(7 pages)

S.No. 7338 **RNENS 8**

(For candidates admitted from 2006–2007 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2023.

Mathematics

NUMERICAL METHODS

Time : Three hours Maximum : 100 marks

Subdivisions A, B and C in each question carry 4, 6 and 10 marks respectively

1. (A) Describe Muller's method shortly.

(B) (i) Derive the formula for finding the complex zero of $f^*(z) = \dfrac{f(z)}{z - z_1}$.

Or

(ii) Explain multipoint iteration method.

(C) (i) Perform five iterations of the Muller method to find the root of the equation $f(x) = \cos x - xe^x = 0$. Computations are performed using the initial approximations $x_0 = -1.0$, $x_2 = 0.0$ and $x_2 = 1.0$.

Or

(ii) Find all the roots of the polynomial $x^3 - 6x^2 + 11x - 6 = 0$ using the Newton-Raphson method.

2. (A) Define partial pivoting.

(B) (i) Solve the equations

$$x_1 + x_2 + x_3 = 6$$
$$3x_1 + (3+\epsilon)x_2 + 4x_3 = 20$$
$$2x_1 + x_2 + 3x_3 = 13$$

using the Gauss elimination method, where ϵ is small such that $1 \pm \epsilon^2 \approx 1$.

Or

(ii) Find the inverse of the coefficient matrix of the system

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \\ 4 \end{bmatrix}$$

by the Gauss-Jordan method with partial pivoting and hence solve the system.

(C) (i) Consider the equations

$$2x_1 - x_2 + 0x_3 = 7$$

$$-x_1 + 2x_2 - x_3 = 1$$

$$0x_1 - x_2 + 2x_3 = 1$$

Use the Gauss–Seidel iterative method and perform three iterations.

Or

(ii) State and prove Brauer's theorem.

3. (A) (i) Define interpolating polynomial.

(ii) Obtain polynomial approximation $P(x)$ to $f(x) = e^{-x}$ using the Taylor's expansion about $x_0 = 0$ and determine x when the error in $P(x)$ obtained from the first four terms only is to be less than 10^{-6} after rounding.

(B) (i) Using $\sin(0.1) = 0.09983$ and $\sin(0.2) = 0.19867$, find an approximate value of $\sin(0.15)$ by Lagrange interpolation. Obtain a bound on the truncation error.

Or

(ii) Explain about piecewise linear interpolation.

(C) (i) Construct the difference table for the sequence of values $f(x) = (0, 0, 0, \in, 0, 0, 0)$ where $\in$ is an error. Also show that (1) the error spreads and increases in magnitude as the order of the differences is increased, (2) the errors in each column have binomial coefficients.

Or

(ii) Given the following values of $f(x)$ and $f'(x)$

x	$f(x)$	$f'(x)$
–1	1	–5
0	1	1
1	3	7

estimate the values of $f(-0.5)$ and $f(0.5)$ using the Hermite interpolation.

4. (A) (i) Define weight function.

(ii) Define weights of the Quadrature formula.

(B) (i) Describe about the Richardson's extrapolation method.

Or

(ii) Evaluate the integral

$$I = \int_0^1 \frac{dx}{1+x}$$

using Gauss-Legendre three point formula.

(C) (i) Find the approximate value of

$$I = \int_0^1 \frac{dx}{1+x} \text{ using}$$

(1) trapezoidal rule and

(2) Simpson's rule. Obtain a bound for the errors. The exact value of $I = \log 2 = 0.693147$ correct to six decimal places.

Or

(ii) Evaluate the integral

$$I = \int_0^1 \frac{dx}{1+x} \text{ using}$$

(1) composite trapezoidal rule

(2) composite Simpson's rule, with 2, 4 and 8 equal subintervals.

5. (A) (i) Define increment function.

(ii) Given the initial value problem $u' = t^2 + u^2$, $u(0) = 0$. Determine the first three non-zero terms in the Taylor series for $u(t)$ and hence obtain the value for $u(1)$. Also determine t, the error in $u(t)$ obtained from the first two non–zero terms is to be less than 10^{-6} after rounding.

(B) (i) Solve the system of equations

$$u' = -3u + 2v,\ u(0) = 0$$

$$v' = 3u - 4v,\ v(0) = \frac{1}{2}$$

with $h = 0.2$ on the interval [0, 1]. Use the Euler—Cauchy method.

Or

(ii) Given $\sigma(\zeta) = \left(23\zeta^2 - 16\zeta + 5\right)/12$; find $\rho(\zeta)$ and write down the explicit linear multistep method.

(C) (i) Find the solution of the two dimensional heat conduction equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ subject to the initial condition $u(x, y, 0) = \sin \pi x \sin \pi y$, $0 \le x, y \le 1$ and the boundary conditions $u = 0$, on the boundaries, $t \ge 0$ using the explicit method with $h = \frac{1}{3}$ and $\lambda = \frac{1}{8}$. Integrate upto two time levels.

Or

(ii) Find the solution of the two dimensional heat conduction equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial^2 x} + \frac{\partial^2 u}{\partial y^2}$

subject to the initial condition $u(x, y, 0) = \sin \pi x \sin \pi y, 0 \le x, y \le 1$ and the boundary conditions $u = 0$, on the boundary for $t \ge 0$ using the Peaceman-Rachford ADI method. Assume $h = \frac{1}{4}$, $\lambda = \frac{1}{8}$ and integrate for one time step.
