

AN INTENSIVE SEARCH ON DISTINCT DOMINATION AND EDGE SEQUENCE IN VARIOUS FUZZY GRAPHS



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This thesis entitled **AN INTENSIVE SEARCH ON DISTINCT DOMINATION AND EDGE SEQUENCE IN VARIOUS FUZZY GRAPHS** submitted by P. Geethanjali in fulfillment of the degree of Doctor of Philosophy in Mathematics is a record of original work done by her under my guidance. It has not previously formed the basis for the award of any Degree, Diploma, Associateship, Fellowship or any such similar title of any University or Institution.

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ABSTRACT

The concept of graph theory was initially developed by Euler in 1736. In 1965, Lotfi. A. Zadeh introduced the concept of fuzzy sets, which has since been successfully applied to several uncertain real-world decision problems. Berge introduced dominance as a theoretical term in graph theory in 1958. In 1983, Atanassov presented intuitionistic fuzzy sets as a generalization of fuzzy sets. As described by Zadeh in 1975, interval-valued fuzzy sets are an extension of fuzzy sets in which the values of the membership degrees are intervals of numbers rather than integers. In 1993, Gau and Buehrer proposed a vague set theory, which is a generalization of Zadeh's fuzzy set theory. Picture fuzzy set is a modified version of fuzzy set and intuitionistic fuzzy set presented by Cuong and Kreinovich. Pathinathan and Jon Arockiyaraj presented the hesitancy fuzzy graph, a new fuzzy graph with different theoretical features and validations. This research work is focused on various kinds of domination and edge sequences in different fuzzy graphs. In this research, we applied real-life applications to the above domination concept and we get the best result from our all concepts.

PREFACE

In 1736 the concept of graph theory was introduced by Euler. Fuzzy set theory was first introduced by Zadeh in 1965. Rosenfield developed the structure of fuzzy graphs and obtained analogs of several graph's theoretical concepts. Mordeson proposed concepts in Fuzzy Incidence Graphs. Ore and Berge introduced the concept of domination in 1962. Cockayne further developed the concept of domination. Domination in graphs has been examined further by Haynes. Domination in fuzzy graphs using effective edges was introduced by Somasundaram. Parvathi (2010) introduced the concept of domination number in intuitionistic fuzzy graphs. Manjusha has also expressed strong domination. Irfan Nazeer has established dominance in fuzzy incidence graphs. AM Ismayil has also expressed accurate split domination in fuzzy graphs. AN Shain has discussed the inverse dominating set of interval-valued fuzzy graphs. Mathew and Sunil Mathew introduced the concept of sequence in fuzzy graphs. This thesis "An intensive search on distinct domination and edge sequence in various fuzzy graphs" Comprises ten chapters.

Chapter 1 contains a brief history of the theory of graphs, fuzzy graphs, domination in fuzzy graphs, intuitionistic fuzzy graphs, interval-valued fuzzy graphs, vague fuzzy graph, and picture fuzzy graphs. Some basic definitions and theorems on fuzzy graphs which are needed for the subsequent chapters have been presented.

Chapter 2 focuses on a kind of strong domination constant number in pseudo regular fuzzy graph and complete fuzzy graph were discussed. Definitions, results, and properties of strong domination constant number are presented. The relationship between the strong domination constant number of a pseudo regular fuzzy graph and the complete fuzzy graph is also examined. Further, the concept of a strong domination constant number is discussed with the inclusion of the relationship between pseudo regular and totally pseudo regular fuzzy graph.

Chapter 3 explores a dual strong domination in vertex squared and vertex squared split intuitionistic fuzzy graphs. Direct product, semi-strong product, and semi-product of two vertex squared intuitionistic fuzzy graphs and join product of two vertex squared split intuitionistic fuzzy graphs are explained. The properties and theorems related to these parameters are compared with other known domination parameters.

Chapter 4 starts with the concept of split domination in vertex squared interval-valued fuzzy graphs namely n - split dominating set, n - split domination number. Moreover, this chapter consists vertex

squared cardinality, vertex squared independent set.

Chapter 5 introduces the new concept of perfect domination in the cartesian product of two interval-valued fuzzy incidence graphs and tensor product of two interval-valued fuzzy incidence graphs utilizing incidence pairs. Also, the concept of perfect domination number is used to discover which countries (country) have the best education policies among various countries.

Chapter 6 focuses on some kinds of strong and weak domination in the composition of two vague fuzzy incidence graphs. The strong edge incidentally dominating set, and weak edge incidentally dominating set, strong edge incidentally domination number, and weak edge incidentally domination number has been discussed.

Chapter 7 explores new kinds of strong and weak domination in complete intuitionistic fuzzy incidence graphs. For different classes of complete intuitionistic fuzzy incidence graphs, we compute the intuitionistic fuzzy incidence domination number, strong intuitionistic fuzzy incidence domination number, and weak intuitionistic fuzzy incidence domination number, and some theorems have been explained.

Chapter 8 introduce various types of dominating sets in product picture fuzzy graphs, such as the fixed vertex dominating set, fixed edge dominating set, total fixed edge dominating set, and fixed edge restrained dominating set. The properties with examples has been presented.

Chapter 9 explores a new concept of twin perfect domination in hesitancy fuzzy graphs and Omicron products of two hesitancy fuzzy graphs offering some interesting results have been included. The properties with examples has been presented.

Chapter 10 starts with the concept of edge sequences in regular fuzzy graphs and pseudo regular fuzzy graphs are explained by theorems with examples. In addition, a comparison of regular fuzzy graphs and totally regular fuzzy graphs are also discussed.

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Chapter 1

Introduction

1. Introduction

1.1 Graph Theory

In the Seventeenth Century, great Mathematician Leonard Euler created the basic idea of graph theory.



Figure 1.1.1: LEONARD EULER

Euler in 1736 first introduced the concept of graph theory. Graph theory is an important branch of Mathematics. A graph is a useful tool for describing information regarding object relationships. Vertices, relations, and edges are used to show the objects. In recent years, Graph Theory has seen a surge in

Research activity.

1.2 Königsberg Bridge Problem

Euler's work on the Königsberg Bridge problem can be traced back to the beginnings of graph theory. The City of Königsberg was located on the Pregel river in Prussia. The city occupied the island of Kneiphopf plus areas on both banks. These regions were linked by seven bridges. The objective was to find a closed path that ran across each of Königsberg's seven bridges exactly once. Euler demonstrated that there is no solution to this problem. Königsberg is now called Kaliningrad and is in Lithuania which recently separated from U.S.S.R.

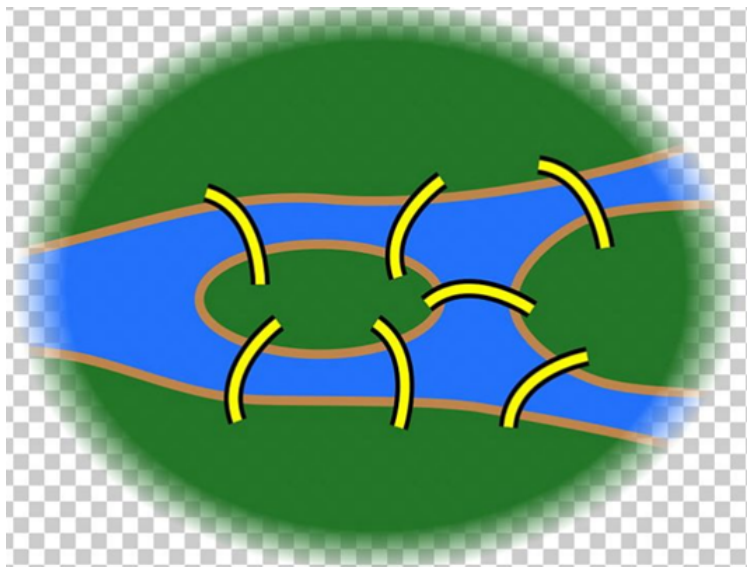


Figure 1.2.1: KÖNIGSBERG BRIDGE PROBLEM

1.3 Symposium on Graph Theory

A symposium on graph theory was organized in Delhi in 1973. A symposium on graph theory was held at I.S.I. Calcutta in December 1976, and the proceedings were published by Macmillan Company India Ltd. A conference on Combinatorics and graph theory was organized at I.S.I. Calcutta in February 1980.

Springer-Verlag, Berlin, published the proceedings. I.S.I. Calcutta held a 'Seminar on Combinatorics and Applications' in December 1982. Graph Theory and Combinatorics now account for one-third of all Mathematics research papers published on a global scale. Due to its wide range of applications in many domains such as Engineering, Social and Biological sciences, graph theory has grown dramatically.

1.4 Fuzzy Graph Theory

Several Mathematicians have discovered a new concept of graph theory. The new concepts like labeling, coloring, and domination are helpful to all field. One of the most important Mathematical breakthroughs of the twentieth century was a fuzzy set theory. Lotfi. A. Zadeh established the concept of fuzzy sets in 1965, and it has since been successfully used for a variety of real-life decision problems that are typically ambiguous. A fuzzy set is a variant of a crisp set in which the set's elements have various degrees of membership. The crisp set is made up of two truth values 0 (false) and 1 (truth), and it is unable to deal with ambiguous real-world problems.

Zadeh created a Mathematical theory that could deal with uncertainty and imprecision. The advantage of substituting classical sets with fuzzy sets is that it improves accuracy. A fuzzy set model is, therefore, more efficient than a classical model in systems with imprecision. Natural variables and traits such as intelligence, beauty, and consistency can be studied efficiently using fuzzy sets.

1.5 Roman Domination

The Roman Empire was under attack some 1700 years ago, and Emperor Constantine had to determine where to station his four field army units to defend eight regions. His strategy was to position the army units so that each region was either defended by its own army (one to two units) or by a neighbour with two army units, one of which could be moved directly to the undefended region if a conflict arose. Constantine stationed two army units in Rome and two in Constantinople, his new capital. As a result, only Britain could be reached in a single step. Constantine's successors ended up losing control of Britain. The causes were undoubtedly more complicated than our basic model could describe.

Apart from the placement of Roman army units, the same Mathematics can be applied to optimize

the position of dwindling British fleets at the end of the Nineteenth Century or American military army units during the Cold War. In addition to army placement, the same type of math is important when determining the optimal location for a new hospital, fire station, or fast food restaurant in a town. Roman dominion or its variants can often be used to model similar optimization Challenges. Cockayne formally defined Roman dominant functions. Due to historical causes deriving from the Ancient Roman Empire, one notable variation of domination in graphs is Roman domination.

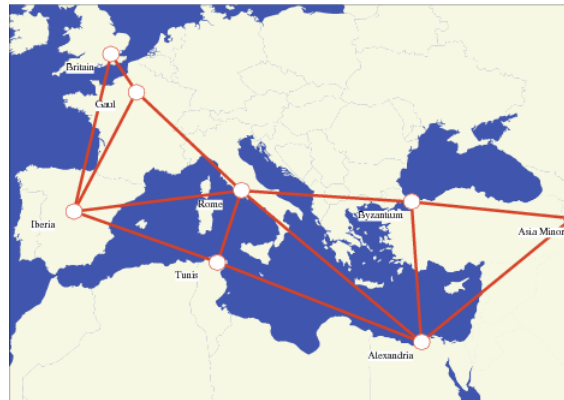


Figure 1.5.1: ROMAN DOMINATION

1.6 Chess Domination

The game of chess, which was popular in ancient India, inspired the introduction of dominance. Around 1850, the study of domination in graphs began with the goal of putting the fewest number of queens on an $n \times n$ chessboard while covering or dominating every square with at least one queen. These problems' solutions are simply dominating sets in a graph, with the vertices representing chessboard squares and the edges indicating the queen's possible moves. Berge introduced dominance as a theoretical term in graph theory in 1958.

In coding theory, the term "dominance" is frequently employed. We defined various new domination and edge sequences in fuzzy graphs.



Figure 1.6.1: CHESS DOMINATION

1.7 Intuitionistic Fuzzy Graph

As a generalization of fuzzy sets, Atanassov introduced intuitionistic fuzzy sets in 1983. In the concept of fuzzy set, Atanassov included a new component like the degree of non-membership. The degree of membership of an element in a given set is given by fuzzy sets, whereas intuitionistic fuzzy sets give both a degree of membership and a degree of non-membership that are more-or-less independent of one another, with the only requirement that the sum of these two degrees is not greater than 1. Interval-valued fuzzy sets are an extension of fuzzy sets in which the values of the membership degrees are intervals of numbers rather than integers, as presented by Zadeh in 1975. Interval-valued fuzzy sets are consequently important in applications like fuzzy control.

1.8 Vague Fuzzy Graph

Gau and Buehrer established the concept of vague set theory in 1993, which is a generalization of Zadeh's fuzzy set theory. The existence of the faulty membership degree may be explained quite well with a vague set. When paired with systems that run on fuzzy graphs, a vague fuzzy graph is a generalized form of a

fuzzy graph that provides more exactness, flexibility, and compatibility to a system. A vague fuzzy graph can focus on determining the uncertainty, as well as the inconsistent and indeterminate information, of any real-world scenario, when fuzzy graphs may not provide sufficient results.

1.9 Picture Fuzzy Graph

Picture fuzzy set is a modified version of fuzzy set and intuitionistic fuzzy set presented by Cuong and Kreinovich. To deal with real-life circumstances containing information kinds such as yes, abstention, no, and rejection, the picture fuzzy graph is more exact, adaptable, and compatible than the intuitionistic fuzzy graph.

1.10 Hesitancy Fuzzy Graph

Pathinathan and Jon Arockiaraj introduced a new fuzzy graph labeled hesitancy fuzzy graphs and discussed their various theoretical properties and validations. The concept of hesitancy fuzzy graph is applied for choosing a time minimized emergency route to transport the accident victims to the preferred hospital.

- A new kind of strong domination constant number in pseudo regular fuzzy graph and complete fuzzy graph are discussed.
- A unique type of dual strong domination is established in vertex squared and vertex squared split intuitionistic fuzzy graphs.
- A new concept of split domination is proposed for vertex squared interval-valued fuzzy graphs.
- A novel concept for perfect domination in cartesian product of two interval valued fuzzy incidence graphs and tensor product of two interval valued fuzzy incidence graphs is offered using incidence pair.
- In the composition of two vague fuzzy incidence graphs, a new concept of strong and weak domination is introduced.

- In complete intuitionistic fuzzy incidence graphs, a new thing of strong and weak domination is established.
- A specific type of fixed domination is proposed in product picture fuzzy graphs.
- A new kind of twin perfect domination in hesitancy fuzzy graph and Omicron product of two hesitancy fuzzy graphs are discussed.
- A new concept of edge sequences is offered in regular fuzzy graphs and pseudo regular fuzzy graphs.

PRELIMINARIES

The chapter contains the basic definitions and theorems required to develop the subsequent chapter of this thesis.

1.11 Basic concepts in fuzzy graph theory

Definition 1.11.1 A **graph** is a finite non empty set of objects called vertices together with a set of unordered pair of distinct vertices of G , called edges. The vertex set and the edge set of G are respectively denoted by $V(G)$ and $E(G)$. A graph G with vertex set and edge set is denoted by $G = (V, E)$.

Definition 1.11.2 Let X be a non empty set. Then a **fuzzy set** A in X (that is a fuzzy subset A of X) is characterized by a function of the form $\mu_A : X \rightarrow [0, 1]$ such a function μ_A is called the membership function and for each $x \in X$, $\mu_A(x)$ is the degree of membership of x (membership grade of x) in the fuzzy set A . In other words, $A = \{(x, \mu_A(x)) / x \in X\}$ where $\mu_A : X \rightarrow [0, 1]$.

Definition 1.11.3 A **fuzzy graph** is denoted by $G : (V, \sigma, \mu)$, where V is a node set, σ and μ are mappings defined as $\sigma : V \rightarrow [0, 1]$ and $\mu : V \times V$, where σ and μ denote the membership values of a node and an arc respectively. For any fuzzy graph, $\mu(x, y) \leq \min\{\sigma(x), \sigma(y)\}$.

Definition 1.11.4 Two nodes u and v in a fuzzy graph G are said to be **adjacent** if, $\mu(u, v) > 0$.

Definition 1.11.5 The **order** p and **size** q of a fuzzy graph $G : (V, \sigma, \mu)$ are defined to be $p = \sum_{x \in V} \sigma(x)$ and $q = \sum_{(x,y) \in V \times V} \mu(x, y)$.

Definition 1.11.6 An arc (u, v) of a fuzzy graph is called an **effective arc** (M -strong arc) if $\mu(u, v) = \sigma(u) \wedge \sigma(v)$.

Definition 1.11.7 A fuzzy graph $G : (V, \sigma, \mu)$ is **connected** if for every x, y in σ^* , $CONN_G(x, y) > 0$.

Definition 1.11.8 Let $G : (V, \sigma, \mu)$ be a fuzzy graph and $S \subseteq V$. Then the **scalar cardinality** of S is defined to be $\sum_{v \in S} \sigma(v)$ and it is denoted by $|S|$. Let p denotes the scalar cardinality of V , also called the order of G .

Definition 1.11.9 The **strength of connectedness** between two nodes of x and y is defined as the maximum of the strengths of all paths between x and y and is denoted by $CONN_G(x, y)$.

Definition 1.11.10 An arc of a fuzzy graph $G : (V, \sigma, \mu)$ is called **strong** if its weight is at least as great as the strength of connectedness of its end nodes should be deleted.

Definition 1.11.11 A path P is called strong path if P contains only strong arcs. If $\mu(u, v) > 0$, then u and v are called **neighbours**. The set of all neighbors of u is denoted by $N(u)$.

Definition 1.11.12 A node u is said to be **isolated** if $\mu(u, v) = 0$ for all $v \neq u$.

Definition 1.11.13 The fuzzy graph G is called a **strong fuzzy graph** if each arc in G is a strong arc.

Definition 1.11.14 An arc (x, y) in G is α - **strong** if $\mu(x, y) > CONN_{G-(x,y)}(x, y)$. An arc (x, y) in G is β - **strong** if $\mu(x, y) = CONN_{G-(x,y)}(x, y)$. An arc (x, y) in G is δ - **arc** if $\mu(x, y) < CONN_{G-(x,y)}(x, y)$. An arc (x, y) is a **strong** if it is either α - strong or β - strong. Also y is called **strong neighbour** of x if arc (x, y) is strong.

Definition 1.11.15 Let $G : (\sigma, \mu)$ be a fuzzy graph $G^* : (V, E)$. The **degree** of a vertex u is $d_G(u) = \sum_{u \neq v} \mu(u, v)$. The minimum degree of G is $\delta(u) = \min\{d_G(v), \forall v \in V\}$ and the maximum degree of G is $\Delta(u) = \max\{d_G(v), \forall v \in V\}$.

Definition 1.11.16 Let $G : (\sigma, \mu)$ be a fuzzy graph $G^* : (V, E)$. If $d(v) = k$ for all $v \in V$, then G is said to be a **regular fuzzy graph** of degree k .

Definition 1.11.17 The total degree of a vertex $u \in V$ is defined by $td_G(u) = \sum_{u \neq v} \mu(u, v) + \sigma(u) = d_G(u) + \sigma(u)$. If each vertex of G has the same degree k , then G is said to be a **totally regular fuzzy graph** of total degree k or k -totally regular fuzzy graph.

Definition 1.11.18 Let $G : (\sigma, \mu)$ be a fuzzy graph $G^* : (V, E)$. The **2-degree** of a vertex v in G is defined as the sum of degrees of the vertices adjacent to v and is denoted by $t_G(v)$. That is, $t_G(v) = \sum d_G(u)$, where $d_G(u)$ is the degree of the vertex u which is adjacent with the vertex v .

Definition 1.11.19 Let $G : (\sigma, \mu)$ be a fuzzy graph $G^* : (V, E)$. A **pseudo (average) degree** of a vertex v in fuzzy graph G is denoted by $d_a(v)$ and is defined by $d_a(v) = \frac{t_G(v)}{d_G^*(v)}$, where $d_G^*(v)$ is the number of edges incident at v .

Definition 1.11.20 Let $G : (\sigma, \mu)$ be a fuzzy graph $G^* : (V, E)$. If $d_a(v) = k$, for all v in V then G is called **k -pseudo regular fuzzy graph**.

Definition 1.11.21 Let $G : (\sigma, \mu)$ be a fuzzy graph $G^* : (V, E)$. The **total pseudo degree** of a vertex v in G is denoted by $td_a(v)$ and is defined as $td_a(v) = d_a(v) + \sigma(v)$ for all $v \in V$.

Definition 1.11.22 Let $G : (\sigma, \mu)$ be a fuzzy graph $G^* : (V, E)$. If all the vertices of G have the same total pseudo degree k , then G is said to be a **totally k -pseudo regular fuzzy graph**.

Definition 1.11.23 A fuzzy graph $G : (V, \sigma, \mu)$ is said to be **complete** if $\mu(u, v) = \sigma(u) \wedge \sigma(v)$ for all $u, v \in \sigma^*$.

Definition 1.11.24 Let X be a given set. An **intuitionistic fuzzy set** A in X is given by $A = \{(x, \mu_A(x), \gamma_A(x)) / x \in X\}$, where $\mu_A : X \rightarrow [0, 1]$, $\gamma_A : X \rightarrow [0, 1]$ and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$, where $\mu_A(x)$ is the degree of membership of the element x in A and $\gamma_A(x)$ is the degree of non membership of the element x in A .

Definition 1.11.25 An **intuitionistic fuzzy graph** is of the form $G : (V, E)$ where

(i) $V = \{v_1, v_2, \dots, v_n\}$ such that $\mu_1 : V \rightarrow [0, 1]$ and $\gamma_1 : V \rightarrow [0, 1]$ denote the degree of membership and non membership of the element $v_i \in V$ respectively and $0 \leq \mu_1(v_i) + \gamma_1(v_i) \leq 1$ for every $v_i \in V (i = 1, 2, \dots, n)$

(ii) $E \subseteq V \times V$ where $\mu_2 : V \times V \rightarrow [0, 1]$ and $\gamma_2 : V \times V \rightarrow [0, 1]$ are such that

$$\mu_2(v_i, v_j) \leq \min(\mu_1(v_i), \mu_1(v_j))$$

$$\gamma_2(v_i, v_j) \leq \max(\gamma_1(v_i), \gamma_1(v_j))$$

$$0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 1 \text{ for every } (v_i, v_j) \in E \quad (i, j = 1, 2, \dots, n)$$

Definition 1.11.26 If $v_i, v_j \in V \subseteq G$, the μ - **strength of connectedness** between v_i and v_j is $\mu_2^\infty(v_i, v_j) = \sup\{\mu_2^k(v_i, v_j) \mid k = 1, 2, \dots, n\}$ and γ - **strength of connectedness** between v_i and v_j is $\gamma_2^\infty(v_i, v_j) = \inf\{\gamma_2^k(v_i, v_j) \mid k = 1, 2, \dots, n\}$.

Definition 1.11.27 An intuitionistic fuzzy graph $G : (V, E)$ is said to be **strong intuitionistic fuzzy graph** if $\mu_2(v_i, v_j) = \min(\mu_1(v_i), \mu_1(v_j))$ and $\gamma_2(v_i, v_j) = \max(\gamma_1(v_i), \gamma_1(v_j))$ for every $v_i, v_j \in E$.

Definition 1.11.28 An edge (u, v) is said to be **strong edge** if $\mu_2(u, v) \geq \mu_2^\infty(u, v)$ and $\gamma_2(u, v) \geq \gamma_2^\infty(u, v)$.

Definition 1.11.29 If all the edges are strong edge in an intuitionistic fuzzy graph then it is called **strengthen intuitionistic fuzzy graph**.

Definition 1.11.30 The **order** p and **size** q of a intuitionistic fuzzy graph $G : (V, E)$ are defined to be

$$p = \sum_{v_i \in V} \left[\frac{1 + \mu_1(v_i) - \gamma_1(v_i)}{2} \right] \text{ and}$$

$$q = \sum_{v_i, v_j \in E} \left[\frac{1 + \mu_2(v_i, v_j) - \gamma_2(v_i, v_j)}{2} \right].$$

Definition 1.11.31 An intuitionistic fuzzy graph $G : (V, E)$ is said to be **complete intuitionistic fuzzy graph** if $\mu_2(v_i, v_j) = \min(\mu_1(v_i), \mu_1(v_j))$ and $\gamma_2(v_i, v_j) = \max(\gamma_1(v_i), \gamma_1(v_j))$ for every $v_i, v_j \in V$.

Definition 1.11.32 An interval-valued fuzzy set A on a set V is defined by $A = \{x, ([\mu_A^-(x), \mu_A^+(x)]) : x \in V\}$ where μ_A^- and μ_A^+ are fuzzy subsets of V such that $\mu_A^-(x) \leq \mu_A^+(x)$ for all $x \in V$. If $G^* : (V, E)$ is a crisp graph, then by an interval-valued fuzzy relation B on V we mean an interval-valued fuzzy set on E such that $\mu_B^-(xy) \leq \min\{\mu_A^-(x), \mu_A^-(y)\}$ and $\mu_B^+(xy) \leq \max\{\mu_A^+(x), \mu_A^+(y)\}$ for all $xy \in E$ and we write $B = \{xy, ([\mu_B^-(xy), \mu_B^+(xy)]) : xy \in E\}$.

Definition 1.11.33 An interval-valued fuzzy graph of a graph $G^* : (V, E)$ is a pair $G : (A, B)$, where $A = [\mu_A^-, \mu_A^+]$ is an interval-valued fuzzy set on V and $B = [\mu_B^-, \mu_B^+]$ is an interval-valued fuzzy relation on V .

Definition 1.11.34 Let A be a picture fuzzy set. A in X is defined by $A = \{x, \mu_A(x), \eta_A(x), \gamma_A(x) / x \in X\}$, where $\mu_A(x), \eta_A(x)$ and $\gamma_A(x)$ follow the condition $0 \leq \mu_A(x) + \eta_A(x) + \gamma_A(x) \leq 1$. The $\mu_A(x), \eta_A(x), \gamma_A(x) \in [0, 1]$, denote respectively the positive membership degree, neutral membership degree and negative membership degree of the element x in the set A . For each picture fuzzy set A in X , the refusal membership degree is described as $\pi_A(x) = 1 - \{\mu_A(x) + \eta_A(x) + \gamma_A(x)\}$.

Definition 1.11.35 Let $G^* = (V, E)$ is a graph. A pair $G = (A, B)$ is called a picture fuzzy graph on G^* where $A = \{\mu_A, \eta_A, \gamma_A\}$ is a PFS on V and $B = \{\mu_B, \eta_B, \gamma_B\}$ is picture fuzzy set on $E \subseteq V \times V$ such that for each edge $uv \in E$.

$$\mu_B(uv) \leq \min(\mu_A(u), \mu_A(v)),$$

$$\eta_B(uv) \leq \min(\eta_A(u), \eta_A(v)),$$

$$\gamma_B(uv) \geq \max(\gamma_A(u), \gamma_A(v)).$$

Definition 1.11.36 A picture fuzzy graph $G = (A, B)$ is said to be strong picture fuzzy graph if

$$\mu_B(uv) = \min(\mu_A(u), \mu_A(v)),$$

$$\eta_B(uv) = \min(\eta_A(u), \eta_A(v)),$$

$$\gamma_B(uv) = \max(\gamma_A(u), \gamma_A(v)), \forall uv \in E.$$

Definition 1.11.37 A picture fuzzy graph $G = (A, B)$ is said to be **complete picture fuzzy graph** if

$$\begin{aligned}\mu_B(uv) &= \min(\mu_A(u), \mu_A(v)), \\ \eta_B(uv) &= \min(\eta_A(u), \eta_A(v)), \\ \gamma_B(uv) &= \max(\gamma_A(u), \gamma_A(v)), \quad \forall uv \in V.\end{aligned}$$

Definition 1.11.38 If $\mu_B(uv) \geq \mu_B^\infty(uv)$, $\eta_B(uv) \geq \eta_B^\infty(uv)$ and $\gamma_B(uv) \leq \gamma_B^\infty(uv)$ for every $uv \in V$, an edge uv is called a **strong edge**, where $\mu_B^\infty(uv)$, $\eta_B^\infty(uv)$ and $\gamma_B^\infty(uv)$ are the strength of the connectedness between u and v in the picture fuzzy graph produced from G by removing the edge uv .

Definition 1.11.39 Let $G = (V, E)$ be a graph. Then, $G = (V, E, I)$ is named as an **incidence graph**, where $I \subseteq V \times E$.

Definition 1.11.40 Let $G = (V, E)$ be a graph. μ be a fuzzy subset of V , and γ be a fuzzy subset of $V \times V$ and Let ψ be a fuzzy subset of $V \times E$. If $\psi(w_1, w_1w_2) \leq \min\{\mu(w_1), \gamma(w_1w_2)\}$ for every $w_1 \in V, w_1w_2 \in E$, then ψ is a **fuzzy incidence of G** .

Definition 1.11.41 Let G be a graph and (μ, γ) is a fuzzy sub graph of G . If ψ is a fuzzy incidence of G , then $G = (\mu, \gamma, \psi)$ is named as **fuzzy incidence graph of G** .

Definition 1.11.42 An intuitionistic fuzzy incidence graph is of the form $G = (V, E, I, \rho, \phi, \chi)$ where $\rho = (\rho_1, \rho_2)$, $\phi = (\phi_1, \phi_2)$, $\chi = (\chi_1, \chi_2)$ and $V = \{x_0, x_1, \dots, x_n\}$ such that $\rho_1 : V \rightarrow [0, 1]$ and $\rho_2 : V \rightarrow [0, 1]$ represent the degree of membership and non membership of the vertex $x_1 \in V$ respectively and $0 \leq \rho_1 + \rho_2 \leq 1$ for each $x_i \in V (i = 1, 2, \dots, n)$, $\phi_1 : V \times V \rightarrow [0, 1]$ and $\phi_2 : V \times V \rightarrow [0, 1]$; $\phi_1(x_1, x_2)$ and $\phi_2(x_1, x_2)$ show the degree of membership and non membership of the edge (x_1, x_2) , respectively, such that $\phi_1(x_1, x_2) \leq \min\{\rho_1(x_1), \rho_1(x_2)\}$ and $\phi_2(x_1, x_2) \leq \max\{\rho_2(x_1), \rho_2(x_2)\}$, $0 \leq \phi_1(x_1, x_2) + \phi_2(x_1, x_2) \leq 1$ for every (x_1, x_2) . $\chi_1 : V \times E \rightarrow [0, 1]$ and $\chi_2 : V \times E \rightarrow [0, 1]$; $\chi_1(x_1, x_1x_2)$ and $\chi_2(x_1, x_1x_2)$ show the degree of membership and non membership of the incidence pair respectively, such that $\chi_1(x_1, x_1x_2) \leq \min\{\rho_1(x_1), \phi_1(x_1x_2)\}$ and $\chi_2(x_1, x_1x_2) \leq \max\{\rho_2(x_1), \phi_2(x_1x_2)\}$, $0 \leq \chi_1(x_1, x_1x_2) + \chi_2(x_1, x_1x_2) \leq 1$ for every (x_1, x_1x_2) .

Definition 1.11.43 A vague set A is a pair (t_A, f_A) on set V where t_A and f_A are taken as real valued functions which can be defined on $V \rightarrow [0, 1]$, so that $t_A(m) + f_A(m) \leq 1$, for all m belongs V . The interval $[t_A(m), 1 - f_A(m)]$ is known as the vague value of m is A .

Definition 1.11.44 A pair $G = (A, B)$ is said to be a **vague graph** on a crisp graph $G = (V, E)$, where $A = (t_A, f_A)$ is a vague set on V and $B = (t_B, f_B)$ is a vague set on $E \subseteq V \times V$ such that

$$\begin{aligned} t_B(mn) &\leq \min(t_A(m), t_A(n)), \\ f_B(mn) &\geq \max(f_A(m), f_A(n)), \text{ for each edge } mn \in E. \end{aligned}$$

Definition 1.11.45 Let $\zeta = (A, B, C)$ is called a **vague incidence graph** of underlying crisp incidence graph $G^* = (V, E, I)$ if

$$\begin{aligned} A &= \{ \langle t_A(v), f_A(v) \rangle / v \in V \}, \\ B &= \{ \langle t_B(mn), f_B(mn) \rangle / mn \in E \}, \\ C &= \{ \langle t_C(v, mn), f_C(v, mn) \rangle / (v, mn) \in I \} \end{aligned}$$

such that

$$\begin{aligned} t_B(mn) &\leq t_A(m) \wedge t_A(n), \quad f_B(mn) \geq f_A(m) \vee f_A(n), \\ t_C(v, mn) &\leq t_A(v) \wedge t_B(mn), \quad f_C(v, mn) \geq f_A(v) \vee f_B(mn), \forall v \in V, mn \in E. \end{aligned}$$

and

$$0 \leq t_A(v) + f_A(v) \leq 1, 0 \leq t_B(mn) + f_B(mn) \leq 1, 0 \leq t_C(v, mn) + f_C(v, mn) \leq 1$$

Definition 1.11.46 A **hesitancy fuzzy graph** is of the form $G = (V, E)$, where $V = \{v_1, v_2, v_3, \dots, v_n\}$ such that $\mu_1 : V \rightarrow [0, 1]$, $\gamma_1 : V \rightarrow [0, 1]$ and $\beta_1 : V \rightarrow [0, 1]$ denote the degree of membership, non-membership and hesitancy of the element $v_1 \in V$ respectively and $\mu_1(v_i) + \gamma_1(v_i) + \beta_1(v_i) = 1$ for every $v_i \in V$, where $\beta_1(v_1) = 1 - [\mu_1(v_i) + \gamma_1(v_i)]$ and $E \subseteq V \times V$ where $\mu_2 : V \times V \rightarrow [0, 1]$,

$\gamma_2 : V \times V \rightarrow [0, 1]$ and $\beta_2 : V \times V \rightarrow [0, 1]$ are such that

$$\mu_2(v_i, v_j) \leq \min[\mu_1(v_i), \mu_1(v_j)]$$

$$\gamma_2(v_i, v_j) \leq \max[\gamma_1(v_i), \gamma_1(v_j)]$$

$$\beta_2(v_i, v_j) \leq \min[\beta_1(v_i), \beta_1(v_j)]$$

and $0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) + \beta_2(v_i, v_j) \leq 1$ for every $(v_i, v_j) \in E$.

Definition 1.11.47 A set of vertices in G is **independent** if no two vertices in the set are adjacent.

Definition 1.11.48 An independent set K of G is named as **retrained independent** set if all nodes of K have the same degrees.

Definition 1.11.49 A set S of vertices of G is a **dominating set** of G if every vertex of $V(G) - S$ is adjacent to some vertex in S . A dominating set S of G is referred to as minimal dominating set in no proper subset of S is a dominating set.

Definition 1.11.50 A minimum dominating set in a graph G is a dominating set of minimum cardinality. The cardinality of a minimum dominating set is called the **domination number** of G and is denoted by $\gamma(G)$.

Definition 1.11.51 Let $G : (\sigma, \mu)$ be a fuzzy graph with node set V . Let u and v be any two nodes of G . We say that u dominates v if (u, v) is a strong arc. A subset D of v is called a **dominating set** of G if for every $v \notin D$, there exists $u \in D$ such that u dominates v .

Definition 1.11.52 A dominating set D is called a minimal dominating set if no proper subset of D is a dominating set. The smallest number of nodes in any dominating set of G is called its **domination number** and is denoted by $\gamma(G)$.

Definition 1.11.53 A set D of nodes of G is a **strong dominating set** of G if every vertex of $V(G) - D$ is a strong neighbour of some node in D .

Definition 1.11.54 A minimum strong dominating set as a strong dominating set of minimum scalar cardinality. The scalar cardinality of a minimum strong dominating set is called the **strong domination number** of G .

Definition 1.11.55 The **weight of a strong dominating set** D is defined as $W(D) = \sum_{u \in D} \mu(u, v)$, where $\mu(u, v)$ is the minimum of the membership values (weight) of the strong arcs incident on u . The strong domination number of a fuzzy graph G is defined as the minimum weight of strong dominating sets of G and it is denoted by $\gamma_S(G)$.

Definition 1.11.56 Let G be a fuzzy graph and e_i and e_j be two adjacent edges of G . We say that e_j if e_i is an effective edge. An edge subset K of E in a G is called an **edge dominating set** if, for each edge $e_i \in E - K$, there are effective edge $e_j \in K$ so that e_j and e_i .

Definition 1.11.57 An edge dominating set K of a G is said to be a minimal edge dominating set if for each edge $e \in K$, $K - \{e\}$ is not an edge dominating set. The minimum cardinality between all minimal edge dominating sets is called an **edge dominating number** and is denoted by $\gamma(G)$.

1.12 List of Abbreviation

CFG	- Complete Fuzzy Graph
RFG	- Regular Fuzzy Graph
PRFG	- Pseudo Regular Fuzzy Graph
VSIFG	- Vertex Squared Intuitionistic Fuzzy Graph
VSSIFG	- Vertex Squared Split Intuitionistic Fuzzy Graph
VSIVFG	- Vertex Squared Interval-Valued Fuzzy Graph
PPFG	- Product Picture Fuzzy Graph
HFG	- Hesitancy Fuzzy Graph
IVFIG	- Interval-Valued Fuzzy Incidence Graph
VFIG	- Vague Fuzzy Incidence Graph
CT-VFIG	- Composition of Two Vague Fuzzy Incidence Graph
CIFIG	- Complete Intuitionistic Fuzzy Incidence Graph
SDCN	- Strong Domination Constant Number
DSDN	- Dual Strong Domination Number
n -SDN	- n -Split Domination Number
PDN	- Perfect Dominating Number
EIDN	- Edge Incidentally Dominating Number
SIFIDN	- Strong Intuitionistic Fuzzy Incidence Domination Number
FVDN	- Fixed Vertex Domination Number
TPDN	- Twin Perfect Domination Number

1.13 Literature Review

In 1736 the concept of graph theory was introduced by Euler. Graph theory is an important branch of Mathematics. In recent years, graph theory has seen a surge in Research activity. Fuzzy set theory was first introduced by Zadeh in 1965 [Zad65,Zad75]. The first definition of fuzzy graph was introduced by

Haufmann in 1973 based on Zadeh's fuzzy relations in 1971. The fuzzy relations between fuzzy sets were also considered by Rosenfield. Rosenfield developed the structure of fuzzy graphs and obtained analogues of several graphs theoretical concepts. Generalized theory and fuzzy logic have been focused on by Zadeh and Yeh [Zad08, YB75]. Fuzzy graphs were introduced by Rosenfeld, who has described the fuzzy analogue of different graph theoretic concepts like paths, cycles, trees and connectedness and established some of their properties [Ros71]. Some important works in fuzzy graph theory were discussed by Mordeson [MN00, MP94][48,49]. Sunil Mathew [MM16, SMY19, MS09, MS10, MS13] defined and classified different types of arcs in fuzzy graphs. Samanta [SP11] have also expressed various fuzzy graphs. Pathinathan [PR14, TPR15] defined relationship between different types of arcs in both regular and totally regular fuzzy graph. Santhi Maheswari [MS16] introduced on pseudo regular fuzzy graphs. Mahalakshmi have also expressed fuzzy strong graphs [KM17, KM18]. Bhutani [BR03a, BR03b] have introduced the concept of strong arcs. Kalaiaarasi [Kal11a, Kal11b] defined Optimization of fuzzy integrated vendor-buyer inventory models in 2011.

In 1983 Atanassov [Ata15] introduced the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. The first definition of intuitionistic fuzzy relations and intuitionistic fuzzy graphs were introduced by Atanassov (1999). Intuitionistic fuzzy sets have been applied in a wide variety of fields including Computer Science, Engineering, Mathematics, Medicine, Chemistry and Economics [SDR01]. Parvathi [RPA09, PK06] have initiated the idea of intuitionistic fuzzy graphs. In intuitionistic fuzzy graphs, Gani [NB10] established the concepts of degree, order, and size. Products in intuitionistic fuzzy graphs were discussed by Sahoo [SP15][69]. Pal [SP16], [SP16, SP17a] researched some types of fuzzy graphs. Sahoo [SP17b] initiated new ideas in intuitionistic fuzzy graphs. Different types of intuitionistic fuzzy graphs and their applications can be found in different research papers.

The concept of fuzzy sets was discussed by Turksen [Tur86]. We summarise Gorzalczy's work on interval-valued fuzzy sets [Gor89] fuzzy relations because interval-valued fuzzy sets are frequently applied. Hossein Rashmanlou have also expressed interval-valued sub semigroups and subgroups [HL09]. Akram [AD11, MAP17] has given the idea that fuzzy graphs. Rashmanlou [RP13] recommended irregular interval-valued fuzzy graphs. Hongmei have also expressed interval-valued sub semigroups and subgroups [HL09]. Sahoo [SSS20] presented a fuzzy graph with application. The product of the new graph was produced by Irfan Nazeer [ING21] in 2021.

Fuzzy incidence graphs were discussed by Dinesh [Din16]. Borzooei [JMB18] developed incidence cuts in fuzzy incidence graphs. Mordeson [MM17, SMY19] proposed concepts in Fuzzy Incidence Graphs. Cuong [Cuo14] proposed a picture fuzzy set ranking method as well as a set of picture fuzzy set attributes. Singh [Sin15] campaigned for picture fuzzy sets. Peng [PD17] suggested and implemented an algorithmic technique for picture fuzzy set in a decision-making situation. Wei [Wei17] has presented a strategy for determining decisions. Wei discussed how to measure picture fuzzy sets. Mean operators and their applications have been extended by Cen Zuo [CZD19]. Notions of picture fuzzy graph were discussed by Rukhshanda [RAG21]. Bipolar picture fuzzy graphs have been created by Babir Ali since 2021 [WAKT21]. Vague sets were first proposed by Gau [GB93]. The concept of vague graphs was developed by Ramakrishna [Ram09]. Akram [MAS14] proposed vague hyper graphs. Degree of vertices in vague graphs were proposed by Borzooei [BR15]. Pal [RBP16] suggested and implemented regularity of vague graphs. Properties of vague graphs extended by Rao [YRS20]. Pathinathan and Jesintha [TPR15] introduced the hesitancy fuzzy graphs. Pathinathan and Jesintha [TPR15] introduced the hesitancy fuzzy graphs. Graph theory was discussed by Arumugam [AR01]. Begum [SS17b] is also expressed by different fuzzy graphs. Bhattacharya [Bha87] presented some remarks on fuzzy graphs. Bustince [HBG10, HBP] has introduced the concept of fuzzy relations and operators. Cao [Cao98] established the bounds on eigen values. An automorphism of fuzzy graphs was proposed by Butani [Bhu89]. De and Srinivasan [SDR01, SS17a] investigated intuitionistic fuzzy graphs. Harary [Har73] were discussed graph theory. Salen [Sal12] has also expressed inter-valued fuzzy topological space. Son [Son16] presented a picture fuzzy graph with applications. Isomorphism on inter-valued fuzzy graphs was discussed by Talebi [TR13]. The concept of the spectral radius of graphs was discussed by Tian [AYT04].

Ore and Berge introduced the concept of domination in 1962. Cockayne [CH77] further developed the concept of domination. Domination in fuzzy graphs using effective edges were introduced by Somasundaram [SS98]. Domination in graphs has been examined further by Haynes [THS98]. Parvathi (2010) introduced the concept of domination number in intuitionistic fuzzy graphs. Xavier [DXC13] was discussed about domination in fuzzy graphs. Equitable domination number for fuzzy graphs was introduced by Revathi in [RH14]. Manjusha have also expressed strong domination [MS15]. Manjusha [MS19] has discussed paired domination in fuzzy graphs. Sarala has also expressed (1,2)-domination for fuzzy graphs [SK16]. Chandrasekaran was discussed about strong arcs [NC16, ANR21]. Dharmalingam

has also expressed domination parameters for fuzzy graphs [DN17]. Pradip Debnath has given the characterization for a minimal dominating set [Deb13]. Irfan Nazeer [ING21] have established dominance in fuzzy incidence graphs. AM Ismayil [IH21] have also expressed accurate split domination in fuzzy graphs. Bhimani [BB21] presented new definition of corona product with another path graph P_{n-1} . AN Shain [SS21] has discussed inverse dominating set of an interval-valued fuzzy graphs. Selvam [SP21] investigated domination in join of fuzzy graphs. Begum [NB10, NR10] was discussed about the extension of fuzzy graphs. Sriram Kalyan and Sunitha [Sri21, SM15, SV99, SV02, SV05] presented various fuzzy graphs with different domination. Yongsheng Roa [YRK21] defined domination in vague graphs.

1.14 Genesis of the Thesis

Lotfi. A. Zadeh established the concept of fuzzy sets in 1965, and it has since been successfully used for a variety of real-life decision problems that are typically ambiguous. A.Nagoorgani and V.T.Chandrasekaran introduced fuzzy domination as a theoretical term in fuzzy graph theory in 2006. In coding theory, the term dominance is frequently employed. So the present research focuses its attention on the domination and edge sequence through pseudo regular fuzzy graph, complete fuzzy graph, vertex squared intuitionistic fuzzy graphs, vertex squared split intuitionistic fuzzy graphs, vertex squared interval-valued fuzzy graphs, cartesian product of two interval-valued fuzzy incidence graphs, tensor product of two interval-valued fuzzy incidence graphs, composition of two vague fuzzy incidence graphs, complete intuitionistic fuzzy incidence graphs, Omicron product of two hesitancy fuzzy graphs, product picture fuzzy graphs. Finally, we performed real-life applications to find exact results for it.

1.15 Motivations and Scope of Research work

- The features of a pseudo regular fuzzy graph and a complete fuzzy graph has described. The concept of strong domination constant number in pseudo regular and complete fuzzy graphs, offering some interesting results have been included.
- In vertex squared and vertex squared split intuitionistic fuzzy graphs, a novel type of dual strong domination is established. The properties and theorems related to these parameters are compared

with other known domination parameters.

- In vertex squared interval-valued fuzzy graphs, a new concept of split domination has been proposed. With the support of the split domination concept, vertex squared interval-valued fuzzy graphs to choose which oxygen cylinder agencies have the lot of oxygen cylinder among various oxygen cylinder agencies inspected.
- Using incidence pair, a unique idea for perfect domination in cartesian product of two interval-valued fuzzy incidence graphs and tensor product of two interval-valued fuzzy incidence graphs were proposed. Eventually, the concept of perfect domination number is used to discover which countries (country) have the best education policies among various countries.
- The special concept of strong and weak domination in vague fuzzy incidence graphs helps to identify the maximum percentage of progress and minimum percentage of non-progress in various journal publications.
- With the support of the strong and weak domination concept, complete intuitionistic fuzzy incidence graphs were used to choose the best treatment facility accessible clinic in various clinics.
- In product picture fuzzy graphs, a specific type of fixed domination were proposed and using these fixed domination concept were proposed to find the nearest hospital for a emergency time in highway roads.
- There are some fascinating results from a new notion of twin perfect domination in hesitancy fuzzy graphs and Omicron products of two hesitancy fuzzy graphs.
- In regular fuzzy graphs and pseudo regular fuzzy graphs, a new concept of edge sequences were offered. In addition, with distinct categories of edge sequences, an analogy has been conducted between pseudo regular fuzzy graphs and totally pseudo regular fuzzy graphs.

1.16 Organization of the Thesis

The entire dissertation consists of ten chapters and each chapter is subdivided into a number of sections.

CHAPTER I contains a brief history of the theory of graphs, fuzzy graphs, domination in fuzzy graphs, intuitionistic fuzzy graphs, interval-valued fuzzy graphs, vague fuzzy graph, and picture fuzzy graphs. Some basic definitions and theorems on fuzzy graphs which are needed for the subsequent chapters have been presented.

CHAPTER II focuses on a kind of strong domination constant number in pseudo regular fuzzy graph and complete fuzzy graph were discussed. Definitions, results, and properties of strong domination constant number are presented. The relationship between the strong domination constant number of a pseudo regular fuzzy graph and the complete fuzzy graph is also examined. Further, the concept of a strong domination constant number is discussed with the inclusion of the relationship between pseudo regular and totally pseudo regular fuzzy graph.

CHAPTER III explores a dual strong domination in vertex squared and vertex squared split intuitionistic fuzzy graphs. Direct product, semi-strong product, and semi-product of two vertex squared intuitionistic fuzzy graphs and join product of two vertex squared split intuitionistic fuzzy graphs are explained. The properties and theorems related to these parameters are compared with other known domination parameters.

CHAPTER IV starts with the concept of split domination in vertex squared interval-valued fuzzy graphs namely n - split dominating set, n - split domination number. Moreover, this chapter consists vertex squared cardinality, vertex squared independent set.

CHAPTER V introduces the new concept of perfect domination in the cartesian product of two interval-valued fuzzy incidence graphs and tensor product of two interval-valued fuzzy incidence graphs utilizing incidence pairs. Also, the concept of perfect domination number is used to discover which countries

(country) have the best education policies among various countries.

CHAPTER VI focuses on some kinds of strong and weak domination in the composition of two vague fuzzy incidence graphs. The strong edge incidentally dominating set, and weak edge incidentally dominating set, strong edge incidentally domination number, and weak edge incidentally domination number has been discussed.

CHAPTER VII explores new kinds of strong and weak domination in complete intuitionistic fuzzy incidence graphs. For different classes of complete intuitionistic fuzzy incidence graphs, we compute the intuitionistic fuzzy incidence domination number, strong intuitionistic fuzzy incidence domination number, and weak intuitionistic fuzzy incidence domination number, and some theorems have been explained.

CHAPTER VIII introduce various types of dominating sets in product picture fuzzy graphs, such as the fixed vertex dominating set, fixed edge dominating set, total fixed edge dominating set, and fixed edge restrained dominating set. The properties with examples has been presented.

CHAPTER IX explores a new concept of twin perfect domination in hesitancy fuzzy graphs and Omicron products of two hesitancy fuzzy graphs offering some interesting results have been included. The properties with examples has been presented.

CHAPTER X starts with the concept of edge sequences in regular fuzzy graphs and pseudo regular fuzzy graphs are explained by theorems with examples. In addition, a comparison of regular fuzzy graphs and totally regular fuzzy graphs are also discussed.

Chapter 2

Strong Domination in Pseudo Regular and Complete Fuzzy Graphs

A wide range of practical difficulties can be modeled and solved using fuzzy graph algorithms. In general, fuzzy graph theory has a wide range of applications in a variety of domains. An expert must model these issues using a fuzzy network since ambiguous information is a prevalent real-life problem that is frequently uncertain. This chapter establishes the concept of strong domination constant number by using membership values of strong arcs in fuzzy graphs. The strong domination constant number of a pseudo regular fuzzy graph and a complete fuzzy graph is found. In addition, with reference to the strong dominating constant number, a comparison study is conducted between pseudo regular and totally pseudo regular fuzzy graphs. The relationship between the strong domination constant number of a pseudo regular fuzzy graph and the complete fuzzy graph is also examined, as well as theorems relating to these ideas.

2.1 Strong Domination Constant Number in Pseudo Regular Fuzzy Graph

In this section, the new concept of strong domination constant number in pseudo regular fuzzy graph (PRFG) is defined and discussed notation of $\gamma_{SC}(G_{PR})$.

Definition 2.1.1 Let $G_{PR} : (\sigma_{PR}, \mu_{PR})$ be a pseudo regular fuzzy graph on G_{PR}^* . A set D_{PR} vertices of

G_{PR} is a strong dominating set of G_{PR} if every vertex of $\sigma_{PR} - D_{PR}$ is a strong neighbour of some vertex in D_{PR} .

Definition 2.1.2 Let $G_{PR} : (\sigma_{PR}, \mu_{PR})$ be a pseudo regular fuzzy graph on G_{PR}^* . The weight of a strong dominating set (SDS) D_{PR} is defined as $W(D_{PR}) = \sum_{m_{11} \in D_{PR}} \mu_{PR}(m_{11}, m_{22})$, where $\mu_{PR}(m_{11}, m_{22})$ is the minimum of the membership value (MV) (weight) of the strong arcs incident on m_{11} . The strong domination number of a G_{PR} is defined as the minimum weight of strong dominating sets of G_{PR} and it is denoted by $\gamma_S(G_{PR})$.

Definition 2.1.3 Let $G_{PR} : (\sigma_{PR}, \mu_{PR})$ be a pseudo regular fuzzy graph on G_{PR}^* . The weight $W(D_{PR}) = C(\text{constant})$, if each strong dominating sets having equal number of vertices. Then C is labeled strong domination constant number (SDCN). It is denoted by $\gamma_{SC}(G_{PR})$.

Example 2.1.1 Let $G_{PR} : (\sigma_{PR}, \mu_{PR})$ be a PRFG with the vertices $k_{11}, l_{11}, m_{11}, n_{11}$ and edges $(k_{11}, l_{11}), (k_{11}, n_{11}), (n_{11}, m_{11}), (m_{11}, l_{11})$.

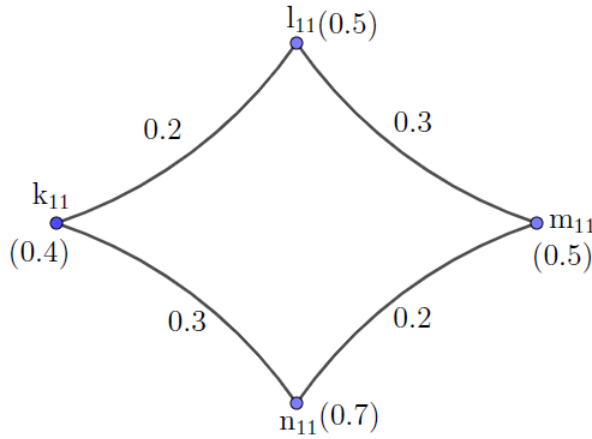


Figure 2.1.1: Strong Domination Constant Number in PRFG

In figure 2.1.1, the strong arcs are $(k_{11}, l_{11}), (k_{11}, n_{11}), (n_{11}, m_{11})$ and (m_{11}, l_{11}) .

The two vertices of SDSs are $D_{11} = (k_{11}, l_{11}), D_{22} = (k_{11}, m_{11}), D_{33} = (k_{11}, n_{11}), D_{44} = (n_{11}, m_{11})$,

$D_{55} = (n_{11}, l_{11}), D_{66} = (m_{11}, l_{11})$ where

$$W(D_{11}) = 0.2 + 0.2 = 0.4, W(D_{22}) = 0.2 + 0.2 = 0.4, W(D_{33}) = 0.2 + 0.2 = 0.4$$

$$W(D_{44}) = 0.2 + 0.2 = 0.4, W(D_{55}) = 0.2 + 0.2 = 0.4, W(D_{66}) = 0.2 + 0.2 = 0.4$$

Hence $\gamma_{SC}(G_{PR}) = 0.4$.

The three vertices of SDSs are $D_{77} = (k_{11}, l_{11}, m_{11}), D_{88} = (l_{11}, m_{11}, n_{11}), D_{99} = (m_{11}, n_{11}, k_{11}),$
 $D_{10} = (k_{11}, l_{11}, n_{11})$ where

$$W(D_{77}) = 0.2 + 0.2 + 0.2 = 0.6, W(D_{88}) = 0.2 + 0.2 + 0.2 = 0.6$$

$$W(D_{99}) = 0.2 + 0.2 + 0.2 = 0.6, W(D_{10}) = 0.2 + 0.2 + 0.2 = 0.6$$

Hence, $\gamma_{SC}(G_{PR}) = 0.6$.

Therefore, the above example having two SDCN.

That is $\gamma_{SC}(G_{PR}) = 0.4$ and $\gamma_{SC}(G_{PR}) = 0.6$.

Remark 2.1.1 *If all the vertices are isolated, then σ_{PR} is the only strong dominating set of G_{PR} of order p_{PR} and $\gamma_{SC}(G_{PR}) = 0$.*

Remark 2.1.2 *A strong domination constant number of fuzzy graph need not be a pseudo regular fuzzy graph.*

Example 2.1.2 *Consider G_{FG} be a fuzzy graph with the vertices $e_{11}, f_{11}, g_{11}, h_{11}$ and edges $(e_{11}, f_{11}), (e_{11}, g_{11}), (e_{11}, h_{11}), (f_{11}, g_{11})$ and (g_{11}, h_{11}) .*

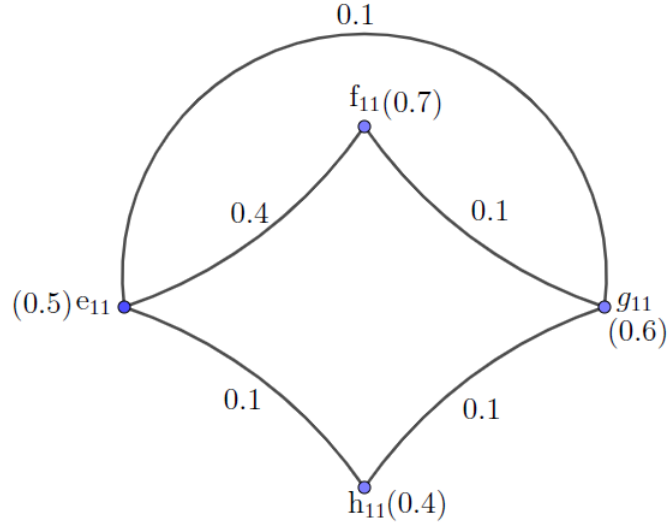


Figure 2.1.2: Strong Domination Constant Number of Fuzzy Graph

In figure 2.1.2, strong arcs are (e_{11}, f_{11}) , (e_{11}, g_{11}) , (e_{11}, h_{11}) , (f_{11}, g_{11}) and (g_{11}, h_{11}) .

One vertex of SDSs are $D_{11} = (e_{11})$, $D_{22} = (g_{11})$ where $W(D_{11}) = 0.1$, $W(D_{22}) = 0.1$. Hence $\gamma_{SC}(G_{PR}) = 0.1$.

The two vertices of SDSs are $D_{33} = (e_{11}, f_{11})$, $D_{44} = (e_{11}, g_{11})$, $D_{55} = (e_{11}, h_{11})$, $D_{66} = (f_{11}, g_{11})$, $D_{77} = (f_{11}, h_{11})$, and $D_{88} = (g_{11}, h_{11})$ where

$$W(D_{33}) = 0.1 + 0.1 = 0.2, W(D_{44}) = 0.1 + 0.1 = 0.2, W(D_{55}) = 0.1 + 0.1 = 0.2$$

$$W(D_{66}) = 0.1 + 0.1 = 0.2, W(D_{77}) = 0.1 + 0.1 = 0.2, W(D_{88}) = 0.1 + 0.1 = 0.2$$

Hence $\gamma_{SC}(G_{PR}) = 0.2$.

The three vertices of SDSs are $D_{99} = (e_{11}, f_{11}, g_{11})$, $D_{10} = (e_{11}, g_{11}, h_{11})$, $D_{11} = (f_{11}, g_{11}, h_{11})$, $D_{12} = (e_{11}, h_{11}, f_{11})$ where

$$W(D_{99}) = 0.1 + 0.1 + 0.1 = 0.3, W(D_{10}) = 0.1 + 0.1 + 0.1 = 0.3$$

$$W(D_{11}) = 0.1 + 0.1 + 0.1 = 0.3, W(D_{12}) = 0.1 + 0.1 + 0.1 = 0.3$$

Hence, $\gamma_{SC}(G_{PR}) = 0.3$.

Therefore, the above example having three SDCN.

That is $\gamma_{SC}(G_{PR}) = 0.1$, $\gamma_{SC}(G_{PR}) = 0.2$ and $\gamma_{SC}(G_{PR}) = 0.3$.

The fuzzy graph G_{FG} is SDCN of fuzzy graph. But $d_a(e_{11}) \neq d_a(h_{11})$. Hence G_{FG} is not a PRFG.

Theorem 2.1.1 *If $G_{PR} : (\sigma_{PR}, \mu_{PR})$ be a pseudo regular fuzzy graph and each strong dominating sets having exactly n vertices, then $\gamma_{SC}(G_{PR}) = n\{\min(\mu_{PR}(m_{11}, m_{22})/m_{11}, m_{22} \in \sigma_{PR}^*)\}$.*

Proof. Since G_{PR} is a PRFG, all arcs are strong and some node is adjacent to all other nodes. Hence, $D_{PR} = \{m_{11}, m_{22}, \dots, m_{nn}\}$ is a SDSs for each $m_{11}, m_{22}, \dots, m_{nn} \in \sigma_{PR}^*$.

Hence $\gamma_{SC}(G_{PR}) = n\{\min(\mu_{PR}(m_{11}, m_{22})/m_{11}, m_{22} \in \sigma_{PR}^*)\}$.

Example 2.1.3 (i) *Let $G_{PR} : (\sigma_{PR}, \mu_{PR})$ be a PRFG with the vertices $i_{11}, j_{11}, k_{11}, l_{11}$ and edges (i_{11}, j_{11}) , (j_{11}, k_{11}) , (k_{11}, l_{11}) , (i_{11}, l_{11}) .*

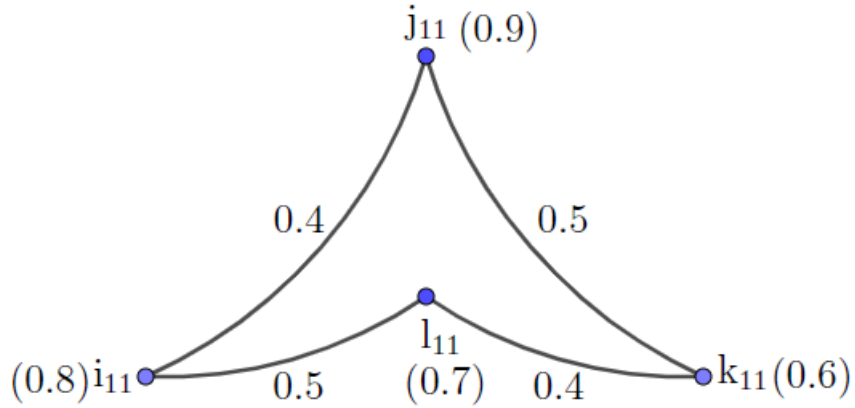


Figure 2.1.3: PRFG with Strong Domination Constant Number

In figure 2.1.3, G_{PR} be a PRFG and each arcs are strong.

For $n = 2$, the SDSs are $D_{11} = (i_{11}, j_{11})$, $D_{22} = (j_{11}, k_{11})$, $D_{33} = (k_{11}, l_{11})$, $D_{44} = (i_{11}, l_{11})$, $D_{55} =$

$(j_{11}, l_{11}), D_{66} = (i_{11}, k_{11})$ where

$$W(D_{11}) = 0.4 + 0.4 = 0.8, W(D_{22}) = 0.4 + 0.4 = 0.8,$$

$$W(D_{33}) = 0.4 + 0.4 = 0.8, W(D_{44}) = 0.4 + 0.4 = 0.8,$$

$$W(D_{55}) = 0.4 + 0.4 = 0.8, W(D_{66}) = 0.4 + 0.4 = 0.8$$

Therefore $\gamma_{SC}(G_{PR}) = 0.8$.

Hence,

$$\begin{aligned} \gamma_{SC}(G_{PR}) &= 2\{\min(\mu_{PR}(m_{11}, m_{22})/m_{11}, m_{22} \in \sigma_{PR}^*)\} \\ &= 2\{\min(0.4, 0.5, 0.4, 0.5)\} \\ &= 0.8 \end{aligned}$$

Hence the result.

Example 2.1.4 (ii) Let G_{PR} be a PRFG with the vertices $i_{11}, j_{11}, k_{11}, l_{11}$ and edges $(i_{11}, j_{11}), (j_{11}, k_{11}), (k_{11}, l_{11}), (i_{11}, l_{11}), (i_{11}, k_{11}), (j_{11}, l_{11})$. Here each arcs are strong.

For $n = 3$, the SDSs are $D_{11} = (i_{11}, j_{11}, k_{11}), D_{22} = (i_{11}, k_{11}, l_{11}), D_{33} = (i_{11}, j_{11}, l_{11}), D_{44} = (j_{11}, k_{11}, l_{11})$ where

$$W(D_{11}) = 0.4 + 0.4 + 0.4 = 1.2, W(D_{22}) = 0.4 + 0.4 + 0.4 = 1.2,$$

$$W(D_{33}) = 0.4 + 0.4 + 0.4 = 1.2, W(D_{44}) = 0.4 + 0.4 + 0.4 = 1.2$$

Therefore $\gamma_{SC}(G_{PR}) = 1.2$.

Here,

$$\begin{aligned} \gamma_{SC}(G_{PR}) &= 3\{\min(\mu_{PR}(m_{11}, m_{22})/m_{11}, m_{22} \in \sigma_{PR}^*)\} \\ &= 3\{\min(0.5, 0.5, 0.5, 0.5, 0.4, 0.4)\} \\ &= 1.2 \end{aligned}$$

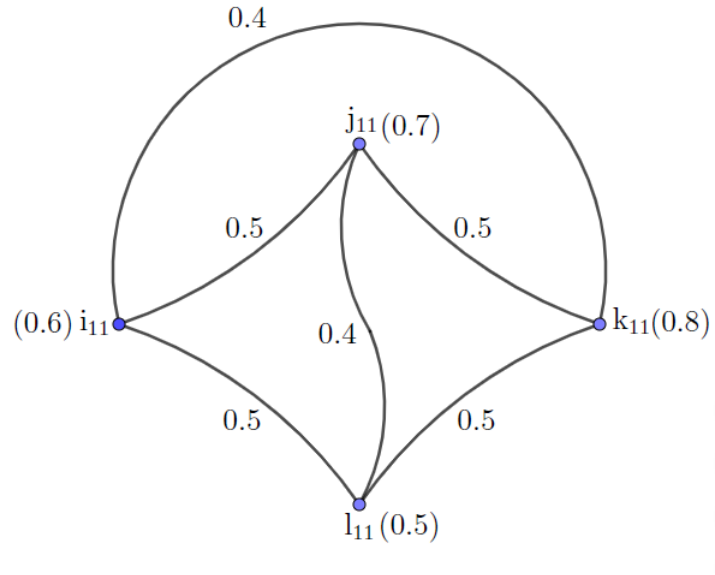


Figure 2.1.4: PRFG with Strong Domination Constant Number

Hence the result.

Remark 2.1.3 *The above result is also true for totally pseudo regular fuzzy graph (TPRFG).*

Example 2.1.5 (i) *Consider G_{TPR} be a TPRFG with the vertices $e_{11}, f_{11}, g_{11}, h_{11}$ and edges $(e_{11}, f_{11}), (e_{11}, h_{11}), (f_{11}, g_{11}), (g_{11}, h_{11})$.*

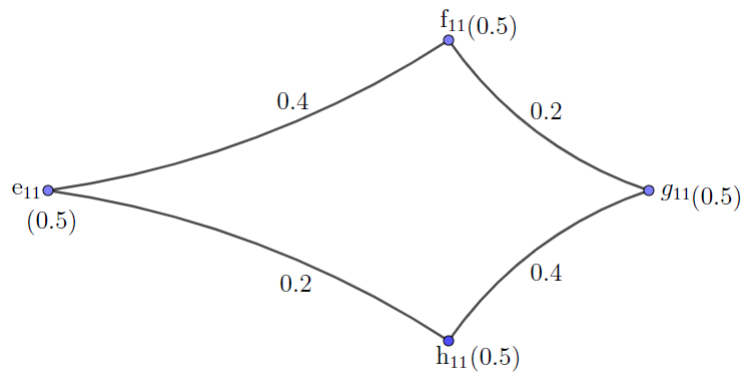


Figure 2.1.5: TPRFG with Strong Domination Constant Number

Hence the result.

In figure 2.1.5, G_{TPR} be a TPRFG and each arcs are strong.

For $n = 2$, the SDSs are $D_{11} = (e_{11}, f_{11})$, $D_{22} = (f_{11}, g_{11})$, $D_{33} = (g_{11}, h_{11})$, $D_{44} = (e_{11}, h_{11})$, $D_{55} = (f_{11}, h_{11})$, $D_{66} = (e_{11}, g_{11})$ where

$$W(D_{11}) = 0.2 + 0.2 = 0.4, W(D_{22}) = 0.2 + 0.2 = 0.4,$$

$$W(D_{33}) = 0.2 + 0.2 = 0.4, W(D_{44}) = 0.2 + 0.2 = 0.4$$

Therefore, $\gamma_{SC}(G_{PR}) = 0.4$.

Here

$$\begin{aligned} \gamma_{SC}(G_{PR}) &= 2\{\min(\mu_{PR}(m_{11}, m_{22}))/m_{11}, m_{22} \in \sigma_{TPR}^*\} \\ &= 2\{\min(0.2, 0.4, 0.2, 0.4)\} \\ &= 0.4 \end{aligned}$$

Hence the result.

Example 2.1.6 (ii) Let G_{TPR} be a TPRFG with the vertices $i_{11}, j_{11}, k_{11}, l_{11}$ and edges (i_{11}, j_{11}) , (j_{11}, k_{11}) , (k_{11}, l_{11}) , (i_{11}, l_{11}) , (i_{11}, k_{11}) , (j_{11}, l_{11}) .

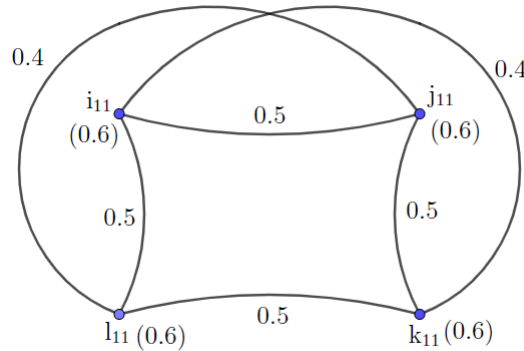


Figure 2.1.6: TPRFG with Strong Dominating Constant Number

In fig 2.1.6, G_{TPR} be a TPRFG and each arcs are strong.

For $n = 3$, the SDSs are $D_{11} = (i_{11}, j_{11}, k_{11})$, $D_{22} = (i_{11}, k_{11}, l_{11})$, $D_{33} = (i_{11}, j_{11}, l_{11})$, $D_{44} = (j_{11}, k_{11}, l_{11})$ where

$$W(D_{11}) = 0.4 + 0.4 + 0.4 = 1.2, W(D_{22}) = 0.4 + 0.4 + 0.4 = 1.2,$$

$$W(D_{33}) = 0.4 + 0.4 + 0.4 = 1.2, W(D_{44}) = 0.4 + 0.4 + 0.4 = 1.2$$

Here $\gamma_{SC}(G_{TPR}) = 1.2$.

Therefore,

$$\begin{aligned} \gamma_{SC}(G_{TPR}) &= 3\{\min(\mu_{PR}(m_{11}, m_{22})/m_{11}, m_{22} \in \sigma_{PR}^*)\} \\ &= 3\{\min(0.5, 0.5, 0.5, 0.5, 0.4, 0.4)\} \\ &= 1.2 \end{aligned}$$

Hence the result.

Theorem 2.1.2 Let $G_{PR} : (\sigma_{PR}, \mu_{PR})$ be a pseudo regular fuzzy graph of size q_{PR} . Then $\gamma_{SC}(G_{PR}) = \frac{q_{PR}}{2}$ iff

- (i) all edges have equal membership value
- (ii) All strong dominating sets having exactly two vertices

Proof. If all edges have equal MV and all SDS of G_{PR} is a set D_{PR} containing exactly two vertices.

Hence, SDCN is exactly $\gamma_{SC}(G_{PR}) = \sum_{m_{11} \in D_{PR}} \mu_{PR}(m_{11}, m_{22}) = \frac{q_{PR}}{2}$.

Contrarily, suppose that $\gamma_{SC}(G_{PR}) = \frac{q_{PR}}{2}$. To prove that all edges have equal MV and all SDSs having exactly two vertices. If the alternative edges have equal MV and all SDSs having more than two vertices, then $\gamma_{SC}(G_{PR}) \neq \frac{q_{PR}}{2}$, which is a contradiction. Hence, all conditions are sufficient.

Example 2.1.7 Let G_{PR} be a PRFG with the vertices $v_{11}, w_{11}, x_{11}, y_{11}$ and edges $(v_{11}, w_{11}), (w_{11}, x_{11}), (x_{11}, y_{11}), (v_{11}, y_{11})$. Here each arcs are strong.

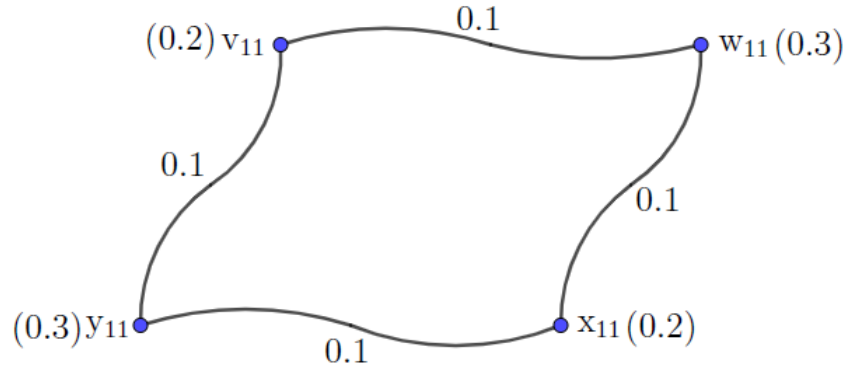


Figure 2.1.7: PRFG with Strong Domination Constant Number

The two vertices of SDSs are $D_{11} = (v_{11}, w_{11})$, $D_{22} = (w_{11}, x_{11})$, $D_{33} = (x_{11}, y_{11})$, $D_{44} = (v_{11}, y_{11})$, $D_{55} = (v_{11}, x_{11})$, $D_{66} = (w_{11}, y_{11})$, where

$$W(D_{11}) = 0.1 + 0.1 = 0.2, W(D_{22}) = 0.1 + 0.1 = 0.2,$$

$$W(D_{33}) = 0.1 + 0.1 = 0.2, W(D_{44}) = 0.1 + 0.1 = 0.2,$$

$$W(D_{55}) = 0.1 + 0.1 = 0.2, W(D_{66}) = 0.1 + 0.1 = 0.2$$

Therefore $\gamma_{SC}(G_{PR}) = 0.2$.

Here,

$$\gamma_{SC}(G_{PR}) = \frac{q_{PR}}{2} = \frac{0.4}{2} = 0.2$$

Hence the result.

Remark 2.1.4 The above condition is also true for TPRFG.

Example 2.1.8 Let G_{TPR} be a TPRFG with the vertices $v_{11}, w_{11}, x_{11}, y_{11}$ and edges (v_{11}, w_{11}) , (w_{11}, x_{11}) , (x_{11}, y_{11}) , (v_{11}, y_{11}) . Here each arcs are strong.

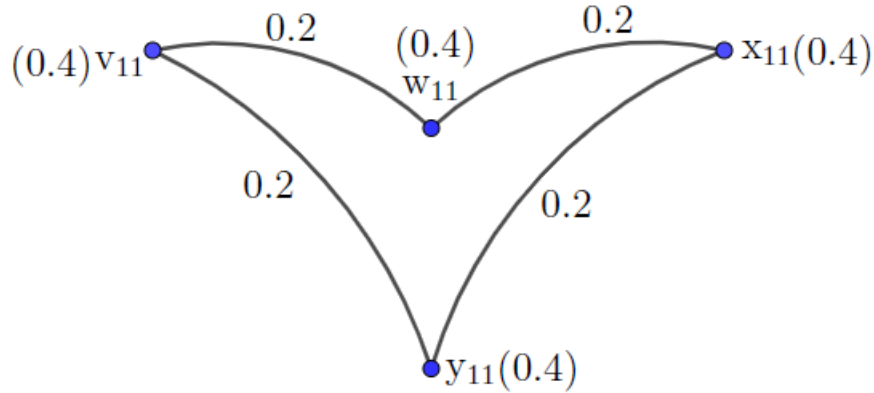


Figure 2.1.8: TPRFG with SDCN

The two vertices of SDSs are $D_{11} = (v_{11}, w_{11})$, $D_{22} = (w_{11}, x_{11})$, $D_{33} = (x_{11}, y_{11})$, $D_{44} = (v_{11}, y_{11})$, $D_{55} = (v_{11}, x_{11})$, $D_{66} = (w_{11}, y_{11})$, where

$$W(D_{11}) = 0.2 + 0.2 = 0.4, W(D_{22}) = 0.2 + 0.2 = 0.4,$$

$$W(D_{33}) = 0.2 + 0.2 = 0.4, W(D_{44}) = 0.2 + 0.2 = 0.4,$$

$$W(D_{55}) = 0.2 + 0.2 = 0.4, W(D_{66}) = 0.2 + 0.2 = 0.4$$

Therefore $\gamma_{SC}(G_{TPR}) = 0.4$.

Here,

$$\gamma_{SC}(G_{TPR}) = \frac{q_{PR}}{2} = \frac{0.8}{2} = 0.4$$

Hence the result.

Theorem 2.1.3 A pseudo regular fuzzy graph $G_{PR} : (\sigma_{PR}, \mu_{PR})$ with its crisp graph G_{PR}^* as even cycle is both pseudo regular fuzzy graph and totally pseudo regular fuzzy graph then G_{PR} contains strong domination constant number.

Proof. Let $G_{PR} : (\sigma_{PR}, \mu_{PR})$ be a PRFG. Then its crisp graph G_{PR}^* as even cycle and G_{PR} be both PRFG and TPRFG. Here are two cases that arise.

Case (i)

Let G_{PR} be both PRFG and TPRFG with stable values in σ_{PR} and μ_{PR} . In G_{PR} all arcs are strong and some SDSs of G_{PR} having equal number of vertices. Then by the 2.1.3 definition G_{PR} contains SDCN.

Example 2.1.9 Let G_{PR} be both PRFG and TPRFG with the vertices $e_{11}, f_{11}, g_{11}, h_{11}$ and edges $(e_{11}, f_{11}), (e_{11}, h_{11}), (f_{11}, g_{11}), (h_{11}, g_{11})$. Here all arcs are strong.

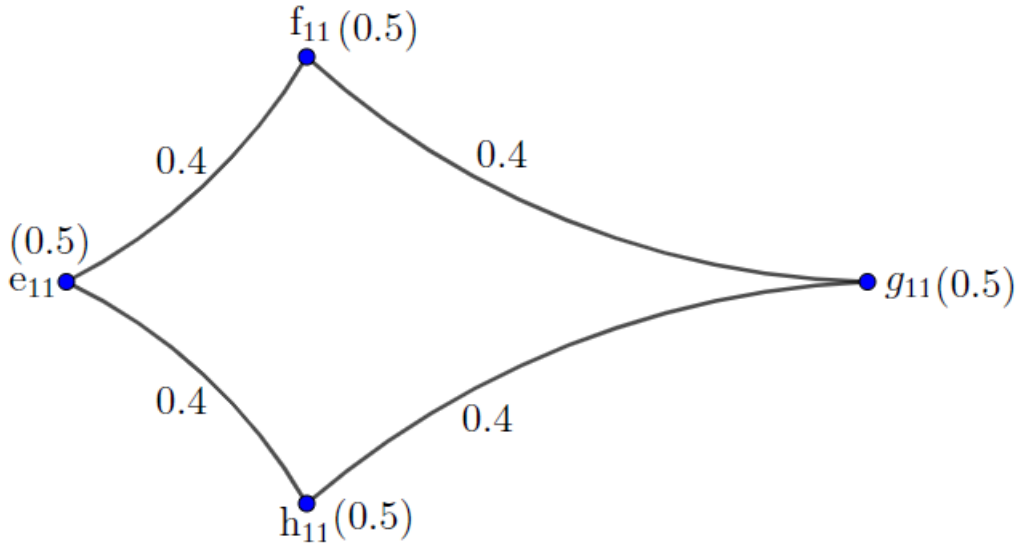


Figure 2.1.9: PRFG and TPRFG with Strong Domination Constant Numbers

$$\gamma_{SC}(G_{PR}) = 0.8 \text{ and } \gamma_{SC}(G_{PR}) = 1.2.$$

The graph G_{PR} is PRFG and TPRFG with SDCNs.

Case (ii)

Let G_{PR} be both PRFG and TPRFG with stable values in σ_{PR} and with equal alternative values in μ_{PR} . In all arcs are strong and some SDSs of G_{PR} having equal number of vertices. Then by the 2.1.3 definition G_{PR} contains SDCN.

Example 2.1.10 Let G_{PR} be both PRFG and TPRFG with the vertices $e_{11}, f_{11}, g_{11}, h_{11}$ and edges $(e_{11}, f_{11}), (e_{11}, h_{11}), (f_{11}, g_{11}), (g_{11}, h_{11})$. Here all arcs are strong.

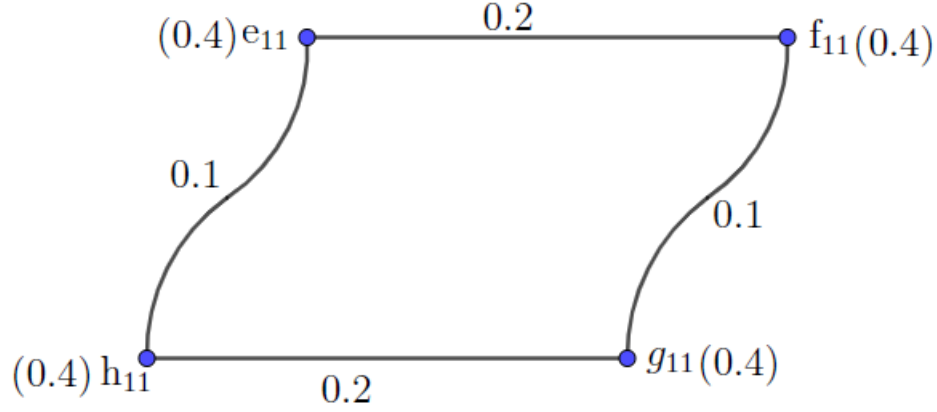


Figure 2.1.10: PRFG and TPRFG with Strong Domination Constant Number

$\gamma_{SC}(G_{PR}) = 0.2$ and $\gamma_{SC}(G_{PR}) = 0.3$.

The graph G_{PR} is PRFG and TPRFG with SDCNs.

Theorem 2.1.4 Let $G_{PR} : (\sigma_{PR}, \mu_{PR})$ be a PRFG, $\gamma_{SC} = \frac{p_{PR}}{2}$ iff the following conditions hold

- (i) G_{PR} is a TPRFG
- (ii) All vertices and edges having same membership value
- (iii) All strong dominating sets having exactly two vertices.

Proof. If G_{PR} is a TPRFG and all vertices and edges having same membership value, then all SDS of G_{PR} is a set D_{PR} containing exactly two vertices. Hence, SDCN is exactly $\gamma_{SC}(G) = \frac{p_{PR}}{2}$.

Contrarily, suppose that $\gamma_{SC}(G) = \frac{p_{PR}}{2}$. To prove that G_{PR} is a TPRFG and all vertices and edges having same membership value. If possible and some nodes say m_{11} and m_{22} have different weights, then the arc weight corresponding to these vertices is $\mu_{PR}(m_{11}, m_{22}) \leq \sigma_{PR}(m_{11}) \wedge \sigma_{PR}(m_{22})$.

If $\mu_{PR}(m_{11}, m_{22}) < \sigma_{PR}(m_{11}) \wedge \sigma_{PR}(m_{22})$, then obviously $\gamma_{SC}(G) \neq \frac{p_{PR}}{2}$, a contradiction and if

$\mu_{PR}(m_{11}, m_{22}) = \sigma_{PR}(m_{11}) \wedge \sigma_{PR}(m_{22})$, then clearly $\gamma_{SC}(G) \neq \frac{p_{PR}}{2}$, a contradiction. Hence, all the conditions are sufficient.

Example 2.1.11 Consider G_{PR} be both PRFG and TPRFG with the vertices $u_{11}, v_{11}, w_{11}, x_{11}$ and edges $(u_{11}, v_{11}), (v_{11}, w_{11}), (w_{11}, x_{11}), (u_{11}, x_{11})$. Here all are strong arcs.

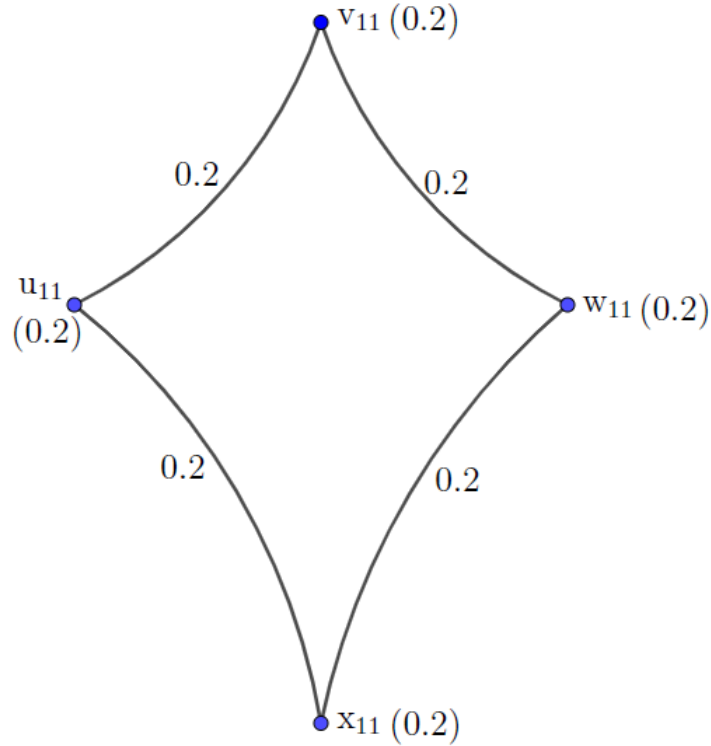


Figure 2.1.11: PRFG with SDCN $\gamma_{SC}(G) = 0.4$

The graph G_{PR} is PRFG and TPRFG with SDCN $\gamma_{SC}(G) = 0.4$.

Here, $\gamma_{SC}(G_{PR}) = \frac{p_{PR}}{2} = \frac{0.8}{2} = 0.4$. Hence the result.

2.2 Complete Fuzzy Graph with Strong Domination Constant Number

In this section the new concept of strong domination constant number in complete fuzzy graph (CFG) is defined and related theorems are discussed.

Definition 2.2.1 Let $G_C : (\sigma_C, \mu_C)$ be a complete fuzzy graph on G_C^* . A set D_C vertices of G_C is a strong domination set of G_C if every vertex of $\sigma_C - D_C$ is a strong neighbour of some vertex in D_C .

Definition 2.2.2 Let $G_C : (\sigma_C, \mu_C)$ be a complete fuzzy graph on G_C^* . The weight of a strong dominating set D_C is defined as $W(D_C) = \sum_{m_{11} \in D_C} \mu_C(m_{11}, m_{22})$, where $\mu_C(m_{11}, m_{22})$ is the minimum of the membership value (weight) of the strong arcs incident on m_{11} . The strong domination number of a G_C is defined as the minimum weight of strong dominating sets of G_C and it is denoted by $\gamma_S(G_C)$.

Definition 2.2.3 Let $G_C : (\sigma_C, \mu_C)$ be a complete fuzzy graph on G_C^* . The weight $W(D_C) = C(\text{constant})$, if each strong dominating sets having equal number of vertices. Then C is labeled strong domination constant number. It is denoted by $\gamma_{SC}(G_C)$.

Example 2.2.1 Let $G_C : (\sigma_C, \mu_C)$ be a CFG with the vertices $w_{11}, x_{11}, y_{11}, z_{11}$ and edges $(w_{11}, x_{11}), (x_{11}, y_{11}), (z_{11}, y_{11}), (w_{11}, z_{11})$. Here each arcs are strong.

In fig 2.2.1, the strong arcs are $(w_{11}, x_{11}), (w_{11}, z_{11}), (z_{11}, y_{11})$ and (y_{11}, x_{11}) .

The two vertices of SDSs are $D_{11} = (w_{11}, x_{11}), D_{22} = (w_{11}, y_{11}), D_{33} = (w_{11}, z_{11}), D_{44} = (z_{11}, y_{11}), D_{55} = (z_{11}, x_{11}), D_{66} = (y_{11}, x_{11})$ where

$$W(D_{11}) = 0.1 + 0.1 = 0.2, W(D_{22}) = 0.1 + 0.1 = 0.2, W(D_{33}) = 0.1 + 0.1 = 0.2$$

$$W(D_{44}) = 0.1 + 0.1 = 0.2, W(D_{55}) = 0.1 + 0.1 = 0.2, W(D_{66}) = 0.1 + 0.1 = 0.2$$

Hence $\gamma_{SC}(G_C) = 0.2$.

The three vertices of SDSs are $D_{77} = (w_{11}, x_{11}, y_{11}), D_{88} = (w_{11}, y_{11}, z_{11}), D_{99} = (x_{11}, y_{11}, z_{11}), D_{10} =$

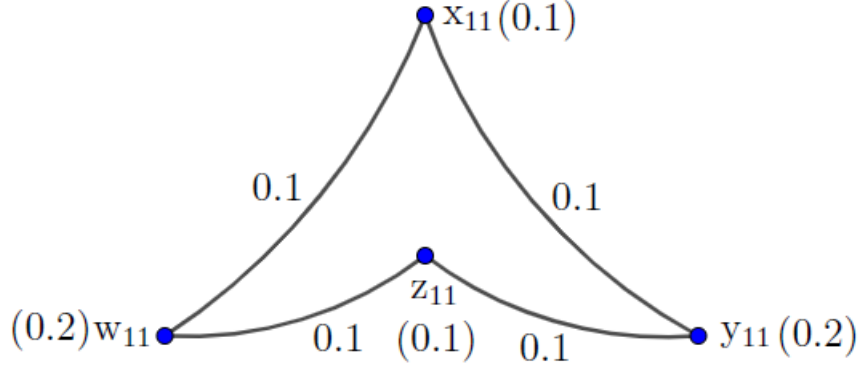


Figure 2.2.1: SDCN in CFG

(x_{11}, w_{11}, z_{11}) where

$$W(D_{77}) = 0.1 + 0.1 + 0.1 = 0.3, W(D_{88}) = 0.1 + 0.1 + 0.1 = 0.3$$

$$W(D_{99}) = 0.1 + 0.1 + 0.1 = 0.3, W(D_{10}) = 0.1 + 0.1 + 0.1 = 0.3$$

Hence, $\gamma_{SC}(G_C) = 0.3$.

Therefore, the above example having two SDCN.

Remark 2.2.1 A complete fuzzy graph need not be a strong domination constant number of fuzzy graph.

Example 2.2.2 Let $G_C : (\sigma_C, \mu_C)$ be a CFG with the vertices $w_{11}, x_{11}, y_{11}, z_{11}$ and edges $(w_{11}, x_{11}), (x_{11}, y_{11}), (z_{11}, y_{11}), (w_{11}, z_{11})$. Here each arcs are strong.

In figure 2.2.2, the strong arcs are $(w_{11}, x_{11}), (w_{11}, z_{11}), (z_{11}, y_{11})$ and (y_{11}, x_{11}) .

The two vertices of SDSs are $D_{11} = (w_{11}, x_{11}), D_{22} = (w_{11}, y_{11}), D_{33} = (w_{11}, z_{11}), D_{44} = (z_{11}, y_{11})$,

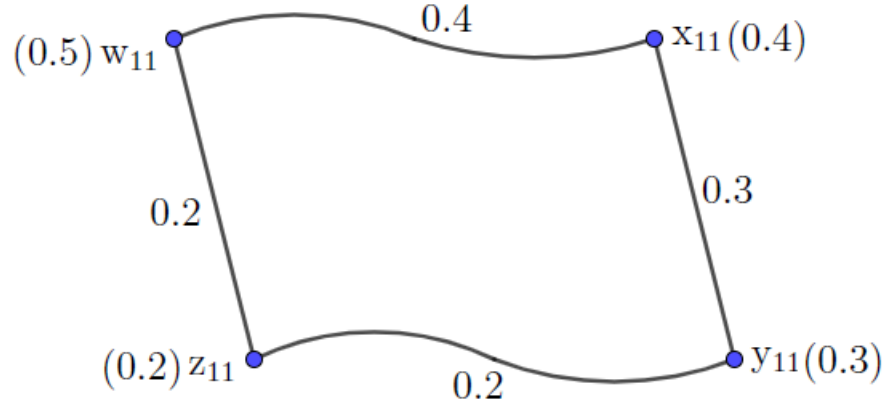


Figure 2.2.2: CFG without Strong Domination Constant Number

$D_{55} = (z_{11}, x_{11}), D_{66} = (y_{11}, x_{11})$ where

$$W(D_{11}) = 0.2 + 0.3 = 0.5, W(D_{22}) = 0.2 + 0.2 = 0.4, W(D_{33}) = 0.2 + 0.2 = 0.4$$

$$W(D_{44}) = 0.2 + 0.2 = 0.4, W(D_{55}) = 0.2 + 0.3 = 0.5, W(D_{66}) = 0.2 + 0.3 = 0.5$$

Here the weight of all SDSs of G_C are not constant.

The three vertices of SDSs are $D_{77} = (w_{11}, x_{11}, y_{11}), D_{88} = (w_{11}, y_{11}, z_{11}), D_{99} = (x_{11}, y_{11}, z_{11}), D_{10} = (x_{11}, w_{11}, z_{11})$ where

$$W(D_{77}) = 0.2 + 0.3 + 0.2 = 0.7, W(D_{88}) = 0.2 + 0.2 + 0.2 = 0.6$$

$$W(D_{99}) = 0.3 + 0.2 + 0.2 = 0.7, W(D_{10}) = 0.2 + 0.3 + 0.2 = 0.7$$

Here the weight of all SDSs of G_C are not constant.

Hence G_C is not a SDCN of fuzzy graph.

Remark 2.2.2 A strong domination constant number of fuzzy graph need not be a complete fuzzy graph.

Example 2.2.3 Let G_{FG} be a fuzzy graph with the vertices $e_{11}, f_{11}, g_{11}, h_{11}$ and edges $(e_{11}, f_{11}),$

$(f_{11}, g_{11}), (g_{11}, h_{11}), (e_{11}, h_{11})$. Here each arcs are strong.

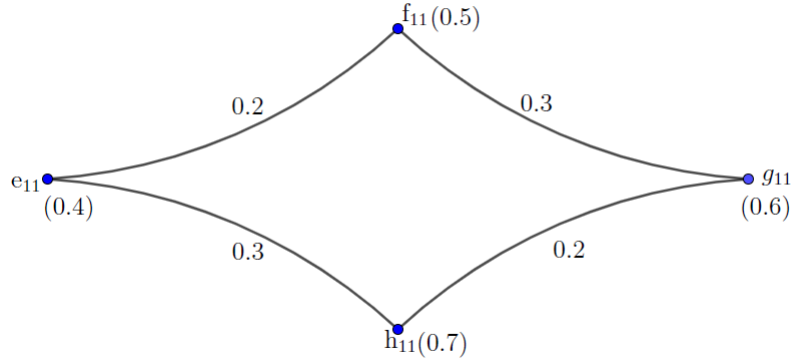


Figure 2.2.3: Strong Domination Constant Number of Fuzzy Graph

In figure 2.2.3, the strong arcs are $(e_{11}, f_{11}), (e_{11}, h_{11}), (h_{11}, g_{11})$ and (g_{11}, f_{11}) .

The two vertices of SDSs are $D_{11} = (e_{11}, f_{11}), D_{22} = (e_{11}, g_{11}), D_{33} = (e_{11}, h_{11}), D_{44} = (h_{11}, g_{11}), D_{55} = (h_{11}, f_{11}), D_{66} = (g_{11}, f_{11})$ where

$$W(D_{11}) = 0.2 + 0.2 = 0.4, W(D_{22}) = 0.2 + 0.2 = 0.4, W(D_{33}) = 0.2 + 0.2 = 0.4$$

$$W(D_{44}) = 0.2 + 0.2 = 0.4, W(D_{55}) = 0.2 + 0.2 = 0.4, W(D_{66}) = 0.2 + 0.2 = 0.4$$

Hence $\gamma_{SC}(G_C) = 0.4$.

The three vertices of SDSs are $D_{77} = (e_{11}, f_{11}, g_{11}), D_{88} = (e_{11}, g_{11}, h_{11}), D_{99} = (f_{11}, g_{11}, h_{11}), D_{10} = (h_{11}, e_{11}, f_{11})$ where

$$W(D_{77}) = 0.2 + 0.2 + 0.2 = 0.6, W(D_{88}) = 0.2 + 0.2 + 0.2 = 0.6$$

$$W(D_{99}) = 0.2 + 0.2 + 0.2 = 0.6, W(D_{10}) = 0.2 + 0.2 + 0.2 = 0.6$$

Hence $\gamma_{SC}(G_C) = 0.6$.

The fuzzy graph G_{FG} is SDCN of fuzzy graph. But G_{FG} is not a CFG.

Theorem 2.2.1 *Let $G_C : (\sigma_C, \mu_C)$ be a complete fuzzy graph of size q_C . Then $\gamma_{SC}(G_C) = \frac{q_C}{2}$ if and only if alternative vertices have equal weight.*

Proof. If alternative vertices have equal weight, then all the edges have equal MV and all arcs are strong, then the SDS of G_C is a set D_C containing two nodes. Hence, SDCN is exactly

$$\gamma_{SC}(G_C) = \sum_{m_{11} \in D_C} \mu_C(m_{11}, m_{22}) = \frac{q_C}{2}.$$

Conversely, suppose that $\gamma_{SC}(G_C) = \frac{q_C}{2}$. To prove that alternative nodes have equal weight. If possible all nodes have different weight and all arcs are strong, then the arcs weight corresponding to nodes is $\mu_C(m_{11}, m_{22}) \leq \sigma_C(m_{11}) \wedge \sigma_C(m_{22})$.

If $\mu_C(m_{11}, m_{22}) < \sigma_C(m_{11}) \wedge \sigma_C(m_{22})$, then obviously $\gamma_{SC}(G_C) = 0$ or $\gamma_{SC}(G_C) < \frac{q_C}{2}$, but G_C is not a CFG which is a contradiction and if $\mu_C(m_{11}, m_{22}) = \sigma_C(m_{11}) \wedge \sigma_C(m_{22})$, then clearly $\gamma_{SC}(G_C) = 0$, which is a contradiction. Hence, alternative vertices have equal weight.

Example 2.2.4 *Consider $G_C : (\sigma_C, \mu_C)$ be a CFG with the vertices $w_{11}, x_{11}, y_{11}, z_{11}$ and edges (w_{11}, x_{11}) , (x_{11}, y_{11}) , $(z_{11}, y_{11}), (w_{11}, z_{11})$. Here each arcs are strong.*

In figure 2.2.4, $\gamma_{SC}(G_C) = 0.6$ and $q_C = 1.2$.

Then $\frac{q_C}{2} = \frac{1.2}{2} = 0.6$.

Therefore $\gamma_{SC}(G_C) = \frac{q_C}{2}$.

Theorem 2.2.2 *Let $G_C : (\sigma_C, \mu_C)$ be a CFG, $\gamma_{SC}(G_C) \geq \frac{p_C}{3}$ iff the following conditions hold*

(i) *Alternative nodes have equal weight.*

(ii) *All edges have equal membership value.*

Proof. If alternative vertices have equal weight and all edges have equal MV, then the SDS of G_C is a set D_C containing two vertices. Hence, SDCN $\gamma_{SC}(G_C) \geq \frac{p_C}{3}$.

Contrarily, suppose that $\gamma_{SC}(G_C) \geq \frac{p_C}{3}$. To prove that alternative nodes have equal weight and all edges have equal MV. If all nodes have equal weight and alternative edges have equal MV then $\gamma_{SC}(G_C) \geq \frac{p_C}{3}$ or $\gamma_{SC}(G_C) < \frac{p_C}{3}$, but G_C is not a CFG, which is a contradiction. Hence, all the conditions are sufficient.

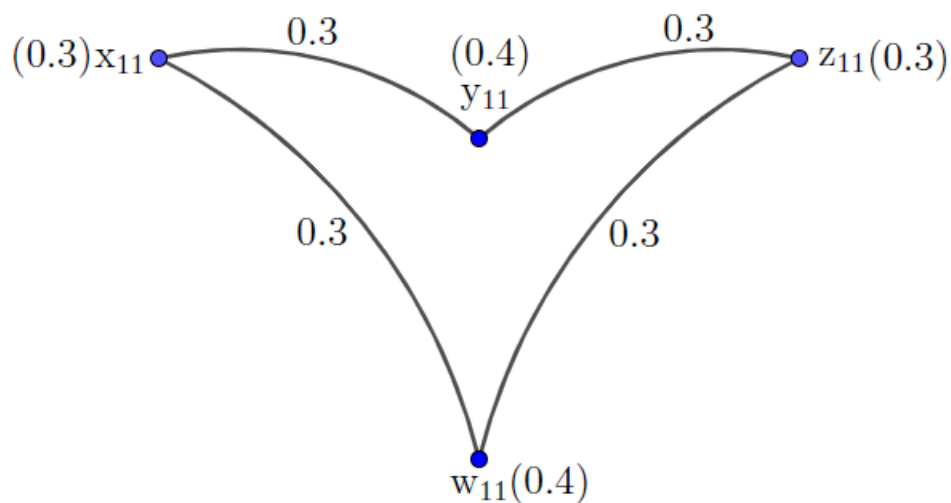


Figure 2.2.4: CFG with SDCN

Example 2.2.5 Let $G_C : (\sigma_C, \mu_C)$ be a CFG with the vertices $a_{11}, b_{11}, c_{11}, d_{11}$ and edges (a_{11}, b_{11}) , (b_{11}, c_{11}) , (c_{11}, d_{11}) , (a_{11}, d_{11}) . Here each arcs are strong.

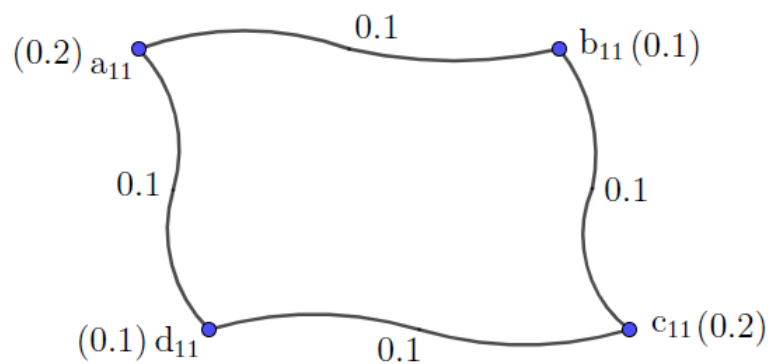


Figure 2.2.5: CFG with Strong Domination Constant Number

In figure 2.2.5, $\gamma_{SC}(G_C) = 0.2$ and $p_C = 0.6$.

Then $\frac{p_C}{3} = \frac{0.6}{3} = 0.2$.

Therefore $\gamma_{SC}(G_C) = \frac{p_C}{3}$.

Example 2.2.6 Let $G_C : (\sigma_C, \mu_C)$ be a CFG with the vertices $w_{11}, x_{11}, y_{11}, z_{11}$ and edges $(w_{11}, x_{11}), (x_{11}, y_{11}), (y_{11}, z_{11}), (z_{11}, w_{11})$. Here each arcs are strong.

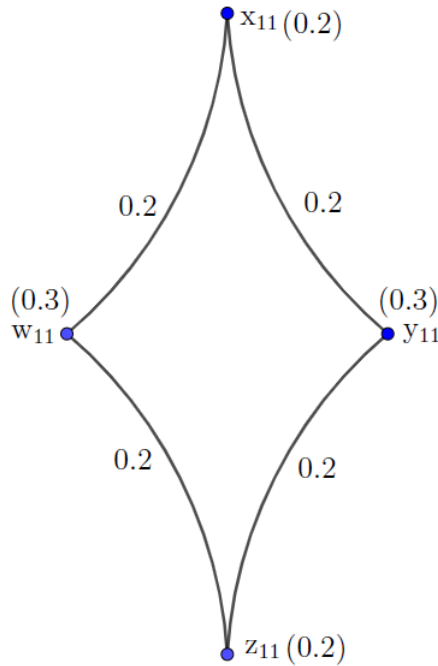


Figure 2.2.6: CFG with Strong Domination Constant Number

In figure 2.2.6, $\gamma_{SC}(G_C) = 0.4$ and $p_C = 1$.

Then $\frac{p_C}{3} = \frac{1}{3} = 0.33$.

Therefore $\gamma_{SC}(G_C) > \frac{p_C}{3}$.

Domination theory survey is exciting because of the wide range of applications and domination traits that can be established. The goal of this chapter is to introduce a study of domination theory in the context of pseudo regular and complete fuzzy graphs. We introduced a definition of the weight of strong domination set using strong arcs. The perception of strong domination constant number for pseudo regular

fuzzy graph and complete fuzzy graph has been introduced in this chapter, and some intriguing results have been demonstrated. In addition, with reference to strong domination constant number, a comparison study is conducted between pseudo regular fuzzy graph and totally pseudo regular fuzzy graph.

Chapter 3

Dual Strong Domination in Vertex Squared and Vertex Squared Split Intuitionistic Fuzzy Graph

The concept of connectivity plays an important role in both theory and applications of fuzzy graphs. An intuitionistic fuzzy set is a generalization of the notion of a fuzzy set. Intuitionistic fuzzy models give more precision, flexibility and compatibility to the system as compared to the classic and fuzzy models. In this chapter, we introduce the notion of vertex squared intuitionistic fuzzy graph, and we define direct product, semi-strong product and semi product. Also we introduce vertex squared split intuitionistic fuzzy graph and we define join product. We generalize the concept of dual strong domination in vertex squared and vertex squared split intuitionistic fuzzy graph and we introduced a definition of weight of strong dominating set using strong arcs. We determine the dual strong domination number for several classes of vertex squared and vertex squared split intuitionistic fuzzy graphs and establish some of their properties. In addition we investigated many interesting results regarding the concept.

3.1 Vertex Squared Intuitionistic Fuzzy Graph

Definition 3.1.1 *An vertex squared intuitionistic fuzzy graph (VSIFG) is of the form $G_{VSIFG} : (V_{VSIFG}, E_{VSIFG})$ where*

- (i) the vertex set $V_{VSIFG} = \{v_{11}, v_{22}, \dots, v_{nn}\}$ such that $\mu_{V_{VSIFG}}^2 : V_{VSIFG} \rightarrow [0, 1]$ and $\gamma_{V_{VSIFG}}^2 : V_{VSIFG} \rightarrow [0, 1]$ denote the degree of membership value and non membership value of the element $v_{ii} \in V_{VSIFG}$ respectively and $0 \leq \mu_{V_{VSIFG}}^2(v_{ii}) + \gamma_{V_{VSIFG}}^2(v_{ii}) \leq 1$ for every $v_{ii} \in V_{VSIFG}$ ($i = 1, 2, \dots, n$).
- (ii) $E_{VSIFG} \subseteq V_{VSIFG} \times V_{VSIFG}$ where $\mu_{E_{VSIFG}} : V_{VSIFG} \times V_{VSIFG} \rightarrow [0, 1]$ and $\gamma_{E_{VSIFG}} : V_{VSIFG} \times V_{VSIFG} \rightarrow [0, 1]$ are such that

$$\mu_{E_{VSIFG}}(v_{ii}, v_{jj}) \leq \min(\mu_{V_{VSIFG}}^2(v_{ii}), \mu_{V_{VSIFG}}^2(v_{jj}))$$

$$\gamma_{E_{VSIFG}}(v_{ii}, v_{jj}) \leq \max(\gamma_{V_{VSIFG}}^2(v_{ii}), \gamma_{V_{VSIFG}}^2(v_{jj}))$$

and $\mu_{E_{VSIFG}}(v_{ii}, v_{jj}) + \gamma_{E_{VSIFG}}(v_{ii}, v_{jj}) \leq 1$ for every $(v_{ii}, v_{jj}) \in E_{VSIFG}$, ($i, j = 1, 2, \dots, n$).

Example 3.1.1 Let $G_{VSIFG} : (V_{VSIFG}, E_{VSIFG})$ be a VSIFG with $m_{11} = (0.7^2, 0.4^2)$, $m_{22} = (0.9^2, 0.8^2)$, $m_{33} = (0.9^2, 0.2^2)$, $m_{44} = (0.3^2, 0.2^2)$, $m_{55} = (0.2^2, 0.2^2)$, $m_{11}, m_{33} = (0.49, 0.16)$, $m_{11}, m_{44} = (0.09, 0.16)$, $m_{22}, m_{33} = (0.70, 0.60)$, $m_{22}, m_{55} = (0.02, 0.60)$, $m_{33}, m_{44} = (0.09, 0.04)$, $m_{33}, m_{55} = (0.04, 0.04)$.

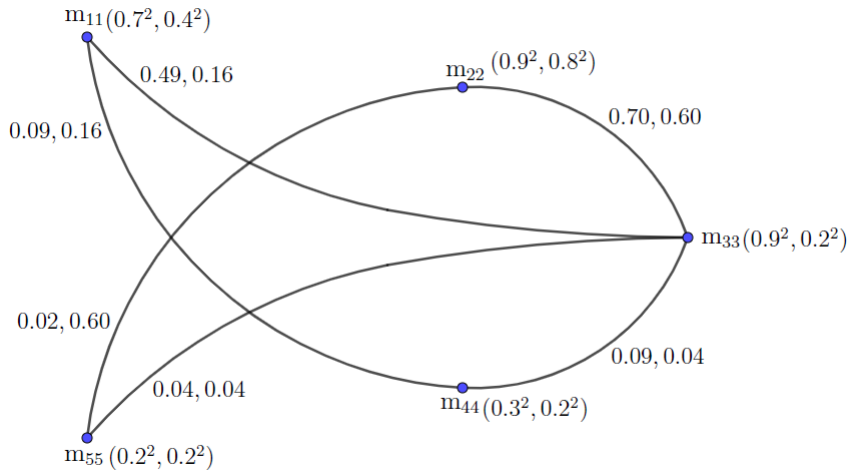


Figure 3.1.1: VSIFG

Definition 3.1.2 A vertex squared intuitionistic fuzzy graph $G_{VSIFG} : (V_{VSIFG}, E_{VSIFG})$ is said to be strong if

$$\begin{aligned}\mu_{E_{VSIFG}}(v_{ii}, v_{jj}) &= \min(\mu_{V_{VSIFG}}^2(v_{ii}), \mu_{V_{VSIFG}}^2(v_{jj})) \\ \gamma_{E_{VSIFG}}(v_{ii}, v_{jj}) &= \max(\gamma_{V_{VSIFG}}^2(v_{ii}), \gamma_{V_{VSIFG}}^2(v_{jj})) \text{ for all } (v_{ii}, v_{jj}) \in V_{VSIFG}\end{aligned}$$

Definition 3.1.3 A vertex squared intuitionistic fuzzy graph $G_{VSIFG} : (V_{VSIFG}, E_{VSIFG})$ is said to be complete if

$$\begin{aligned}\mu_{E_{VSIFG}}(v_{ii}, v_{jj}) &= \min(\mu_{V_{VSIFG}}^2(v_{ii}), \mu_{V_{VSIFG}}^2(v_{jj})) \\ \gamma_{E_{VSIFG}}(v_{ii}, v_{jj}) &= \max(\gamma_{V_{VSIFG}}^2(v_{ii}), \gamma_{V_{VSIFG}}^2(v_{jj})) \text{ for all } (v_{ii}, v_{jj}) \in V_{VSIFG}\end{aligned}$$

Definition 3.1.4 If all the edges are strong edge in an vertex squared intuitionistic fuzzy graph then it is called strengthened VSIFG.

Example 3.1.2 Let $G_{VSIFG} : (V_{VSIFG}, E_{VSIFG})$ be a VSIFG with $m_{11} = (0.3^2, 0.2^2)$, $m_{22} = (0.1^2, 0.6^2)$, $m_{33} = (0.5^2, 0.3^2)$, $m_{44} = (0.3^2, 0.4^2)$, $m_{55} = (0.4^2, 0.5^2)$, $m_{66} = (0.6^2, 0.2^2)$, $m_{11}, m_{22} = (0.01, 0.36)$, $m_{11}, m_{66} = (0.09, 0.04)$, $m_{22}, m_{66} = (0.01, 0.36)$, $m_{33}, m_{55} = (0.16, 0.25)$, $m_{33}, m_{44} = (0.09, 0.16)$, $m_{44}, m_{55} = (0.09, 0.25)$.

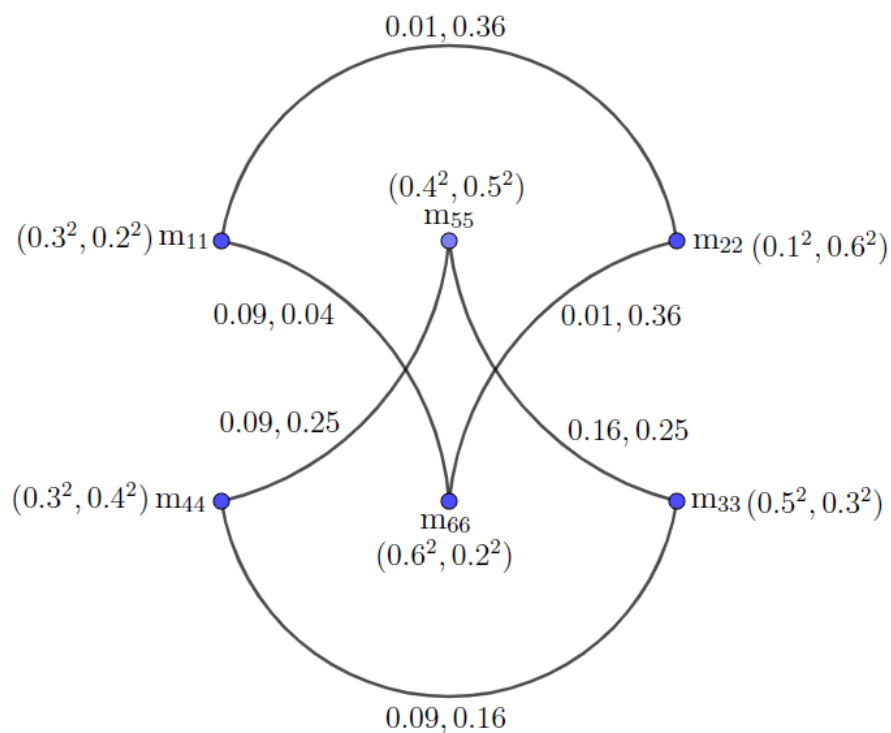


Figure 3.1.2: Strengthened VSIFG

3.2 Dual Strong Domination In Vertex Squared Intuitionistic Fuzzy Graph

Definition 3.2.1 Let G_{VSIFG} be VSIFG. A set D_{VSIFG} of nodes of G_{VSIFG} is a dual strong dominating set (DSDS) of G_{VSIFG} if every node of $V_{VSIFG} - D_{VSIFG}$ is a strong neighbour of atleast two nodes in D_{VSIFG} .

Definition 3.2.2 *The weight of a dual strong dominating set D_{VSIFG} of VSIFG is defined as*

$$W(D_{VSIFG}) = \sum_{m_{11} \in D_{VSIFG}} \left[\frac{1 + |\mu_2(m_{11}, m_{22}) - \gamma_2(m_{11}, m_{22})|}{2} \right],$$

where $|\mu_2(m_{11}, m_{22}) - \gamma_2(m_{11}, m_{22})|$ is the minimum of the membership value and non membership value of the strong arcs incident on m_{11} . The dual strong domination number (DSDN) of a VSIFG

G_{VSIFG} is defined as the minimum weight of dual strong dominating set of G_{VSIFG} and it is denoted by $\gamma_{DSD}(G_{VSIFG})$.

Example 3.2.1 Let $G_{VSIFG} : (V_{VSIFG}, E_{VSIFG})$ be a VSIFG with each edges are strong.

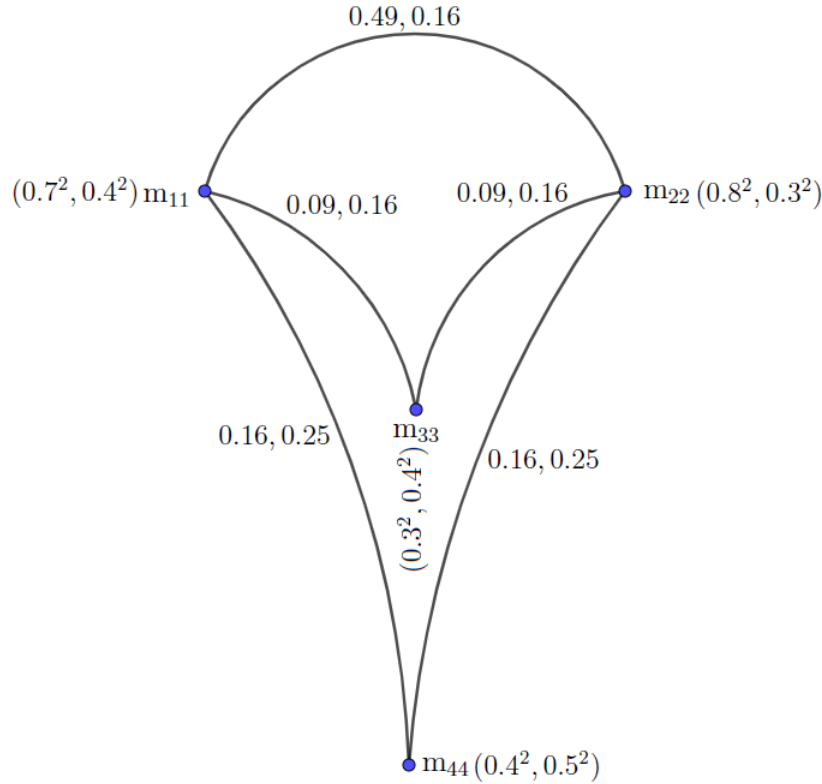


Figure 3.2.1: VSIFG with $\gamma_{DSD}(G_{VSIFG})$

The DSDSs are $S_{11} = \{m_{11}, m_{22}\}$, $S_{22} = \{m_{33}, m_{44}\}, \dots$ After calculating the weight of SDSs are $S_{11} = 0.14$ and $S_{22} = 0.14$. Therefore $\gamma_{DSD}(G_{VSIFG}) = 0.6$.

Theorem 3.2.1 If $G_{VSIFG} : (V_{VSIFG}, E_{VSIFG})$ is a strengthened vertex squared intuitionistic fuzzy graph, then equal number of vertices of dual strong dominating sets of G_{VSIFG} having equal weight.

Proof. Assume that $G_{VSIFG} : (V_{VSIFG}, E_{VSIFG})$ is a strengthened VSIFG. Then every edge of G_{VSIFG} is strong edge. If possible let (m_{11}, m_{22}) be an edge of G_{VSIFG} which is not strong edge. Then equal

number of vertices DSDSs of G_{VSIFG} having equal weight. But G_{VSIFG} is not a strengthened VSIFG, which is a contradiction. Hence all DSDSs of G_{VSIFG} having equal weight.

Example 3.2.2 Let G_{VSIFG} be a strengthened VSIFG and all edges are strong. Here all DSDSs of G_{VSIFG} having equal weight.

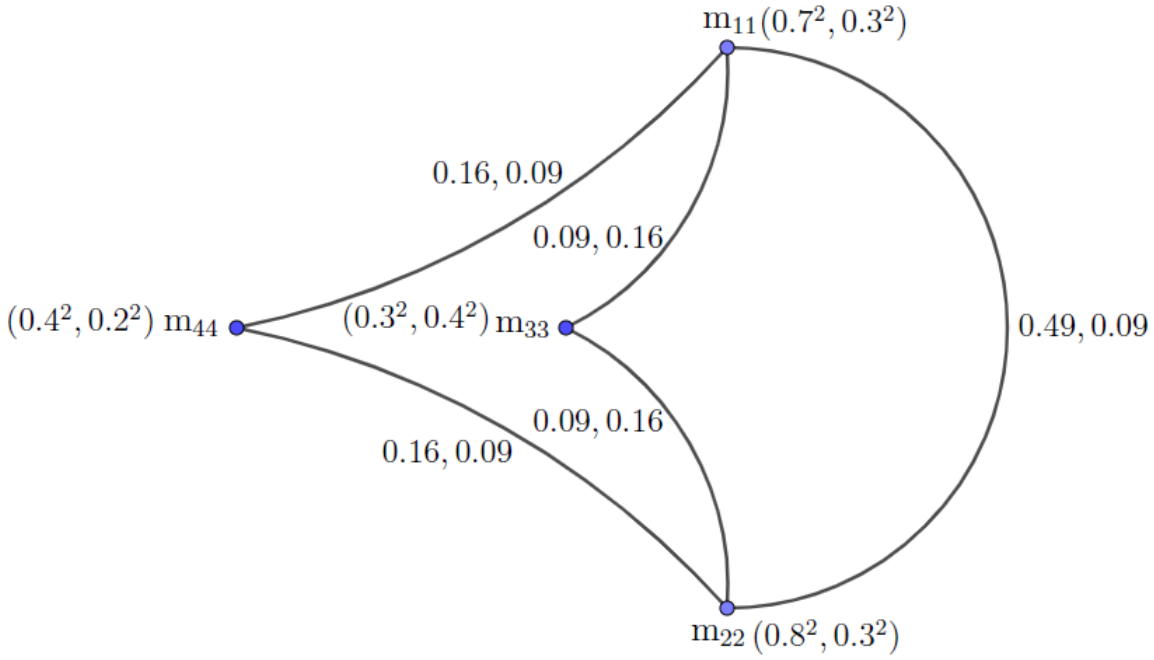


Figure 3.2.2: Strengthened VSIFG

Theorem 3.2.2 Let $G_{VSIFG} : (V_{VSIFG}, E_{VSIFG})$ be an vertex squared intuitionistic fuzzy graph and if G_{VSIFG} contains only one strong arc, then $\gamma_{DSD}(G_{VSIFG}) = 0$.

Proof. Let $G_{VSIFG} : (V_{VSIFG}, E_{VSIFG})$ be an VSIFG. If G_{VSIFG} contains only one strong arc, then $\gamma_{DSD}(G_{VSIFG}) = 0$. Conversely, suppose that $\gamma_{DSD}(G_{VSIFG}) = 0$. To prove that G_{VSIFG} contains only one strong arc. If possible G_{VSIFG} contains more than one strong arc, then $\gamma_{DSD}(G) \neq 0$ which is a contradiction. Hence G_{VSIFG} contains only one strong arc.

Example 3.2.3 Let G_{VSIFG} be a strengthened VSIFG. Here $\{m_{11}, m_{22}\}$ is a only strong arc.

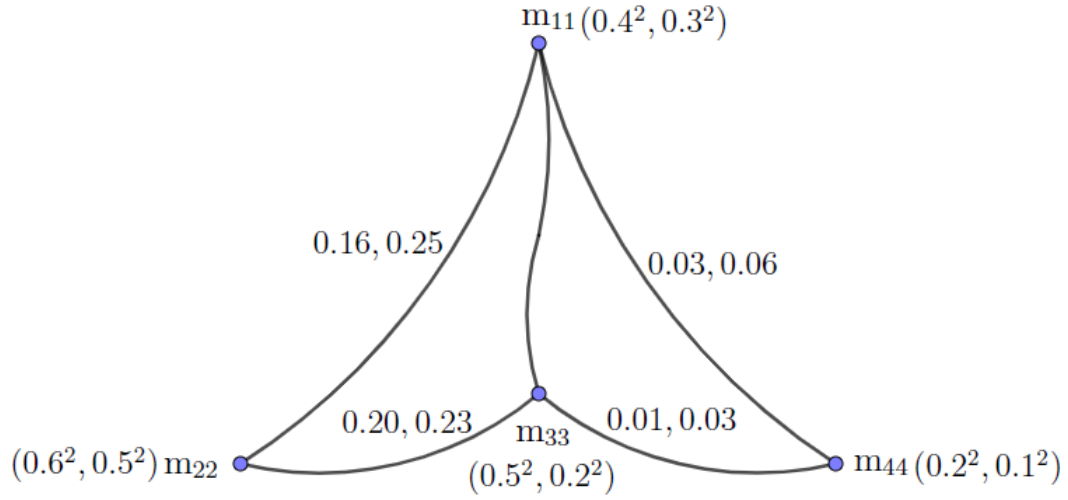


Figure 3.2.3: VSIFG

Theorem 3.2.3 Let $G_{VSIFG} : (V_{VSIFG}, E_{VSIFG})$ be an vertex squared intuitionistic fuzzy graph and if G_{VSIFG} is complete vertex squared intuitionistic fuzzy graph then all dual strong dominating sets of G_{VSIFG} have equal value.

Proof. Assume that G_{VSIFG} is complete VSIFG. Then by the definition of complete VSIFG, $\mu_2(m_{11}, m_{22}) = \min(\mu_1(m_{11}), \mu_1(m_{22}))$ and $\gamma_2(m_{11}, m_{22}) = \max(\gamma_1(m_{11}), \gamma_1(m_{22}))$ for every $m_{11}, m_{22} \in V_{VSIFG}$. Suppose G_{VSIFG} has at least one weak (m_{11}, m_{22}) edge then, $\mu_2(m_{11}, m_{22}) < \mu_2^\infty(m_{11}, m_{22})$ and $\gamma_2(m_{11}, m_{22}) < \gamma_2^\infty(m_{11}, m_{22})$ which implies that $\mu_2(m_{11}, m_{22}) < \min(\mu_1(m_{11}), \mu_1(m_{22}))$ and $\gamma_2(m_{11}, m_{22}) < \max(\gamma_1(m_{11}), \gamma_1(m_{22}))$, for some $m_{11}, m_{22} \in V_{IFG}$. This contradicts our assumption that G_{VSIFG} is complete VSIFG. Hence all DSDSs have equal value.

3.3 Direct Product of Two Vertex Squared Intuitionistic Fuzzy Graph

Definition 3.3.1 The direct product of two vertex squared intuitionistic fuzzy graph $G_1 = (V_1, E_1, \sigma', \mu')$ and $G_2 = (V_2, E_2, \sigma'', \mu'')$ such that $V_1 \cap V_2 = \phi$ is defined to be the VSIFG $G_1 \Pi G_2 = (V, E, \sigma' \Pi \sigma'', \mu' \Pi \mu'')$

where

$$V = V_1 \times V_2,$$

$$E = \{[(x_1, y_1), (x_2, y_2)] / (x_1, x_2) \in E_1, (y_1, y_2) \in E_2\}.$$

The MV and NMV of the vertex (x, y) in $G_1 \Pi G_2$ are given by

$$(\sigma'_1 \Pi \sigma''_1)(x, y) = \min\{\sigma_1'^2(x), \sigma_1''^2(y)\}$$

$$(\sigma'_2 \Pi \sigma''_2)(x, y) = \max\{\sigma_2'^2(x), \sigma_2''^2(y)\}$$

The MV and NMV of the edge $(x_1, y_1), (x_2, y_2)$ in $G_1 \Pi G_2$ are given by

$$(\mu'_1 \Pi \mu''_1)\{(x_1, y_1), (x_2, y_2)\} = \min\{\mu_1'(x_1, x_2), \mu_1''(y_1, y_2)\}$$

$$(\mu'_2 \Pi \mu''_2)\{(x_1, y_1), (x_2, y_2)\} = \max\{\mu_2'(x_1, x_2), \mu_2''(y_1, y_2)\}$$

Example 3.3.1 Let G_1 be a VSIFG with $m_{11} = (0.5^2, 0.3^2)$, $m_{22} = (0.2^2, 0.4^2)$, $m_{11}, m_{22} = (0.04, 0.0625)$

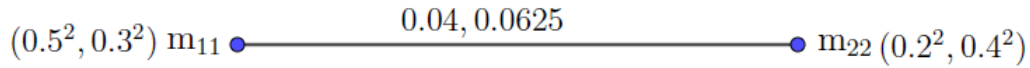
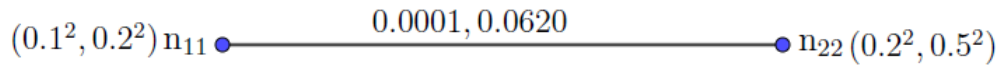
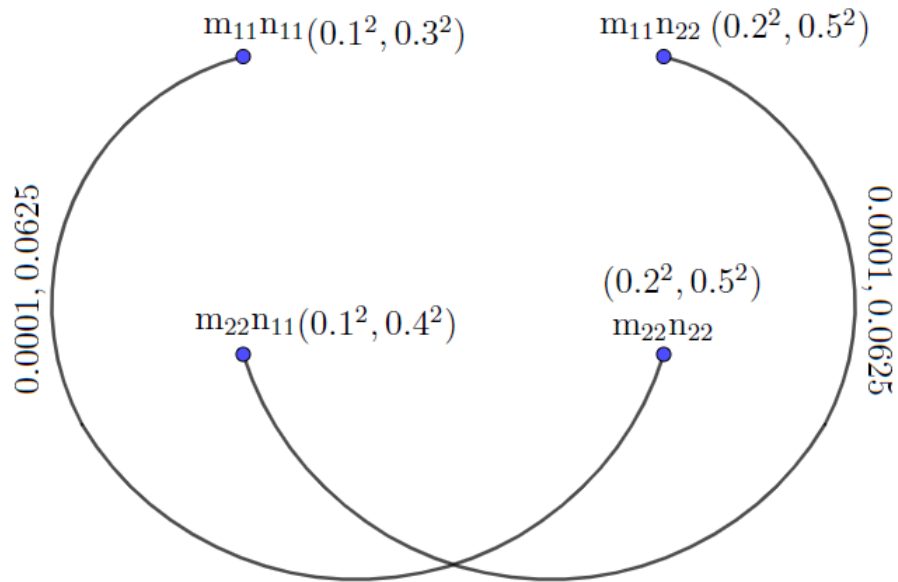


Figure 3.3.1: VSIFG G_1

Let G_2 be a VSIFG with $n_{11} = (0.1^2, 0.2^2)$, $n_{22} = (0.2^2, 0.5^2)$, $n_{11}, n_{22} = (0.0001, 0.0620)$

Figure 3.3.2: VSIFG G_2

Let $G_1 \Pi G_2$ be a VSIFG with $m_{11}n_{11} = (0.1^2, 0.3^2)$, $m_{11}n_{22} = (0.2^2, 0.5^2)$, $m_{22}n_{11} = (0.1^2, 0.4^2)$, $m_{22}n_{22} = (0.2^2, 0.5^2)$, $(m_{11}n_{11}, m_{22}n_{22}) = (0.0001, 0.0625)$, $(m_{11}n_{22}, m_{22}n_{11}) = (0.0001, 0.0625)$.

Figure 3.3.3: VSIFG $G_1 \Pi G_2$

Theorem 3.3.1 If $G_1 = (V_1, E_1, \sigma', \mu')$ and $G_2 = (V_2, E_2, \sigma'', \mu'')$ are vertex squared intuitionistic fuzzy graph, then $G_1 \Pi G_2$ is also vertex squared intuitionistic fuzzy graph.

Proof. Let $G_1 = (V_1, E_1, \sigma', \mu')$ and $G_2 = (V_2, E_2, \sigma'', \mu'')$ are VSIFG, then

$$\mu'_1(x_1, x_2) = \min\{\sigma'^2_1(x_1), \sigma''^2_1(x_2)\}$$

$$\mu'_2(x_1, x_2) = \max\{\sigma'^2_2(x_1), \sigma''^2_2(x_2)\}$$

$$\mu''_1(y_1, y_2) = \min\{\sigma'^2_1(y_1), \sigma''^2_1(y_2)\}$$

$$\mu''_2(y_1, y_2) = \max\{\sigma'^2_2(y_1), \sigma''^2_2(y_2)\} \text{ for all } (x_1, x_2) \in E_1 \text{ and } (y_1, y_2) \in E_2.$$

Now

$$\begin{aligned} (\mu'_1 \Pi \mu''_1)\{(x_1, y_1), (x_2, y_2)\} &= \mu'_1(x_1, x_2) \wedge \mu''_1(y_1, y_2) \\ &= \min\{[\min\{\sigma'^2_1(x_1), \sigma''^2_1(x_2)\}], [\min\{\sigma'^2_1(y_1), \sigma''^2_1(y_2)\}]\} \\ &= \min\{[\min\{\sigma'^2_1(x_1), \sigma'^2_1(y_1)\}], [\min\{\sigma''^2_1(x_2), \sigma''^2_1(y_2)\}]\} \\ &= \{(\sigma'_1 \Pi \sigma''_1)(x_1, y_1)\} \wedge \{(\sigma'_1 \Pi \sigma''_1)(x_2, y_2)\} \\ (\mu'_1 \Pi \mu''_1)\{(x_1, y_1), (x_2, y_2)\} &= \{(\sigma'_1 \Pi \sigma''_1)(x_1, y_1)\} \wedge \{(\sigma'_1 \Pi \sigma''_1)(x_2, y_2)\} \end{aligned}$$

In addition

$$\begin{aligned} (\mu'_2 \Pi \mu''_2)\{(x_1, y_1), (x_2, y_2)\} &= \{\mu'_2(x_1, x_2) \vee \mu''_2(y_1, y_2)\} \\ &= \max\{[\max\{\sigma'^2_2(x_1), \sigma''^2_2(x_2)\}], [\max\{\sigma'^2_2(y_1), \sigma''^2_2(y_2)\}]\} \\ &= \max\{[\max\{\sigma'^2_2(x_1), \sigma'^2_2(y_1)\}], [\max\{\sigma''^2_2(x_2), \sigma''^2_2(y_2)\}]\} \\ &= \{(\sigma'_2 \Pi \sigma''_2)(x_1, y_1)\} \vee \{(\sigma'_2 \Pi \sigma''_2)(x_2, y_2)\} \\ (\mu'_2 \Pi \mu''_2)\{(x_1, y_1), (x_2, y_2)\} &= \{(\sigma'_2 \Pi \sigma''_2)(x_1, y_1)\} \vee \{(\sigma'_2 \Pi \sigma''_2)(x_2, y_2)\} \end{aligned}$$

Therefore G_1 and G_2 are VSIFG, then $G_1 \Pi G_2$ is also a VSIFG.

Theorem 3.3.2 *If $G_1 = (V_1, E_1, \sigma', \mu')$ and $G_2 = (V_2, E_2, \sigma'', \mu'')$ are strengthen VSIFG, then $G_1 \Pi G_2$ is a strengthen vertex squared intuitionistic fuzzy graph.*

Remark 3.3.1 *If $G_1 = (V_1, E_1, \sigma', \mu')$ and $G_2 = (V_2, E_2, \sigma'', \mu'')$ are strong VSIFG, then $G_1 \Pi G_2$ is not a strong vertex squared intuitionistic fuzzy graph.*

3.4 Semi-Strong Product of Two Vertex Squared Intuitionistic Fuzzy Graphs

Definition 3.4.1 The semi strong product of two vertex squared intuitionistic fuzzy graph $G_1 = (V_1, E_1, \sigma', \mu')$ and $G_2 = (V_2, E_2, \sigma'', \mu'')$ such that $V_1 \cap V_2 = \phi$ is defined to be the vertex squared intuitionistic fuzzy graph $G_1 \bullet G_2 = (V, E, \sigma' \bullet \sigma'', \mu' \bullet \mu'')$ where

$$V = V_1 \times V_2, \text{ and}$$

$$E = \{[(x, y_1), (x, y_2)] / x \in E_1, (y_1, y_2) \in E_2\} \cup \{[(x_1, y_1), (x_2, y_2)] / (x_1, x_2) \in E_1, (y_1, y_2) \in E_2\}.$$

The MV and NMV of the vertex (x, y) in $G_1 \bullet G_2$ are given by

$$(\sigma_1'^2 \bullet \sigma_1''^2)(x, y) = \sigma_1'^2(x) \wedge \sigma_1''^2(y)$$

$$(\sigma_2'^2 \bullet \sigma_2''^2)(x, y) = \sigma_2'^2(x) \wedge \sigma_2''^2(y)$$

In addition, the MV and NMV of the edge in $G_1 \bullet G_2$ are given by

$$(\mu_1' \bullet \mu_1'')((x, y_1), (x, y_2)) = \sigma_1'^2(x) \wedge \mu_1''(y_1, y_2)\}$$

$$(\mu_1' \bullet \mu_1'')((x_1, y_1), (x_2, y_2)) = \mu_1'(x_1, x_2) \wedge \mu_1''(y_1, y_2)\}$$

$$(\mu_2' \bullet \mu_2'')((x, y_1), (x, y_2)) = \sigma_2'^2(x) \vee \mu_2''(y_1, y_2)$$

$$(\mu_2' \bullet \mu_2'')((x_1, y_1), (x_2, y_2)) = \mu_2'(x_1, x_2) \vee \mu_2''(y_1, y_2)$$

Example 3.4.1 Let G_1 be a VSIFG with $m_{11} = (0.5^2, 0.3^2), m_{22} = (0.2^2, 0.4^2), (m_{11}, m_{22}) = (0.04, 0.16)$.

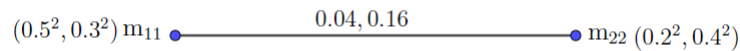


Figure 3.4.1: VSIFG G_1

Let G_2 be a VSIFG with $n_{11} = (0.1^2, 0.2^2), n_{22} = (0.2^2, 0.5^2), (n_{11}, n_{22}) = (0.01, 0.25)$.

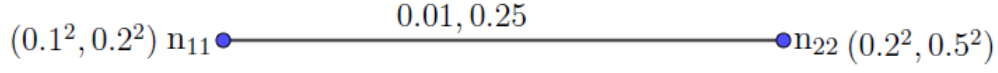


Figure 3.4.2: VSIFG G_2

Let $G_1 \bullet G_2$ be a VSIFG with $m_{11}n_{11} = (0.1^2, 0.3^2), m_{11}n_{22} = (0.2^2, 0.5^2), m_{22}n_{11} = (0.1^2, 0.4^2), m_{22}n_{22} = (0.2^2, 0.5^2), (m_{11}n_{11}, m_{22}n_{22}) = (0.01, 0.25), (m_{11}n_{22}, m_{22}n_{11}) = (0.01, 0.25)$.

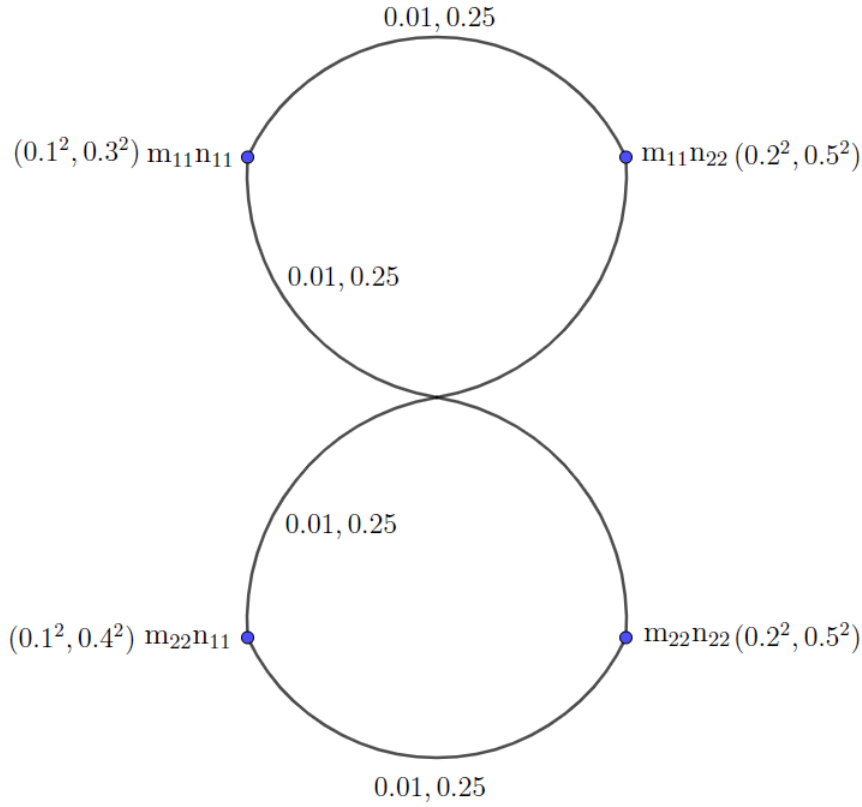


Figure 3.4.3: VSIFG $G_1 \bullet G_2$

Theorem 3.4.1 *If $G_1 = (V_1, E_1, \sigma', \mu')$ and $G_2 = (V_2, E_2, \sigma'', \mu'')$ are strong vertex squared intuitionistic fuzzy graph, then $G_1 \bullet G_2$ is also a strong vertex squared intuitionistic fuzzy graph.*

Proof. Because $G_1 = (V_1, E_1, \sigma', \mu')$ and $G_2 = (V_2, E_2, \sigma'', \mu'')$ are strong VSIFG, then

$$\mu'_1(x_1, x_2) = \sigma'^2_1(x_1) \wedge \sigma'^2_1(x_2)$$

$$\mu'_2(x_1, x_2) = \sigma'^2_2(x_1) \vee \sigma'^2_2(x_2)$$

$$\mu''_1(y_1, y_2) = \sigma''^2_1(y_1) \wedge \sigma''^2_1(y_2)$$

$$\mu''_2(y_1, y_2) = \sigma''^2_2(y_1) \vee \sigma''^2_2(y_2) \text{ for all } (x_1, x_2) \in E_1 \text{ and } (y_1, y_2) \in E_2$$

If $((x, y_1), (x, y_2)) \in E$, then

$$\begin{aligned} (\mu'_1 \bullet \mu''_1)((x, y_1), (x, y_2)) &= \sigma'^2_1(x) \wedge \mu''_1(y_1, y_2) \\ &= \sigma'^2_1(x) \wedge [\sigma''^2_1(y_1) \wedge \sigma''^2_1(y_2)] \\ &= [\sigma'^2_1(x) \wedge \sigma''^2_1(y_1)] \wedge [\sigma'^2_1(x) \wedge \sigma''^2_1(y_2)] \\ &= \{[\sigma'^2_1 \bullet \sigma''^2_1](x, y_1)\} \wedge \{[\sigma'^2_1 \bullet \sigma''^2_1](x, y_2)\} \end{aligned}$$

Similarly, we can show that

$$(\mu'_2 \bullet \mu''_2)((x, y_1), (x, y_2)) = \{[\sigma'^2_2 \bullet \sigma''^2_2](x, y_1)\} \vee \{[\sigma'^2_2 \bullet \sigma''^2_2](x, y_2)\}$$

Again if $((x_1, y_1), (x_2, y_2)) \in E$, then

$$\begin{aligned} (\mu'_1 \bullet \mu''_1)\{(x_1, y_1), (x_2, y_2)\} &= \{\mu'_1(x_1, x_2) \wedge \mu''_1(y_1, y_2)\} \\ &= [\sigma'^2_1(x_1) \wedge \sigma'^2_1(x_2)] \wedge [\sigma''^2_1(y_1) \wedge \sigma''^2_1(y_2)] \\ &= [\sigma'^2_1(x_1) \wedge \sigma''^2_1(y_1)] \wedge [\sigma'^2_1(x_2) \wedge \sigma''^2_1(y_2)] \\ &= \{[\sigma'^2_1 \bullet \sigma''^2_1](x_1, y_1)\} \wedge \{[\sigma'^2_1 \bullet \sigma''^2_1](x_2, y_2)\} \end{aligned}$$

Similarly,

$$(\mu'_2 \bullet \mu''_2)\{(x_1, y_1), (x_2, y_2)\} = \{(\sigma'^2_2 \bullet \sigma''^2_2)(x_1, y_1)\} \vee \{(\sigma'^2_2 \bullet \sigma''^2_2)(x_2, y_2)\}$$

Therefore $G_1 \bullet G_2$ is also strong VSIFG.

Theorem 3.4.2 *If $G_1 = (V_1, E_1, \sigma', \mu')$ and $G_2 = (V_2, E_2, \sigma'', \mu'')$ are vertex squared intuitionistic fuzzy graph, then $G_1 \bullet G_2$ is also a vertex squared intuitionistic fuzzy graph.*

Remark 3.4.1 *If $G_1 = (V_1, E_1, \sigma', \mu')$ and $G_2 = (V_2, E_2, \sigma'', \mu'')$ are non strong VSIFG, then $G_1 \bullet G_2$ is a VSIFG.*

3.5 Semi Product of Two Vertex Squared Intuitionistic Fuzzy Graphs

Definition 3.5.1 *The semi-product of two vertex squared intuitionistic fuzzy graph $G_1 = (V_1, E_1, \sigma', \mu')$ and $G_2 = (V_2, E_2, \sigma'', \mu'')$ such that $V_1 \cap V_2 = \phi$ is defined to be the vertex squared intuitionistic fuzzy graph $G_1 \otimes G_2 = (V, E, \sigma' \otimes \sigma'', \mu' \otimes \mu'')$ where*

$$V = V_1 \times V_2, \text{ and}$$

$$E = \{[(x, y_1), (x, y_2)]/x \in E_1, (y_1, y_2) \in E_2\} \cup \{[(x_1, y), (x_2, y)]/(x_1, x_2) \in E_1, y \in E_2\} \\ \cup \{[(x_1, y_1), (x_2, y_2)]/(x_1, x_2) \in E_1, (y_1, y_2) \in E_2\}$$

The MV and NMV of the vertex (x, y) in $G_1 \otimes G_2$ are given by

$$(\sigma_1'^2 \otimes \sigma_1''^2)(x, y) = \sigma_1'^2(x) \wedge \sigma_1''^2(y)$$

$$(\sigma_2'^2 \otimes \sigma_2''^2)(x, y) = \sigma_2'^2(x) \wedge \sigma_2''^2(y)$$

In addition, the MV and NMV of the edge in $G_1 \otimes G_2$ are given by

$$(\mu_1' \otimes \mu_1'')((x, y_1), (x, y_2)) = \sigma_1'^2(x) \wedge \mu_1''(y_1, y_2)\}$$

$$(\mu_1' \otimes \mu_1'')((x_1, y), (x_2, y)) = \mu_1'(x_1, x_2) \wedge \sigma_1''^2(y)\}$$

$$(\mu_1' \otimes \mu_1'')((x_1, y_1), (x_2, y_2)) = \mu_1'(x_1, x_2) \wedge \mu_1''(y_1, y_2)\}$$

$$\begin{aligned}
(\mu'_2 \otimes \mu''_2)((x, y_1), (x, y_2)) &= \sigma_2'^2(x) \vee \mu_2''(y_1, y_2) \\
(\mu'_2 \otimes \mu''_2)((x_1, y), (x_2, y)) &= \mu_2'(x_1, x_2) \vee \sigma_2''^2(y) \\
(\mu'_2 \otimes \mu''_2)((x_1, y_1), (x_2, y_2)) &= \mu_2'(x_1, x_2) \vee \mu_2''(y_1, y_2)
\end{aligned}$$

Example 3.5.1 Consider G_1 be a VSIFG with $m_{11} = (0.5^2, 0.3^2)$, $m_{22} = (0.2^2, 0.4^2)$, $m_{11}, m_{22} = (0.04, 0.16)$

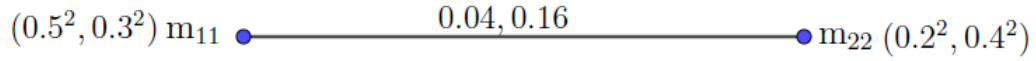


Figure 3.5.1: VSIFG G_1

Consider G_2 be a VSIFG with $n_{11} = (0.1^2, 0.2^2)$, $n_{22} = (0.2^2, 0.5^2)$, $n_{11}, n_{22} = (0.01, 0.25)$

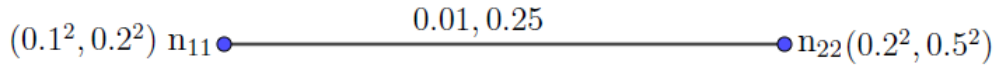
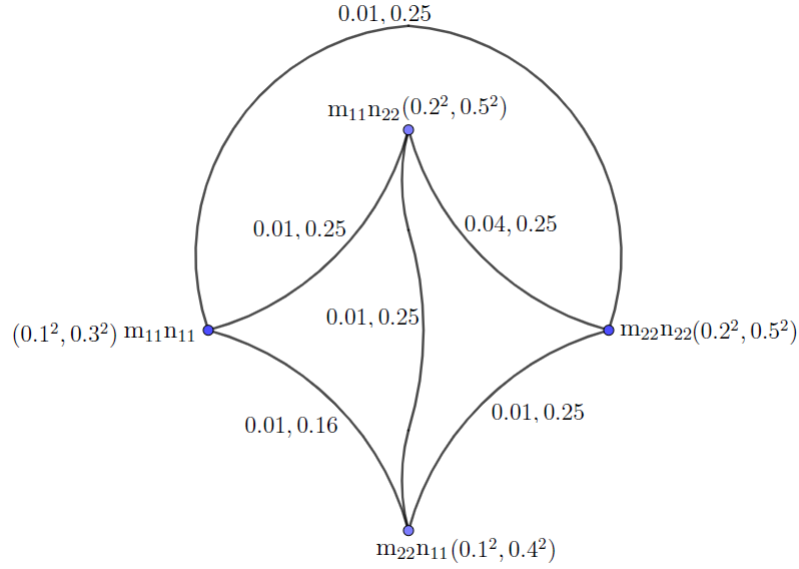


Figure 3.5.2: VSIFG G_2

Let $G_1 \otimes G_2$ be a VSIFG with $m_{11}n_{11} = (0.1^2, 0.3^2)$, $m_{11}n_{22} = (0.2^2, 0.5^2)$, $m_{22}, n_{11} = (0.1^2, 0.4^2)$, $m_{22}n_{22} = (0.2^2, 0.5^2)$, $(m_{11}n_{11}, m_{22}n_{22}) = (0.01, 0.25)$, $(m_{11}n_{22}, m_{22}n_{11}) = (0.01, 0.25)$, $(m_{11}n_{11}, m_{11}n_{22}) = (0.01, 0.25)$, $(m_{11}n_{22}, m_{22}n_{22}) = (0.04, 0.25)$, $(m_{22}n_{11}, m_{22}n_{22}) = (0.01, 0.25)$, $(m_{11}n_{11}, m_{22}n_{11}) = (0.01, 0.16)$.

Theorem 3.5.1 If $G_1 = (V_1, E_1, \sigma', \mu')$ and $G_2 = (V_2, E_2, \sigma'', \mu'')$ are complete vertex squared intuitionistic fuzzy graph, then $G_1 \otimes G_2$ is complete vertex squared intuitionistic fuzzy graph.

Figure 3.5.3: VSIFG $G_1 \otimes G_2$

Proof. Because $G_1 = (V_1, E_1, \sigma', \mu')$ and $G_2 = (V_2, E_2, \sigma'', \mu'')$ are complete VSIFG, then

$$\mu'_1(x_1, x_2) = \sigma'^2_1(x_1) \wedge \sigma'^2_1(x_2)$$

$$\mu'_2(x_1, x_2) = \sigma'^2_2(x_1) \vee \sigma'^2_2(x_2)$$

$$\mu''_1(y_1, y_2) = \sigma''^2_1(y_1) \wedge \sigma''^2_1(y_2)$$

$$\mu''_2(y_1, y_2) = \sigma''^2_2(y_1) \vee \sigma''^2_2(y_2) \text{ for all } (x_1, x_2) \in E_1 \text{ and } (y_1, y_2) \in E_2$$

If $((x, y_1), (x, y_2)) \in E$, then

$$\begin{aligned} (\mu'_1 \otimes \mu''_1)((x, y_1), (x, y_2)) &= \sigma'^2_1(x) \wedge \mu''_1(y_1, y_2) \\ &= \sigma'^2_1(x) \wedge [\sigma''^2_1(y_1) \wedge \sigma''^2_1(y_2)] \\ &= [\sigma'^2_1(x) \wedge \sigma''^2_1(y_1)] \wedge [\sigma'^2_1(x) \wedge \sigma''^2_1(y_2)] \\ &= \{[\sigma'^2_1 \otimes \sigma''^2_1](x, y_1)\} \wedge \{[\sigma'^2_1 \otimes \sigma''^2_1](x, y_2)\} \end{aligned}$$

Similarly, we can show that

$$(\mu'_2 \otimes \mu''_2)((x, y_1), (x, y_2)) = \{[\sigma'^2_2 \otimes \sigma''^2_2](x, y_1)\} \vee \{[\sigma'^2_2 \otimes \sigma''^2_2](x, y_2)\}$$

If $((x_1, y), (x_2, y)) \in E$, then

$$\begin{aligned} (\mu'_1 \otimes \mu''_1)\{(x_1, y), (x_2, y)\} &= \{\mu'_1(x_1, x_2) \wedge \sigma''^2_1(y)\} \\ &= [\sigma'^2_1(x_1) \wedge \sigma'^2_1(x_2)] \wedge \sigma''^2_1(y) \\ &= [\sigma'^2_1(x_1) \wedge \sigma''^2_1(y)] \wedge [\sigma'^2_1(x_2) \wedge \sigma''^2_1(y)] \\ &= \{[\sigma'_1 \otimes \sigma''_1](x_1, y)\} \wedge \{[\sigma'_1 \otimes \sigma''_1](x_2, y)\} \end{aligned}$$

Again if $((x_1, y_1), (x_2, y_2)) \in E$, then

$$\begin{aligned} (\mu'_1 \otimes \mu''_1)\{(x_1, y_1), (x_2, y_2)\} &= \{\mu'_1(x_1, x_2) \wedge \mu''_1(y_1, y_2)\} \\ &= [\sigma'^2_1(x_1) \wedge \sigma'^2_1(x_2)] \wedge [\sigma''^2_1(y_1) \wedge \sigma''^2_1(y_2)] \\ &= [\sigma'^2_1(x_1) \wedge \sigma''^2_1(y_1)] \wedge [\sigma'^2_1(x_2) \wedge \sigma''^2_1(y_2)] \\ &= \{[\sigma'^2_1 \otimes \sigma''^2_1](x_1, y_1)\} \wedge \{[\sigma'^2_1 \otimes \sigma''^2_1](x_2, y_2)\} \end{aligned}$$

Similarly,

$$(\mu'_2 \otimes \mu''_2)((x_1, y_1), (x_2, y_2)) = \{(\sigma'^2_2 \otimes \sigma''^2_2)(x_1, y_1)\} \vee \{(\sigma'^2_2 \otimes \sigma''^2_2)(x_2, y_2)\}$$

Therefore $G_1 \otimes G_2$ is also complete VSIFG.

Theorem 3.5.2 *Let G_{1VSIFG} and G_{2VSIFG} be two vertex squared intuitionistic fuzzy graph and if G_{1VSIFG} and G_{2VSIFG} are strong VSIFG, then the semi - product of two vertex squared intuitionistic fuzzy graph contains dual strong domination number.*

Proof. Let G_{1VSIFG} and G_{2VSIFG} be two VSIFG . If G_{1VSIFG} and G_{2VSIFG} are strong SMIFG, then the semi - product of two SMIFG contains dual strong domination number.

Conversely , Suppose that the semi - product of two SMIFG contains SDSN. To prove that G_{1VSIFG} and

G_{2VSIFG} are strong VSIFG. If possible G_{1VSIFG} and G_{2VSIFG} are non strong VSIFG, then all arc of G_{1VSIFG} and G_{2VSIFG} are non strong arc. Therefore the semi - product of two VSIFG does not have DSDN, which is a contradiction. Hence G_{1VSIFG} and G_{2VSIFG} are strong VSIFG.

Example 3.5.2 In figure 3.5.1 and 3.5.2 are strong VSIFG. Then by figure 3.5.3 $G_1 \otimes G_2$ having all arcs are strong. Therefore $G_1 \otimes G_2$ contains DSDN.

3.6 Vertex Squared Split Intuitionistic Fuzzy Graph

The new concept of dual strong domination in VSSIFG are defined in this section, along with related results.

Definition 3.6.1 An vertex squared split intuitionistic fuzzy graph is of the form $G_{VSSIFG} : (V_{VSSIFG}, E_{VSSIFG})$ where

(i) the vertex set $V_{VSSIFG} = \{v_{11}, v_{22}, \dots, v_{nn}\}$ such that $\mu_{VSSIFG}^2 : V_{VSSIFG} \rightarrow [0, 1]$ and $\gamma_{VSSIFG}^2 : V_{VSSIFG} \rightarrow [0, 1]$ denote the degree of membership value and non membership value of the element $v_{ii} \in V_{VSSIFG}$ respectively and $0 \leq \mu_{VSSIFG}^2(v_{ii}) + \gamma_{VSSIFG}^2(v_{ii}) \leq 1$ for every $v_{ii} \in V_{VSSIFG}$ ($i = 1, 2, \dots, n$).

(ii) $E_{VSSIFG} \subseteq V_{VSSIFG} \times V_{VSSIFG}$ where $\mu_{E_{VSSIFG}} : V_{VSSIFG} \times V_{VSSIFG} \rightarrow [0, 1]$ and $\gamma_{E_{VSSIFG}} : V_{VSSIFG} \times V_{VSSIFG} \rightarrow [0, 1]$ are such that

$$\mu_{E_{VSSIFG}}(v_{ii}, v_{jj}) \leq \min \left(\frac{\mu_{VSSIFG}^2(v_{ii})}{2}, \frac{\mu_{VSSIFG}^2(v_{jj})}{2} \right)$$

$$\gamma_{E_{VSSIFG}}(v_{ii}, v_{jj}) \leq \max \left(\frac{\gamma_{VSSIFG}^2(v_{ii})}{2}, \frac{\gamma_{VSSIFG}^2(v_{jj})}{2} \right)$$

and $0 \leq \mu_{E_{VSSIFG}}(v_{ii}, v_{jj}) + \gamma_{E_{VSSIFG}}(v_{ii}, v_{jj}) \leq 1$ for every $(v_{ii}, v_{jj}) \in E_{VSSIFG}$, ($i, j = 1, 2, \dots, n$).

Example 3.6.1 Let G_{VSSIFG} be a VSSIFG with

$$\begin{aligned} m_{11} &= \left(\frac{0.7^2}{2}, \frac{0.2^2}{2} \right), m_{22} = \left(\frac{0.3^2}{2}, \frac{0.4^2}{2} \right), m_{33} = \left(\frac{0.2^2}{2}, \frac{0.7^2}{2} \right), m_{44} = \left(\frac{0.5^2}{2}, \frac{0.6^2}{2} \right) \\ m_{55} &= \left(\frac{0.3^2}{2}, \frac{0.4^2}{2} \right), (m_{11}, m_{22}) = (0.045, 0.08), (m_{11}, m_{33}) = (0.02, 0.245), \\ (m_{11}, m_{55}) &= (0.030, 0.10), (m_{22}, m_{33}) = (0.02, 0.245), (m_{22}, m_{44}) = (0.045, 0.18), \\ (m_{33}, m_{44}) &= (0.02, 0.245), (m_{44}, m_{55}) = (0.030, 0.10) \end{aligned}$$

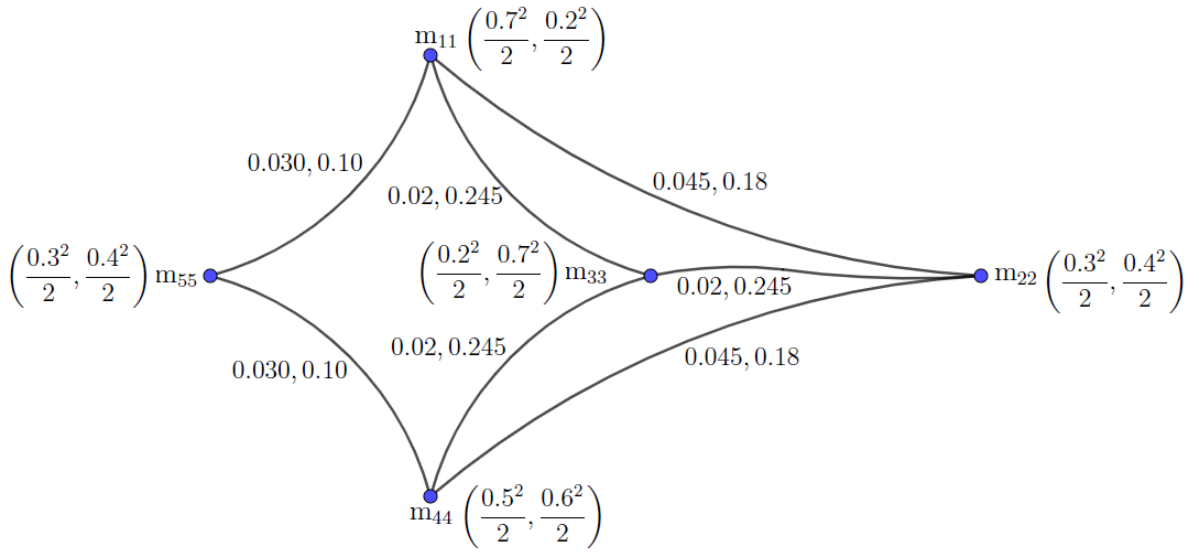


Figure 3.6.1: VSSIFG

Definition 3.6.2 A vertex squared split intuitionistic fuzzy graph $G_{VSSIFG} : (V_{VSSIFG}, E_{VSSIFG})$ is said to be strong if

$$\begin{aligned} \mu_{E_{VSSIFG}}(v_{ii}, v_{jj}) &= \min \left(\frac{\mu_{VSSIFG}^2(v_{ii})}{2}, \frac{\mu_{VSSIFG}^2(v_{jj})}{2} \right) \\ \gamma_{E_{VSSIFG}}(v_{ii}, v_{jj}) &= \max \left(\frac{\gamma_{VSSIFG}^2(v_{ii})}{2}, \frac{\gamma_{VSSIFG}^2(v_{jj})}{2} \right) \text{ for all } (v_{ii}, v_{jj}) \in E_{VSSIFG} \end{aligned}$$

Definition 3.6.3 A vertex squared split intuitionistic fuzzy graph $G_{VSSIFG} : (V_{VSSIFG}, E_{VSSIFG})$ is said to be complete if

$$\begin{aligned}\mu_{E_{VSSIFG}}(v_{ii}, v_{jj}) &= \min \left(\frac{\mu_{V_{VSSIFG}}^2(v_{ii})}{2}, \frac{\mu_{V_{VSSIFG}}^2(v_{jj})}{2} \right) \\ \gamma_{E_{VSSIFG}}(v_{ii}, v_{jj}) &= \max \left(\frac{\gamma_{V_{VSSIFG}}^2(v_{ii})}{2}, \frac{\gamma_{V_{VSSIFG}}^2(v_{jj})}{2} \right) \text{ for all } (v_{ii}, v_{jj}) \in E_{VSSIFG}\end{aligned}$$

3.7 The Join Product of Two Vertex Squared Split Intuitionistic Fuzzy Graph

In this section the join product of two vertex squared split intuitionistic fuzzy graph are defined and related theorems are discussed.

Definition 3.7.1 The join product of two VSSIFG $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ denoted by

$$G_1 + G_2 = [V_1 \cup V_2, E_1 \cup E_2 \cup E^N]$$

where E^N is the new edge joining V_1 and V_2 , its MV and NMV are defined as follows

$$(\mu_{V_1} + \mu_{V_2})(u) = \begin{cases} \mu_{V_1}(u) & \text{if } u \in V_1 \\ \mu_{V_2}(u) & \text{if } u \in V_2 \end{cases}$$

$$(\gamma_{V_1} + \gamma_{V_2})(u) = \begin{cases} \gamma_{V_1}(u) & \text{if } u \in V_1 \\ \gamma_{V_2}(u) & \text{if } u \in V_2 \end{cases}$$

$$(\mu_{E_1} + \mu_{E_2})(uv) = \begin{cases} \mu_{E_1}(uv) & \text{if } uv \in E_1 \\ \mu_{E_2}(uv) & \text{if } uv \in E_2 \\ \min \left\{ \frac{\mu_{V_1}^2(u)}{2}, \frac{\mu_{V_2}^2(v)}{2} \right\} & \text{if } uv \in E^N \end{cases}$$

$$(\gamma_{E_1} + \gamma_{E_2})(uv) = \begin{cases} \gamma_{E_1}(uv) & \text{if } uv \in E_1 \\ \gamma_{E_2}(uv) & \text{if } uv \in E_2 \\ \max \left\{ \frac{\gamma_{V_1}^2(u)}{2}, \frac{\gamma_{V_2}^2(v)}{2} \right\} & \text{if } uv \in E^N \end{cases}$$

Example 3.7.1 Let G_1 be a VSSIFG with

$$m_{11} = \left(\frac{0.7^2}{2}, \frac{0.2^2}{2} \right), m_{22} = \left(\frac{0.3^2}{2}, \frac{0.4^2}{2} \right), (m_{11}, m_{22}) = (0.045, 0.08).$$

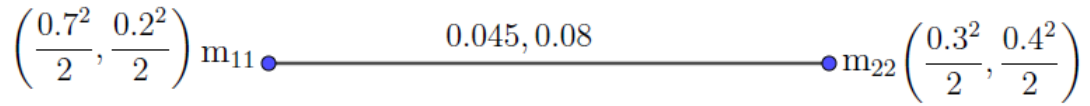


Figure 3.7.1: VSSIFG G_1

Let G_2 be a VSSIFG with $n_{11} = \left(\frac{0.5^2}{2}, \frac{0.6^2}{2} \right), n_{22} = \left(\frac{0.3^2}{2}, \frac{0.4^2}{2} \right), (n_{11}, n_{22}) = (0.030, 0.10)$.

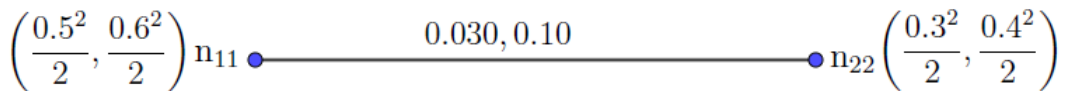


Figure 3.7.2: VSSIFG G_2

Let $G_1 + G_2$ be a VSSIFG with $m_{11} = \left(\frac{0.7^2}{2}, \frac{0.2^2}{2}\right)$, $m_{22} = \left(\frac{0.3^2}{2}, \frac{0.4^2}{2}\right)$, $n_{11} = \left(\frac{0.5^2}{2}, \frac{0.6^2}{2}\right)$, $n_{22} = \left(\frac{0.3^2}{2}, \frac{0.4^2}{2}\right)$, $(m_{11}, m_{22}) = (0.045, 0.08)$, $(n_{11}, n_{22}) = (0.030, 0.10)$, $(m_{11}, n_{11}) = (0.125, 0.18)$, $(m_{22}, n_{22}) = (0.045, 0.08)$, $(m_{11}, n_{22}) = (0.045, 0.08)$, $(m_{22}, n_{11}) = (0.045, 0.08)$

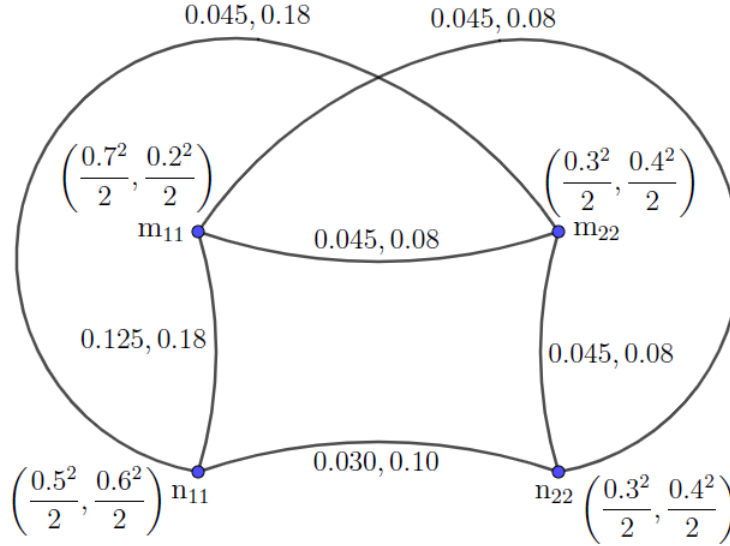


Figure 3.7.3: VSSIFG $G_1 + G_2$

Theorem 3.7.1 *The join of two vertex squared split intuitionistic fuzzy graph is again an vertex squared split intuitionistic fuzzy graph.*

Proof. Let $G_1 = [V_1, E_1]$ and $G_2 = [V_2, E_2]$ be VSSIFG, we have to prove that the join of G_1 and G_2 denoted by $G_1 + G_2 = [V_1 \cup V_2, E_1 \cup E_2 \cup E^N]$ where E^N is the new edge joining V_1 and V_2 is an VSSIFG. By definition, we have if $x \in V_1$ then

$$(\mu_{V_1} + \mu_{V_2})(x) = \mu_{V_1}(x) \&$$

$$(\gamma_{V_1} + \gamma_{V_2})(x) = \gamma_{V_1}(x)$$

$$\text{therefore, } 0 \leq (\mu_{V_1} + \mu_{V_2})(x) + (\gamma_{V_1} + \gamma_{V_2})(x) \leq 1$$

Similarly, if $x \in V_2$ then

$$(\mu_{V_1} + \mu_{V_2})(x) = \mu_{V_2}(x) \ \&$$

$$(\gamma_{V_1} + \gamma_{V_2})(x) = \gamma_{V_2}(x)$$

$$\text{therefore, } 0 \leq (\mu_{V_1} + \mu_{V_2})(x) + (\gamma_{V_1} + \gamma_{V_2})(x) \leq 1$$

and if $xy \in E_1$, then

$$(\mu_{E_1} + \mu_{E_2})(x, y) = \mu_{E_1}(xy)$$

$$\begin{aligned} (\mu_{E_1} + \mu_{E_2})(x, y) &\leq \min \left(\frac{\mu_{V_1}^2(x)}{2}, \frac{\mu_{V_1}^2(y)}{2} \right) \\ &\leq \min \left(\frac{(\mu_{V_1} + \mu_{V_2})^2(x)}{2}, \frac{(\mu_{V_1} + \mu_{V_2})^2(y)}{2} \right) \end{aligned}$$

$$\text{and } (\gamma_{E_1} + \gamma_{E_2})(x, y) = \gamma_{E_1}(xy)$$

$$\begin{aligned} (\gamma_{E_1} + \gamma_{E_2})(x, y) &\leq \max \left(\frac{\gamma_{V_1}^2(x)}{2}, \frac{\gamma_{V_1}^2(y)}{2} \right) \\ &\leq \max \left(\frac{(\gamma_{V_1} + \gamma_{V_2})^2(x)}{2}, \frac{(\gamma_{V_1} + \gamma_{V_2})^2(y)}{2} \right) \end{aligned}$$

Similarly, if $xy \in E_2$

$$(\mu_{E_1} + \mu_{E_2})(x, y) \leq \min \left(\frac{(\mu_{V_1} + \mu_{V_2})^2(x)}{2}, \frac{(\mu_{V_1} + \mu_{V_2})^2(y)}{2} \right)$$

$$\text{and } (\gamma_{E_1} + \gamma_{E_2})(x, y) \leq \max \left(\frac{(\gamma_{V_1} + \gamma_{V_2})^2(x)}{2}, \frac{(\gamma_{V_1} + \gamma_{V_2})^2(y)}{2} \right)$$

If $xy \in E^N$,

$$\begin{aligned}
 (\mu_{E_1} + \mu_{E_2})(x, y) &= \min \left(\frac{\mu_{V_1}^2(x)}{2}, \frac{\mu_{V_1}^2(y)}{2} \right) \\
 &= \min \left(\frac{(\mu_{V_1} + \mu_{V_2})^2(x)}{2}, \frac{(\mu_{V_1} + \mu_{V_2})^2(y)}{2} \right) \\
 (\gamma_{E_1} + \gamma_{E_2})(x, y) &= \max \left(\frac{\gamma_{V_1}^2(x)}{2}, \frac{\gamma_{V_1}^2(y)}{2} \right) \\
 &= \max \left(\frac{(\gamma_{V_1} + \gamma_{V_2})^2(x)}{2}, \frac{(\gamma_{V_1} + \gamma_{V_2})^2(y)}{2} \right)
 \end{aligned}$$

Therefore $G_1 + G_2$ is also an VSSIFG.

Theorem 3.7.2 *If G_1 and G_2 are strong vertex squared split intuitionistic fuzzy graph, then their join denoted by $G_1 + G_2$ is again strong vertex squared split intuitionistic fuzzy graph.*

Proof. Since G_1 and G_2 are strong VSSIFG

$$\begin{aligned}
 \mu_{E_1}(x, y) &= \min \left(\frac{\mu_{V_1}^2(x)}{2}, \frac{\mu_{V_1}^2(y)}{2} \right) \text{ and} \\
 \gamma_{E_1}(x, y) &= \max \left(\frac{\gamma_{V_1}^2(x)}{2}, \frac{\gamma_{V_1}^2(y)}{2} \right) \quad \forall x, y \in E_1 \\
 \text{and } \mu_{E_2}(x, y) &= \min \left(\frac{\mu_{V_2}^2(x)}{2}, \frac{\mu_{V_2}^2(y)}{2} \right) \text{ and} \\
 \gamma_{E_2}(x, y) &= \max \left(\frac{\gamma_{V_2}^2(x)}{2}, \frac{\gamma_{V_2}^2(y)}{2} \right) \quad \forall x, y \in E_2 \\
 (\mu_{E_1} + \mu_{E_2})(x, y) &= \begin{cases} \mu_{E_1}(x, y) & \text{if } xy \in E_1 \\ \mu_{E_2}(x, y) & \text{if } xy \in E_2 \end{cases} \\
 &= \begin{cases} \min \left\{ \frac{\mu_{V_1}^2(x)}{2}, \frac{\mu_{V_1}^2(y)}{2} \right\} & \text{if } xy \in E_1 \\ \min \left\{ \frac{\mu_{V_2}^2(x)}{2}, \frac{\mu_{V_2}^2(y)}{2} \right\} & \text{if } xy \in E_2 \end{cases} \\
 (\gamma_{E_1} + \gamma_{E_2})(x, y) &= \begin{cases} \gamma_{E_1}(x, y) & \text{if } xy \in E_1 \\ \gamma_{E_2}(x, y) & \text{if } xy \in E_2 \end{cases}
 \end{aligned}$$

$$= \begin{cases} \max \left\{ \frac{\gamma_{V_1}^2(x)}{2}, \frac{\gamma_{V_1}^2(y)}{2} \right\} \\ \max \left\{ \frac{\gamma_{V_2}^2(x)}{2}, \frac{\gamma_{V_2}^2(y)}{2} \right\} \end{cases}$$

and if $x, y \in E^N$

$$\begin{aligned} (\mu_{E_1} + \mu_{E_2})(x, y) &= \min \left(\frac{\mu_{V_1}^2(x)}{2}, \frac{\mu_{V_2}^2(y)}{2} \right) \\ (\gamma_{E_1} + \gamma_{E_2})(x, y) &= \max \left(\frac{\gamma_{V_1}^2(x)}{2}, \frac{\gamma_{V_2}^2(y)}{2} \right) \end{aligned}$$

Therefore $G_1 + G_2$ is strong VSSIFG.

Theorem 3.7.3 *If G_1 and G_2 are complete vertex squared split intuitionistic fuzzy graph, then their join product denoted by $G_1 + G_2$ is again complete vertex squared split intuitionistic fuzzy graph.*

Proof. Since G_1 and G_2 are complete VSSIFG

$$\begin{aligned} \mu_{E_1}(x, y) &= \min \left(\frac{\mu_{V_1}^2(x)}{2}, \frac{\mu_{V_1}^2(y)}{2} \right) \text{ and} \\ \gamma_{E_1}(x, y) &= \max \left(\frac{\gamma_{V_1}^2(x)}{2}, \frac{\gamma_{V_1}^2(y)}{2} \right) \quad \forall x, y \in V_1 \\ \text{and } \mu_{E_2}(x, y) &= \min \left(\frac{\mu_{V_2}^2(x)}{2}, \frac{\mu_{V_2}^2(y)}{2} \right) \text{ and} \\ \gamma_{E_2}(x, y) &= \max \left(\frac{\gamma_{V_2}^2(x)}{2}, \frac{\gamma_{V_2}^2(y)}{2} \right) \quad \forall x, y \in V_2 \\ (\mu_{E_1} + \mu_{E_2})(x, y) &= \begin{cases} \mu_{E_1}(x, y) & \text{if } xy \in E_1 \\ \mu_{E_2}(x, y) & \text{if } xy \in E_2 \end{cases} \\ &= \begin{cases} \min \left\{ \frac{\mu_{V_1}^2(x)}{2}, \frac{\mu_{V_1}^2(y)}{2} \right\} \\ \min \left\{ \frac{\mu_{V_2}^2(x)}{2}, \frac{\mu_{V_2}^2(y)}{2} \right\} \end{cases} \quad \forall x, y \in V_1 \end{aligned}$$

$$\begin{aligned}
(\gamma_{E_1} + \gamma_{E_2})(x, y) &= \begin{cases} \gamma_{E_1}(x, y) & \text{if } xy \in E_1 \\ \gamma_{E_2}(x, y) & \text{if } xy \in E_2 \end{cases} \\
&= \begin{cases} \max \left\{ \frac{\gamma_{V_1}^2(x)}{2}, \frac{\gamma_{V_1}^2(y)}{2} \right\} \\ \max \left\{ \frac{\gamma_{V_2}^2(x)}{2}, \frac{\gamma_{V_2}^2(y)}{2} \right\} \end{cases} \quad \forall x, y \in V_2
\end{aligned}$$

and if $x, y \in E^N$

$$\begin{aligned}
(\mu_{E_1} + \mu_{E_2})(x, y) &= \min \left(\frac{\mu_{V_1}^2(x)}{2}, \frac{\mu_{V_2}^2(y)}{2} \right) \quad \forall x, y \in V_1 \\
(\gamma_{E_1} + \gamma_{E_2})(x, y) &= \max \left(\frac{\gamma_{V_1}^2(x)}{2}, \frac{\gamma_{V_2}^2(y)}{2} \right) \quad \forall x, y \in V_2
\end{aligned}$$

Therefore $G_1 + G_2$ is complete VSSIFG.

3.8 Dual Strong Domination in Vertex Squared Split Intuitionistic Fuzzy Graph

In this section, the concept of dual strong domination in VSSIFG is defined and discussed the notion of $\gamma_{DSD}(G_{VSSIFG})$.

Definition 3.8.1 Let G_{VSSIFG} be a VSSIFG. A set D_{VSSIFG} of nodes of G_{VSSIFG} is a dual strong dominating set of G_{VSSIFG} if every node of $V_{VSSIFG} - D_{VSSIFG}$ is a strong neighbor of at least two node in D_{VSSIFG} .

Definition 3.8.2 The weight of a dual strong dominating set D_{VSSIFG} of VSSIFG is defined as

$$W(D_{VSSIFG}) = \sum_{m_{11} \in D_{VSSIFG}} \left[\frac{1 + |\mu_2(m_{11}, m_{22}) - \gamma_2(m_{11}, m_{22})|}{2} \right],$$

where $|\mu_2(m_{11}, m_{22}) - \gamma_2(m_{11}, m_{22})|$ is the minimum of the membership value and non membership value of the strong arcs incident on m_{11} . The dual strong domination number of a VSIFG G_{VSSIFG} is defined as the minimum weight of dual strong dominating set of G_{VSSIFG} and it is denoted by $\gamma_{DSD}(G_{VSSIFG})$.

Example 3.8.1 Let G_{VSSIFG} be a VSSIFG with all edges are strong.

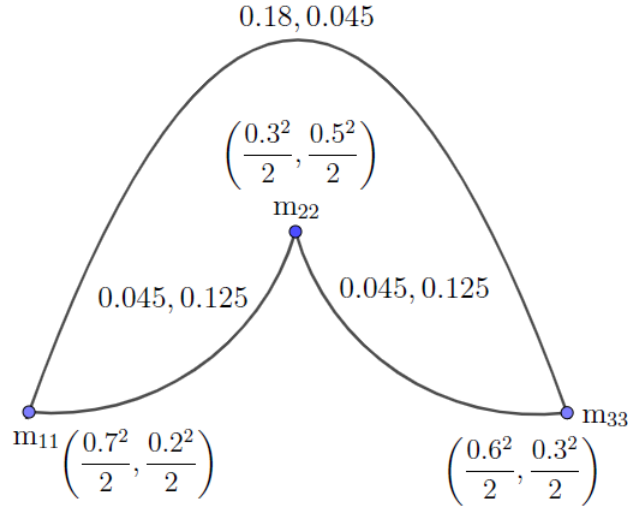


Figure 3.8.1: VSSIFG with $\gamma_{DSD}(G_{VSSIFG})$

The DSDSs are $S_{11} = \{m_{11}\}$, $S_{22} = \{m_{22}\}$ and $S_{33} = \{m_{33}\}$. After calculating the weight of SDSs are $S_{11} = 0.08$ and $S_{22} = 0.08$ and $S_{33} = 0.08$. Therefore $\gamma_{DSD}(G_{VSSIFG}) = 0.08$.

Theorem 3.8.1 Let $G_1 + G_2$ be an VSSIFG and if $G_1 + G_2$ having at most two non strong arcs, then $G_1 + G_2$ contains dual strong dominating set.

Proof. Let $G_1 + G_2$ be an VSSIFG. If $G_1 + G_2$ having at most two non strong arcs, then $G_1 + G_2$ contains dual strong dominating set.

Conversely, suppose that $G_1 + G_2$ contains DSDS. To prove that $G_1 + G_2$ having at most two non strong arcs. If possible $G_1 + G_2$ contains more than two non strong arcs, then $G_1 + G_2$ does not contain a DSDS, which is a contradiction. Hence $G_1 + G_2$ having at most two non strong arcs.

Remark 3.8.1 If $G_1 + G_2$ contains dual strong domination number then G_1 and G_2 need not be strong VSSIFG.

In this chapter, we introduce the notion of vertex squared intuitionistic fuzzy graph and we define strong vertex squared intuitionistic fuzzy graph, complete vertex squared intuitionistic fuzzy graph. Also,

a direct product of two vertex squared intuitionistic fuzzy graph, semi-strong product and semi product of two vertex squared intuitionistic fuzzy graph are defined, and many interesting results involving these concepts are investigated. We develop the concept of vertex squared split intuitionistic fuzzy graph, strong vertex squared split intuitionistic fuzzy graph, complete vertex squared split intuitionistic fuzzy graph and join product of two vertex squared split intuitionistic fuzzy graph are defined, and many interesting results involving these concepts are investigated. The relationship between the vertex squared intuitionistic fuzzy graph of a dual strong domination and theorems relating to these notions are also analyzed. Moreover, we discuss dual strong domination number and investigated their many interesting results.

Chapter 4

Split Domination in Vertex Squared Interval-Valued Fuzzy Graphs

In this chapter we study different concepts like vertex squared interval-valued fuzzy graph, vertex squared cardinality, vertex squared independent set, n - split dominating set, n -split domination number. We likewise, investigate a relationship between n - split dominating set and vertex squared independent set for vertex squared interval-valued fuzzy graphs. The vertex squared interval-valued fuzzy graphs are more adaptable and viable than fuzzy graphs because of the way that they have numerous applications in networks. This work will be useful to concentrate enormous vertex squared interval-valued fuzzy graphs as a mix of little vertex squared interval-valued fuzzy graphs. Vertex squared interval-valued fuzzy graph hypothesis is presently developing and growing its applications. The theoretical improvement in this space is talked about here. Eventually, genuine utilization of n -split domination number to choose which oxygen cylinder agencies have the lot of oxygen cylinder among various oxygen cylinder agencies is inspected.

4.1 Vertex Squared Interval-Valued Fuzzy Graph

Definition 4.1.1 *An vertex squared interval-valued fuzzy set (VSIVFS) X_{IV} on a set V_{IV} is denoted by $X_{IV} = \{(i_{11}, [(\sigma_{X_{IV}}^-(i_{11}))^2, (\sigma_{X_{IV}}^+(i_{11}))^2]), i_{11} \in V_{IV}\}$, where $(\sigma_{X_{IV}}^-)^2$ and $(\sigma_{X_{IV}}^+)^2$ are fuzzy subsets of V_{IV} such that $(\sigma_{X_{IV}}^-(i_{11}))^2 \leq (\sigma_{X_{IV}}^+(i_{11}))^2$ for all $i_{11} \in V_{IV}$. If $G_{IV}^* = (V_{IV}, E_{IV})$ is a crisp graph, then by an vertex squared interval-valued fuzzy relation Y_{IV} on V_{IV} we mean an VSIVFS on E_{IV} such that*

$\sigma_{Y_{IV}}^-(i_{11}i_{22}) \leq \min\{(\sigma_{X_{IV}}^-(i_{11}))^2, (\sigma_{X_{IV}}^-(i_{22}))^2\}$ and $\sigma_{Y_{IV}}^+(i_{11}i_{22}) \leq \max\{(\sigma_{X_{IV}}^+(i_{11}))^2, (\sigma_{X_{IV}}^+(i_{22}))^2\}$ for all $i_{11}i_{22} \in E_{IV}$ and we write $Y_{IV} = \{(i_{11}i_{22}, [\sigma_{Y_{IV}}^-(i_{11}i_{22}), \sigma_{Y_{IV}}^+(i_{11}i_{22})]), i_{11}i_{22} \in E_{IV}\}$.

Definition 4.1.2 An vertex squared interval-valued fuzzy graph of a graph $G_{IV}^* = (V_{IV}, E_{IV})$ is a pair $G_{IV} = (X_{IV}, Y_{IV})$, where $X_{IV} = [(\sigma_{X_{IV}}^-)^2, (\sigma_{X_{IV}}^+)^2]$ is an VSIVFS on V_{IV} and $Y_{IV} = [\sigma_{Y_{IV}}^-, \sigma_{Y_{IV}}^+]$ is an vertex squared interval-valued fuzzy relation on V_{IV} .

Example 4.1.1

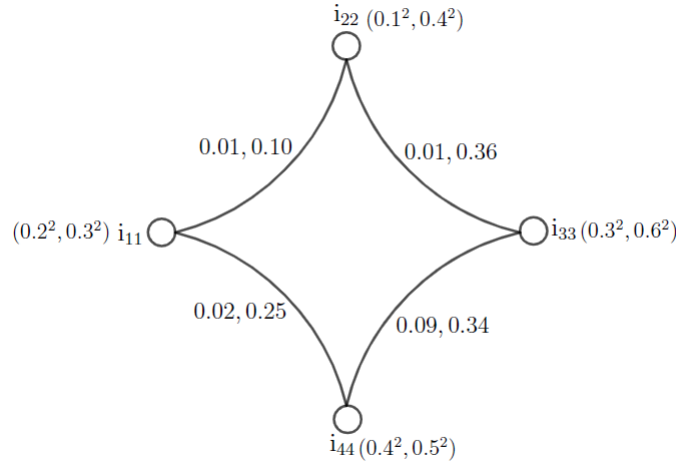


Figure 4.1.1: VSIVFG (G_{IV})

In the above figure,

$$V_{IV} = \{i_{11}, i_{22}, i_{33}, i_{44}\}$$

$$E_{IV} = \{i_{11}i_{22}, i_{22}i_{33}, i_{33}i_{44}, i_{44}i_{11}\}$$

Here we take X_{IV} be an VSIVFS on V_{IV} and Y_{IV} be an VSIVFS on $E_{IV} \subseteq V_{IV} \times V_{IV}$ defined by

$$X_{IV} = \left\langle \left(\frac{i_{11}}{(0.2)^2}, \frac{i_{22}}{(0.1)^2}, \frac{i_{33}}{(0.3)^2}, \frac{i_{44}}{(0.4)^2} \right) \right\rangle \left\langle \left(\frac{i_{11}}{(0.3)^2}, \frac{i_{22}}{(0.4)^2}, \frac{i_{33}}{(0.6)^2}, \frac{i_{44}}{(0.5)^2} \right) \right\rangle$$

$$Y_{IV} = \left\langle \left(\frac{i_{11}i_{22}}{0.01}, \frac{i_{22}i_{33}}{0.01}, \frac{i_{33}i_{44}}{0.09}, \frac{i_{44}i_{11}}{0.02} \right) \right\rangle \left\langle \left(\frac{i_{11}i_{22}}{0.10}, \frac{i_{22}i_{33}}{0.36}, \frac{i_{33}i_{44}}{0.34}, \frac{i_{44}i_{11}}{0.25} \right) \right\rangle$$

Then $G_{IV} = (X_{IV}, Y_{IV})$ is an VSIVFG.

Definition 4.1.3 The order p_{IV} and size q_{IV} of an VSIVFG $G_{IV} = (X_{IV}, Y_{IV})$ of a graph $G_{IV}^* = (V_{IV}, E_{IV})$ are denoted by

$$p_{IV} = \sum_{i_{11} \in V_{IV}} \frac{1 + (\sigma_{X_{IV}}^+(i_{11}))^2 - (\sigma_{X_{IV}}^-(i_{11}))^2}{2}$$

and

$$q_{IV} = \sum_{i_{11}i_{22} \in V_{IV}} \frac{1 + \sigma_{Y_{IV}}^+(i_{11}i_{22}) - \sigma_{Y_{IV}}^-(i_{11}i_{22})}{2}$$

Definition 4.1.4 Let $G_{IV} = (X_{IV}, Y_{IV})$ be an VSIVFG on $G_{IV}^* = (V_{IV}, E_{IV})$ and $S_{IV} \subseteq V_{IV}$. Then the vertex squared cardinality of S_{IV} is defined to be

$$\sum_{i_{11} \in V_{IV}} \frac{1 + (\sigma_{X_{IV}}^+(i_{11}))^2 - (\sigma_{X_{IV}}^-(i_{22}))^2}{2}$$

Definition 4.1.5 An arc $e_{IV} = i_{11}i_{22}$ of the VSIVFG is called a vertex squared effective edge if

$$\sigma_{Y_{IV}}^-(i_{11}i_{22}) = \min\{(\sigma_{X_{IV}}^-(i_{11}))^2, (\sigma_{X_{IV}}^-(i_{22}))^2\}$$

and

$$\sigma_{Y_{IV}}^+(i_{11}i_{22}) = \max\{(\sigma_{X_{IV}}^+(i_{11}))^2, (\sigma_{X_{IV}}^+(i_{22}))^2\}$$

Definition 4.1.6 A set S_{IV} of vertices of the VSIVFG is called the vertex squared independent set (VSIS) if

$$\sigma_{Y_{IV}}^-(i_{11}i_{22}) < \min\{(\sigma_{X_{IV}}^-(i_{11}))^2, (\sigma_{X_{IV}}^-(i_{22}))^2\}$$

and

$$\sigma_{Y_{IV}}^+(i_{11}i_{22}) < \max\{(\sigma_{X_{IV}}^+(i_{11}))^2, (\sigma_{X_{IV}}^+(i_{22}))^2\}$$

for all $i_{11}, i_{22} \in S_{IV}$.

4.2 n -Split Domination in Vertex Squared Interval-Valued Fuzzy Graph

Definition 4.2.1 Let $G_{IV} = (X_{IV}, Y_{IV})$ be an vertex squared interval-valued fuzzy graph on V_{IV} and $i_{11}, i_{22} \in V_{IV}$. We say ' i_{11} ' n -split dominates ' i_{22} ' if

$$\sigma_{Y_{IV}}^-(i_{11}i_{22}) = \min \left\{ \frac{(\sigma_{X_{IV}}^-(i_{11}))^2}{n}, \frac{(\sigma_{X_{IV}}^-(i_{22}))^2}{n} \right\}$$

$$\text{and } \sigma_{Y_{IV}}^+(i_{11}i_{22}) = \max \left\{ \frac{(\sigma_{X_{IV}}^+(i_{11}))^2}{n}, \frac{(\sigma_{X_{IV}}^+(i_{22}))^2}{n} \right\}.$$

Example 4.2.1

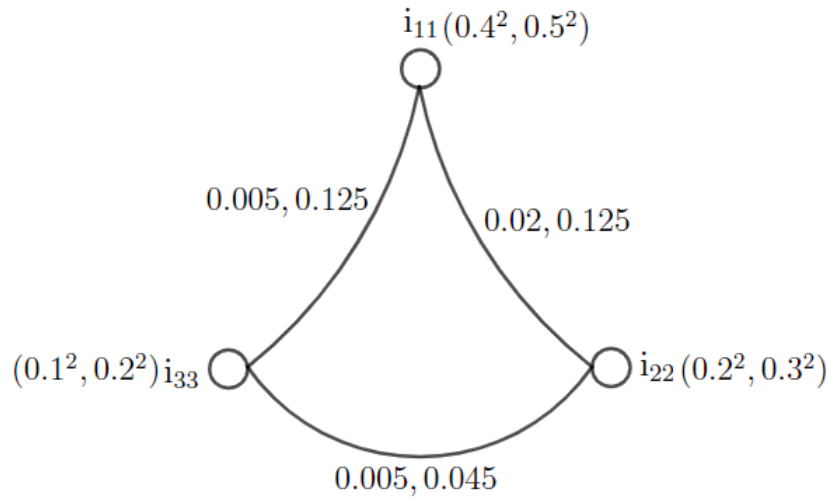


Figure 4.2.1: VSIVFG (G_{IV}) with 2-Split Dominates

In the above figure,

$$V_{IV} = \{i_{11}, i_{22}, i_{33}\}$$

$$E_{IV} = \{i_{11}i_{22}, i_{22}i_{33}, i_{33}i_{11}\}$$

Here we take X_{IV} be an VSIVFS on V_{IV} and Y_{IV} be an VSIVFS on $E_{IV} \subseteq V_{IV} \times V_{IV}$ denoted by

$$X_{IV} = \left\langle \left(\frac{i_{11}}{(0.4)^2}, \frac{i_{22}}{(0.2)^2}, \frac{i_{33}}{(0.1)^2} \right) \right\rangle \left\langle \left(\frac{i_{11}}{(0.5)^2}, \frac{i_{22}}{(0.3)^2}, \frac{i_{33}}{(0.2)^2} \right) \right\rangle$$

$$Y_{IV} = \left\langle \left(\frac{i_{11}i_{22}}{0.02}, \frac{i_{22}i_{33}}{0.005}, \frac{i_{33}i_{11}}{0.005} \right) \right\rangle \left\langle \left(\frac{i_{11}i_{22}}{0.125}, \frac{i_{22}i_{33}}{0.045}, \frac{i_{33}i_{11}}{0.125} \right) \right\rangle$$

Then $G_{IV} = (X_{IV}, Y_{IV})$ is an VSIVFG.

In this figure 4.2.1 for $n = 2$,

$$\begin{aligned} \sigma_{Y_{IV}}^-(i_{11}i_{22}) &= \min \left\{ \frac{(\sigma_{X_{IV}}^-(i_{11}))^2}{n}, \frac{(\sigma_{X_{IV}}^-(i_{22}))^2}{n} \right\} \\ &= \min \left\{ \frac{(0.4)^2}{2}, \frac{(0.2)^2}{2} \right\} \\ &= 0.02 \end{aligned}$$

$$\begin{aligned} \sigma_{Y_{IV}}^+(i_{11}i_{22}) &= \max \left\{ \frac{(\sigma_{X_{IV}}^+(i_{11}))^2}{n}, \frac{(\sigma_{X_{IV}}^+(i_{22}))^2}{n} \right\} \\ &= \max \left\{ \frac{(0.5)^2}{2}, \frac{(0.3)^2}{2} \right\} \\ &= 0.125 \end{aligned}$$

Similarly, $\sigma_{Y_{IV}}^-(i_{22}i_{33}) = 0.005$ and $\sigma_{Y_{IV}}^+(i_{22}i_{33}) = 0.045$,

$\sigma_{Y_{IV}}^-(i_{11}i_{33}) = 0.005$ and $\sigma_{Y_{IV}}^+(i_{11}i_{33}) = 0.125$.

Therefore all edges are 2- split dominate edges.

Definition 4.2.2 A subset S_{IV} of V_{IV} is called a n -split dominating set (n -SDS) in VSIVFG if for every $i_{22} \notin S_{IV}$, there exist $i_{11} \in S_{IV}$ such that i_{11} n -split dominates i_{22} . A n -split dominating set R_{IV} of a VSIVFG is called the minimal n -split dominating set if no proper subset of R_{IV} is a n -split dominating set of VSIVFG.

Definition 4.2.3 The minimal vertex squared cardinality of a n -split dominating set in VSIVFG is said to be n -split domination number of VSIVFG and is denoted by $\gamma_{nSPD}(G_{IV})$.

Example 4.2.2 In Figure 4.2.2, $V_{IV} = \{i_{11}, i_{22}, i_{33}\}$

$$E_{IV} = \{i_{11}i_{22}, i_{22}i_{33}, i_{33}i_{11}\}$$

Here we take X_{IV} be an VSIVFS on V_{IV} and Y_{IV} be an VSIVFS on $E_{IV} \subseteq V_{IV} \times V_{IV}$ denoted by

$$X_{IV} = \left\langle \left(\frac{i_{11}}{(0.4)^2}, \frac{i_{22}}{(0.3)^2}, \frac{i_{33}}{(0.2)^2} \right) \right\rangle \left\langle \left(\frac{i_{11}}{(0.5)^2}, \frac{i_{22}}{(0.4)^2}, \frac{i_{33}}{(0.4)^2} \right) \right\rangle$$

$$Y_{IV} = \left\langle \left(\frac{i_{11}i_{22}}{0.045}, \frac{i_{22}i_{33}}{0.02}, \frac{i_{33}i_{11}}{0.02} \right) \right\rangle \left\langle \left(\frac{i_{11}i_{22}}{0.125}, \frac{i_{22}i_{33}}{0.08}, \frac{i_{33}i_{11}}{0.125} \right) \right\rangle$$

Then $G_{IV} = (X_{IV}, Y_{IV})$ is an VSIVFG.

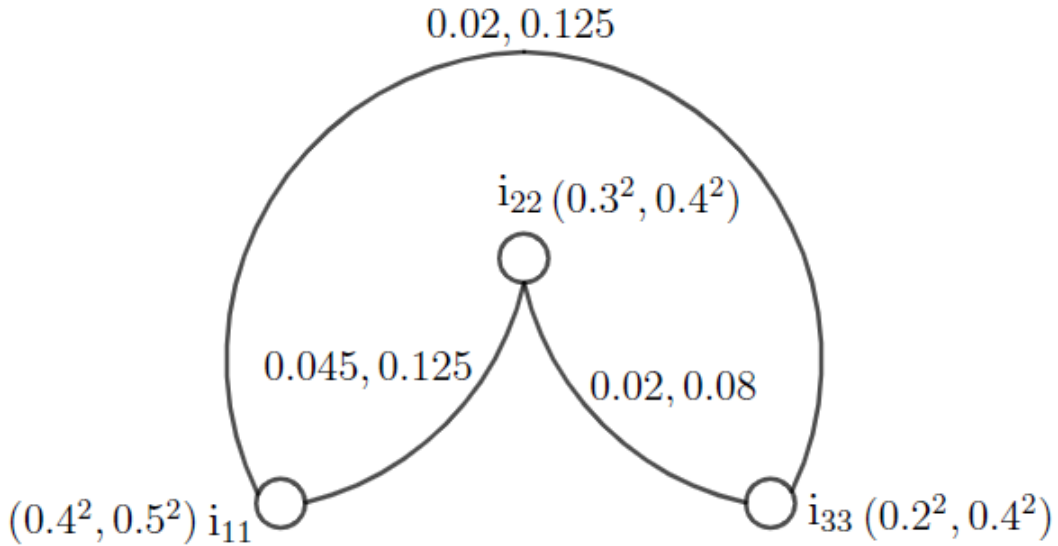


Figure 4.2.2: VSIVFG (G_{IV}) with 2-Split Domination Number

For $n=2$, the 2-split dominating sets (2-SDSs) are

$$\begin{aligned}
 D_1 = \{i_{11}\} &= \frac{1 + (0.5)^2 - (0.4)^2}{2} = 0.55 \\
 D_2 = \{i_{22}\} &= \frac{1 + (0.4)^2 - (0.3)^2}{2} = 0.54 \\
 D_3 = \{i_{33}\} &= \frac{1 + (0.4)^2 - (0.2)^2}{2} = 0.56 \\
 D_4 = \{i_{11}, i_{22}\} &= \frac{1 + (0.5)^2 - (0.4)^2}{2} + \frac{1 + (0.4)^2 - (0.3)^2}{2} = 1.09 \\
 D_5 = \{i_{22}, i_{33}\} &= \frac{1 + (0.4)^2 - (0.3)^2}{2} + \frac{1 + (0.4)^2 - (0.2)^2}{2} = 1 \\
 D_6 = \{i_{11}, i_{33}\} &= \frac{1 + (0.5)^2 - (0.4)^2}{2} + \frac{1 + (0.4)^2 - (0.2)^2}{2} = 1.11
 \end{aligned}$$

Then the minimal vertex squared cardinality of a 2-SDS is $\{i_{22}\}$ and $\gamma_{2SPD}(G_{IV}) = 0.54$.

Theorem 4.2.1 A vertex squared independent set is a maximal vertex squared independent set of a vertex squared interval-valued fuzzy graph iff it is a vertex squared independent set and n -split dominating set.

Proof. Let S_{IV} is a maximal vertex squared independent set of a VSIVFG. Thus for each $x \in V_{IV} - S_{IV}$, the set $S_{IV} \cup \{x\}$ is not vertex squared independent set. In this way, for each vertex $x \in V_{IV} - S_{IV}$, there is a vertex $y \in S_{IV}$ to such an extent that y is n -split dominated by x .

Consequently S_{IV} is a n -SDS. Hence S_{IV} is an vertex squared independent and n -SDS.

Conversely, let S_{IV} be vertex squared independent set and n -SDS. If conceivable, assume S_{IV} is not a maximal vertex squared independent set. Then there exists $x \in V_{IV} - S_{IV}$ to such an extent that the set $S_{IV} \cup \{x\}$ is vertex squared independent set. Then no vertex in S_{IV} is n -split dominated by x . Hence S_{IV} cannot be a n -SDS, which is a contradiction. Hence S_{IV} should be a maximal vertex squared independent set.

Example 4.2.3

In the above figure the maximal vertex squared independent sets are $S_1 = \{i_{11}, i_{22}, i_{33}\}$, $S_2 = \{i_{22}, i_{33}, i_{44}\}$. This S_1 and S_2 is also a vertex squared independent set and 2-SDS.

Theorem 4.2.2 In a vertex squared interval-valued fuzzy graph, every maximal vertex squared independent set is a minimal n -split dominating set.

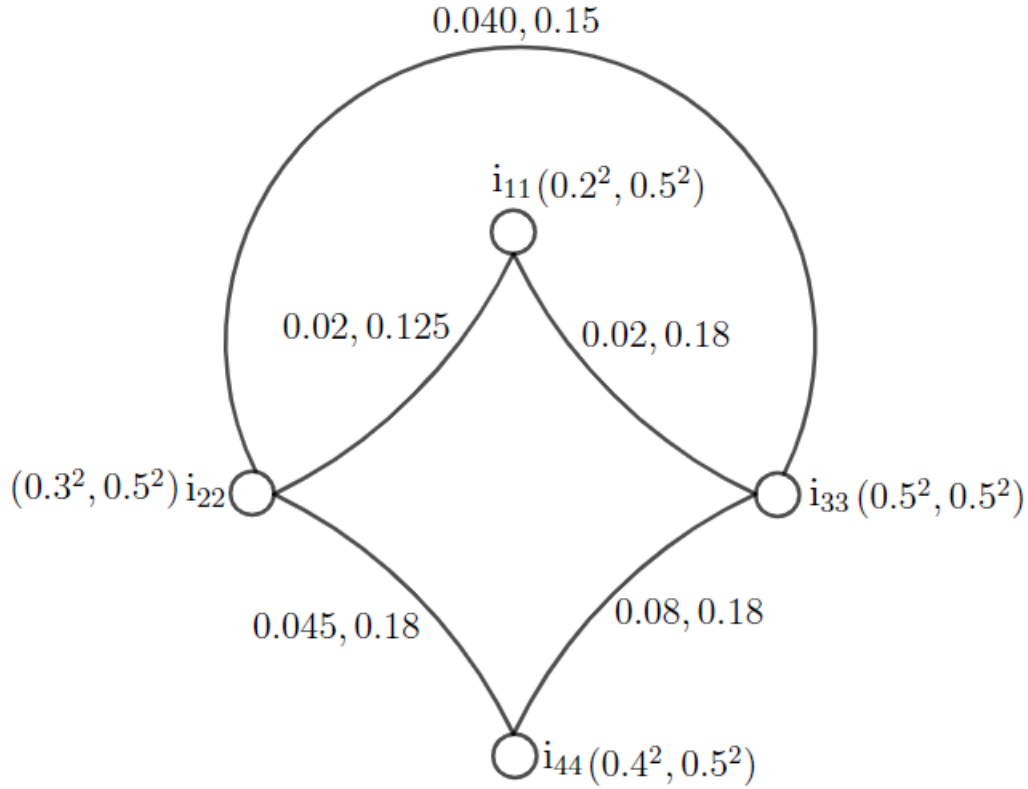
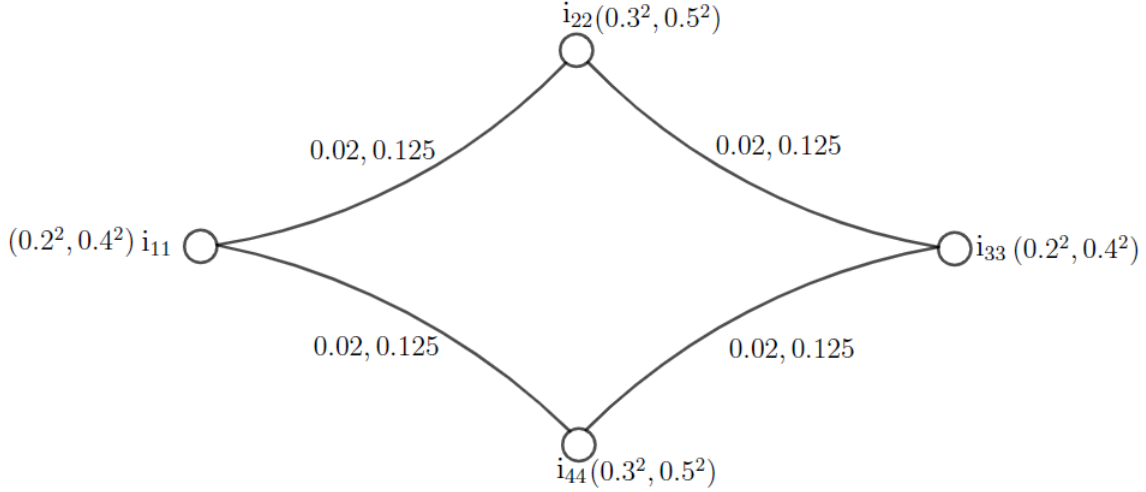


Figure 4.2.3: VSIVFG (G_{IV}) with 2-Split Dominating Set

Proof. Let S_{IV} be a maximal vertex squared independent set in VSIVFG. By the theorem 4.2.1, S_{IV} is a n -SDS. Assume S_{IV} be not a minimal n -split dominating set. Then there exists somewhere around one vertex $x \in S_{IV}$ for which $S_{IV} - \{x\}$ is a n -SDS. Yet, if $S_{IV} - \{x\}$ n -split dominates $V_{IV} - (S_{IV} - \{x\})$ then at least one vertex in $S_{IV} - \{x\}$ must n -split dominate x . This contradicts the way that S_{IV} is a vertex squared independent set of VSIVFG. Hence S_{IV} should be a minimal n -split dominating set.

Example 4.2.4Figure 4.2.4: VSIVFG (G_{IV}) with 2-Split Dominating Set

In the above figure the maximal vertex squared independent sets are $S_1 = \{i_{11}, i_{22}\}$, $S_2 = \{i_{22}, i_{33}\}$, $S_3 = \{i_{33}, i_{44}\}$, $S_4 = \{i_{11}, i_{44}\}$ and 2-SDSs are

$$\begin{aligned}
 D_1 = \{i_{11}, i_{22}\} &= \frac{1 + (0.4)^2 - (0.2)^2}{2} + \frac{1 + (0.5)^2 - (0.3)^2}{2} = 1.2 \\
 D_2 = \{i_{11}, i_{33}\} &= \frac{1 + (0.4)^2 - (0.2)^2}{2} + \frac{1 + (0.4)^2 - (0.2)^2}{2} = 1.2 \\
 D_3 = \{i_{11}, i_{44}\} &= \frac{1 + (0.4)^2 - (0.2)^2}{2} + \frac{1 + (0.5)^2 - (0.3)^2}{2} = 1.2 \\
 D_4 = \{i_{22}, i_{33}\} &= \frac{1 + (0.5)^2 - (0.3)^2}{2} + \frac{1 + (0.4)^2 - (0.2)^2}{2} = 1.2 \\
 D_5 = \{i_{22}, i_{44}\} &= \frac{1 + (0.5)^2 - (0.3)^2}{2} + \frac{1 + (0.5)^2 - (0.3)^2}{2} = 1.2 \\
 D_6 = \{i_{33}, i_{44}\} &= \frac{1 + (0.4)^2 - (0.2)^2}{2} + \frac{1 + (0.5)^2 - (0.3)^2}{2} = 1.2
 \end{aligned}$$

This shows that every maximal vertex squared independent set is a minimal 2-SDS.

Theorem 4.2.3 Let G_{IV} be a vertex squared interval-valued fuzzy graph with n -split dominate edges. If S_{IV} is a minimal n -split dominating set, then $V_{IV} - S_{IV}$ is a n -split dominating set.

Proof. Let S_{IV} be a minimal n -split dominating set of VSIVFG. Assume $V_{IV} - S_{IV}$ is not n -split dominating set. Then there exist a vertex to $x \in S_{IV}$ such an extent that x is not n -split dominated by anyone vertex in $V_{IV} - S_{IV}$. Since G_{IV} has n -split dominate edges, x is a n -split dominate of somewhere around one vertex in $S_{IV} - \{x\}$. Then $S_{IV} - \{x\}$ is a n -split dominating set, which contradicts the minimality of S_{IV} . Subsequently, every vertex in S_{IV} is a n -split dominate of no less than one vertex in $V_{IV} - S_{IV}$. Hence $V_{IV} - S_{IV}$ is a n -split dominating set.

Example 4.2.5

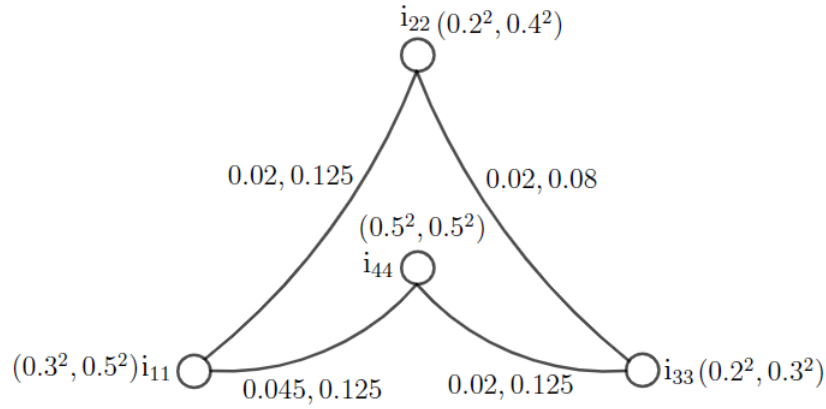


Figure 4.2.5: VSIVFG (G_{IV}) with 2-Split Dominating Set

For $n=2$, the 2-split dominating sets are

$$\begin{aligned}
 D_1 = \{i_{11}, i_{22}\} &= \frac{1 + (0.5)^2 - (0.3)^2}{2} + \frac{1 + (0.4)^2 - (0.2)^2}{2} = 1.2 \\
 D_2 = \{i_{11}, i_{33}\} &= \frac{1 + (0.5)^2 - (0.3)^2}{2} + \frac{1 + (0.3)^2 - (0.2)^2}{2} = 1.1 \\
 D_3 = \{i_{11}, i_{44}\} &= \frac{1 + (0.5)^2 - (0.3)^2}{2} + \frac{1 + (0.5)^2 - (0.5)^2}{2} = 1.1 \\
 D_4 = \{i_{22}, i_{33}\} &= \frac{1 + (0.4)^2 - (0.2)^2}{2} + \frac{1 + (0.3)^2 - (0.2)^2}{2} = 1.1 \\
 D_5 = \{i_{22}, i_{44}\} &= \frac{1 + (0.4)^2 - (0.2)^2}{2} + \frac{1 + (0.5)^2 - (0.5)^2}{2} = 1.1 \\
 D_6 = \{i_{33}, i_{44}\} &= \frac{1 + (0.2)^2 - (0.3)^2}{2} + \frac{1 + (0.5)^2 - (0.5)^2}{2} = 1
 \end{aligned}$$

Then the minimal vertex squared cardinality of a 2-SDS is $\{i_{33}, i_{44}\}$ and $\gamma_{2SPD}(G_{IV}) = 1$. Here $V_{IV} = \{i_{11}, i_{22}, i_{33}, i_{44}\}$, then $V_{IV} - D_6 = \{i_{11}, i_{22}\}$ is also a 2-SDS.

4.3 Application

Many individuals today have been contaminated with the Corona virus infection and now and again passed on. Lamentably, the oxygen cylinder insufficiency in agencies has expanded the demise rate. Hence we have attempted to recognize the lot oxygen cylinder accessible agencies with the assistance of 2-split dominating sets to save both expense and time. For this reason, assume that five unique oxygen cylinder agencies are working in a city for providing oxygen cylinders to the medical clinics.

Consider the oxygen cylinder agencies $A_{11}, A_{22}, A_{33}, A_{44}$ and A_{55} . In VSIVFGs, the vertices show the agencies, and edges show the collaboration of one agency with another agency and non collaboration with one another. The diagram 2-split dominating sets is the arrangement of agencies that produce the oxygen cylinder independently.

The vertex $A_{44}((0.3)^2, (0.4)^2)$ implies that it has 30% of the oxygen cylinder accessible for the agencies and 40% of the oxygen cylinder inaccessible for the agencies. The edge $A_{11}, A_{22}(0.02, 0.125)$ show that there is just 0.02% of the connection between the two agencies, and because of the contest, and 0.125% on the conflict between them.

The 2-split dominating sets for the figure 4.3.1 are

$$\begin{aligned} D_1 &= \{A_{44}\}, D_2 = \{A_{11}, A_{22}\}, D_3 = \{A_{11}, A_{33}\}, D_4 = \{A_{11}, A_{44}\}, D_5 = \{A_{22}, A_{44}\}, \\ D_6 &= \{A_{22}, A_{55}\}, D_7 = \{A_{33}, A_{44}\}, D_8 = \{A_{33}, A_{55}\}, D_9 = \{A_{44}, A_{55}\} \end{aligned}$$

after calculating the vertex squared cardinality of $D_1, D_2, \dots, D_9, \dots$

$$\begin{aligned} D_1 &= 0.535, D_2 = 1.07, D_3 = 1.145, D_4 = 1.08, D_5 = 1.06, \\ D_6 &= 1.085, D_7 = 1.135, D_8 = 1.16, D_9 = 1.095 \end{aligned}$$

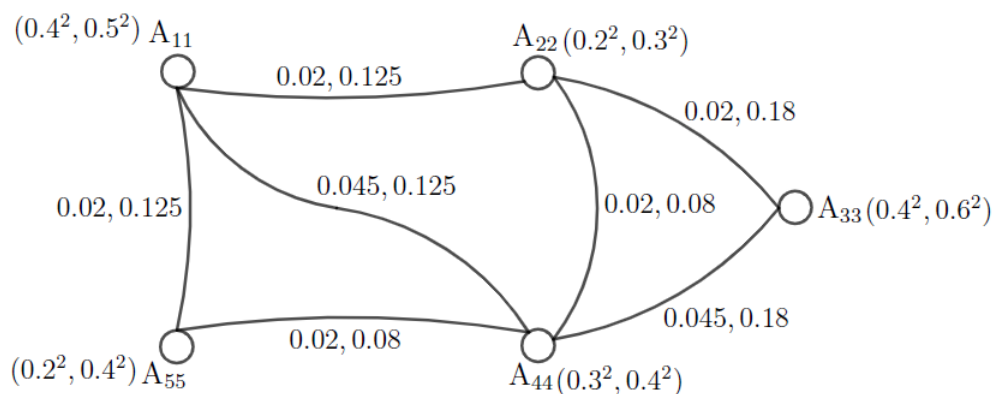


Figure 4.3.1: VSIVFG (G_{IV}) with 2-Split Dominating Set

Obviously D_1 has the base vertex squared cardinality among other 2-split dominating sets, so we presume that it tends to be the most ideal decision since it will set aside time and cash for patients. Along these lines, the excess agencies should expand oxygen cylinder creation, with the goal that patients can be distinguished at the earliest opportunity and don't need to go to various agencies and went through a huge load of cash.

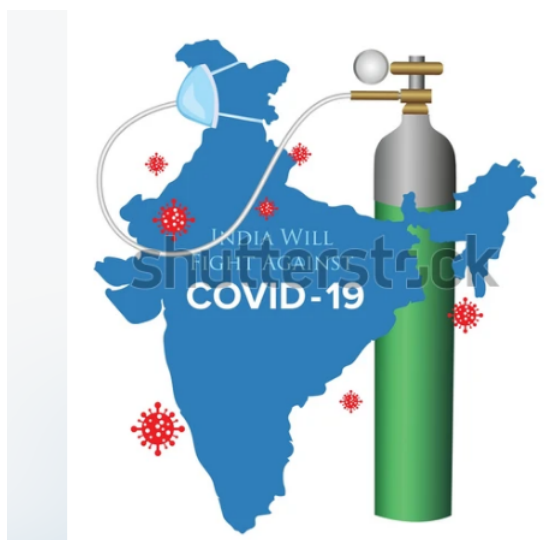


Figure 4.3.2: Oxygen Cylinder

4.4 Motivation and Comparative Analysis

Our examination will be gainful to grasp the further qualities of VSIVFGs exhaustively. We have applied the model on VSIVFGs given in figure 4.4.1 and interval-valued fuzzy graphs gave in figure 4.4.2 and we get the accompanying outcome. In figure 4.4.1 $\gamma_{2SPD} = 0.525$ and figure 4.4.2 $\gamma = 0.55$. Here $\gamma_{2SPD} < \gamma$. Because of this explanation, the current model isn't useful to choose which agencies have the lot of oxygen cylinder. Thusly our proposed strategy is superior to the current technique.

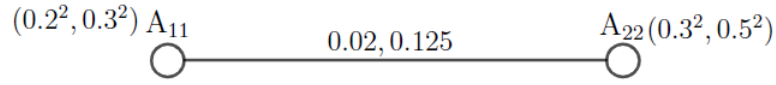


Figure 4.4.1: VSIVFG (G_{IV}) with 2-Split Dominating Set

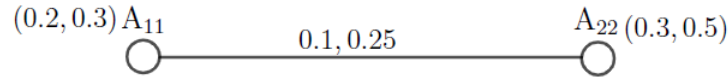


Figure 4.4.2: IVFG with Dominating Set

The dominance theory survey is intriguing because of the wide range of applications and dominant features that can be established. The new thought has been explained in this chapter for vertex squared cardinality, vertex squared effective edge, n -split dominating set, and n -split domination number. Theorems identified with this concept are inferred and the relation between n -split domination set and vertex squared independent set are set up. Vertex squared interval-valued fuzzy graphs are more adaptable and practical than fuzzy graphs because of their many applications in networks. The fuzzy graph hypothesis with vertex squared interval-valued is actively being explored and modified. We trust our investigation will empower us to expand fuzzy graph classes, for example, interval-valued fuzzy incidence graphs and intuitionistic fuzzy incidence graphs.

Chapter 5

Perfect Domination in Product of Interval-Valued Fuzzy Incidence Graphs

Fuzzy graphs, also known as fuzzy incidence graphs, are a useful and well-organized tool for encapsulating and resolving a variety of real-world situations involving ambiguous data and information. In this investigation article, we introduced the chance of interval-valued fuzzy incidence graphs alongside their specific properties. The operations of Cartesian Product (CP), Tensor Product (TP) in interval-valued fuzzy incidence graphs are additionally examined. The technique to compute the degree of interval-valued fuzzy incidence graphs acquired by Cartesian Product and Tensor Product is examined. Some significant hypotheses to figure the degree (DG) of the vertices of interval-valued fuzzy incidence graphs gained by Cartesian Product and Tensor Product are explained. An innovative idea of perfect domination in Cartesian product of two interval-valued fuzzy incidence graphs and Tensor Product of two interval-valued fuzzy incidence graphs utilizing incidence pair are presented and gotten the connection between them. Eventually, genuine utilization of perfect domination number to discover which countries (country) have the best education policies among various countries is inspected.

5.1 Degree of a Vertex in Cartesian Product of Two Interval-Valued Fuzzy Incidence Graphs

Definition 5.1.1 An interval-valued fuzzy incidence graph is of the form $G_{IVI} = (V_{IVI}, E_{IVI}, I_{IVI}, \mu_K, \mu_L, \mu_M)$ where $\mu_K = (\mu_K^-, \mu_K^+)$, $\mu_L = (\mu_L^-, \mu_L^+)$, $\mu_M = (\mu_M^-, \mu_M^+)$ and $V_{IVI} = \{w_0, w_1, \dots, w_n\}$ such that $\mu_K^- : V_{IVI} \rightarrow [0, 1]$ and $\mu_K^+ : V_{IVI} \rightarrow [0, 1]$ represent the degree of membership (MS) and non membership (NMS) of the vertex $w_{ii} \in V_{IVI}$ respectively, and $\mu_K^-(w_{11}) \leq \mu_K^+(w_{11})$, $0 \leq \mu_K^- + \mu_K^+ \leq 1$ for each $w_{ii} \in V_{IVI} (i = 1, 2, \dots, n)$, $\mu_L^- : V_{IVI} \times V_{IVI} \rightarrow [0, 1]$ and $\mu_L^+ : V_{IVI} \times V_{IVI} \rightarrow [0, 1]$, $\mu_L^-(w_{11}, w_{22})$ and $\mu_L^+(w_{11}, w_{22})$ show the degree of membership and non membership of the edge (w_{11}, w_{22}) respectively, such that $\mu_L^-(w_{11}, w_{22}) \leq \min\{\mu_K^-(w_{11}), \mu_K^-(w_{22})\}$ and $\mu_L^+(w_{11}, w_{22}) \leq \max\{\mu_K^+(w_{11}), \mu_K^+(w_{22})\}$, $0 \leq \mu_L^-(w_{11}, w_{22}) + \mu_L^+(w_{11}, w_{22}) \leq 1$ for every (w_{11}, w_{22}) , $\mu_M^- : V_{IVI} \times E_{IVI} \rightarrow [0, 1]$ and $\mu_M^+ : V_{IVI} \times E_{IVI} \rightarrow [0, 1]$ and $\mu_M^-(w_{11}, w_{11}w_{22})$ and $\mu_M^+(w_{11}, w_{11}w_{22})$ show the degree of membership and non membership of the incidence pair respectively, such that $\mu_M^-(w_{11}, w_{11}w_{22}) \leq \min\{\mu_K^-(w_{11}), \mu_L^-(w_{11}, w_{22})\}$ and $\mu_M^+(w_{11}, w_{11}w_{22}) \leq \max\{\mu_K^+(w_{11}), \mu_L^+(w_{11}, w_{22})\}$, $0 \leq \mu_M^-(w_{11}, w_{11}w_{22}) + \mu_M^+(w_{11}, w_{11}w_{22}) \leq 1$ for every $(w_{11}, w_{11}w_{22})$.

Definition 5.1.2 Let $G_{IVI} = (V_{IVI}, E_{IVI}, I_{IVI}, \mu_K, \mu_L, \mu_M)$ is an interval-valued fuzzy incidence graph and $w_{11} \in V_{IVI}$, then its degree is represented by $d_{G_{IVI}}(w_{11}) = (d_{1G_{IVI}}(w_{11}), d_{2G_{IVI}}(w_{11}))$ and defined by

$$d_{1G_{IVI}}(w_{11}) = \sum_{w_{11} \neq w_{22}} (w_{11}, w_{11}w_{22}) \in I_{IVI}$$

$$\text{and } d_{2G_{IVI}}(w_{11}) = \sum_{w_{11} \neq w_{22}} (w_{11}, w_{11}w_{22}) \in I_{IVI}$$

Definition 5.1.3 The Cartesian Product of two interval-valued fuzzy incidence graphs $G_{IVI}^1 = (V_{IVI}^1, E_{IVI}^1, I_{IVI}^1, \mu_K^1, \mu_L^1, \mu_M^1)$ and

$G_{IVI}^2 = (V_{IVI}^2, E_{IVI}^2, I_{IVI}^2, \mu_K^2, \mu_L^2, \mu_M^2)$ is defined as an interval-valued fuzzy incidence graph

$$G_{IVI} = G_{IVI}^1 \times G_{IVI}^2 = (V_{IVI}, E_{IVI}, I_{IVI}, \mu_K^1 \times \mu_K^2, \mu_L^1 \times \mu_L^2, \mu_M^1 \times \mu_M^2)$$

where $V_{IVI} = V_{IVI}^1 \times V_{IVI}^2$, and

$$E_{IVI} = \{((m_1n_1), (m_2, n_2))/m_1 = m_2, (n_1, n_2) \in E_{IVI}^2 \text{ or } n_1 = n_2, (m_1, m_2) \in E_{IVI}^1\}$$

$$I_{IVI} = \{(m_1n_1), (m_1, n_1)(m_1, n_2)/m_1 = m_2, (n_1, n_1n_2) \in I_{IVI}^2, (n_2, n_1n_2) \in I_{IVI}^2 \text{ or}$$

$$n_1 = n_2, (m_1, m_1m_2) \in I_{IVI}^1, (m_2, m_1m_2) \in I_{IVI}^1\} \text{ with}$$

$$\left(\mu_K^{-1} \times \mu_K^{-2}\right)(m_1, n_1) = \min\{\mu_K^{-1}(m_1), \mu_K^{-2}(n_1)\} \forall (m_1, n_1) \in V_{IVI}^1 \times V_{IVI}^2,$$

$$\left(\mu_K^{+1} \times \mu_K^{+2}\right)(m_1, n_1) = \max\{\mu_K^{+1}(m_1), \mu_K^{+2}(n_1)\} \forall (m_1, n_1) \in V_{IVI}^1 \times V_{IVI}^2,$$

$$\left(\mu_L^{-1} \times \mu_L^{-2}\right)((m_1, n_1)(m_2, n_2)) = \begin{cases} \min\{\mu_K^{-1}(m_1), \mu_L^{-2}(n_1, n_2)\} & \text{if } m_1 = m_2, (n_1, n_2) \in E_{IVI}^2 \\ \min\{\mu_L^{-1}(m_1, m_2), \mu_K^{-2}(n_1)\} & \text{if } n_1 = n_2, (m_1, m_2) \in E_{IVI}^1 \end{cases}$$

$$\left(\mu_L^{+1} \times \mu_L^{+2}\right)((m_1, n_1)(m_2, n_2)) = \begin{cases} \max\{\mu_K^{+1}(m_1), \mu_L^{+2}(n_1, n_2)\} & \text{if } m_1 = m_2, (n_1, n_2) \in E_{IVI}^2 \\ \max\{\mu_L^{+1}(m_1, m_2), \mu_K^{+2}(n_1)\} & \text{if } n_1 = n_2, (m_1, m_2) \in E_{IVI}^1 \end{cases}$$

$$\left(\mu_M^{-1} \times \mu_M^{-2}\right)[(m_1, n_1), (m_1, n_1)(m_1, n_2)] = \min\{\mu_K^{-1}(m_1), \mu_M^{-2}(n_1, n_1n_2)\}$$

$$\text{if } m_1 = m_2, (n_1, n_1n_2) \in I_{IVI}^2$$

$$\left(\mu_M^{-1} \times \mu_M^{-2}\right)[(m_1, n_2), (m_1, n_1)(m_1, n_2)] = \min\{\mu_K^{-1}(m_1), \mu_M^{-2}(n_2, n_1n_2)\}$$

$$\text{if } m_1 = m_2, (n_2, n_1n_2) \in I_{IVI}^2$$

$$\left(\mu_M^{-1} \times \mu_M^{-2}\right)[(m_1, n_1), (m_1, n_1)(m_2, n_1)] = \min\{\mu_M^{-1}(m_1, m_1m_2), \mu_K^{-2}(n_1)\}$$

$$\text{if } n_1 = n_2, (m_1, m_1m_2) \in I_{IVI}^1$$

$$\left(\mu_M^{-1} \times \mu_M^{-2}\right)[(m_2, n_1), (m_1, n_1)(m_2, n_1)] = \min\{\mu_M^{-1}(m_2, m_1m_2), \mu_K^{-2}(n_1)\}$$

$$\text{if } n_1 = n_2, (m_2, m_1m_2) \in I_{IVI}^1$$

$$\left(\mu_M^{-1} \times \mu_M^{-2}\right)[(m_1, n_2), (m_1, n_2)(m_2, n_2)] = \min\{\mu_M^{-1}(m_1, m_1m_2), \mu_K^{-2}(n_2)\}$$

$$\text{if } n_1 = n_2, (m_1, m_1m_2) \in I_{IVI}^1$$

$$\left(\mu_M^{-1} \times \mu_M^{-2}\right)[(m_2, n_2), (m_1, n_2)(m_2, n_2)] = \min\{\mu_M^{-1}(m_2, m_1m_2), \mu_K^{-2}(n_2)\}$$

$$\text{if } n_1 = n_2, (m_2, m_1m_2) \in I_{IVI}^1$$

$$\left(\mu_M^{-1} \times \mu_M^{-2}\right)[(m_2, n_1), (m_2, n_1)(m_2, n_2)] = \min\{\mu_M^{-2}(n_1, n_1n_2), \mu_K^{-1}(m_2)\}$$

$$\text{if } m_1 = m_2, (n_1, n_1n_2) \in I_{IVI}^2$$

$$\left(\mu_M^{-1} \times \mu_M^{-2}\right)[(m_2, n_2), (m_2, n_1)(m_2, n_2)] = \min\{\mu_M^{-2}(n_2, n_1n_2), \mu_K^{-1}(m_2)\}$$

$$\text{if } m_1 = m_2, (n_2, n_1n_2) \in I_{IVI}^2$$

$$\left(\mu_M^{+1} \times \mu_M^{+2}\right)[(m_1, n_1), (m_1, n_1)(m_1, n_2)] = \max\{\mu_K^{+1}(m_1), \mu_M^{+2}(n_1, n_1n_2)\}$$

$$\text{if } m_1 = m_2, (n_1, n_1n_2) \in I_{IVI}^2$$

$$\left(\mu_M^{+1} \times \mu_M^{+2}\right)[(m_1, n_2), (m_1, n_1)(m_1, n_2)] = \max\{\mu_K^{+1}(m_1), \mu_M^{+2}(n_2, n_1n_2)\}$$

$$\begin{aligned}
& \text{if } m_1 = m_2, (n_2, n_1 n_2) \in I_{IVI}^2 \\
& \left(\mu_M^{+1} \times \mu_M^{+2} \right) [(m_1, n_1), (m_1, n_1)(m_2, n_1)] = \max\{\mu_M^{+1}(m_1, m_1 m_2), \mu_K^{+2}(n_1)\} \\
& \text{if } n_1 = n_2, (m_1, m_1 m_2) \in I_{IVI}^1 \\
& \left(\mu_M^{+1} \times \mu_M^{+2} \right) [(m_2, n_1), (m_1, n_1)(m_2, n_1)] = \max\{\mu_M^{+1}(m_2, m_1 m_2), \mu_K^{+2}(n_1)\} \\
& \text{if } n_1 = n_2, (m_2, m_1 m_2) \in I_{IVI}^1 \\
& \left(\mu_M^{+1} \times \mu_M^{+2} \right) [(m_1, n_2), (m_1, n_2)(m_2, n_2)] = \max\{\mu_M^{+1}(m_1, m_1 m_2), \mu_K^{+2}(n_2)\} \\
& \text{if } n_1 = n_2, (m_1, m_1 m_2) \in I_{IVI}^1 \\
& \left(\mu_M^{+1} \times \mu_M^{+2} \right) [(m_2, n_2), (m_1, n_2)(m_2, n_2)] = \max\{\mu_M^{+1}(m_2, m_1 m_2), \mu_K^{+2}(n_2)\} \\
& \text{if } n_1 = n_2, (m_2, m_1 m_2) \in I_{IVI}^1 \\
& \left(\mu_M^{+1} \times \mu_M^{+2} \right) [(m_2, n_1), (m_2, n_1)(m_2, n_2)] = \max\{\mu_M^{+2}(n_1, n_1 n_2), \mu_K^{+1}(m_2)\} \\
& \text{if } m_1 = m_2, (n_1, n_1 n_2) \in I_{IVI}^2 \\
& \left(\mu_M^{+1} \times \mu_M^{+2} \right) [(m_2, n_2), (m_2, n_1)(m_2, n_2)] = \max\{\mu_M^{+2}(n_2, n_1 n_2), \mu_K^{+1}(m_2)\} \\
& \text{if } m_1 = m_2, (n_2, n_1 n_2) \in I_{IVI}^2
\end{aligned}$$

Example 5.1.1 Let $G_{IVI}^1 = (V_{IVI}^1, E_{IVI}^1, I_{IVI}^1, \mu_K^1, \mu_L^1, \mu_M^1)$ be a IVFIG

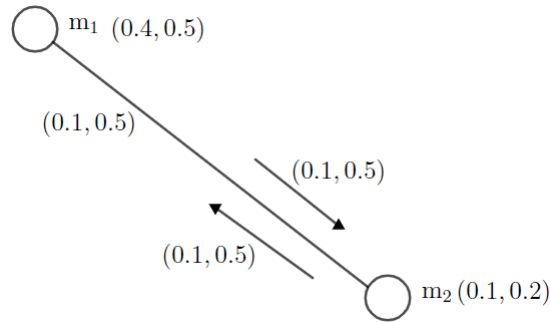
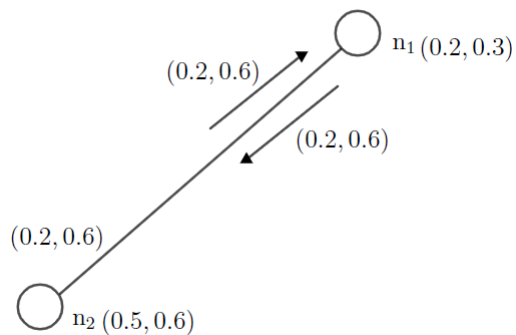


Figure 5.1.1: G_{IVI}^1

Figure 5.1.1 $\mu_K^1(m_1) = (0.4, 0.5)$, $\mu_K^1(m_2) = (0.1, 0.2)$, $\mu_L^1(m_1 m_2) = (0.1, 0.5)$, $\mu_M^1(m_1, m_1 m_2) = (0.1, 0.5)$, $\mu_M^1(m_2, m_1 m_2) = (0.1, 0.5)$ and satisfies the IVFIG conditions.

Let $G_{IVI}^2 = (V_{IVI}^2, E_{IVI}^2, I_{IVI}^2, \mu_K^2, \mu_L^2, \mu_M^2)$ be a IVFIG. Figure 5.1.2 $\mu_K^1(n_1) = (0.2, 0.3)$, $\mu_K^1(n_2) = (0.5, 0.6)$, $\mu_L^1(n_1 n_2) = (0.2, 0.6)$, $\mu_M^1(n_1, n_1 n_2) = (0.2, 0.6)$, $\mu_M^1(n_2, n_1 n_2) = (0.2, 0.6)$ and satisfies the

Figure 5.1.2: G_{IVI}^2

IVFIG conditions.

Let $G_{IVI}^1 \times G_{IVI}^2 = (V_{IVI}, E_{IVI}, I_{IVI}, \mu_K^1 \times \mu_K^2, \mu_L^1 \times \mu_L^2, \mu_M^1 \times \mu_M^2)$ be a CP of two IVFIGs.

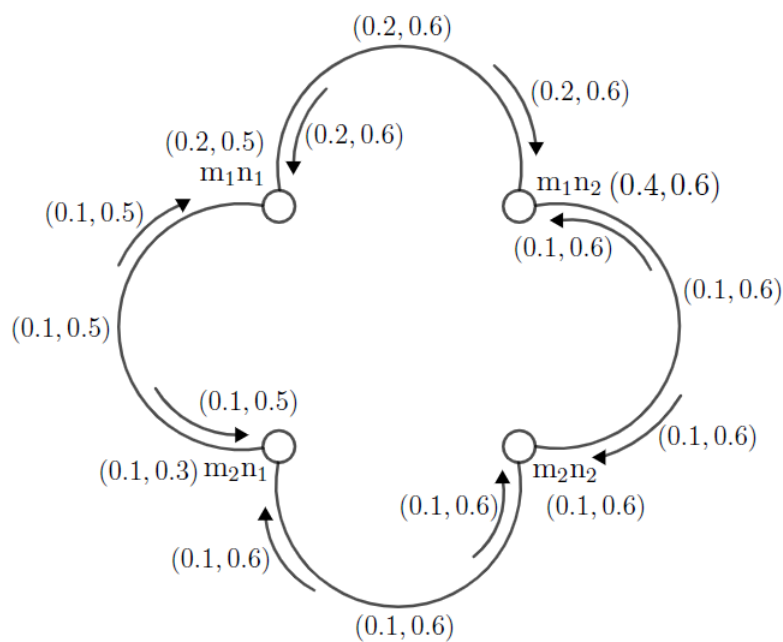
Figure 5.1.3: $G_{IVI}^1 \times G_{IVI}^2$ of Figure 5.1.1 and 5.1.2

Figure 5.1.3,

$$\begin{aligned}
(\mu_K^1 \times \mu_K^2)(m_1 n_1) &= (0.2, 0.5), (\mu_K^1 \times \mu_K^2)(m_1 n_2) = (0.4, 0.6), \\
(\mu_K^1 \times \mu_K^2)(m_2 n_1) &= (0.1, 0.3), (\mu_K^1 \times \mu_K^2)(m_2 n_2) = (0.1, 0.6), \\
(\mu_L^1 \times \mu_L^2)(m_1 n_1, m_1 n_2) &= (0.2, 0.6), (\mu_L^1 \times \mu_L^2)(m_1 n_1, m_2 n_1) = (0.1, 0.5), \\
(\mu_L^1 \times \mu_L^2)(m_1 n_2, m_2 n_2) &= (0.1, 0.6), (\mu_L^1 \times \mu_L^2)(m_2 n_1, m_2 n_2) = (0.1, 0.6), \\
(\mu_M^1 \times \mu_M^2)(m_1 n_1, m_1 n_1 m_1 n_2) &= (0.2, 0.6), (\mu_M^1 \times \mu_M^2)(m_1 n_2, m_1 n_1 m_1 n_2) = (0.2, 0.6), \\
(\mu_M^1 \times \mu_M^2)(m_1 n_2, m_1 n_2 m_2 n_2) &= (0.1, 0.6), (\mu_M^1 \times \mu_M^2)(m_2 n_2, m_1 n_2 m_2 n_2) = (0.1, 0.6), \\
(\mu_M^1 \times \mu_M^2)(m_2 n_1, m_2 n_1 m_2 n_2) &= (0.1, 0.6), (\mu_M^1 \times \mu_M^2)(m_2 n_2, m_2 n_1 m_2 n_2) = (0.1, 0.6), \\
(\mu_M^1 \times \mu_M^2)(m_1 n_1, m_1 n_1 m_2 n_1) &= (0.1, 0.5), (\mu_M^1 \times \mu_M^2)(m_2 n_1, m_1 n_1 m_2 n_1) = (0.1, 0.5).
\end{aligned}$$

Definition 5.1.4 Let $G_{IVI} = G_{IVI}^1 \times G_{IVI}^2 = (V_{IVI}, E_{IVI}, I_{IVI}, \mu_K^1 \times \mu_K^2, \mu_L^1 \times \mu_L^2, \mu_M^1 \times \mu_M^2)$ be the Cartesian Product of two IVFIGs $G_{IVI}^1 = (V_{IVI}^1, E_{IVI}^1, I_{IVI}^1, \mu_K^1, \mu_L^1, \mu_M^1)$ and $G_{IVI}^2 = (V_{IVI}^2, E_{IVI}^2, I_{IVI}^2, \mu_K^2, \mu_L^2, \mu_M^2)$. Then the degree of $V_{IVI} = (m_1, n_1)$ is represented by $d_{G_{IVI}^1 \times G_{IVI}^2}(m_1, n_1) = (d_{1G_{IVI}^1 \times G_{IVI}^2}(m_1, n_1), d_{2G_{IVI}^1 \times G_{IVI}^2}(m_1, n_1))$ and defined by

$$\begin{aligned}
d_{1G_{IVI}^1 \times G_{IVI}^2}(m_1, n_1) &= \sum_{m_1=m_2, (n_1, n_1 n_2) \in I^2} \min\{\mu_K^{-1}(m_1), \mu_M^{-2}(n_1, n_1 n_2)\} \\
&+ \sum_{n_1=n_2, (m_1, m_1 m_2) \in I^1} \min\{\mu_M^{-1}(m_1, m_1 m_2), \mu_K^{-2}(n_1)\} \\
d_{2G_{IVI}^1 \times G_{IVI}^2}(m_1, n_1) &= \sum_{m_1=m_2, (n_1, n_1 n_2) \in I^2} \max\{\mu_K^{+1}(m_1), \mu_M^{+2}(n_1, n_1 n_2)\} \\
&+ \sum_{n_1=n_2, (m_1, m_1 m_2) \in I^1} \max\{\mu_M^{+1}(m_1, m_1 m_2), \mu_K^{+2}(n_1)\}
\end{aligned}$$

Theorem 5.1.1 Let $G_{IVI}^1 = (V_{IVI}^1, E_{IVI}^1, I_{IVI}^1, \mu_K^1, \mu_L^1, \mu_M^1)$ and $G_{IVI}^2 = (V_{IVI}^2, E_{IVI}^2, I_{IVI}^2, \mu_K^2, \mu_L^2, \mu_M^2)$ be two IVFIGs. If $\mu_K^{-1} \leq \mu_K^{+1}$, $\mu_K^{-1} \geq \mu_M^{-2}$, $\mu_K^{+1} \leq \mu_M^{+2}$ and $\mu_K^{-2} \leq \mu_K^{+2}$, $\mu_K^{-2} \geq \mu_M^{-1}$, $\mu_K^{+2} \leq \mu_M^{+1}$ then $d_{G_{IVI}^1 \times G_{IVI}^2}(m_1, n_1) = (d_{G_{IVI}^1}(m_1) + d_{G_{IVI}^2}(n_1))$

Proof. In Cartesian Product by the definition of the degree of a vertex, we have

$$\begin{aligned}
d_{1G_{IVI}^1 \times G_{IVI}^2}(m_1, n_1) &= \sum_{m_1=m_2, (n_1, n_1 n_2) \in I^2} \min\{\mu_K^{-1}(m_1), \mu_M^{-2}(n_1, n_1 n_2)\} \\
&+ \sum_{n_1=n_2, (m_1, m_1 m_2) \in I^1} \min\{\mu_M^{-1}(m_1, m_1 m_2), \mu_K^{-2}(n_1)\} \\
&= \sum_{(n_1, n_1 n_2) \in I^2} \mu_M^{-2}(n_1, n_1 n_2) + \sum_{(m_1, m_1 m_2) \in I^1} \mu_M^{-1}(m_1, m_1 m_2) \\
&\quad \text{since } \mu_K^{-1} \leq \mu_K^{+1}, \mu_K^{-1} \geq \mu_M^{-2}, \mu_K^{-2} \geq \mu_M^{-1} \\
&= \sum_{(m_1, m_1 m_2) \in I^1} \mu_M^{-1}(m_1, m_1 m_2) + \sum_{(n_1, n_1 n_2) \in I^2} \mu_M^{-2}(n_1, n_1 n_2) \\
&= d_{1G_{IVI}^1}(m_1) + d_{1G_{IVI}^2}(n_1) \\
\\
d_{2G_{IVI}^1 \times G_{IVI}^2}(m_1, n_1) &= \sum_{m_1=m_2, (n_1, n_1 n_2) \in I^2} \max\{\mu_K^{+1}(m_1), \mu_M^{+2}(n_1, n_1 n_2)\} \\
&+ \sum_{n_1=n_2, (m_1, m_1 m_2) \in I^1} \max\{\mu_M^{+1}(m_1, m_1 m_2), \mu_K^{+2}(n_1)\} \\
&= \sum_{(n_1, n_1 n_2) \in I^2} \mu_M^{+2}(n_1, n_1 n_2) + \sum_{(m_1, m_1 m_2) \in I^1} \mu_M^{+1}(m_1, m_1 m_2) \\
&\quad \text{since } \mu_K^{-2} \leq \mu_K^{+2}, \mu_K^{+1} \leq \mu_M^{+2}, \mu_K^{+2} \leq \mu_M^{+1} \\
&= \sum_{(m_1, m_1 m_2) \in I^1} \mu_M^{+1}(m_1, m_1 m_2) + \sum_{(n_1, n_1 n_2) \in I^2} \mu_M^{+2}(n_1, n_1 n_2) \\
&= d_{2G_{IVI}^1}(m_1) + d_{2G_{IVI}^2}(n_1) \\
d_{G_{IVI}^1 \times G_{IVI}^2}(m_1, n_1) &= (d_{G_{IVI}^1}(m_1) + d_{G_{IVI}^2}(n_1))
\end{aligned}$$

Therefore the degree of each vertex of Cartesian Product of two IVFIGs is equal to sum of corresponding vertices of G_{IVI}^1 and G_{IVI}^2 .

Example 5.1.2 Let G_{IVI}^1 and G_{IVI}^2 be two IVFIGs as shown in figures 5.1.1 and 5.1.2, and their CP is provided in figure 5.1.3 with

$$\mu_K^{-1} \leq \mu_K^{+1}, \mu_K^{-1} \geq \mu_M^{-2}, \mu_K^{+1} \leq \mu_M^{+2} \text{ and } \mu_K^{-2} \leq \mu_K^{+2}, \mu_K^{-2} \geq \mu_M^{-1}, \mu_K^{+2} \leq \mu_M^{+1}.$$

Then, by theorem 5.1.1, we have $d_{1G_{IVI}^1 \times G_{IVI}^2}(m_1, n_1) = d_{1G_{IVI}^1}(m_1) + d_{1G_{IVI}^2}(n_1) = 0.1 + 0.2 = 0.3$,
 $d_{2G_{IVI}^1 \times G_{IVI}^2}(m_1, n_1) = d_{2G_{IVI}^1}(m_1) + d_{2G_{IVI}^2}(n_1) = 0.5 + 0.6 = 1.1$. So $d_{G_{IVI}^1 \times G_{IVI}^2}(m_1, n_1) = (0.3, 1.1)$

5.2 Degree of a Vertex in Tensor Product of Two Interval-Valued Fuzzy Incidence Graphs

Definition 5.2.1 The Tensor Product of two IVFIGs $G_{IVI}^1 = (V_{IVI}^1, E_{IVI}^1, I_{IVI}^1, \mu_K^1, \mu_L^1, \mu_M^1)$ and $G_{IVI}^2 = (V_{IVI}^2, E_{IVI}^2, I_{IVI}^2, \mu_K^2, \mu_L^2, \mu_M^2)$ is defined as an IVFIG, $G_{IVI} = G_{IVI}^1 \diamond G_{IVI}^2 = (V_{IVI}, E_{IVI}, I_{IVI}, \mu_K^1 \diamond \mu_K^2, \mu_L^1 \diamond \mu_L^2, \mu_M^1 \diamond \mu_M^2)$, where $V_{IVI} = V_{IVI}^1 \times V_{IVI}^2$,

$$E_{IVI} = \{((m_1, n_1), (m_2, n_2)) / (m_1, m_2) \in E_{IVI}^1, (n_1, n_2) \in E_{IVI}^2\}$$

and

$$I_{IVI} = \{(m_1, n_1), (m_1, n_1)(m_1, n_2) / (m_1, m_1 m_2) \in I_{IVI}^1, (m_2, m_1 m_2) \in I_{IVI}^1, \\ (n_1, n_1 n_2) \in I_{IVI}^2, (n_2, n_1 n_2) \in I_{IVI}^2\}$$

$$\text{with } (\mu_K^{-1} \diamond \mu_K^{-2})(m_1, n_1) = \min\{\mu_K^{-1}(m_1), \mu_K^{-2}(n_1)\} \forall (m_1, n_1) \in V_{IVI}^1 \diamond V_{IVI}^2$$

$$(\mu_K^{+1} \diamond \mu_K^{+2})(m_1, n_1) = \max\{\mu_K^{+1}(m_1), \mu_K^{+2}(n_1)\} \forall (m_1, n_1) \in V_{IVI}^1 \diamond V_{IVI}^2$$

$$(\mu_L^{-1} \diamond \mu_L^{-2})((m_1, n_1)(m_2, n_2)) = \min\{\mu_L^{-1}(m_1, m_2), \mu_L^{-2}(n_1, n_2)\} \\ \forall (m_1, m_2) \in E_{IVI}^1, (n_1, n_2) \in E_{IVI}^2$$

$$\begin{aligned} \left(\mu_L^{+1} \diamond \mu_L^{+2} \right) ((m_1, n_1)(m_2, n_2)) &= \max\{\mu_L^{+1}(m_1, m_2), \mu_L^{+2}(n_1, n_2)\} \\ \forall (m_1, m_2) \in E_{IVI}^1, (n_1, n_2) \in E_{IVI}^2 \end{aligned}$$

$$\begin{aligned} \left(\mu_M^{-1} \diamond \mu_M^{-2} \right) [(m_1, n_1), (m_1, n_1)(m_2, n_2)] &= \min\{\mu_M^{-1}(m_1, m_1m_2), \mu_M^{-2}(n_1, n_1n_2)\} \\ \forall (m_1, m_1m_2) \in I_{IVI}^1, (n_1, n_1n_2) \in I_{IVI}^2 \end{aligned}$$

$$\begin{aligned} \left(\mu_M^{-1} \diamond \mu_M^{-2} \right) [(m_2, n_2), (m_1, n_1)(m_2, n_2)] &= \min\{\mu_M^{-1}(m_2, m_1m_2), \mu_M^{-2}(n_2, n_1n_2)\} \\ \forall (m_2, m_1m_2) \in I_{IVI}^1, (n_2, n_1n_2) \in I_{IVI}^2 \end{aligned}$$

$$\begin{aligned} \left(\mu_M^{-1} \diamond \mu_M^{-2} \right) [(m_1, n_2), (m_1, n_2)(m_2, n_1)] &= \min\{\mu_M^{-1}(m_1, m_1m_2), \mu_M^{-2}(n_2, n_1n_2)\} \\ \forall (m_1, m_1m_2) \in I_{IVI}^1, (n_2, n_1n_2) \in I_{IVI}^2 \end{aligned}$$

$$\begin{aligned} \left(\mu_M^{-1} \diamond \mu_M^{-2} \right) [(m_2, n_1), (m_1, n_2)(m_2, n_1)] &= \min\{\mu_M^{-1}(m_2, m_1m_2), \mu_M^{-2}(n_1, n_1n_2)\} \\ \forall (m_2, m_1m_2) \in I_{IVI}^1, (n_1, n_1n_2) \in I_{IVI}^2 \end{aligned}$$

$$\begin{aligned} \left(\mu_M^{+1} \diamond \mu_M^{+2} \right) [(m_1, n_1), (m_1, n_1)(m_2, n_2)] &= \max\{\mu_M^{+1}(m_1, m_1m_2), \mu_M^{+2}(n_1, n_1n_2)\} \\ \forall (m_1, m_1m_2) \in I_{IVI}^1, (n_1, n_1n_2) \in I_{IVI}^2 \end{aligned}$$

$$\begin{aligned} \left(\mu_M^{+1} \diamond \mu_M^{+2} \right) [(m_2, n_2), (m_1, n_1)(m_2, n_2)] &= \max\{\mu_M^{+1}(m_2, m_1m_2), \mu_M^{+2}(n_2, n_1n_2)\} \\ \forall (m_2, m_1m_2) \in I_{IVI}^1, (n_2, n_1n_2) \in I_{IVI}^2 \end{aligned}$$

$$\begin{aligned} \left(\mu_M^{+1} \diamond \mu_M^{+2} \right) [(m_1, n_2), (m_1, n_2)(m_2, n_1)] &= \max\{\mu_M^{+1}(m_1, m_1m_2), \mu_M^{+2}(n_2, n_1n_2)\} \\ \forall (m_1, m_1m_2) \in I_{IVI}^1, (n_2, n_1n_2) \in I_{IVI}^2 \end{aligned}$$

$$\begin{aligned} \left(\mu_M^{+1} \diamond \mu_M^{+2} \right) [(m_2, n_1), (m_1, n_2)(m_2, n_1)] &= \max\{\mu_M^{+1}(m_2, m_1m_2), \mu_M^{+2}(n_1, n_1n_2)\} \\ \forall (m_2, m_1m_2) \in I_{IVI}^1, (n_1, n_1n_2) \in I_{IVI}^2 \end{aligned}$$

Example 5.2.1 Let $G_{IVI}^1 = (V_{IVI}^1, E_{IVI}^1, I_{IVI}^1, \mu_K^1, \mu_L^1, \mu_M^1)$ be a IVFIG.

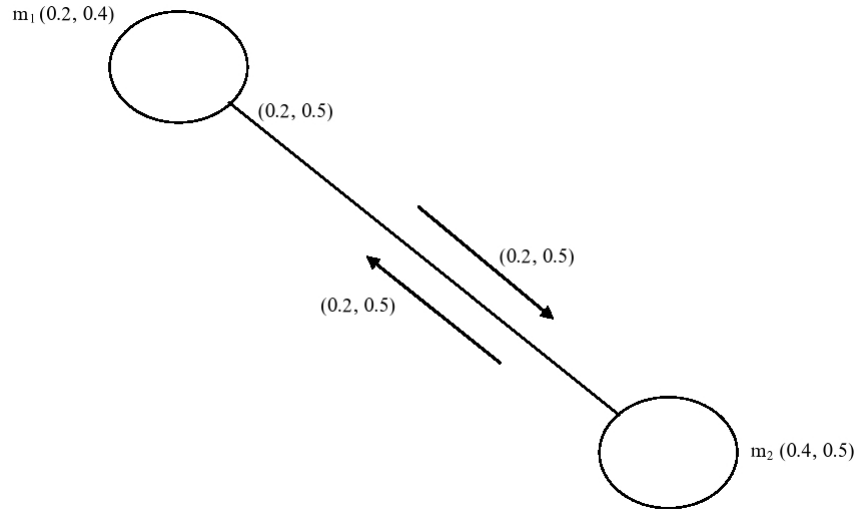
Figure 5.2.1: G_{IVI}^1

Figure 5.2.1, $\mu_K^1(m_1) = (0.2, 0.4)$, $\mu_K^1(m_2) = (0.4, 0.5)$, $\mu_L^1(m_1m_2) = (0.2, 0.5)$, $\mu_M^1(m_1, m_1m_2) = (0.2, 0.5)$, $\mu_M^1(m_2, m_1m_2) = (0.2, 0.5)$ and satisfies the IVFIG conditions.

Let $G_{IVI}^2 = (V_{IVI}^2, E_{IVI}^2, I_{IVI}^2, \mu_K^2, \mu_L^2, \mu_M^2)$ be a IVFIG.

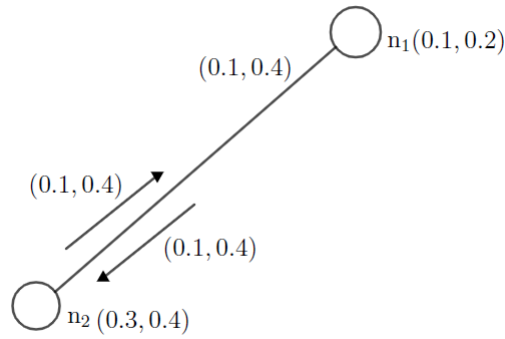
Figure 5.2.2: G_{IVI}^2

Figure 5.2.2, $\mu_K^1(n_1) = (0.1, 0.2)$, $\mu_K^1(n_2) = (0.3, 0.4)$, $\mu_L^1(n_1n_2) = (0.1, 0.4)$, $\mu_M^1(n_1, n_1n_2) = (0.1, 0.4)$, $\mu_M^1(n_2, n_1n_2) = (0.1, 0.4)$ and satisfies the IVFIG conditions.

Let $G_{IVI}^1 \diamond G_{IVI}^2 = (V_{IVI}, E_{IVI}, I_{IVI}, \mu_K^1 \diamond \mu_K^2, \mu_L^1 \diamond \mu_L^2, \mu_M^1 \diamond \mu_M^2)$ be a TP of two IVFIGs.

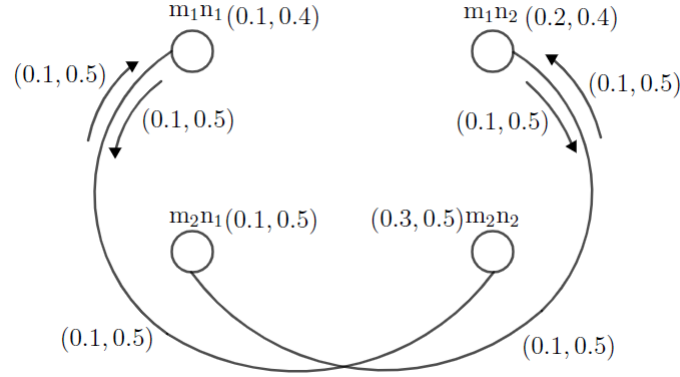
Figure 5.2.3: $G_{IVI}^1 \diamond G_{IVI}^2$ of Figure 5.2.1 and 5.2.2

Figure 5.2.3 indicates a TP of two IVFIGs

$$\begin{aligned}
 (\mu_K^1 \diamond \mu_K^2)(m_1n_1) &= (0.1, 0.4), (\mu_K^1 \diamond \mu_K^2)(m_1n_2) = (0.2, 0.4), \\
 (\mu_K^1 \diamond \mu_K^2)(m_2n_1) &= (0.1, 0.5), (\mu_K^1 \diamond \mu_K^2)(m_2n_2) = (0.3, 0.5), \\
 (\mu_L^1 \diamond \mu_L^2)(m_1n_1, m_2n_2) &= (0.1, 0.5), (\mu_L^1 \diamond \mu_L^2)(m_1n_2, m_2n_1) = (0.1, 0.5), \\
 (\mu_M^1 \diamond \mu_M^2)(m_1n_1, m_1n_1m_2n_2) &= (0.1, 0.5), (\mu_M^1 \diamond \mu_M^2)(m_2n_2, m_1n_1m_2n_2) = (0.1, 0.5), \\
 (\mu_M^1 \diamond \mu_M^2)(m_1n_2, m_1n_2m_2n_1) &= (0.1, 0.5), (\mu_M^1 \diamond \mu_M^2)(m_2n_1, m_1n_2m_2n_1) = (0.1, 0.5)
 \end{aligned}$$

Definition 5.2.2 Let $G_{IVI} = G_{IVI}^1 \diamond G_{IVI}^2 = (V_{IVI}, E_{IVI}, I_{IVI}, \mu_K^1 \diamond \mu_K^2, \mu_L^1 \diamond \mu_L^2, \mu_M^1 \diamond \mu_M^2)$ be the Tensor Product of two IVFIGs $G_{IVI}^1 = (V_{IVI}^1, E_{IVI}^1, I_{IVI}^1, \mu_K^1, \mu_L^1, \mu_M^1)$ and $G_{IVI}^2 = (V_{IVI}^2, E_{IVI}^2, I_{IVI}^2, \mu_K^2, \mu_L^2, \mu_M^2)$. Then the DG of $V_{IVI} = (m_1, n_1)$ is represented by $d_{G_{IVI}^1 \diamond G_{IVI}^2}(m_1, n_1) = (d_{1G_{IVI}^1 \diamond G_{IVI}^2}(m_1, n_1), d_{2G_{IVI}^1 \diamond G_{IVI}^2}(m_1, n_1))$ and defined by

$$\begin{aligned}
 d_{1G_{IVI}^1 \diamond G_{IVI}^2}(m_1, n_1) &= \sum_{(m_1, m_1m_2) \in I^1, (n_1, n_1n_2) \in I^2} \min\{\mu_M^{-1}(m_1, m_1m_2), \mu_M^{-2}(n_1, n_1n_2)\} \\
 d_{2G_{IVI}^1 \diamond G_{IVI}^2}(m_1, n_1) &= \sum_{(m_1, m_1m_2) \in I^1, (n_1, n_1n_2) \in I^2} \max\{\mu_M^{+1}(m_1, m_1m_2), \mu_M^{+2}(n_1, n_1n_2)\}
 \end{aligned}$$

Theorem 5.2.1 Let $G_{IVI}^1 = (V_{IVI}^1, E_{IVI}^1, I_{IVI}^1, \mu_K^1, \mu_L^1, \mu_M^1)$ and $G_{IVI}^2 = (V_{IVI}^2, E_{IVI}^2, I_{IVI}^2, \mu_K^2, \mu_L^2, \mu_M^2)$

be two IVFIGs. If $\mu_K^{-1} \leq \mu_K^{+1}$, $\mu_M^{-2} \geq \mu_M^{-1}$, $\mu_M^{+2} \leq \mu_M^{+1}$, then $d_{G_{IVI}^1 \diamond G_{IVI}^2}(m_1, n_1) = d_{G_{IVI}^1}(m_1)$ and if $\mu_K^{-2} \leq \mu_K^{+2}$, $\mu_M^{-1} \geq \mu_M^{-2}$, $\mu_M^{+1} \leq \mu_M^{+2}$ then $d_{G_{IVI}^1 \diamond G_{IVI}^2}(m_1, n_1) = d_{G_{IVI}^2}(n_1)$.

Proof. Suppose $\mu_K^{-1} \leq \mu_K^{+1}$, $\mu_M^{-2} \geq \mu_M^{-1}$, $\mu_M^{+2} \leq \mu_M^{+1}$, then

$$\begin{aligned} d_{1G_{IVI}^1 \diamond G_{IVI}^2}(m_1, n_1) &= \sum_{(m_1, m_1 m_2) \in I^1, (n_1, n_1 n_2) \in I^2} \min\{\mu_M^{-1}(m_1, m_1 m_2), \mu_M^{-2}(n_1, n_1 n_2)\} \\ &= \sum \mu_M^{-1}(m_1, m_1 m_2) = d_{1G_{IVI}^1}(m_1) \\ d_{2G_{IVI}^1 \diamond G_{IVI}^2}(m_1, n_1) &= \sum_{(m_1, m_1 m_2) \in I^1, (n_1, n_1 n_2) \in I^2} \max\{\mu_M^{+1}(m_1, m_1 m_2), \mu_M^{+2}(n_1, n_1 n_2)\} \\ &= \sum \mu_M^{+1}(m_1, m_1 m_2) = d_{2G_{IVI}^1}(m_1) \end{aligned}$$

This implies $d_{G_{IVI}^1 \diamond G_{IVI}^2}(m_1, n_1) = d_{G_{IVI}^1}(m_1)$. Similarly if $\mu_K^{-2} \leq \mu_K^{+2}$, $\mu_M^{-1} \geq \mu_M^{-2}$, $\mu_M^{+1} \leq \mu_M^{+2}$, then

$$\begin{aligned} d_{1G_{IVI}^1 \diamond G_{IVI}^2}(m_1, n_1) &= \sum_{(m_1, m_1 m_2) \in I^1, (n_1, n_1 n_2) \in I^2} \min\{\mu_M^{-1}(m_1, m_1 m_2), \mu_M^{-2}(n_1, n_1 n_2)\} \\ &= \sum \mu_M^{-2}(n_1, n_1 n_2) = d_{1G_{IVI}^2}(n_1) \\ d_{2G_{IVI}^1 \diamond G_{IVI}^2}(m_1, n_1) &= \sum_{(m_1, m_1 m_2) \in I^1, (n_1, n_1 n_2) \in I^2} \max\{\mu_M^{+1}(m_1, m_1 m_2), \mu_M^{+2}(n_1, n_1 n_2)\} \\ &= \sum \mu_M^{+2}(n_1, n_1 n_2) = d_{2G_{IVI}^2}(n_1) \end{aligned}$$

This implies $d_{G_{IVI}^1 \diamond G_{IVI}^2}(m_1, n_1) = d_{G_{IVI}^2}(n_1)$.

Therefore, if $\mu_K^{-1} \leq \mu_K^{+1}$, $\mu_M^{-2} \geq \mu_M^{-1}$, $\mu_M^{+2} \leq \mu_M^{+1}$, then the degree of each vertex TP of two IVFIGs is equal to corresponding vertex of G_{IVI}^1 and if $\mu_K^{-2} \leq \mu_K^{+2}$, $\mu_M^{-1} \geq \mu_M^{-2}$, $\mu_M^{+1} \leq \mu_M^{+2}$, then the degree of each vertex TP of two IVFIGs is equal to corresponding vertex of G_{IVI}^2 .

Example 5.2.2 In Figure 5.2.1 and 5.2.2 $\mu_K^{-1} \leq \mu_K^{+1}$, $\mu_M^{-2} \geq \mu_M^{-1}$, $\mu_M^{+2} \leq \mu_M^{+1}$ and $\mu_K^{-2} \leq \mu_K^{+2}$, $\mu_M^{-1} \geq \mu_M^{-2}$, $\mu_M^{+1} \leq \mu_M^{+2}$. Then, by theorem 5.2.1, we have

$$\begin{aligned} d_{1G_{IVI}^1 \diamond G_{IVI}^2}(m_1, n_1) &= 0.1 = d_{1G_{IVI}^1}(m_1), \\ d_{2G_{IVI}^1 \diamond G_{IVI}^2}(m_1, n_1) &= 0.5 = d_{2G_{IVI}^2}(n_1). \end{aligned}$$

Hence $d_{G_{IV}^1 \diamond G_{IV}^2}(m_1, n_1) = (0.1, 0.5)$.

5.3 Perfect Domination in Cartesian Product and Tensor Product of Two Interval-Valued Fuzzy Incidence Graphs

Definition 5.3.1 A vertex w_{11} in an G_{IV} dominates to vertex w_{22} if $\mu_L^-(w_{11}, w_{22}) = \min\{\mu_K^-(w_{11}), \mu_K^-(w_{22})\}$ and $\mu_M^+(w_{11}, w_{22}) = \max\{\mu_K^+(w_{11}), \mu_K^+(w_{22})\}$. Then (w_{11}, w_{22}) edge is called dominates edge.

Definition 5.3.2 A subset W_{IV} of V_{IV} is said to be a perfect dominating set (PDS) if for each vertex w_{11} not in W_{IV} , w_{11} is dominates exactly one vertex of W_{IV} .

Definition 5.3.3 A perfect dominating set W_{IV} of the G_{IV} is said to be a minimal perfect dominating set if each vertex w_{11} in W_{IV} , $W_{IV} - \{w_{11}\}$ is not a perfect dominating set.

Definition 5.3.4 A PDS with the lowest vertex cardinality is called a minimum PDS.

Definition 5.3.5 A vertex cardinality of a minimum perfect dominating set is called perfect domination number (PDN) of the G_{IV} . It is denoted by γ_{PIV}

Example 5.3.1

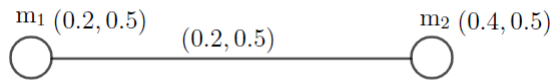


Figure 5.3.1: G_{IV}

Figure 5.3.1 indicates a $G_{IV} = (V_{IV}, E_{IV}, \mu_K, \mu_L)$, $\mu_K(m_1) = (0.2, 0.5)$, $\mu_K(m_2) = (0.4, 0.5)$, $\mu_L(m_1 m_2) = (0.2, 0.5)$. The above figure 5.3.1, the dominates edge is $\{m_1, m_2\}$ and the PDSs are $S_{11} = \{m_1\}$, $S_{22} = \{m_2\}$. After calculating the vertex cardinality of S_{11} and S_{22} , we obtain $|S_{11}| = 0.7$, $|S_{22}| = 0.6$. The vertex cardinality of a minimum PDS is $|S_{22}| = 0.6$ and $\gamma_{PIV} = 0.6$.

Definition 5.3.6 A vertex w_{11} in an $G_{IVI}^1 \times G_{IVI}^2$ (or $G_{IVI}^1 \diamond G_{IVI}^2$) incidentally dominates to vertex w_{22} if $\mu_M^-(w_{11}, w_{11}w_{22}) = \min\{\mu_K^-(w_{11}), \mu_L^-(w_{11}, w_{22})\}$ and $\mu_M^+(w_{11}, w_{11}w_{22}) = \max\{\mu_K^+(w_{11}), \mu_L^+(w_{11}, w_{22})\}$. Then (w_{11}, w_{22}) edge is called incidentally dominates edge.

Definition 5.3.7 A subset W_{IVI} of V_{IVI} is said to be a perfect dominating set if for each vertex w_{11} not in W_{IVI} , w_{11} is incidentally dominates exactly one vertex of W_{IVI} .

Definition 5.3.8 A PDS W_{IVI} of the $G_{IVI}^1 \times G_{IVI}^2$ (or $G_{IVI}^1 \diamond G_{IVI}^2$) is said to be a minimal PDS if each vertex w_{11} in W_{IVI} , $W_{IVI} - \{w_{11}\}$ is not a PDS.

Definition 5.3.9 A perfect dominating set with the lowest vertex cardinality is called a minimum perfect dominating set.

Definition 5.3.10 A vertex cardinality of a minimum perfect dominating set is called perfect dominating number of the $G_{IVI}^1 \times G_{IVI}^2$ (or $G_{IVI}^1 \diamond G_{IVI}^2$). It is denoted by γ_{PIVI} .

Example 5.3.2 In figure 5.1.3, the incidentally dominates edge are $\{m_1n_1, m_1n_2\}$, $\{m_1n_2, m_2n_2\}$, $\{m_2n_2, m_2n_1\}$, $\{m_1n_1, m_2n_1\}$ and the PDSs are $S_{11} = \{m_1n_1, m_1n_2\}$, $S_{22} = \{m_1n_2, m_2n_2\}$, $S_{33} = \{m_2n_2, m_2n_1\}$, $S_{44} = \{m_1n_1, m_2n_1\}$, $S_{55} = \{m_1n_1, m_2n_2\}$, $S_{66} = \{m_1n_2, m_2n_1\}$.

After calculating the vertex cardinality of $S_{11}, S_{22}, \dots, S_{66}$, we obtain $|S_{11}| = 1.3$, $|S_{22}| = 1.4$, $|S_{33}| = 1.4$, $|S_{44}| = 1.3$, $|S_{55}| = 1.5$, $|S_{66}| = 1.2$. The vertex cardinality of a minimum PDS is $|S_{66}| = 1.2$ and $\gamma_{PIVI} = 1.2$.

In figure 5.2.3, the incidentally dominates edge are $\{m_1n_1, m_2n_2\}$, $\{m_1n_2, m_2n_1\}$ and the PDSs are $S_{11} = \{m_1n_1, m_1n_2\}$, $S_{22} = \{m_1n_2, m_2n_2\}$, $S_{33} = \{m_2n_2, m_2n_1\}$, $S_{44} = \{m_1n_1, m_2n_1\}$.

After calculating the vertex cardinality of $S_{11}, \dots, S_{44}$, we obtain $|S_{11}| = 1.3$, $|S_{22}| = 1.2$, $|S_{33}| = 1.3$, $|S_{44}| = 1.4$. The vertex cardinality of a minimum PDS is $|S_{22}| = 1.2$ and $\gamma_{PIVI} = 1.2$.

Theorem 5.3.1 If $G_{IVI}^1 \times G_{IVI}^2$ be a Cartesian Product of two IVFIGs without isolated vertices and W_{IVI} is the minimal perfect dominating set in $G_{IVI}^1 \times G_{IVI}^2$, then $V_{IVI} - W_{IVI}$ is a perfect dominating set.

Proof. Assume W_{IVI} is any minimal PDS of $G_{IVI}^1 \times G_{IVI}^2$ and vertex $w_{11} \in W_{IVI}$ is not incidentally dominated by any vertex in $V_{IVI} - W_{IVI}$. Since $G_{IVI}^1 \times G_{IVI}^2$ has no isolated vertex, w_{11} must incidentally

be dominated by at least one vertex in $W_{IVI} - \{w_{11}\}$, then $W_{IVI} - \{w_{11}\}$ is a PDS, which is a contradiction with the minimality of W_{IVI} . Therefore any vertex in W_{IVI} incidentally dominated by at least one vertex in $V_{IVI} - W_{IVI}$ and so $V_{IVI} - W_{IVI}$ is a PDS.

Example 5.3.3 Let $G_{IVI}^1 \times G_{IVI}^2$ be a CP of two IVFIGs shown in figure 5.1.3 with the incidentally dominates edges are $\{m_1n_1, m_1n_2\}$, $\{m_1n_2, m_2n_2\}$, $\{m_2n_2, m_2n_1\}$, $\{m_1n_1, m_2n_1\}$ and the PDSs are $S_{11} = \{m_1n_1, m_1n_2\}$, $S_{22} = \{m_1n_2, m_2n_2\}$, $S_{33} = \{m_2n_2, m_2n_1\}$, $S_{44} = \{m_1n_1, m_2n_1\}$, $S_{55} = \{m_1n_1, m_2n_2\}$, $S_{66} = \{m_1n_2, m_2n_1\}$. After calculating the vertex cardinality of $S_{11}, S_{22}, \dots, S_{66}$, we obtain $|S_{11}| = 1.3$, $|S_{22}| = 1.4$, $|S_{33}| = 1.4$, $|S_{44}| = 1.3$, $|S_{55}| = 1.5$, $|S_{66}| = 1.2$. The vertex cardinality of a minimum PDS is S_{66} , then $V_{IVI} - S_{66}$ is also a PDS.

Remark 5.3.1 The above theorem is also true for Tensor Product of two interval-valued fuzzy incidence graphs.

Example 5.3.4 Let $G_{IVI}^1 \diamond G_{IVI}^2$ be a TP of two IVFIGs shown in figure 5.2.3 with the incidentally dominates edge are $\{m_1n_1, m_2n_2\}$, $\{m_1n_2, m_2n_1\}$ and the PDSs are $S_{11} = \{m_1n_1, m_1n_2\}$, $S_{22} = \{m_1n_2, m_2n_2\}$, $S_{33} = \{m_2n_2, m_2n_1\}$, $S_{44} = \{m_1n_1, m_2n_1\}$. After calculating the vertex cardinality of $S_{11}, \dots, S_{44}$, we obtain $|S_{11}| = 1.3$, $|S_{22}| = 1.2$, $|S_{33}| = 1.3$, $|S_{44}| = 1.4$. The vertex cardinality of a minimum PDS is S_{22} , then $V_{IVI} - S_{22}$ is also a PDS.

Theorem 5.3.2 For a $G_{IVI}^1 \diamond G_{IVI}^2$ without isolated vertices, then $\gamma_{P_{IVI}} \leq \frac{P_{IVI}}{2}$.

Proof. If W_{IVI} is a minimal PDS of $G_{IVI}^1 \diamond G_{IVI}^2$, then $V_{IVI} - W_{IVI}$ is a PDS. Therefore $P_{IVI} = |V_{IVI}| = |W_{IVI}| + |V_{IVI} - W_{IVI}|$. Thus, at least one of the sets W_{IVI} or $V_{IVI} - W_{IVI}$ has the cardinality equal $\frac{P_{IVI}}{2}$ or less.

Example 5.3.5 (i) Let $G_{IVI}^1 \diamond G_{IVI}^2$ be a TP of two IVFIGs with

$$\begin{aligned} (\mu_K^1 \diamond \mu_K^2)(m_1n_1) &= (0.1, 0.4), (\mu_K^1 \diamond \mu_K^2)(m_1n_2) = (0.2, 0.4), \\ (\mu_K^1 \diamond \mu_K^2)(m_2n_1) &= (0.1, 0.5), (\mu_K^1 \diamond \mu_K^2)(m_2n_2) = (0.3, 0.5), \\ (\mu_L^1 \diamond \mu_L^2)(m_1n_1, m_2n_2) &= (0.1, 0.5), (\mu_L^1 \diamond \mu_L^2)(m_1n_2, m_2n_1) = (0.1, 0.5), \\ (\mu_M^1 \diamond \mu_M^2)(m_1n_1, m_1n_1m_2n_2) &= (0.1, 0.5), (\mu_M^1 \diamond \mu_M^2)(m_2n_2, m_1n_1m_2n_2) = (0.1, 0.5), \\ (\mu_M^1 \diamond \mu_M^2)(m_1n_2, m_1n_2m_2n_1) &= (0.1, 0.5), (\mu_M^1 \diamond \mu_M^2)(m_2n_1, m_1n_2m_2n_1) = (0.1, 0.5), \end{aligned}$$

the incidentally dominates edge are $\{m_1n_1, m_2n_2\}, \{m_1n_2, m_2n_1\}$ and the PDSs are $S_{11} = \{m_1n_1, m_1n_2\}$, $S_{22} = \{m_1n_2, m_2n_2\}$, $S_{33} = \{m_2n_2, m_2n_1\}$, $S_{44} = \{m_1n_1, m_2n_1\}$. After calculating the vertex cardinality of $S_{11}, \dots, S_{44}$, we obtain $|S_{11}| = 1.3$, $|S_{22}| = 1.2$, $|S_{33}| = 1.3$, $|S_{44}| = 1.4$. The vertex cardinality of a minimum PDS is S_{22} with $\gamma_{P_{IVI}} = 1.2$ and vertex cardinality (P_{IVI}) of $G_{IVI}^1 \diamond G_{IVI}^2$ is 5.2, then $\gamma_{P_{IVI}} \leq \frac{P_{IVI}}{2}$ that is $1.2 < 2.6$.

(ii) Let $G_{IVI}^1 \diamond G_{IVI}^2$ be a TP of two IVFIGs with

$$\begin{aligned} (\mu_K^1 \diamond \mu_K^2)(m_1n_1) &= (0.1, 0.5), (\mu_K^1 \diamond \mu_K^2)(m_1n_2) = (0.4, 0.7), \\ (\mu_K^1 \diamond \mu_K^2)(m_2n_1) &= (0.1, 0.4), (\mu_K^1 \diamond \mu_K^2)(m_2n_2) = (0.3, 0.7), \\ (\mu_L^1 \diamond \mu_L^2)(m_1n_1, m_2n_2) &= (0.1, 0.7), (\mu_L^1 \diamond \mu_L^2)(m_1n_2, m_2n_1) = (0.1, 0.7), \\ (\mu_M^1 \diamond \mu_M^2)(m_1n_1, m_1n_1m_2n_2) &= (0.1, 0.7), (\mu_M^1 \diamond \mu_M^2)(m_2n_2, m_1n_1m_2n_2) = (0.1, 0.7), \\ (\mu_M^1 \diamond \mu_M^2)(m_1n_2, m_1n_2m_2n_1) &= (0.1, 0.7), (\mu_M^1 \diamond \mu_M^2)(m_2n_1, m_1n_2m_2n_1) = (0.1, 0.7), \end{aligned}$$

the incidentally dominates edge are $\{m_1n_1, m_2n_2\}, \{m_1n_2, m_2n_1\}$ and the PDSs are $S_{11} = \{m_1n_1, m_1n_2\}$, $S_{22} = \{m_1n_2, m_2n_2\}$, $S_{33} = \{m_2n_2, m_2n_1\}$, $S_{44} = \{m_1n_1, m_2n_1\}$. After calculating the vertex cardinality of $S_{11}, \dots, S_{44}$, we obtain $|S_{11}| = 1.4$, $|S_{22}| = 1.4$, $|S_{33}| = 1.4$, $|S_{44}| = 1.4$. Here all vertex cardinality of PDS is equal with $\gamma_{P_{IVI}} = 1.4$ and vertex cardinality (P_{IVI}) of $G_{IVI}^1 \diamond G_{IVI}^2$ is 2.8, then $\gamma_{P_{IVI}} = \frac{P_{IVI}}{2}$ that is $1.4 = 1.4$.

Theorem 5.3.3 Let $G_{IVI}^1 \times G_{IVI}^2$ be a Cartesian Product of two IVFIGs and if anyone G_{IVI}^1 or G_{IVI}^2 must having incidentally dominates edges, then the Cartesian Product of two IVFIGs contains $\gamma_{P_{IVI}}$.

Proof. Let $G_{IVI}^1 \times G_{IVI}^2$ be a CP of two IVFIGs. If anyone G_{IVI}^1 or G_{IVI}^2 must having incidentally dominated edges, then the CP of two IVFIG contains $\gamma_{P_{IVI}}$.

Conversely, suppose that the CP of two IVFIG contains $\gamma_{P_{IVI}}$. To prove that anyone G_{IVI}^1 or G_{IVI}^2 must have incidentally dominates edges. If possible G_{IVI}^1 or G_{IVI}^2 does not have incidentally dominates edges, then $G_{IVI}^1 \times G_{IVI}^2$ dose not having $\gamma_{P_{IVI}}$, which is a contradiction. Hence anyone G_{IVI}^1 or G_{IVI}^2 must having incidentally dominates edges.

Example 5.3.6 Let G_{IVI}^1 be a IVFIG with $\mu_K^1(m_1) = (0.4, 0.5)$, $\mu_K^1(m_2) = (0.2, 0.3)$, $\mu_L^1(m_1, m_2) = (0.2, 0.5)$, $\mu_M^1(m_1, m_1m_2) = (0.2, 0.5)$, $\mu_M^1(m_2, m_1m_2) = (0.2, 0.5)$ and let G_{IVI}^2 be a IVFIG with $\mu_K^1(n_1) = (0.2, 0.3)$, $\mu_K^1(n_2) = (0.5, 0.6)$, $\mu_L^1(n_1n_2) = (0.2, 0.6)$, $\mu_M^1(n_1, n_1n_2) = (0.1, 0.4)$, $\mu_M^1(n_2, n_1n_2) = (0.1, 0.3)$. Here G_{IVI}^1 having incidentally dominates edge, but G_{IVI}^2 does not have an incidentally dominates edge.

Assume $G_{IVI}^1 \times G_{IVI}^2$ is a CP of two IVFIGs with

$$\begin{aligned} (\mu_K^1 \times \mu_K^2)(m_1n_1) &= (0.2, 0.5), (\mu_K^1 \times \mu_K^2)(m_1n_2) = (0.4, 0.6), \\ (\mu_K^1 \times \mu_K^2)(m_2n_1) &= (0.2, 0.3), (\mu_K^1 \times \mu_K^2)(m_2n_2) = (0.2, 0.6), \\ (\mu_L^1 \times \mu_L^2)(m_1n_1, m_1n_2) &= (0.2, 0.6), (\mu_L^1 \times \mu_L^2)(m_1n_1, m_2n_1) = (0.2, 0.5), \\ (\mu_L^1 \times \mu_L^2)(m_1n_2, m_2n_2) &= (0.2, 0.6), (\mu_L^1 \times \mu_L^2)(m_2n_1, m_2n_2) = (0.2, 0.6), \\ (\mu_M^1 \times \mu_M^2)(m_1n_1, m_1n_1m_1n_2) &= (0.1, 0.5), (\mu_M^1 \times \mu_M^2)(m_1n_2, m_1n_1m_1n_2) = (0.1, 0.5) \\ (\mu_M^1 \times \mu_M^2)(m_1n_2, m_1n_2m_2n_2) &= (0.2, 0.6), (\mu_M^1 \times \mu_M^2)(m_2n_2, m_1n_2m_2n_2) = (0.2, 0.6), \\ (\mu_M^1 \times \mu_M^2)(m_2n_1, m_2n_1m_2n_2) &= (0.2, 0.6), (\mu_M^1 \times \mu_M^2)(m_2n_2, m_2n_1m_2n_2) = (0.1, 0.3), \\ (\mu_M^1 \times \mu_M^2)(m_1n_1, m_1n_1m_2n_1) &= (0.2, 0.5), (\mu_M^1 \times \mu_M^2)(m_2n_1, m_1n_1m_2n_1) = (0.2, 0.5). \end{aligned}$$

Here the incidentally dominates edges are $\{m_1n_2, m_2n_2\}$, $\{m_1n_1, m_2n_1\}$ and the PDSs are $S_{11} = \{m_1n_1, m_1n_2\}$, $S_{22} = \{m_2n_2, m_2n_1\}$, $S_{33} = \{m_1n_1, m_2n_2\}$, $S_{44} = \{m_1n_2, m_2n_1\}$.

After calculating the vertex cardinality of $S_{11}, \dots, S_{44}$, we obtain $|S_{11}| = 1.3$, $|S_{22}| = 1.3$, $|S_{33}| = 1.4$, $|S_{44}| = 1.2$. The vertex cardinality of a minimum PDS is S_{44} and $\gamma_{PIVI} = 1.2$. Therefore $G_{IVI}^1 \times G_{IVI}^2$ contains γ_{PIVI} .

Theorem 5.3.4 Let $G_{IVI}^1 \diamond G_{IVI}^2$ be a Tensor Product of two IVFIGs and if G_{IVI}^1 and G_{IVI}^2 both having incidentally dominates edges, then the Tensor Product of two IVFIGs contains γ_{PIVI} .

Proof. Let $G_{IVI}^1 \diamond G_{IVI}^2$ be a TP of two IVFIGs. If G_{IVI}^1 and G_{IVI}^2 both having incidentally dominates edges, then the TP of two IVFIGs contains γ_{PIVI} .

Conversely, suppose that the TP of two IVFIGs contains γ_{PIVI} . To prove that G_{IVI}^1 and G_{IVI}^2 both having incidentally dominates edges. If possible G_{IVI}^1 does not having incidentally dominant edges, then

the TP of two IVFIGs does not contains γ_{PIVI} , which is a contradiction. Hence G_{IVI}^1 and G_{IVI}^2 must having incidentally dominated edges.

Example 5.3.7 In figure 5.2.1 and 5.2.2 is a interval-valued fuzzy incidence graphs with incidentally dominated edges and figure 5.2.3 contains PDSs are $S_{11} = \{m_1n_1, m_1n_2\}$, $S_{22} = \{m_1n_2, m_2n_2\}$, $S_{33} = \{m_2n_2, m_2n_1\}$, $S_{44} = \{m_1n_1, m_2n_1\}$. After calculating the vertex cardinality of $S_{11}, \dots, S_{44}$, we obtain $|S_{11}| = 1.3$, $|S_{22}| = 1.2$, $|S_{33}| = 1.3$, $|S_{44}| = 1.4$. The vertex cardinality of a minimum PDS is $|S_{22}| = 1.2$ and $\gamma_{PIVI} = 1.2$. Therefore $G_{IVI}^1 \diamond G_{IVI}^2$ contains γ_{PIVI} .

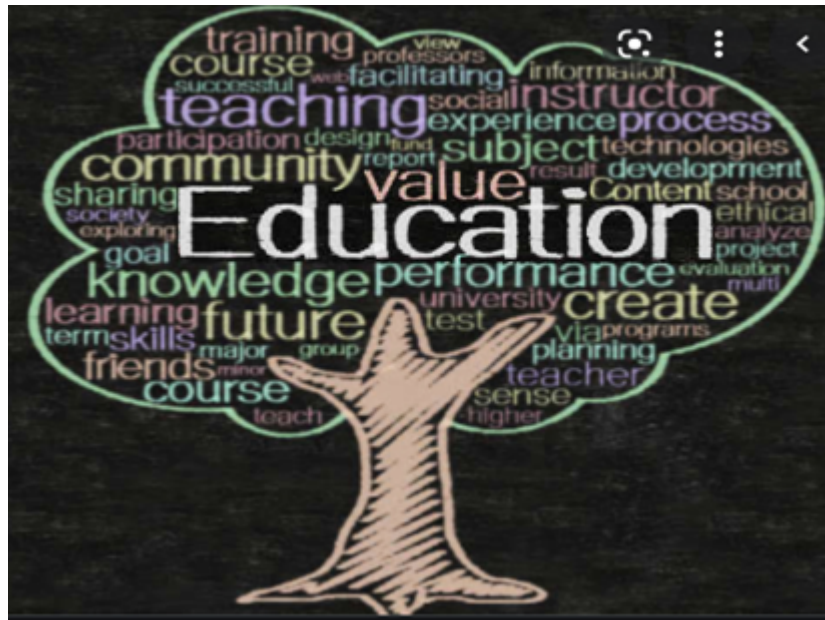
5.4 Application

Here we, incorporate a genuine use of perfect domination number in a matter of education policies among various countries. As an outline case, consider an network $G_{IVI}^1 \times G_{IVI}^2$ of four vertices addressing four distinct countries $C_1(m_1n_1)$, $C_2(m_1n_2)$, $C_3(m_2n_2)$ and $C_4(m_2n_1)$ as displayed in figure 5.1.3. The MS value of the vertices shows the percentage of people who are educated and the NMS value of the vertices demonstrates the percentage of those people who are uneducated. The MS value of the edges communicates the cooperation of one country with another country to enhance the percentage of educated people and the NMS value indicates the non cooperation with one another. The MS value of the incidence pair means the education policies among these countries and the NMS value of the incidence pair indicates the un education policies among these countries. With the assistance of the perfect domination number, we will want to discover which country (countries) have the best education policies.

In figure 5.1.3, the PDSs are $S_{11} = \{C_1, C_2\}$, $S_{22} = \{C_2, C_3\}$, $S_{33} = \{C_3, C_4\}$, $S_{44} = \{C_1, C_4\}$, $S_{55} = \{C_1, C_3\}$, $S_{66} = \{C_2, C_4\}$.

After calculating the vertex cardinality of $S_{11}, S_{22}, \dots, S_{66}$, we obtain $|S_{11}| = 1.3$, $|S_{22}| = 1.4$, $|S_{33}| = 1.4$, $|S_{44}| = 1.3$, $|S_{55}| = 1.5$, $|S_{66}| = 1.2$. The vertex cardinality of a minimum PDS is $|S_{66}| = 1.2$ and $\gamma_{PIVI} = 1.2$.

It is obvious that S_{66} has the minimum perfect dominating set between other perfect dominating sets, hence we conclude that C_2 and C_4 countries have best education policies among all other countries.



5.5 Comparative Analysis

In figure 5.1.3 a $G_{IVI}^1 \times G_{IVI}^2$ indicating four different countries C_1, C_2, C_3 and C_4 and we get minimum PDS $S_{66} = \{C_2, C_4\}$ with $\gamma_{PIVI} = 1.2$. But in figure 5.1.3 if we remove all the incidence pairs we get interval valued fuzzy graph (IVFG). In the case of IVFG, we find the all PDSs. All possible PDSs of the IVFG are $W_{11} = \{C_1, C_2\}$, $W_{22} = \{C_2, C_3\}$, $W_{33} = \{C_3, C_4\}$, $W_{44} = \{C_1, C_4\}$, $W_{55} = \{C_1, C_3\}$, $W_{66} = \{C_2, C_4\}$ with vertex cardinality $|W_{11}| = 1.3$, $|W_{22}| = 1.4$, $|W_{33}| = 1.4$, $|W_{44}| = 1.3$, $|W_{55}| = 1.5$, $|W_{66}| = 1.2$. The vertex cardinality of a minimum PDS is $|W_{66}| = 1.2$ with $\gamma_{PIV} = 1.2$. By applying the model on the $G_{IVI}^1 \diamond G_{IVI}^2$ given in figure 5.2.3, we get minimum PDS $S_{22} = \{C_2, C_3\}$ with $\gamma_{PIVI} = 1.2$. But in figure 5.2.3 if we remove all the incidence pairs we get IVFG. In the case of IVFG, we find the all PDSs. All possible PDSs of the IVFG are $M_{11} = \{C_1, C_2\}$, $M_{22} = \{C_2, C_3\}$, $M_{33} = \{C_3, C_4\}$, $M_{44} = \{C_1, C_4\}$ with vertex cardinality $|M_{11}| = 1.3$, $|M_{22}| = 1.2$, $|M_{33}| = 1.3$, $|M_{44}| = 1.3$. The vertex cardinality of a minimum PDS is $|M_{22}| = 1.2$ with $\gamma_{PIV} = 1.2$. Here $G_{IVI}^1 \times G_{IVI}^2$ and $G_{IVI}^1 \diamond G_{IVI}^2$ both the models $\gamma_{PIV} = \gamma_{PIVI}$, however, on account of IVFG, we can't discuss best education policies because of the non-accessibility of incidence pairs. IVFGs can show the relationship among various countries yet quiet to discuss education policies among various countries. In this way, IVFIGs are more advantageous

and compelling IVFGs.

In this exploration chapter, Cartesian Product and Tensor Product in IVFIGs are presented and we inspected the degree of the vertices of the IVFIGs $G_{IVI}^1 \times G_{IVI}^2$ and $G_{IVI}^1 \diamond G_{IVI}^2$ under specific agreements and showed them with different models. We additionally settled some new outcomes on the degree of a vertex as far as hypotheses. The idea of perfect domination in IVFIGs utilizing incidence pairs is additionally considered. The perfect domination number of IVFIGs is determined. It is also possible to use perfect domination number in the context of education policies in different countries.

Chapter 6

Strong And Weak Domination in Vague Fuzzy Incidence Graphs

Fuzzy graphs also known as fuzzy incidence graphs, are a well-organized and useful tool for capturing and resolving a range of real-world scenarios involving ambiguous data and information. In this chapter, we define the composition of two vague fuzzy incidence graphs and use incidence pairs to extend the idea of fuzzy graph dominance to composition of two vague fuzzy incidence graphs. Examples are used to clarify the concepts of edge incidentally dominating set, strong edge incidentally dominating set, and weak edge incidentally dominating set. CT-VFIGs have an edge incidentally domination number, a strong edge incidentally domination number, and a weak edge incidentally domination number. In the research field, CT-VIFGs are used to find the finest consolidations of journal publications that express the most progress and the least amount of non - progress. The results of our investigation are compared to those of other studies. Our research will help us fully appreciate and comprehend the additional properties of CT-VFIGs. Another benefit of our research is that it will aid in determining the maximum percentage of progress and the minimum percentage of non- progress in various journal publications.

6.1 Composition of Two Vague Fuzzy Incidence Graphs

Definition 6.1.1 *The composition of two VFIGs (CT-VFIGs) $G_{VI}^1 = (V_{VI}^1, E_{VI}^1, I_{VI}^1, A_{VIP}^1, B_{VIL}^1, C_{VII}^1)$ and $G_{VI}^2 = (V_{VI}^2, E_{VI}^2, I_{VI}^2, A_{VIP}^2, B_{VIL}^2, C_{VII}^2)$ is defined as an vague fuzzy incidence graph (VFIG)*

$G_{CVI} = G_{VI}^1 \odot G_{VI}^2 = (V_{VI}, E_{VI}, I_{VI}, A_{VIP}^1 \odot A_{VIP}^2, B_{VIL}^1 \odot B_{VIL}^2, C_{VII}^1 \odot C_{VII}^2)$ where $V_{CVI} = V_{VI}^1 \odot V_{VI}^2$ and $E_{CVI} = \{((m_{11}, n_{11}), (m_{22}, n_{22}))/m_{11} = m_{22}, (n_{11}, n_{22}) \in E_{VI}^2 \text{ or } n_{11} = n_{22}, (m_{11}, m_{22}) \in E_{VI}^1\}$

$I_{CVI} = \{(m_{11}, n_{11}), (m_{11}, n_{11})(m_{11}, n_{22})/m_{11} = m_{22}, (n_{11}, n_{11}n_{22}) \in I_{VI}^2, (n_{22}, n_{11}n_{22}) \in I_{VI}^2 \text{ or } n_{11} = n_{22}, (m_{11}, m_{11}m_{22}) \in I_{VI}^1, (m_{22}, m_{11}m_{22}) \in I_{VI}^1\}$ with

$$(A_{VIP}^1 \odot A_{VIP}^2)(m_{11}, n_{11}) = \min\{A_{VIP}^1(m_{11}), A_{VIP}^2(n_{11})\} \forall (m_{11}, n_{11}) \in V_{VI}^1 \odot V_{VI}^2,$$

$$(A_{VIP}^1 \odot A_{VIP}^2)(m_{11}, n_{11}) = \max\{A_{VIP}^1(m_{11}), A_{VIP}^2(n_{11})\} \forall (m_{11}, n_{11}) \in V_{VI}^1 \odot V_{VI}^2$$

$$(B_{VIL}^1 \odot B_{VIL}^2)((m_{11}, n_{11})(m_{22}, n_{22}))$$

$$= \begin{cases} \min\{A_{VIP}^1(m_{11}), B_{VIL}^2(n_{11}, n_{22})\}, & \text{if } m_{11} = m_{22}, (n_{11}, n_{22}) \in E_{VI}^2 \\ \min\{B_{VIL}^1(m_{11}, m_{22}), A_{VIP}^2(n_{11})\}, & \text{if } n_{11} = n_{22}, (m_{11}, m_{22}) \in E_{VI}^1 \\ \min\{B_{VIL}^1(m_{11}, m_{22}), A_{VIP}^1(n_{11}), A_{VIP}^2(n_{22})\}, & \text{if } n_{11} \neq n_{22}, (m_{11}, m_{22}) \in E_{VI}^1 \end{cases}$$

$$(B_{VIL}^1 \odot B_{VIL}^2)((m_{11}, n_{11})(m_{22}, n_{22}))$$

$$= \begin{cases} \max\{A_{VIP}^1(m_{11}), B_{VIL}^2(n_{11}, n_{22})\}, & \text{if } m_{11} = m_{22}, (n_{11}, n_{22}) \in E_{VI}^2 \\ \max\{B_{VIL}^1(m_{11}, m_{22}), A_{VIP}^2(n_{11})\}, & \text{if } n_{11} = n_{22}, (m_{11}, m_{22}) \in E_{VI}^1 \\ \max\{B_{VIL}^1(m_{11}, m_{22}), A_{VIP}^1(n_{11}), A_{VIP}^2(n_{22})\}, & \text{if } n_{11} \neq n_{22}, (m_{11}, m_{22}) \in E_{VI}^1 \end{cases}$$

$$(C_{VII}^1 \odot C_{VII}^2)[(m_{11}, n_{11}), (m_{11}, n_{11})(m_{11}, n_{22})] = \min\{A_{VIP}^1(m_{11}), C_{VII}^2(n_{11}, n_{11}n_{22})\}$$

$$\text{if } m_{11} = m_{22}, (n_{11}, n_{11}n_{22}) \in I_{VI}^2$$

$$(C_{VII}^1 \odot C_{VII}^2)[(m_{11}, n_{22}), (m_{11}, n_{11})(m_{11}, n_{22})] = \min\{A_{VIP}^1(m_{11}), C_{VII}^2(n_{22}, n_{11}n_{22})\}$$

$$\text{if } m_{11} = m_{22}, (n_{22}, n_{11}n_{22}) \in I_{VI}^2$$

$$(C_{VII}^1 \odot C_{VII}^2)[(m_{11}, n_{11}), (m_{11}, n_{11})(m_{22}, n_{11})] = \min\{C_{VII}^1(m_{11}, m_{11}m_{22}), A_{VIP}^2(n_{11})\}$$

$$\text{if } n_{11} = n_{22}, (m_{11}, m_{11}m_{22}) \in I_{VI}^1$$

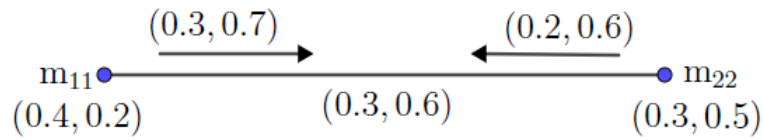
$$(C_{VII}^1 \odot C_{VII}^2)[(m_{22}, n_{11}), (m_{11}, n_{11})(m_{22}, n_{11})] = \min\{C_{VII}^1(m_{22}, m_{11}m_{22}), A_{VIP}^2(n_{11})\}$$

$$\text{if } n_{11} = n_{22}, (m_{22}, m_{11}m_{22}) \in I_{VI}^1$$

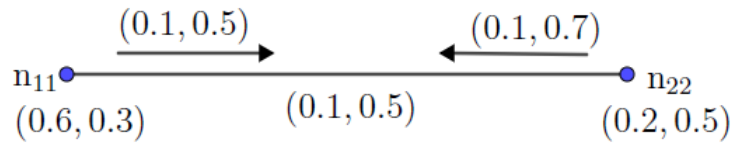
$$\begin{aligned}
& (C_{1VII}^1 \odot C_{1VII}^2)[(m_{11}, n_{11}), (m_{11}, n_{11})(m_{22}, n_{22})] = \min\{C_{1VII}^1(m_{11}, m_{11}m_{22}), A_{1VIP}^2(n_{11}), A_{1VIP}^2(n_{22})\}, \\
& \text{if } m_{11} \neq m_{22}, n_{11} \neq n_{22}, (m_{11}, m_{11}m_{22}) \in I_{VI}^1 \\
& (C_{1VII}^1 \odot C_{1VII}^2)[(m_{22}, n_{22}), (m_{11}, n_{11})(m_{22}, n_{22})] = \min\{C_{1VII}^1(m_{22}, m_{11}m_{22}), A_{1VIP}^2(n_{11}), A_{1VIP}^2(n_{22})\}, \\
& \text{if } m_{11} \neq m_{22}, n_{11} \neq n_{22}, (m_{22}, m_{11}m_{22}) \in I_{VI}^1 \\
& (C_{1VII}^1 \odot C_{1VII}^2)[(m_{11}, n_{22}), (m_{11}, n_{22})(m_{22}, n_{11})] = \min\{C_{1VII}^1(m_{11}, m_{11}m_{22}), A_{1VIP}^2(n_{11}), A_{1VIP}^2(n_{22})\}, \\
& \text{if } m_{11} \neq m_{22}, n_{11} \neq n_{22}, (m_{11}, m_{11}m_{22}) \in I_{VI}^1 \\
& (C_{1VII}^1 \odot C_{1VII}^2)[(m_{22}, n_{11}), (m_{11}, n_{22})(m_{22}, n_{11})] = \min\{C_{1VII}^1(m_{22}, m_{11}m_{22}), A_{1VIP}^2(n_{11}), A_{1VIP}^2(n_{22})\}, \\
& \text{if } m_{11} \neq m_{22}, n_{11} \neq n_{22}, (m_{22}, m_{11}m_{22}) \in I_{VI}^1 \\
& (C_{2VII}^1 \odot C_{2VII}^2)[(m_{11}, n_{11}), (m_{11}, n_{11})(m_{11}, n_{22})] = \max\{A_{2VIP}^2(m_{11}), C_{2VII}^2(n_{11}, n_{11}n_{22})\}, \\
& \text{if } m_{11} = m_{22}, (n_{11}, n_{11}n_{22}) \in I_{VI}^2 \\
& (C_{2VII}^1 \odot C_{2VII}^2)[(m_{11}, n_{22}), (m_{11}, n_{11})(m_{11}, n_{22})] = \max\{A_{2VIP}^2(m_{11}), C_{2VII}^2(n_{22}, n_{11}n_{22})\}, \\
& \text{if } m_{11} = m_{22}, (n_{22}, n_{11}n_{22}) \in I_{VI}^2 \\
& (C_{2VII}^1 \odot C_{2VII}^2)[(m_{11}, n_{11}), (m_{11}, n_{11})(m_{22}, n_{11})] = \max\{C_{2VII}^1(m_{11}, m_{11}m_{22}), A_{2VIP}^2(n_{11})\}, \\
& \text{if } n_{11} = n_{22}, (m_{11}, m_{11}m_{22}) \in I_{VI}^1 \\
& (C_{2VII}^1 \odot C_{2VII}^2)[(m_{22}, n_{11}), (m_{11}, n_{11})(m_{22}, n_{11})] = \max\{C_{2VII}^1(m_{22}, m_{11}m_{22}), A_{2VIP}^2(n_{11})\}, \\
& \text{if } n_{11} = n_{22}, (m_{22}, m_{11}m_{22}) \in I_{VI}^1 \\
& (C_{2VII}^1 \odot C_{2VII}^2)[(m_{11}, n_{11}), (m_{11}, n_{11})(m_{22}, n_{22})] = \max\{C_{2VII}^1(m_{11}, m_{11}m_{22}), A_{1VIP}^2(n_{11}), A_{1VIP}^2(n_{22})\}, \\
& \text{if } m_{11} \neq m_{22}, n_{11} \neq n_{22}, (m_{11}, m_{11}m_{22}) \in I_{VI}^1 \\
& (C_{2VII}^1 \odot C_{2VII}^2)[(m_{22}, n_{22}), (m_{11}, n_{11})(m_{22}, n_{22})] = \max\{C_{2VII}^1(m_{22}, m_{11}m_{22}), A_{1VIP}^2(n_{11}), A_{1VIP}^2(n_{22})\}, \\
& \text{if } m_{11} \neq m_{22}, n_{11} \neq n_{22}, (m_{22}, m_{11}m_{22}) \in I_{VI}^1 \\
& (C_{2VII}^1 \odot C_{2VII}^2)[(m_{11}, n_{22}), (m_{11}, n_{22})(m_{22}, n_{11})] = \max\{C_{2VII}^1(m_{11}, m_{11}m_{22}), A_{1VIP}^2(n_{11}), A_{1VIP}^2(n_{22})\}, \\
& \text{if } m_{11} \neq m_{22}, n_{11} \neq n_{22}, (m_{11}, m_{11}m_{22}) \in I_{VI}^1 \\
& (C_{2VII}^1 \odot C_{2VII}^2)[(m_{22}, n_{11}), (m_{11}, n_{22})(m_{22}, n_{11})] = \max\{C_{2VII}^1(m_{22}, m_{11}m_{22}), A_{1VIP}^2(n_{11}), A_{1VIP}^2(n_{22})\}, \\
& \text{if } m_{11} \neq m_{22}, n_{11} \neq n_{22}, (m_{22}, m_{11}m_{22}) \in I_{VI}^1
\end{aligned}$$

Example 6.1.1

Let $G_{VI}^1 = (V_{VI}^1, E_{VI}^1, I_{VI}^1, A_{VIP}^1, B_{VIL}^1, C_{VII}^1)$ be a VFIG with $A_{VIP}^1(m_{11}) = (0.4, 0.2)$, $A_{VIP}^1(m_{22}) = (0.3, 0.5)$, $B_{VIL}^1(m_{11}m_{22}) = (0.3, 0.6)$, $C_{VII}^1(m_{11}, m_{11}m_{22}) = (0.3, 0.7)$, $C_{VII}^1(m_{22}, m_{11}m_{22}) = (0.2, 0.6)$.

Figure 6.1.1: VFIG G_{VI}^1

Let $G_{VI}^2 = (V_{VI}^2, E_{VI}^2, I_{VI}^2, A_{VIP}^2, B_{VIL}^2, C_{VII}^2)$ be a VFIG with $A_{VIP}^2(n_{11}) = (0.6, 0.3)$, $A_{VIP}^2(n_{22}) = (0.2, 0.5)$, $B_{VIL}^2(n_{11}n_{22}) = (0.1, 0.5)$, $C_{VII}^2(n_{11}, n_{11}n_{22}) = (0.1, 0.5)$, $C_{VII}^2(n_{22}, n_{11}n_{22}) = (0.1, 0.7)$.

Figure 6.1.2: VFIG G_{VI}^2

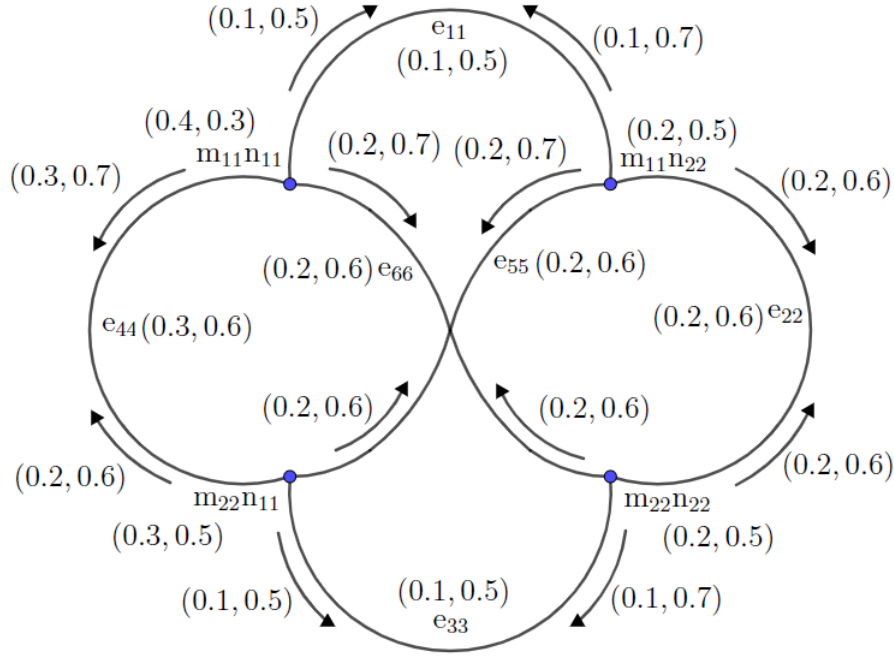


Figure 6.1.3: Composition of figure 6.1.1 and figure 6.1.2

Let $G_{VI}^1 \odot G_{VI}^2 = (V_{VI}, E_{VI}, I_{VI}, A_{VIP}^1 \odot A_{VIP}^2, B_{VIL}^1 \odot B_{VIL}^2, C_{VII}^1 \odot C_{VII}^2)$ be a CT-VFIGs with

$$A_{VIP}^1 \odot A_{VIP}^2(m_{11}, n_{11}) = (0.4, 0.3), (A_{VIP}^1 \odot A_{VIP}^2)(m_{11}, n_{22}) = (0.2, 0.5)$$

$$A_{VIP}^1 \odot A_{VIP}^2(m_{22}, n_{11}) = (0.3, 0.5), (A_{VIP}^1 \odot A_{VIP}^2)(m_{22}, n_{22}) = (0.2, 0.5)$$

$$(B_{VIL}^1 \odot B_{VIL}^2)((m_{11}, n_{11})(m_{11}, n_{22})) = (0.1, 0.5),$$

$$(B_{VIL}^1 \odot B_{VIL}^2)((m_{11}, n_{22})(m_{22}, n_{22})) = (0.2, 0.6)$$

$$(B_{VIL}^1 \odot B_{VIL}^2)((m_{22}, n_{11})(m_{22}, n_{22})) = (0.1, 0.5),$$

$$(B_{VIL}^1 \odot B_{VIL}^2)((m_{11}, n_{11})(m_{22}, n_{11})) = (0.3, 0.6)$$

$$(B_{VIL}^1 \odot B_{VIL}^2)((m_{11}, n_{11})(m_{22}, n_{22})) = (0.2, 0.6),$$

$$(B_{VIL}^1 \odot B_{VIL}^2)((m_{11}, n_{22})(m_{22}, n_{11})) = (0.2, 0.6)$$

$$(C_{VII}^1 \odot C_{VII}^2)[(m_{11}, n_{11}), (m_{11}, n_{11})(m_{11}, n_{22})] = (0.1, 0.5),$$

$$(C_{VII}^1 \odot C_{VII}^2)[(m_{11}, n_{22}), (m_{11}, n_{11})(m_{11}, n_{22})] = (0.1, 0.7),$$

$$(C_{VII}^1 \odot C_{VII}^2)[(m_{11}, n_{22}), (m_{11}, n_{22})(m_{22}, n_{22})] = (0.2, 0.7),$$

$$(C_{VII}^1 \odot C_{VII}^2)[(m_{22}, n_{22}), (m_{11}, n_{22})(m_{22}, n_{22})] = (0.2, 0.6),$$

$$\begin{aligned}
(C_{VII}^1 \odot C_{VII}^2)[(m_{22}, n_{11}), (m_{22}, n_{11})(m_{22}, n_{22})] &= (0.1, 0.5), \\
(C_{VII}^1 \odot C_{VII}^2)[(m_{22}, n_{22}), (m_{22}, n_{11})(m_{22}, n_{22})] &= (0.1, 0.7), \\
(C_{VII}^1 \odot C_{VII}^2)[(m_{11}, n_{11}), (m_{11}, n_{11})(m_{22}, n_{11})] &= (0.3, 0.7), \\
(C_{VII}^1 \odot C_{VII}^2)[(m_{22}, n_{11}), (m_{11}, n_{11})(m_{22}, n_{11})] &= (0.2, 0.6), \\
(C_{VII}^1 \odot C_{VII}^2)[(m_{11}, n_{11}), (m_{11}, n_{11})(m_{22}, n_{22})] &= (0.2, 0.7), \\
(C_{VII}^1 \odot C_{VII}^2)[(m_{22}, n_{22}), (m_{11}, n_{11})(m_{22}, n_{22})] &= (0.2, 0.6), \\
(C_{VII}^1 \odot C_{VII}^2)[(m_{22}, n_{11}), (m_{22}, n_{11})(m_{11}, n_{22})] &= (0.2, 0.6), \\
(C_{VII}^1 \odot C_{VII}^2)[(m_{11}, n_{22}), (m_{22}, n_{11})(m_{11}, n_{22})] &= (0.2, 0.7)
\end{aligned}$$

In figure 6.1.3 satisfies the condition of VFIG. Therefore $G_{VI}^1 \odot G_{VI}^2$ is also a VFIG.

Definition 6.1.2 Let G_{CVI} be a composition of two vague fuzzy incidence graphs

(i) G_{CVI} cardinality is determined by

$$\begin{aligned}
|G_{CVI}| &= \sum_{w_{11} \in V_{VI}} \frac{1 + t_{A_{VIP}}(w_{11}) - f_{A_{VIP}}(w_{11})}{2} + \sum_{w_{11}w_{22} \in E_{VI}} \frac{1 + t_{B_{VIL}}(w_{11}w_{22}) - f_{B_{VIL}}(w_{11}w_{22})}{2} \\
&+ \sum_{w_{11}, w_{11}w_{22} \in I_{VI}} \frac{1 + t_{C_{VII}}(w_{11}, w_{11}w_{22}) - f_{C_{VII}}(w_{11}, w_{11}w_{22})}{2}
\end{aligned}$$

(ii) G_{CVI} vertex cardinality is determined by

$$|V_{CVI}| = \sum_{w_{11} \in V_{CVI}} \frac{1 + t_{A_{VIP}}(w_{11}) - f_{A_{VIP}}(w_{11})}{2} \quad \forall w_{11} \in V_{CVI}$$

(iii) G_{CVI} edge cardinality is specified by

$$|E_{CVI}| = \sum_{w_{11}w_{22} \in E_{CVI}} \frac{1 + t_{B_{VIL}}(w_{11}w_{22}) - f_{B_{VIL}}(w_{11}w_{22})}{2} \quad \forall w_{11}w_{22} \in E_{CVI}$$

(iv) G_{CVI} incidence pair cardinality is specified by

$$|I_{CVI}| = \sum_{w_{11}, w_{11}w_{22} \in I_{CVI}} \frac{1 + t_{CVI}(w_{11}, w_{11}w_{22}) - f_{CVI}(w_{11}, w_{11}w_{22})}{2} \quad \forall w_{11}, w_{11}w_{22} \in I_{CVI}$$

Example 6.1.2 In figure 6.1.3, $|V_{CVI}| = 1.65$, $|E_{CVI}| = 1.85$, $|I_{CVI}| = 3.25$ and $|G_{CVI}| = |V_{CVI}| + |E_{CVI}| + |I_{CVI}| = 1.65 + 1.85 + 3.25 = 6.75$.

6.2 Relationship between order and size of composition of two vague fuzzy incidence graphs

Definition 6.2.1 Assume G_{CVI} is a CT-VFIGs. Then

$$O_{CVI}(G_{CVI}) = \sum_{w_{11} \neq w_{22}, w_{11}, w_{22} \in V_{CVI}} \left(\frac{1 + t_{CVI}(w_{11}, w_{11}w_{22}) - f_{CVI}(w_{11}, w_{11}w_{22})}{2} \right)$$

is called order of G_{CVI} and

$$S_{CVI}(G_{CVI}) = \sum_{w_{11}, w_{22} \in E_{CVI}} \left(\frac{1 + t_{CVI}(w_{11}, w_{22}) - f_{CVI}(w_{11}w_{22})}{2} \right)$$

is called size of G_{CVI} .

Definition 6.2.2 The edge degree of a e_{1VI} in a CT-VFIGs is defined as the sum of the weights of edges incident to e_{1VI} . It is defined by $|d_{G_{CVI}}(e_{1VI})| = \{deg^t(e_{1VI}), deg^f(e_{1VI})\}$. The minimum cardinality of edge degree of G_{CVI} is $\delta_{CVI}(G_{CVI}) = \min\{d_{G_{CVI}}(e_{1VI})/e_{1VI} \in E_{CVI}\}$. The maximum cardinality of edge degree of G_{CVI} is $\Delta_{CVI}(G_{CVI}) = \max\{d_{G_{CVI}}(e_{1VI})/e_{1VI} \in E_{CVI}\}$.

Proposition 6.2.1 In a composition of two vague fuzzy incidence graphs $O_{CVI}(G_{CVI}) \geq S_{CVI}(G_{CVI})$.

Proof. Let G_{CVI} be a CT-VFIGs with one node. Then $O_{CVI}(G_{CVI}) = S_{CVI}(G_{CVI}) = 0$. That is

$$O_{CVI}(G_{CVI}) = S_{CVI}(G_{CVI}) \tag{6.1}$$

It is a frivolous case. Assume G_{CVI} with more than one nodes. $O_{CVI}(G_{CVI})$ is the sum of all incidence pairs cardinality of G_{CVI} . Since incidence pairs are two times of edges. Therefore, the total sum of all the incidence pairs cardinality will invariably greater than the total sum of all the edge cardinality.

$$O_{CVI}(G_{CVI}) > S_{CVI}(G_{CVI}) \quad (6.2)$$

From equations (6.1) and (6.2), we get $O_{CVI}(G_{CVI}) \geq S_{CVI}(G_{CVI})$.

Example 6.2.1 Suppose any CT-VFIGs with one node. Then $O_{CVI}(G_{CVI}) = S_{CVI}(G_{CVI}) = 0$.

That is $O_{CVI}(G_{CVI}) = S_{CVI}(G_{CVI})$

In figure 6.1.3, $O_{CVI}(G_{CVI}) = 3.25$ and $S_{CVI}(G_{CVI}) = 1.85$.

That is $O_{CVI}(G_{CVI}) > S_{CVI}(G_{CVI})$

Therefore $O_{CVI}(G_{CVI}) \geq S_{CVI}(G_{CVI})$.

Proposition 6.2.2 For any composition of two vague fuzzy incidence graphs the following inequality holds

$$\delta_{CVI}(G_{CVI}) < \Delta_{CVI}(G_{CVI}) < S_{CVI}(G_{CVI}) < O_{CVI}(G_{CVI}).$$

Proof. Assume G_{CVI} is a CT-VFIGs with non empty node set. Since $\delta_{CVI}(G_{CVI})$ represents lowest edge degree and $\Delta_{CVI}(G_{CVI})$ denotes highest edge degree of G_{CVI} .

$$\delta_{CVI}(G_{CVI}) < \Delta_{CVI}(G_{CVI}) \quad (6.3)$$

We know

$$O_{CVI}(G_{CVI}) = \sum_{w_{11} \neq w_{22}, w_{11}, w_{22} \in V_{CVI}} \left(\frac{1 + t_{CVI}(w_{11}, w_{11}w_{22}) - f_{CVI}(w_{11}, w_{11}w_{22})}{2} \right)$$

and

$$S_{CVI}(G_{CVI}) = \sum_{w_{11}, w_{22} \in E_{CVI}} \left(\frac{1 + t_{CVI}(w_{11}, w_{22}) - f_{CVI}(w_{11}w_{22})}{2} \right).$$

By definition of size of G_{CVI} ,

$$S_{CVI}(G_{CVI}) = \sum_{w_{11}, w_{22} \in E_{CVI}} \left(\frac{1 + t_{B_{CVI}}(w_{11}, w_{22}) - f_{B_{CVI}}(w_{11}w_{22})}{2} \right) > \max\{d_{G_{CVI}}(e_{1VI})/e_{1VI} \in E_{CVI}\}$$

That is

$$S_{CVI}(G_{CVI}) > \Delta_{CVI}(G_{CVI}) \quad (6.4)$$

Also, in a CT-VFIGs, G_{CVI} by proposition 6.2.1

$$O_{CVI}(G_{CVI}) > S_{CVI}(G_{CVI}) \quad (6.5)$$

From inequalities (6.3), (6.4) and (6.5), we obtained

$$\delta_{CVI}(G_{CVI}) < \Delta_{CVI}(G_{CVI}) < S_{CVI}(G_{CVI}) < O_{CVI}(G_{CVI}).$$

Example 6.2.2 In figure 6.1.3, $\delta_{CVI}(G_{CVI}) = 0.25$

$$\Delta_{CVI}(G_{CVI}) = 0.3$$

$$S_{CVI}(G_{CVI}) = 1.85 \text{ and}$$

$$O_{CVI}(G_{CVI}) = 3.25.$$

That is $0.25 < 0.3 < 1.85 < 3.25$.

Therefore $\delta_{CVI}(G_{CVI}) < \Delta_{CVI}(G_{CVI}) < S_{CVI}(G_{CVI}) < O_{CVI}(G_{CVI})$.

6.3 Domination in Composition of Two Vague Fuzzy Incidence Graphs

Definition 6.3.1 A edge e_{VI} in an CT-VFIGs G_{CVI} is called incidentally dominate edge if

$$t_{C_{CVI}}(w_{11}, w_{11}w_{22}) = \min\{t_{A_{CVI}}(w_{11}), t_{B_{CVI}}(w_{11}, w_{22})\}$$

and

$$f_{C_{CVI}}(w_{11}, w_{11}w_{22}) = \max\{f_{A_{CVI}}(w_{11}), f_{B_{CVI}}(w_{11}, w_{22})\}$$

Definition 6.3.2 A edge e_{1VI} in an CT-VFIGs G_{CVI} dominates to edge e_{2VI} if they are incidentally dominate edges.

Definition 6.3.3 A subset R_{CVI} of E_{CVI} is said to be edge incidentally dominating set (EIDS) if for each edge e_{1VI} not in R_{CVI} , e_{1VI} is dominate at least one edge in R_{CVI} .

Definition 6.3.4 A edge incidentally dominating set R_{CVI} of the CT-VFIGs G_{CVI} is said to be a minimal EIDS of CT-VFIGs G_{CVI} if each edge in R_{CVI} , the set $R_{CVI} - \{e_{1VI}\}$ is not a EIDS.

Definition 6.3.5 A EIDS with the lowest edge cardinality is labeled a minimum EIDS. The edge cardinality of a minimum EIDS is called edge incidentally dominating number (EIDN) of the CT-VFIGs G_{CVI} . It is denoted by $\gamma_{VI}(G_{CVI})$.

Example 6.3.1 In figure 6.1.3, the incidentally dominating edges are $\{e_{11}\}, \{e_{22}\}, \{e_{33}\}, \{e_{44}\}, \{e_{55}\}, \{e_{66}\}$ and the EIDSs are $S_{11} = \{e_{11}e_{22}\}, S_{22} = \{e_{11}e_{33}\}, S_{33} = \{e_{11}e_{44}\}, S_{44} = \{e_{11}e_{55}\}, S_{55} = \{e_{11}e_{66}\}, \dots$. After calculating the edge cardinality of $S_{11}, S_{22}, S_{33}, S_{44}, \dots$, we obtain $|S_{11}| = 0.6, |S_{22}| = 0.6, |S_{33}| = 0.65, |S_{44}| = 0.6, |S_{55}| = 0.6, \dots$. The edge cardinality of a minimum EIDS is $|S_{11}| = 0.6$ and $\gamma_{G_{CVI}} = 0.6$.

Theorem 6.3.1 Let $G_{VI}^1 = (A_{VIP}^1, B_{VIL}^1, C_{VII}^1)$ and $G_{VI}^2 = (A_{VIP}^2, B_{VIL}^2, C_{VII}^2)$ be two vague fuzzy incidence graphs. Then $\gamma_{VI}(G_{CVI}) = \min\{A_{VIP}^1(m_{11}), A_{VIP}^2(n_{11})\}$ where $m_{11} \in G_{VI}^1$ and $n_{11} \in G_{VI}^2$.

Proof. Assume $G_{VI}^1 = (A_{VIP}^1, B_{VIL}^1, C_{VII}^1)$ and $G_{VI}^2 = (A_{VIP}^2, B_{VIL}^2, C_{VII}^2)$ are two VFIGs. Since G_{VI}^1 and G_{VI}^2 are two VFIGs, then $G_{VI}^1 \odot G_{VI}^2$ will be a VFIGs. So, each two edges in $G_{VI}^1 \odot G_{VI}^2$ will dominates remaining edges. Then by definition of EIDN, $\gamma_{VI}(G_{CVI}) = \min\{\text{cardinality of } (A_{VIP}^1(m_{11}), A_{VIP}^2(n_{11}))\}$.

Example 6.3.2 In figure 6.1.1 and 6.1.2, $\min\{\text{cardinality of } (A_{VIP}^1(m_{11}), A_{VIP}^2(n_{11}))\} = 0.6$. In figure 6.1.3, the EIDSs are $|S_{11}| = 0.6, |S_{22}| = 0.6, |S_{33}| = 0.6, |S_{44}| = 0.6, |S_{55}| = 0.6, \dots$

The edge cardinality of a minimum EIDS is $|S_{11}| = 0.6$ and $\gamma_{VI}(G_{CVI}) = 0.6$.

Therefore $\gamma_{VI}(G_{CVI}) = \min\{\text{cardinality of } (A_{VIP}^1(m_{11}), A_{VIP}^2(n_{11}))\}$.

Theorem 6.3.2 Let $G_{VI}^1 = (A_{VIP}^1, B_{VIL}^1, C_{VII}^1)$ and $G_{VI}^2 = (A_{VIP}^2, B_{VIL}^2, C_{VII}^2)$ be two vague fuzzy incidence graphs with $k \geq 2$ and $l \geq 2$, where k and l are representing the number of vertices in G_{VI}^1 and G_{VI}^2 , respectively. Then $\frac{\gamma_{VI}(G_{CVI})}{2} = \min\{\text{cardinality of } (B_{VIL}^1(m_{11}m_{22}), B_{VIL}^2(n_{11}n_{22}))\}$.

Proof. Consider $G_{VI}^1 = (A_{VIP}^1, B_{VIL}^1, C_{VII}^1)$ and $G_{VI}^2 = (A_{VIP}^2, B_{VIL}^2, C_{VII}^2)$ are two VFIGs. Since G_{VI}^1 and G_{VI}^2 are VFIGs. Then $G_{VI}^1 \odot G_{VI}^2$ will also a VFIG with $\frac{\gamma_{VI}(G_{CVI})}{2} = \min\{\text{cardinality of } (B_{VIL}^1(m_{11}m_{22}), B_{VIL}^2(n_{11}n_{22}))\}$ because each two edges in $G_{VI}^1 \odot G_{VI}^2$ dominates to all remaining edges.

Example 6.3.3 In figure 6.1.1 and 6.1.2, $\min\{\text{cardinality of } (B_{VIL}^1(m_{11}m_{22}), B_{VIL}^2(n_{11}n_{22}))\} = 0.3$. In figure 6.1.3, the EIDSs are $|S_{11}| = 0.6, |S_{22}| = 0.6, |S_{33}| = 0.6, |S_{44}| = 0.6, |S_{55}| = 0.6, \dots$

The edge cardinality of a minimum EIDS is $|S_{11}| = 0.6, \gamma_{VI}(G_{CVI}) = 0.6$ and $\frac{\gamma_{VI}(G_{CVI})}{2} = 0.3$.

Therefore $\frac{\gamma_{VI}(G_{CVI})}{2} = \min\{\text{cardinality of } (B_{VIL}^1(m_{11}m_{22}), B_{VIL}^2(n_{11}n_{22}))\}$.

6.4 Strong and Weak Domination in Composition of Two Vague Fuzzy Incidence Graphs

The strong and weak domination in graph theory was introduced by Sampathkumar and Pushpalatha in 1996.

Definition 6.4.1 Let G_{CVI} be a CT-VFIGs. For any two edges $e_{1VI}, e_{2VI} \in E_{CVI}$, e_{1VI} strongly dominates e_{2VI} in CT-VFIGs G_{CVI} if

- (i) they are incidentally dominate edges
- (ii) $\deg^t(e_{1VI}) \geq \deg^t(e_{2VI}), \deg^f(e_{1VI}) \leq \deg^f(e_{2VI})$

Similarly e_{1VI} weakly dominates e_{2VI} if

- (i) they are incidentally dominate edges
- (ii) $\deg^t(e_{2VI}) \geq \deg^t(e_{1VI}), \deg^f(e_{2VI}) \leq \deg^f(e_{1VI})$

Definition 6.4.2 An edge incidentally dominating set $R_{CVI} \subseteq E_{CVI}$ is labeled a strong (weak) edge incidentally dominating set (SEIDS, WEIDS) of G_{CVI} if, for each $e_{1VI} \in E_{CVI} - R_{CVI}$, there exist at least one edge $e_{2VI} \in R_{CVI}$, so that e_{1VI} strongly (weakly) dominates e_{2VI} . The strong (weak) edge incidentally domination number of G_{CVI} denoted by $\gamma_{SVI}(G_{CVI})$ ($\gamma_{WVI}(G_{CVI})$), is called as the minimum cardinality of a strong (weak) edge incidentally dominating set of G_{CVI} .

Example 6.4.1 In figure 6.1.3, the incidentally dominating edges are $\{e_{11}\}, \{e_{22}\}, \{e_{33}\}, \{e_{44}\}, \{e_{55}\}, \{e_{66}\}$ and the SEIDS are $S_{11} = \{e_{11}e_{22}\}, S_{22} = \{e_{11}e_{44}\}, S_{33} = \{e_{22}e_{33}\}, S_{44} = \{e_{33}e_{44}\}$. After calculating the edge cardinality of $S_{11}, S_{22}, S_{33}, S_{44}$ we obtain $|S_{11}| = 0.6, |S_{22}| = 0.65, |S_{33}| = 0.6, |S_{44}| = 0.6$. The edge cardinality of a minimum SEIDS is $|S_{11}| = 0.6$ and $\gamma_{SVI}(G_{CVI}) = 0.6$. The WEIDSs are $S_{55} = \{e_{11}e_{55}\}, S_{66} = \{e_{11}e_{66}\}, S_{77} = \{e_{33}e_{66}\}$. After calculating the edge cardinality of S_{55}, S_{66}, S_{77} we obtain $|S_{55}| = 0.6, |S_{66}| = 0.6, |S_{77}| = 0.6$. The edge cardinality of a minimum WEIDS is $|S_{55}| = 0.6$ and $\gamma_{WVI}(G_{CVI}) = 0.6$.

Theorem 6.4.1 Let G_{CVI} be a CT-VFIGs without single node and R_{CVI} be a minimum strong edge incidentally dominating set of G_{CVI} , then $E_{CVI} - R_{CVI}$ is an strong edge incidentally dominating set of CT-VFIGs.

Proof. Let G_{CVI} be a CT-VFIGs with minimum SEIDS, then for each edge $e_{2VI} \in R_{CVI}$, there is at least one edge $e_{1VI} \in E_{CVI} - N_{CVI}$ so that $\deg^t(e_{1VI}) \geq \deg^t(e_{2VI}), \deg^f(e_{1VI}) \leq \deg^f(e_{2VI})$ and

$$t_{G_{CVI}}(w_{11}, w_{11}w_{22}) = \min\{t_{A_{CVI}}(w_{11}), t_{B_{CVI}}(w_{11}, w_{22})\}$$

and

$$f_{G_{CVI}}(w_{11}, w_{11}w_{22}) = \max\{f_{A_{CVI}}(w_{11}), f_{B_{CVI}}(w_{11}, w_{22})\}.$$

Hence, $E_{CVI} - R_{CVI}$ strongly dominates each edge of R_{CVI} . So, $E_{CVI} - R_{CVI}$ is an SEIDS of CT-VFIGs.

Example 6.4.2 In figure 6.1.3, the SEIDSs are $S_{11} = \{e_{11}e_{22}\}, S_{22} = \{e_{11}e_{44}\}, S_{33} = \{e_{22}e_{33}\}, S_{44} = \{e_{33}e_{44}\}$. After calculating the edge cardinality of $S_{11}, S_{22}, S_{33}, S_{44}$ we obtain $|S_{11}| = 0.6, |S_{22}| = 0.65, |S_{33}| = 0.6, |S_{44}| = 0.6$. The edge cardinality of a minimum SEIDS is $|S_{11}| = 0.6$, then $E_{CVI} - S_{11} = \{e_{33}, e_{44}, e_{55}, e_{66}\}$ is also a SEIDS.

Theorem 6.4.2 Let G_{CVI} be a CT-VFIGs without single node and R_{CVI} be a minimum weak edge incidentally dominating set of G_{CVI} , then $E_{CVI} - R_{CVI}$ is an weak edge incidentally dominating set of CT-VFIGs.

Theorem 6.4.3 For any CT-VFIGs with $t_{G_{CVI}}(w_{11}, w_{11}w_{22}) = \min\{t_{A_{CVI}}(w_{11}), t_{B_{CVI}}(w_{11}, w_{22})\}$ and $f_{G_{CVI}}(w_{11}, w_{11}w_{22}) = \max\{f_{A_{CVI}}(w_{11}), f_{B_{CVI}}(w_{11}, w_{22})\}$ for all $w_{11} \in V_{CVI}, w_{11}w_{22} \in E_{CVI}$, then $\gamma_{SVI} = \gamma_{WVI}$.

Proof. Let G_{CVI} be a CT-VFIGs with $t_{G_{CVI}}(w_{11}, w_{11}w_{22}) = \min\{t_{A_{CVI}}(w_{11}), t_{B_{CVI}}(w_{11}, w_{22})\}$ and $f_{G_{CVI}}(w_{11}, w_{11}w_{22}) = \max\{f_{A_{CVI}}(w_{11}), f_{B_{CVI}}(w_{11}, w_{22})\}$. Assume for every node have same or different value. Since G_{CVI} is CT-VFIGs with $t_{G_{CVI}}(w_{11}, w_{22}) = \min\{t_{A_{CVI}}(w_{11}), t_{A_{CVI}}(w_{22})\}$ and $f_{G_{CVI}}(w_{11}, w_{22}) = \max\{f_{A_{CVI}}(w_{11}), f_{A_{CVI}}(w_{22})\}$ for all $w_{11}, w_{22} \in V_{CVI}$ and $t_{G_{CVI}}(w_{11}, w_{11}w_{22}) = \min\{t_{A_{CVI}}(w_{11}), t_{B_{CVI}}(w_{11}, w_{22})\}$ and $f_{G_{CVI}}(w_{11}, w_{11}w_{22}) = \max\{f_{A_{CVI}}(w_{11}), f_{B_{CVI}}(w_{11}, w_{22})\}$ for all $w_{11} \in V_{CVI}, w_{11}w_{22} \in E_{CVI}$. Thus every $e_{1VI}e_{2VI} \in E_{CVI}$ is SEIDS as well as WEIDS. Therefore, $\gamma_{SVI} = \gamma_{WVI}$.

Example 6.4.3 In figure 6.1.3, the incidentally dominating edges are $\{e_{11}\}, \{e_{22}\}, \{e_{33}\}, \{e_{44}\}, \{e_{55}\}, \{e_{66}\}$ and the SEIDSs are $S_{11} = \{e_{11}e_{22}\}, S_{22} = \{e_{11}e_{44}\}, S_{33} = \{e_{22}e_{33}\}, S_{44} = \{e_{33}e_{44}\}$. After calculating the edge cardinality of $S_{11}, S_{22}, S_{33}, S_{44}$ we obtain $|S_{11}| = 0.6, |S_{22}| = 0.65, |S_{33}| = 0.6, |S_{44}| = 0.6$ and $\gamma_{SVI}(G_{CVI}) = 0.6$. The WEIDSs are $S_{55} = \{e_{11}e_{55}\}, S_{66} = \{e_{11}e_{66}\}, S_{77} = \{e_{33}e_{66}\}$. After calculating the edge cardinality of S_{55}, S_{66}, S_{77} we obtain $|S_{55}| = 0.6, |S_{66}| = 0.6, |S_{77}| = 0.6$ and $\gamma_{WVI}(G_{CVI}) = 0.6$. Therefore $\gamma_{SVI} = \gamma_{WVI}$.

Theorem 6.4.4 For a composition of two vague fuzzy incidence graphs, the below inequalities are true.

- (i) $\gamma_{VI} \leq \gamma_{SVI} \leq O_{CVI}(G_{CVI}) - \text{maximum } d_{G_{CVI}} \text{ of } G_{CVI}$.
- (ii) $\gamma_{VI} \leq \gamma_{WVI} \leq O_{CVI}(G_{CVI}) - \text{minimum } d_{G_{CVI}} \text{ of } G_{CVI}$.

Proof.

- (i) From definition 6.4.1 and 6.4.2 we have

$$\gamma_{VI} \leq \gamma_{SVI} \tag{6.6}$$

We know $O_{CVI}(G_{CVI}) =$ the sum of the incidence pair of CT-VFIGs. Also $O_{CVI}(G_{CVI}) -$ not including the maximum $d_{G_{CVI}}$ of CT-VFIGs

$$= O_{CVI}(G_{CVI}) - \Delta_{CVI}(G_{CVI}) \tag{6.7}$$

From equation (6.6) and (6.7),

$$\gamma_{VI} \leq \gamma_{SVI} \leq O_{CVI}(G_{CVI}) - \text{maximum } d_{G_{CVI}} \text{ of } G_{CVI}.$$

- (ii) From definition 6.4.1 and 6.4.2 domination number γ_{VI} of CT-VFIGs is less than or equal to the γ_{WVI} of CT-VFIGs, because the edges of WEIDS M_{CVI} , it weakly dominates any one of the edges of $E_{CVI} - M_{CVI}$. Therefore,

$$\gamma_{CVI}(G_{CVI}) \geq \gamma_{VI}(G_{CVI}) \quad (6.8)$$

Also $O_{CVI}(G_{CVI})$ – not including the minimum $d_{G_{CVI}}$ of CT-VFIGs

$$= O_{CVI}(G_{CVI}) - \delta_{CVI}(G_{CVI}) \quad (6.9)$$

From equation (6.8) and (6.9), we get

$$\gamma_{VI} \leq \gamma_{WVI} \leq O_{CVI}(G_{CVI}) - \text{minimum } d_{G_{CVI}} \text{ of } G_{CVI}$$

6.5 Real-Life Application of CT-VFIGs

An application of CT-VFIGs is included here. Consider two networks (CT-VFIGs) G_{VI}^1 and G_{VI}^2 , which have two and two vertices, respectively, and show distinct journal publications from different journals of a research field. The vertices membership (MS) value indicates the percentage of accepted research papers in journal publishing, while the non membership (NMS) value represents the rejected research papers. The MS value of the edges indicates that the journal publications are mutually collaborative, whereas the NMS value indicates that the journal publications are not mutually collaborative. The MS value of the incidence pairs represents the percentage of progress, whereas the NMS value represents the percentage of journal publications that have not progressed. As in figure 6.1.3 composition of G_{VI}^1 and G_{VI}^2 show the percentage of progress of journal publication m_{11} with journal publications n_{11} and n_{22} has the maximum MS value and the percentage of non progress of journal publication m_{11} with journal publications n_{11} and n_{22} has the lesser NMS value. As a result, the best suited combinations of journal

publications demonstrating the largest percentage of progress and the lowest percentage of non-progress in the research field exist.



Figure 6.5.1: Journal Publications

6.6 Comparative Analysis

In figure 6.1.3, calculate the edge cardinality of each edge, we get all the edges have same value. In our study the edge degree cardinality of the CT-VIFGs $|d_{G_{CVI}}(e_{1VI})| = 0.2$ and $|d_{G_{CVI}}(e_{2VI})| = 0.3$ are not all the same. It can be observed that the edge degree of the edges $|d_{G_{CVI}}(e_{1VI})| = \{0.9, 2.4\}$ shows the percentage of progress of journal publication m_{11} with journal publications n_{11} and n_{22} has the maximum MS value and the percentage of non progress of journal publication m_{11} with journal publications n_{11} and n_{22} has the lesser NMS value. As a result, the current method is ineffective in determining which journal publications have the highest percentage of progress and the lowest percentage of non-progress.

The current method is useful for single networks, but it is insufficient to explain the overall impact of different networks' products. However, we may use composition to discuss the overall impact of combining multiple networks in our strategy. Our technique works with several networks as well as a single network. This allows us to discuss the overall influence of various networks products. As a result, our proposed strategy outperforms the existing one.

CT-VFIGs are extremely useful tools for researching a variety of computational intelligence and computer science topics. CT-VFIGs are used in a variety of fields, including natural networks and operations research. We introduced three new CT-VIFG concepts in this chapter edge incidentally dominating set, strong edge incidentally dominating set and weak edge incidentally dominating set. In the CT-VFIGs, some advantageous and instrumental theorems of domination are also explained. A study of the makeup of VFIGs in the field of research is also included. We plan to expand our research into CT-VFIG coloring, Hamiltonian CT-VFIGs, and CT-VFIG chromaticity in the future.

Chapter 7

Strong And Weak Domination in Complete Intuitionistic Fuzzy Incidence Graphs

Fuzzy graphs, also known as fuzzy incidence graphs, are a well-organized and useful tool for capturing and resolving a range of real-world scenarios involving ambiguous data and information. The concept of complete intuitionistic fuzzy incidence graphs was presented in this chapter of the investigation. Complete intuitionistic fuzzy incidence graphs are characterised in terms of order, size, degree cardinality, strong intuitionistic fuzzy incidence dominating set and weak intuitionistic fuzzy incidence dominating set. For different classes of complete intuitionistic fuzzy incidence graphs, we compute the intuitionistic fuzzy incidence domination number, strong intuitionistic fuzzy incidence domination number, and weak intuitionistic fuzzy incidence domination number, and some theorems are explored. With the help of various outline models, we can better understand these concepts. In addition, for the delineation, the application of domination for complete intuitionistic fuzzy incidence graph to determine the best treatment facility accessible clinic is discussed.

7.1 Complete Intuitionistic Fuzzy Incidence Graph

Definition 7.1.1 The support of intuitionistic fuzzy incidence graph (IFIG) $G_{FI} = (R, S, T)$ is $\text{supp}(G_{FI}) = \{\text{supp}(R), \text{supp}(S), \text{supp}(T)\}$ so that

$$\begin{aligned}\text{supp}(R) &= \{x_{11}/\rho_1(x_{11}) > 0, \rho_2(x_{11}) > 0\} \\ \text{supp}(S) &= \{x_{11}x_{22}/\phi_1(x_{11}x_{22}) > 0, \phi_2(x_{11}x_{22}) > 0\} \\ \text{supp}(T) &= \{(x_{11}, x_{11}x_{22})/\chi_1(x_{11}, x_{11}x_{22}) > 0, \chi_2(x_{11}, x_{11}x_{22}) > 0\}\end{aligned}$$

ρ^*, ϕ^* and χ^* are representing support of ρ, ϕ and χ respectively.

Definition 7.1.2 A IFIG is said to be complete intuitionistic fuzzy incidence graph if $\chi_1(x_{11}, x_{11}x_{22}) = \min\{\rho_1(x_{11}), \phi_1(x_{11}x_{22})\}$ and $\chi_2(x_{11}, x_{11}x_{22}) = \max\{\rho_2(x_{11}), \phi_2(x_{11}x_{22})\}$, for each $\chi_1(x_{11}, x_{11}x_{22}), \chi_2(x_{11}, x_{11}x_{22}) \in \chi^*$.

Remark 7.1.1 Every CIFIG is a intuitionistic fuzzy incidence graph but not conversely.

Definition 7.1.3 Assume $G_{IFI} = (\rho_{IFI}, \phi_{IFI}, \chi_{IFI})$ is a CIFIG. Then

$$O(G_{IFI}) = \sum_{x_{11} \neq x_{22}, x_{11}x_{22} \in V_{IFI}} \left(\frac{1 + \chi_1(x_{11}, x_{11}x_{22}) - \chi_2(x_{11}, x_{11}x_{22})}{2} \right)$$

is called order of G_{IFI} and

$$S(G_{IFI}) = \sum_{x_{11}x_{22} \in \phi^*} \left(\frac{1 + \phi_1(x_{11}x_{22}) - \phi_2(x_{11}x_{22})}{2} \right)$$

is called size of G_{IFI} .

7.2 Domination in Complete Intuitionistic Fuzzy Incidence Graph

Definition 7.2.1 A vertex x_{11} in a complete intuitionistic fuzzy incidence graph dominates to vertex x_{22} if

$$\chi_1(x_{11}, x_{11}x_{22}) = \min\{\rho_1(x_{11}), \phi_1(x_{11}x_{22})\} \text{ and } \chi_2(x_{11}, x_{11}x_{22}) = \max\{\rho_2(x_{11}), \phi_2(x_{11}x_{22})\}.$$

Remark 7.2.1 For any $x_{11}, x_{22} \in V_{IFI}$, if x_{11} dominates x_{22} then x_{22} also dominates x_{11} .

Definition 7.2.2 A set $M_{IFI} \subseteq V_{IFI}$ is a intuitionistic fuzzy incidence dominating set (IFIDS) if each nodes in $V_{IFI} - M_{IFI}$ is dominated by atleast one node in M_{IFI} .

Definition 7.2.3 The lowest intuitionistic fuzzy incidence cardinality of a IFIDS is uttered as the intuitionistic fuzzy incidence domination number and it is represented by $\gamma_{IFI}(G_{IFI})$ or γ_{IFI} .

Definition 7.2.4 Consider $G_{IFI} = (V_{IFI}, E_{IFI}, I_{IFI}, \rho_{IFI}, \phi_{IFI}, \chi_{IFI})$ is an CIFIG and $x_{11} \in V_{IFI}$ then its degree is expressed by $d_{G_{IFI}}(x_{11}) = (d_{1G_{IFI}}(x_{11}), d_{2G_{IFI}}(x_{11}))$ and represented by $d_{1G_{IFI}}(x_{11}) = \sum_{x_{11} \neq x_{22}} (x_{11}, x_{11}x_{22}) \in I_{IFI}$ and $d_{2G_{IFI}}(x_{11}) = \sum_{x_{11} \neq x_{22}} (x_{11}, x_{11}x_{22}) \in I_{IFI}$.

7.3 Strong and Weak Domination in CIFIGs

Definition 7.3.1 Let G_{IFI} be a complete intuitionistic fuzzy incidence graph. Then the degree cardinality of $d_{G_{IFI}}(x_{11})$ is represented to be $|d_{G_{IFI}}(x_{11})| = \frac{1 + d_{1G_{IFI}}(x_{11}) - d_{2G_{IFI}}(x_{11})}{2}$. The lowest degree cardinality of G_{IFI} is defined by $\delta(G_{IFI}) = \min\{d_{G_{IFI}}(x_{11})/x_{11} \in V_{IFI}\}$ and highest degree cardinality of G_{IFI} is defined by $\Delta(G_{IFI}) = \max\{d_{G_{IFI}}(x_{11})/x_{11} \in V_{IFI}\}$.

Definition 7.3.2 Assume G_{IFI} is a complete intuitionistic fuzzy incidence graph and let x_{11} and x_{22} be the nodes of G_{IFI} . Then x_{11} strongly dominates x_{22} or x_{22} weakly dominates x_{11} if $d_i(x_{11}) \geq d_i(x_{22})$ and $\chi_1(x_{11}, x_{11}x_{22}) = \min\{\rho_1(x_{11}), \phi_1(x_{11}x_{22})\}$, $\chi_2(x_{11}, x_{11}x_{22}) = \max\{\rho_2(x_{11}), \phi_2(x_{11}x_{22})\}$. We call x_{22} strongly dominates x_{11} or x_{11} weakly dominates x_{22} if $d_i(x_{22}) \geq d_i(x_{11})$ and $\chi_1(x_{22}, x_{11}x_{22}) = \min\{\rho_1(x_{22}), \phi_1(x_{11}x_{22})\}$ and $\chi_2(x_{22}, x_{11}x_{22}) = \max\{\rho_2(x_{22}), \phi_2(x_{11}x_{22})\}$,

Definition 7.3.3 A set $S_{IFI} \subseteq V_{IFI}$ is a strong intuitionistic fuzzy incidence dominating set (SIFIDS) if every vertex in $V_{IFI} - S_{IFI}$ is strongly fuzzy incidence dominated by at least one vertex in S_{IFI} . Similarly, S_{IFI} is labeled a WIFIDS if every vertex in $V_{IFI} - S_{IFI}$ is weakly fuzzy incidence dominated by at least one vertex in S_{IFI} .

Definition 7.3.4 The lowest intuitionistic fuzzy incidence cardinality of a SIFIDS is uttered as the SIFIDN and it is represented by $\gamma_{SIFI}(G_{IFI})$ or γ_{SIFI} and the lowest intuitionistic fuzzy incidence cardinality of

a weak intuitionistic fuzzy incidence dominating set (WIFIDS) is uttered as the weak intuitionistic fuzzy incidence domination number (WIFIDN) and it is represented by $\gamma_{WIFI}(G_{IFI})$ or γ_{WIFI} .

Example 7.3.1

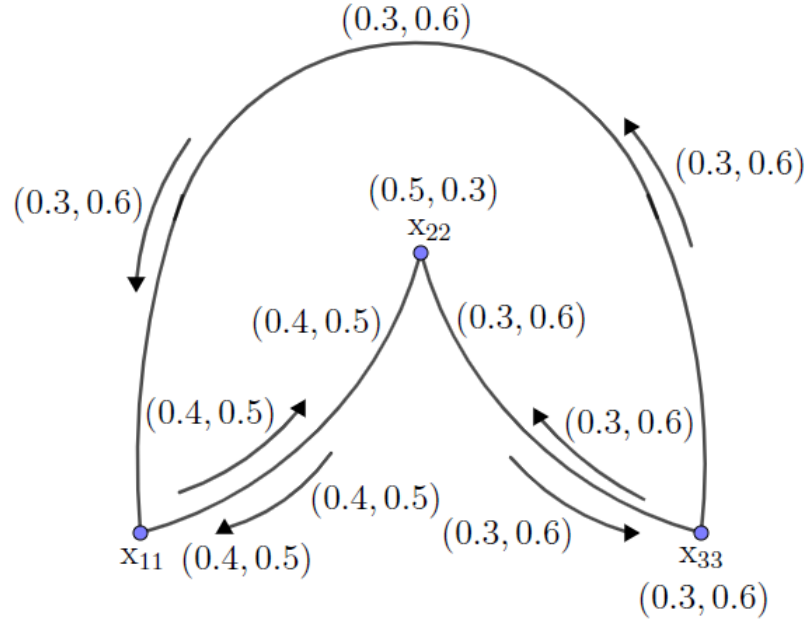


Figure 7.3.1: CIFIG with $\gamma_{SIFI} = 0.5$ and $\gamma_{WIFI} = 0.4$

Assume $G_{IFI} = (\rho_{IFI}, \phi_{IFI}, \chi_{IFI})$ is an CIFIG given in above figure 7.3.1 having $V_{IFI} = (x_{11}, x_{22}, x_{33})$ and

$$\begin{aligned} \rho(x_{11}) &= (0.4, 0.5), \rho(x_{22}) = (0.5, 0.3), \rho(x_{33}) = (0.3, 0.6) \\ \phi(x_{11}, x_{22}) &= (0.4, 0.5), \phi(x_{22}, x_{33}) = (0.3, 0.6), \phi(x_{33}, x_{11}) = (0.3, 0.6) \\ \chi(x_{11}, x_{11}x_{22}) &= (0.4, 0.5), \chi(x_{22}, x_{11}x_{22}) = (0.4, 0.5), \chi(x_{22}, x_{22}x_{33}) = (0.3, 0.6), \\ \chi(x_{33}, x_{22}x_{33}) &= (0.3, 0.6), \chi(x_{11}, x_{11}x_{33}) = (0.3, 0.6), \chi(x_{33}, x_{11}x_{33}) = (0.3, 0.6) \end{aligned}$$

Assume $D_{IFI} = \{x_{33}\}$. We have $V_{IFI} - D_{IFI} = \{x_{11}, x_{22}\}$. Here x_{33} weakly fuzzy incidence dominates x_{11}, x_{22} because $d_{G_{IFI}}(x_{33}) = 0.2$ is less than the $d_{G_{IFI}}$ of all the remaining vertices. That is $d_{G_{IFI}}(x_{11}) =$

0.3, $d_{G_{IFI}}(x_{33}) = 0.3$. There is no other weak intuitionistic fuzzy incidence dominating sets. Thus the only weak intuitionistic fuzzy incidence dominating set is $D_{IFI} = \{x_{33}\}$. Therefore $\gamma_{WIFI} = 0.4$. We have strong IFIDS is $D_{IFI} = \{x_{11}\}$ with $\gamma_{SIFI} = 0.5$.

Theorem 7.3.1 *For any CFIG with $\chi_1(x_{11}, x_{11}x_{22}) = \min\{\rho_1(x_{11}), \phi_1(x_{11}x_{22})\}$ and $\chi_2(x_{11}, x_{11}x_{22}) = \max\{\rho_2(x_{11}), \phi_2(x_{11}x_{22})\}$ for all $x_{11} \in V_{IFI}, x_{11}x_{22} \in E_{IFI}$, then*

$$(i) \quad \gamma_{SIFI} = \gamma_{WIFI}$$

$$(ii) \quad \gamma_{SIFI} > \gamma_{WIFI}$$

Proof. Let $G_{IFI} = (\rho_{IFI}, \phi_{IFI}, \chi_{IFI})$ be a CFIG with $\chi_1(x_{11}, x_{11}x_{22}) = \min\{\rho_1(x_{11}), \phi_1(x_{11}x_{22})\}$ and $\chi_2(x_{11}, x_{11}x_{22}) = \max\{\rho_2(x_{11}), \phi_2(x_{11}x_{22})\}$. Assume for all $x_{11} \in V_{IFI}, (\rho_1(x_{11}), \rho_2(x_{11}))$ have same value. Since G_{IFI} is CFIG with $\phi_1(x_{11}x_{22}) = \min\{\rho_1(x_{11}), \rho_2(x_{11})\}$ and $\phi_2(x_{11}x_{22}) = \max\{\rho_2(x_{11}), \rho_2(x_{11})\}$ for all $x_{11}, x_{22} \in V_{IFI}$ and $\chi_1(x_{11}, x_{11}x_{22}) = \min\{\rho_1(x_{11}), \phi_1(x_{11}x_{22})\}$ and $\chi_2(x_{11}, x_{11}x_{22}) = \max\{\rho_2(x_{11}), \phi_2(x_{11}x_{22})\}$ for all $x_{11} \in V_{IFI}, x_{11}x_{22} \in E_{IFI}$. Thus every $x_{11} \in V_{IFI}$ is SIFIDS as well as WIFIDS. Therefore $\gamma_{WIFI} = \gamma_{SIFI}$.

Assume for all $x_{11} \in V_{IFI}, (\rho_1(x_{11}), \rho_2(x_{11}))$ have different value. In a CFIG with $d_{G_{IFI}}(x_{11}) \geq d_{G_{IFI}}(x_{22})$ from all the nodes one of them strongly dominates all the remaining nodes, if it is smallest among all the nodes then the IFIDS with that node is called WIFIDN, that is $\gamma_{WIFI} = (\rho_1(x_{11}), \rho_2(x_{11}))$ with $d_{G_{IFI}}(x_{11}) \leq d_{G_{IFI}}(x_{22})$ for all $x_{11}x_{22} \in V_{IFI}$ and $\chi_1(x_{11}, x_{11}x_{22}) = \min\{\rho_1(x_{11}), \phi_1(x_{11}x_{22})\}$ and $\chi_2(x_{11}, x_{11}x_{22}) = \max\{\rho_2(x_{11}), \phi_2(x_{11}x_{22})\}$ for all $x_{11} \in V_{IFI}, x_{11}x_{22} \in E_{IFI}$. Certainly, the strong IFIDS has a node set other than the that node set. This implies $\gamma_{SIFI} > \gamma_{WIFI}$.

Theorem 7.3.2 *For a CFIG, the below inequalities are true.*

$$(i) \quad \gamma_{IFI} \leq \gamma_{SIFI} \leq O(G_{IFI}) - \text{maximum } d_{G_{IFI}} \text{ of } G_{IFI}$$

$$(ii) \quad \gamma_{IFI} \leq \gamma_{WIFI} \leq O(G_{IFI}) - \text{minimum } d_{G_{IFI}} \text{ of } G_{IFI}$$

Proof.

(i) From definition 7.3.2, 7.3.3 and 7.3.4, we have

$$\gamma_{IFI} \leq \gamma_{SIFI} \quad (7.1)$$

We know $O(G_{IFI})$ = the sum of the incidence pair of CFIG.

Also $O(G_{IFI})$ - not including the maximum $d_{G_{IFI}}$ of CFIG

$$= O(G_{IFI}) - \Delta(G_{IFI}) \quad (7.2)$$

From equation (7.1) and (7.2),

$$\gamma_{IFI} \leq \gamma_{SIFI} \leq O(G_{IFI}) - \text{maximum } d_{G_{IFI}} \text{ of } G_{IFI}$$

(ii) From definition 7.3.2, 7.3.3 and 7.3.4 domination number γ_{IFI} of CFIG is less than or equal to the γ_{WIFI} of CFIG, because the vertices of WIFIDS M_{IFI} , it weakly dominates any one of the vertices of $V_{IFI} - M_{IFI}$. Therefore

$$\gamma_{WIFI}(G_{IFI}) \geq \gamma_{IFI}(G_{IFI}) \quad (7.3)$$

Also $O(G_{IFI})$ - not including the minimum $d_{G_{IFI}}$ of CFIG

$$= O(G_{IFI}) - \delta(G_{IFI}) \quad (7.4)$$

From equation (7.3) and (7.4), we get

$$\gamma_{IFI} \leq \gamma_{WIFI} \leq O(G_{IFI}) - \text{minimum } d_{G_{IFI}} \text{ of } G_{IFI}$$

7.4 Application

Here, we incorporate an every day life model. Assume there are five multispeciality clinics are working (24 hours) in a city for giving crisis treatment to individuals. Here in our examination we are not referencing the original names of these clinics in this manner think about the clinics $h_{11}, h_{22}, h_{33}, h_{44}$ and h_{55} . In CIFIGs, the vertices show the clinics and edges show the contract conditions between the clinics to share the facilities. The incidence pairs show the transferring of patients from one clinic to another because of the lack of resources. The vertex $h_{11}(0.4, 0.6)$ means that it has 40% of the necessary facilities for treatment and unfortunately lacks 60% of the equipment. The edge $h_{11}h_{22}(0.14, 0.86)$ shows that there is only 14% of the interaction and relationship between the two clinics, and due to financial issues, there is 86% on the conflict between them. IFIDS ruling arrangements of the graph is the arrangement of clinics which give the crisis treatment autonomously. Along these lines, we can save the time of patients and conquer the long going of patients by giving the couple of offices to the remainder of the clinics.



Figure 7.4.1: MULTISPECIALITY CLINIC

Assume $G_{IFI} = (V_{IFI}, E_{IFI}, I_{IFI}, \rho_{IFI}, \phi_{IFI}, \chi_{IFI})$ is a CIFIG show in figure 7.4.2 having $V_{IFI} =$

$(h_{11}, h_{22}, h_{33}, h_{44}, h_{55})$ and

$$\begin{aligned} \rho(h_{11}) &= (0.4, 0.6), \rho(h_{22}) = (0.14, 0.86), \rho(h_{33}) = (0.52, 0.48), \rho(h_{44}) = (0.24, 0.76), \\ \rho(h_{55}) &= (0.24, 0.76), \phi(h_{11}, h_{22}) = (0.14, 0.86), \phi(h_{11}, h_{33}) = (0.4, 0.6), \phi(h_{11}, h_{44}) = (0.24, 0.76), \\ \phi(h_{33}, h_{44}) &= (0.24, 0.76), \phi(h_{44}, h_{55}) = (0.24, 0.76), \chi(h_{11}, h_{11}h_{22}) = (0.14, 0.86), \\ \chi(h_{22}, h_{11}h_{22}) &= (0.14, 0.86), \chi(h_{11}, h_{11}h_{33}) = (0.4, 0.6), \chi(h_{33}, h_{11}h_{33}) = (0.4, 0.6), \\ \chi(h_{11}, h_{11}h_{44}) &= (0.24, 0.76), \chi(h_{44}, h_{11}h_{44}) = (0.24, 0.76), \chi(h_{33}, h_{33}h_{44}) = (0.24, 0.76), \\ \chi(h_{44}, h_{33}h_{44}) &= (0.24, 0.76), \chi(h_{44}, h_{44}h_{55}) = (0.24, 0.76), \chi(h_{55}, h_{44}h_{55}) = (0.24, 0.76) \end{aligned}$$

Example 7.4.1

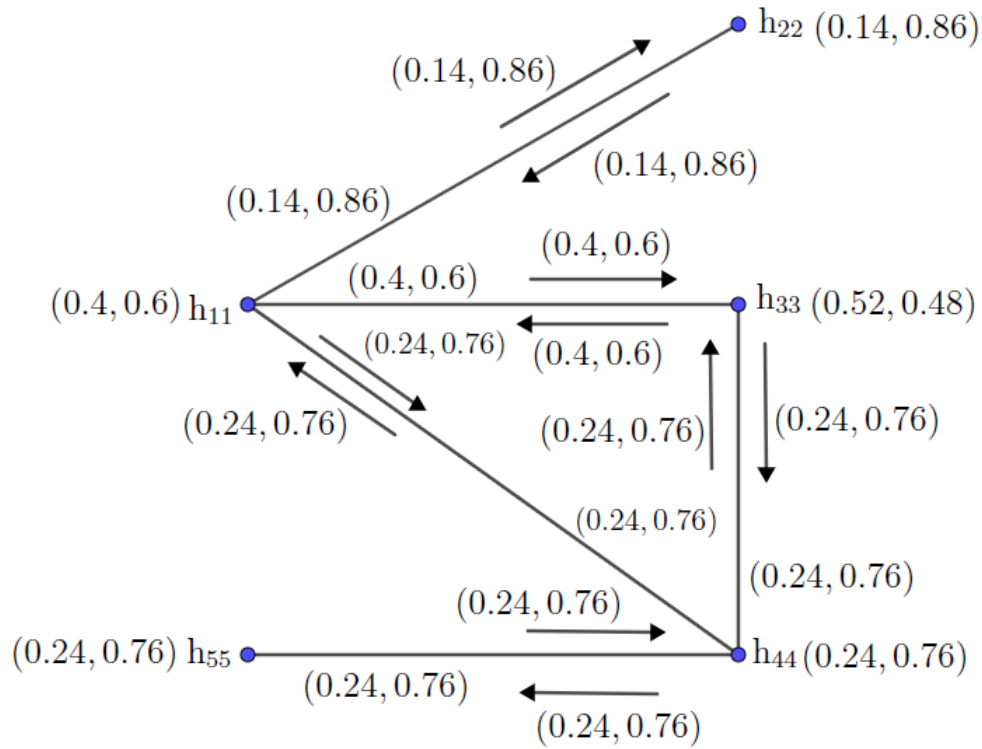


Figure 7.4.2: CIFIG with $\gamma_{IFI} = 0.38$

In figure having intuitionistic fuzzy incidence dominating set are $D_{IFI} = \{h_{22}, h_{44}\}$ and $\gamma_{IFI} = 0.38$.

This shows that patients can visit any one of the clinics from this set. The rest of the clinics upgrade their facilities to provide better treatment to the people.

The dominance theory survey is intriguing because of the wide range of applications and dominant qualities that can be established. Domination in complete intuitionistic fuzzy incidence graphs is essential from both a religious and an application standpoint. The feasibility of a complete intuitionistic fuzzy incidence graph, as well as strong and weak intuitionistic fuzzy incidence dominating sets and strong and weak intuitionistic fuzzy incidence dominating numbers, is examined in this research. We compute the intuitionistic fuzzy incidence domination number, strong intuitionistic fuzzy incidence domination number, and weak intuitionistic fuzzy incidence domination number for distinct classifications of complete intuitionistic fuzzy incidence graphs, as well as associated theorems. The relationship between the complete intuitionistic fuzzy incidence graph of an intuitionistic fuzzy incidence domination number, strong intuitionistic fuzzy incidence domination number, order, and maximum degree cardinality is examined.

Chapter 8

Fixed Domination in Product Picture Fuzzy Graphs

Fuzzy graph algorithms can be used to model and solve a wide range of practical issues. Fuzzy graph theory, in general, has a wide range of applications in a wide range of domains. Because ambiguous information is a common real-life problem that is frequently uncertain, an expert must model these challenges using a fuzzy graph. A useful mathematical model for dealing with uncertain real-world circumstances is the picture fuzzy set. The picture fuzzy set (PFS) is a variant of the traditional fuzzy set. It can be particularly effective in ambiguous settings that require more yes, no, abstain, and rejection responses. This research introduces the concept of a product picture fuzzy graph. Some varieties of product picture fuzzy graphs are discussed, including strong product picture fuzzy graph and complete product picture fuzzy graph, as well as their features. One of the most extensively utilized notions in numerous areas is dominance in fuzzy graphs theory. Many current research investigations are attempting to uncover new uses for dominance in their sector if there is enough interest. As a result, in this chapter, we introduce various types of dominating sets in product picture fuzzy graphs, such as the fixed vertex dominating set, fixed edge dominating set, total fixed edge dominating set, and fixed edge restrained dominating set, and try to represent their properties using examples. Finally, we give a medical example to demonstrate the importance of fixed vertex domination (FVD) in product picture fuzzy graphs.

8.1 Product Picture Fuzzy Graphs

Definition 8.1.1 Let R_{PP} be product picture fuzzy set (PPFS). R_{PP} in X_{PP} is defined by $R_{PP} = \{x_{PP}, \mu_{R_{PP}}(x_{PP}), \eta_{R_{PP}}(x_{PP}), \gamma_{R_{PP}}(x_{PP})/x_{PP} \in X_{PP}\}$, where $\mu_{R_{PP}}(x_{PP})$, $\eta_{R_{PP}}(x_{PP})$ and $\gamma_{R_{PP}}(x_{PP})$ follow the condition $0 \leq \mu_{R_{PP}}(x_{PP}) \times \eta_{R_{PP}}(x_{PP}) \times \gamma_{R_{PP}}(x_{PP}) \leq 1$. The $\mu_{R_{PP}}(x_{PP})$, $\eta_{R_{PP}}(x_{PP})$, $\gamma_{R_{PP}}(x_{PP}) \in [0, 1]$, denote respectively the positive membership degree, neutral membership degree and negative membership degree of the element x_{PP} in the set R_{PP} . For each PPFS R_{PP} in X_{PP} , the refusal membership degree is described as

$$\pi_{R_{PP}}(x_{PP}) = 1 - \{\mu_{R_{PP}}(x_{PP}) \times \eta_{R_{PP}}(x_{PP}) \times \gamma_{R_{PP}}(x_{PP})\}.$$

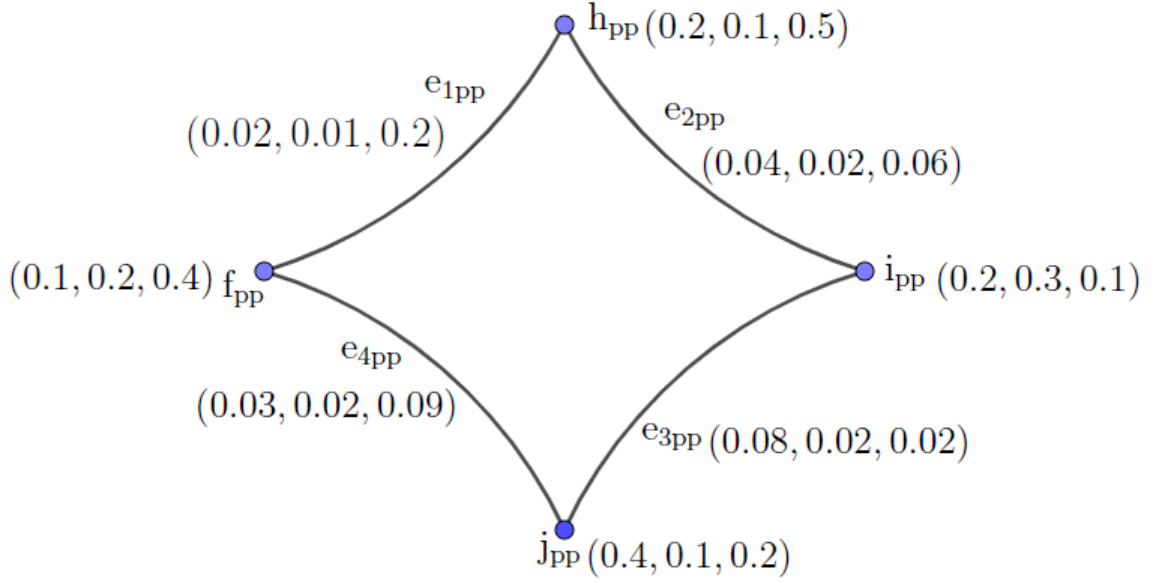
Definition 8.1.2 Assume $G_{PP}^* = (V_{PP}, E_{PP})$ is a graph. A pair $\xi_{PP} = (R_{PP}, S_{PP})$ is referred a product picture fuzzy graph on G_{PP}^* where $R_{PP} = \{\mu_{R_{PP}}, \eta_{R_{PP}}, \gamma_{R_{PP}}\}$ is a PPFS on V_{PP} and $S_{PP} = \{\mu_{S_{PP}}, \eta_{S_{PP}}, \gamma_{S_{PP}}\}$ is a PPFS on $E_{PP} \subseteq V_{PP} \times V_{PP}$ such that for each edge $f_{PP}h_{PP} \in E_{PP}$.

$$\mu_{S_{PP}}(f_{PP}h_{PP}) \leq \mu_{R_{PP}}(f_{PP}) \times \mu_{R_{PP}}(h_{PP}),$$

$$\eta_{S_{PP}}(f_{PP}h_{PP}) \leq \eta_{R_{PP}}(f_{PP}) \times \eta_{R_{PP}}(h_{PP}),$$

$$\gamma_{S_{PP}}(f_{PP}h_{PP}) \geq \gamma_{R_{PP}}(f_{PP}) \times \gamma_{R_{PP}}(h_{PP}).$$

Example 8.1.1 Consider a PPFG ξ_{PP} as in Figure 8.1.1, such that $V_{PP} = \{f_{PP}, h_{PP}, i_{PP}, j_{PP}\}$ $E_{PP} = \{f_{PP}h_{PP}, h_{PP}i_{PP}, i_{PP}j_{PP}, j_{PP}f_{PP}\}$

Figure 8.1.1: PPFG ξ_{PP}

Note 8.1.1 There is no edge between f_{PP} and h_{PP} when $\mu_{S_{PP}}(f_{PP}h_{PP}) = \eta_{S_{PP}}(f_{PP}h_{PP}) = \gamma_{S_{PP}}(f_{PP}h_{PP}) = 0$.

Remark 8.1.1 A product picture fuzzy graph is not necessarily a picture fuzzy graph.

Example 8.1.2 In Figure 8.1.1, it is easy to show that PPFG but not PFG.

Definition 8.1.3 A product picture fuzzy graph $\xi_{PP} = (R_{PP}, S_{PP})$ is said to be strong product picture fuzzy graph if

$$\mu_{S_{PP}}(f_{PP}h_{PP}) = \mu_{R_{PP}}(f_{PP}) \times \mu_{R_{PP}}(h_{PP}),$$

$$\eta_{S_{PP}}(f_{PP}h_{PP}) = \eta_{R_{PP}}(f_{PP}) \times \eta_{R_{PP}}(h_{PP}),$$

$$\gamma_{S_{PP}}(f_{PP}h_{PP}) = \gamma_{R_{PP}}(f_{PP}) \times \gamma_{R_{PP}}(h_{PP}) \quad \forall f_{PP}h_{PP} \in E_{PP}.$$

Example 8.1.3 Consider a strong PPFG ξ_{PP} as in Figure 8.1.2, such that $V_{PP} = \{f_{PP}, h_{PP}, i_{PP}, j_{PP}\}$
 $E_{PP} = \{f_{PP}h_{PP}, h_{PP}i_{PP}, i_{PP}j_{PP}, j_{PP}f_{PP}, f_{PP}i_{PP}\}$.

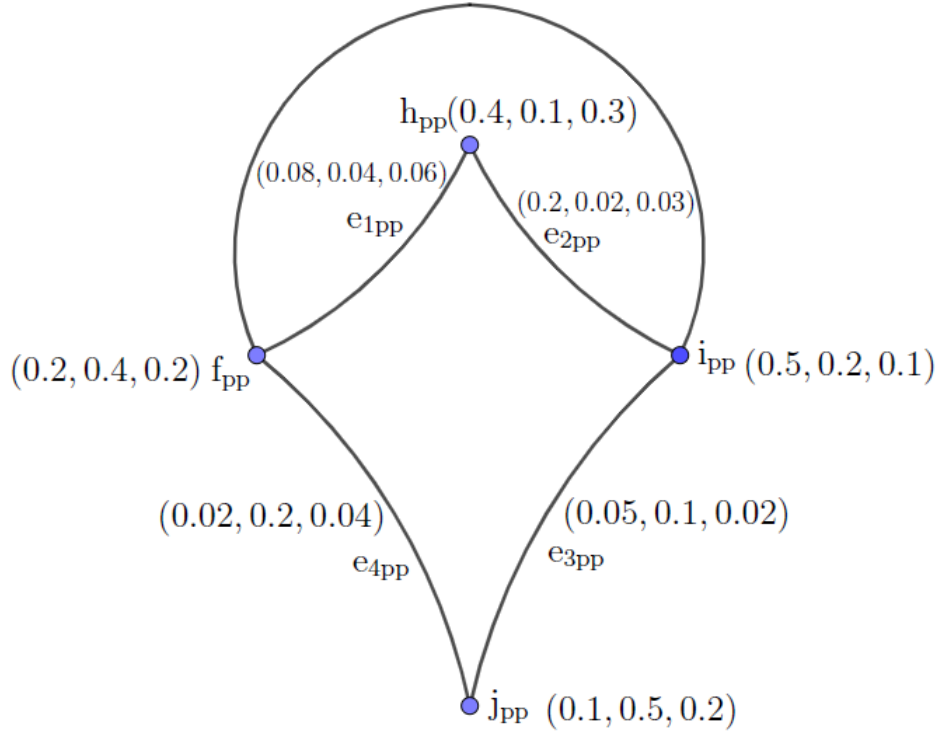
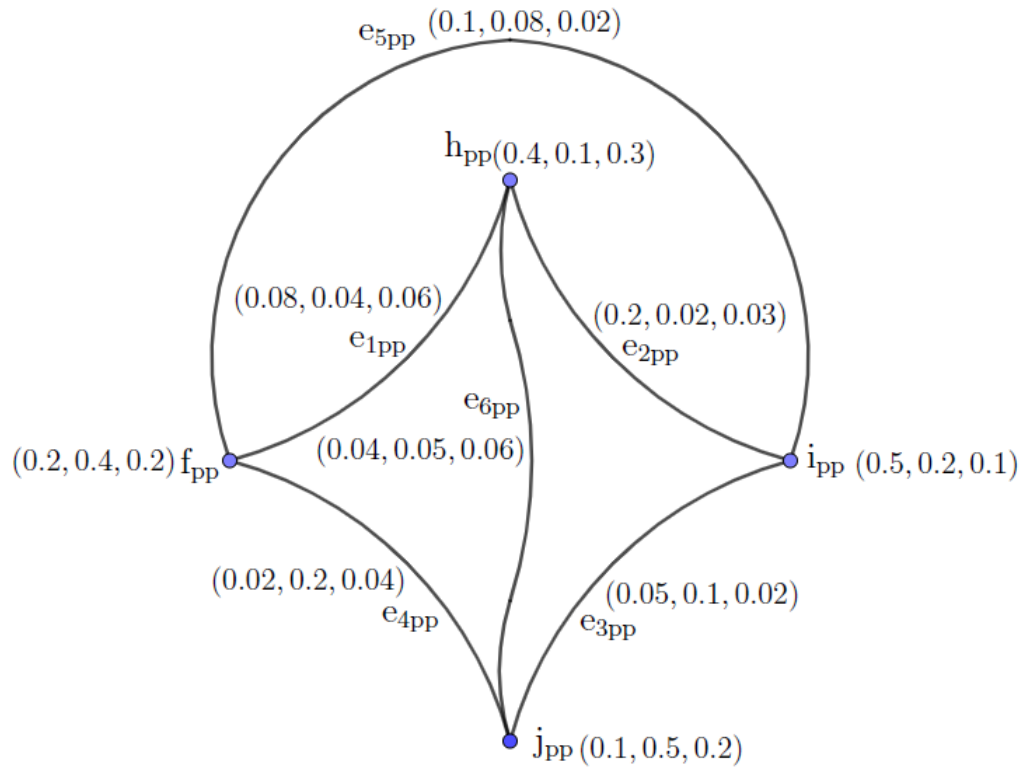


Figure 8.1.2: Strong PPFG ξ_{PP}

Definition 8.1.4 A product picture fuzzy graph $\xi_{PP} = (R_{PP}, S_{PP})$ is defined as complete PPFG if

$$\begin{aligned}\mu_{S_{PP}}(f_{PP}, h_{PP}) &= \mu_{R_{PP}}(f_{PP}) \times \mu_{R_{PP}}(h_{PP}), \\ \eta_{S_{PP}}(f_{PP}h_{PP}) &= \eta_{R_{PP}}(f_{PP}) \times \eta_{R_{PP}}(h_{PP}), \\ \gamma_{S_{PP}}(f_{PP}h_{PP}) &= \gamma_{R_{PP}}(f_{PP}) \times \gamma_{R_{PP}}(h_{PP}) \quad \forall f_{PP}h_{PP} \in E_{PP}.\end{aligned}$$

Example 8.1.4 Consider a complete PPFG ξ_{PP} as in Figure 8.1.3, such that $V_{PP} = \{f_{PP}, h_{PP}, i_{PP}, j_{PP}\}$
 $E_{PP} = \{f_{PP}h_{PP}, h_{PP}i_{PP}, i_{PP}j_{PP}, j_{PP}f_{PP}, f_{PP}i_{PP}, h_{PP}j_{PP}\}$.

Figure 8.1.3: Complete PPFG ξ_{PP}

Remark 8.1.2 Every complete product picture fuzzy graph is a strong product picture fuzzy graph but not conversely.

Example 8.1.5 In Figure 8.1.2, it is simple to demonstrate that ξ_{PP} is a strong PPFG but not a complete PPFG.

Definition 8.1.5 Let $\xi_{PP} = (R_{PP}, S_{PP})$ be a product picture fuzzy graph

1. ξ_{PP} cardinality is determined by

$$|\xi_{PP}| = \sum_{f_{iPP} \in V_{PP}} \frac{1 + \mu_{R_{PP}}(f_{iPP}) - \eta_{R_{PP}}(f_{iPP}) - \gamma_{R_{PP}}(f_{iPP})}{2} + \sum_{f_{iPP}, h_{iPP} \in E_{PP}} \frac{1 + \mu_{S_{PP}}(f_{iPP}, h_{iPP}) - \eta_{S_{PP}}(f_{iPP}, h_{iPP}) - \gamma_{S_{PP}}(f_{iPP}, h_{iPP})}{2}$$

2. ξ_{PP} vertex cardinality is determined by

$$|V_{PP}| = \sum_{f_{iPP} \in V_{PP}} \frac{1 + \mu_{R_{PP}}(f_{iPP}) - \eta_{R_{PP}}(f_{iPP}) - \gamma_{R_{PP}}(f_{iPP})}{2} \forall f_{iPP} \in V_{PP},$$

is referred the order of a PPFG ξ_{PP} , and it is denoted by $p(\xi_{PP})$.

3. ξ_{PP} edge cardinality is specified by

$$|E_{PP}| = \sum_{f_{iPP}, h_{iPP} \in E_{PP}} \frac{1 + \mu_{S_{PP}}(f_{iPP}, h_{iPP}) - \eta_{S_{PP}}(f_{iPP}, h_{iPP}) - \gamma_{S_{PP}}(f_{iPP}, h_{iPP})}{2},$$

$\forall f_{iPP}, h_{iPP} \in E_{PP}$ is referred the size of a PPFG ξ_{PP} , and it is denoted by $q(\xi_{PP})$.

Example 8.1.6 In Figure 8.1.1,

$$|V_{PP}| = 0.25 + 0.3 + 0.4 + 0.55 = 1.5$$

$$|E_{PP}| = 0.405 + 0.48 + 0.52 + 0.46 = 1.865$$

$$|\xi_{PP}| = 1.5 + 1.865 = 3.365$$

Definition 8.1.6 An edge f_{PP}, h_{PP} in a product picture fuzzy graph $\xi_{PP} = (R_{PP}, S_{PP})$ is called the strong edge if

$$\mu_{S_{PP}}(f_{PP}, h_{PP}) \geq \mu_{S_{PP}}^{\infty}(f_{PP}, h_{PP}),$$

$$\eta_{S_{PP}}(f_{PP}, h_{PP}) \geq \eta_{S_{PP}}^{\infty}(f_{PP}, h_{PP}),$$

$$\gamma_{S_{PP}}(f_{PP}, h_{PP}) \leq \gamma_{S_{PP}}^{\infty}(f_{PP}, h_{PP})$$

Example 8.1.7 In Figure 8.1.1, $e_{2PP}, e_{3PP}, e_{4PP}$ are strong edges.

8.2 Fixed Vertex Domination in Product Picture Fuzzy Graphs

Definition 8.2.1 In a product picture fuzzy graph, two vertices f_{PP} and h_{PP} are considered to be neighbors if one of the following conditions holds.

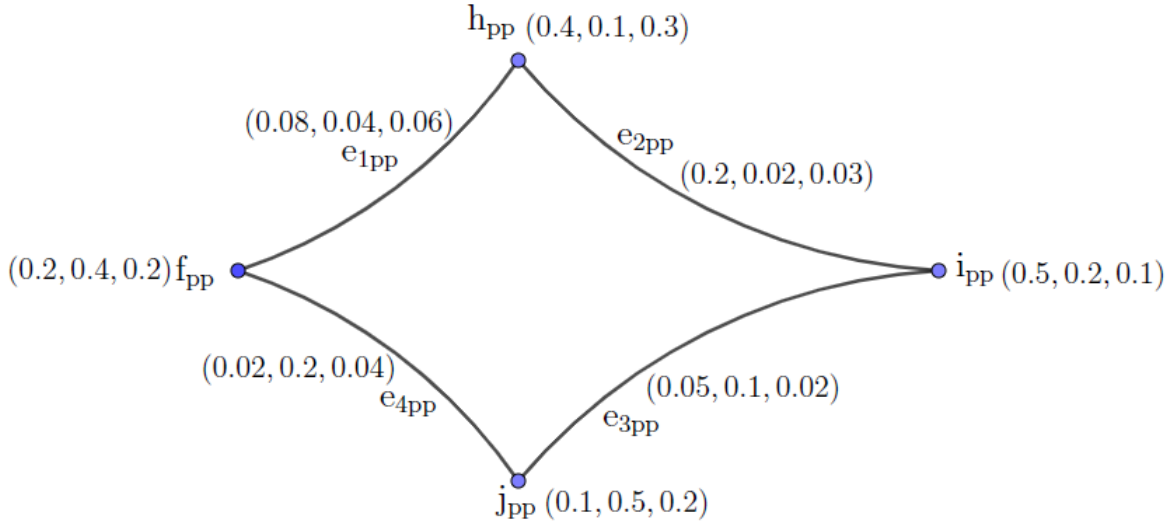
1. $\mu_{S_{PP}}(f_{PP}h_{PP}) > 0, \eta_{S_{PP}}(f_{PP}h_{PP}) > 0, \gamma_{S_{PP}}(f_{PP}h_{PP}) > 0$
2. $\mu_{S_{PP}}(f_{PP}h_{PP}) = 0, \eta_{S_{PP}}(f_{PP}h_{PP}) \geq 0, \gamma_{S_{PP}}(f_{PP}h_{PP}) > 0$
3. $\mu_{S_{PP}}(f_{PP}h_{PP}) > 0, \eta_{S_{PP}}(f_{PP}h_{PP}) = 0, \gamma_{S_{PP}}(f_{PP}h_{PP}) > 0$
4. $\mu_{S_{PP}}(f_{PP}h_{PP}) \geq 0, \eta_{S_{PP}}(f_{PP}h_{PP}) > 0, \gamma_{S_{PP}}(f_{PP}h_{PP}) > 0, \forall f_{PP}h_{PP} \in V_{PP}.$

Definition 8.2.2 In a product picture fuzzy graph ξ_{PP} , the two vertices f_{PP} and h_{PP} are considered to be strong neighbours if

$$\begin{aligned}\mu_{S_{PP}}(f_{PP}h_{PP}) &= \mu_{R_{PP}}(f_{PP}) \times \mu_{R_{PP}}(h_{PP}) \\ \eta_{S_{PP}}(f_{PP}h_{PP}) &= \eta_{R_{PP}}(f_{PP}) \times \eta_{R_{PP}}(h_{PP}) \\ \gamma_{S_{PP}}(f_{PP}h_{PP}) &= \gamma_{R_{PP}}(f_{PP}) \times \gamma_{R_{PP}}(h_{PP})\end{aligned}$$

Definition 8.2.3 Let ξ_{PP} be a PPFG and f_{PP} and h_{PP} are neighbors of ξ_{PP} . We say that f_{PP} dominates h_{PP} if they are strong neighbors. An vertex subset M_{PP} of V_{PP} in a PPFG ξ_{PP} is called an fixed vertex dominating set (FVDS), if for each vertex $V_{PP} - M_{PP}$ is dominates exactly one vertex in M_{PP} . An fixed vertex dominating set M_{PP} of a PPFG ξ_{PP} is said to be a minimal fixed vertex dominating set if for each edge $f_{PP} \in M_{PP}$, $M_{PP} - \{f_{PP}\}$ is not an fixed vertex dominating set. An fixed vertex domination number (FVDN) of ξ_{PP} is the smallest cardinality between all minimal fixed vertex dominating sets, and it is described by $\gamma_{V_{PP}}(\xi_{PP})$ or simply $\gamma_{V_{PP}}$.

Example 8.2.1 Consider the PPFG ξ_{PP} as in figure 8.2.1, $D_{11} = \{f_{PP}, h_{PP}\}$, $D_{22} = \{h_{PP}, i_{PP}\}$, $D_{33} = \{i_{PP}, j_{PP}\}$, $D_{44} = \{f_{PP}, j_{PP}\}$ is a fixed vertex dominating sets and $\gamma_{V_{PP}}(\xi_{PP}) = 0.5$.

Figure 8.2.1: PPFG ξ_{PP} with FVDS

8.3 Fixed Edge Domination in Product Picture Fuzzy Graphs

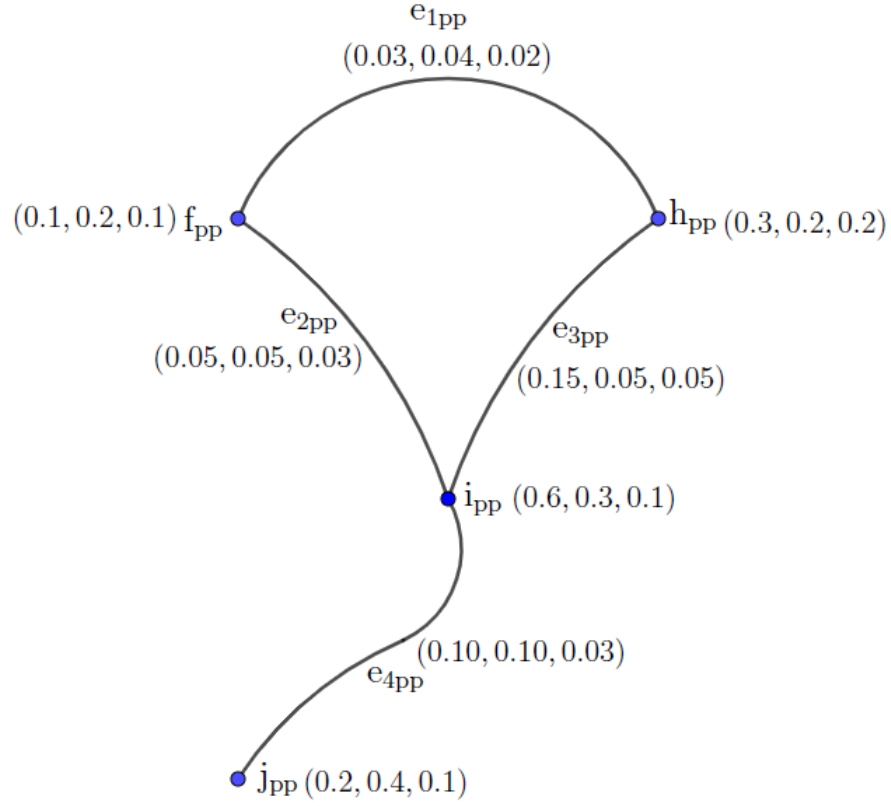
Definition 8.3.1 If two edges e_{1PP} and e_{2PP} in a PPFG ξ_{PP} are neighbours, they are said to be adjacent.

Definition 8.3.2 An edge subset L_{PP} of E_{PP} in a PPFG ξ_{PP} is referred an edge independent set (EIS) if

$$\begin{aligned}\mu_{S_{PP}}(f_{PP}h_{PP}) &< \mu_{R_{PP}}(f_{PP}) \times \mu_{R_{PP}}(h_{PP}) \\ \eta_{S_{PP}}(f_{PP}h_{PP}) &< \eta_{R_{PP}}(f_{PP}) \times \eta_{R_{PP}}(h_{PP}) \\ \gamma_{S_{PP}}(f_{PP}h_{PP}) &> \gamma_{R_{PP}}(f_{PP}) \times \gamma_{R_{PP}}(h_{PP}) \forall f_{PP}h_{PP} \in L_{PP}.\end{aligned}$$

The edge independent number (EIN) is the highest cardinality among all maximal edge independent set (EIS) in ξ_{PP} , and it is indicated by $\beta_{IPP}(\xi_{PP})$ or β_{IPP} .

Example 8.3.1 In figure 8.3.1, $\{e_{2PP}, e_{3PP}\}$, $\{e_{2PP}, e_{4PP}\}$, $\{e_{3PP}, e_{4PP}\}$, $\{e_{2PP}, e_{3PP}, e_{4PP}\}$ are EISs in ξ_{PP} and $\beta_{IPP}(\xi_{PP}) = 1.495$.

Figure 8.3.1: PPFG ξ_{PP} with EISs

Definition 8.3.3 If an edge e_{1PP} and a vertex k_{PP} in a product picture fuzzy graph ξ_{PP} are incident, they are said to cover each other.

Definition 8.3.4 An edge subset L_{PP} of E_{PP} in a PPFG ξ_{PP} , which covers all nodes in ξ_{PP} , is termed a edge cover set (ECS) of ξ_{PP} . The edge cover number (ECN) of ξ_{PP} is the lowest cardinality among all edge cover set (ECS), and it is denoted by $\alpha_{CPP}(\xi_{PP})$ or simply α_{CPP} .

Example 8.3.2 Consider the PPFG ξ_{PP} in figure 8.1.1.

Here $\{e_{1PP}, e_{3PP}\}$ and $\{e_{2PP}, e_{4PP}\}$ are ECSs and $\alpha_{CPP}(\xi_{PP}) = 0.925$.

Theorem 8.3.1 An edge subset $L_{PP} \subseteq E_{PP}$ in a product picture fuzzy graph ξ_{PP} is an edge independent set in ξ_{PP} if $E_{PP} - L_{PP}$ is an edge cover set of ξ_{PP} .

Proof. L_{PP} is an EIS if and only if no two of its edges are adjacent, if and only if each of its edges is incident with at least one vertex of $E_{PP} - L_{PP}$, and if and only if $E_{PP} - L_{PP}$ is an ECS of ξ_{PP} .

Example 8.3.3 Consider the PPFG ξ_{PP} as in figure 8.3.2. It is easy to show that $L_{PP} = \{e_{2PP}, e_{4PP}\}$ is an EIS and $E_{PP} - L_{PP} = \{e_{1PP}, e_{3PP}\}$ is an ECS.

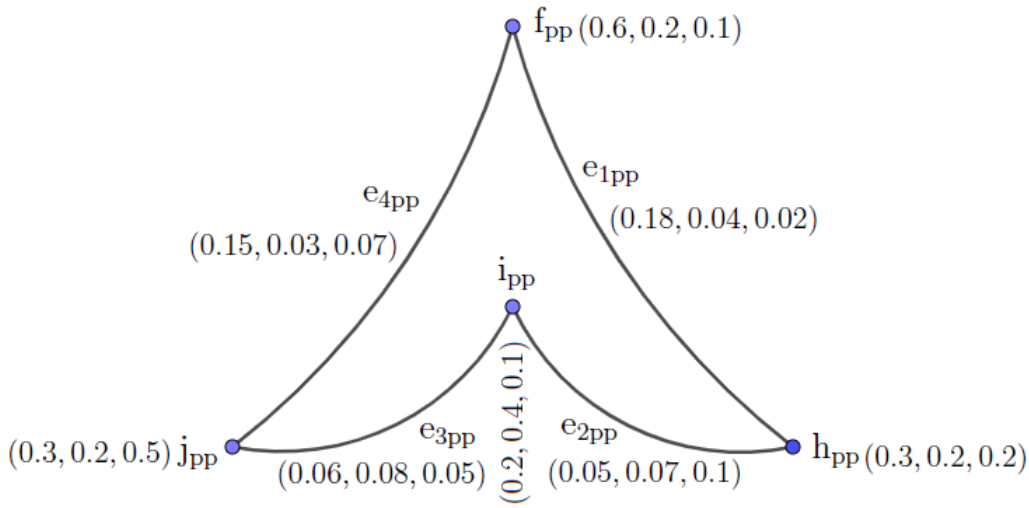


Figure 8.3.2: PPFG ξ_{PP} with EIS and ECS

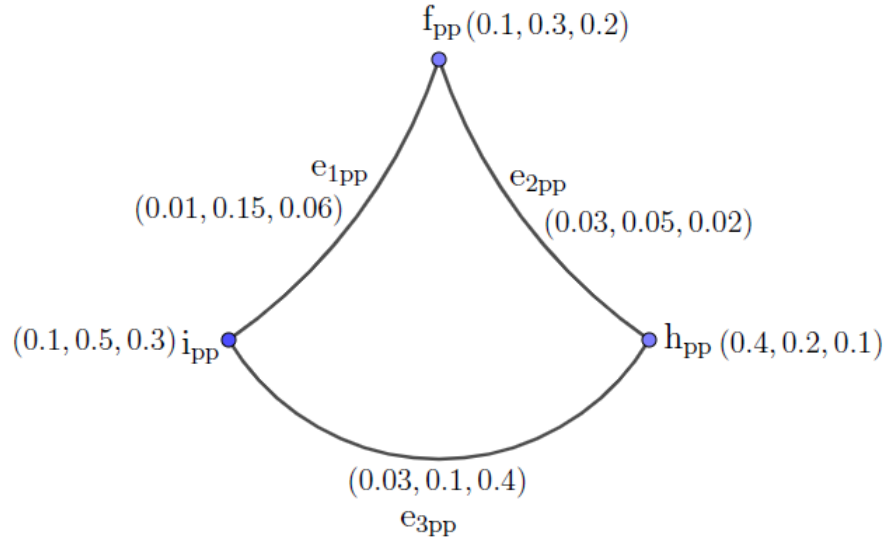
Definition 8.3.5 An edge $f_{PP}h_{PP}$ in a product picture fuzzy graphs ξ_{PP} is labeled an effective edge if

$$\mu_{S_{PP}}(f_{PP}h_{PP}) = \mu_{R_{PP}}(f_{PP}) \times \mu_{R_{PP}}(h_{PP})$$

$$\eta_{S_{PP}}(f_{PP}h_{PP}) = \eta_{R_{PP}}(f_{PP}) \times \eta_{R_{PP}}(h_{PP})$$

$$\gamma_{S_{PP}}(f_{PP}h_{PP}) = \gamma_{R_{PP}}(f_{PP}) \times \gamma_{R_{PP}}(h_{PP}).$$

Example 8.3.4 Consider a PPFG ξ_{PP} as in figure 8.3.3, such that $V_{PP} = \{f_{PP}, h_{PP}, i_{PP}\}$
 $E_{PP} = \{f_{PP}h_{PP}, h_{PP}i_{PP}, i_{PP}f_{PP}\}$.

Figure 8.3.3: PPFG ξ_{PP} with effective edge

Here e_{1PP} is an effective edge.

$$\mu_{S_{PP}}(f_{PP}i_{PP}) = 0.1 \times 0.1 = 0.01$$

$$\eta_{S_{PP}}(f_{PP}i_{PP}) = 0.3 \times 0.5 = 0.15$$

$$\gamma_{S_{PP}}(f_{PP}i_{PP}) = 0.2 \times 0.3 = 0.06.$$

Definition 8.3.6 An edge e_{PP} of a PPFG ξ_{PP} is said to be an isolated edge (IE) if no effective edge are incident with the vertices of e_{PP} . As a result, no other edge in ξ_{PP} is dominated by an isolated edge.

Example 8.3.5 In figure 8.3.2, the edges e_{1PP} and e_{3PP} are isolated edges.

Theorem 8.3.2 For any PPFG $\xi_{PP} = (R_{PP}, S_{PP})$ with isolated edges, $\alpha_{CPP}(\xi_{PP}) + \beta_{IPP}(\xi_{PP}) = q_{PP}$.

Proof. Let L_{PP} be an edge independent set in ξ_{PP} and M_{PP} be an edge cover set in ξ_{PP} so that $|L_{PP}| = \beta_{IPP}(\xi_{PP})$ and $|M_{PP}| = \alpha_{CPP}(\xi_{PP})$. Then, by theorem 8.3.1, $E_{PP} - L_{PP}$ is an edge cover set of ξ_{PP} .

Therefore, $|M_{PP}| \leq |E_{PP} - L_{PP}|$ and $\alpha_{CPP}(\xi_{PP}) \leq q_{PP} - \beta_{IPP}(\xi_{PP})$ or

$$\alpha_{CPP}(\xi_{PP}) + \beta_{IPP}(\xi_{PP}) \leq q_{PP} \quad (8.1)$$

Furthermore, by theorem 8.3.1, $E_{PP} - M_{PP}$ is an EIS in ξ_{PP} , so $|L_{PP}| \geq |E_{PP} - M_{PP}|$. Therefore,

$$\beta_{IPP}(\xi_{PP}) \geq q_{PP} - \alpha_{CPP}(\xi_{PP}) \text{ or } \alpha_{CPP}(\xi_{PP}) + \beta_{IPP}(\xi_{PP}) \geq q_{PP} \quad (8.2)$$

From (8.1) and (8.2), we obtain $\alpha_{CPP}(\xi_{PP}) + \beta_{IPP}(\xi_{PP}) = q_{PP}$.

Example 8.3.6 In figure 8.3.2, $\alpha_{CPP}(\xi_{PP}) = 1.025$, $\beta_{IPP}(\xi_{PP}) = 0.965$ and $q_{PP} = 1.99$.

Definition 8.3.7 Let e_{PP} be any edge in a PPFG ξ_{PP} . Then, $N(e_{PP}) = \{m_{PP} \in E_{PP} : m_{PP} \text{ is an effective edge incident with the nodes of } e_{PP}\}$ and is called the open degree neighborhood set of e_{PP} . $N[e_{PP}] = N(e_{PP}) \cup \{e_{PP}\}$ is named the closed neighborhood set of e_{PP} .

Definition 8.3.8 Let e_{PP} be any edge in a PPFG ξ_{PP} . Then, $d_N(e_{PP}) = \sum_{l_{PP} \in N(e_{PP})} |l_{PP}|$ is termed the edge neighborhood degree of e_{PP} . The minimum edge neighborhood degree of a PPFG ξ_{PP} is $\delta_N(\xi_{PP}) = \min\{d_N(e_{PP})/e_{PP} \in E_{PP}\}$. The maximum edge neighborhood degree of a PPFG ξ_{PP} is $\Delta_N(\xi_{PP}) = \max\{d_N(e_{PP})/e_{PP} \in E_{PP}\}$.

Example 8.3.7 Consider the PPFG ξ_{PP} as in figure 8.1.2. It is obvious that $N(e_{1PP}) = \{e_{2PP}, e_{4PP}, e_{5PP}\}$ and $d_N(e_{1PP}) = 1.465$.

Definition 8.3.9 Two edges e_{1PP} and e_{2PP} in a product picture fuzzy graph ξ_{PP} , are said to be strong neighbor if they are effective edges.

Definition 8.3.10 Let ξ_{PP} be a PPFG and e_{1PP} and e_{2PP} be two edges of ξ_{PP} . We say that e_{1PP} dominates e_{2PP} if e_{1PP} is effective edge and they are adjacent. An edge subset L_{PP} of E_{PP} in a PPFG ξ_{PP} is named an fixed edge dominating set (FEDS) if, for each edge $E_{PP} - L_{PP}$ is dominates exactly one edge in L_{PP} . An fixed edge dominating set L_{PP} of a PPFG ξ_{PP} is said to be a minimal fixed edge dominating set if for each edge $e_{PP} \in L_{PP}$, $L_{PP} - \{e_{PP}\}$, is not an fixed edge dominating set. An fixed edge domination

number (FEDN) of ξ_{PP} is the least cardinality between all minimal fixed edge dominating sets and is denoted by $\gamma_{E_{PP}}(\xi_{PP})$ or simply $\gamma_{E_{PP}}$. A fixed edge dominating set L_{PP} of a PPFG ξ_{PP} is said to be independent if

$$\begin{aligned}\mu_{S_{PP}}(f_{PP}h_{PP}) &< \mu_{R_{PP}}(f_{PP}) \times \mu_{R_{PP}}(h_{PP}), \\ \eta_{S_{PP}}(f_{PP}h_{PP}) &< \eta_{R_{PP}}(f_{PP}) \times \eta_{R_{PP}}(h_{PP}), \\ \gamma_{S_{PP}}(f_{PP}h_{PP}) &> \gamma_{R_{PP}}(f_{PP}) \times \gamma_{R_{PP}}(h_{PP}), \quad \forall f_{PP}h_{PP} \in L_{PP}.\end{aligned}$$

Example 8.3.8

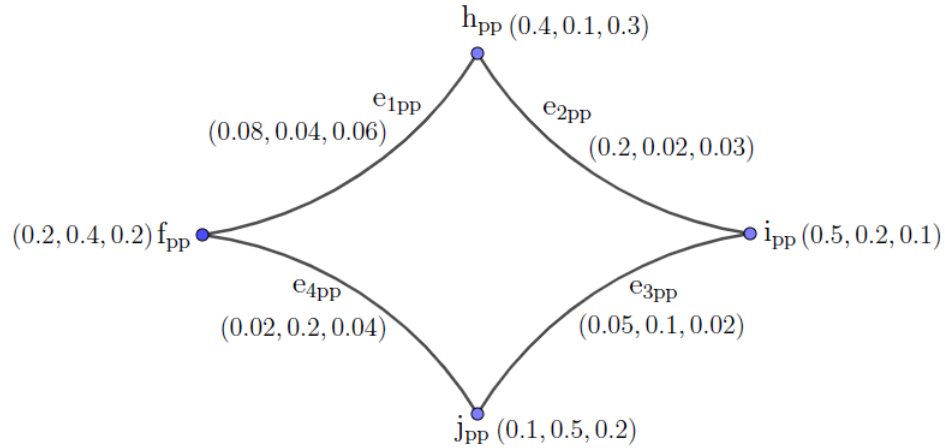


Figure 8.3.4: PPFG ξ_{PP} with FEDSs

Consider a PPFG ξ_{PP} as in figure 8.3.4, $D_{11} = \{e_{1PP}, e_{2PP}\}$, $D_{22} = \{e_{2PP}, e_{3PP}\}$, $D_{33} = \{e_{3PP}, e_{4PP}\}$ and $D_{44} = \{e_{1PP}, e_{4PP}\}$ are FEDSs and $\gamma_{E_{PP}}(\xi_{PP}) = 0.88$.

Theorem 8.3.3 For any product picture fuzzy graph without isolated edges, $\gamma_{E_{PP}}(\xi_{PP}) \leq \frac{q_{PP}}{2}$.

Proof. Any product picture fuzzy graph without isolated edges has two disjoint fixed edge dominating sets and hence $\gamma_{E_{PP}}(\xi_{PP}) \leq \frac{q_{PP}}{2}$.

Example 8.3.9 Consider the PPFG ξ_{PP} as in figure 8.3.4 with $q_{PP} = 1.92$ and $\gamma_{E_{PP}}(\xi_{PP}) = 0.88 < \frac{q_{PP}}{2} = 0.96$.

Theorem 8.3.4 *An edge independent set L_{PP} of a PPFG ξ_{PP} is a maximal edge independent set iff it is an edge independent set and fixed edge dominating set.*

Proof. Let L_{PP} be a maximal EIS in a PPFG ξ_{PP} and, hence for each edge $e_{PP} \in E_{PP} - L_{PP}$, the set $L_{PP} \cup \{e_{PP}\}$ is not independent. For each edge $e_{PP} \in E_{PP} - L_{PP}$ is dominated exactly one edge in L_{PP} . Hence, L_{PP} is an FEDS. Therefore, L_{PP} is both an FEDS and EIS.

Conversely, assume L_{PP} is both independent and an FEDS. Suppose that L_{PP} is not a maximal EIS, then there exist an edge $e_{PP} \in E_{PP} - L_{PP}$, and the set $L_{PP} \cup \{e_{PP}\}$ is independent. If $L_{PP} \cup \{e_{PP}\}$ is independent, then no effective edge in L_{PP} is strong neighbor to e_{PP} . Therefore, L_{PP} cannot be an FEDS, which is a contradiction. Thus, ξ_{PP} is a maximal EIS.

Example 8.3.10 *Consider a PPFG ξ_{PP} as in figure 8.3.5, $\{e_{2PP}, e_{3PP}\}$ is a maximal EIS that is both an EIS and FEDS.*

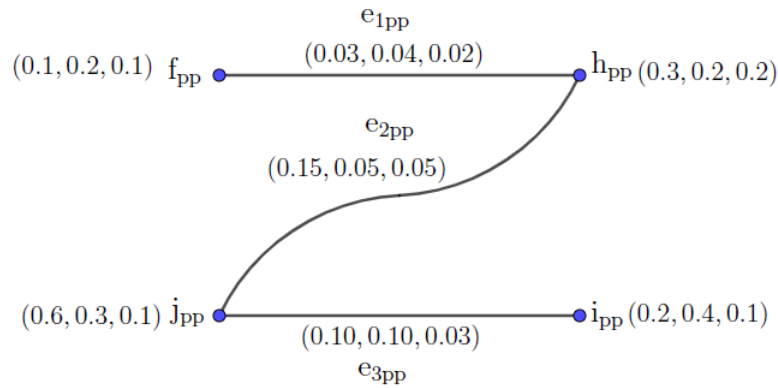


Figure 8.3.5: PPFG ξ_{PP} with EIS and FEDS

Theorem 8.3.5 *Every maximal edge independent set L_{PP} in a PPFG ξ_{PP} is a minimal fixed edge dominating set.*

Proof. Let L_{PP} be a maximal EIS in a PPFG ξ_{PP} . By theorem 8.3.4, L_{PP} is an FEDS. Assume L_{PP} is not a minimal FEDS. There exist at least one edge $e_{PP} \in L_{PP}$ for which $L_{PP} - \{e_{PP}\}$ is an FEDS. However, if $E_{PP} - \{L_{PP} - \{e_{PP}\}\}$ dominates $L_{PP} - \{e_{PP}\}$, then at least one edge in $E_{PP} - \{L_{PP} - \{e_{PP}\}\}$

must be strong neighbor to e_{PP} . This contradicts the fact that L_{PP} is an EIS in ξ_{PP} . Hence, L_{PP} must be a minimal FEDS.

8.4 Total Fixed Edge Domination in Product Picture Fuzzy Graph

In this section, the concept of total fixed edge domination in PPFG is defined and discussed notation of $\gamma_{TPP}(\xi_{PP})$.

Definition 8.4.1 Let $\xi_{PP} = (R_{PP}, S_{PP})$ be a product picture fuzzy graphs without isolated edges. An edge subset L_{PP} of E_{PP} is said to be total fixed edge dominating set (TFEDS) if for each edge $e_{PP} \in E_{PP}$, there exist an edge $g_{PP} \in L_{PP}$, $g_{PP} \neq e_{PP}$, so that e_{PP} dominates exactly one edge in L_{PP} and the corresponding vertex for each edge in L_{PP} have same degree.

Definition 8.4.2 The total fixed edge domination number (TFEDN) of ξ_{PP} is represented by $\gamma_{TPP}(\xi_{PP})$ and is the smallest cardinality among all total fixed edge dominating sets.

Theorem 8.4.1 Let $\xi_{PP} = (R_{PP}, S_{PP})$ be any product picture fuzzy graph without isolated edges. Then, for each minimal total fixed edge dominating set L_{PP} , $E_{PP} - L_{PP}$, is also an total fixed edge dominating set.

Proof. Let e_{PP} be any edge in L_{PP} . Since ξ_{PP} has no IEs, there is an edge $g_{PP} \in N(e_{PP})$ and $g_{PP} \in E_{PP} - L_{PP}$. Hence, each element of $E_{PP} - L_{PP}$ is dominated exactly one edge in L_{PP} and the corresponding vertex for each edge in L_{PP} have same degree. Thus $E_{PP} - L_{PP}$ is an TFEDS in ξ_{PP} .

8.5 Fixed Edge Restrained Domination in Product Picture Fuzzy Graph

In this section, the concept of fixed edge restrained dominating set and edge restrained independent set in PPFG are discussed.

Definition 8.5.1 Let $\xi_{PP} = (R_{PP}, S_{PP})$ be a PPFG. An edge subset $L_{PP} \subseteq E_{PP}$ is called fixed edge restrained dominating set (FERDS) if

1. Each edge in $E_{PP} - L_{PP}$ dominates exactly one edge in L_{PP} .
2. In L_{PP} , all of the edges have the equal degree.

Example 8.5.1 In figure 8.5.1, $D_{11} = \{e_{1PP}, e_{2PP}\}$, $D_{22} = \{e_{2PP}, e_{3PP}\}$, $D_{33} = \{e_{3PP}, e_{4PP}\}$ and $D_{44} = \{e_{1PP}, e_{4PP}\}$ are FERDSs.

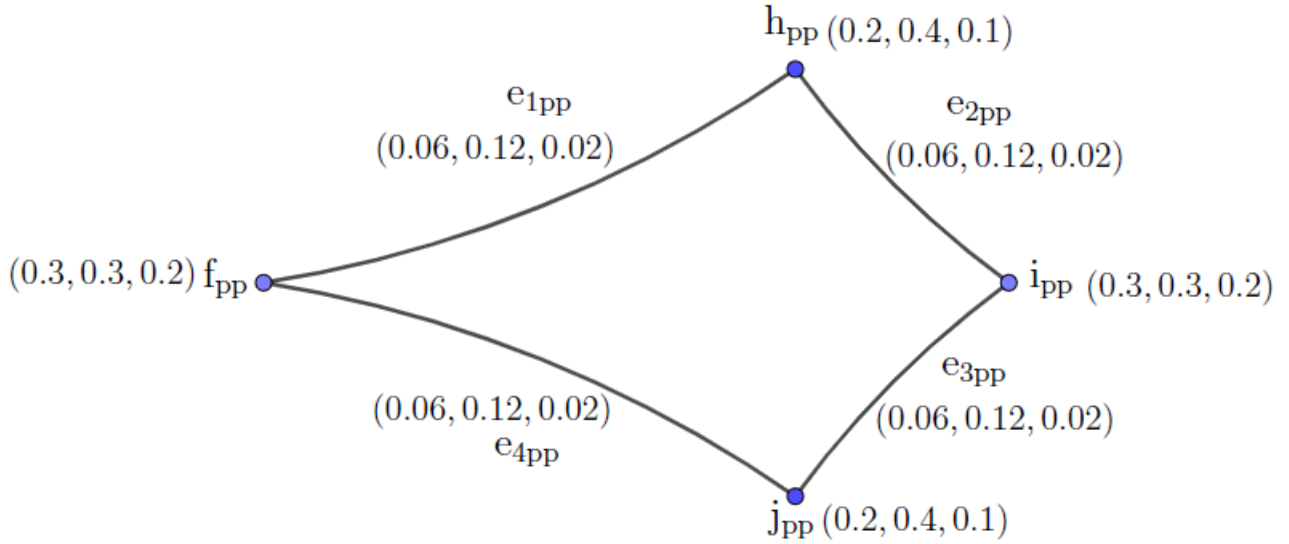
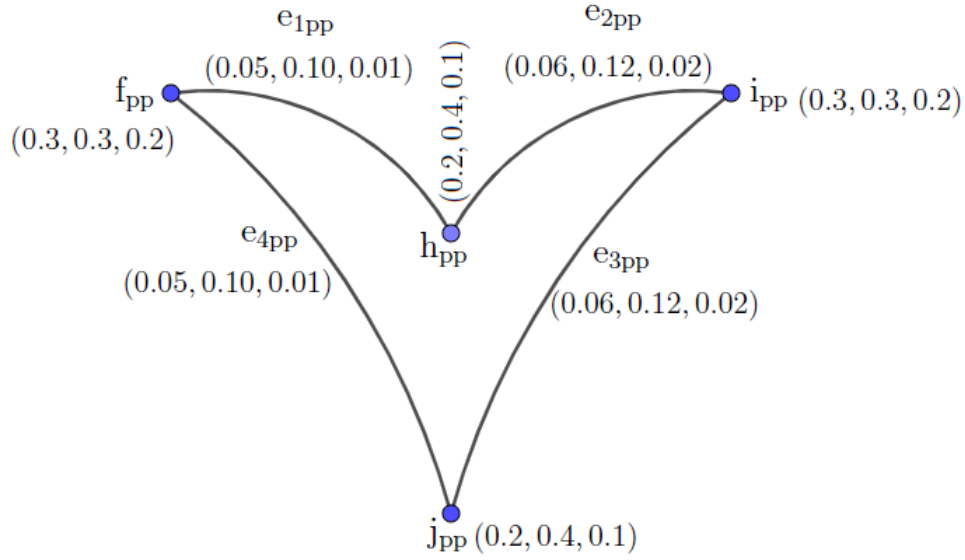


Figure 8.5.1: PPFG ξ_{PP} with FERDS

Definition 8.5.2 An edge independent set L_{PP} of a PPFG ξ_{PP} is labeled an edge restrained independent set (ERIS) if all the edges of L_{PP} have the equal degrees. L_{PP} is a maximal edge restrained independent set if for every $f_{PP} \in V_{PP} - L_{PP}$, and the set $L_{PP} \cup \{f_{PP}\}$ is not an edge restrained independent set.

Example 8.5.2 Consider the PPFG ξ_{PP} as in figure 8.5.2, $D_{11} = \{e_{1PP}, e_{4PP}\}$ is a ERIS.

Figure 8.5.2: PPFG ξ_{PP} with ERIS

Theorem 8.5.1 *An edge restrained independent set is a maximal edge restrained independent set of a PPFG ξ_{PP} iff it is an edge restrained independent set and FERDS.*

Proof. Let L_{PP} be a maximal ERIS in a PPFG ξ_{PP} , then for each $t_{PP} \in E_{PP} - L_{PP}$, the set $L_{PP} \cup \{t_{PP}\}$ is not an independent set, that is for every $t_{PP} \in E_{PP} - L_{PP}$, there exist a edge $n_{PP} \in L_{PP}$ so that t_{PP} dominates n_{PP} . Therefore, L_{PP} is a FERDS of ξ_{PP} and also an ERIS of ξ_{PP} . Therefore, L_{PP} is an ERIS and FERDS.

Conversely, assume that L_{PP} is both an ERIS and FERDS of ξ_{PP} . We have to prove that L_{PP} is a maximal ERIS. Suppose that L_{PP} is not a maximal independent set. Then, there exist a edge $t_{PP} \notin L_{PP}$ so that $L_{PP} \cup \{t_{PP}\}$ is an independent set, there is no edge in L_{PP} strong neighbor to t_{PP} , and hence, t_{PP} is not dominated any edge in L_{PP} . Thus, L_{PP} cannot be a FERDS of ξ_{PP} , which is a contradiction. Therefore, L_{PP} is a maximal ERIS.

8.6 Application

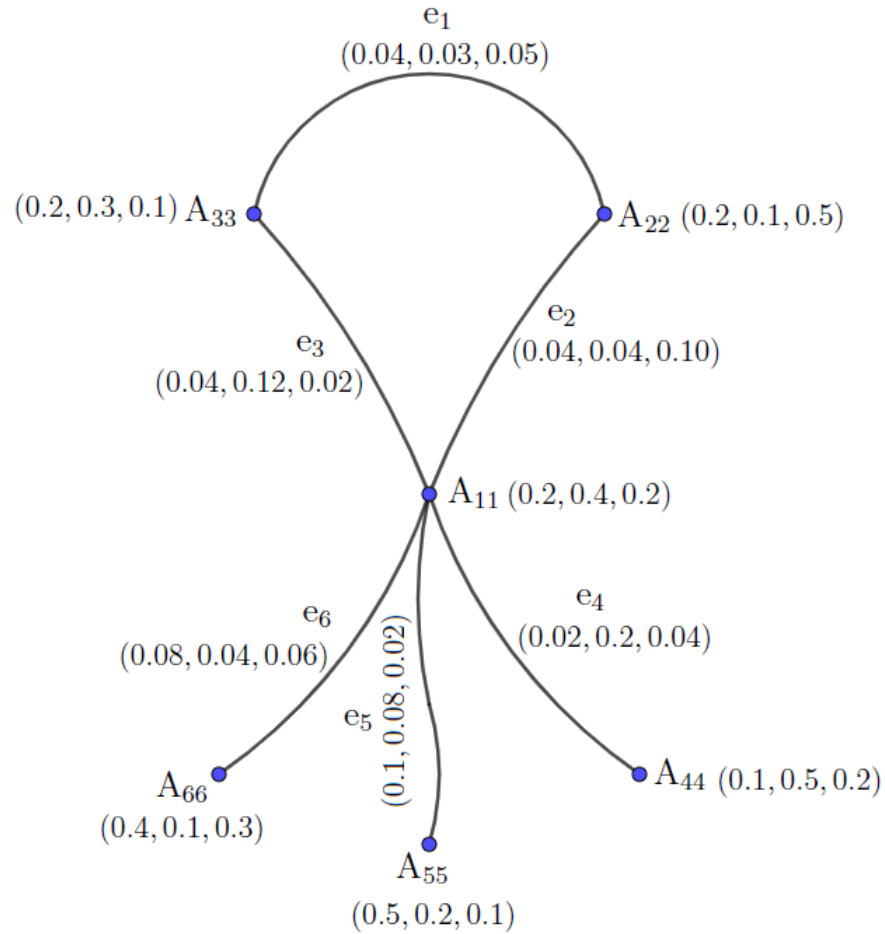
Many emergency accident patients have died in the past as a result of transportation delays to the hospital. One of the elements driving this delay is traffic congestion in cities. As a result, we attempted to find the closest hospitals in our study based on distance, traffic load, and patient suggestions. We evaluate five hospitals located in diverse locations along the by-pass road for this purpose. The hospital is depicted as $A_{22}, A_{33}, A_{44}, A_{55}, A_{66}$. In this PPFG, one vertex (A_{11}) represents the accident site, while the other vertices correspond to hospitals located throughout the bypass road.

The vertex $A_{22}(0.2, 0.1, 0.1)$ indicates that it has 20% of the essential amenities for treating the patient, but only 10% of the required equipment and only 10% of patient referrals to the proper hospital. The edge $A_{11}A_{22}$ denotes a 4% distance between the accident site and the hospital, a 4% low traffic load on the patient's ambulance transport route to the hospital, and a 2% heavy traffic load on the patient's ambulance transport route to the hospital. The fixed vertex dominating sets for figure 8.6.1 are as follows.

$$D_{1PP} = \{A_{11}A_{44}\}, D_{2PP} = \{A_{11}A_{33}\}, D_{3PP} = \{A_{11}A_{55}\}, D_{4PP} = \{A_{11}A_{66}\}, \\ D_{5PP} = \{A_{11}A_{22}A_{33}\}$$

After calculating the cardinality of $D_{1PP}, D_{2PP}, D_{3PP}, D_{4PP}$ and D_{5PP} we obtain

$$|D_{1PP}| = 0.5, |D_{2PP}| = 0.6, |D_{3PP}| = 0.9, |D_{4PP}| = 0.8, |D_{5PP}| = 1$$

Figure 8.6.1: PPFG ξ_{PP}

Because D_{1PP} has the smallest size among the other fixed vertex dominating set, we conclude that it is the best option because it allows the ambulance to travel from the accident scene to the hospital A_{44} with more free space, allowing it to transport patients to their desired location faster, saving our lives, time, and money. Second, hospital A_{44} offers a tremendous medical services than some other hospitals. As a result, the government should invest more money on improving intercity routes and traffic control so that ambulances can deliver patients to specialist hospitals swiftly.



Figure 8.6.2: BY-PASS ROAD AMBULANCE AND HOSPITAL

8.7 Analytical Comparison

Our investigation will be fruitful in fully comprehending the additional properties of fixed vertex domination in PPFG. We applied the model to fixed vertex domination in PPFG (fig 8.6.1) and domination in picture fuzzy graph (PFG)(example 8.7.1) and obtained the following results. In figure 8.6.1, $\gamma_{V_{PP}}(\xi_{PP}) = 0.5$ and example 8.7.1, $\gamma_P(G_P) = 0.6$. Here $\gamma_{V_{PP}} < \gamma_P$. As a result of this explanation, the

current model is useful for estimating the best hospital in approximate. However, our method is effective in determining the best hospital in accurate. As a result, our proposed strategy outperforms the current method.

Example 8.7.1 Let G_P be a PFG with $A_{11} = (0.2, 0.4, 0.2)$, $A_{22} = (0.2, 0.1, 0.5)$, $A_{33} = (0.2, 0.3, 0.1)$, $A_{44} = (0.1, 0.5, 0.2)$, $A_{55} = (0.5, 0.2, 0.1)$, $A_{66} = (0.4, 0.1, 0.3)$, $e_{1P} = (0.2, 0.1, 0.5)$, $e_{2P} = (0.2, 0.1, 0.5)$, $e_{3P} = (0.2, 0.3, 0.2)$, $e_{4P} = (0.1, 0.4, 0.2)$, $e_{5P} = (0.2, 0.2, 0.2)$, $e_{6P} = (0.2, 0.1, 0.3)$ and the PFDSs are $D_{1P} = \{A_{11}A_{22}\}$, $D_{2P} = \{A_{11}A_{33}\}$, $D_{3P} = \{A_{22}A_{33}\}$ with $\gamma_P(G_P) = 0.6$.

In a range of domains, a fuzzy graph is a useful tool for replicating a variety of uncertain real-world decision-making difficulties. A direct extension of fuzzy set and picture fuzzy set is the product picture fuzzy set. We also go through some of the many forms of PPFG, such as strong PPFG and complete PPFG. When compared to traditional fuzzy graph models, the PPFG can boost flexibility, efficiency, precision, and comparability when modeling complicated real-world settings. One of the most commonly discussed subjects in numerous sciences, artificial intelligence, and other fields is dominance in fuzzy graphs. As a result, we describe numerous types of dominating sets in PPFGs in this study, such as fixed vertex dominating set, fixed edge dominating set, total fixed edge dominating set, and fixed edge restrained dominating set. We also establish the relationship between edge independent sets and edge cover sets by presenting the attributes of each through numerous examples. Finally, we discussed how dominance can be used in the transportation system. The concept of a PPFG can be used to database systems, transportation networks, and image processing, among other things. The examination of new concepts of product picture bridges, product picture cycles, and product picture competition graphs, as well as their applications in medical sciences, will be the focus of future research.

Chapter 9

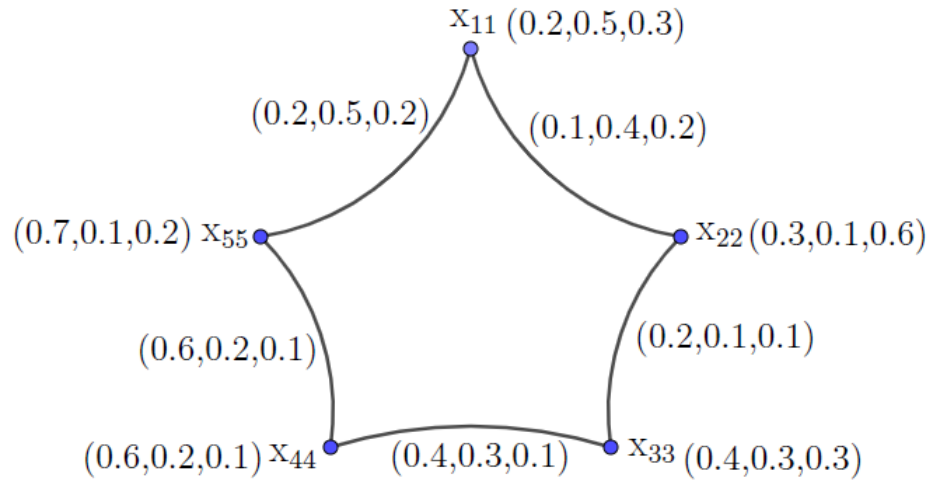
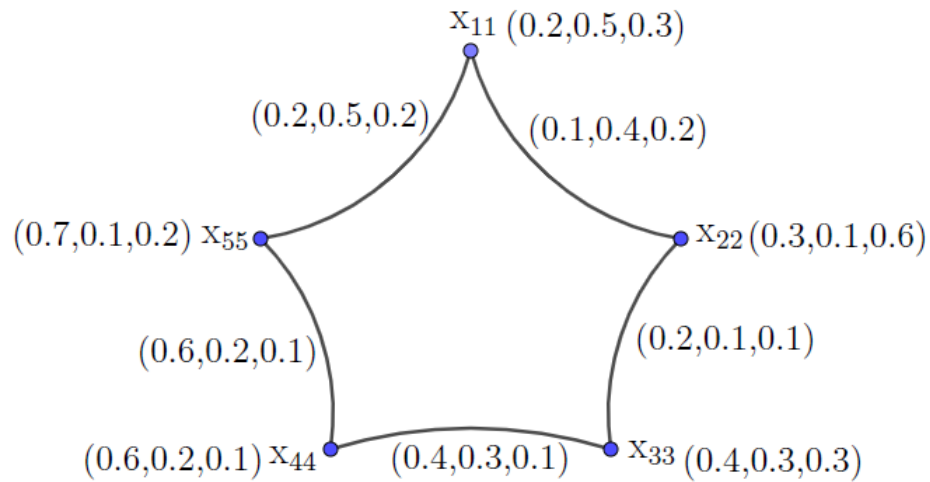
Twin Perfect Domination in Omicron Product of Two Hesitancy Fuzzy Graphs

A wide range of logistical challenges can be modeled and solved using fuzzy graph algorithms. In general, fuzzy graph theory has a wide range of applications in a variety of domains. An expert must model these issues using a fuzzy network since ambiguous information is a prevalent real-life problem that is frequently uncertain. The Omicron Product of two hesitancy fuzzy graphs is defined in this chapter. The connection between an innovative notion of twin perfect domination in hesitancy fuzzy graph and the Omicron Product of two hesitancy fuzzy graphs is provided. For numerous kinds of Omicron Product of two hesitancy fuzzy graphs, the twin perfect domination number is determined. The twin perfect domination number of a hesitancy fuzzy graph and the Omicron Product of two hesitancy fuzzy graphs are also presented, as well as their features and bounds.

9.1 Duplicate Hesitancy Fuzzy Graph

Definition 9.1.1 A duplicate hesitancy fuzzy graph is denoted by $\zeta = (\nu, \mu)$ with set of vertices and edges.

Example 9.1.1 Let us observe HFG ζ with 6 vertices then their duplicate of a hesitancy fuzzy graph ζ' showing in the following figures 9.1.1 and 9.1.2.

Figure 9.1.1: ζ Figure 9.1.2: ζ'

9.2 Omicron Product of Two Hesitancy Fuzzy Graphs

Definition 9.2.1 The Omicron Product of two hesitancy fuzzy graphs ζ_1 and ζ_2 is the hesitancy fuzzy graph denoted by $\zeta_1 \odot \zeta_2$, is the hesitancy fuzzy graph obtained by taking one duplicate of ζ_1 and $|\nu(\zeta_1)|$

duplicates of ζ_2 such that m^{th} vertex of the duplicate of ζ_1 is adjacent to each vertex of the m^{th} duplicate of ζ_2 .

Example 9.2.1 Consider the following figure 9.2.1, a HFG ζ_1 with 9 vertices and figure 9.2.2, a HFG ζ_2 with 3 vertices now we obtain the Omicron Product of these two HFGs $\zeta_1 \odot \zeta_2$ showing in the following figure 9.2.3

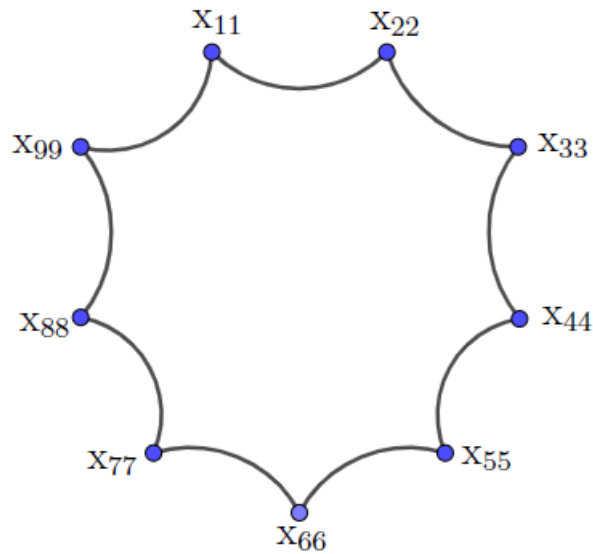


Figure 9.2.1: ζ_1

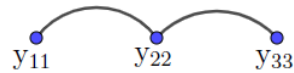
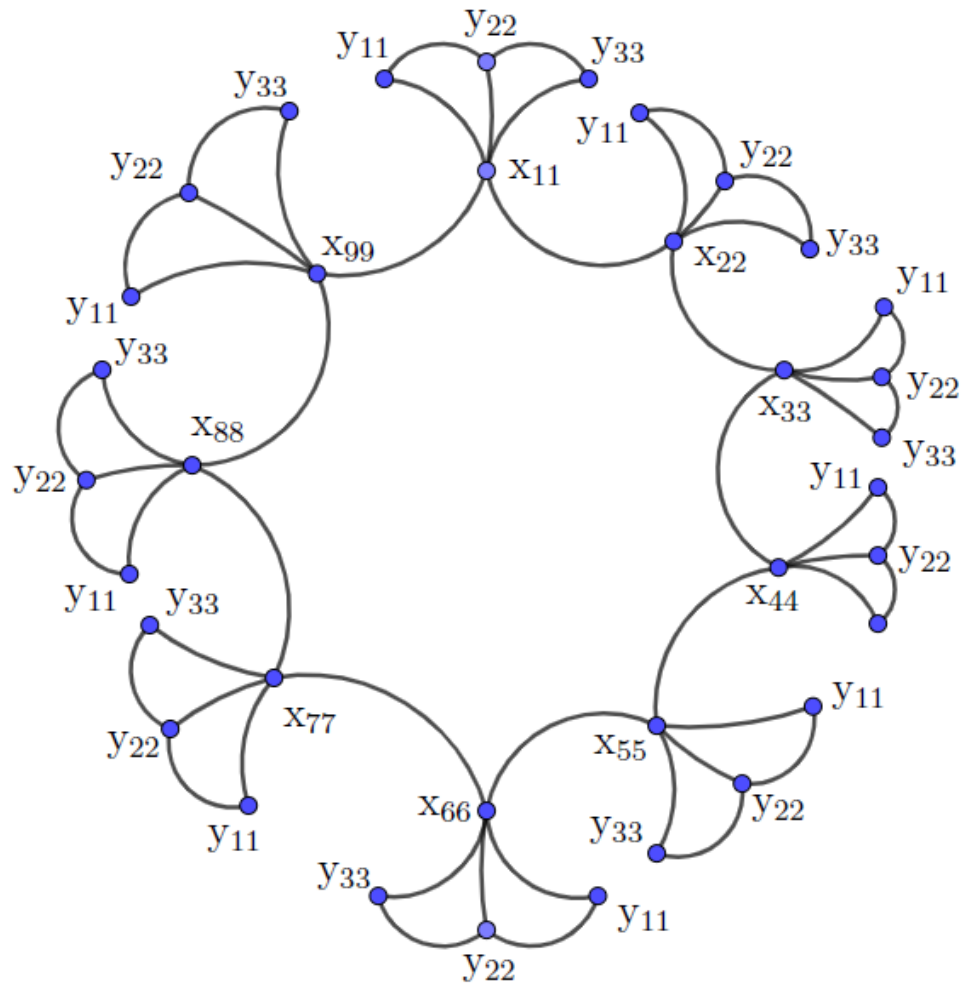


Figure 9.2.2: ζ_2

Figure 9.2.3: Omicron Product of hesitancy fuzzy graphs ζ_1 and ζ_2

Vertex	Value	Vertex	Value
x_{11}	0.2, 0.5, 0.3	x_{77}	0.5, 0.3, 0.2
x_{22}	0.6, 0.3, 0.1	x_{88}	0.1, 0.4, 0.5
x_{33}	0.3, 0.4, 0.3	x_{99}	0.3, 0.3, 0.4
x_{44}	0.2, 0.6, 0.2	y_{11}	0.5, 0.2, 0.3
x_{55}	0.4, 0.2, 0.4	y_{22}	0.4, 0.3, 0.3
x_{66}	0.7, 0.2, 0.1	y_{33}	0.2, 0.4, 0.4

Table 9.2.1: Vertex values of Figure 9.2.1, Figure 9.2.2, Figure 9.2.3

Edge	Value	Edge	Value
x_{11}, x_{22}	0.2, 0.5, 0.1	x_{44}, y_{22}	0.1, 0.3, 0.2
x_{22}, x_{33}	0.3, 0.4, 0.1	x_{44}, y_{33}	0.2, 0.5, 0.2
x_{33}, x_{44}	0.2, 0.5, 0.1	x_{55}, y_{11}	0.4, 0.2, 0.3
x_{44}, x_{55}	0.1, 0.5, 0.2	x_{55}, y_{22}	0.2, 0.3, 0.1
x_{55}, x_{66}	0.4, 0.1, 0.1	x_{55}, y_{33}	0.2, 0.4, 0.2
x_{66}, x_{77}	0.5, 0.3, 0.1	x_{66}, y_{11}	0.5, 0.2, 0.1
x_{77}, x_{88}	0.1, 0.4, 0.2	x_{66}, y_{22}	0.4, 0.3, 0.1
x_{88}, x_{99}	0.1, 0.4, 0.4	x_{66}, y_{33}	0.2, 0.4, 0.1
x_{11}, x_{99}	0.2, 0.5, 0.1	x_{77}, y_{11}	0.4, 0.3, 0.1
x_{11}, y_{11}	0.1, 0.4, 0.2	x_{77}, y_{22}	0.3, 0.2, 0.2
x_{11}, y_{22}	0.2, 0.3, 0.2	x_{77}, y_{33}	0.2, 0.4, 0.2
x_{11}, y_{33}	0.2, 0.4, 0.1	x_{88}, y_{11}	0.1, 0.4, 0.3
x_{22}, y_{11}	0.4, 0.3, 0.1	x_{88}, y_{22}	0.1, 0.3, 0.3
x_{22}, y_{22}	0.4, 0.2, 0.1	x_{88}, y_{33}	0.1, 0.4, 0.4
x_{22}, y_{33}	0.2, 0.4, 0.1	x_{99}, y_{11}	0.3, 0.3, 0.3
x_{33}, y_{11}	0.3, 0.2, 0.3	x_{99}, y_{22}	0.2, 0.3, 0.1
x_{33}, y_{22}	0.3, 0.3, 0.1	x_{99}, y_{33}	0.2, 0.4, 0.3
x_{33}, y_{33}	0.2, 0.4, 0.1	y_{11}, y_{22}	0.4, 0.3, 0.2
x_{44}, y_{11}	0.2, 0.4, 0.2	y_{22}, y_{33}	0.2, 0.4, 0.3

Table 9.2.2: Edge values of Figure 9.2.1, Figure 9.2.2, Figure 9.2.3

9.3 Twin Perfect Domination in Hesitancy Fuzzy Graph

Definition 9.3.1 A subset M of $v(\zeta)$ is said to be a twin perfect dominating set if for every vertex x_{11} not in M , x_{11} is adjacent to exactly two vertex of M .

Example 9.3.1 Consider the following figure 9.3.1 with four vertices say x_{11}, x_{22}, x_{33} and x_{44} in which the set $M_1 = \{x_{11}, x_{33}\}$ and $M_2 = \{x_{22}, x_{44}\}$ are twin perfect dominating sets.

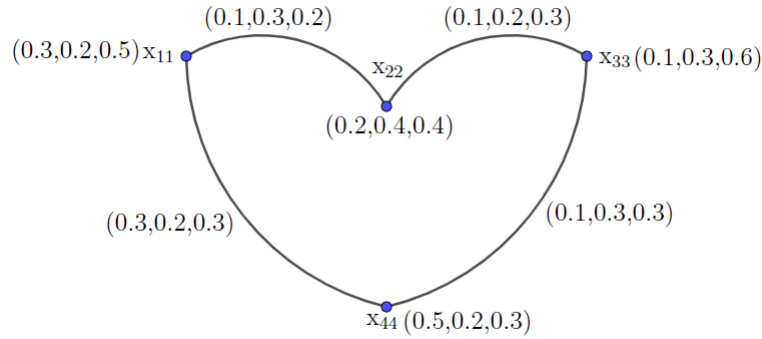


Figure 9.3.1: Hesitancy Fuzzy Graph with Twin Perfect Dominating Sets

Definition 9.3.2 A twin perfect dominating set M of the hesitancy fuzzy graph ζ is said to be minimal twin perfect dominating set if each vertex x_{11} in M , $M = \{x_{11}\}$ is not a twin perfect dominating set.

Example 9.3.2 Consider a following figure 9.3.2, a HGF with 6 vertices say $x_{11}, x_{22}, x_{33}, x_{44}, x_{55}, x_{66}$. The set $M_1 = \{x_{11}, x_{33}\}$ is a minimal twin perfect dominating set of $\nu(\zeta)$ and also a twin perfect dominating set.

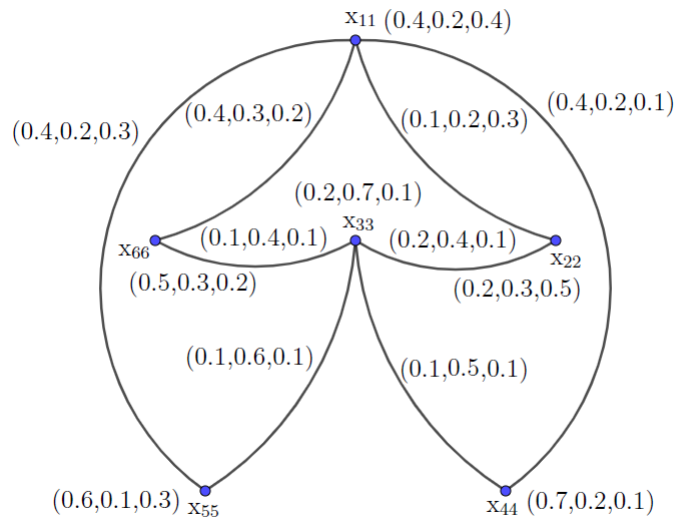


Figure 9.3.2: Hesitancy Fuzzy Graph with Minimal Twin Perfect Dominating Set

Definition 9.3.3 A twin perfect dominating set with smallest cardinality is called minimum twin perfect dominating set.

Definition 9.3.4 A cardinality of minimum twin perfect dominating set is called twin perfect domination number of ζ and it is denoted by $\gamma_{tpd}(\zeta)$.

Example 9.3.3 In the following figure 9.3.3, a HFG with 4 vertices say $x_{11}, x_{22}, x_{33}, x_{44}$. We observe that the twin perfect domination number of HFG, $\gamma_{tpd}(\zeta) = 2$.

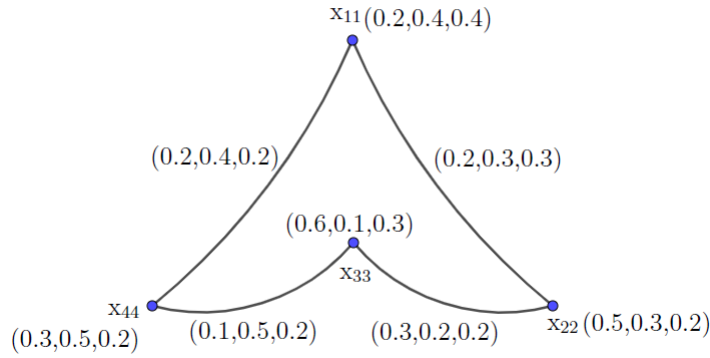


Figure 9.3.3: Hesitancy Fuzzy Graph with Twin Perfect Domination Number

Let $\zeta = (\nu, \mu)$ be any HFG where ν indicates set of vertices and μ indicates set of edges. Through out the result we are consider HFG ζ_p with p number of vertices and $p - 1$ number of edges in which we invented some new results regarding the twin perfect domination number of HFG ζ_p and its Omicron Product with the ζ_{p-1} .

Theorem 9.3.1 Let ζ_p be any hesitancy fuzzy graph with p number of vertices and $p - 1$ number of edges the twin perfect domination number of hesitancy fuzzy graph ζ_p is given by

$$\gamma_{tpd}(\zeta_p) = \begin{cases} \frac{p}{3}, & \text{where } p = 3n, n \in \mathbb{N} \\ \frac{p+1}{3}, & \text{where } p = 3n - 1, n \in \mathbb{N} \\ \frac{p+2}{3}, & \text{where } p = 3n - 2, n \in \mathbb{N} \end{cases}$$

where N indicates set of natural numbers.

Proof

Here we are proving this theorem by using Principle of Mathematical Induction theory. First we are trying to prove that the given result in the hypothesis is true for $p = 1$. Now for $p = 1$ is suitable in the order of $3n - 2$ so that we take $n = 1$.

Therefore $\gamma_{tpd}(\zeta_1) = \frac{1+2}{3} = \frac{3}{3} = 1$. Since the twin perfect domination number of isolated vertex is must be 1 and hence it is true for $p = 1$.

Now we assume that above result is true for $p = r$, where r be any positive integer then we get

$$\gamma_{tpd}(\zeta_r) = \begin{cases} \frac{r}{3}, & \text{where } r = 3n, n \in N \\ \frac{r+1}{3}, & \text{where } r = 3n - 1, n \in N \\ \frac{r+2}{3}, & \text{where } r = 3n - 2, n \in N \end{cases}$$

Next we trying to prove that above result given in the hypothesis is true for $p = r + 1$, where r be any positive integer.

To prove

$$\gamma_{tpd}(\zeta_{r+1}) = \begin{cases} \frac{r+1}{3}, & \text{where } r + 1 = 3n, n \in N \\ \frac{r+2}{3}, & \text{where } r + 1 = 3n - 1, n \in N \\ \frac{r+3}{3}, & \text{where } r + 1 = 3n - 2, n \in N \end{cases}$$

Case (i)

If $r + 1 = 3n$, this implies $r + 1$ is a multiple of 3. Here r be any positive integer.

Therefore $r + 1 = \alpha$ is must be a positive integer for some $n \in N$ follows $\alpha = 3n$ so according to above result

$$\Rightarrow \gamma_{tpd}(\zeta_\alpha) = \frac{\alpha}{3} \text{ but } \alpha = r + 1, \text{ then } \gamma_{tpd}(\zeta_{r+1}) = \frac{r + 1}{3}$$

Case (ii)

If $r + 1 = 3n - 1$, this implies $r + 1$ is a multiple of 3 minus 1(one). Here r be any positive integer.

Therefore $r + 1 = \beta$ is must be a positive integer for some $n \in N$ follows $\beta = 3n - 1$ so according to above result

$$\Rightarrow \gamma_{tpd}(\zeta_\beta) = \frac{\beta + 1}{3} \text{ but } \beta = r + 1, \text{ then } \gamma_{tpd}(\zeta_{r+1}) = \frac{r + 2}{3}$$

Case (iii)

If $r + 1 = 3n - 2$, this implies $r + 1$ is a multiple of 3 minus 2(two). Here r be any positive integer.

Therefore $r + 1 = \delta$ is must be a positive integer for some $n \in N$ follows $\delta = 3n - 2$ so according to above result

$$\Rightarrow \gamma_{tpd}(\zeta_\delta) = \frac{\delta + 2}{3} \text{ but } \delta = r + 1, \text{ then } \gamma_{tpd}(\zeta_{r+1}) = \frac{r + 3}{3}$$

Hence the result.

Example 9.3.4 Consider the following HFG, showing in the figure 9.3.4, then $4 = 3n - 2$, where $n = 2 \in N$ then the theorem 9.3.1, the twin perfect domination number of HFG ζ_4 is

$$\gamma_{tpd}(\zeta_4) = \frac{p + 2}{3} = \frac{4 + 2}{3} = \frac{6}{3} = 2$$

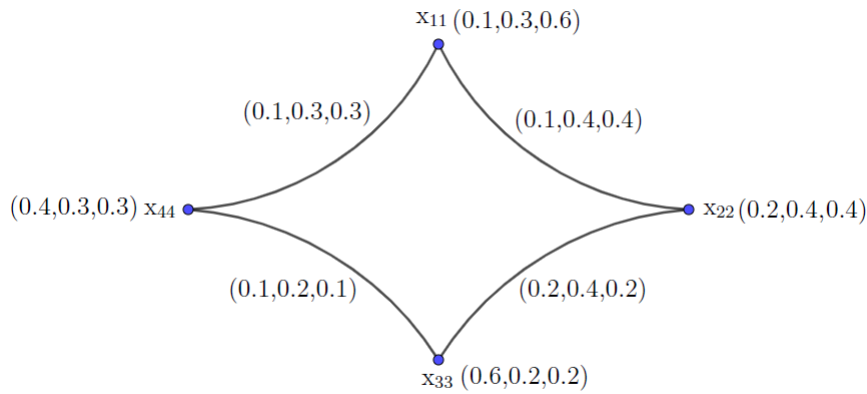


Figure 9.3.4: HFG with 4-vertices

9.4 Bounds of Twin Perfect Domination Number of Hesitancy Fuzzy Graphs

Theorem 9.4.1 *Let ζ_p be any hesitancy fuzzy graph with p -vertices and $\gamma_{tpd}(\zeta_p)$ indicates the twin perfect domination number of hesitancy fuzzy graph ζ_p then $p - 3 < \gamma_{tpd}(\zeta_p) < p - 1$, where $p \geq 3 \in N$.*

Proof.

Here we can try to prove the above result by contradictory way.

To prove $p - 3 < \gamma_{tpd}(\zeta_p) < p - 1$, where $p \in N$. Now we partitioned an interval $(p - 3, p - 1)$ in to two parts.

(i) $p - 3 < \gamma_{tpd}(\zeta_p)$ and

(ii) $\gamma_{tpd}(\zeta_p) < p - 1$

Now conversely we assume that $p - 3 > \gamma_{tpd}(\zeta_p)$ and $\gamma_{tpd}(\zeta_p) > p - 1$. By the above theorem 9.3.1, we divide the hypothesis into three cases as follows

Case (i)

If $p = 3n$, for some $n \in N$ then $\gamma_{tpd}(\zeta_p) = \frac{p}{3}$

Now let $p - 3 > \gamma_{tpd}(\zeta_p)$

$$\Rightarrow p - 3 > \frac{p}{3}$$

$$\Rightarrow 3p - 9 > p$$

$$\Rightarrow 2p > 9$$

$\Rightarrow p > \frac{9}{2}$ is contradict with our assumption because if we take $n = 1$ follows $p = 3$ it gives $3 > \frac{9}{2}$ this is conflict. So our assumption is wrong. Therefore $p - 3 < \gamma_{tpd}(\zeta_p)$.

Now let $\gamma_{tpd}(\zeta_p) > p - 1$,

$$\Rightarrow \frac{p}{3} > p - 1$$

$$\Rightarrow p > 3p - 3$$

$$\Rightarrow 3 > 2p, \text{ this is contradiction,}$$

It is true for all $p = 3n, n \in N$. Since $n = 1$ follows $p = 3$ this gives $3 > 6$ is conflict, therefore our assumption is wrong and hence $\gamma_{tpd}(\zeta_p) < p - 1$. So in this case (i) the result is true for all $p \in N$.

Case (ii)

If $p = 3n - 1$, for some $n \in N$ then $\gamma_{tpd}(\zeta_p) = \frac{p+1}{3}$.

Now let $p - 3 > \gamma_{tpd}(\zeta_p)$

$$\Rightarrow p - 3 > \frac{p+1}{3}$$

$$\Rightarrow 3p - 9 > p + 1$$

$\Rightarrow 2p > 10$ is contradict with our assumption because if we take $n = 1$ follows $p = 2$ it gives $4 > 10$ this is conflict. So our assumption is wrong. Therefore $p - 3 < \gamma_{tpd}(\zeta_p)$.

Let $\gamma_{tpd}(\zeta_p) > p - 1$,

$$\Rightarrow \frac{p+1}{3} > p - 1$$

$$\Rightarrow p + 1 > 3p - 3$$

$$\Rightarrow 4 > 2p.$$

It is true for all $p = 3n - 1, n \in N$. Since $n = 1$ follows $p = 2$ this gives $4 > 4$ is conflict, therefore our assumption is wrong and hence $\gamma_{tpd}(\zeta_p) < p - 1$. So in this case (ii) the result is true for all $p \in N$.

Case (iii)

If $p = 3n - 2$, for some $n \in N$ then $\gamma_{tpd}(\zeta_p) = \frac{p+2}{3}$.

Now let $p - 3 > \gamma_{tpd}(\zeta_p)$

$$\Rightarrow p - 3 > \frac{p+2}{3}$$

$$\Rightarrow 3p - 9 > p + 2$$

$\Rightarrow 2p > 11$ is contradict with our assumption because if we take $n = 1$ follows $p = 1$ it gives $2 > 11$ this is conflict. So our assumption is wrong. Therefore $p - 3 < \gamma_{tpd}(\zeta_p)$.

Let $\gamma_{tpd}(\zeta_p) > p - 1$,

$$\Rightarrow \frac{p+2}{3} > p - 1$$

$$\Rightarrow p + 2 > 3p - 3$$

$$\Rightarrow 5 > 2p$$

$$\Rightarrow \frac{5}{2} > p.$$

It is true for all $p = 3n - 2, n \in N$. Since $n = 2$ follows $p = 4$ this gives $\frac{5}{2} > 4$ is conflict, therefore our assumption is wrong and hence $\gamma_{tpd}(\zeta_p) < p - 1$. So in this case (iii) the result is true for all $p \in N$.

Theorem 9.4.2 Let ζ_{p-1} be any hesitancy fuzzy graph with $p - 1$ vertices and $\gamma_{tpd}(\zeta_{p-1})$ indicates the

twin perfect domination number of hesitancy fuzzy graph ζ_{p-1} then $p - 4 < \gamma_{tpd}(\zeta_{p-1}) < p - 2$, where $p \geq 4 \in N$.

Proof. Here we can try to prove the above result by contradictory way.

To prove $p - 4 < \gamma_{tpd}(\zeta_p) < p - 2$, where $p \in N$. Now we partitioned an interval $(p - 4, p - 2)$ in to two parts.

(i) $p - 4 < \gamma_{tpd}(\zeta_{p-1})$ and

(ii) $\gamma_{tpd}(\zeta_{p-1}) < p - 2$

Now, conversely we assume that $p - 4 > \gamma_{tpd}(\zeta_{p-1})$ and $\gamma_{tpd}(\zeta_{p-1}) > p - 2$. By the above theorem 9.3.1, we divide the hypothesis into three cases as follows

Case (i)

If $p = 3n$, for some $n \in N$ then $\gamma_{tpd}(\zeta_{p-1}) = \frac{p}{3}$.

Now let $p - 4 > \gamma_{tpd}(\zeta_{p-1})$

$$\Rightarrow p - 4 > \frac{p}{3}$$

$$\Rightarrow 3p - 12 > p$$

$$\Rightarrow 2p > 12$$

$\Rightarrow p > 6$ is contradict with our assumption because if we take $n = 1$ follows $p = 3$ it gives $6 > 12$ this is conflict. So our assumption is wrong. Therefore $p - 4 < \gamma_{tpd}(\zeta_{p-1})$.

Let $\gamma_{tpd}(\zeta_{p-1}) > p - 2$,

$$\Rightarrow \frac{p}{3} > p - 2$$

$$\Rightarrow p > 3p - 6$$

$$\Rightarrow 6 > 2p.$$

It is true for all $p = 3n, n \in N$. Since $n = 1$ follows $p = 3$ this gives $6 > 6$ is conflict, therefore our assumption is wrong and hence $\gamma_{tpd}(\zeta_{p-1}) < p - 2$. So in this case (i) the result is true for all $p \in N$.

Case (ii)

If $p = 3n - 1$, for some $n \in N$ then $\gamma_{tpd}(\zeta_{p-1}) = \frac{p+1}{3}$.

Now let $p - 4 > \gamma_{tpd}(\zeta_{p-1})$

$$\Rightarrow p - 4 > \frac{p+1}{3}$$

$$\Rightarrow 3p - 12 > p + 1$$

$\Rightarrow 2p > 13$ is contradict with our assumption because if we take $n = 1$ follows $p = 2$ it gives $4 > 13$ this is conflict. So our assumption is wrong. Therefore $p - 4 < \gamma_{tpd}(\zeta_{p-1})$.

$$\text{Let } \gamma_{tpd}(\zeta_{p-1}) > p - 2,$$

$$\Rightarrow \frac{p+1}{3} > p - 2$$

$$\Rightarrow p + 1 > 3p - 6$$

$$\Rightarrow 7 > 2p.$$

It is true for all $p = 3n - 1, n \in N$. Since $n = 2$ follows $p = 5$ this gives $7 > 10$ is conflict, therefore our assumption is wrong and hence $\gamma_{tpd}(\zeta_{p-1}) < p - 2$. So in this case (ii) the result is true for all $p \in N$.

Case (iii)

$$\text{If } p = 3n - 2, \text{ for some } n \in N \text{ then } \gamma_{tpd}(\zeta_{p-1}) = \frac{p+2}{3}.$$

$$\text{Now let } p - 4 > \gamma_{tpd}(\zeta_{p-1})$$

$$\Rightarrow p - 4 > \frac{p+2}{3}$$

$$\Rightarrow 3p - 12 > p + 2$$

$\Rightarrow 2p > 14$ is contradict with our assumption because if we take $n = 1$ follows $p = 1$ it gives $2 > 14$ this is conflict. So our assumption is wrong. Therefore $p - 4 < \gamma_{tpd}(\zeta_{p-1})$.

$$\text{Let } \gamma_{tpd}(\zeta_{p-1}) > p - 2,$$

$$\Rightarrow \frac{p+2}{3} > p - 2$$

$$\Rightarrow p + 2 > 3p - 6$$

$$\Rightarrow 8 > 2p$$

$$\Rightarrow 4 > p.$$

It is true for all $p = 3n - 2, n \in N$. Since $n = 2$ follows $p = 4$ this gives $4 > 4$ is conflict, therefore our assumption is wrong and hence $\gamma_{tpd}(\zeta_{p-1}) < p - 2$. So in this case (iii) the result is true for all $p \in N$.

9.5 Bounds of Twin Perfect Domination Number of Omicron Products of Two Hesitancy Fuzzy Graphs ζ_p and ζ_{p-1} .

Lemma 9.5.1 *The twin perfect domination number of Omicron Product of two hesitancy fuzzy graphs ζ_p and ζ_{p-1} is given by $\gamma_{tpd}(\zeta_p \odot \zeta_{p-1})$ and it is bounded by $0 < \gamma_{tpd}(\zeta_p \odot \zeta_{p-1}) < p^2$, where $p \geq 3$.*

Proof. Here we want to prove that $0 < \gamma_{tpd}(\zeta_p \odot \zeta_{p-1}) < p^2$, where $p \geq 3$.

That can be written as

$$0 < \gamma_{tpd}(\zeta_p \odot \zeta_{p-1}) \quad \text{and} \quad (9.1)$$

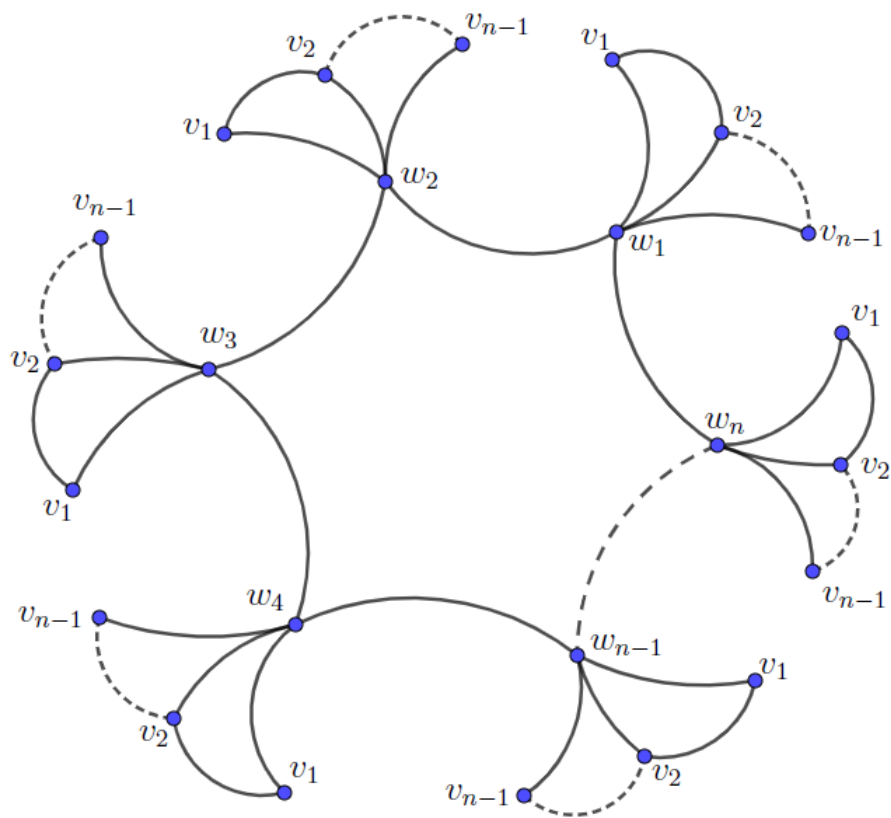
$$\gamma_{tpd}(\zeta_p \odot \zeta_{p-1}) < p^2 \quad (9.2)$$

Since twin perfect domination number is always greater than zero.

So the above inequality (9.1) is obvious. For the further proof of result (9.2) observe the following figure 9.5.1.

Let us consider HFG ζ_p with p vertices say $\omega_1, \omega_2, \omega_3, \dots, \omega_{p-1}, \omega_p$ and another HFG ζ_{p-1} with $p - 1$ vertices namely $v_1, v_2, v_3, \dots, v_{p-1}, v_p$. Now take a omicron product of them, we get at most p^2 vertices. Since each vertex of HFG ζ_p is adjacent with every vertex of HFG ζ_{p-1} . So as per the definition of minimum twin perfect dominating set it contains at least two vertices and most p^2 vertices.

In figure 9.5.1, the twin perfect domination number of omicron product of HFGs ζ_p and ζ_{p-1} is almost p^2 and always greater than zero that is $0 < \gamma_{tpd}(\zeta_p \odot \zeta_{p-1}) < p^2$, where $p \geq 3$. Hence the result.

Figure 9.5.1: Omicron Product of $\zeta_p \odot \zeta_{p-1}$

Vertex	Value
ω_1	0.6,0.3,0.1
ω_2	0.4,0.3,0.3
ω_3	0.7,0.2,0.1
ω_4	0.2,0.6,0.2
ω_{n-1}	0.2,0.5,0.3
ω_n	0.4,0.4,0.2
v_1	0.3,0.2,0.5
v_2	0.4,0.3,0.3
v_3	0.1,0.5,0.4

Table 9.5.1: Vertex values of Figure 9.5.1

Edge	Value	Edge	Value
ω_1, ω_2	0.4,0.3,0.1	ω_3, v_2	0.4,0.3,0.1
ω_2, ω_3	0.4,0.3,0.1	ω_3, v_{n-1}	0.1,0.5,0.1
ω_3, ω_4	0.2,0.6,0.1	ω_4, v_1	0.2,0.6,0.1
ω_4, ω_{n-1}	0.1,0.6,0.2	ω_4, v_2	0.2,0.4,0.1
ω_{n-1}, ω_n	0.2,0.5,0.2	ω_4, v_{n-1}	0.1,0.5,0.2
ω_n, ω_1	0.4,0.4,0.1	ω_{n-1}, v_1	0.2,0.5,0.2
ω_1, v_1	0.3,0.3,0.1	ω_{n-1}, v_2	0.2,0.4,0.1
ω_1, v_2	0.4,0.3,0.1	ω_{n-1}, v_{n-1}	0.1,0.5,0.3
ω_1, v_{n-1}	0.1,0.5,0.1	ω_n, v_1	0.3,0.4,0.2
ω_2, v_1	0.3,0.3,0.3	ω_n, v_2	0.4,0.3,0.2
ω_2, v_2	0.4,0.3,0.2	ω_n, v_{n-1}	0.1,0.2,0.2
ω_2, v_{n-1}	0.1,0.5,0.3	v_1, v_2	0.3,0.3,0.3
ω_3, v_1	0.3,0.2,0.1	v_2, v_{n-1}	0.1,0.5,0.3

Table 9.5.2: Edge values of Figure 9.5.1

9.6 The Twin Perfect Domination Number of Omicron Products of Two Hesitancy Fuzzy Graphs ζ_p and ζ_{p-1}

Theorem 9.6.1 Let ζ_p , hesitancy fuzzy graph with p vertices and ζ_{p-1} , hesitancy fuzzy graph with $p - 1$ vertices where $p \geq 3 \in N$ then $\gamma_{tpd}(\zeta_p \odot \zeta_{p-1}) \geq \gamma_{tpd}(\zeta_p) + \gamma_{tpd}(\zeta_{p-1}) + p$ for some $p \geq 3$.

Proof. The given hypothesis is proving by using contradictory method as follows. Now by theorem 9.4.1, theorem 9.4.2, and lemma 9.5.1, we have

- (i) $p - 3 < \gamma_{tpd}(\zeta_p) < p - 1$, where $p \geq 3 \in N$
- (ii) $p - 4 < \gamma_{tpd}(\zeta_{p-1}) < p - 2$, where $p \geq 4 \in N$
- (iii) $0 < \gamma_{tpd}(\zeta_p \odot \zeta_{p-1}) < p^2$, where $p \geq 3$

Now construct (i) + (ii), we get

$$\begin{aligned}
 2p - 7 &< \gamma_{tpd}(\zeta_p) + \gamma_{tpd}(\zeta_{p-1}) < p - 1 + p - 2 \\
 \Rightarrow 2p - 7 + p &< \gamma_{tpd}(\zeta_p) + \gamma_{tpd}(\zeta_{p-1}) + p < p - 1 + p - 2 + p \\
 \Rightarrow 3p - 7 &< \gamma_{tpd}(\zeta_p) + \gamma_{tpd}(\zeta_{p-1}) < 3p - 3 \text{ where } 3p \geq 8 \in N
 \end{aligned} \tag{9.3}$$

Conversely, assume that

$$\gamma_{tpd}(\zeta_p \odot \zeta_{p-1}) < \gamma_{tpd}(\zeta_p) + \gamma_{tpd}(\zeta_{p-1}) + p$$

This can be rewritten as

$$\begin{aligned}
 0 &< \gamma_{tpd}(\zeta_p \odot \zeta_{p-1}) < \gamma_{tpd}(\zeta_p) + \gamma_{tpd}(\zeta_{p-1}) + p \\
 \Rightarrow 0 &< \gamma_{tpd}(\zeta_p \odot \zeta_{p-1}) < p^2 < \gamma_{tpd}(\zeta_p) + \gamma_{tpd}(\zeta_{p-1}) + p < 3p - 3 \\
 \Rightarrow p^2 &< 3p - 3 \\
 \Rightarrow p^2 - 3p &< -3 \\
 \Rightarrow p(p - 3) &< -3 \\
 \Rightarrow \text{Either } p &< -3 \text{ and } p - 3 < -3 \\
 \Rightarrow p < -3 \text{ and } p < 0, &\text{ is conflict with values of } p, p \geq 3
 \end{aligned}$$

Therefore our assumption is wrong and hence proved the theorem.

That is $\gamma_{tpd}(\zeta_p \odot \zeta_{p-1}) \geq \gamma_{tpd}(\zeta_p) + \gamma_{tpd}(\zeta_{p-1}) + p$ for some $p \geq 3, p \in N$.

Example 9.6.1 Let ζ_4 HFG having 4 vertices with $\gamma_{tpd}(\zeta_4) = 2$ and ζ_3 HFG having 3 vertices with $\gamma_{tpd}(\zeta_3) = 2$. In figure 9.6.1, the twin perfect domination number of Omicron Product of two HFGs ζ_4 and ζ_3 is $\gamma_{tpd}(\zeta_4 \odot \zeta_3) = 8$.

Here

$$p = 4, \gamma_{tpd}(\zeta_4) = 2, \gamma_{tpd}(\zeta_3) = 2 \text{ and } \gamma_{tpd}(\zeta_4 \odot \zeta_3) = 8$$

$$\Rightarrow 8 \geq 2 + 2 + 4$$

$$\Rightarrow 8 = 8$$

Therefore $\gamma_{tpd}(\zeta_p \odot \zeta_{p-1}) \geq \gamma_{tpd}(\zeta_p) + \gamma_{tpd}(\zeta_{p-1}) + p$. Hence the result.

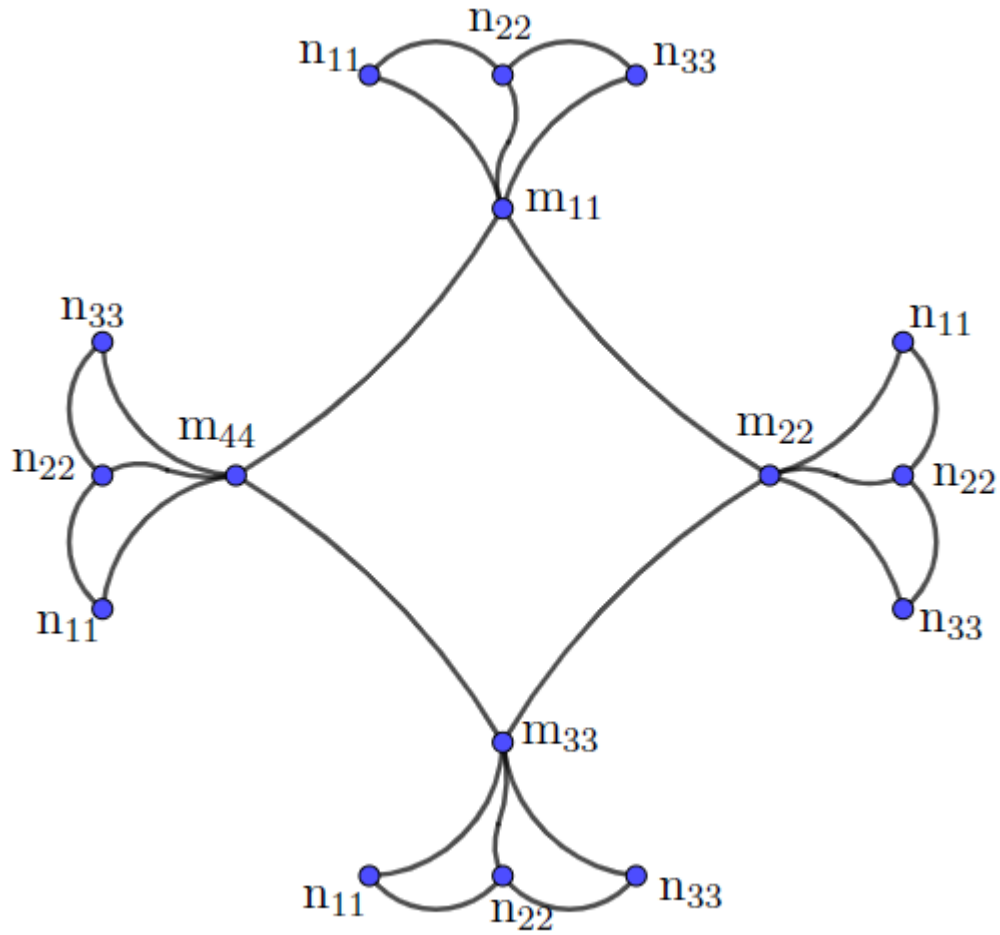


Figure 9.6.1: $(\zeta_4 \odot \zeta_3)$ with Twin Perfect Domination Number

Vertex	Value
m_{11}	0.2, 0.4, 0.4
m_{22}	0.1, 0.3, 0.6
m_{33}	0.4, 0.5, 0.1
m_{44}	0.3, 0.5, 0.2
n_{11}	0.1, 0.4, 0.5
n_{22}	0.3, 0.6, 0.1
n_{33}	0.5, 0.3, 0.2

Table 9.6.1: vertex values of Figure 9.6.1

Edge	Value	Edge	Value
m_{11}, m_{22}	0.1, 0.4, 0.4	m_{22}, n_{33}	0.1, 0.3, 0.2
m_{22}, m_{33}	0.1, 0.5, 0.1	m_{33}, n_{11}	0.1, 0.5, 0.1
m_{33}, m_{44}	0.3, 0.5, 0.1	m_{33}, n_{22}	0.2, 0.4, 0.1
m_{44}, m_{11}	0.2, 0.4, 0.1	m_{33}, n_{33}	0.3, 0.4, 0.1
m_{11}, n_{11}	0.1, 0.4, 0.4	m_{44}, n_{11}	0.1, 0.5, 0.2
m_{11}, n_{22}	0.2, 0.6, 0.1	m_{44}, n_{22}	0.3, 0.4, 0.1
m_{11}, n_{33}	0.2, 0.4, 0.2	m_{44}, n_{33}	0.2, 0.4, 0.2
m_{22}, n_{11}	0.1, 0.4, 0.3	n_{11}, n_{22}	0.1, 0.6, 0.1
m_{22}, n_{22}	0.1, 0.6, 0.1	n_{33}, n_{11}	0.2, 0.4, 0.1

Table 9.6.2: Edge values of Figure 9.6.1

Example 9.6.2 Let ζ_5 HFG having 5 vertices with $\gamma_{tpd}(\zeta_5) = 2$ and ζ_4 HFG having 4 vertices with $\gamma_{tpd}(\zeta_4) = 2$. In figure 9.6.2, the twin perfect domination number of Omicron Product of two HFGs ζ_5 and ζ_4 is $\gamma_{tpd}(\zeta_5 \odot \zeta_4) = 15$.

Here

$$p = 5, \gamma_{tpd}(\zeta_5) = 2, \gamma_{tpd}(\zeta_4) = 2 \text{ and } \gamma_{tpd}(\zeta_4 \Theta \zeta_3) = 15$$

$$\Rightarrow 15 \geq 2 + 2 + 5$$

$$\Rightarrow 15 > 9$$

Therefore $\gamma_{tpd}(\zeta_p \odot \zeta_{p-1}) \geq \gamma_{tpd}(\zeta_p) + \gamma_{tpd}(\zeta_{p-1}) + p$. Hence the result.

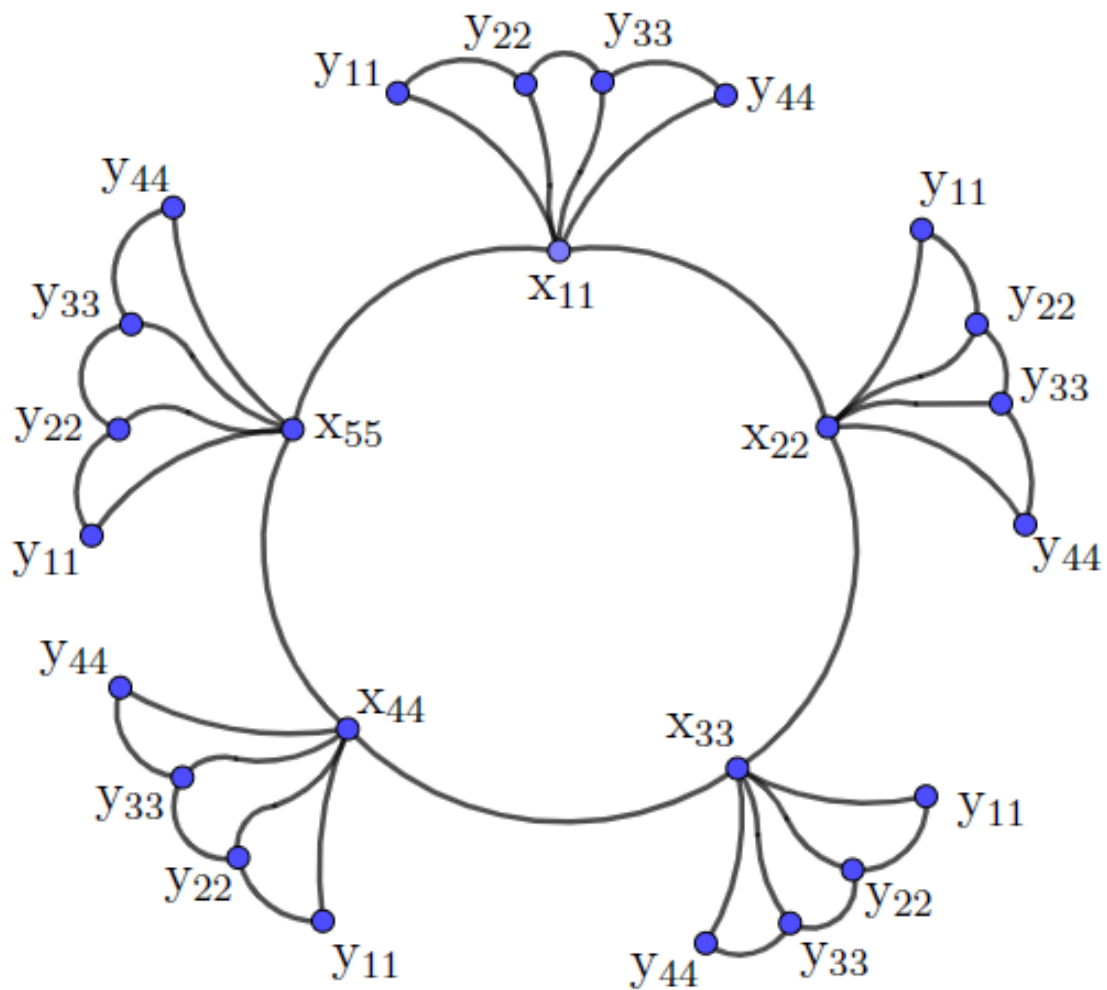


Figure 9.6.2: $(\zeta_5 \odot \zeta_4)$ with Twin Perfect Domination Number

Vertex	Value
x_{11}	0.2, 0.6, 0.4
x_{22}	0.7, 0.2, 0.1
x_{33}	0.6, 0.3, 0.1
x_{44}	0.3, 0.4, 0.3
x_{55}	0.4, 0.3, 0.4
y_{11}	0.2, 0.5, 0.3
y_{22}	0.5, 0.3, 0.2
y_{33}	0.4, 0.3, 0.3
y_{44}	0.3, 0.2, 0.5

Table 9.6.3: Vertex values of Figure 9.6.2

Edge	Value	Edge	Value
x_{11}, x_{22}	0.2, 0.6, 0.1	x_{33}, y_{22}	0.5, 0.3, 0.1
x_{22}, x_{33}	0.3, 0.3, 0.1	x_{33}, y_{33}	0.4, 0.3, 0.1
x_{33}, x_{44}	0.3, 0.4, 0.1	x_{33}, y_{44}	0.3, 0.3, 0.1
x_{44}, x_{55}	0.2, 0.4, 0.1	x_{44}, y_{11}	0.2, 0.5, 0.1
x_{55}, x_{11}	0.2, 0.5, 0.1	x_{44}, y_{22}	0.3, 0.4, 0.2
x_{11}, y_{11}	0.2, 0.3, 0.2	x_{44}, y_{33}	0.3, 0.4, 0.1
x_{11}, y_{22}	0.1, 0.4, 0.2	x_{44}, y_{44}	0.3, 0.4, 0.2
x_{11}, y_{33}	0.2, 0.5, 0.1	x_{55}, y_{11}	0.2, 0.4, 0.2
x_{11}, y_{44}	0.2, 0.4, 0.2	x_{55}, y_{22}	0.4, 0.3, 0.2
x_{22}, y_{11}	0.2, 0.5, 0.1	x_{55}, y_{33}	0.4, 0.2, 0.1
x_{22}, y_{22}	0.5, 0.3, 0.1	x_{55}, y_{44}	0.3, 0.3, 0.2
x_{22}, y_{33}	0.4, 0.3, 0.1	y_{11}, y_{22}	0.2, 0.5, 0.2
x_{22}, y_{44}	0.3, 0.2, 0.1	y_{22}, y_{33}	0.4, 0.3, 0.2
x_{33}, y_{11}	0.2, 0.5, 0.1	y_{33}, y_{44}	0.3, 0.3, 0.3

Table 9.6.4: Edge values of Figure 9.6.2

Theorem 9.6.2 Let ζ_1 and ζ_2 are two hesitancy fuzzy graph. Let M_{11} and M_{22} be the twin perfect dominating sets of ζ_1 and ζ_2 respectively. Then the twin perfect domination number of $\gamma_{tpd}(\zeta_1 \odot \zeta_2) > |M_{11}| + |M_{22}|$.

Proof. Let ζ_1 and ζ_2 are two HFG. Assume M_{11} and M_{22} be the twin perfect dominating sets of ζ_1 and ζ_2 respectively. If every vertex $x_{11} \in \zeta_1 \odot \zeta_2$ this implies $x_{11} \in \zeta_1$ or $x_{11} \in \zeta_2$, therefore there is a vertices $x_{22}, x_{33} \in M_{11}$ or $x_{22}, x_{33} \in M_{22}$ such that x_{11} is adjacent to exactly two vertices in M_{11} or M_{22} . Since M_{11} and M_{22} be the twin perfect dominating sets of ζ_1 and ζ_2 respectively. Then the twin perfect domination number of $\gamma_{tpd}(\zeta_1 \odot \zeta_2) > |M_{11}| + |M_{22}|$.

Example 9.6.3 Let ζ_4 HFG having 4 vertices with twin perfect dominating sets of $M_{11} = (m_{11}, m_{22}, m_{33})$ and ζ_3 HFG having 3 vertices with $M_{22} = (n_{11}, n_{33})$. In figure 9.6.1, the twin perfect domination number of Omicron Product of two HFGs ζ_4 and ζ_3 is $\gamma_{tpd}(\zeta_4 \odot \zeta_3) = 8$.

Here

$$\begin{aligned} |M_{11}| &= 3, |M_{22}| = 2 \text{ and } \gamma_{tpd}(\zeta_4 \odot \zeta_3) = 8 \\ \Rightarrow 8 &\geq 3 + 2 \\ \Rightarrow 8 &> 5 \end{aligned}$$

Therefore $\gamma_{tpd}(\zeta_1 \odot \zeta_2) > |M_{11}| + |M_{22}|$. Hence the result.

Theorem 9.6.3 Let ζ_1 and ζ_2 are two hesitancy fuzzy graph. If $|v(\zeta_1)| = |v(\zeta_2)|$, then $|v(\zeta_1)| + |v(\zeta_2)| \leq \gamma_{tpd}(\zeta_1 \odot \zeta_2)$.

Proof. Let ζ_1 and ζ_2 are two HFG. Let $|v(\zeta_1)|$ and $|v(\zeta_2)|$ be the cardinality in vertices of ζ_1 and ζ_2 respectively. In $\zeta_1 \odot \zeta_2$ every vertex in ζ_1 is adjacent to every vertices in ζ_2 . If $|v(\zeta_1)| = |v(\zeta_2)|$, then the minimal twin perfect domination number of $\zeta_1 \odot \zeta_2$ is $|v(\zeta_1)| + |v(\zeta_2)| \leq \gamma_{tpd}(\zeta_1 \odot \zeta_2)$.

Example 9.6.4 Let ζ_1 HFG having 4 vertices and ζ_2 HFG having 4 vertices. In figure 9.6.3, the twin perfect domination number of Omicron Product of two HFGs ζ_1 and ζ_2 is $\gamma_{tpd}(\zeta_1 \odot \zeta_2) = 12$.

Here

$$\begin{aligned} |v(\zeta_1)| &= 4, |v(\zeta_2)| = 4 \text{ and } \gamma_{tpd}(\zeta_1 \odot \zeta_2) = 12 \\ \Rightarrow 4 + 4 &< 12 \\ \Rightarrow 8 &< 12 \end{aligned}$$

Therefore $\zeta_1 \odot \zeta_2$ is $|v(\zeta_1)| + |v(\zeta_2)| < \gamma_{tpd}(\zeta_1 \odot \zeta_2)$. Hence the result.

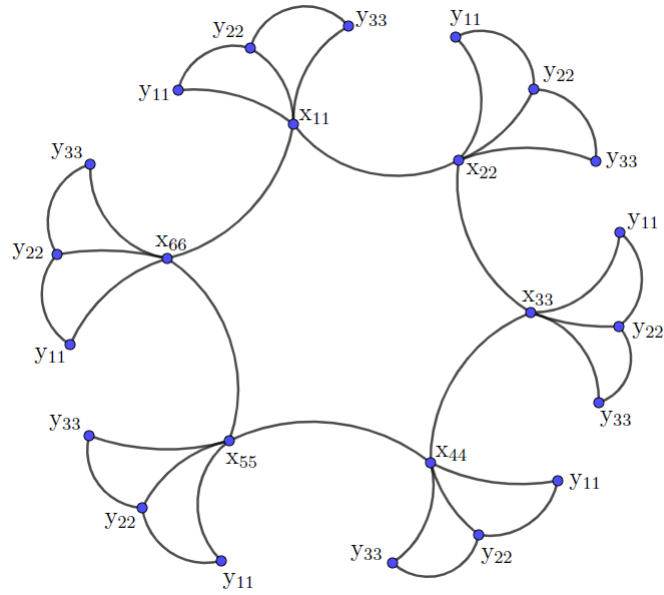


Figure 9.6.3: $(\zeta_1 \odot \zeta_2)$ with Twin Perfect Domination Number

Vertex	Value
x_{11}	0.2, 0.5, 0.3
x_{22}	0.6, 0.3, 0.1
x_{33}	0.3, 0.4, 0.3
x_{44}	0.2, 0.6, 0.2
x_{55}	0.4, 0.2, 0.4
x_{66}	0.7, 0.2, 0.1
y_{11}	0.5, 0.2, 0.3
y_{22}	0.4, 0.3, 0.3
y_{33}	0.2, 0.4, 0.4

Table 9.6.5: Vertex values of Figure 9.6.3

Edge	Value	Edge	Value
x_{11}, x_{22}	0.2, 0.5, 0.1	x_{33}, y_{22}	0.3, 0.3, 0.1
x_{22}, x_{33}	0.3, 0.4, 0.1	x_{33}, y_{33}	0.2, 0.4, 0.1
x_{33}, x_{44}	0.2, 0.5, 0.1	x_{44}, y_{11}	0.2, 0.4, 0.2
x_{44}, x_{55}	0.1, 0.5, 0.2	x_{44}, y_{22}	0.1, 0.3, 0.2
x_{55}, x_{66}	0.4, 0.1, 0.1	x_{44}, y_{33}	0.2, 0.5, 0.2
x_{66}, x_{11}	0.2, 0.3, 0.1	x_{55}, y_{11}	0.4, 0.2, 0.3
x_{11}, y_{11}	0.1, 0.4, 0.2	x_{55}, y_{22}	0.2, 0.3, 0.1
x_{11}, y_{22}	0.2, 0.3, 0.2	x_{55}, y_{33}	0.2, 0.4, 0.2
x_{11}, y_{33}	0.2, 0.4, 0.1	x_{66}, y_{11}	0.5, 0.2, 0.1
x_{22}, y_{11}	0.4, 0.3, 0.1	x_{66}, y_{22}	0.4, 0.3, 0.1
x_{22}, y_{22}	0.4, 0.2, 0.1	x_{66}, y_{33}	0.2, 0.4, 0.1
x_{22}, y_{33}	0.2, 0.4, 0.1	y_{11}, y_{22}	0.4, 0.3, 0.2
x_{33}, y_{11}	0.3, 0.2, 0.3	y_{22}, y_{33}	0.2, 0.4, 0.3

Table 9.6.6: Edge values of Figure 9.6.3

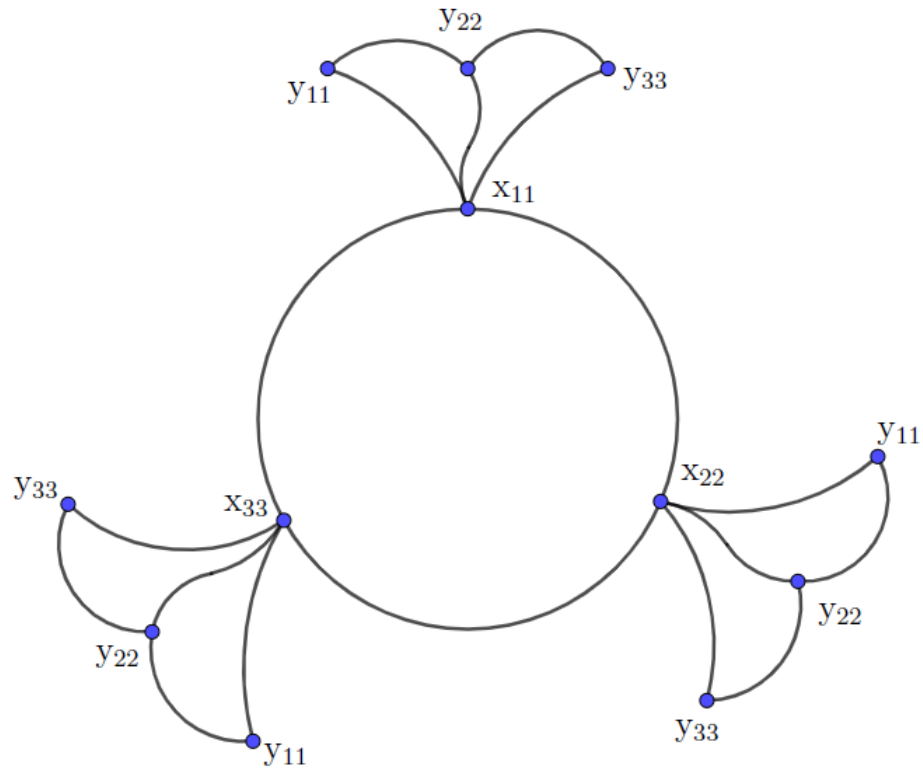
Example 9.6.5 Let ζ_1 HFG having 3 vertices and ζ_2 HFG having 3 vertices. In figure 9.6.4, the twin perfect domination number of Omicron Product of two HFGs ζ_1 and ζ_2 is $\gamma_{tpd}(\zeta_1 \odot \zeta_2) = 6$.

Here

$$|v(\zeta_1)| = 3, |v(\zeta_2)| = 3 \text{ and } \gamma_{tpd}(\zeta_1 \odot \zeta_2) = 6$$

$$\Rightarrow 3 + 3 = 6$$

Therefore $|v(\zeta_1)| + |v(\zeta_2)| = \gamma_{tpd}(\zeta_1 \odot \zeta_2)$. Hence the result.

Figure 9.6.4: $(\zeta_1 \odot \zeta_2)$ with Twin Perfect Domination Number

Vertex	Value
x_{11}	0.6, 0.3, 0.1
x_{22}	0.5, 0.3, 0.2
x_{33}	0.4, 0.3, 0.3
y_{11}	0.2, 0.6, 0.2
y_{22}	0.7, 0.2, 0.1
y_{33}	0.6, 0.3, 0.1

Table 9.6.7: Vertex values of Figure 9.6.4

Edge	Value	Edge	Value
x_{11}, x_{22}	0.2, 0.3, 0.1	x_{22}, y_{22}	0.4, 0.2, 0.1
x_{22}, x_{33}	0.4, 0.3, 0.2	x_{22}, y_{33}	0.3, 0.3, 0.1
x_{33}, x_{11}	0.4, 0.3, 0.1	x_{33}, y_{11}	0.4, 0.3, 0.1
x_{11}, y_{11}	0.2, 0.4, 0.1	x_{33}, y_{22}	0.3, 0.3, 0.1
x_{11}, y_{22}	0.6, 0.2, 0.1	x_{33}, y_{33}	0.4, 0.3, 0.1
x_{11}, y_{33}	0.5, 0.2, 0.1	y_{11}, y_{22}	0.2, 0.6, 0.1
x_{22}, y_{11}	0.2, 0.5, 0.2	y_{22}, y_{33}	0.4, 0.3, 0.1

Table 9.6.8: Edge Values of Figure 9.6.4

The Technical Advisory Group on SARS-CoV-2 Virus Evolution (TAG-VE) is an independent group of experts that periodically monitors and evaluates the evolution of SARS-CoV-2 and assesses if specific mutations and combinations of mutations alter the behavior of the virus. The TAG-VE was convened on 26 November 2021 to assess the SARS-CoV-2 variant B.1.1.529. The B.1.1.529 variant was first reported to WHO from South Africa on 24 November 2021 and has been named Omicron.

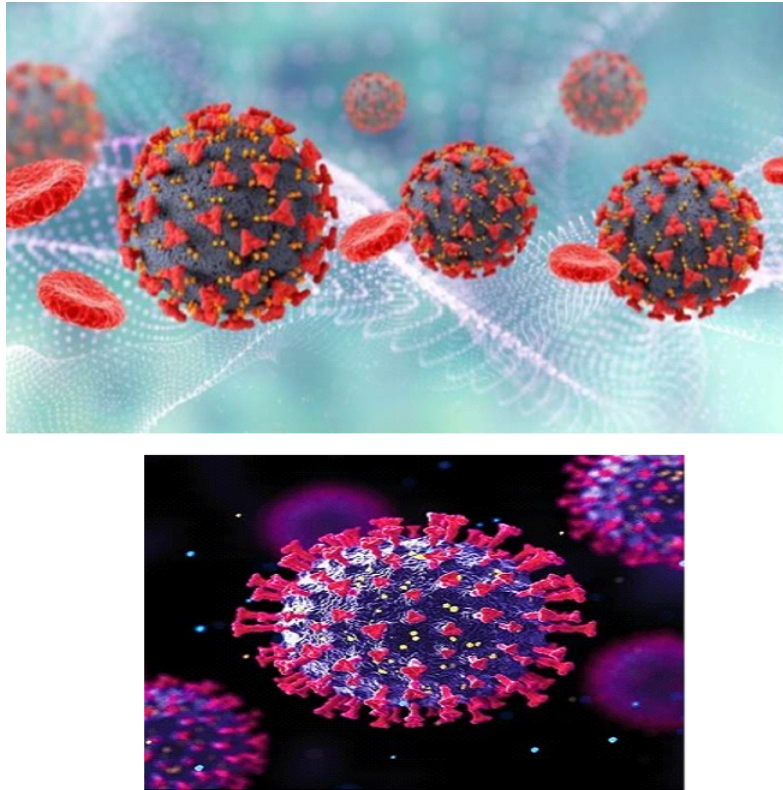


Figure 9.6.5: Omicron Virus

In this chapter the Omicron Product of two hesitancy fuzzy graphs is defined. In hesitancy fuzzy graphs, the concept of domination has a lot of theoretical and practical implications. The concept of twin perfect domination number in hesitancy fuzzy graphs and the Omicron Product of two hesitancy fuzzy graphs are introduced in this chapter. The bounds of the twin perfect domination number of the Omicron Product of two hesitancy fuzzy graphs are determined. In a hesitancy fuzzy graph, those vertices that are in the minimum twin perfect dominant set operate as a special node, server, or controller. The Omicron Product of two hesitancy fuzzy graphs creates a complex network, and their twin perfect dominating set represents the number of vertices that operate as a server. As a result, we can quickly identify our best server within the complex network.

Chapter 10

Distinct Categories of Edge Sequence in Regular and Pseudo Regular Fuzzy Graphs

The concept of applications of fuzzy graphs with connectedness is crucial. This chapter divides the edge sequence of a fuzzy graph into separate types based on the strength of an edge. The concept of edge sequences in regular fuzzy graphs and pseudo regular fuzzy graphs are explained by theorems and examples. These ideas are inspired by the concept of fuzzy graph connectedness. In addition, a comparison of regular fuzzy graphs and total regular fuzzy graphs is made about distinct categories of edge sequences in fuzzy graphs. In terms of edge sequence, a required condition for a graph to be regular fuzzy graph and total regular fuzzy graph are explained, and the concepts of pseudo regular fuzzy graphs, total pseudo regular fuzzy graphs, and distinct categories of edge sequence are examined. Also we identify strong edge sequence in regular fuzzy graphs and pseudo regular fuzzy graphs. In addition, with distinct categories of edge sequences, an analogy is conducted between pseudo regular fuzzy graphs and total pseudo regular fuzzy graphs.

10.1 Edge Sequence in Regular Fuzzy Graphs

The new concepts of α - edge sequence, β - edge sequence and δ - edge sequence in regular fuzzy graphs are defined in this section, along with related theorems.

Definition 10.1.1 Let $G_R : (\sigma_R, \mu_R)$ be a regular fuzzy graph (RFG) with $\sigma_R^* = \{v_{11}, v_{22}, \dots, v_{qq}\}$ in any form. Then a finite sequence $\alpha_{ES}(G_R) = (n_{11}, n_{22}, \dots, n_{qq})$ is named the α -edge sequence (ES) of G_R if

$$n_k = \begin{cases} r, & \text{where } r \text{ is number of } \alpha\text{-strong edges incident on } v_k \\ 0, & \text{otherwise} \end{cases}$$

Example 10.1.1 Consider $G_R : (\sigma_R, \mu_R)$ where

$$\sigma_R(m_{11}) = 0.4, \sigma_R(m_{22}) = 0.5, \sigma_R(m_{33}) = 0.5, \sigma_R(m_{44}) = 0.4$$

$$\mu_R(m_{11}, m_{22}) = 0.4, \mu_R(m_{22}, m_{44}) = 0.3, \mu_R(m_{11}, m_{33}) = 0.3 \text{ and } \mu_R(m_{33}, m_{44}) = 0.4$$

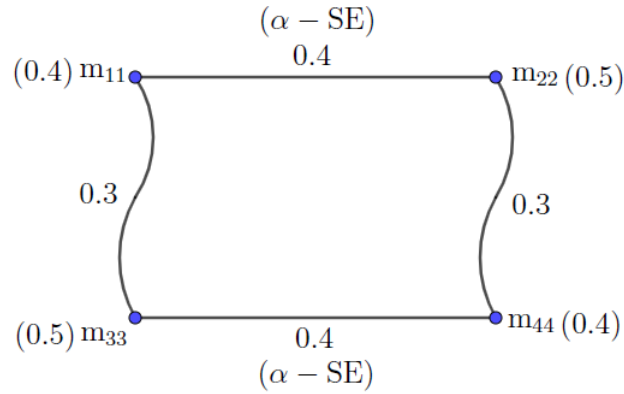


Figure 10.1.1: RFG with α -edge sequence

$$\alpha_{ES}(G_R) = (1, 1, 1, 1)$$

Here (m_{11}, m_{22}) and (m_{33}, m_{44}) are α -strong edges. Then every vertex having one α -ES. Therefore $\alpha_{ES}(G_R) = (1, 1, 1, 1)$.

Definition 10.1.2 Let $G_R : (\sigma_R, \mu_R)$ be a regular fuzzy graph with $\sigma_R^* = \{v_{11}, v_{22}, \dots, v_{qq}\}$ in any form.

Then a finite sequence $\beta_{SE}(G_R) = (n_{11}, n_{22}, \dots, n_{qq})$ is labeled the β -edge sequence of G_R if

$$n_k = \begin{cases} r, & \text{where } r \text{ is number of } \beta - \text{strong edges incident on } v_k \\ 0, & \text{otherwise} \end{cases}$$

Example 10.1.2 Consider $G_R : (\sigma_R, \mu_R)$ where

$$\sigma_R(m_{11}) = 0.3, \sigma_R(m_{22}) = 0.2, \sigma_R(m_{33}) = 0.3, \sigma_R(m_{44}) = 0.2$$

$$\mu_R(m_{11}, m_{22}) = 0.2, \mu_R(m_{22}, m_{44}) = 0.2, \mu_R(m_{11}, m_{33}) = 0.2 \text{ and } \mu_R(m_{33}, m_{44}) = 0.2$$

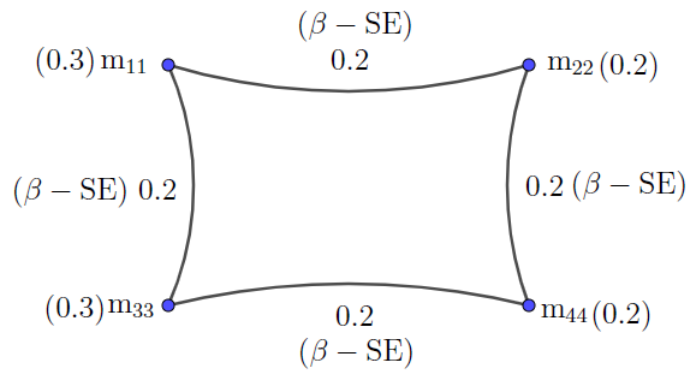


Figure 10.1.2: RFG with β -edge sequence

$$\beta_{ES}(G_R) = (2, 2, 2, 2)$$

Here (m_{11}, m_{22}) , (m_{22}, m_{44}) , (m_{11}, m_{33}) and (m_{33}, m_{44}) are β -SEs. Then every vertex having two β -ES. Therefore $\beta_{ES}(G_R) = (2, 2, 2, 2)$.

Definition 10.1.3 Let $G_R : (\sigma_R, \mu_R)$ be a regular fuzzy graph with $\sigma_R^* = \{v_{11}, v_{22}, \dots, v_{qq}\}$ in any form. Then a finite sequence $\delta_{SE}(G_R) = (n_{11}, n_{22}, \dots, n_{qq})$ is termed the δ -edge sequence of G_R if

$$n_k = \begin{cases} r, & \text{where } r \text{ is number of } \delta - \text{edges incident on } v_k \\ 0, & \text{otherwise} \end{cases}$$

Example 10.1.3 Consider $G_R : (\sigma_R, \mu_R)$ where

$$\begin{aligned} \sigma_R(m_{11}) &= 0.5, \sigma_R(m_{22}) = 0.4, \sigma_R(m_{33}) = 0.4, \sigma_R(m_{44}) = 0.6 \\ \mu_R(m_{11}, m_{22}) &= 0.4, \mu_R(m_{22}, m_{44}) = 0.2, \mu_R(m_{11}, m_{33}) = 0.2, \mu_R(m_{33}, m_{44}) = 0.4, \\ \mu_R(m_{11}, m_{44}) &= 0.1 \text{ and } \mu_R(m_{22}, m_{33}) = 0.1 \end{aligned}$$

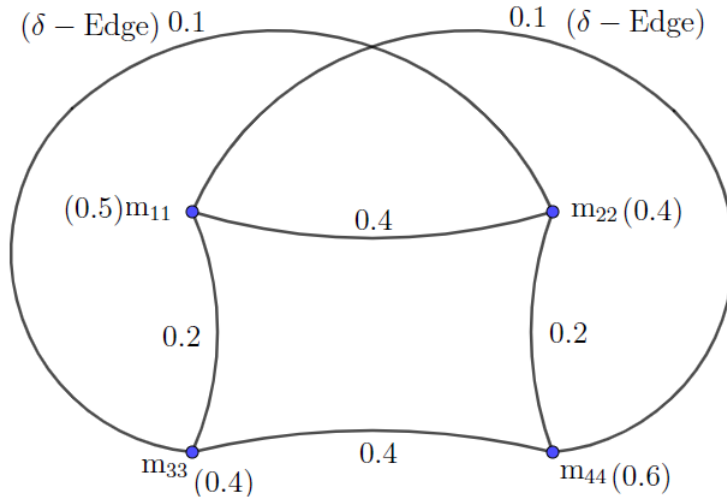


Figure 10.1.3: RFG with δ -edge sequence

$$\delta_{ES}(G_R) = (1, 1, 1, 1)$$

Here (m_{11}, m_{44}) and (m_{22}, m_{33}) are δ -edges. Then every vertex having one δ -ES. Therefore $\delta_{ES}(G_R) = (1, 1, 1, 1)$.

Definition 10.1.4 Let $G_R : (\sigma_R, \mu_R)$ be a regular fuzzy graph with $\sigma_R^* = \{v_{11}, v_{22}, \dots, v_{qq}\}$ in any form. Then a finite sequence $S_{ES}(G_R) = (n_{11}, n_{22}, \dots, n_{qq})$ is labeled the strong edge sequence (SES) of G_R if

$$n_k = \begin{cases} r, & \text{where } r \text{ is number of } \alpha\text{-strong edges and } \beta\text{-strong edges incident on } v_k \\ 0, & \text{otherwise} \end{cases}$$

Example 10.1.4 Consider $G_R : (\sigma_R, \mu_R)$ where

$$\begin{aligned} \sigma_R(m_{11}) &= 0.2, \sigma_R(m_{22}) = 0.2, \sigma_R(m_{33}) = 0.4, \sigma_R(m_{44}) = 0.6, \sigma_R(m_{55}) = 0.4 \\ \mu_R(m_{11}, m_{22}) &= 0.1, \mu_R(m_{11}, m_{33}) = 0.1, \mu_R(m_{11}, m_{44}) = 0.1, \mu_R(m_{11}, m_{55}) = 0.1, \\ \mu_R(m_{22}, m_{33}) &= 0.3 \text{ and } \mu_R(m_{44}, m_{55}) = 0.3 \end{aligned}$$

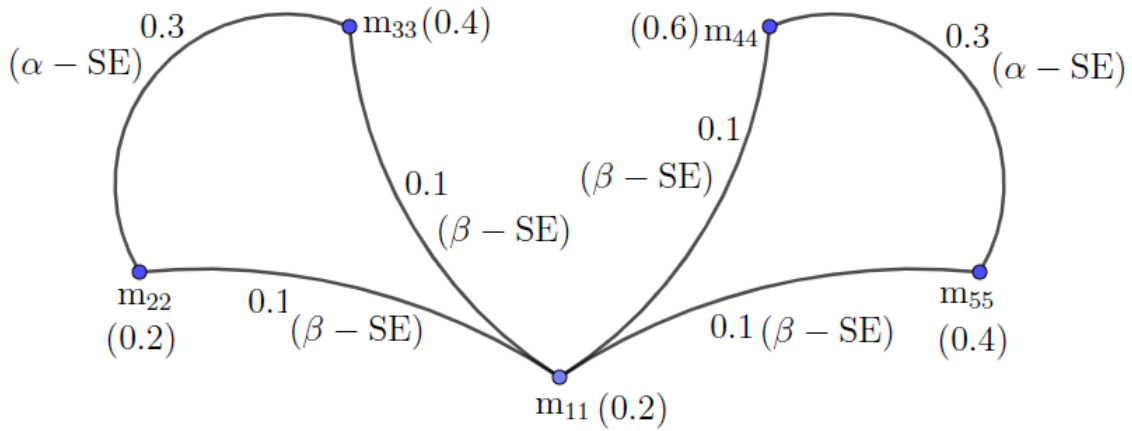


Figure 10.1.4: RFG with Strong Edge Sequence

$$\alpha_{ES}(G_R) = (0, 1, 1, 1, 1), \beta_{ES}(G_R) = (4, 1, 1, 1, 1), S_{ES}(G_R) = (4, 2, 2, 2, 2).$$

Here $(m_{22}, m_{33}), (m_{44}, m_{55})$ are α -SEs. $(m_{11}, m_{22}), (m_{11}, m_{33}), (m_{11}, m_{44}), (m_{11}, m_{55})$ are β -SEs and therefore

$$\alpha_{ES}(G_R) = (0, 1, 1, 1, 1)$$

$$\beta_{ES}(G_R) = (4, 1, 1, 1, 1)$$

$$S_{ES}(G_R) = (4, 2, 2, 2, 2)$$

Remark 10.1.1 Let $G_R : (\sigma_R, \mu_R)$ be a RFG with all types of edge sequence

Example 10.1.5 Consider $G_R : (\sigma_R, \mu_R)$ where

$$\begin{aligned} \sigma_R(m_{11}) &= 0.5, \sigma_R(m_{22}) = 0.5, \sigma_R(m_{33}) = 0.5, \sigma_R(m_{44}) = 0.5 \\ \mu_R(m_{11}, m_{22}) &= 0.5, \mu_R(m_{22}, m_{44}) = 0.3, \mu_R(m_{11}, m_{33}) = 0.3, \mu_R(m_{33}, m_{44}) = 0.5, \\ \mu_R(m_{11}, m_{44}) &= 0.1 \text{ and } \mu_R(m_{22}, m_{33}) = 0.1 \end{aligned}$$

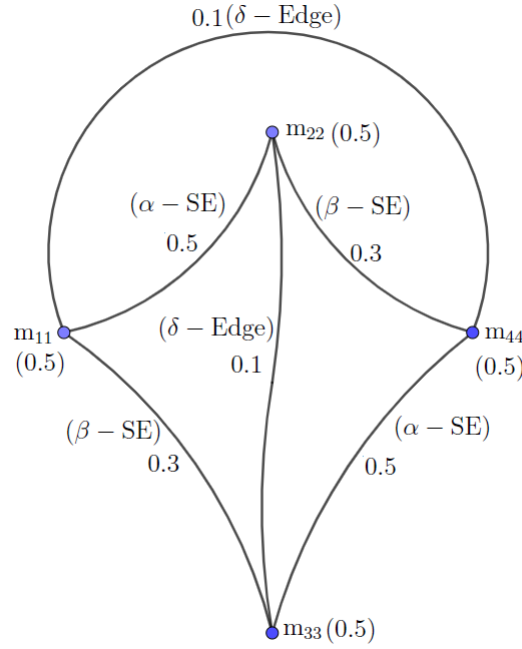


Figure 10.1.5: RFG with all types of Edge Sequences

$$\alpha_{ES}(G_R) = (1, 1, 1, 1), \beta_{ES}(G_R) = (1, 1, 1, 1), \delta_{ES}(G_R) = (1, 1, 1, 1), S_{ES}(G_R) = (2, 2, 2, 2).$$

Here $(m_{11}, m_{22}), (m_{33}, m_{44})$ are α -SEs. $(m_{22}, m_{44}), (m_{11}, m_{33})$ are β -SEs and $(m_{11}, m_{44}), (m_{22}, m_{33})$ are δ -edges. Then each vertex having one α -ES, one β -ES and one δ -ES.

That is $\alpha_{ES}(G_R) = (1, 1, 1, 1), \beta_{ES}(G_R) = (1, 1, 1, 1)$ and $\delta_{ES}(G_R) = (1, 1, 1, 1)$.

Therefore $S_{ES}(G_R) = (2, 2, 2, 2)$.

Definition 10.1.5 A zero sequence is a real sequence having only '0'. It is represented by (0).

Theorem 10.1.1 A fuzzy graph G whose crisp graph is an odd cycle is regular if and only if μ is a constant function. [NR10]

Theorem 10.1.2 A fuzzy graph G whose crisp graph is an even cycle is regular if and only if μ is a constant function or alternative edges will have same values. [NR10]

Theorem 10.1.3 Let $G_R : (\sigma_R, \mu_R)$ be a regular fuzzy graph such that crisp graph G_R^* is an odd cycle. Then G_R is a regular fuzzy graph iff $\alpha_{ES}(G_R) = (0)$ and $\delta_{ES}(G_R) = (0)$.

Proof. Suppose that $\alpha_{ES}(G_R) = (0)$ and $\delta_{ES}(G_R) = (0)$. This means G_R contains only $\beta_{ES}(G_R)$. Then by 10.1.2 definition, we have $\mu_R(m_{11}, m_{22}) = \text{CONN}_{G_R - (m_{11}, m_{22})}(m_{11}, m_{22})$. Thus all the edges in G_R will have the equal membership value (MV). Then by the 10.1.1 theorem, we get G_R as a RFG.

Contrarily, suppose that G_R be a RFG. Then by the 10.1.1 theorem, the membership value is a constant function. Thus the removals of any edge in G_R will not change the strength of connectedness of any $m_{11} - m_{44}$ path in G_R .

That is, $\mu_R(m_{11}, m_{22}) = \text{CONN}_{G_R - (m_{11}, m_{22})}(m_{11}, m_{22}) \forall (m_{11}, m_{22}) \in G_R$.

That is G_R contains only $\beta_{ES}(G_R)$.

Thus $\alpha_{ES}(G_R) = (0)$ and $\delta_{ES}(G_R) = (0)$.

Example 10.1.6

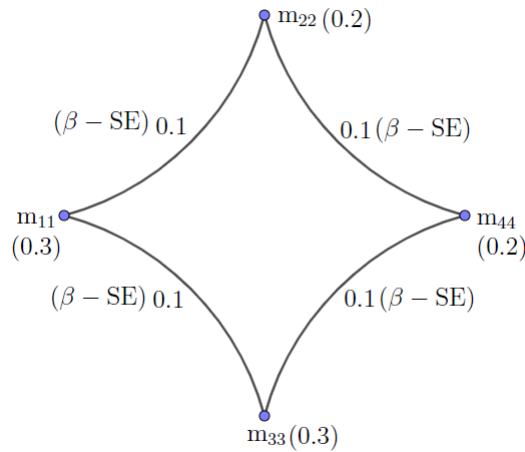


Figure 10.1.6: RFG without $\alpha_{ES}(G_R)$ and $\delta_{ES}(G_R)$

In the above example G_R is a RFG with β -SEs.

Here (m_{11}, m_{22}) , (m_{22}, m_{44}) , (m_{33}, m_{44}) and m_{33}, m_{44} are β -SEs. Then each vertex having two β -ESs.

Therefore $\alpha_{ES}(G_R) = (0, 0, 0, 0)$ and $\delta_{ES}(G_R) = (0, 0, 0, 0)$.

Remark 10.1.2 The above result does not hold for a totally regular fuzzy graph (TRFG).

Example 10.1.7 Consider $G_{TR} : (\sigma_{TR}, \mu_{TR})$ where

$$\begin{aligned} \sigma_{TR}(m_{11}) &= 0.6, \sigma_{TR}(m_{22}) = 0.7, \sigma_{TR}(m_{33}) = 0.7 \\ \mu_{TR}(m_{11}, m_{22}) &= 0.6, \mu_{TR}(m_{11}, m_{33}) = 0.6 \text{ and } \mu_{TR}(m_{22}, m_{33}) = 0.5 \end{aligned}$$

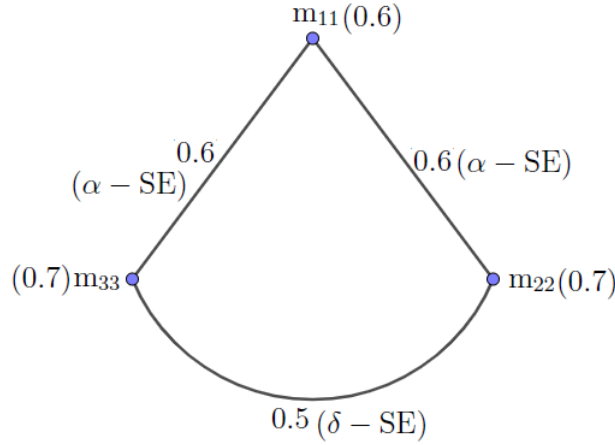


Figure 10.1.7: TRFG without $\alpha_{ES}(G_R)$ and $\delta_{ES}(G_R)$

Here G_{TR} is a TRFG. But it has $\alpha_{ES}(G_{TR})$ and $\delta_{ES}(G_{TR})$.

Theorem 10.1.4 A regular fuzzy graph $G_R : (\sigma_R, \mu_R)$ whose even cycle is the crisp graph G_R^* contains $\alpha_{ES}(G_R)$ and $\beta_{ES}(G_R)$. Also $\delta_{ES}(G_R) = (0)$.

Proof. Assume $\delta_{ES}(G_R)$ is a zero sequence. That is $\delta_{ES}(G_R) = (0)$. Then by the 10.1.1 and 10.1.2 definitions, we have $\mu_R(m_{11}, m_{22}) \geq \text{CONN}_{G_R - (m_{11}, m_{22})}(m_{11}, m_{22})$ which implies that $MV \mu_R$ has either stable or alternative edges will have equal values. Then by the 10.1.2 theorem, we get G_R as an RFG.

Contrarily, Let G_R be an RFG. Then by the 10.1.2 theorem, the MV μ_R is either stable or alternative edges will have equal values.

That is, $\mu_R(m_{11}, m_{22}) \geq CONN_{G_R - (m_{11}, m_{22})}(m_{11}, m_{22})$.

This implies $\delta_{ES}(G_R)$ is a zero edge sequence. That is $\delta_{ES}(G_R) = (0)$.

Example 10.1.8 Consider $G_R : (\sigma_R, \mu_R)$ where

$$\sigma_R(m_{11}) = 0.2, \sigma_R(m_{22}) = 0.6, \sigma_R(m_{33}) = 0.3, \sigma_R(m_{44}) = 0.5$$

$$\mu_R(m_{11}, m_{22}) = 0.2, \mu_R(m_{22}, m_{44}) = 0.3, \mu_R(m_{11}, m_{33}) = 0.3 \text{ and } \mu_R(m_{33}, m_{44}) = 0.2$$

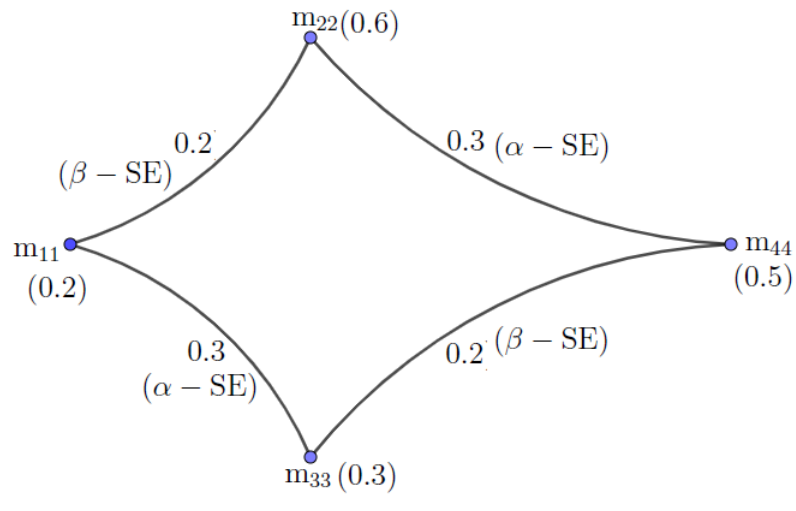


Figure 10.1.8: RFG without $\delta_{ES}(G_R)$

Here $(m_{11}, m_{22}), (m_{33}, m_{44})$ are β -SEs and $(m_{22}, m_{44}), (m_{11}, m_{33})$ are α -SEs. Then each vertex having one α -ES and one β -ES.

That is $\alpha_{ES}(G_R) = (1, 1, 1, 1), \beta_{ES}(G_R) = (1, 1, 1, 1)$ and $\delta_{ES}(G_R) = (0, 0, 0, 0)$.

Remark 10.1.3 The above result need not be true for a totally regular fuzzy graph.

Example 10.1.9 Consider $G_{TR} : (\sigma_{TR}, \mu_{TR})$ where

$$\begin{aligned} \sigma_{TR}(m_{11}) &= 0.4, \sigma_{TR}(m_{22}) = 0.4, \sigma_{TR}(m_{33}) = 0.6, \sigma_{TR}(m_{44}) = 0.4 \\ \mu_{TR}(m_{11}, m_{22}) &= 0.3, \mu_{TR}(m_{22}, m_{44}) = 0.3, \mu_{TR}(m_{11}, m_{33}) = 0.2, \\ \mu_{TR}(m_{11}, m_{44}) &= 0.1 \text{ and } \mu_{TR}(m_{33}, m_{44}) = 0.2. \end{aligned}$$

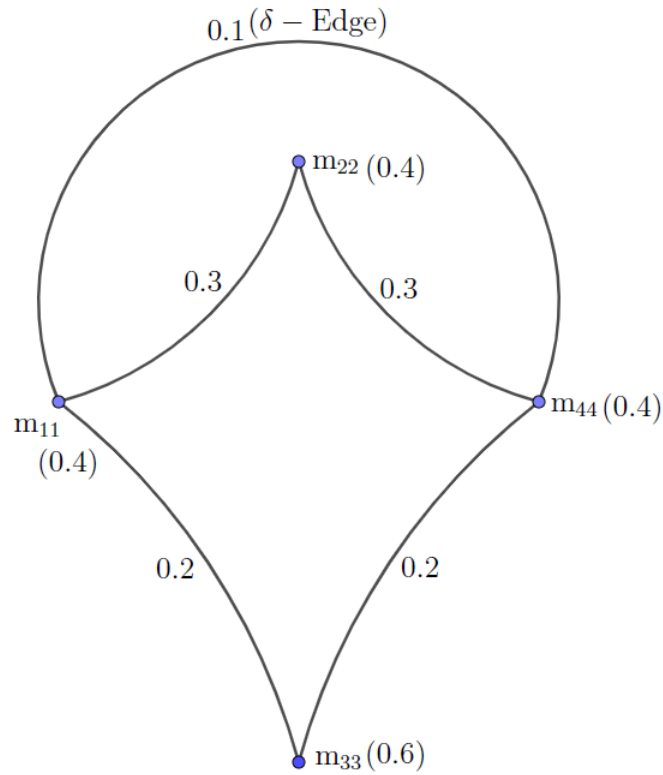


Figure 10.1.9: TRFG with $\delta_{ES}(G_R)$

Here G_{TR} is a TRFG but it has $\delta_{ES}(G_{TR})$.

Theorem 10.1.5 A regular fuzzy graph $G_R : (\sigma_R, \mu_R)$ with its crisp graph G_R^* as even cycle is both regular fuzzy graph and totally regular fuzzy graph then $\delta_{ES}(G_R) = (0)$.

Proof. Let $G_R : (\sigma_R, \mu_R)$ be an RFG. Then its crisp graph G_R^* is an even cycle and G_R be both RFG and TRFG. Here are two cases that arise.

Case (i)

Let G_R be both RFG and TRFG with stable values in σ_R and μ_R then by 10.1.2 definition, this means G_R contains only $\beta_{ES}(G_R)$.

Example 10.1.10 Let G_R is an RFG and TRFG without $\delta_{ES}(G_R)$. That is G_R having β -SEs. Therefore $\delta_{ES}(G_R) = (0, 0, 0, 0)$.

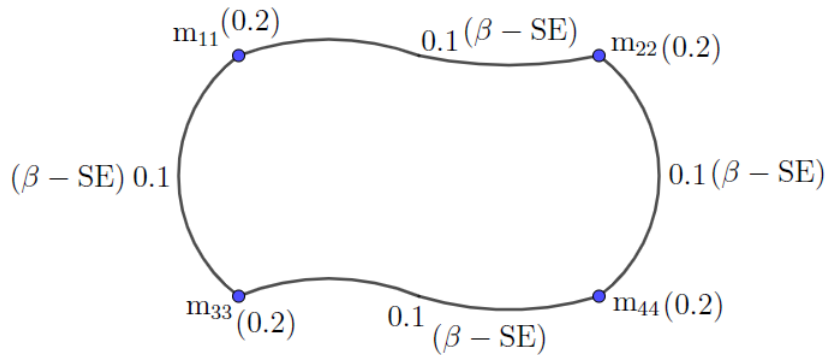
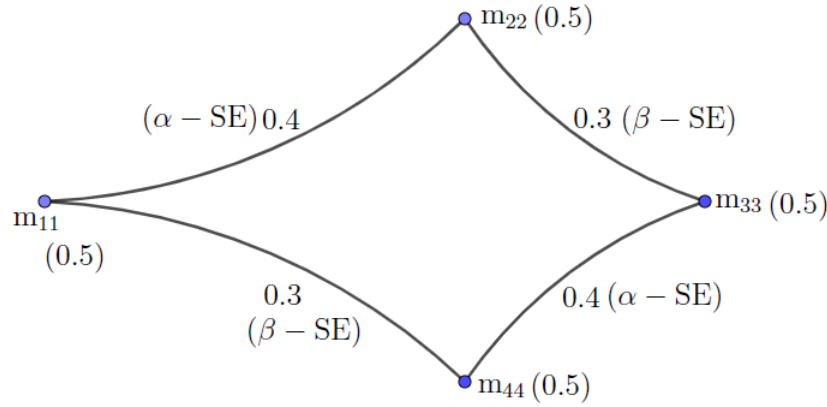


Figure 10.1.10: RFG and TRFG without $\delta_{ES}(G_R)$

Case (ii)

Let G_R be both RFG and TRFG with stable values in σ_R and with equal alternative values in μ_R then by 10.1.1 and 10.1.2 definitions, G_R contains only $\alpha_{ES}(G_R)$ and $\beta_{ES}(G_R)$. This means $\delta_{ES}(G_R) = (0)$.

Example 10.1.11 Let G_R is an RFG and TRFG without $\delta_{ES}(G_R)$. That is G_R having α -SEs and β -SEs. Therefore $\delta_{ES}(G_R) = (0, 0, 0, 0)$.

Figure 10.1.11: RFG and TRFG without $\delta_{ES}(G_R)$

10.2 Edge Sequence in Pseudo Regular Fuzzy Graphs

In this section, the new idea of edge sequence in a pseudo regular fuzzy graph (PRFG) is defined and discussed notation of α -edge sequence, β -edge sequence, and δ -edge sequence.

Definition 10.2.1 Let $G_{PR} : (\sigma_{PR}, \mu_{PR})$ be a pseudo regular fuzzy graph with $\sigma_{PR}^* = \{v_{11}, v_{22}, \dots, v_{ll}\}$ in any form. Then a finite sequence $\alpha_{ES}(G_{PR}) = (n_{11}, n_{22}, \dots, n_{ll})$ is named the α -edge sequence of G_{PR} if

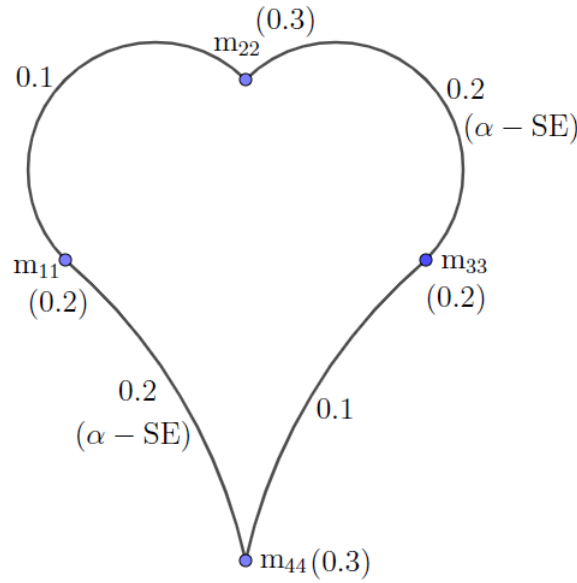
$$n_i = \begin{cases} r, & \text{where } r \text{ is number of } \alpha - \text{strong edges incident on } v_i \\ 0, & \text{otherwise} \end{cases}$$

Example 10.2.1 Consider $G_{PR} : (\sigma_{PR}, \mu_{PR})$ where

$$\sigma_{PR}(m_{11}) = 0.2, \sigma_{PR}(m_{22}) = 0.3, \sigma_{PR}(m_{33}) = 0.2, \sigma_{PR}(m_{44}) = 0.3$$

$$\mu_{PR}(m_{11}, m_{22}) = 0.1, \mu_{PR}(m_{22}, m_{33}) = 0.2, \mu_{PR}(m_{11}, m_{44}) = 0.2 \text{ and } \mu_{PR}(m_{33}, m_{44}) = 0.1$$

$$\alpha_{ES}(G_{PR}) = (1, 1, 1, 1)$$

Figure 10.2.1: PRFG with α -Edge Sequence

Here (m_{11}, m_{44}) and (m_{22}, m_{33}) are α -SEs. Then every vertex having one α -ES. Therefore $\alpha_{ES}(G_{PR}) = (1, 1, 1, 1)$.

Definition 10.2.2 Let $G_{PR} : (\sigma_{PR}, \mu_{PR})$ be a pseudo regular fuzzy graph with $\sigma_{PR}^* = \{v_{11}, v_{22}, \dots, v_{ll}\}$ in any form. Then a finite sequence $\beta_{SE}(G_{PR}) = (n_{11}, n_{22}, \dots, n_{ll})$ is labeled the β -edge sequence of G_{PR} if

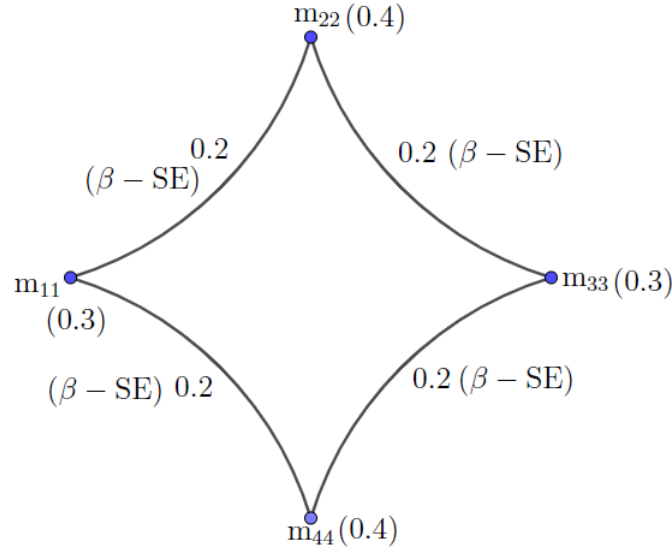
$$n_i = \begin{cases} r, & \text{where } r \text{ is number of } \beta\text{-strong edges incident on } v_i \\ 0, & \text{otherwise} \end{cases}$$

Example 10.2.2 Consider $G_{PR} : (\sigma_{PR}, \mu_{PR})$ where

$$\sigma_{PR}(m_{11}) = 0.3, \sigma_{PR}(m_{22}) = 0.4, \sigma_{PR}(m_{33}) = 0.3, \sigma_{PR}(m_{44}) = 0.4$$

$$\mu_{PR}(m_{11}, m_{22}) = 0.2, \mu_{PR}(m_{22}, m_{33}) = 0.2, \mu_{PR}(m_{11}, m_{44}) = 0.2 \text{ and } \mu_{PR}(m_{33}, m_{44}) = 0.2$$

$$\beta_{ES}(G_{PR}) = (2, 2, 2, 2)$$

Figure 10.2.2: PRFG with β -Edge Sequence

Here (m_{11}, m_{22}) , (m_{22}, m_{33}) , (m_{33}, m_{44}) and (m_{11}, m_{44}) are β -SEs. Then every vertex having two β -ESs. Therefore $\beta_{ES}(G_{PR}) = (2, 2, 2, 2)$.

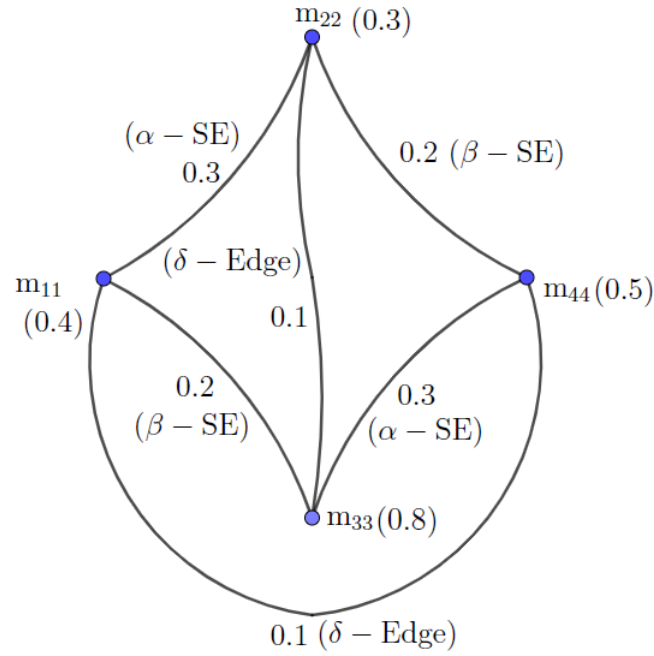
Definition 10.2.3 Let $G_{PR} : (\sigma_{PR}, \mu_{PR})$ be a pseudo regular fuzzy graph with $\sigma_{PR}^* = \{v_{11}, v_{22}, \dots, v_{ll}\}$ in any form. Then a finite sequence $\delta_{ES}(G_{PR}) = (n_{11}, n_{22}, \dots, n_{ll})$ is termed the δ -edge sequence of G_{PR} if

$$n_i = \begin{cases} r, & \text{where } r \text{ is number of } \delta\text{-edges incident on } v_i \\ 0, & \text{otherwise} \end{cases}$$

Example 10.2.3 Consider $G_{PR} : (\sigma_{PR}, \mu_{PR})$ where

$$\begin{aligned} \sigma_{PR}(m_{11}) &= 0.4, \sigma_{PR}(m_{22}) = 0.3, \sigma_{PR}(m_{33}) = 0.3, \sigma_{PR}(m_{44}) = 0.5 \\ \mu_{PR}(m_{11}, m_{22}) &= 0.3, \mu_{PR}(m_{22}, m_{44}) = 0.2, \mu_{PR}(m_{11}, m_{33}) = 0.2, \mu_{PR}(m_{33}, m_{44}) = 0.3, \\ \mu_{PR}(m_{11}, m_{44}) &= 0.1 \text{ and } \mu_{PR}(m_{22}, m_{33}) = 0.1 \end{aligned}$$

$$\delta_{ES}(G_{PR}) = (1, 1, 1, 1)$$

Figure 10.2.3: PRFG with δ -ES

Here (m_{11}, m_{44}) and (m_{22}, m_{33}) are δ -edges. Then each vertex having one δ -ES. Therefore $\delta_{ES}(G_{PR}) = (1, 1, 1, 1)$.

Definition 10.2.4 Let $G_{PR} : (\sigma_{PR}, \mu_{PR})$ be a pseudo regular fuzzy graph with $\sigma_{PR}^* = \{v_{11}, v_{22}, \dots, v_{ll}\}$ in any form. Then a finite sequence $S_{ES}(G_{PR}) = (n_{11}, n_{22}, \dots, n_{ll})$ is labeled the strong edge sequence of G_{PR} if

$$n_i = \begin{cases} r, & \text{where } r \text{ is number of } \alpha - \text{strong edges and } \beta - \text{strong edges incident on } v_i \\ 0, & \text{otherwise} \end{cases}$$

Example 10.2.4 Consider $G_{PR} : (\sigma_{PR}, \mu_{PR})$ where

$$\begin{aligned} \sigma_{PR}(m_{11}) &= 0.5, \sigma_{PR}(m_{22}) = 0.4, \sigma_{PR}(m_{33}) = 0.5, \sigma_{PR}(m_{44}) = 0.3, \sigma_{PR}(m_{55}) = 0.6 \\ \mu_{PR}(m_{11}, m_{22}) &= 0.1, \mu_{PR}(m_{11}, m_{33}) = 0.1, \mu_{PR}(m_{11}, m_{44}) = 0.1, \mu_{PR}(m_{11}, m_{55}) = 0.1, \\ \mu_{PR}(m_{22}, m_{33}) &= 0.3 \text{ and } \mu_{PR}(m_{44}, m_{55}) = 0.3 \end{aligned}$$

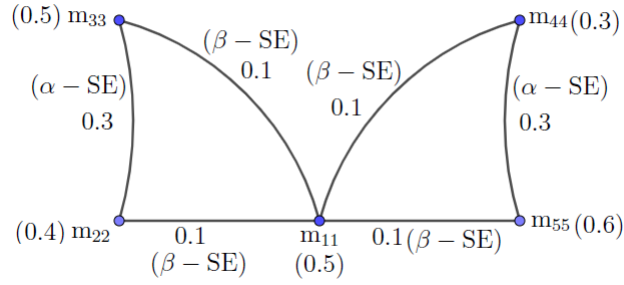


Figure 10.2.4: PRFG with Strong Edge Sequence

$$\alpha_{ES}(G_R) = (0, 1, 1, 1, 1), \beta_{ES}(G_R) = (4, 1, 1, 1, 1), S_{ES}(G_R) = (4, 2, 2, 2, 2)$$

Here $(m_{22}, m_{33}), (m_{44}, m_{55})$ are α -SEs.

$(m_{11}, m_{22}), (m_{11}, m_{33}), (m_{11}, m_{44}), (m_{11}, m_{55})$ are β -SEs.

Therefore $\alpha_{ES}(G_{PR}) = (0, 1, 1, 1, 1), \beta_{ES}(G_{PR}) = (4, 1, 1, 1, 1)$ and $S_{ES}(G_{PR}) = (4, 2, 2, 2, 2)$.

Remark 10.2.1 A pseudo regular fuzzy graph need not contain a strong edge sequence.

Example 10.2.5 The graph G_{PR} is a PRFG. But $\alpha_{ES}(G_{PR}) = (0, 0, 0, 0)$. Hence G_{PR} does not contain an SES.

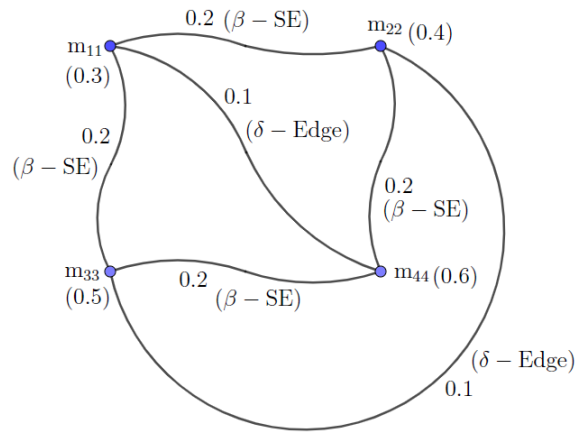


Figure 10.2.5: PRFG without Strong Edge Sequence

$$\beta_{ES}(G_{PR}) = (2, 2, 2, 2) \quad \delta_{ES}(G_{PR}) = (1, 1, 1, 1).$$

Remark 10.2.2 An strong edge sequence of fuzzy graph need not be a pseudo regular fuzzy graph.

Example 10.2.6 The graph G_{FG} is an SES of fuzzy graph. But $d_a(m_{11}) \neq d_a(m_{22})$. Hence G_{FG} is not a PRFG.

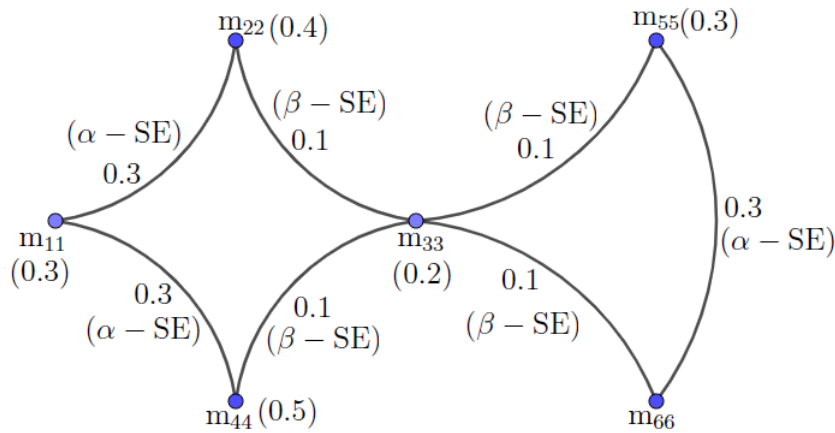


Figure 10.2.6: Strong Edge Sequence with Fuzzy Graph

$$\alpha_{ES}(G_{PR}) = (2, 1, 0, 1, 1, 1), \beta_{ES}(G_{PR}) = (0, 1, 4, 1, 1, 1) \text{ and } S_{ES}(G_{PR}) = (2, 2, 4, 2, 2, 2).$$

Theorem 10.2.1 Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$, a cycle of length n . If μ is a constant function, then G is a pseudo regular fuzzy graph. [MS16]

Theorem 10.2.2 Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$, an even cycle of length n . If the alternative edges have same membership values, then G is a pseudo regular fuzzy graph. [MS16]

Theorem 10.2.3 A pseudo regular fuzzy graph $G_{PR} : (\sigma_{PR}, \mu_{PR})$ whose a cycle is the crisp graph G_{PR}^* have only β - edge sequence, but no α - edge sequence and δ - edge sequence.

Proof. If PRFG $G_{PR} : (\sigma_{PR}, \mu_{PR})$ have only β -ES. This means G_{PR} does not have α -ES and δ -ES. Then by 10.2.2 definition, we have $\mu_{PR}(m_{11}, m_{22}) = CONN_{G_{PR}-(m_{11}, m_{22})}(m_{11}, m_{22})$. Thus all the edges in G_{PR} will have equal MV. Then by the 10.2.1 theorem, we get G_{PR} as a PRFG.

Contrarily, Let G_{PR} be a PRFG. Then by the 10.2.1 theorem, the $MV \mu_{PR}$ is a constant function. Thus the removals of any edge in G_{PR} will not change the strength of connectedness of an $m_{11} - m_{44}$ path in G_{PR} . That is, $\mu_{PR}(m_{11}, m_{22}) = CONN_{G_{PR}-(m_{11}, m_{22})}(m_{11}, m_{22}) \forall (m_{11}, m_{22}) \in G_{PR}$. Thus G_{PR} have only β -ES.

Remark 10.2.3 The above result is also true for a totally pseudo regular fuzzy graph (TPRFG).

Example 10.2.7 The graph is a TPRFG with β -SEs. Then each vertex having two β -ESs. Therefore $S_{ES}(G_{TPR}) = (2, 2, 2, 2)$.

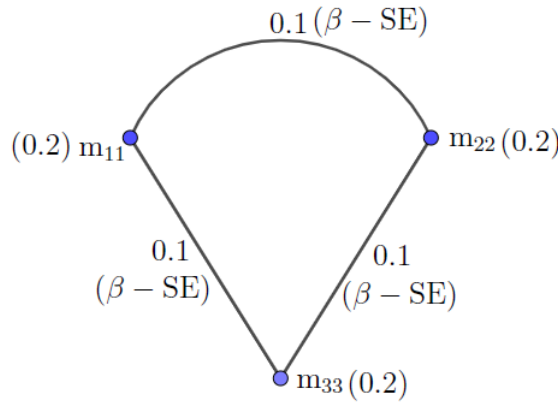


Figure 10.2.7: TPRFG with β -Edge Sequence

$$\beta_{ES}(G_{TPR}) = (2, 2, 2)$$

Theorem 10.2.4 Let $G_{PR} : (\sigma_{PR}, \mu_{PR})$ be a pseudo regular fuzzy graph on $G_{PR}^* : (V, E)$, an even cycle. If μ_{PR} is an alternative edges have equal membership value, then G_{PR} have only strong edge sequence, but no δ - edge sequence.

Proof. Let $G_{PR} : (\sigma_{PR}, \mu_{PR})$ be a PRFG on $G_{PR}^* : (V, E)$, an even cycle. We want to prove that G_{PR} has an strong edge sequence. That is G_{PR} have only α -ES and β -ES. If G_{PR} have only SEs. That is G_{PR} has no δ -ES. Then by 10.2.1 and 10.2.2 definition, we have $\mu_{PR}(m_{11}, m_{22}) \geq CONN_{G_{PR}-(m_{11}, m_{22})}(m_{11}, m_{22})$. This means alternative edges have equal MV . Then by the 10.2.2 theorem, we get G_{PR} as a PRFG.

Contrarily, Let G_{PR} be a PRFG. Then by the 10.2.2 theorem, the alternative edges have equal MV . That

is, $\mu_{PR}(m_{11}, m_{22}) \geq \text{CONN}_{G_{PR}-(m_{11}, m_{22})}(m_{11}, m_{22})$. Thus G_{PR} have α -ES and β -ES. This means G_{PR} have only strong edge sequence.

Remark 10.2.4 The above result is also true for a totally pseudo regular fuzzy graph.

Example 10.2.8 The graph is a TPRFG with α -SEs and β -SEs. Then each vertex having one α -ESs and one β -ES. That is $\alpha_{ES}(G_{TPR}) = (1, 1, 1, 1)$ and $\beta_{ES}(G_{TPR}) = (1, 1, 1, 1)$. Therefore $S_{ES}(G_{TPR}) = (2, 2, 2, 2)$.

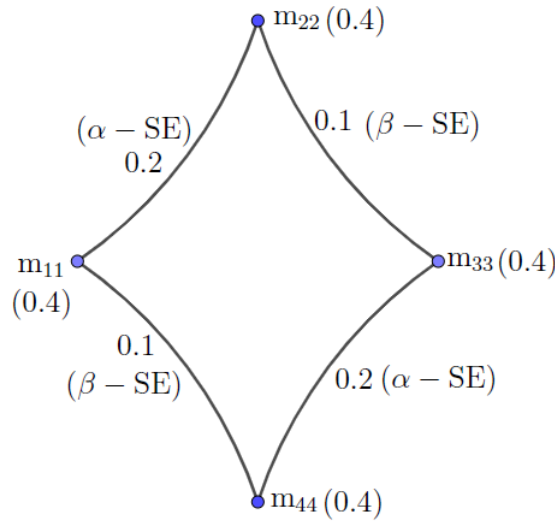


Figure 10.2.8: TPRFG with Strong Edge Sequence

$$\alpha_{ES}(G_{TPR}) = (1, 1, 1, 1), \beta_{ES}(G_{TPR}) = (1, 1, 1, 1), S_{ES}(G_{TPR}) = (2, 2, 2, 2).$$

Theorem 10.2.5 A pseudo regular fuzzy graph $G_{PR} : (\sigma_{PR}, \mu_{PR})$ with its crisp graph $G_{PR}^* : (V, E)$ as even cycle is both pseudo regular fuzzy graph and totally pseudo regular fuzzy graph if G_{PR} have either β -edge sequence or strong edge sequence.

Proof. Let $G_{PR} : (\sigma_{PR}, \mu_{PR})$ be a PRFG. Then its crisp graph $G_{PR}^* : (V, E)$ as even cycle and G_{PR} be both PRFG and TPRFG. Here are two cases that arise.

Case (i) Let G_{PR} be both PRFG and TPRFG with stable values in σ_{PR} and μ_{PR} . Then by 10.2.2 definition, G_{PR} have only β -ES.

Example 10.2.9 Let G_{PR} is an PRFG and TPRFG without $\delta_{ES}(G_{PR})$. That is G_{PR} having β -SEs. Therefore $\delta_{ES}(G_{PR}) = (0, 0, 0, 0)$.

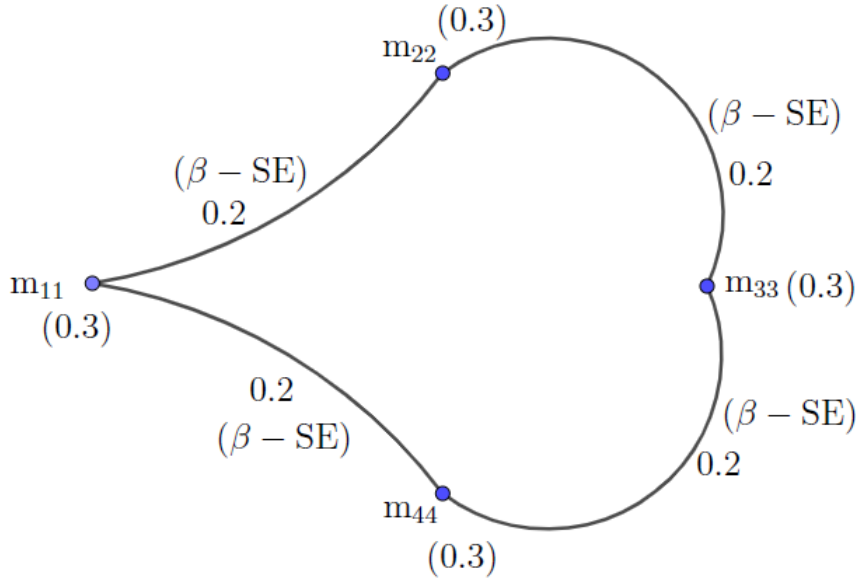


Figure 10.2.9: PRFG and TPRFG

$$\beta_{ES}(G_{PR}) = (2, 2, 2, 2)$$

Case (ii) Let G_{PR} be both PRFG and TPRFG with stable values in σ_{PR} and with equal alternative values in μ_{PR} . Then by 10.2.1 and 10.2.2 definitions, G_{PR} have α -ES and β -ES. Thus G_{PR} have a strong edge sequence.

Example 10.2.10 Let G_{PR} is an RFG and TRFG without $\delta_{ES}(G_{PR})$. That is G_{PR} having α -SEs and β -SEs. Therefore $\delta_{ES}(G_{PR}) = (0, 0, 0, 0)$.

$$\alpha_{ES}(G_{PR}) = (1, 1, 1, 1), \beta_{ES}(G_{PR}) = (1, 1, 1, 1), S_{ES}(G_{PR}) = (2, 2, 2, 2).$$

This chapter covered the structural features of fuzzy graphs. Distinct categories of edge sequences in fuzzy graphs have discussed and their concepts are based on edge classification. To identify regular fuzzy graphs and totally regular fuzzy graphs, we exploited the features of distinct types of edge sequences.

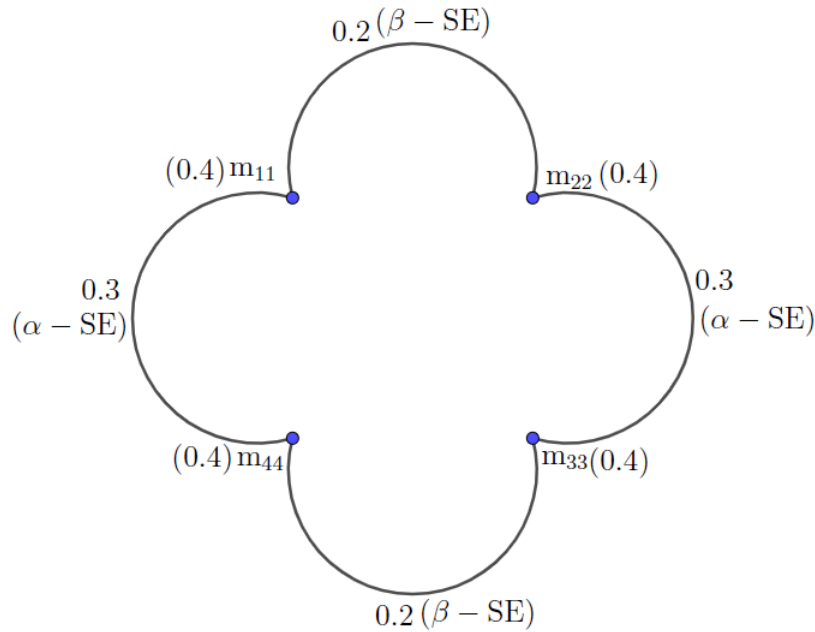


Figure 10.2.10: PRFG and TPRFG with Strong Edge Sequence

In both pseudo regular fuzzy graphs and totally pseudo regular fuzzy graphs, the links between distinct types of edge sequence are investigated. These many sorts of edge sequence classification aid in fully comprehending the underlying structure of a fuzzy graph. Regular fuzzy graphs, Pseudo regular fuzzy graphs, totally regular fuzzy graphs, and totally pseudo regular fuzzy graphs were classified based on the features of different types of edge sequences. More investigation could lead to a better understanding of the nature of fuzzy graphs.

Conclusion

This research work is focused on various kinds of domination and edge sequences in different fuzzy graphs. The concept of strong domination constant number in pseudo regular fuzzy graph and complete fuzzy graph were discussed. We introduced the notion of vertex squared and vertex squared split intuitionistic fuzzy graph based on dual strong domination.

Split domination is discussed on vertex squared interval-valued fuzzy graphs. Specific kinds of perfect domination, both cartesian product of two interval-valued fuzzy incidence graphs and tensor product of two interval-valued fuzzy incidence graphs are defined.

Strong and weak domination is found for the composition of two vague fuzzy incidence graphs. We have explained the concept of strong and weak domination with the help of complete intuitionistic fuzzy incidence graphs.

Some domination parameters such as fixed vertex domination, fixed edge domination, total fixed edge domination, and fixed edge restrained domination are applied on picture fuzzy graphs, and results are derived from them. The twin perfect domination number is determined for a variety of Omicron Products of two hesitancy fuzzy graphs. The concept of edge sequences in regular fuzzy graphs and pseudo regular fuzzy graphs are explained by theorems with examples.

We plan to expand our research into various edge sequences of Picture fuzzy incidence graphs, Hamiltonian fuzzy incidence graphs, interval-valued fuzzy incidence graphs, and Intuitionistic fuzzy incidence graphs in the future.

In future research, the new different dominations of product complete fuzzy graphs, product regular fuzzy graphs, product vague graphs, and product interval-valued fuzzy graphs, as well as their applications in the shopping malls, transport systems, textile industries, medical sciences, will be applied.

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List of Papers Published and Presented

A part of the work in the thesis has been published/presented/communicated to the proceedings/Journals given below

Papers Published

1. Arc-Sequence in Complete and Regular Fuzzy Graphs, *International Journal of Current Research*, Vol. 9, Issue 07, July 2017, Pg.no:54502-54507, ISSN NO: 0975-833X, Impact Factor : 7.617
2. Different types of edge sequence in pseudo regular fuzzy graphs, *International Journal of Pure and Applied Mathematics*, Vol. 118, No. 6, Special Issue, Feb 2018, Pg.no: 95-104, ISSN NO : 1314-3395, Impact Factor : 7.19
3. Strong Domination in Pseudo Regular and Complete fuzzy graphs, *International Journal of Pure and Applied Mathematics*, Vol. 120, No. 5, Special Issue, Sep 2018, Pg.no:1273-1294, ISSN NO : 1314-3395, Impact Factor : 7.19
4. Strong and Weak Domination in Intuitionistic Fuzzy Graph, *International Journal of Research and Analytical Reviews*, Vol. 06, Issue 1, Feb 2019, Pg.no: 265-269, ISSN NO: 2349-5138 Impact Factor : 5.75
5. Various Product and Dual Strong Domination in Mixed and Square Mixed Intuitionistic Fuzzy Graphs, *Adalya Journal*, Vol. 08, Issue 8, August 2019, Pg.no: 259-269, ISSN NO: 1301-2746, Impact Factor : 5.3
6. The Join Product and Dual Strong Domination in Mixed Split Intuitionistic Fuzzy Graph, *Parishodh Journal*, Vol IX, Issue III, March 2020, Pg.no:779-791, ISSN NO: 2347-6648, Impact Factor : 6.3
7. An Application of Domination in Vague Fuzzy Incidence Graphs, *Journal of Mathematical Control Science and Applications*, Vol. 7, No. 1, June 2021, Pg no: 107-120, ISSN NO: 0974-0570, Impact Factor : 6.24. **(Scopus)**

8. Domination in Complete Intuitionistic Fuzzy Incidence Graphs with Application, *Journal of Mathematical Control Science and Applications*, Vol. 7, No. 2 December 2021, Pg no: 189-199, ISSN NO: 0974-0570, Impact Factor : 6.24. (**Scopus**)
9. n -Split Domination in Vertex Squared Interval-Valued Fuzzy Graphs, *Advances and Applications in Mathematical Sciences*, Vol. 21, Issue 3, January 2022, Pg no: 1579-1588, ISSN NO: 0974-6803. (**Web of Science**)

Papers Presented

- ✿ Different types of edge sequence in pseudo regular fuzzy graphs, International Conference on Mathematical Methods and Computations, organized by Department of Mathematics, Jamal Mohamed College, Trichy on December 11, 2017.
- ✿ Strong Domination in Pseudo Regular and Complete fuzzy graphs, International Conference on Analysis and Applied Mathematics, organized by Department of Mathematics, National Institute of Technology, Trichy on July 02, 2018.

List of Papers Accepted

- ✿ K. Kalaiarasi and P. Geethanjali, "A Study on Domination in Product Picture Fuzzy Graph and its Application "(Web of Science).

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AN APPLICATION OF DOMINATION IN VAGUE FUZZY INCIDENCE GRAPHS

K.KALAIARASI *, P.GEETHANJALI

Abstract. Fuzzy Graphs (FGs), also known as Fuzzy Incidence Graphs (FIGs), are a well-organized and useful tool for capturing and resolving a range of real-world scenarios involving ambiguous data and information. In this paper, the Composition of Two Vague Fuzzy Incidence Graphs (CT-VFIGs) and use incidence pairs to extend the idea of FG dominance to CT-VFIGs defined. Examples are used to clarify the concepts of Edge Incidentally Dominating Set (EIDS), Strong Edge Incidentally Dominating Set (SEIDS), and Weak Edge Incidentally Dominating Set (WEIDS). CT-VFIGs have an Edge Incidentally Domination Number (EIDN), a Strong Edge Incidentally Domination Number (SEIDN), and a Weak Edge Incidentally Domination Number (WEIDN). In the research field, CT-VFIGs are used to find the best combinations of journal publications that express the most progress and the least amount of non-progress. The results of our investigation are compared to those of other studies. Our research will help us fully appreciate and comprehend the additional properties of CT-VFIGs. Another benefit of our research is that it will aid in determining the maximum percentage of progress and the minimum percentage of non-progress in various journal publications.

KEYWORDS: Vague Fuzzy Incidence Graph, Composition of two VFIGs, Strong Edge Incidentally Dominating Set, Weak Edge Incidentally Dominating Set.

1. Introduction

Zadeh [40] [42] [43] introduced fuzzy set theory and related fuzzy logic as a technique of dealing with and addressing a wide range of situations in which variables, parameters, and relationships are only approximated, necessitating the employment of approximate reasoning systems. This is true of practically all nontrivial occurrences, processes, and systems that exist in reality, and standard binary logic mathematics cannot sufficiently characterize them. We summarise Gorzalczy's work on interval-valued fuzzy sets (IVFSs) [8] and Roy *et al.* [29] works on fuzzy relations because interval-valued fuzzy graphs (IVFGs) are commonly employed. Vague sets (VSs) were first proposed by W.L Gau and D.J Buehrer [7]. FG operations were investigated by R. Parvathi *et al.* [22]. In vague graphs (VGs), N. Ramakrishna [6] developed the concepts. In IFGs, A. N. Gani [9] developed the concepts of degree, order, and size. S. Samanta and M. Pal [30] have also expressed many FGs. H. Rashmanlou and M. Pal [26] advised irregular IVFGs. Akram. M [2] proposed vague hyper graphs. Degree of vertices in VGs were proposed by Borzooei [3]. Dinesh [5] looked at the topic of FIGs. Borzooei *et al.* [4] suggested and implemented regularity of VGs. Kalaiarasi & Mahalakshmi have also articulated and Kalaiarasi & Gopinath discussed fuzzy strong graphs. Akram *et al.* [1] proposed the concept Cayley VGs. S. Mathew and J.N.

Mordeson [17] proposed concepts in FIGs. Mordeson *et al.* [19] talked about VFIGs. Properties of VGs extended by Rao *et al.* [27].

Ore and Berge were the first to introduce dominance. IrfanNazeer *et al.* [11], developed the new graph's product. Haynes and Hedetniemi [10] looked into dominance in graphs further. Somasundaram and Somasundaram [33] have gained supremacy in FGs by utilising effective edges. In FGs, Xavior *et al.* [38] suggested dominance. PradipDebnath [23] has also shown dominance in IVFGs. For FGs, Revathi and Harinarayanan [28] developed an equitable domination number. Sunitha & Manjusha [34] have also declared that they have a stronghold. Nagoorgani & Chandrasekaran [21] have also demonstrated dominance in a FG. Sarala & Kavitha [35] have also expressed (1,2)-domination for FGs. Dharmalingam & Nithya [6] have also provided dominance values for FGs. Manjusha *et al.* [18] have studied paired domination. In FIGs, IrfanNazeer *et al.* [12] have achieved dominance. AN Shain and MMQ Shubatah [36] advocated the inverted dominating set of IVFGs. Kalaiarasi & Sabina have also expressed fuzzy inventory EOQ optimization mathematical model [15]. Kalaiarasi & Gopinath suggested fuzzy inventory order EOQ model with machine learning [16]. A new path graph definition was proposed by Tushar *et al.* [32]. A. Nagoor Gani *et al.* [10] addressed domination in FGs. AM Ismayil and HS Begum [4] have both accurately depicted split dominance. In ambiguous graphs, Yongsheng Rao *et al.* [39] established dominance. Shanmugavadivu and Gopinath suggested non homogeneous ternary five degrees equation [31]. Shanmugavadivu and Gopinath have also expressed on the homogeneous five degree equation [32]. Priyadharshini *et al.* have also expressed a fuzzy MCDM approach for measuring the business impact of employee selection [24]. and Mapreduce Methodology for Elliptical Curve Discrete Logarithmic Problems [41].

Section 2 gives some preliminary findings that are required in order to understand the rest of the paper. A definition of CT-VFIGs is given in section 3. In section 4, we look at the relationship between CT-VFIG order and size. Domination in CT-VFIGs is discussed in Section 5. Strong and weak domination in CT-VFIGs is discussed in section 6. The application of CT-VFIGs is discussed in section 7. A comparative analysis is offered in section 8.

2. Preliminaries

Definition 2.1[12]

Assume $G_I = (V_I, E_I)$ is a graph. Then, $G_I = (V_I, E_I, I_I)$ is named as an incidence graph, where $I_I \subseteq V_I \times E_I$.

Definition 2.2[12]

Assume $G_{FS} = (V_{FS}, E_{FS})$ is a graph, μ_{FS} is a fuzzy subset of V_{FS} , and γ_{FS} is a fuzzy subset of $V_{FS} \times V_{FS}$. Let ψ_{FS} be a fuzzy subset of $V_{FS} \times E_{FS}$. If $\psi_{FS}(w_{11}, w_{11}w_{22}) \leq \min\{\mu_{FS}(w_{11}), \gamma_{FS}(w_{11}w_{22})\}$ for every $w_{11} \in V_{FS}, w_{11}w_{22} \in E_{FS}$, then ψ_{FS} is a fuzzy incidence of G_{FS} .

Definition 2.3[12]

Assume G_I is a graph and (μ_I, γ_I) is a fuzzy sub graph of G_I . If ψ_I is a fuzzy incidence of G_I , then $G_I = (\mu_I, \gamma_I, \psi_I)$ is named as FIG of G_I .

Definition 2.4 [4]

A VS A is a pair (t_{VA}, f_{VA}) on set V where t_{VA} and f_{VA} are taken as real valued functions which can be defined on $V \rightarrow [0,1]$, so that $t_{VA}(w_{11}) + f_{VA}(w_{11}) \leq 1$, for all w_{11} belongs V . The interval $[t_{VA}(w_{11}), 1 - f_{VA}(w_{11})]$ is known as the vague value of w_{11} is A .

Definition 2.5[6]

A pair $G_V = (A, B)$ is said to be a VG on a crisp graph $G = (V, E)$, where $A = (t_{VA}, f_{VA})$ is a VS on V and $B = (t_{VB}, f_{VB})$ is a VS on $E \subseteq V \times V$ such that $t_{VB}(w_{11}w_{22}) \leq \min(t_{VA}(w_{11}), t_{VA}(w_{22}))$ and $f_{VB}(w_{11}w_{22}) \geq \max(f_{VA}(w_{11}), f_{VA}(w_{22}))$, for each edge $w_{11}w_{22} \in E$

Definition 2.6

An VFIG is of the form $G_{VI} = (V_{VI}, E_{VI}, I_{VI}, A_{VI}, B_{VI}, C_{VI})$ where $A_{VI} = (t_{A_{VI}}, f_{A_{VI}})$, $B_{VI} = (t_{B_{VI}}, f_{B_{VI}})$, $C_{VI} = (t_{C_{VI}}, f_{C_{VI}})$ and $V_{VI} = \{w_0, w_1, \dots, w_n\}$ such that $t_{A_{VI}}: V_{VI} \rightarrow [0,1]$ and $f_{A_{VI}}: V_{VI} \rightarrow [0,1]$ represent the degree (DG) of membership (MS) and non membership (NMS) of the vertex $w_{ii} \in V_{VI}$ respectively, and $0 \leq t_{A_{VI}} + f_{A_{VI}} \leq 1$ for each $w_{ii} \in V_{VI} (i = 1, 2, \dots, n)$, $t_{B_{VI}}: V_{VI} \times V_{VI} \rightarrow [0,1]$ and $f_{B_{VI}}: V_{VI} \times V_{VI} \rightarrow [0,1]$ show the DG of MS and NMS of the edge (w_{11}, w_{22}) respectively, such that $t_{B_{VI}}(w_{11}, w_{22}) \leq \min\{t_{A_{VI}}(w_{11}), t_{A_{VI}}(w_{22})\}$ and $f_{B_{VI}}(w_{11}, w_{22}) \geq \max\{f_{A_{VI}}(w_{11}), f_{A_{VI}}(w_{22})\}$, $0 \leq t_{B_{VI}}(w_{11}, w_{22}) + f_{B_{VI}}(w_{11}, w_{22}) \leq 1$ for every (w_{11}, w_{22}) , $t_{C_{VI}}: V_{VI} \times E_{VI} \rightarrow [0,1]$ and $f_{C_{VI}}: V_{VI} \times E_{VI} \rightarrow [0,1]$, $t_{C_{VI}}(w_{11}, w_{11}w_{22})$ and $f_{C_{VI}}(w_{11}, w_{11}w_{22})$ show the DG of MS and NMS of the incidence pair respectively, such that $t_{C_{VI}}(w_{11}, w_{11}w_{22}) \leq \min\{t_{A_{VI}}(w_{11}), t_{B_{VI}}(w_{11}, w_{22})\}$ and $f_{C_{VI}}(w_{11}, w_{11}w_{22}) \geq \max\{f_{A_{VI}}(w_{11}), f_{B_{VI}}(w_{11}, w_{22})\}$, $0 \leq t_{C_{VI}}(w_{11}, w_{11}w_{22}) + f_{C_{VI}}(w_{11}, w_{11}w_{22}) \leq 1$ for every $(w_{11}, w_{11}w_{22})$.

3. Composition of two VFIGs

Definition 3.1

The Composition of two VFIGs (CT-VFIGs) $G_{VI}^1 = (V_{VI}^1, E_{VI}^1, I_{VI}^1, A_{VI}^1, B_{VI}^1, C_{VI}^1)$ and $G_{VI}^2 = (V_{VI}^2, E_{VI}^2, I_{VI}^2, A_{VI}^2, B_{VI}^2, C_{VI}^2)$ is defined as an VFIG

$$G_{CVI} = G_{VI}^1 \Theta G_{VI}^2 = (V_{VI}, E_{VI}, I_{VI}, A_{VI}^1 \Theta A_{VI}^2, B_{VI}^1 \Theta B_{VI}^2, C_{VI}^1 \Theta C_{VI}^2) \text{ where } V_{CVI} = V_{VI}^1 \Theta V_{VI}^2$$

$$\text{and } E_{CVI} = \{((m_{11}, n_{11}), (m_{22}, n_{22}))/m_{11} = m_{22}, (n_{11}, n_{22}) \in E_{VI}^2 \text{ or } n_{11} = n_{22}, (m_{11}, m_{22}) \in E_{VI}^1\}$$

$$I_{CVI} =$$

$$\{(m_{11}, n_{11}), (m_{11}, n_{11})(m_{11}, n_{22})/m_{11} = m_{22}, (n_{11}, n_{11}n_{22}) \in I_{VI}^2, (n_{22}, n_{11}n_{22}) \in I_{VI}^1 \text{ or } n_{11} = n_{22}, (m_{11}, m_{11}m_{22}) \in I_{VI}^1, (m_{22}, m_{11}m_{22}) \in I_{VI}^1\} \text{ with}$$

$$(A_{VI}^1 \Theta A_{VI}^2)(m_{11}, n_{11}) = \min\{A_{VI}^1(m_{11}), A_{VI}^2(n_{11})\} \forall (m_{11}, n_{11}) \in V_{VI}^1 \Theta V_{VI}^2,$$

$$(A_{VI}^1 \boxtimes A_{VI}^2)(m_{11}, n_{11}) = \max\{A_{VI}^1(m_{11}), A_{VI}^2(n_{11})\} \forall (m_{11}, n_{11}) \in V_{VI}^1 \boxtimes V_{VI}^2$$

$$\begin{aligned}
& (B_{1VIL}^1 \boxtimes B_{1VIL}^2)((m_{11}, n_{11})(m_{22}, n_{22})) \\
&= \begin{cases} \min\{A_{1VIP}^1(m_{11}), B_{1VIL}^2(n_{11}, n_{22})\}, & \text{if } m_{11} = m_{22}, (n_{11}, n_{22}) \in E_{VI}^2 \\ \min\{B_{1VIL}^1(m_{11}, m_{22}), A_{1VIP}^2(n_{11})\}, & \text{if } n_{11} = n_{22}, (m_{11}, m_{22}) \in E_{VI}^1 \\ \min\{B_{1VIL}^1(m_{11}, m_{22}), A_{1VIP}^2(n_{11}), A_{1VIP}^2(n_{22})\}, & \text{if } n_{11} \neq n_{22}, (m_{11}, m_{22}) \in E_{VI}^1 \end{cases} \\
& (B_{2VIL}^1 \boxtimes B_{2VIL}^2)((m_{11}, n_{11})(m_{22}, n_{22})) \\
&= \begin{cases} \max\{A_{2VIP}^1(m_{11}), B_{2VIL}^2(n_{11}, n_{22})\}, & \text{if } m_{11} = m_{22}, (n_{11}, n_{22}) \in E_{VI}^2 \\ \max\{B_{2VIL}^1(m_{11}, m_{22}), A_{2VIP}^2(n_{11})\}, & \text{if } n_{11} = n_{22}, (m_{11}, m_{22}) \in E_{VI}^1 \\ \max\{B_{1VIL}^1(m_{11}, m_{22}), A_{1VIP}^2(n_{11}), A_{1VIP}^2(n_{22})\}, & \text{if } n_{11} \neq n_{22}, (m_{11}, m_{22}) \in E_{VI}^1 \end{cases} \\
& (C_{1VII}^1 \boxtimes C_{1VII}^2)[(m_{11}, n_{11}), (m_{11}, n_{11})(m_{11}, n_{22})] \\
&= \min\{A_{1VIP}^1(m_{11}), C_{1VII}^2(n_{11}, n_{11}n_{22})\} \text{ if } m_{11} \\
&= m_{22}, (n_{11}, n_{11}n_{22}) \in I_{VI}^2 \\
& (C_{1VII}^1 \boxtimes C_{1VII}^2)[(m_{11}, n_{22}), (m_{11}, n_{11})(m_{11}, n_{22})] \\
&= \min\{A_{1VIP}^1(m_{11}), C_{1VII}^2(n_{22}, n_{11}n_{22})\} \text{ if } m_{11} \\
&= m_{22}, (n_{22}, n_{11}n_{22}) \in I_{VI}^2 \\
& (C_{1VII}^1 \boxtimes C_{1VII}^2)[(m_{11}, n_{11}), (m_{11}, n_{11})(m_{22}, n_{11})] \\
&= \min\{C_{1VII}^1(m_{11}, m_{11}m_{22}), A_{1VIP}^2(n_{11})\} \text{ if } n_{11} \\
&= n_{22}, (m_{11}, m_{11}m_{22}) \in I_{VI}^1 \\
& (C_{1VII}^1 \boxtimes C_{1VII}^2)[(m_{22}, n_{11}), (m_{11}, n_{11})(m_{22}, n_{11})] \\
&= \min\{C_{1VII}^1(m_{22}, m_{11}m_{22}), A_{1VIP}^2(n_{11})\} \text{ if } n_{11} \\
&= n_{22}, (m_{22}, m_{11}m_{22}) \in I_{VI}^1 \\
& (C_{1VII}^1 \boxtimes C_{1VII}^2)[(m_{11}, n_{11}), (m_{11}, n_{11})(m_{22}, n_{22})] \\
&= \min\{C_{1VII}^1(m_{11}, m_{11}m_{22}), A_{1VIP}^2(n_{11}), A_{1VIP}^2(n_{22})\}, \text{ if } m_{11} \\
&\neq m_{22}, n_{11} \neq n_{22}, (m_{11}, m_{11}m_{22}) \in I_{VI}^1 \\
& (C_{1VII}^1 \boxtimes C_{1VII}^2)[(m_{22}, n_{22}), (m_{11}, n_{11})(m_{22}, n_{22})] \\
&= \min\{C_{1VII}^1(m_{22}, m_{11}m_{22}), A_{1VIP}^2(n_{11}), A_{1VIP}^2(n_{22})\}, \text{ if } m_{11} \\
&\neq m_{22}, n_{11} \neq n_{22}, (m_{22}, m_{11}m_{22}) \in I_{VI}^1 \\
& (C_{1VII}^1 \boxtimes C_{1VII}^2)[(m_{11}, n_{22}), (m_{11}, n_{22})(m_{22}, n_{11})] \\
&= \min\{C_{1VII}^1(m_{11}, m_{11}m_{22}), A_{1VIP}^2(n_{11}), A_{1VIP}^2(n_{22})\}, \text{ if } m_{11} \\
&\neq m_{22}, n_{11} \neq n_{22}, (m_{11}, m_{11}m_{22}) \in I_{VI}^1 \\
& (C_{1VII}^1 \boxtimes C_{1VII}^2)[(m_{22}, n_{11}), (m_{11}, n_{22})(m_{22}, n_{11})] \\
&= \min\{C_{1VII}^1(m_{22}, m_{11}m_{22}), A_{1VIP}^2(n_{11}), A_{1VIP}^2(n_{22})\}, \text{ if } m_{11} \\
&\neq m_{22}, n_{11} \neq n_{22}, (m_{22}, m_{11}m_{22}) \in I_{VI}^1 \\
& (C_{2VII}^1 \boxtimes C_{2VII}^2)[(m_{11}, n_{11}), (m_{11}, n_{11})(m_{11}, n_{22})] \\
&= \max\{A_{2VIP}^1(m_{11}), C_{2VII}^2(n_{11}, n_{11}n_{22})\} \text{ if } m_{11} \\
&= m_{22}, (n_{11}, n_{11}n_{22}) \in I_{VI}^2 \\
& (C_{2VII}^1 \boxtimes C_{2VII}^2)[(m_{11}, n_{22}), (m_{11}, n_{11})(m_{11}, n_{22})] \\
&= \max\{A_{2VIP}^1(m_{11}), C_{2VII}^2(n_{22}, n_{11}n_{22})\} \text{ if } m_{11} \\
&= m_{22}, (n_{22}, n_{11}n_{22}) \in I_{VI}^2 \\
& (C_{2VII}^1 \boxtimes C_{2VII}^2)[(m_{11}, n_{11}), (m_{11}, n_{11})(m_{22}, n_{11})] \\
&= \max\{C_{2VII}^1(m_{11}, m_{11}m_{22}), A_{2VIP}^2(n_{11})\} \text{ if } n_{11} \\
&= n_{22}, (m_{11}, m_{11}m_{22}) \in I_{VI}^1 \\
& (C_{2VII}^1 \boxtimes C_{2VII}^2)[(m_{22}, n_{11}), (m_{11}, n_{11})(m_{22}, n_{11})] \\
&= \max\{C_{2VII}^1(m_{22}, m_{11}m_{22}), A_{2VIP}^2(n_{11})\} \text{ if } n_{11} \\
&= n_{22}, (m_{22}, m_{11}m_{22}) \in I_{VI}^1
\end{aligned}$$

$$\begin{aligned}
 & (C_{2VII}^1 \boxtimes C_{2VII}^2)[(m_{11}, n_{11}), (m_{11}, n_{11})(m_{22}, n_{22})] \\
 & \quad = \max\{C_{2VII}^1(m_{11}, m_{11}m_{22}), A_{1VIP}^2(n_{11}), A_{1VIP}^2(n_{22})\}, \text{ if } m_{11} \\
 & \quad \neq m_{22}, n_{11} \neq n_{22}, (m_{11}, m_{11}m_{22}) \in I_{VI}^1 \\
 & (C_{2VII}^1 \boxtimes C_{2VII}^2)[(m_{22}, n_{22}), (m_{11}, n_{11})(m_{22}, n_{22})] \\
 & \quad = \max\{C_{2VII}^1(m_{22}, m_{11}m_{22}), A_{1VIP}^2(n_{11}), A_{1VIP}^2(n_{22})\}, \text{ if } m_{11} \\
 & \quad \neq m_{22}, n_{11} \neq n_{22}, (m_{22}, m_{11}m_{22}) \in I_{VI}^1 \\
 & (C_{2VII}^1 \boxtimes C_{2VII}^2)[(m_{11}, n_{22}), (m_{11}, n_{22})(m_{22}, n_{11})] \\
 & \quad = \max\{C_{2VII}^1(m_{11}, m_{11}m_{22}), A_{1VIP}^2(n_{11}), A_{1VIP}^2(n_{22})\}, \text{ if } m_{11} \\
 & \quad \neq m_{22}, n_{11} \neq n_{22}, (m_{11}, m_{11}m_{22}) \in I_{VI}^1 \\
 & (C_{2VII}^1 \boxtimes C_{2VII}^2)[(m_{22}, n_{11}), (m_{11}, n_{22})(m_{22}, n_{11})] \\
 & \quad = \max\{C_{2VII}^1(m_{22}, m_{11}m_{22}), A_{1VIP}^2(n_{11}), A_{1VIP}^2(n_{22})\}, \text{ if } m_{11} \\
 & \quad \neq m_{22}, n_{11} \neq n_{22}, (m_{22}, m_{11}m_{22}) \in I_{VI}^1
 \end{aligned}$$

Example 3.2

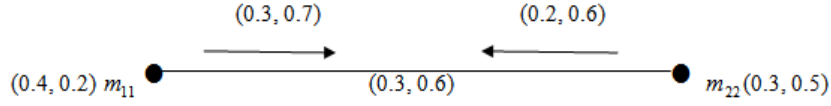


Figure1. VFVG G_{VI}^1

Figure 1 indicates a VFVG $G_{VI}^1 = (V_{VI}^1, E_{VI}^1, I_{VI}^1, A_{VIP}^1, B_{VIL}^1, C_{VII}^1)$ with
 $A_{VIP}^1(m_{11}) = (0.4, 0.2), A_{VIP}^1(m_{22}) = (0.3, 0.5), B_{VIL}^1(m_{11}m_{22}) = (0.3, 0.6),$
 $C_{VII}^1(m_{11}, m_{11}m_{22}) = (0.3, 0.7), C_{VII}^1(m_{22}, m_{11}m_{22}) = (0.2, 0.6)$

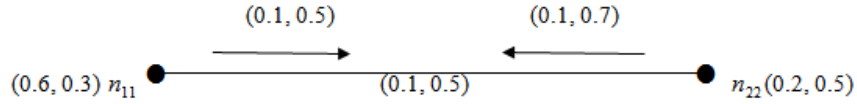


Figure2. VFVG G_{VI}^2

Figure 2 indicates a VFVG $G_{VI}^2 = (V_{VI}^2, E_{VI}^2, I_{VI}^2, A_{VIP}^2, B_{VIL}^2, C_{VII}^2)$ with
 $A_{VIP}^2(n_{11}) = (0.6, 0.3), A_{VIP}^2(n_{22}) = (0.2, 0.5), B_{VIL}^2(n_{11}n_{22}) =$
 $(0.1, 0.5), C_{VII}^2(n_{11}, n_{11}n_{22}) = (0.1, 0.5), C_{VII}^2(n_{22}, n_{11}n_{22}) = (0.1, 0.7).$

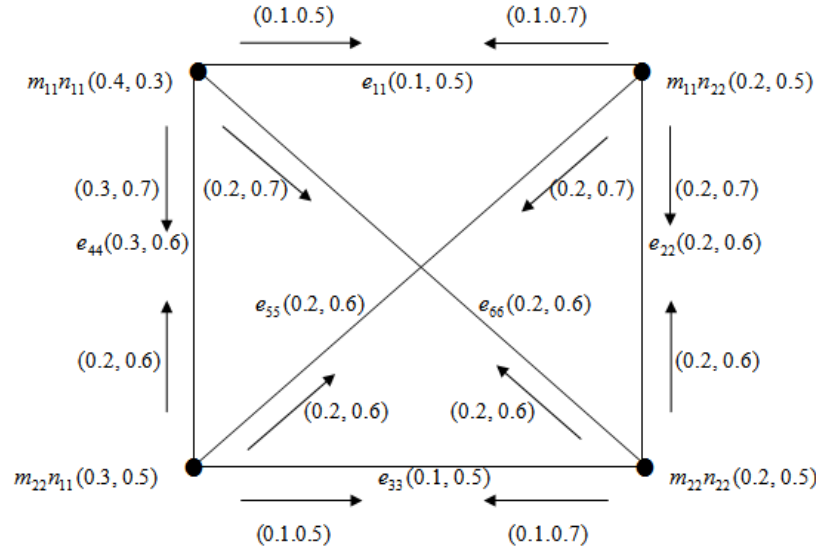


Figure3. Composition of figure 1 and figure 2

Figure 3 indicates a CT-VFIGs

$$\begin{aligned}
 G_{VI}^1 \boxtimes G_{VI}^2 &= (V_{VI}, E_{VI}, I_{VI}, A_{VIP}^1 \boxtimes A_{VIP}^2, B_{VIL}^1 \boxtimes B_{VIL}^2, C_{VII}^1 \boxtimes C_{VII}^2) \\
 (A_{VIP}^1 \boxtimes A_{VIP}^2)(m_{11}, n_{11}) &= (0.4, 0.3), (A_{VIP}^1 \boxtimes A_{VIP}^2)(m_{11}, n_{22}) = (0.2, 0.5) \\
 (A_{VIP}^1 \boxtimes A_{VIP}^2)(m_{22}, n_{11}) &= (0.3, 0.5), (A_{VIP}^1 \boxtimes A_{VIP}^2)(m_{22}, n_{22}) = (0.2, 0.5) \\
 (B_{VIL}^1 \boxtimes B_{VIL}^2)((m_{11}, n_{11})(m_{11}, n_{22})) &= \\
 0.1, 0.5, (B_{VIL}^1 \boxtimes B_{VIL}^2)((m_{11}, n_{22})(m_{22}, n_{22})) &= 0.2, 0.6, \\
 (B_{VIL}^1 \boxtimes B_{VIL}^2)((m_{22}, n_{11})(m_{22}, n_{22})) &= 0.1, 0.5, \\
 (B_{VIL}^1 \boxtimes B_{VIL}^2)((m_{11}, n_{11})(m_{22}, n_{11})) &= 0.3, 0.6, \\
 (B_{VIL}^1 \boxtimes B_{VIL}^2)((m_{11}, n_{11})(m_{22}, n_{22})) &= 0.2, 0.6, \\
 (B_{VIL}^1 \boxtimes B_{VIL}^2)((m_{11}, n_{22})(m_{22}, n_{11})) &= 0.2, 0.6 \\
 (C_{VII}^1 \boxtimes C_{VII}^2)[(m_{11}, n_{11}), (m_{11}, n_{11})(m_{11}, n_{22})] &= (0.1, 0.5) \\
 (C_{VII}^1 \boxtimes C_{VII}^2)[(m_{11}, n_{22}), (m_{11}, n_{11})(m_{11}, n_{22})] &= \\
 (0.1, 0.7), (C_{VII}^1 \boxtimes C_{VII}^2)[(m_{11}, n_{22}), (m_{11}, n_{22})(m_{22}, n_{22})] &= (0.2, 0.7), \\
 (C_{VII}^1 \boxtimes C_{VII}^2)[(m_{22}, n_{22}), (m_{11}, n_{22})(m_{22}, n_{22})] &= (0.2, 0.6), \\
 (C_{VII}^1 \boxtimes C_{VII}^2)[(m_{22}, n_{11}), (m_{22}, n_{11})(m_{22}, n_{22})] &= (0.1, 0.5), \\
 (C_{VII}^1 \boxtimes C_{VII}^2)[(m_{22}, n_{22}), (m_{22}, n_{11})(m_{22}, n_{22})] &= (0.1, 0.7), \\
 (C_{VII}^1 \boxtimes C_{VII}^2)[(m_{11}, n_{11}), (m_{11}, n_{11})(m_{22}, n_{11})] &= (0.3, 0.7), \\
 (C_{VII}^1 \boxtimes C_{VII}^2)[(m_{22}, n_{11}), (m_{11}, n_{11})(m_{22}, n_{11})] &= (0.2, 0.6), \\
 (C_{VII}^1 \boxtimes C_{VII}^2)[(m_{11}, n_{11}), (m_{11}, n_{11})(m_{22}, n_{22})] &= (0.2, 0.7), \\
 (C_{VII}^1 \boxtimes C_{VII}^2)[(m_{22}, n_{22}), (m_{11}, n_{11})(m_{22}, n_{22})] &= (0.2, 0.6), \\
 (C_{VII}^1 \boxtimes C_{VII}^2)[(m_{22}, n_{11}), (m_{22}, n_{11})(m_{11}, n_{22})] &= (0.2, 0.6), \\
 (C_{VII}^1 \boxtimes C_{VII}^2)[(m_{11}, n_{22}), (m_{22}, n_{11})(m_{11}, n_{22})] &= (0.2, 0.7)
 \end{aligned}$$

Definition 3.3

Let G_{CVI} be a CT-VFIGs

- (i) G_{CVI} cardinality is determined by

$$\begin{aligned}
 |G_{CVI}| &= \sum_{w_{11} \in V_{VI}} \frac{1 + t_{A_{VIP}}(w_{11}) - f_{A_{VIP}}(w_{11})}{2} \\
 &\quad + \sum_{w_{11}w_{22} \in E_{VI}} \frac{1 + t_{B_{VIL}}(w_{11}w_{22}) - f_{B_{VIL}}(w_{11}w_{22})}{2} + \\
 &\quad \sum_{w_{11}, w_{11}w_{22} \in H_{VI}} \frac{1 + t_{C_{VII}}(w_{11}, w_{11}w_{22}) - f_{C_{VII}}(w_{11}, w_{11}w_{22})}{2} \\
 \text{(ii)} \quad G_{CVI} \text{ vertex cardinality is determined by } |V_{CVI}| &= \sum_{w_{11} \in V_{CVI}} \frac{1 + t_{A_{VIP}}(w_{11}) - f_{A_{VIP}}(w_{11})}{2} \forall w_{11} \in V_{CVI} \\
 \text{(iii)} \quad G_{CVI} \text{ edge cardinality is specified by } |E_{CVI}| &= \sum_{w_{11}w_{22} \in E_{CVI}} \frac{1 + t_{B_{VIL}}(w_{11}w_{22}) - f_{B_{VIL}}(w_{11}w_{22})}{2} \forall w_{11}w_{22} \in E_{CVI} \\
 \text{(iv)} \quad G_{CVI} \text{ incidence pair cardinality is specified by } & \\
 |I_{CVI}| &= \sum_{\substack{w_{11}, w_{11}w_{22} \in I_{CVI} \\ \in I_{CVI}}} \frac{1 + t_{C_{VII}}(w_{11}, w_{11}w_{22}) - f_{C_{VII}}(w_{11}, w_{11}w_{22})}{2} \forall w_{11}, w_{11}w_{22}
 \end{aligned}$$

4. Relationship between order and size of CT-VFIGs

Definition 4.1

Assume G_{CVI} is a CT-VFIGs. Then $O_{CVI}(G_{CVI}) = \sum_{w_{11} \neq w_{22}, w_{11}, w_{22} \in V_{CVI}} \left(\frac{1 + t_{C_{CVI}}(w_{11}, w_{11}w_{22}) - f_{C_{CVI}}(w_{11}, w_{11}w_{22})}{2} \right)$ is called order of G_{CVI} and $S_{CVI}(G_{CVI}) = \sum_{w_{11}, w_{22} \in E_{CVI}} \left(\frac{1 + t_{B_{CVI}}(w_{11}, w_{11}w_{22}) - f_{B_{CVI}}(w_{11}, w_{11}w_{22})}{2} \right)$ is called size of G_{CVI} .

Definition 4.2

The edge degree of ae_{1VI} in a CT-VFIGs is defined as the sum of the weights of edges incident to e_{1VI} . It is defined by $|d_{G_{CVI}}(e_{1VI})| = \{deg^t(e_{1VI}), deg^f(e_{1VI})\}$. The minimum cardinality of edge degree of G_{CVI} is $\delta_{CVI}(G_{CVI}) = \min\{d_{G_{CVI}}(e_{1VI})/e_{1VI} \in E_{CVI}\}$. The maximum cardinality of edge degree of G_{CVI} is $\Delta_{CVI}(G_{CVI}) = \max\{d_{G_{CVI}}(e_{1VI})/e_{1VI} \in E_{CVI}\}$

Proposition 4.3

In a CT-VFIGs $O_{CVI}(G_{CVI}) \geq S_{CVI}(G_{CVI})$

Proof. Let G_{CVI} be a CT-VFIGs with one node. Then $O_{CVI}(G_{CVI}) = S_{CVI}(G_{CVI}) = 0$.

That is $O_{CVI}(G_{CVI}) = S_{CVI}(G_{CVI})$ (1)

It is a trivial case. Assume G_{CVI} with more than one nodes. $O_{CVI}(G_{CVI})$ is the sum of all incidence pairs cardinality of G_{CVI} . Since incidence pairs are two times of edges. Therefore, the total sum of all the incidence pairs cardinality will always greater than the total sum of all the edge cardinality.

$$O_{CVI}(G_{CVI}) > S_{CVI}(G_{CVI}) \quad (2)$$

From equations (1) and (2), we get $O_{CVI}(G_{CVI}) \geq S_{CVI}(G_{CVI})$.

Proposition 4.4

For any CT-VFIGs the following inequality holds

$$\delta_{CVI}(G_{CVI}) \leq \mathbb{Q}_{CVI}(G_{CVI}) \leq S_{CVI}(G_{CVI}) \leq O_{CVI}(G_{CVI}).$$

Proof. Assume G_{CVI} is a CT-VFIGs with non empty node set.

Since $\delta_{CVI}(G_{CVI})$ represents lowest edge degree and $\Delta_{CVI}(G_{CVI})$ denotes highest edge degree of G_{CVI} .

$$\delta_{CVI}(G_{CVI}) \leq \mathbb{Q}_{CVI}(G_{CVI}) \quad (3)$$

We know $O_{CVI}(G_{CVI}) = \sum_{w_{11} \neq w_{22}, w_{11}, w_{22} \in V_{CVI}} \left(\frac{1+t_{C_{CVI}}(w_{11}, w_{11}w_{22})-f_{C_{CVI}}(w_{11}, w_{11}w_{22})}{2} \right)$

$$\text{and } S_{CVI}(G_{CVI}) = \sum_{w_{11}, w_{22} \in E_{CVI}} \left(\frac{1+t_{B_{CVI}}(w_{11}, w_{22})-f_{B_{CVI}}(w_{11}, w_{22})}{2} \right)$$

$$\text{By definition of size of } G_{CVI}, \quad S_{CVI}(G_{CVI}) = \sum_{w_{11}, w_{22} \in E_{CVI}} \left(\frac{1+t_{B_{CVI}}(w_{11}, w_{22})-f_{B_{CVI}}(w_{11}, w_{22})}{2} \right) \geq \max\{d_{G_{CVI}}(e_{1VI})/e_{1VI} \in E_{CVI}\}$$

$$\text{That is} \quad S_{CVI}(G_{CVI}) \geq \mathbb{Q}_{CVI}(G_{CVI}) \quad (4)$$

Also, in a CT-VFIGs, G_{CVI} by 4.3 proposition

$$O_{CVI}(G_{CVI}) \geq S_{CVI}(G_{CVI}) \quad (5)$$

From inequalities (3), (4) and (5), we obtained $\delta_{CVI}(G_{CVI}) \leq \mathbb{Q}_{CVI}(G_{CVI}) \leq S_{CVI}(G_{CVI}) \leq O_{CVI}(G_{CVI})$

5. Domination in CT-VFIGs

Definition 5.1

A edge e_{VI} in an CT-VFIGs G_{CVI} is called incidentally dominate edge if $t_{C_{CVI}}(w_{11}, w_{11}w_{22}) = \min\{t_{A_{CVI}}(w_{11}), t_{B_{CVI}}(w_{11}, w_{22})\}$ and $f_{C_{CVI}}(w_{11}, w_{11}w_{22}) = \max\{f_{A_{CVI}}(w_{11}), f_{B_{CVI}}(w_{11}, w_{22})\}$

Definition 5.2

A edge e_{1VI} in an CT-VFIGs G_{CVI} dominates to edge e_{2VI} if they are incidentally dominate edges.

Definition 5.3

A subset R_{CVI} of E_{CVI} is said to be edge incidentally dominating set (EIDS) if for each edge e_{1VI} not in R_{CVI} , e_{1VI} is dominate at least one edge in R_{CVI} .

Definition 5.4

A edge incidentally dominating set R_{CVI} of the CT-VFIGs G_{CVI} is said to be a minimal EIDS of CT-VFIGs G_{CVI} if each edge in R_{CVI} , the set $R_{CVI} - \{e_{1VI}\}$ is not a EIDS.

Definition 5.5

AEIDS with the lowest edge cardinality is called a minimum EIDS. The edge cardinality of a minimum EIDS is called edge incidentally dominating number of the CT-VFIGs G_{CVI} . It is denoted by $\gamma_{VI}(G_{CVI})$

Example 5.6

In figure 3, the incidentally dominating edges are $\{e_{11}\}, \{e_{22}\}, \{e_{33}\}, \{e_{44}\}, \{e_{55}\}, \{e_{66}\}$ and the EIDSs are $S_{11} = \{e_{11}e_{22}\}, S_{22} = \{e_{11}e_{33}\}, S_{33} = \{e_{11}e_{44}\}, S_{44} = \{e_{11}e_{55}\}, S_{55} = \{e_{11}e_{66}\}, \dots$. After calculating the edge cardinality of $S_{11}, S_{22}, S_{33}, S_{44}, \dots$, we obtain $|S_{11}| = 0.6, |S_{22}| = 0.6, |S_{33}| = 0.65, |S_{44}| = 0.6, |S_{55}| = 0.6, \dots$. The edge cardinality of a minimum EIDS is $|S_{11}| = 0.6$ and $\gamma_{VI}(G_{CVI}) = 0.6$.

Theorem 5.7

Let $G_{VI}^1 = (A_{VIP}^1, B_{VIL}^1, C_{VII}^1)$ and $G_{VI}^2 = (A_{VIP}^2, B_{VIL}^2, C_{VII}^2)$ be two VFIGs. Then $\gamma_{VI}(G_{CVI}) = \min\{A_{VIP}^1(m_{11}), A_{VIP}^2(n_{11})\}$ where $m_{11} \in G_{VI}^1$ and $n_{11} \in G_{VI}^2$

Proof. Assume $G_{VI}^1 = (A_{VIP}^1, B_{VIL}^1, C_{VII}^1)$ and $G_{VI}^2 = (A_{VIP}^2, B_{VIL}^2, C_{VII}^2)$ are two VFigs. Since G_{VI}^1 and G_{VI}^2 are two VFigs, then $G_{VI}^1 \theta G_{VI}^2$ will be a VFigs. So, each two edges in $G_{VI}^1 \theta G_{VI}^2$ will dominates remaining edges. Then by definition of EIDN, $\gamma_{VI}(G_{CVI}) = \min\{\text{cardinalityof}(A_{VIP}^1(m_{11}), A_{VIP}^2(n_{11}))\}$.

Theorem 5.8

Let $G_{VI}^1 = (A_{VIP}^1, B_{VIL}^1, C_{VII}^1)$ and $G_{VI}^2 = (A_{VIP}^2, B_{VIL}^2, C_{VII}^2)$ be two VFigs with $k \geq 2$ and $l \geq 2$, where k and l are representing the number of vertices in G_{VI}^1 and G_{VI}^2 , respectively. Then $\frac{\gamma_{VI}(G_{CVI})}{2} = \min\{\text{cardinalityof}(B_{VIL}^1(m_{11}m_{22}), B_{VIL}^2(n_{11}n_{22}))\}$.

Proof. Consider $G_{VI}^1 = (A_{VIP}^1, B_{VIL}^1, C_{VII}^1)$ and $G_{VI}^2 = (A_{VIP}^2, B_{VIL}^2, C_{VII}^2)$ are two VFigs. Since G_{VI}^1 and G_{VI}^2 are VFigs. Then $G_{VI}^1 \theta G_{VI}^2$ will also a VFig with $\frac{\gamma_{VI}(G_{CVI})}{2} = \min\{\text{cardinalityof}(B_{VIL}^1(m_{11}m_{22}), B_{VIL}^2(n_{11}n_{22}))\}$ because each two edges in $G_{VI}^1 \theta G_{VI}^2$ dominates to all remaining edges.

6. Strong and Weak Domination in CT-VFigs

Definition 6.1

Let G_{CVI} be a CT-VFigs. For any two edges $e_{1VI}, e_{2VI} \in E_{CVI}$, e_{1VI} strongly dominates e_{2VI} in CT-VFigs G_{CVI} if

- (i) they are incidentally dominate edges
- (ii) $\deg^t(e_{1VI}) \geq \deg^t(e_{2VI}), \deg^f(e_{1VI}) \leq \deg^f(e_{2VI})$

Similarly e_{1VI} weakly dominates e_{2VI} if

- (i) they are incidentally dominate edges
- (ii) $\deg^t(e_{2VI}) \geq \deg^t(e_{1VI}), \deg^f(e_{2VI}) \leq \deg^f(e_{1VI})$

Definition 6.2

An edge incidentally dominating set $R_{CVI} \subseteq E_{CVI}$ is called a strong (weak) edge incidentally dominating set (SEIDS, WEIDS) of G_{CVI} if, for each $e_{1VI} \in E_{CVI} - R_{CVI}$, there exist at least one edge $e_{2VI} \in R_{CVI}$, so that e_{1VI} strongly (weakly) dominates e_{2VI} . The strong (weak) edge incidentally domination number of G_{CVI} denoted by $\gamma_{SVI}(G_{CVI})$ ($\gamma_{WVI}(G_{CVI})$), is called as the minimum cardinality of a strong (weak) edge incidentally dominating set of G_{CVI} .

Example 6.3

In figure 3, the incidentally dominating edges are $\{e_{11}\}, \{e_{22}\}, \{e_{33}\}, \{e_{44}\}, \{e_{55}\}, \{e_{66}\}$ and the SEIDSs are $S_{11} = \{e_{11}e_{22}\}, S_{22} = \{e_{11}e_{44}\}, S_{33} = \{e_{22}e_{33}\}, S_{44} = \{e_{33}e_{44}\}$. After calculating the edge cardinality of $S_{11}, S_{22}, S_{33}, S_{44}$ we obtain $|S_{11}| = 0.6, |S_{22}| = 0.65, |S_{33}| = 0.6, |S_{44}| = 0.6$. The edge cardinality of a minimum SEIDS is $|S_{11}| = 0.6$ and $\gamma_{SVI}(G_{CVI}) = 0.6$. The WEIDSs are $S_{55} = \{e_{11}e_{55}\}, S_{66} = \{e_{11}e_{66}\}, S_{77} = \{e_{33}e_{66}\}$. After calculating the edge cardinality of S_{55}, S_{66}, S_{77} we obtain $|S_{55}| = 0.6, |S_{66}| = 0.6, |S_{77}| = 0.6$. The edge cardinality of a minimum WEIDS is $|S_{55}| = 0.6$ and $\gamma_{WVI}(G_{CVI}) = 0.6$.

Theorem 6.4

Let G_{CVI} be a CT-VFigs without single node and R_{CVI} be a minimum SEIDS of G_{CVI} , then $E_{CVI} - R_{CVI}$ is an SEIDS of CT-VFigs.

Proof. Let G_{CVI} be a CT-VFIGS with minimum SEIDS, then for each edge $e_{2VI} \in R_{CVI}$, there is at least one edge $e_{1VI} \in E_{CVI} - N_{CVI}$ so that $\deg^t(e_{1VI}) \geq \deg^t(e_{2VI})$, $\deg^f(e_{1VI}) \leq \deg^f(e_{2VI})$ and $t_{c_{CVI}}(w_{11}, w_{11}w_{22}) = \min\{t_{a_{CVI}}(w_{11}), t_{b_{CVI}}(w_{11}, w_{22})\}$, $f_{c_{CVI}}(w_{11}, w_{11}w_{22}) = \max\{f_{a_{CVI}}(w_{11}), f_{b_{CVI}}(w_{11}, w_{22})\}$. Hence $E_{CVI} - R_{CVI}$ strongly dominates each edge of R_{CVI} . So, $E_{CVI} - R_{CVI}$ is an SEIDS of CT-VFIGS.

Theorem 6.5

Let G_{CVI} be a CT-VFIGS without single node and R_{CVI} be a minimum WEIDS of G_{CVI} , then $E_{CVI} - R_{CVI}$ is an WEIDS of CT-VFIGS.

Theorem 6.6

For any CT-VFIGS with $t_{c_{CVI}}(w_{11}, w_{11}w_{22}) = \min\{t_{a_{CVI}}(w_{11}), t_{b_{CVI}}(w_{11}, w_{22})\}$ and $f_{c_{CVI}}(w_{11}, w_{11}w_{22}) = \max\{f_{a_{CVI}}(w_{11}), f_{b_{CVI}}(w_{11}, w_{22})\}$ for all $w_{11} \in V_{CVI}$, $w_{11}w_{22} \in E_{CVI}$, then $\gamma_{SVI} = \gamma_{WVI}$.

Proof. Let G_{CVI} be a CT-VFIGS with $t_{c_{CVI}}(w_{11}, w_{11}w_{22}) = \min\{t_{a_{CVI}}(w_{11}), t_{b_{CVI}}(w_{11}, w_{22})\}$ and $f_{c_{CVI}}(w_{11}, w_{11}w_{22}) = \max\{f_{a_{CVI}}(w_{11}), f_{b_{CVI}}(w_{11}, w_{22})\}$. Assume for every node have same or different value. Since G_{CVI} is CT-VFIGS with $t_{b_{CVI}}(w_{11}, w_{22}) = \min\{t_{a_{CVI}}(w_{11}), t_{a_{CVI}}(w_{22})\}$ and $f_{b_{CVI}}(w_{11}, w_{22}) = \max\{f_{a_{CVI}}(w_{11}), f_{a_{CVI}}(w_{22})\}$ for all $w_{11}, w_{22} \in V_{CVI}$ and $t_{c_{CVI}}(w_{11}, w_{11}w_{22}) = \min\{t_{a_{CVI}}(w_{11}), t_{b_{CVI}}(w_{11}, w_{22})\}$ and $f_{c_{CVI}}(w_{11}, w_{11}w_{22}) = \max\{f_{a_{CVI}}(w_{11}), f_{b_{CVI}}(w_{11}, w_{22})\}$ for all $w_{11} \in V_{CVI}$, $w_{11}w_{22} \in E_{CVI}$. Thus every $e_{1VI}e_{2VI} \in E_{CVI}$ is SEIDS as well as WEIDS. Therefore $\gamma_{SVI} = \gamma_{WVI}$.

Theorem 6.7

For a CT-VFIGS, the below inequalities are true.

(i) $\gamma_{VI} \leq \gamma_{SVI} \leq O_{CVI}(G_{CVI}) - \max i mum d_{G_{CVI}} of G_{CVI}$.

(ii) $\gamma_{VI} \leq \gamma_{WVI} \leq O_{CVI}(G_{CVI}) - \min i mum d_{G_{CVI}} of G_{CVI}$.

Proof. (i) From definition 6.1 and 6.2 we have $\gamma_{VI} \leq \gamma_{SVI}$ (6)

We know $O_{CVI}(G_{CVI})$ = the sum of the incidence pair of CT-VFIGS.

Also $O_{CVI}(G_{CVI})$ - not including the maximum $d_{G_{CVI}}$ of CT-VFIGS

$$= O_{CVI}(G_{CVI}) - \mathbb{Q}_{CVI}(G_{CVI})$$

(7)

From equation (6) and (7)

$$\gamma_{VI} \leq \gamma_{SVI} \leq O_{CVI}(G_{CVI}) - \max i mum d_{G_{CVI}} of G_{CVI}$$

(ii) From definition 6.1 and 6.2 domination number γ_{VI} of CT-VFIGS is less than or equal to the γ_{WVI} of CT-VFIGS, because the edges of WEIDSM $_{CVI}$, it weakly dominates any one of the edges of $E_{CVI} - M_{CVI}$.

Therefore

$$\gamma_{CVI}(G_{CVI}) \geq \gamma_{VI}(G_{CVI})$$

(8)

Also $O_{CVI}(G_{CVI})$ -not including the minimum $d_{G_{CVI}}$ of CT-VFIGS

$$=O_{CVI}(G_{CVI}) - \delta_{CVI}(G_{CVI})$$

(9)

From equation (8) and (9), we get

$$\gamma_{VI} \leq \gamma_{WVI} \leq O_{CVI}(G_{CVI}) - \min i \text{ mum } d_{G_{CVI}} \text{ of } G_{CVI}$$

7. Real-Life Application of CT-VFIGs

An application of CT-VFIGs is included here. Consider two networks (CT-VFIGs) G_{VI}^1 and G_{VI}^2 , which have two and two vertices, respectively, and show distinct journal publications from different journals of a research filed. The vertices MS value indicates the percentage of accepted research papers in journal publishing, while the NMS value represents the rejected research papers. The MS value of the edges indicates that the journal publications are mutually collaborative, whereas the NMS value indicates that the journal publications are not mutually collaborative. The MS value of the incidence pairs represents the percentage of progress, whereas the NMS value represents the percentage of journal publications that have not progressed. As in figure 3 composition of G_{VI}^1 and G_{VI}^2 show the percentage of progress of journal publication m_{11} with journal publications n_{11} and n_{22} has the maximum MS value and the percentage of non progress of journal publication m_{11} with journal publications n_{11} and n_{22} has the lesser NMS value. As a result, the best suited combinations of journal publications demonstrating the largest percentage of progress and the lowest percentage of non-progress in the research field exist.

8. Comparative Analysis

In figure 3, calculate the edge cardinality of each edge, we get all the edges have same value. In our study the edge degree cardinality of the CT-VIFGs $|d_{G_{CVI}}(e_{1VI})| = 0.2$ and $|d_{G_{CVI}}(e_{2VI})| = 0.3$ are not all the same. It can be observed that the edge degree of the edges $|d_{G_{CVI}}(e_{1VI})| = \{0.9, 2.4\}$ shows the percentage of progress of journal publication m_{11} with journal publications n_{11} and n_{22} has the maximum MS value and the percentage of non progress of journal publication m_{11} with journal publications n_{11} and n_{22} has the lesser NMS value. As a result, the current method is ineffective in determining which journal publications have the highest percentage of progress and the lowest percentage of non-progress. The current method is useful for single networks, but it is insufficient to explain the overall impact of different networks' products. However, we may use composition to discuss the overall impact of combining multiple networks in our strategy. Our technique works with several networks as well as a single network. This allows us to discuss the overall influence of various networks products. As a result, our proposed strategy outperforms the existing one.

9. Conclusion

CT-VFIGs are extremely useful tools for researching a variety of computational intelligence and computer science topics. CT-VFIGs are used in a variety of fields, including natural networks and operations research. Three new CT-VIFG concepts in this article EIDS, SEIDS, and WEIDS. In the CT-VFIGs, some advantageous and

instrumental theorems of domination are also explained. A study of the makeup of VFIGs in the field of research is also included. Our research into CT-VFIG coloring, Hamiltonian CT-VFIGs, and CT-VFIG chromaticity in the future. The results of future research on these concepts will be revealed in upcoming papers.

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DOMINATION IN COMPLETE INTUITIONISTIC FUZZY INCIDENCE GRAPHS WITH APPLICATION

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Abstract. In this exploration article, the possibility of Complete Intuitionistic Fuzzy Incidence Graphs (CIFIG). Degree cardinality, strong and weak domination for complete intuitionistic fuzzy incidence graphs is characterized. The author clarifies these ideas with some outline models. Besides, a use of domination for Complete Intuitionistic Fuzzy Incidence Graph (CIFIG) to choose the best treatment facility accessible hospital is talked about for the delineation.

AMS Subject Classification: 05C12, 03E72, 03F55

KEYWORDS: Complete Intuitionistic Fuzzy Incidence Graph, Degree Cardinality, Strong Intuitionistic Fuzzy Incidence Domination Number(SIFIDN), Weak Intuitionistic Fuzzy Incidence Domination Number(WIFIDN).

1. Introduction

Zadeh[28] [30] [31] [32] have initiated fuzzy sets. Parvathi and Karunambigai[13] have initiated the idea of Intuitionistic Fuzzy Graphs (IFGs). Gani and Begum [5] talked about the extension of fuzzy graphs. Products in IFGs were discussed by Sahoo & Pal [17]. Sahoo and Pal [18,19] studied some types of fuzzy graphs. Sahoo et al [21] initiated new ideas in intuitionistic fuzzy graphs. Kalaierasi and Mahalakshmi have also expressed fuzzy strong graphs [8]. Shanmugavadivu and Gopinath, suggested non homogeneous ternary five degrees equation [24]. Shanmugavadivu and Gopinath, have also expressed on the homogeneous five degree equation [25], Bozhenyuk et al[2] has talked about dominating set and Mapreduce Methodology for Elliptical Curve Discrete Logarithmic Problems [29].

Ore and Berge introduced the concept of domination in 1962. Cockayne and Hedetniemi have further studied about domination in graphs[6]. Somasundaram and Somasundaram have initiated domination in fuzzy graphs by making use of effective edges[23]. Xavier et al. [27] has talked about domination in fuzzy graphs but differently. Dharmalingam and Nithya have also expressed domination parameters for fuzzy graphs[3]. Equitable domination number for fuzzy graphs was introduced by Revathi and Harinarayanan in [16]. Sarala and Kavitha have also expressed (1,2)-domination for fuzzy graphs[22]. Gani and Chandrasekaran have talked about strong arcs[12]. Sunitha and Manjusha have also expressed strong domination [26]. Kalaierasi and Mahalakshmi have also expressed fuzzy inventory EOQ optimization mathematical model [9]. Kalaierasi and Gopinath suggested fuzzy inventory order EOQ model with machine learning [10]. Fuzzy Incidence Graphs (FIGS) discussed by Dinesh [4]. Mordeson talked about incidence cuts in FIGS [11]. Priyadharshini et

al.[18] have also expressed a fuzzy MCDM approach for measuring the business impact of employee selection [15].

The design of this article in section 2 provides some preliminary results which are required to understand the remaining part of the article. In section 3 CIFIG is defined. In section 4 conveys meaning domination in CIFIG. In section 5 we examine Strong Intuitionistic Fuzzy Incidence Dominating Set (SIFIDS) and SIFIDN and Weak Intuitionistic Fuzzy Incidence Dominating Set (WIFIDS) and WIFIDN. In section 6 application of intuitionistic fuzzy incidence domination number is given.

2. Preliminaries

Definition 2.1[17]

An intuitionistic fuzzy graph is of the form $G_{IF} = (V_{IF}, E_{IF}, \rho_{IF}, \phi_{IF})$ where $\rho_{IF} = (\rho_1, \rho_2)$, $\phi_{IF} = (\phi_1, \phi_2)$ and $V_{IF} = \{x_0, x_1, x_2, \dots, x_n\}$ such that $\rho_1 : V_{IF} \rightarrow [0,1]$ and $\rho_2 : V_{IF} \rightarrow [0,1]$ represent the degree of membership and non membership of the vertex $x_{11} \in V_{IF}$, respectively and $0 \leq \rho_1 + \rho_2 \leq 1$ for each $x_{ii} \in V_{IF} (i = 1, 2, \dots, n)$, $\phi_1 : V_{IF} \times V_{IF} \rightarrow [0,1]$ and $\phi_2 : V_{IF} \times V_{IF} \rightarrow [0,1]$ show the degree of membership and non membership of the edge (x_{11}, x_{22}) , respectively, such that $\phi_1(x_{11}, x_{22}) \leq \min \{\rho_1(x_{11}), \rho_1(x_{22})\}$ and $\phi_2(x_{11}, x_{22}) \leq \max \{\rho_2(x_{11}), \rho_2(x_{22})\}$, $0 \leq \phi_1(x_{11}, x_{22}) + \phi_2(x_{11}, x_{22}) \leq 1$ for every (x_{11}, x_{22}) .

Definition 2.2[4]

Assume $G_I = (V_I, E_I)$ is a graph. Then, $G_I = (V_I, E_I, I_I)$ is named as an incidence graph, where $I_I \subseteq V_I \times E_I$.

Definition 2.3[4]

Assume $G_{FS} = (V_{FS}, E_{FS})$ is a graph, μ_{FS} is a fuzzy subset of V_{FS} , and γ_{FS} is a fuzzy subset of $V_{FS} \times V_{FS}$. Let ψ_{FS} be a fuzzy subset of $V_{FS} \times E_{FS}$. If $\psi_{FS}(w_{11}, w_{11}w_{22}) \leq \min \{\mu_{FS}(w_{11}), \gamma_{FS}(w_{11}w_{22})\}$ for every $w_{11} \in V_{FS}, w_{11}w_{22} \in E_{FS}$, then ψ_{FS} is a fuzzy incidence of G_{FS} .

Definition 2.4[4]

Assume G_I is a graph and (μ_I, γ_I) is a fuzzy sub graph of G_I . If ψ_I is a fuzzy incidence of G_I , then $G_I = (\mu_I, \gamma_I, \psi_I)$ is named as FIG of G_I .

Definition 2.5[7]

An intuitionistic fuzzy incidence graph(IFIG) is of the form $G_{FI} = (V_{FI}, E_{FI}, I_{FI}, \rho_{FI}, \phi_{FI}, \chi_{FI})$ where

$\rho_{FI} = (\rho_1, \rho_2)$, $\phi_{FI} = (\phi_1, \phi_2)$, $\chi_{FI} = (\chi_1, \chi_2)$ and

$V_{FI} = \{x_0, x_1, x_2, \dots, x_n\}$ such that $\rho_1 : V_{FI} \rightarrow [0,1]$ and $\rho_2 : V_{FI} \rightarrow [0,1]$

represent the degree of membership and non membership of the vertex $x_{11} \in V_{FI}$, respectively and $0 \leq \rho_1 + \rho_2 \leq 1$ for each $x_{ii} \in V_{FI} (i=1,2,\dots,n)$,

$\phi_1 : V_{FI} \times V_{FI} \rightarrow [0,1]$ and $\phi_2 : V_{FI} \times V_{FI} \rightarrow [0,1]$; $\phi_1(x_{11}, x_{22})$ and $\phi_2(x_{11}, x_{22})$ show the degree of membership and non membership of the edge (x_{11}, x_{22}) , respectively, such that $\phi_1(x_{11}, x_{22}) \leq \min \{\rho_1(x_{11}), \rho_1(x_{22})\}$ and $\phi_2(x_{11}, x_{22}) \leq \max \{\rho_2(x_{11}), \rho_2(x_{22})\}$, $0 \leq \phi_1(x_{11}, x_{22}) + \phi_2(x_{11}, x_{22}) \leq 1$ for every (x_{11}, x_{22}) .

$\chi_1 : V_{FI} \times E_{FI} \rightarrow [0,1]$ and $\chi_2 : V_{FI} \times E_{FI} \rightarrow [0,1]$; $\chi_1(x_{11}, x_{11}x_{22})$ and $\chi_2(x_{11}, x_{11}x_{22})$ show the degree of membership and non membership of the incidence pair respectively, such that $\chi_1(x_{11}, x_{11}x_{22}) \leq \min \{\rho_1(x_{11}), \phi_1(x_{11}x_{22})\}$ and $\chi_2(x_{11}, x_{11}x_{22}) \leq \max \{\rho_2(x_{11}), \phi_2(x_{11}x_{22})\}$, $0 \leq \chi_1(x_{11}, x_{11}x_{22}) + \chi_2(x_{11}, x_{11}x_{22}) \leq 1$ for every $(x_{11}, x_{11}x_{22})$.

3. Complete Intuitionistic Fuzzy Incidence Graph

Definition 3.1

The support of IFIG $G_{FI} = (R, S, T)$ is $\text{supp}(G_{FI}) = \{\text{supp}(R), \text{supp}(S), \text{supp}(T)\}$ so

$$\text{that } \text{supp}(R) = \{x_{11} / \rho_1(x_{11}) > 0, \rho_2(x_{11}) > 0\}$$

$$\text{supp}(S) = \{x_{11}x_{22} / \phi_1(x_{11}x_{22}) > 0, \phi_2(x_{11}x_{22}) > 0\}$$

$$\text{supp}(T) = \{(x_{11}, x_{11}x_{22}) / \chi_1(x_{11}, x_{11}x_{22}) > 0, \chi_2(x_{11}, x_{11}x_{22}) > 0\}$$

ρ^*, ϕ^* and χ^* are representing support of ρ, ϕ and χ respectively.

Definition 3.2

A IFIG is said to be complete intuitionistic fuzzy incidence graph if

$$\chi_1(x_{11}, x_{11}x_{22}) = \min \{\rho_1(x_{11}), \phi_1(x_{11}x_{22})\} \text{ and}$$

$$\chi_2(x_{11}, x_{11}x_{22}) = \max \{\rho_2(x_{11}), \phi_2(x_{11}x_{22})\}, \text{ for each}$$

$$\chi_1(x_{11}, x_{11}x_{22}), \chi_2(x_{11}, x_{11}x_{22}) \in \chi^*.$$

Remark 3.3

Every CIFIG is a IFIG but not conversely.

Definition 3.4

Assume $G_{IFI} = (\rho_{IFI}, \phi_{IFI}, \chi_{IFI})$ is a CIFIG. Then

$$O(G_{IFI}) = \sum_{x_{11} \neq x_{22}, x_{11}x_{22} \in V_{IFI}} \left(\frac{1 + \chi_1(x_{11}, x_{11}x_{22}) - \chi_2(x_{11}, x_{11}x_{22})}{2} \right)$$

is called

$$S(G_{IFI}) = \sum_{x_{11}x_{22} \in \phi^*} \left(\frac{1 + \phi_1(x_{11}x_{22}) - \phi_2(x_{11}x_{22})}{2} \right)$$

order of G_{IFI} and

size of G_{IFI} is called

4. Domination in CIFIGs

Definition 4.1

Definition 4.1

A vertex x_{11} in a CIFIG dominates to vertex x_{22} if

$$\chi_1(x_{11}, x_{11}x_{22}) = \min \{ \rho_1(x_{11}), \phi_1(x_{11}x_{22}) \} \text{ and}$$

$$\chi_2(x_{11}, x_{11}x_{22}) = \max \{ \rho_2(x_{11}), \phi_2(x_{11}x_{22}) \}.$$

Remark 4.2

For any $x_{11}, x_{22} \in V_{IFI}$, if x_{11} dominates x_{22} then x_{22} also dominates x_{11} .

Definition 4.3

A set $M_{IFI} \subseteq V_{IFI}$ is a intuitionistic fuzzy incidence dominating set (IFIDS) if each nodes in $V_{IFI} - M_{IFI}$ is dominated by atleast one node in M_{IFI} .

Definition 4.4

The lowest intuitionistic fuzzy incidence cardinality of a IFIDS is uttered as the intuitionistic fuzzy incidence domination number and it is represented by $\gamma_{IFI}(G_{IFI})$ or γ_{IFI} .

Definition 4.5

Consider $G_{IFI} = (V_{IFI}, E_{IFI}, I_{IFI}, \rho_{IFI}, \phi_{IFI}, \chi_{IFI})$ is an CIFIG and $x_{11} \in V_{IFI}$ then its degree is expressed by $d_{G_{IFI}}(x_{11}) = (d_{1G_{IFI}}(x_{11}), d_{2G_{IFI}}(x_{11}))$ and represented by $d_{1G_{IFI}}(x_{11}) = \sum_{x_{11} \neq x_{22}, (x_{11}, x_{11}x_{22}) \in I_{IFI}}$ and $d_{2G_{IFI}}(x_{11}) = \sum_{x_{11} \neq x_{22}, (x_{11}, x_{11}x_{22}) \in I_{IFI}}$

5. Strong and Weak Domination in CIFIGs

Definition 5.1

Let G_{IFI} be aCIFIG. Then the degree cardinality of $d_{G_{IFI}}(x_{11})$ is represented to be

$$|d_{G_{IFI}}(x_{11})| = \frac{1 + d_{1G_{IFI}}(x_{11}) - d_{2G_{IFI}}(x_{11})}{2} .$$

The lowest degree cardinality of G_{IFI} is defined by $\delta(G_{IFI}) = \min\{d_{G_{IFI}}(x_{11}) / x_{11} \in V_{IFI}\}$ and highest degree cardinality of G_{IFI} is defined by $\Delta(G_{IFI}) = \max\{d_{G_{IFI}}(x_{11}) / x_{11} \in V_{IFI}\}$.

Definition 5.2

Assume G_{IFI} is a CIFIG and let x_{11} and x_{22} be the nodes of G_{IFI} . Then x_{11} strongly dominates x_{22} or x_{22} weakly dominates x_{11} if $d_i(x_{11}) \geq d_i(x_{22})$ and $\chi_1(x_{11}, x_{11}x_{22}) = \min\{\rho_1(x_{11}), \phi_1(x_{11}x_{22})\}$, $\chi_2(x_{11}, x_{11}x_{22}) = \max\{\rho_2(x_{11}), \phi_2(x_{11}x_{22})\}$.

We call x_{22} strongly dominates x_{11} or x_{11} weakly dominates x_{22} if $d_i(x_{22}) \geq d_i(x_{11})$ and $\chi_1(x_{22}, x_{11}x_{22}) = \min\{\rho_1(x_{22}), \phi_1(x_{11}x_{22})\}$, $\chi_2(x_{22}, x_{11}x_{22}) = \max\{\rho_2(x_{22}), \phi_2(x_{11}x_{22})\}$.

Definition 5.3

A set $S_{IFI} \subseteq V_{IFI}$ is a SIFIDS if every vertex in $V_{IFI} - S_{IFI}$ is strongly fuzzy incidence dominated by atleast one vertex in S_{IFI} . Similarly, S_{IFI} is labeled a WIFIDS if every vertex in $V_{IFI} - S_{IFI}$ is weakly fuzzy incidence dominated by at least one vertex in S_{IFI} .

Definition 5.4

The lowest intuitionistic fuzzy incidence cardinality of a SIFIDS is uttered as the SIFIDN and it is represented by $\gamma_{SIFI}(G_{IFI})$ or γ_{SIFI} and the lowest intuitionistic fuzzy incidence cardinality of a WIFIDS is uttered as the WIFIDN and it is represented by $\gamma_{WIFI}(G_{IFI})$ or γ_{WIFI} .

Example 5.5

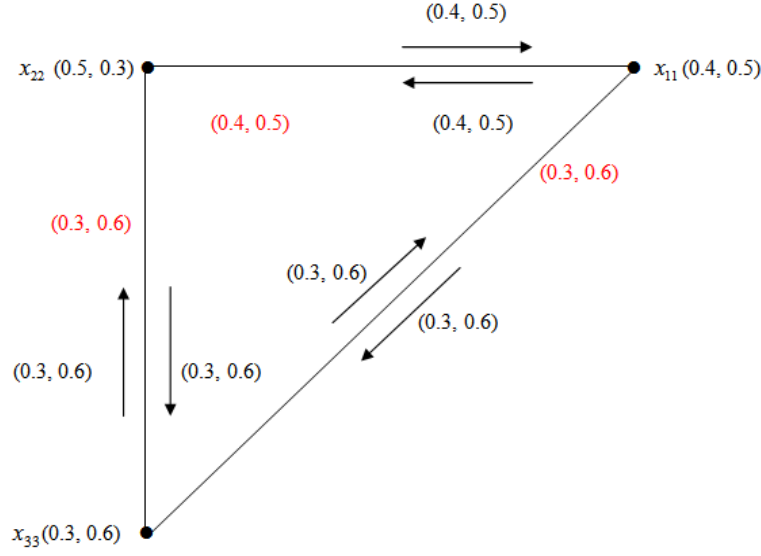


Fig: 1CIFIG with $\gamma_{SIFI} = 0.5$ and $\gamma_{WIFI} = 0.4$

Assume $G_{IFI} = (\rho_{IFI}, \phi_{IFI}, \chi_{IFI})$ is anCIFIG given in above figure having $V_{IFI} = (x_{11}, x_{22}, x_{33})$ and

$$\begin{aligned} \rho(x_{11}) &= (0.4, 0.5), \rho(x_{22}) = (0.5, 0.3), \rho(x_{33}) = (0.3, 0.6), \\ \phi(x_{11}, x_{22}) &= (0.4, 0.5), \phi(x_{22}, x_{33}) = (0.3, 0.6), \phi(x_{33}, x_{11}) = (0.3, 0.6) \\ \chi(x_{11}, x_{11}x_{22}) &= (0.4, 0.5), \chi(x_{22}, x_{11}x_{22}) = (0.4, 0.5), \\ \chi(x_{22}, x_{22}x_{33}) &= (0.3, 0.6), \chi(x_{33}, x_{22}x_{33}) = (0.3, 0.6), \\ \chi(x_{11}, x_{11}x_{33}) &= (0.3, 0.6), \chi(x_{33}, x_{11}x_{33}) = (0.3, 0.6) \end{aligned}$$

Assume $D_{IFI} = \{x_{33}\}$. We have $V_{IFI} - D_{IFI} = \{x_{11}, x_{22}\}$. Here x_{33} weakly fuzzy incidence dominates x_{11}, x_{22} because $d_{G_{IFI}}(x_{33}) = 0.2$ is less than the $d_{G_{IFI}}$ of all the remaining vertices. That is $d_{G_{IFI}}(x_{11}) = 0.3$, $d_{G_{IFI}}(x_{33}) = 0.3$. There is no other weak intuitionistic fuzzy incidence dominating sets. Thus the only weak intuitionistic fuzzy incidence dominating set is $D_{IFI} = \{x_{33}\}$. Therefore $\gamma_{WIFI} = 0.4$. We have strong IFIDS is $D_{IFI} = \{x_{11}\}$ with $\gamma_{SIFI} = 0.5$.

Theorem 5.6

For anyCIFIG with $\chi_1(x_{11}, x_{11}x_{22}) = \min \{\rho_1(x_{11}), \phi_1(x_{11}x_{22})\}$ and $\chi_2(x_{11}, x_{11}x_{22}) = \max \{\rho_2(x_{11}), \phi_2(x_{11}x_{22})\}$ for all $x_{11} \in V_{IFI}, x_{11}x_{22} \in E_{IFI}$, then

$$(i) \quad \gamma_{SIFI} = \gamma_{WIFI}$$

$$(ii) \quad \gamma_{SIFI} > \gamma_{WIFI}$$

Proof

Let $G_{IFI} = (\rho_{IFI}, \phi_{IFI}, \chi_{IFI})$ be a CIFIG with
 $\chi_1(x_{11}, x_{11}x_{22}) = \min \{\rho_1(x_{11}), \phi_1(x_{11}x_{22})\}$ and
 $\chi_2(x_{11}, x_{11}x_{22}) = \max \{\rho_2(x_{11}), \phi_2(x_{11}x_{22})\}$. Assume for all $x_{11} \in V_{IFI}$,
 $(\rho_1(x_{11}), \rho_2(x_{11}))$ have same value. Since G_{IFI} is CIFIG with
 $\phi_1(x_{11}x_{22}) = \min \{\rho_1(x_{11}), \rho_1(x_{22})\}$ and
 $\phi_2(x_{11}x_{22}) = \max \{\rho_2(x_{11}), \rho_2(x_{22})\}$ for all $x_{11}, x_{22} \in V_{IFI}$ and
 $\chi_1(x_{11}, x_{11}x_{22}) = \min \{\rho_1(x_{11}), \phi_1(x_{11}x_{22})\}$ and
 $\chi_2(x_{11}, x_{11}x_{22}) = \max \{\rho_2(x_{11}), \phi_2(x_{11}x_{22})\}$ for all
 $x_{11} \in V_{IFI}, x_{11}x_{22} \in E_{IFI}$. Thus every $x_{11} \in V_{IFI}$ is SIFIDS as well as WIFIDS.

Therefore $\gamma_{WIFI} = \gamma_{SIFI}$.

Assume for all $x_{11} \in V_{IFI}$, $(\rho_1(x_{11}), \rho_2(x_{11}))$ have different value. In a CIFIG
 with $d_{G_{IFI}}(x_{11}) \geq d_{G_{IFI}}(x_{22})$ from all the nodes one of them strongly dominates
 all the remaining nodes, if it is smallest among all the nodes then the IFIDS with that
 node is called WIFIDN, that is $\gamma_{WIFI} = (\rho_1(x_{11}), \rho_2(x_{11}))$ with
 $d_{G_{IFI}}(x_{11}) \leq d_{G_{IFI}}(x_{22})$ for all $x_{11}, x_{22} \in V_{IFI}$ and
 $\chi_1(x_{11}, x_{11}x_{22}) = \min \{\rho_1(x_{11}), \phi_1(x_{11}x_{22})\}$ and
 $\chi_2(x_{11}, x_{11}x_{22}) = \max \{\rho_2(x_{11}), \phi_2(x_{11}x_{22})\}$ for all
 $x_{11} \in V_{IFI}, x_{11}x_{22} \in E_{IFI}$. Certainly, the strong IFIDS has a node set other than

the that node set. This implies $\gamma_{SIFI} > \gamma_{WIFI}$.

Theorem 5.7

For a CIFIG, the below inequalities are true.

$$(i) \quad \gamma_{IFI} \leq \gamma_{SIFI} \leq O(G_{IFI}) - \max \text{imum } d_{G_{IFI}} \text{ of } G_{IFI}.$$

$$(ii) \quad \gamma_{IFI} \leq \gamma_{WIFI} \leq O(G_{IFI}) - \min \text{imum } d_{G_{IFI}} \text{ of } G_{IFI}.$$

Proof

$$(i) \text{ From definition 5.2, 5.3 and 5.4 we have } \gamma_{IFI} \leq \gamma_{SIFI} \quad (1)$$

We know $O(G_{IFI})$ = the sum of the incidence pair of CIFIG.

Also $O(G_{IFI}) -$ not including the maximum $d_{G_{IFI}}$ of CFIG

$$= O(G_{IFI}) - \Delta(G_{IFI}) \quad (2)$$

From equation (1) and (2)

$$\gamma_{IFI} \leq \gamma_{SIFI} \leq O(G_{IFI}) - \max imum d_{G_{IFI}} \text{ of } G_{IFI}$$

(ii) From definition 5.2, 5.3 and 5.4 domination number γ_{IFI} of CFIG is less than or equal to the γ_{WIFI} of CFIG, because the vertices of WIFIDS M_{IFI} , it weakly dominates any one of the vertices of $V_{IFI} - M_{IFI}$.

$$\text{Therefore } \gamma_{WIFI}(G_{IFI}) \geq \gamma_{IFI}(G_{IFI}) \quad (3)$$

Also $O(G_{IFI}) -$ not including the minimum $d_{G_{IFI}}$ of CFIG

$$= O(G_{IFI}) - \delta(G_{IFI}) \quad (4)$$

From equation (3) and (4), we get

$$\gamma_{IFI} \leq \gamma_{WIFI} \leq O(G_{IFI}) - \min imum d_{G_{IFI}} \text{ of } G_{IFI}.$$

6. Application

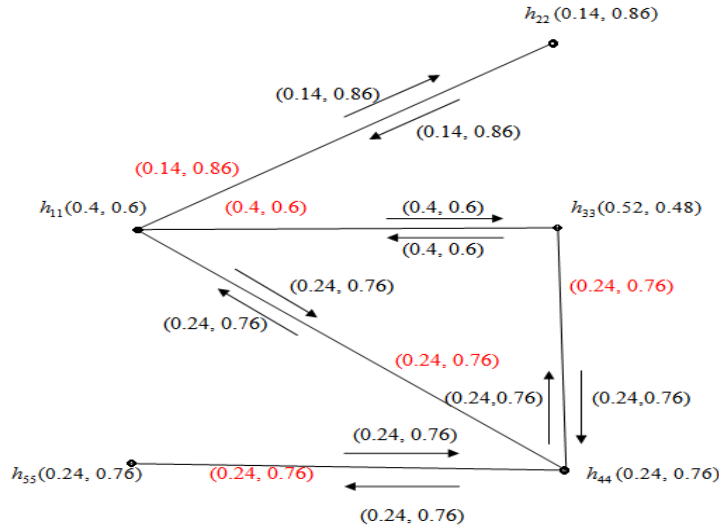
Here, incorporate an every day life model. Assume there are five multispeciality clinics are working (24 hours) in a city for giving crisis treatment to individuals. Here in our examination we are not referencing the original names of these clinics in this manner think about the clinics $h_{11}, h_{22}, h_{33}, h_{44}$ and h_{55} . In CFIGs, the vertices show the clinics and edges show the contract conditions between the clinics to share the facilities. The incidence pairs show the transferring of patients from one clinic to another because of the lack of resources. The vertex $h_{11}(0.4, 0.6)$ means that it has 40% of the necessary facilities for treatment and unfortunately lacks 60% of the equipment. The edge $h_{11}h_{22}(0.14, 0.86)$ shows that there is only 14% of the interaction and relationship between the two clinics, and due to financial issues, there is 86% on the conflict between them. IFIDS ruling arrangements of the graph is the arrangement of clinics which give the crisis treatment autonomously. Along these lines, we can save the time of patients and conquer the long going of patients by giving the couple of offices to the remainder of the clinics.

Assume $G_{IFI} = (V_{IFI}, E_{IFI}, I_{IFI}, \rho_{IFI}, \phi_{IFI}, \chi_{IFI})$ is a CFIG show in figure

having $V_{IFI} = (h_{11}, h_{22}, h_{33}, h_{44}, h_{55})$ and $\rho(h_{11}) = (0.4, 0.6)$,
 $\rho(h_{22}) = (0.14, 0.86)$, $\rho(h_{33}) = (0.52, 0.48)$,

$$\begin{aligned}
 \rho(h_{44}) &= (0.24, 0.76), \rho(h_{55}) = (0.24, 0.76), \phi(h_{11}, h_{22}) = (0.14, 0.86), \\
 \phi(h_{11}, h_{33}) &= (0.4, 0.6), \phi(h_{11}, h_{44}) = (0.24, 0.76), \\
 \phi(h_{33}, h_{44}) &= (0.24, 0.76), \phi(h_{44}, h_{55}) = (0.24, 0.76) \\
 \chi(h_{11}, h_{11}h_{22}) &= (0.14, 0.86), \chi(h_{22}, h_{11}h_{22}) = (0.14, 0.86), \\
 \chi(h_{11}, h_{11}h_{33}) &= (0.4, 0.6), \chi(h_{33}, h_{11}h_{33}) = (0.4, 0.6), \\
 \chi(h_{11}, h_{11}h_{44}) &= (0.24, 0.76), \chi(h_{44}, h_{11}h_{44}) = (0.24, 0.76), \\
 \chi(h_{33}, h_{33}h_{44}) &= (0.24, 0.76), \chi(h_{44}, h_{33}h_{44}) = (0.24, 0.76), \\
 \chi(h_{44}, h_{44}h_{55}) &= (0.24, 0.76), \chi(h_{55}, h_{44}h_{55}) = (0.24, 0.76)
 \end{aligned}$$

Example 6.1


 Fig: 2CIFIG with $\gamma_{IFI} = 0.38$

In figure having intuitionistic fuzzy incidence dominating set are $D_{IFI} = \{h_{22}, h_{44}\}$ and $\gamma_{IFI} = 0.38$.

This shows that patients can visit any one of the clinics from this set. The rest of the clinics upgrade their facilities to provide better treatment to the people.

9. Conclusion

The idea of domination in CIFIGs is imperative from religious just as an applications perspective. In this paper, the possibility of complete intuitionistic fuzzy incidence graph, strong and weak intuitionistic fuzzy incidence dominating set and strong and weak intuitionistic fuzzy incidence domination number is talked about. Further work on these thoughts will be accounted for in impending papers.

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RESEARCH ARTICLE

ARC-SEQUENCE IN COMPLETE AND REGULAR FUZZY GRAPHS

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ABSTRACT

In a fuzzy graph, the arcs are mainly classified in to α , β and δ . In this paper, some arc sequences in fuzzy graphs are introduced, whose concept are based on the classification of arcs. Besides complete in fuzzy graphs, regular in fuzzy graph are obtained. It is shown that α -arc sequence of a complete is a zero-one sequence, δ -arc sequence of a regular fuzzy graph is a zero sequence.

Key words:

α -arc sequence,
 β -arc sequence, δ -arc sequence,
Strong-arc sequence,
Complete, Regular.

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INTRODUCTION

Graph theory has now become a major branch of applied mathematics due to its large variety of applications and effectiveness. Graph theory is a widely used tool for solving combinatorial problems in different areas such as geometry, algebra, number theory, topology, optimization and computer science. In models, when we have an uncertainty about either the set of vertices or the set of edges or both, the models becomes a fuzzy graph. Currently, the theory of fuzzy graphs is an intense area of research. Fuzzy graphs differ from the classical ones in several ways, among them the most prominent one is connectivity. Distance and central concepts also play important roles in applications related with fuzzy graphs. Rosenfeld (1975) gave a mathematical definition for a fuzzy graph in 1975. Bhattacharya (1987) had established some connectivity concepts regarding fuzzy cutnodes and fuzzy bridges. Bhutani (1989) had studied automorphisms on fuzzy graphs and certain properties of complete fuzzy graphs. Pathinathan and Jesintha Rosline (2014) defined relationship between different types of arcs in both regular and totally regular fuzzy graph. Sunil Mathew and Sunitha (Bhutani, 2003; Mathew, 2009; Sunitha, 1999; Sunitha, 2002; Sunitha, 2005) introduced many connectivity concepts in fuzzy graphs. Kalaiaarasi (2011) defined Optimization of fuzzy integrated vendor-buyer inventory models.

In this article, the concept of arc sequence in fuzzy graphs are discussed. These concepts are derived from the notion of connectivity in fuzzy graphs. Also a comparative study is made between regular and totally regular fuzzy graphs with reference to different types of arc sequence in fuzzy graphs. Also a necessary condition for a graph to be regular or totally regular is formulated in terms of arc-sequence

2. Preliminaries

Definition 2.1

A fuzzy graph G is a pair of function $G: (\sigma, \mu)$ where σ is a fuzzy subset of a non empty set V and μ is a symmetric fuzzy relation on σ . The underlying crisp graph of $G: (\sigma, \mu)$ is denoted by $G^*: (V, E)$ where $E \subseteq V \times V$.

Definition 2.2

A fuzzy graph G is complete if $\mu(uv) = \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$, where uv denotes the edge between u and v

Definition 2.3

The strength of connectedness between two nodes x and y is defined as the maximum of the strengths of all paths between x and y and is denoted by $CONN_G(x, y)$.

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Different Types of Edge Sequence in Pseudo Regular Fuzzy Graphs

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Abstract

The concept of connectivity plays an important role in both theory and applications of fuzzy graphs. Depending on the strength of an edge, this paper classifies edge sequence of a fuzzy graph in to different types. We analyze the relation between different types of edge sequence in both pseudo regular and totally pseudo regular fuzzy graphs. Also we identify strong edge sequence in pseudo regular fuzzy graph.

AMS Subject Classification: 05C12, 03E72, 05C72

Key Words and Phrases: Pseudo regular fuzzy graph, Totally pseudo regular fuzzy graph, α - edge sequence, β - edge sequence, δ - edge sequence, Strong edge sequence.

1 Introduction

Euler in 1736 first introduced the concept of graph theory. Fuzzy graph theory is finding numerous application in the fields of information theory, neural network, expert systems, cluster analysis, medical diagnosis, control theory ect. Fuzzy set theory was first introduced by Zadeh in 1965 [19]. The first definition of fuzzy graph was introduced by Haufmann in 1973 based on Zadeh's fuzzy relations in 1971. In 1975, A.Rosenfeld [16] introduced the concept of fuzzy graphs. Sunil Mathew and Sunitha [10] defined different types of arcs in fuzzy graphs and using them classified fuzzy graphs. Pathinathan and Jesintha Rosline [15] defined relationship between different types of arcs in both regular and totally regular fuzzy graph. Santhi Maheswari and Sekar [17] introduced on pseudo regular fuzzy graphs. Butani and Rosenfeld [2] have introduced the concept of strong arcs. Kalaiarasi [8] defined Optimization of fuzzy integrated vendor-buyer inventory models.



STRONG DOMINATION IN PSEUDO REGULAR AND COMPLETE FUZZY GRAPHS

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Abstract

In this article, the perception of strong domination constant number is introduced by using membership values of strong arcs in fuzzy graphs. The strong domination constant number γ_{sc} of pseudo regular fuzzy graph and complete fuzzy graph is determined. Further the relationship between the strong domination constant number of a pseudo regular fuzzy graph and complete fuzzy graph are discussed and theorems related to these concepts are stated and proved.

Keywords:

Pseudo regular fuzzy graph, Totally pseudo regular fuzzy graph, Complete fuzzy graph, Strong arcs, Weight of arcs, Strong domination constant number.

AMS Subject Classification: 05C12, 03E72, 05C72

STRONG AND WEAK DOMINATION IN INTUITIONISTIC FUZZY GRAPH

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Abstract: In this article, we generalize the concept of strong and weak domination in intuitionistic fuzzy graph and we introduced a definition of weight of strong dominating set using strong arcs and weight of weak dominating set using weak arcs of intuitionistic fuzzy graphs. We determine the strong domination number $\gamma_{SIFG}(G)$ and weak domination number $\gamma_{WIFG}(G)$ for several classes of intuitionistic fuzzy graphs and some theorems are discussed.

Keywords: Intuitionistic fuzzy graph, Strong arcs, Weak arcs, Weight of arcs, Strong domination number, Weak domination number.

1.Introduction

In 1983 Atanassov [2] introduced the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets [9]. Atanassov added a new component (which determines the degree of non-membership) in the definition of fuzzy set. The fuzzy sets give the degree of membership of an element in a given set (and the non-membership degree equals one minus the degree of membership), while intuitionistic fuzzy sets give both a degree of membership and a degree of non-membership which are more –or-less independent from each other, the only requirement is that the sum of these two degrees is not greater than 1. Intuitionistic fuzzy sets have been applied in a wide variety of fields including computer science, engineering, mathematics, medicine, chemistry and economics[1,3].

Atanassov [1] introduced the concept of intuitionistic fuzzy relations and intuitionistic fuzzy graphs, and further studied in [8]. In this article, we introduce the notion of strong and weak domination in intuitionistic fuzzy graphs. We discuss strong domination number and weak domination number in intuitionistic fuzzy graphs.

2.Preliminaries

Definition 2.1

An intuitionistic fuzzy graph (IFG) is of the form $G : (V, E)$ where

(i) $V = \{v_1, v_2, \dots, v_n\}$ such that $\mu_1 : V \rightarrow [0,1]$ and $\gamma_1 : V \rightarrow [0,1]$ denote the degree of membership and non membership of the element $v_i \in V$ respectively and $0 \leq \mu_1(v_i) + \gamma_1(v_i) \leq 1$ for every $v_i \in V (i = 1, 2, \dots, n)$

(ii) $E \subseteq V \times V$ where $\mu_2 : V \times V \rightarrow [0,1]$ and $\gamma_2 : V \times V \rightarrow [0,1]$ are such that

$$\mu_2(v_i, v_j) \leq \min(\mu_1(v_i), \mu_1(v_j))$$

$$\gamma_2(v_i, v_j) \leq \max(\gamma_1(v_i), \gamma_1(v_j))$$

and $0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 1$ for every $(v_i, v_j) \in E, (i, j = 1, 2, \dots, n)$

Definition 2.2

If $v_i, v_j \in V \subseteq G$, the μ - strength of connectedness between v_i and v_j is $\mu_2^\infty(v_i, v_j) = \sup\{\mu_2^k(v_i, v_j) \mid k = 1, 2, \dots, n\}$ and γ - strength of connectedness between v_i and v_j is $\gamma_2^\infty(v_i, v_j) = \inf\{\gamma_2^k(v_i, v_j) \mid k = 1, 2, \dots, n\}$.

VARIOUS PRODUCT AND DUAL STRONG DOMINATION IN MIXED AND SQUARE MIXED INTUITIONISTIC FUZZY GRAPHS

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Abstract: In this article, we introduce the notion of mixed intuitionistic fuzzy graph, square mixed intuitionistic fuzzy graph and we define direct product, semi-strong product and semi product. In addition we investigated many interesting results regarding the concept. Finally we define dual strong domination number and some theorems are discussed.

Keyword: Mixed intuitionistic fuzzy graph, Square mixed intuitionistic fuzzy graph, Direct product, Semi-strong product, Semi-product, Weight of dual strong dominating set, Dual strong domination number.

1.Introduction

In 1975, Rosenfield [11] discussed the concept of the fuzzy graph, the basic idea of which was introduced by Kauffman [8] in 1973. The fuzzy relations between fuzzy sets were also considered by Rosenfield, he developed the structure of fuzzy graphs and obtained analogues of several graphs theoretical concepts. The first definition of intuitionistic fuzzy relations and intuitionistic fuzzy graphs were introduced by Atanassov (1999), and further studied in (2009). Different types of intuitionistic fuzzy graphs and their applications can be found in several papers. Parvathi and Thamizhendhi (2010) introduced the concept of domination number in intuitionistic fuzzy graphs.

In this paper, direct product of two mixed intuitionistic fuzzy graphs, semi-strong product and semi product of two square mixed intuitionistic fuzzy graphs are defined, and many interesting results involving these concepts are investigated. Moreover, we defined dual strong domination number and investigated their many interesting results.

2.Preliminaries

Definition 2.1

An intuitionistic fuzzy graph (IFG) is of the form $G : (V, E)$ where

- (i) $V = \{v_1, v_2, \dots, v_n\}$ such that $\mu_1 : V \rightarrow [0,1]$ and $\gamma_1 : V \rightarrow [0,1]$ denote the degree of membership and non membership of the element $v_i \in V$ respectively and $0 \leq \mu_1(v_i) + \gamma_1(v_i) \leq 1$ for every $v_i \in V (i = 1, 2, \dots, n)$
- (ii) $E \subseteq V \times V$ where $\mu_2 : V \times V \rightarrow [0,1]$ and $\gamma_2 : V \times V \rightarrow [0,1]$ are such that

THE JOIN PRODUCT AND DUAL STRONG DOMINATION IN MIXED SPLIT INTUITIONISTIC FUZZY GRAPH

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Abstract

In this paper, we introduce the notion of mixed split intuitionistic fuzzy graph, strong mixed split intuitionistic fuzzy graph, complete mixed split intuitionistic fuzzy graph, join product of two mixed split intuitionistic fuzzy graphs and establish some of their properties. Also we discuss dual strong domination regarding the concept.

Keywords:

Mixed split intuitionistic fuzzy graph, Strong mixed split intuitionistic fuzzy graph, Complete mixed split intuitionistic fuzzy graph, Join product of two mixed split intuitionistic fuzzy graphs .



n - SPLIT DOMINATION IN VERTEX SQUARED INTERVAL-VALUED FUZZY GRAPHS

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Abstract

In this paper we study different concepts like vertex squared interval-valued fuzzy graph, vertex squared cardinality, vertex squared independent set, n -split dominating set, n -split domination number. We likewise, investigate a relationship between n -split dominating set and vertex squared independent set for vertex squared interval-valued fuzzy graphs.

1. Introduction

Fuzzy graphs differ from the classical ones in several ways, among them the most prominent one is connectivity. Distance and central concepts additionally assume important parts in applications related to fuzzy graphs. In 1965 Lotfi. A. Zadeh initiated fuzzy sets and later in 1983 Krassimir T. Bhattacharya [3] has discussed fuzzy graphs. Kalaiarasi and Mahalakshmi have also expressed fuzzy strong graphs [10].

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Keywords: Vertex Squared Interval-Valued Fuzzy Graph (VSIVFG), n -Split dominating set, n -Split domination number, Vertex squared independent set.

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