(6 pages)

S.No. 6871 **P 22 MAE 1 B**

(For candidates admitted from 2022–2023 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2023.

Mathematics – Elective

DISCRETE MATHEMATICS

Time : Three hours Maximum : 75 marks

PART A — (20 marks)

Answer ALL questions.

I. (A) Choose the correct answer : (5 × 1 = 5)

1. A relation R in a set X is ———— if, for every x and y in X, whenever xRy and yRx, then $x = y$.

   (a) reflexive (b) symmetric

   (c) transitive (d) antisymmetric

2. A product of the variables and their negations in a formula is called ————.

   (a) elementary sum

   (b) elementary product

   (c) product of elementary sums

   (d) sum of elementary products

3. $GLB\{a, b\}$ = ————.

   (a) $a * b$ (b) $a + b$

   (c) $a \otimes b$ (d) $a \oplus b$

4. $(a \otimes b)^1$ = ————.

   (a) $a' \otimes b'$ (b) $a' \oplus b'$

   (c) $a' * b'$ (d) $a' + b'$

5. A ———— grammar contains productions of only the form $\alpha \to \beta$ where $|\alpha| < |\beta|$ and $\alpha \in V_n$.

   (a) context-sensitive (b) contest-free

   (c) regular (d) none of these

(B) Fill in the blanks : (5 × 1 = 5)

6. If a binary operation $*$ on $\times$ with the identity e is commutative, then any element that is left-invertible or right-invertible is ————.

7. An atomic formula is a ———— formula.

8. The principle of ———— holds for all distributive lattices.

9. For any Boolean algebra $< B, *, \oplus, ', 0, 1 >$, the subsets $\{0,1\}$ and the set B are both ———— algebras.

10. The rank of a connective O_j is ————, where n is the order of O.

II. Answer the following : $(5 \times 2 = 10)$

11. Let $X = \{1, 2, 3, 4\}$ and $R \equiv \langle x, y \rangle \mid x > y$. Draw the graph of R.

12. Define a simple statement function of one variable.

13. Write the principle of duality of Lattice.

14. Define sub-boolean algebra.

15. Define a regular grammar.

PART B — $(5 \times 5 = 25)$

Answer the following by choosing (a) or (b).

16. (a) Let $R = \{\langle 1, 2\rangle, \langle 3, 4\rangle, \langle 2, 2\rangle\}$ and $S = \{\langle 4, 2\rangle, \langle 2, 5\rangle, \langle 3, 1\rangle, \langle 1, 3\rangle\}$. Find $R \circ S$, $S \circ R$, $R \circ (S \circ R), (R \circ S) \circ R, R \circ R, S \circ S$ AND $R \circ R \circ R$.

Or

(b) Let A be a given finite set and $\rho(A)$ its power set. Let $\subseteq$ be the inclusion relation on the elements of $\rho(A)$. Draw Hasse diagrams of $\langle \rho(A), \subseteq \rangle$ for the following sets

(i) $A = \{a\}$

(ii) $A = \{a, b\}$

(iii) $A = \{a, b, c\}$

17. (a) Show that $S \vee R$ is tautologicaly implied by $(P \vee Q) \wedge (P \to R) \wedge (Q \to S)$.

Or

(b) Show that $P \to (Q \to R) \Leftrightarrow P \to (\neg Q \vee R) \Leftrightarrow (P \wedge Q) \to R$

18. (a) Explain some properties of lattices.

Or

(b) Prove that every chain is a distributive lattice.

19. (a) Explain the sub-boolean algebras with example.

Or

(b) write the following Boolean expression in an equivalent sum of products canonical form in three variable x_1, x_2, x_3 and x_3.

(i) $x_1 * x_2$

(ii) $x_1 \oplus x_2$.

20. (a) Explain the Polish notation.

Or

(b) Let the grammar $G_3 = \langle S, B, C, \{a, b, c\}, S, Q\rangle$ where ϕ consists of the productions $S \to aSBC, S \to aBC, CB \to BC$, $aB \to bb$, $bC \to bc$, $cC \to cc$. Find the derivation of the string $a^2b^2c^2$.

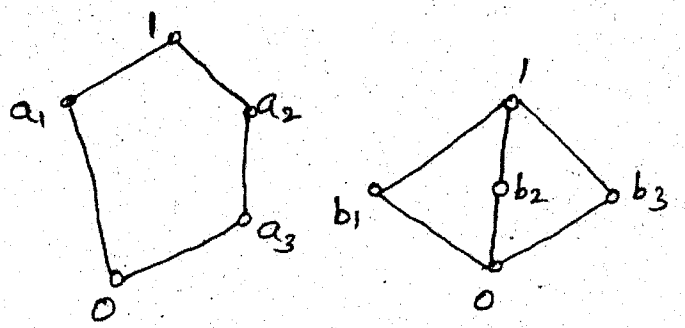
PART C — (3 × 10 = 30)

Answer any THREE questions.

21. Let $X = \{1, 2, ..., 7\}$ and $R\{\langle x, y\rangle / x - y$ is divisible by 3$\}$. Show that R is an equivalence relation. Draw the graph of R.

22. Show that $((P \vee Q) \wedge \neg(\neg P \wedge (\neg Q \vee \neg R)) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$ is a tautology.

23. Explain a distributive lattice with example. Also show that the following lattices given by the diagrams are not distributive.



24. Show that the following Boolean expressions are equivalent to one another. Obtain their sum-of-product canonical form

(a) $(x \oplus y) * (x' \oplus z) * (y \oplus z)$

(b) $(x * z) \oplus (x' * y) \oplus (y * z)$

(c) $(x \oplus y) * (x' \oplus z)$

(d) $(x * z) \oplus (x' * y)$.

25. Explain Notations of Syntax Analysis.