(6 pages)

S.No. 7339 **RNENS 9**

(For candidates admitted from 2006–2007 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2023.

Mathematics

MATHEMATICAL PROGRAMMING

Time : Three hours Maximum : 100 marks

Answer ALL the questions.

Subdivisions (a), (b) and (c) in each questions carries 4, 6 and 10 marks respectively.

1. (a) Ozark Farms uses at least 800 lb of special feed daily. The special feed is a mixture of corn and soybean meal with the following compositions:

lb per lb of feedstuff

Feedstuff	protein	fiber	cost ($./lb)
Corn	.09	.02	.30
Soybean meal	.60	.06	.90

The dietary requirements of the special feed are at least 30% protein and at most 5% fiber. Determine the L.P.P for daily minimum cost feed mix.

(b) (i) Find the graphical solution of maximize $z = 2x_1 + 3x_2$, subject to $2x_1 + x_2 \le 4$, $x_1 + 2x_2 \le 5$, $x_1, x_2 \ge 0$.

Or

(ii) An individual wishes to invest Rs. 5000 over the next year in two types of investment: Investment A yields 5% and investment B yields 8%. Market research recommends an allocation of at least 25% in A and at most 50% in B. Moreover investment in A should be at least half the investment in B. How should the fund be allocated to the two investments?

(c) (i) Solve: Minimize $z = 4x_1 + x_2$ subject to $3x_1 + x_2 = 3$; $4x_1 + 3x_2 \ge 6$; $x_1 + 2x_2 \le 4$; $x_1, x_2 \ge 0$.

Or

(ii) Consider the problem maximize $z = 2x_1 + 4x_2 + 4x_3 - 3x_4$ subject to $x_1 + x_2 + x_3 = 4$; $x_1 + 4x_2 + x_4 = 8$, $x_1, x_2, x_3, x_4 \ge 0$ solve the problem with x_3 and x_4 as the starting basic variable and without using any artificial variables.

2. (a) Write the dual simplex algorithm.

(b) (i) Show that given the optimal primal basis B and its associated objective coefficient vector C_B, the optimal solution of the dual problem is $Y = C_B B^{-1}$.

Or

(ii) Solve minimize $z = 3x_1 + 2x_2 + x_3$ subject to $3x_1 + x_2 + x_3 \geq 3$; $-3x_1 + 3x_2 + x_3 \geq 6$; $x_1 + x_2 + x_3 \leq 3$, $x_1, x_2, x_3 \geq 0$.

(c) (i) Acme manufacturing produces two products. The daily capacity of the manufacturing process is 430 minutes. Product 1 requires 2 minutes per unit, and product 2 requires 1 minute per unit. There is not limit on the amount produced of product 1, but the maximum daily demand for product 2 is 230 units. The unit profit of product 1 is Rs. 2 and that of product 2 is Rs. 5. Find the optimal solution by DP.

Or

(ii) Solve maximize $z = (3-6t)x_1 + x_2 + (2-2t)\ x_2 + (5+5t)x_3$ subject to $x_1 + 2x_2 + x_3 \leq 40$; $3x_1 + 2x_3 \leq 60$; $x_1 + 4x_2 \leq 30$, $x_1, x_2, x_3 \leq 0$.

3. (a) Write the integer programming algorithms.

(b) (i) I have been approached by three telephone companies to subscribe to their long-distance service in the Indian states. A Bell will charge a flat Rs. 16 per month plus Rs. 0.25 a minute. P Bell will charge Rs. 25 a month but will reduce the per-minute cost to Rs.0.21. As for Baby Bell, the flat monthly charge is Rs.18 and the cost per minute is Rs.0.22. I usually make an average of 200 minutes of long distance calls a month. Assuming that I do not pay the flat monthly fee unless I make calls and that I can apportion my calls among all three companies as I please, how should I use the three companies to minimize my monthly telephone bill?

Or

(ii) Jobco is planning to produce at least 2000 widgets on three machines. The minimum lot size on any machine is 600 widgets. The following table gives the pertinent data of the situation.

Machine	Setup cost ($)	Production cost/ unit ($)	Capacity (units)
1	300	2	650
2	100	10	850
3	200	5	1250

Formulate the problem as an ILP and find the optimum solution.

(c) (i) Solve the ILP by cutting plane; maximize $z = 7x_1 + 10x_2$ subject to $-x_1 + 3x_2 \le 6$; $7x_1 + x_2 \le 35$; $x_1, x_2 \ge 0$ and integer.

Or

(ii) Develop the B and B tree for the following problems. For convenience, always select x_1 as the branching variable at node 0. Maximize $z = 3x_1 + 2x_2$ subject to $2x_1 + 5x_2 \le 18$; $4x_1 + 2x_2 \le 18$, $x_1, x_2 \ge 0$ and integer.

4. (a) If $f(x_1, x_2, x_3) = x_1 + 2x_3 + x_2x_3 - x_1^2 - x_2^2 - x_3^2$, show that H / x_0 is negative definite.

(b) (i) Derive the Jacobian matrix and control matrix.

Or

(ii) Determine the extreme points of the following function $f(x) = x^4 + x^2$.

(c) (i) Solve the function $g(x) = (3x - 2)^2 (2x - 3)^2$ by the Newton-Raphson method.

Or

(ii) Find the optimal solution to the problem minimize $f(x) = x_1^2 + 2x_2^2 + 10x_3^2$ subject to $g_1(x) = x_1 + x_2^2 + x_3 - 5 = 0$; $g_2(x) = x_1 + 5x_2 + x_3 - 7 = 0$. Suppose that $g_1(x) = 0.01$ and $g_2(x) = 0.02$ Find the corresponding change in the optimal value of $f(x)$.

5. (a) Derive the necessary conditions in Quadratic programming.

(b) (i) Explain about direct search method.

Or

(ii) Show how the following problem can be made separable. Maximize $z = x_1x_2 + x_3 + x_1x_3$ subject to $x_1x_2 + x_2 + x_1x_3 \le 10$, $x_1, x_2, x_3 \ge 0$.

(c) (i) Solve: Maximize $f(x_1, x_2) = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$ with initial point $X_0 = (1,1)$.

Or

(ii) Consider the problem maximize $z = 6x_1 + 3x_2 - 4x_1x_2 - 2x_1^2 - 3x_2^2$. Subject to $x_1 + x_2 \le 1$; $2x_1 + 3x_2 \le 4$, $x_1, x_2 \ge 0$. Show that z is strictly concave and then solve the problem using the quadratic programming algorithm.