PART C — (3 × 15 = 45)

Answer any THREE questions.

16. Verify Stoke's theorem for the vector $A = (2x - y)\hat{i} - yz^2\hat{j} - yz^2\hat{k}$ over the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$.

17. State and prove Taylor theorem for a complex function.

18. Brief the Legendre differential equation and Legendre functions in detail.

19. What are reducible and irreducible representations of a group? State and prove the five important rules about irreducible representations and use them to derive a character table for C_{3v} point group.

20. Write general qualitative formula for equidistant values of argument x and hence give brief account of following formula

    (a) Trapezoidal rule

    (b) Simpson's one-third rule

    (c) Simpson's three-eighth rule.

———————

(For candidates admitted from 2015–2016 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2023.

Part III – Physics

MATHEMATICAL PHYSICS AND NUMERICAL METHODS

Time : Three hours Maximum : 100 marks

PART A — (10 × 2 = 20)

Answer ALL questions.

1. Define divergence and give its geometrical meaning.

2. What is characteristic equation of a matrix?

3. Define analytic function of a complex variable.

4. Define conformal mapping.

5. Define orthogonality relations.

6. Define β-function.

7. Define abelian group.

8. How many times does each irreducible representation of C_{3v} occur in the regular representation?

9. What is the criterion for the convergence of Newton-Raphson's method?

10. Compare Gauss elimination and Gauss Seidel method.

PART B — (5 × 7 = 35)

Answer ALL questions, choosing either (a) or (b)

11. (a) Find a set of vectors reciprocal to the following set $\left(2\hat{i}+3\hat{j}-\hat{k}\right)$, $\left(\hat{i}-\hat{j}-2\hat{k}\right)$, $\left(-\hat{i}+2\hat{j}+2\hat{k}\right)$.

Or

(b) Define curl of a vector function and give its physical significance. Obtain an expression for curl A is cartesian coordinates.

12. (a) Show that $\log z$ is analytic function except at $z=0$.

Or

(b) State and prove the Cauchy-Riemann equation for a function of a complex variable to be analytic.

13. (a) Explain Beta and Gamma functions in detail.

Or

(b) Prove that $\int_{-1}^{+1}\left(1-x^2\right)\left(\frac{dPn}{dx}\right)^2 dx=\frac{2n(n+1)}{2n+1}$.

14. (a) Derive and explain the product of characters of an irreducible representation.

Or

(b) Define a group. What do you mean by permutation group.

15. (a) Calculate the approximate value of $\int_{-3}^{+3} x^4 dx$ by Simpson's $\frac{1}{3}^{rd}$ rule taking sever equidistant coordinates. Compare it with the exact value and the value obtained by Trapezoidal rule.

Or

(b) Using Runge-Kutta method to find by (0.2) for the equation $\frac{dy}{dx}=\frac{y-x}{y+x}$, $y(0)=1$, take $h=0.2$.