(6 pages)

S.No. 6980 **P 22 PYCC 12**

(For candidates admitted from 2022-23 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2023.

Physics

MATHEMATICAL PHYSICS

Time : Three hours Maximum : 75 marks

SECTION A — (20 marks)

Answer ALL questions.

I. (A) Multiple choice questions: $(5 \times 1 = 5)$

1. If $\vec{a} \cdot (\vec{b} \times \vec{c}) = 0$, then $\vec{a}, \vec{b}\ \vec{c}$ are
   (a) Coplanar vectors
   (b) Collinear vectors
   (c) Rotational vectors
   (d) None of the above

2. If the determinant of the matrix is zero, then the matrix is known as
   (a) Unitary matrix
   (b) Hermitian matrix
   (c) Singular matrix
   (d) Orthogonal matrix

3. A differential equation together with a set of additional constraints, called the ______ conditions.
   (a) Initial value
   (b) Boundary
   (c) Ordinary
   (d) None of the above

4. If $P_n(x)$ is the solution of Legendre's solution, then $P_1(x) =$ ______.
   (a) 1 (b) $\frac{3x^2 - 1}{2}$
   (c) 0 (d) x

5. Let A and B are two events in a probability, then $P(A \text{ or } B) = P(A) + P(B)$ ______.
   (a) $-P(A \text{ and } B)$
   (b) $+P(A \text{ and } B)$
   (c) $+P(A)$
   (d) $-P(A \text{ or } B)$

(B) Fill in the blanks: (5 × 1 = 5)

6. If $\vec{F}$ represents the variable force acting on a particle along arc AB, then the total work done = ________.

7. A square matrix is called skew symmetric matrix, if A^1 = ________.

8. A differential equation involving derivatives with respect to ________ independent variable is called an ordinary differential equation.

9. The indicial equation of Legendre's function gives the roots m = ________ and m = ________.

10. In the case of dependent events the multiplication law of probability is $P(A \text{ and } B) = P(A) \times$ ________.

II. Answer the following questions: (5 × 2 = 10)

11. What is surface integral?

12. Define unitary matrix.

13. What is Wronskian?

14. Write the Strum-Liouville equation.

15. Define statistical probability.

SECTION B — (5 × 5 = 25)

Answer ALL questions, choosing either (a) or (b).

16. (a) Evaluate $\int_c \vec{F}.\vec{dr}$ where $\vec{F} = x^2\hat{i} + xy\hat{j}$ and C is the boundary of the square in the plane $Z = 0$ and bounded by the lines $x = 0$, $y = 0$, $x = a$ and $y = a$.

Or

(b) State and Prove Green's theorem.

17. (a) Show that any square matrix can be expressed as the sum of two matrices, one symmetric and the other anti-symmetric.

Or

(b) Find the eigen value and corresponding eigen vectors of the matrix $A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$.

18. (a) Write a note on superposition principle.

Or

(b) By the elimination of the constant A and B obtain the differential equation of which $xy = Ae^x + Be^x + x^2$ is the solution.

19. (a) Derive the generating function of Legendre's differential equation.

Or

(b) Obtain the recurrence relations of Hermite differential equation.

20. (a) Write a note on probability distribution.

Or

(b) List the important aspects of Binomial distribution.

SECTION C — $(3 \times 10 = 30)$

Answer any THREE questions.

21. State and prove Stoke's theorem.

22. Find the eigen values, eigen vectors, the model matrix and diagonalise the matrix given below

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}.$$

23. Solve the differential equation

$$y \log y dx + (x - \log y) dy = 0.$$

24. Find the solution of Laguerre differential equation.

25. Derive the mean and variance of the Poisson distribution.