fuzzy Semigraphs ", Advances and Applications in Mathematical Sciences,Vol.20, Issue 5, March 2021, 895-904. (UGC CARE LISTJOURNAL)

3. K.Radha. and P,Renganathan "Isomorphic Properties of Fuzzy Semigraphs ", Advances and Applications in Mathematical Sciences, Vol. 21, Issue 6, April 2022, 3371-3382. (UGC CARE LISTJOURNAL)

PRESENTATIONS

1. Presented a Paper Entitled, "On Fuzzy Semigraphs", in the Heber International Conference on Applications of Actuarial Science, Mathematics, Management and Computer Science, held in Bishop Heber College, Trichy, Jan 9th& 10th – 2020.

2. Presented a Paper Entitled, "Effective Fuzzy Semigraphs", in the International Conference on Mathematical methods and Computations, held in Jamal Mohamed College, Trichy, 19thFeb–2020.

3. Presented a Paper Entitled, "On Bipartite Fuzzy Semigraphs", in the National Conference on Mathematical Analysis and Applications (NCMAA), held in National Institute of Technology, Trichy, April 23rd& 24th –2021.

4. Presented a Paper Entitled, "Various Isomorphisms on Fuzzy Semigraphs", in the Heber International Conference on Applied Mathematics, held in Bishop Heber College, Trichy,26TH April 2021.

5. Presented a Paper Entitled, "Various Degrees on Fuzzy Semigraphs and Complete Fuzzy Semigraphs ", in the International Conferences on Mathematical Sciences and Applications (Virtual) ICMSA 2022 held in St. Joseph's College, Trichy, March 11th to 14th 2022.

6. Presented a Paper Entitled, "Isomorphic Properties of Fuzzy Semigraphs", in the Heber International Conference on Applied Mathematics, Management, Computer Science and Statistics held in Bishop Heber College, Trichy,18th March 2022.

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Focus Subject	Mathematics (miscellaneous)
UGC-CARE coverage year	from June-2019 to Present

Advances and Applications in Mathematical Sciences
Volume 20, Issue 5, March 2021, Pages 895-904

EFFECTIVE FUZZY SEMIGRAPHS

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Abstract

In this paper, effective fuzzy and various concepts in fuzzy semigraphs are defined and illustrated through examples. Some results on effective fuzzy semigraphs have been arrived.

1. Introduction

The theory of fuzzy graphs was introduced by Rosanfeld in 1975. characteristics of Fuzzy graphs were dealt by Azriei Rosanfeld [7]. Bhattacharya [1] contributed some useful remarks on fuzzy graphs. Some operations on fuzzy graphs were defined by J. N. Modeson and C. S. Peng [3]. The concept of semigraph was introduced by E. Sampath Kumar [2]. K. Radha [6] introduced the concept of Fuzzy semigraph. Fuzzy semigraphs have applications in road network, railway network and telecommunications. In this paper we have defined effective edge and effective fuzzy semigraphs.

2. Preliminaries

Definition 2.1. A graph G is a pair (V, E) where V is a non empty set of

2010 Mathematics Subject Classification: 03E72, 05C72.

Keywords: effective fuzzy graphs, strong fuzzy graphs fuzzy semigraphs, fuzzy subsemigraphs end vertices middle vertices, middle-end vertices, connected fuzzy graph.

Received January 23, 2020; Accepted May 13, 2020

points which are called vertices and E is a set of ordered pairs of elements of V which are called edges of G.

Definition 2.2. A simple graph is an undirected graph without self loops and parallel edges.

Definition 2.3. A complete graph is a simple graph in which every pair of distinct vertices is connected by an edge. A graph G is connected if there exists a path between every pair of vertices.

Definition 2.4 [7]**.** Let V be a non-empty finite set and $E \subseteq V \times V$. A fuzzy graph $G : (\sigma, \mu)$ is a pair of functions $\sigma : V \to [0, 1]$ and $\mu : E \to [0, 1]$ such that $\mu(xy) \leq \sigma(x) \wedge \sigma(y)$ for all $x, y \in V$. Underlying crisp graph is denoted by $G^* : (V, E)$.

Definition 2.5. G is an effective fuzzy graph if $\mu(xy) = \sigma(x) \wedge \sigma(y)$ for all $xy \in E$.

Definition 2.6. G is a complete fuzzy graph if $\mu(xy) = \sigma(x) \wedge \sigma(y)$ for all $x, y \in V$.

Definition 2.7 [7]**.** If $\mu(xy) > 0,$ then x and y are called neighbours, x and y are said to lie on the edge $e = xy$.

Definition 2.8. A path ρ in a fuzzy graph $G : (\sigma, \mu)$ is a sequence of distinct vertices $v_0, v_1, v_2, \ldots, v_n$ such that $\mu(v_{i-1}, v_i) > 0, 1 \leq i \leq n$. Here '$n$' is called the length of the path. The consecutive pairs (v_{i-1}, v_i) are called arcs of the path.

Definition 2.9 [7]**.** The strength of that path is defined as $\overset{n}{\underset{i=1}{\wedge}} \mu(v_{i-1}v_i)$ i.e., it is the Weight of the weakest edge. If u, v are connected by means of paths of length 'k', then $\mu^k(uv) = \sup\{\mu(uv_1)\mu(v_1v_2)\mu(v_2v_3)\ldots\mu(v_{k-1}v)/u, v_1, v_2, \ldots, v_{k-1} \in V, v_{i-1}v_i \in E\}$.

Definition 2.10 [7]**.** The strength of connectedness between u and v is $\mu^{\infty}(u, v) = \sup\{\mu^k(u, v)/k = 1, 2, 3, \ldots,\}$. A fuzzy graph G is connected if $\mu^{\infty}(u, v) > 0$ for all $u, v \in V$. An edge xy is said to be a strong edge if

$\mu(x, y) \geq \mu^{\infty}(x, y)$. A node x is said to be an isolated node if $\mu(x, y) = 0, \forall y = X$.

Definition 2.11 [8]. A semigraph is a pair (V, X), where V is a non-empty set of elements called vertices and X is a set of n-tuples called edges of distinct vertices for various $n \geq 2$ satisfying the conditions

1. Any two edges have at most one vertex in common

2. Two edges $E_1 = (u_1, u_2, \ldots, u_n), E_2 = (v_1, v_2, \ldots, v_m)$ are considered to be equal if and only if (a) $m = n$ (b) either $u_i = v_i$ for $i = 1$ to n or $u_i = v_{n-i+1}$ for $i = 1$ to n.

In the edge $E = (u_1, u_2, \ldots, u_n), u_1$ and u_n are called called the end vertices and all vertices in between them are called middle vertices. (m-vertices). If a middle vertex is an end vertex of some other edge, it is called middle end vertex.

Definition 2.12 [6]. Consider a semigraph $G^* : (V, X)$. A fuzzy semigraph on $G^* : (V, X)$ is defined as (σ, μ, η) where $\sigma : V \to [0, 1]$, $\mu : V \times V \to [0, 1]$, $\eta : X \to [0, 1]$ are such that

(i) $\mu(u, v) \leq \sigma(u) \wedge \sigma(v) \forall (u, v) \in V \times V$

(ii) $\eta(e) = \mu(u_1, u_2) \wedge \mu(u_2, u_3) \wedge \ldots \wedge \mu(u_{n-1}, u_n) \leq \sigma(u_1) \wedge \sigma(u_n)$ if $e = (u_1, u_2, \ldots, u_n), n \geq 2$ is an edge in G.

Here (σ, μ) is a fuzzy graph.

Example 2.13 [6]. Consider the fuzzy semigraph in Figure 2.1. Here v_1, v_3 and v_5 are end vertices v_2 is a middle vertex v_4 and v_6 are middle-end vertices. Here $E_1 = (v_1, v_6, v_5), E_2 = (v_1, v_2, v_3), E_3 = (v_3, v_4, v_5)$ and $E_4 = (v_4, v_6)$ are the edges of the semigraph.

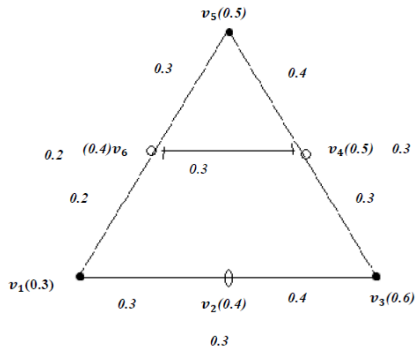

Figure 2.1. Fuzzy semi graph $G:(\sigma, \mu, \eta)$.

Definition 2.14 [6]**.** The fuzzy semigraph $H = (\gamma, \rho, \delta)$ is called a fuzzy subsemigraph of $G = (\sigma, \mu, \eta)$ if

(i) all the edges of H are subedges of G,

(ii) $\gamma(u) \leq \sigma(u)$ for all $u \in V$,

(iii) $\rho(uv) \leq \mu(uv)$ for all $(u, v) \in V \times V$,

(iv) $\delta(e) \leq \eta(e)$ for all $e \in X$.

Definition 2.15 [6]**.** The fuzzy subsemigraph $H = (\gamma, \rho, \delta)$ is called a spanning fuzzy subsemigraph of the fuzzy semi graph $G = (\sigma, \mu, \eta)$ if $\gamma(u) = \sigma(u)$ for all $u \in V$.

In this case the fuzzy semigraph and its spanning fuzzy semigraph differ in the weights of their edges. Spanning fuzzy sub semi graph $H = (\gamma, \rho, \delta)$ of the fuzzy semigraph $G : (\sigma, \mu, \eta)$ is given in Figure 2.

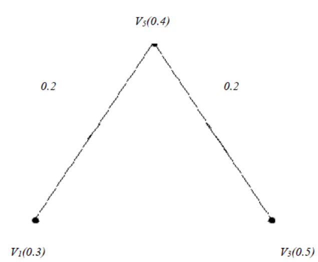

Figure 2.2. Fuzzy subsemigraph of the graph in 2.1.

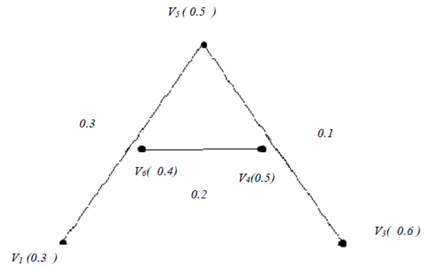

Figure 2.3. Spanning fuzzy subsemigraph of figure 2.1.

Definition 2.16 [6]**.** A subedge (fs-edge) of an edge $E = (v_1, v_2, v_3, \ldots, v_n)$ is a k-tuple $(v_{i_1}, v_{i_2}, \ldots, v_{i_k})$ where $1 \leq i_1 < i_2 < \ldots < i_k \leq n$ or $1 \leq i_k < i_{k-1} < \ldots < i_1 \leq n$.

Definition 2.17 [6]**.** A partial edge (fp-edge) of an edge $E = (v_1, v_2, v_3, \ldots, v_n)$ is a $(j - i + n)$- tuple $E = (v_i, v_{i-1}, \ldots, v_j)$ where $1 \leq i \leq n$.

3. Fuzzy Graphs Associated with Semigraphs

In this section, we define three fuzzy graphs associated with given fuzzy semigraph and discuss some of their properties.

Definition 3.1. End Vertex Fuzzy Graph (e-Fuzzy Graph) G_e :

Let $G : (\sigma, \mu, \eta)$ be a fuzzy semigraph $G^*(V, X)$. The fuzzy graph $G_e : (\sigma_e, \eta_e)$ with vertex set V in which two vertices are adjacent if and only if they are end vertices of an edge in G such that $\sigma_e(u) = \sigma(u)$ for every u in V and $\eta_e(uv) = \eta(uv)$ for every pair of end vertices u and v in G is called the end vertex fuzzy graph associated with G.

Definition 3.2. Adjacency Fuzzy Graph (a-Fuzzy Graph) G_a :

The fuzzy graph $G_a : (\sigma_a, \eta_a)$ with vertex set V in which two vertices are adjacent if and only if they are adjacent in G such that $\sigma_e(u) = \sigma(u)$ for every u in V and $\mu_e(uv) = \mu(uv_i) \wedge \mu(v_i v_{i+1}) \wedge, \ldots, \wedge \mu(v_j v)$ for every pair of adjacent vertices u and v in G, where $(u, v_i, v_{i+1}, \ldots, v_j, v)$ is an edge or a partial edge of G, is called the adjacency vertex fuzzy graph associated with G.

Definition 3.3. Consecutive adjacency fuzzy graph (*ca*-fuzzy graph) G_{ca} :

Let $G : (\sigma, \mu, \eta)$ be a fuzzy semigraph $G^{*}(V, X)$. The fuzzy graph $G_e : (\sigma_{ca}, \mu_{ca})$ with vertex set V in which two vertices are adjacent if and only if they are consecutively adjacent in G such that $\sigma_{ca}(u) = \sigma(u)$ for every u in V and $\mu_{ca}(uv) = \mu(uv)$ for every pair of consecutive adjacent vertices u and v in G is called the consecutive adjacency vertex fuzzy graph associated with G.

4. Various Concepts in Fuzzy Semigraphs

Definition 4.1. Two vertices in a fuzzy semigraph G are said to be adjacent if they belong to the same edge and are consecutively adjacent if in addition they are consecutive in order as well.

Definition 4.2. Any two edges in a fuzzy semigraph are adjacent if they have a vertex in common.

Definition 4.3. Any two edges in a fuzzy semigraph are said to be

(i) *ee*-adjacent if common vertex of the edges is end vertex in both the edges,

(ii) *em*-adjacent if common vertex of the edges is an end vertex of one edge and middle vertex of the other edge and

(iii) *mm*-adjacent if common vertex of the edges is a middle vertex in both the edges.

Definition 4.4. Cardinality of an edge in a semigraph is said to be k if the edge contains k number of vertices.

Definition 4.5. An edge in a semigraph G is said to be an s-edge if its cardinality $K \geq 3$.

Example 4.6. Consider Figure 2.1.

1. v_1 and v_5 are adjacent vertices since they belong to the same edge E_1, v_1 and v_5 are consecutively adjacent.

2. E_1 and E_3 are *ee*-adjacent. E_1 and E_4 are *em*-adjacent.

3. E_1, e_2 and E_3 have cardinality 3. E_4 has cardinality 2.

5. Effective Edges

Definition 5.1. An effective edge in a fuzzy semigraph.

An edge "*e*" of a fuzzy semigraph is called an effective edge if $\eta(e) = \eta(u_1, u_2, \ldots, u_n) = \sigma(u_1) \wedge \sigma(u_n)$ for $\forall e \in X$ and $\mu(uv) = \sigma(u) \wedge \sigma(v)$ for every $uv \in E$. An edge "*e*" of a fuzzy semigraph is called an *e*-effective edge if $\eta(e) = \eta(u_1, u_2, \ldots, u_n) = \sigma(u_1) \wedge \sigma(u_n)$ for $n > 2$.

Definition 5.2. Effective fuzzy semigraph.

A fuzzy semigraph $G : (\sigma, \mu, \eta)$ is said to be an Effective Fuzzy semigraph if all the edges of G are effective edges. In other words $\eta(e) = \eta(u_1, u_2, \ldots, u_n) = \sigma(u_1) \wedge \sigma(u_n) \forall e \in X$ and $\mu(uv) = \sigma(u) \wedge \sigma(v)$ for every edge $uv \in E$.

An effective fuzzy semigraph is shown in Figure 5.1.

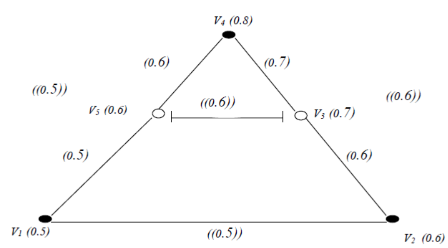

Figure 5.1. Effective semigraph.

Definition 5.3. Fuzzy effective subsemigraph.

A fuzzy subsemigraph H of a fuzzy semigraph $G : (\sigma, \mu, \eta)$ is said to be a fuzzy effective sub semigraph if all its edges are effective edges.

Example 5.4. Consider the fuzzy semigraph in Figure 5.2, the fuzzy subsemigraph of (5.2) is given in the figure 5.3.

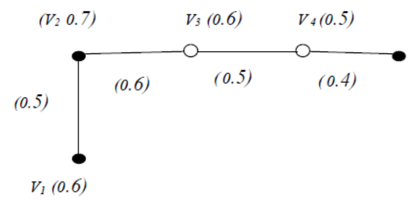

Figure 5.2. Fuzzy semigraph.

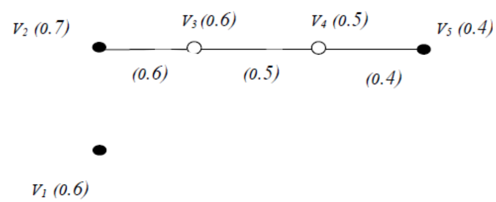

Figure 5.3. Fuzzy effective subsemigraph of the fuzzy semigraph in Figure 5.2.

6. Some Properties

Remark 6.1. Fuzzy sub semigraphs and spanning fuzzy sub semigraphs of an effective fuzzy semigraph need not be effective.

Theorem 6.2. *Induced subsemigraphs of an effective fuzzy semigraph are effective.*

Proof. Since membership values are preserved in induced fuzzy subsemigraphs, induced subsemigraphs of an effective fuzzy semigraph are effective.

Theorem 6.3. *End vertex fuzzy graph of an effective fuzzy semigraph is an effective fuzzy graph.*

Proof. In the end vertex fuzzy graph associated with G, $G_e : (\sigma_e, \eta_e)$ with vertex set V, two vertices are adjacent if and only if they are end vertices of an edge in G such that $\sigma_e(u) = \sigma(u)$ for every u in V and $\mu_e(uv) = \mu(uv)$ for every pair of end vertices u and v in G.

If G is an effective fuzzy semigraph, then $\eta(uv) = \sigma(u) \wedge \sigma(v)$ for every pair of end vertices u and v in G.

Therefore $\eta_e(uv) = \eta(uv) = \sigma(u) \wedge \sigma(v) = \sigma_e(u) \wedge \sigma_e(v)$ for every edge uv in G_e. Hence G_e is an effective fuzzy graph.

Theorem 6.4. *End vertex fuzzy graph of an e-effective fuzzy semigraph is an effective fuzzy graph.*

Proof. If G is an e-effective fuzzy semigraph, then $\eta(uv) = \sigma(u) \wedge \sigma(v)$ for every pair of end vertices u and v in G and hence the theorem follows.

Remark 6.4. Adjacency fuzzy graph (*ea*-Fuzzy Graph) of an e-effective fuzzy semigraph need not be effective.

Remark 6.5. Consecutive adjacency fuzzy graph of an e-effective fuzzy semigraph need not be effective.

Remark 6.6. Adjacency fuzzy graph of an effective fuzzy semigraph need not be effective.

Theorem 6.7. *If G is an effective fuzzy semigraph, then the consecutive adjacency fuzzy graph of G is an effective fuzzy graph.*

Proof. In the consecutive adjacency vertex fuzzy graph associated with G, $G_{ca} : (\sigma_{ca}, \mu_{ca})$ with vertex set V, two vertices are adjacent if and only if they are consecutively adjacent in G.

Also $\sigma_{ca}(u) = \sigma(u)$ for every u in V and $\mu_{ca}(uv) = \mu(uv)$ for every pair of consecutive adjacent vertices u and v in G.

Hence G is effective $\Rightarrow \mu(uv) = \sigma(u) \wedge \sigma(v)$

$$\Rightarrow \mu_{ca}(uv) = \sigma_{ca}(u) \wedge \sigma_{ca}(v)$$

$$\Rightarrow G_{ca} \text{ is effective.}$$

7. Conclusion

In this paper we have introduced the concept of effective fuzzy semigraph. Some properties of effectiveness of the three associated fuzzy graphs of an effective fuzzy semigraphs are also discussed which may be used for future studies and research. Neural networks and transportation networks can be modeled into fuzzy semigraphs and effective fuzzy semigraphs.

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Advances and Applications in Mathematical Sciences
Volume 21, Issue 6, April 2022, Pages 3371-3382

ISOMORPHIC PROPERTIES OF FUZZY SEMIGRAPHS

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Abstract

In this paper, isomorphism, weak isomorphism and co-weak isomorphism of fuzzy semigraphs are introduced and some of their properties are studied. End vertex isomorphism (ev-isomorphism), edge isomorphism (e-isomorphism) and adjacency isomorphism (a-isomorphism) of fuzzy semigraphs are defined. Properties of effective edges and effective fuzzy semigraphs under isomorphism are studied. Also, it is proved that isomorphism is an equivalence relation and week isomorphism is a partial order relation.

1. Introduction

In the year 1975 Rosenfeld introduced the theory of fuzzy graphs. Characteristics of fuzzy graphs were dealt by him. Some wonderful results and remarks on fuzzy graphs were contributed by Bhattacharya. Some operations on fuzzy graphs were defined by Modeson and J. N. Peng. A. Nagoor Gani and K. Radha studied the regularity properties of fuzzy graphs. The concept of semigraph was introduced by E. Sampath Kumar. The

2020 Mathematics Subject Classification: 05C72.
Keywords: Fuzzy graph, Fuzzy semigraph, Homomorphism, Isomorphism, Weak isomorphism, Co-weak isomorphism, End vertex isomorphism, Edge isomorphism, Adjacency isomorphism.
Received March 24, 2022; Accepted April 16, 2022

Concept of fuzzy semigraphs was introduced by K. Radha and P. Renganathan, K. Radha and P. Renganathan defined effective fuzzy semigraph and studied some properties of it. The order, size and the degree of isomorphic fuzzy graphs were studied by A. Nagoor Gani and J. Malarvizhi. The degree sequence and the degree set of fuzzy graphs and their properties were studied by K. Radha and A. Rosemine. Fuzzy K. Radhaand P. Renganathan semigraphs have wide range of applications in Railway network, Road network, telecommunication system, etc. In this paper isomorphism, weak isomorphism and co-weak isomorphism of fuzzy semigraphs are introduced. Also, some isomorphic properties of fuzzy semigraphs are discussed.

2. Preliminaries

Definition 2.1. A graph G is a pair (V, E), where V is a non-empty set of points which are called the vertices and E is a set of ordered pairs of elements of V which are called edges of G.

Definition 2.2. A fuzzy graph $G:(\sigma, \mu)$ on $G:(V, E)$ is a pair of functions $\sigma: V \to [0, 1]$ and $\mu: V \times V \to [0, 1]$ such that $\mu(uv) \leq \sigma(u) \wedge \sigma(v)$, $\forall\, uv \in E$.

Definition 2.3. G is an effective fuzzy graph if $\mu(uv) = \sigma(u) \wedge \sigma(v)$ for all $uv \in E$ and G is a complete fuzzy graph if $\mu(uv) = \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$.

Definition 2.4 [3]**.** A semigraph is a pair (V, X), where V is a non-empty set of elements called vertices and X is a set of n-tuples called edges of distinct vertices for various $n \geq 2$ satsfying the conditions

(1) Any two edges have at most one vertex in common

(2) Two edges $x_1 = (u_1, u_2, \ldots, u_n)$ and $x_2 = (v_1, v_2, \ldots, v_m)$ are considered to be equal if and only if (a) $m = n$ (b) either $u_i = v_i$ for $i = 1, 2, \ldots, n$ or $u_i = v_{n-i+1}$ for $i = 1, 2, \ldots, n$.

In the edge $x = (u_1, u_2, \ldots, u_n)$, u_1 and u_n are called the end vertices and all the vertices in between them are called middle vertices (m-vertices). If a middle vertex is an end vertex of some other edge, then it is called a middle end vertex.

Definition 2.5 [3]**.** A *subedge* (*fs-edge*) of an edge $x = (v_1, v_2, v_3, \ldots, v_n)$ is a k-tuple $x' = (v_{i_1}, v_{i_2}, \ldots, v_{i_k})$ where $1 \leq i_1 < i_2 < \ldots < i_k \leq n$ or $1 \leq i_k < i_{k-1} < \ldots < i_1 \leq n$ and a *partial edge* (*fp-edge*) of an edge $x = (v_1, v_2, v_3, \ldots, v_n)$ is a $(j - i + 1)$-tuple $x'' = (v_i, v_{i+1}, \ldots, v_j)$ where $1 \leq i \leq n$.

If $E \subseteq V \times V$ is taken as the set of all uv which is a partial edge of some edge $x \in X$, then a semigraph can be taken as a triple (V, E, X).

Definition 2.6 [5]**.** A fuzzy semigraph on $G^* : (V, E, X)$ is defined as $G : (\sigma, \mu. \eta)$ where $\sigma : V \to [0, 1], \mu : V \times V \to [0, 1], \eta : X \to [0, 1]$ are such that

(i) $\mu(uv) \leq \sigma(u) \wedge \sigma(v), \forall\, uv \in E$

(ii) $\eta(x) = \mu(u_1u_2) \wedge \ldots \wedge \mu(u_{n-1}u_n) \leq \sigma(u_1) \wedge \sigma(u_n), \forall x = (u_1, u_2, \ldots, u_n) \in X, n \geq 2.$

Definition 2.7 [6]**.** An edge $x = (u_1, u_2, \ldots, u_n)$ of a fuzzy semigraph is called an *effective* edge, if $\eta(x) = \mu(u_1u_2) \wedge \mu(u_2u_3) \wedge \ldots \wedge \mu(u_{n-1}u_n) = \sigma(u_1) \wedge \sigma(u_n)$ and $\mu(u_iu_{i+1}) = \sigma(u_i) \wedge \sigma(u_{i+1})$ for all i.

Definition 2.8 [6]**.** A fuzzy semigraph $G : (\sigma, \mu. \eta)$ is said to be an effective fuzzy semigraph if all the edges of G are effective edges.

Definition 2.9 [4]**.** A homomorphism of fuzzy graphs $f : G \to G'$ is a map $f : V \to V'$ such that $\sigma(u) = \sigma'(f(u))$ for all $u \in V, \mu(uv) \leq \mu'(f(u)f(v))$ for all $u, v \in V$.

Definition 2.10 [4]**.** An isomorphism of fuzzy graphs $f : G \to G'$ is a bijective map $f : V \to V'$ such that $\sigma(u) = \sigma'(f(u)), \forall u \in V, \mu(uv) = \mu'(f(u)f(v)), u, v \in V$.

Definition 2.11 [4]. A weak isomorphism of fuzzy graphs $f: G \to G'$ is a map $f: V \to V'$ which is bijective and satisfies $\sigma(u) = \sigma'(f(u))$ for all $u \in V$ and $\mu(uv) \leq \mu'(f(u)f(v))$ for all $u, v \in V$.

Definition 2.12 [4]. A co-weak isomorphism of fuzzy graphs $f: G \to G'$ is a map $f: V \to V'$ which is bijective and satisfies $\sigma(u) = \sigma'(f(u))$ for all $u \in V$ and $\mu(uv) \leq \mu'(f(u)f(v))$ for all $u, v \in V$.

Definition 2.13 [3]. Let $G_1 : (V_1, X_1)$ and $G_2 : (V_2, X_2)$ be two semigraphs and f is a bijection from V_1 to V_2. Let $x = (v_1, v_2, \ldots, v_n)$ be an edge in G_1, f *is an isomorphism* if $(v_1, v_2, \ldots, v_n)$ is an edge in G_1 then $(f(v_1), f(v_2), \ldots, f(v_n))$ is an edge in G_2, f *is an end vertex isomorphism* (*ev-isomorphism*) if the set $\{f(v_1), f(v_2), \ldots, f(v_n)\}$ forms an edge in G_2 with end vertices $f(v_1)$ and $f(v_n)$. f is an *edge isomorphism* (*e-isomorphism*) if the set $\{f(v_1), f(v_2), \ldots, f(v_n)\}$ forms an edge in G_2 and f is an adjacency isomorphism (a-isomorphism) if the adjacent vertices in G_1 are mapped onto adjacent vertices in G_2.

3. Isomorphisms on Fuzzy Semigraphs

Definition 3.1. Let $G : (\sigma, \mu, \eta)$ and $G' : (\sigma', \mu', \eta')$ be two fuzzy semigraphs with underlying semigraphs $G^* : (V, E, X)$ and $G'^* : (V', E', X')$. A homomorphism of fuzzy semigraphs $f: G \to G'$ is a map denoted by $f: V \to V'$ which satisfies $\sigma(u) \leq \sigma'(f(u))$ for all $u \in V$, $\mu(uv) \leq \mu'(f(u)f(v))$ for all $u, v \in V$ and $\eta(x) \leq \eta'(f(x))$, for all $x \in X$.

Definition 3.2. Let $G : (\sigma, \mu, \eta)$ and $G' : (\sigma', \mu', \eta')$ be two fuzzy semigraphs with underlying semigraphs $G^* : (V, E, X)$ and $G'^* : (V', E', X')$. An *isomorphism of fuzzy semigraphs* $f: G \to G'$ is a bijective map denoted by $f: V \to V'$ which satisfies (1). If $x = (v_1, v_2, \ldots, v_n)$ is an edge in G, then $(f(v_1), f(v_2), \ldots, f(v_n))$ is an edge in G' (2). $\sigma(u) = \sigma'(f(u))$ for all $u \in V$

(3). $\mu(uv) \leq \mu'(f(u)f(v))$ for all $uv \in E$ and 4. $\eta(x) \leq \eta'(f(x))$, for all $x \in X$.

Definition 3.3. An *end vertex isomorphism* (*ev-isomorphism*) of fuzzy semigraphs $f : G \to G'$ is a bijective map $f : V \to V'$ which satisfies (1). If $x = (v_1, v_2, \ldots, v_n)$ is an edge in G, then $\{f(v_1), f(v_2), \ldots, f(v_n)\}$ forms an edge in G' with end vertices $f(v_1)$ and $f(v_n)$. (2). $\sigma(u) = \sigma'(f(u))$ for all $u \in V$ (3). $\mu(uv) \leq \mu'(f(u)f(v))$ for all $uv \in E$, and (4). $\eta(x) \leq \eta'(f(x))$, for all $x \in X$.

Definition 3.4. An *edge isomorphism* (*e-isomorphism*) of fuzzy semigraphs $f : G \to G'$ is a bijective map $f : V \to V'$ such that 1. If $x = (v_1, v_2, \ldots, v_n)$ is an edge in G, then $\{f(v_1), f(v_2), \ldots, f(v_n)\}$ forms an edge in G'. (2) $\sigma(u) = \sigma'(f(u)), \forall u \in V$, 3. $\mu(uv) \leq \mu'(f(u)f(v)), uv \in E$, 4. $\eta(x) \leq \eta'(f(x)), x \in X$.

Definition 3.5. An *adjacency isomorphism* (*a-isomorphism*) of fuzzy semigraphs $f : G \to G'$ is a bijective map $f : V \to V'$ which satisfies (1). The adjacent vertices in G are mapped onto adjacent vertices in G', (2). $\sigma(u) = \sigma'(f(u))$ for all $u \in V$, (3). $\mu(uv) \leq \mu'(f(u)f(v))$ for all $uv \in E$ and (4). $\eta(x) \leq \eta'(f(x))$, for all $x \in X$.

Theorem 3.6. *Isomorphisms between fuzzy semigraphs is an equivalence relation.*

Proof of Theorem 3.6. Let $G : (\sigma, \mu, \eta), G' : (\sigma', \mu', \eta')$ and $G'' : (\sigma'', \mu'', \eta'')$ be fuzzy semigraphs with vertex sets V, V' and V'' respectively.

Let $f : V \to V$ be such that $f(v) = v, \forall v \in V$. This mapping f is a bijection. Also $\sigma(u) = \sigma(f(u))$ for all $u \in V, \mu(uv) = \mu(f(u), \mu(v))$ for all $uv \in E$ and $\eta(x) \leq \eta(f(x))$, for all $x \in X$. Thus f is an isomorphism from G to itself.

Hence isomorphism is a *reflexive relation*.

Let $f : G \to G'$ be an isomorphism between the fuzzy semigraphs G and G' Then the mapping $f : V \to V'$ satisfies

$$\sigma(u) = \sigma'(f(u)) \text{ for all } u \in V \text{ and} \tag{1}$$

$$\mu(uv) = \mu(f(u), \mu(v)) \text{ for all } uv \in E \tag{2}$$

$$\eta(x) \leq \eta'(f(x)), \text{ for all } x \in X. \tag{3}$$

Since f is bijective, for u' in V', there exists u in V such that $f^{-1}(u') = u$

Hence by (1) $\sigma(f^{-1}(u')) = \sigma'(f(u)) = \sigma'(u')$ for all $u' \in V'$. From this it follows that $\eta(x') = \eta'(f^{-1}(x'))$, for all x' in X'.

$$\text{Also } \mu((f^{-1}(u')f^{-1}(v')) = (f(u)f(v)) = \mu'(u'v') \text{ for all } u'v' \in E' \tag{4}$$

Hence we get a 1-1, onto map $f^{-1} : V' \to V$ which is an isomorphism.

Thus G' is isomorphic to G. Hence isomorphism is the symmetric.

Let $f : V \to V'$ and $g : V' \to V''$ be isomorphisms from fuzzy semigraphs G to G' and from G' to G'' respectively.

Then $g \circ f$ is 1-1 and onto map from $V \to V''$ where $(g \circ f)u = g(f(u))$, for all $u \in V$.

Then $\sigma(u) = \sigma'(f(u)) = \sigma''(g(f(u)))$ for all u in V

$\mu(uv) = \mu'(f(u), \mu(v)) = \mu''(g(f(u)), g(\mu(v))), \forall uv \in E$

$\eta(x) = \eta''(g(f(x)))$ for all $x \in X$.

Therefore $g \circ f$ is an isomorphism between G and G''

Hence isomorphism is transitive and hence it is an equivalence relation.

Theorem 3.7. *Let $G : (\sigma, \mu, \eta)$ and $G' : (\sigma', \mu', \eta')$ be two isomorphic fuzzy semigraphs. Then an edge in G is an effective edge if and only if the corresponding image edge in G' is effective.*

Proof of Theorem 3.7. Let $f : G \to G'$ be an isomorphism between the fuzzy semigraphs G and G' with underlying sets V and V'.

x is an effective edge in G

$\Leftrightarrow \eta(x) = \mu(u_1u_2) \wedge \mu(u_2u_3) \wedge \ldots \wedge \mu(u_{n-1}u_n) = \sigma(u_1) \wedge \sigma(u_n),$

$x = (u_1, u_2, \ldots, u_n)$

$\Leftrightarrow \eta'(f(x)) = \mu'(f(u_1) f(u_2)) \wedge \ldots \wedge \mu'(f(u_{n-1}) f(u_n)) = \sigma'(f(u_1)) \wedge \sigma'(f(u_n))$

$\Leftrightarrow f(x)$ is an effective edge in G'.

Also for any $e = uv$ in E,

$$\mu(uv) = \sigma(u) \wedge \wedge\sigma(v) \Leftrightarrow \mu'(f(u)f(v)) = \sigma'(f(u)) \wedge \sigma'(f(v)).$$

Theorem 3.8. *If G and G' are isomorphic fuzzy semigraphs then G is an effective fuzzy semigraph if and only if G' is also effective.*

Proof of Theorem 3.8. Since G is isomorphic to G', there is an isomorphism $f : G \to G'$. Therefore G is an effective fuzzy semigraph $\Leftrightarrow$ Each edge in G is effective $\Leftrightarrow$ The image of each edge in G is effective $\Leftrightarrow$ Each edge in G' is effective $\Leftrightarrow$ G' is an effective fuzzy semigraph.

Definition 3.9. An edge $x = (u_1, u_2, \ldots, u_n)$ of a fuzzy semigraph is called an *e-effective* edge if

$$\eta(x) = \mu(u_1u_2) \wedge \ldots \wedge \mu(u_{n-1}u_n) = \sigma(u_1) \wedge \sigma(u_n) \text{ for } n > 2.$$

An edge $x = (u_1, u_2, \ldots, u_n)$ of a fuzzy semigraph is called a *semi-effective* edge if $\mu(u_iu_{i+1}) = \sigma(u_i) \wedge \sigma(u_{i+1})$ for all i.

Definition 3.10. A fuzzy semigraph $G : (\sigma, \mu, \eta)$ is said to be an e-effective fuzzy semigraph if all the edges of G are e-effective edges.

G is said to be a semi-effective fuzzy semigraph if all the edges of G are semi-effective edges.

Theorem 3.11. *Let $G : (\sigma, \mu, \eta)$ and $G' : (\sigma', \mu', \eta')$ be two isomorphic fuzzy semigraphs. Then an edge in G is an e-effective edge if and only if the corresponding image edge in G' is e-effective.*

Theorem 3.12. *Let $G : (\sigma, \mu, \eta)$ and $G' : (\sigma', \mu', \eta')$ be two isomorphic fuzzy semigraphs. Then an edge in G is a semi-effective edge if and only if the corresponding image edge in G' is semi-effective.*

Theorem 3.13. *Let $G : (\sigma, \mu, \eta)$ and $G' : (\sigma', \mu', \eta')$ be two isomorphic* *fuzzy semigraphs. Then G is an e-effective fuzzy semigraph if and only if G' is e-effective.*

Theorem 3.14. *Let $G : (\sigma, \mu, \eta)$ and $G' : (\sigma', \mu', \eta')$ be two isomorphic fuzzy semigraphs. Then G is a semi-effective fuzzy semigraph if and only G' is semi-effective.*

4. Weak Isomorphisms

Definition 4.1. Let $G : (\sigma, \mu, \eta)$ and $G' : (\sigma', \mu', \eta')$ be two fuzzy semigraphs with underlying semigraphs $G^* : (V, E, X)$ and $G'^* : (V', E', X')$. A *weak isomorphism of fuzzy semigraphs* $f : G \to G'$ is a bijective map $f : V \to V'$ which satisfies (1). If $e = (v_1, v_2, \ldots, v_n)$ is an edge in G, then $\{f(v_1), f(v_2), \ldots, f(v_n)\}$ forms an edge in G'. (2). $\sigma(u) = \sigma'(f(u))$ for all vertices $u \in V$ (3). $\mu(uv) \leq \mu'(f(u)f(v))$ for all $uv \in E$.

A *weak-end vertex isomorphism* (*weak-ev-isomorphism*) of fuzzy semigraphs $f : G \to G'$ is a bijective map $f : V \to V'$ which satisfies, (1). If $e = (v_1, v_2, \ldots, v_n)$ is an edge in G then $\{f(v_1), f(v_2), \ldots, f(v_n)\}$ forms an edge in G' with end vertices $f(v_1)$ and $f(v_n)$, (2). $\sigma(u) = \sigma'(f(u))$ for all $u \in V$, (3). $\mu(uv) \leq \mu'(f(u)f(v))$ for all $uv \in E$.

A *weak-edge isomorphism* (*weak-e isomorphism*) of fuzzy semigraphs $f : G \to G'$ is a bijective map denoted by $f : V \to V'$ and which satisfies (1). If $e = (v_1, v_2, \ldots, v_n)$ is an edge in G, then $\{f(v_1), f(v_2), \ldots, f(v_n)\}$ forms an edge in G' (2). $\sigma(u) = \sigma'(f(u))$ for all $u \in V$, (3). $\mu(uv) \leq \mu'(f(u)f(v))$ for all $u, v \in V$.

A *weak-adjacency isomorphism* (*weak-a isomorphism*) of fuzzy semigraphs $f : G \to G'$ is a bijective map denoted by $f : V \to V'$ and which satisfies (1). If the adjacent vertices in G are mapped onto adjacent vertices in G', (2). $\sigma(u) = \sigma'(f(u))$ for all $u \in V$, (3). $\mu(uv) \leq \mu'(f(u)f(v))$ for all $uv \in E$.

Theorem 4.2. *Weak isomorphism between fuzzy semigraphs is a partial order relation.*

Proof of Theorem 4.2. Let $G : (\sigma, \mu, \eta), G' : (\sigma', \mu', \eta')$ and $G'' : (\sigma'', \mu'', \eta'')$ be fuzzy semigraphs with vertex sets V, V' and V'' respectively.

Let $f : V \to V$ such that $f(v) = v, \forall v \in V$. Then f is a weak isomorphism from G to itself. Thus G is weak isomorphic to itself. Hence weak isomorphism satisfies *reflexive relation*.

Let $f : V \to V'$ and $g : V' \to V$ be weak isomorphisms from fuzzy semigraphs G to G' and from G' to G respectively

Then f and g satisfies $\mu(uv) \le \mu'(f(u)f(v)) \forall uv \in E$.

And $\mu'(u'v') \le \mu(g(f(u))g(f(v)))$, for all $u'v' \in E'$.

This can happen only when G and G' have the same number of edges and the corresponding membership values of the edges are equal. Hence G and G' are identical (apart from the naming of the vertices). Thus Weak isomorphism between fuzzy semigraphs is *anti symmetric*.

Let $f : V \to V'$ and $g : V' \to V''$ be weak isomorphisms on fuzzy semigraphs G to G' and G' to G'' respectively.

Then $\sigma(u) = \sigma'(f(u)) = \sigma''(g(f(u)))$ for all u in V

$$\mu(uv) \le \mu'(f(u)f(v)) \le \mu''(g(f(u))g(f(v))), \forall u, v \in V$$

Thus $g \circ f$ is a weak isomorphism between G and G'' and therefore weak isomorphism between fuzzy semigraphs is *transitive*. Hence the weak isomorphism between fuzzy semigraphs is a *partial order relation*.

Remark 4.3. When $G : (\sigma, \mu, \eta)$ is weak isomorphic to $G'(\sigma', \mu', \eta')$, then the effectiveness of one fuzzy semigraph need not imply the effectiveness of the other. The Fuzzy semigraph G in Figure 4.1 and the Fuzzy semigraph G' in Figure 4.2 are weak isomorphic, G is effective but G' is not effective. Also the Fuzzy semigraph G in Figure 4.3 and the Fuzzy semigraph G' in Figure 4.4 are weak isomorphic, G' is effective but G is not effective.

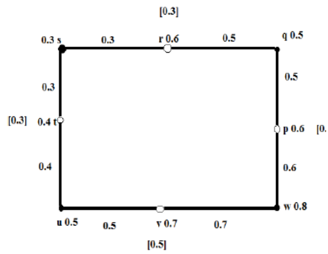

Figure 4.1. $G : (\sigma, \mu, \eta)$.

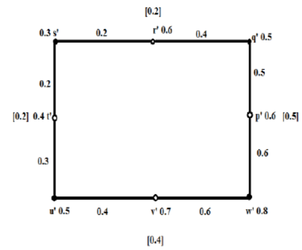

Figure 4.2. $G' : (\sigma', \mu', \eta')$.

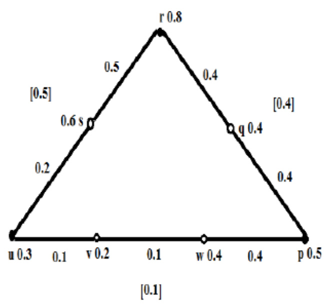

Figure 4.3. $G : (\sigma, \mu, \eta)$.

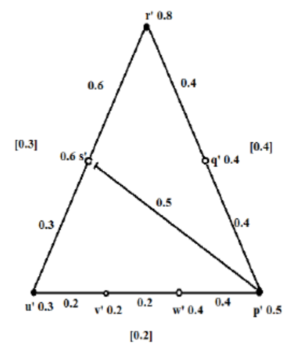

Figure 4.4. $G' : (\sigma', \mu', \eta')$.

Remark 4.4. Weak isomorphism need not preserve the e-effective and the semi-effective properties of the edges.

5. Co-Weak Isomorphisms

Definition 5.1. Let $G : (\sigma, \mu, \eta)$ and $G' : (\sigma', \mu', \eta')$ be two fuzzy semigraphs with underlying semigraphs $G^* : (V, E, X)$ and $G'^* : (V', E', X')$. A *co-weak isomorphism of fuzzy semigraphs* $f : G \to G'$ is a bijective map $f : V \to V'$ which satisfies (1). If $x = (v_1, v_2, \ldots, v_n)$ is an edge in G, then $\{f(v_1), f(v_2), \ldots, f(v_n)\}$ forms an edge in G' (2). $\sigma(u) = \sigma'(f(u))$ for all $u \in V$ (3). $\mu(uv) = \mu'(f(u)f(v)), \forall\, uv \in E, \eta(x) = \eta'(f(x)), \forall\, x \in X$.

A *co-weak end vertex isomorphism* (*co-weak ev-isomorphism*) of fuzzy semigraphs $f : G \to G'$ is a bijective map $f : V \to V'$ and which satisfies, (1). If $x = (v_1, v_2, \ldots, v_n)$ is an edge in G then $\{f(v_1), f(v_2), \ldots, f(v_n)\}$ forms an edge in G' with end vertices $f(v_1)$ and $f(v_n)$, (2). $\sigma(u) = \sigma'(f(u))$ for all $u \in V$, (3). $\mu(uv) \leq \mu'(f(u)f(v))$ for all $uv \in E$, (4). $\eta(x) = \eta'(f(x))$, for all $x \in X$.

A *co-weak edge isomorphism* (*co-weak e-isomorphism*) of fuzzy semigraphs $f : G \to G'$ is a bijective map $f : V \to V'$ and which satisfies (1). If $x = (v_1, v_2, \ldots, v_n)$ is an edge in G, then $\{f(v_1), f(v_2), \ldots, f(v_n)\}$ forms an edge in G' (2). $\sigma(u) = \sigma'(f(u))$ for all $u \in V$, (3). $\mu(uv) \leq \mu'(f(u)f(v))$ for all $uv \in E$ and 4. $\eta(x) = \eta'(f(x))$, for all $x \in X$.

A *co-weak adjacency isomorphism* (*co-weak a-isomorphism*) of fuzzy semigraphs $f : G \to G'$ is a bijective map $f : V \to V'$ which satisfies, (1). If the adjacent vertices in G are mapped onto adjacent vertices in G' (2). $\sigma(u) = \sigma'(f(u))$ for all $u \in V$, (3). $\mu(uv) \leq \mu'(f(u)f(v))$ for all $uv \in E$ and (4). $\eta(x) = \eta'(f(x))$, for all $x \in X$.

Theorem 5.2. *If $f : G \to G'$ is a co-weak isomorphism on fuzzy semigraphs G and G' and if G' is a semi-effective fuzzy semigraph, then G is also a semi-effective fuzzy semigrap*h.

Proof of Theorem 5.2. Since G' is an effective fuzzy semigraph and f is a co-weak isomorphism,

Now $\mu(uv) \leq \mu'(f(u)f(v)) = \sigma'(f(u)) \wedge \sigma'(f(v)) \geq \sigma(u) \wedge \sigma(v)$

But $\mu(uv) \leq \sigma(u) \wedge \sigma(v)$. Hence $\mu(uv) = \sigma(u) \wedge \sigma(v)$, for all $uv \in E$

Thus G is a semi-effective fuzzy semigraph.

Remark 5.3. The semi-effectiveness of G need not imply the semi-effectiveness of G' when G is co-week isomorphic to G'.

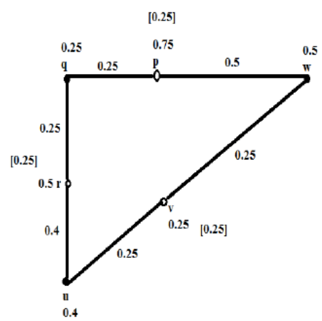

Figure 5.1. $G : (\sigma, \mu, \eta)$.

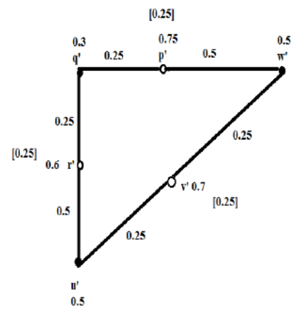

Figure 5.2. $G' : (\sigma', \mu', \eta')$.

Here G is co-weak isomorphic to G'. G is a semi-effective fuzzy semigraph but G' is not semi-effective.

6. Conclusion

In this work, isomorphism, weak isomorphism and co-weak isomorphism of a fuzzy semigraph are introduced and their properties are discussed. Transport networks and telecommunication networks can be modeled as fuzzy semigraphs. Hence our findings may be useful for future Studies and Research.

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