(6 pages)

S.No. 6867 **P 22 MACC 13**

(For candidates admitted from 2022–2023 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2023.

Mathematics

ORDINARY DIFFERENTIAL EQUATIONS

Time : Three hours Maximum : 75 marks

PART A — (20 Marks)

Answer ALL questions.

I. (A) Choose the best answer: $(5 \times 1 = 5)$

1. If two functions $f(x)$ and $g(x)$ are defined on the interval $[a,b]$ and have the property that one is a constant multiple of other, then they are said to be _____

(a) linearly independent

(b) lineraly dependent

(c) such function never exists

(d) none of these

2. For the differential equation $y''+\frac{1}{x^2}y'-\frac{1}{x^3}y=0$, $x = 0$ is

(a) Regular Singular point

(b) Irregular singular point

(c) both (a) and (b)

(d) None of these

3. In Picard's theorem the defined inital value problem has _____

(a) No solution

(b) Only one solution

(c) such theorem doesn't exists

(d) none of these

4. If $y_1(x)$ and $y_2(x)$ are two linearly independent solutions of $y''+P(x)y'+Q(x)y = 0$, then the zeros of these functions are distinct and occur alternatively, then $y_1(x)$ vanishes exactly once between _____

(a) any zeros of y_2

(b) any two successive zeroes of $y_2(x)$

(c) any alternate zeros of $y_2(x)$

(d) none

5. If the roots m_1 and m_2 are real, distinct and of opposite signs, then the critical point is called as

(a) nodes (b) saddle point

(c) spiral (d) none of these

(B) Fill in the blanks $(5 \times 1 = 5)$

6. The function e^x is analytic at _____

7. Write down the Rodrigues formula _____.

8. Write down the general solution of the system $\frac{dx}{dt} = a_1x + b_1y$; $\frac{dy}{dt} = a_2x + b_2y$, where a_1, b_1, a_2, b_2 are constants, when it has distinct real roots _____

9. Write down the Bernoulli's solution for wave equation _____.

10. The critical point (0,0) of the linear system is stable _____

II. Answer ALL questions $(5 \times 2 = 10)$

11. Define non-homogenous differential equation.

12. Write the n-th term of the Legendre polynomial.

13. Define singular point.

14. Write D-Almbert's solution for wave equation.

15. Write the Van-der polar equation.

PART B — $(5 \times 5 = 25)$

Answer ALL questions Choosing either (a) or (b).

16. (a) Solve: $x^2y'' + 2xy' - 2y = 0$

Or

(b) Find the general solution of $y'' + y = 0$ when $y(0) = 2$ and $y'(0) = 3$.

17. (a) State and prove minimax property.

Or

(b) Write a short note on hypergeometric equation.

18. (a) State and prove Picards theorem.

Or

(b) Find the general solution of $\frac{dx}{dt} = x + y$ and $\frac{dy}{dt} = 4x - 2y$.

19. (a) Find the particular solution of $y''-3y'+2y=ex^x$.

Or

(b) Find the particular solution of $y'''-2y''+y=x^4+2x+5$

20. (a) Prove that, the function $E(X,Y)=ax^2+bxy+cy^2$ positive definite if and only if a > 0, $b^2-4ac<0$ and is negative definite if and only if a > 0, $b^2-4ac<0$.

Or

(b) Discuss the behaviour of the pendulum $\frac{d^2x}{dt^2}+\frac{c}{m}\frac{dx}{dt}+\frac{g}{a}\sin x=0$.

PART C — (3 × 10 = 30)

Answer any THREE questions.

21. Prove that, if y_1 and y_2 are any two solution of the equation $y''+P(x)y'(x)+Q(x)y=0$ on $[a,b]$ and their wronskian W is either identically zero or never zero on $[a,b]$

22. Explain in detail about the Bessel's formula.

23. Find the general solution of $\frac{dx}{dt}=3x-4y$ and $\frac{dy}{dt}=x-y$.

24. Discuss the Bernoulli's solution of the Wave equation.

25. Explain borderline case in detail.