(6 pages)

S.No. 7335 **RNENS 5**

(For candidates admitted from 2006–2007 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2023.

Mathematics

ORDINARY AND PARTIAL DIFFERENTIAL EQUATIONS

Time : Three hours Maximum : 100 marks

Answer ALL questions.

Subdivisions (a), (b) and (c) in each questions carry 4, 6 and 10 marks respectively.

1. (a) (i) Show that e^{x} and e^{-x} are linearly independent solutions of $y'' - y = 0$ on any interval.

(ii) Find a particular solution of $y'' - y' - 6y = e^{-x}$.

(b) (i) If $y_1(x)$ and $y_2(x)$ are any two solutions of $y'' + P(x)y' + Q(x)y = 0$ on $[a,b]$, then show that their Wronskian $W = W(y_1, y_2)$ is either identically zero or never zero on $[a,b]$.

Or

(ii) Obtain a power series solution of the form $\sum a_n x^n$ for the difference equation $y' = 2xy$.

(c) (i) Derive hypergeometric series.

Or

(ii) Find the general solution of Legendre's equation in terms of power series in x.

2. (a) (i) Prove that $\Gamma(p+1) = p\Gamma(p)$.

(ii) Write the Bessel function of the second kind.

(b) (i) Prove that for the n^{th} Legendre polynomial $P_n(x) = \dfrac{1}{2^n n!}\dfrac{d^n}{dx^n}(x^2 - 1)^n$.

Or

(ii) Prove that $J_{\frac{1}{2}}(x) = \sqrt{\dfrac{2}{\pi x}}\sin x$.

(c) (i) Derive the Bessel function of the first kind of order P.

Or

(ii) Find the general solution of the system

$$\begin{cases} \dfrac{dx}{dt} = 3x - 4y \\ \dfrac{dy}{dt} = x - y \end{cases}$$

3. (a) (i) Determine whether the function $x^2 - xy - y^2$ is positive definite, negative definite, or neither.

(ii) Find the critical points of the nonlinear system

$$\begin{cases} \dfrac{dx}{dt} = -x \\ \dfrac{dy}{dt} = 2x^2y^2 \end{cases}$$

(b) (i) Show that (0, 0) is an asymptotically stable critical point for the following system

$$\begin{cases} \dfrac{dx}{dt} = -3x^3 - y \\ \dfrac{dy}{dt} = x^5 - 2y^3 \end{cases}$$

Or

(ii) Find the exact solution of the initial value problem $y' = x + y, y(0) = 1$.

(c) (i) State and prove Picard's theorem.

Or

(ii) Describe geometrically about the four main types of critical points.

4. (a) (i) Find the complete integral of the partial differential equation $z = px + qy + \log pq$.

(ii) State the necessary and sufficient conditions for the integrebility of the equation $dz = \phi(x, y, z)dx + \psi(x, y, z)dy$

(b) (i) Find the general solution of $x(y^2 - z^2)p - y(z^2 + x^2)q = (x^2 + y^2)z$.

Or

(ii) Find a complete integral of the partial differential equation $f = x^2p^2 + y^2q^2 - 4 = 0$.

(c) (i) State and prove necessary and sufficient condition that the Pfaffian differential equation $\vec{X}.d\vec{r} = P(x,y,z)dx + Q(x,y,z)$

$dy + R(x,y,z)dz = 0$ be integrable.

Or

(ii) Find the complete integral of the equation $(p^2+q^2)x = pz$ and the integral surface containing the curve $C: x_0 = 0, y_0 = s^2, z_0 = 2s$.

5. (a) (i) Write the solution describing the vibrations of a semi-infinite string.

(ii) Classify the partial differential equation $xu_{xx} + 2\sqrt{xy}u_{xy} + yu_{yy} - u_x = 0$.

(b) (i) Reduce the equation

$$y^2u_{xx} - 2xyu_{xy} + x^2u_{yy} = \frac{y^2}{x}u_x + \frac{x^2}{y}u_y$$ to

a canonical form and solve it.

Or

(ii) Obtain the d'Alembert's solution which describes the vibrations of an infinite string.

(c) (i) Reduce the following equations to the canonical form:

(1) $u_{xx} + x^2u_{yy} = 0$

(2) $(n-1)^2u_{xx} - y^{2n}u_{yy} = ny^{2n-1}u_y$.

Or

(ii) Describe about the properties of characteristics.