20. (a) Show that a graph is planar if and only if it is embeddable on a sphere.

Or

(b) Show that a graph G is planar if and only if each of its blocks is planar.

SECTION C — (3 × 10 = 30)

Answer any THREE questions.

21. Let G and G' be simple connected graphs with isomorphic line graphs. Then prove that G and G' are isomorphic unless one of them is $K_{1,3}$ and the other is K_3.

22. Show that every tree has a center consisting of either a single vertex or two adjacent vertices.

23. Show that a graph G is Eulerian if and only if each edge e of G belongs to an odd number of cycles of G.

24. For any simple graph G, prove that $2\sqrt{n} \leq \chi + \chi^c \leq n+1$ and $n \leq \chi, \chi^c \leq \left(\frac{n+1}{2}\right)^2$.

25. Show that every planar graph is 5-vertex colorable.

S.No. 6870 **P22 MAE 1 A**

(For candidates admitted from 2022–2023 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2023.

Mathematics — Elective

GRAPH THEORY

Time : Three hours Maximum : 75 marks

SECTION A — (20 marks)

Answer ALL questions.

I. (A) Choose the correct answer. (5 × 1 = 5)

1. A graph is said to be ———— if its n vertices are distinguished from one another by levels such as $v_1, v_2, \ldots\ldots, v_n$.

(a) complete (b) labeled
(c) clique (d) degree

2. An edge e of a graph G is said to be ———— if it is deleted from G and its ends are identified.

(a) contracted (b) loop
(c) complete (d) degree

3. If G is a simple graph with $n \geq 3$ and $\delta \geq \frac{n}{2}$, then G is ————

(a) traceable (b) Eulerian
(c) matchings (d) Hamiltonian

4. A graph G is said to be k-colorable if G admits a proper vertex coloring using ————.
   (a) atmost $k-1$ colors
   (b) atleast $k-1$ colors
   (c) atmost k colors
   (d) atleast k colors

5. Every plane triangular of order $n \geq 3$ is ————
   (a) 3-connected (b) 4-connected
   (c) 5-connected (d) none

(B) Fill in the blanks. $(5 \times 1 = 5)$

6. A simple graph G is called ———— if $G \approx G^c$.
7. The ———— of a vertex u of T is the maximum number of edges in any branch at u.
8. The wheel W_n is Hamiltonian for every ————.
9. The ———— of G is the maximum k for which G has a pseudocomplete k-coloring.
10. IF G is a planar graph that contains no triangle, then G is ————.

II. Answer the following questions. $(5 \times 2 = 10)$

11. Define complete graph.
12. Define centroid vertex.
13. Define Hamiltonian and traceable.
14. Define chromatic number.
15. Show that the Petersen graph P is nonplanar.

SECTION B — $(5 \times 5 = 25)$

Answer ALL questions, choosing either (a) or (b).

16. (a) Show that in any group of n persons $(n \geq)$, there are at least two with the same number of friends.

Or

(b) If the simple graphs G_1 and G_2 are isomorphic, then prove that $L(G_1)$ and $L(G_2)$ are isomorphic.

17. (a) Show that an edge $e = xy$ of a connected graph G is a cut edge of G if and only if e belongs to no cycle of G.

Or

(b) Prove that for a simple connected graph G, $L(G)$ is isomorphic to G if only if G is a cycle.

18. (a) Show that for any graph G for which $\delta > 0$, $\alpha' + \beta' = n$.

Or

(b) Let G be a simple graph with $n \geq 3$ vertices. If $d(u) + d(v) \geq n - 1$ for every pair of nonadjacent vertices u and v of G, then prove that G is traceable.

19. (a) In a critical graph G prove that no vertex cut is a clique.

Or

(b) If a graph G admits a complete k-coloring and a complete l-coloring, then show that it admits a complete i-coloring for every i between k and l.