(6 pages)

S.No. 6846 **P 16 MAE 1 C**

(For candidates admitted from 2016–2021 batch)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2023.

Mathematics — Elective

FUZZY SETS AND THEIR APPLICATIONS

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20)

Answer ALL Questions

1. State the Third Decomposition Theorem.

2. Define height of a fuzzy set.

3. Give an example of a fuzzy complement that is continuous but not involutive.

4. State the First Characterization Theorem of Fuzzy Complements.

5. Define the division on Closed intervals.

6. Prove that $A = A.1 = 1.A$.

7. Define the standard complement of a fuzzy relations.

8. Define reflexive fuzzy relations and $\varepsilon-$ reflexive fuzzy relations.

9. Define objective function.

10. Define feasible set.

SECTION B — (5 × 5 = 25)

Answer ALL Questions, choosing either (a) or (b)

11. (a) Prove that a fuzzy set A on $\mathbb{R}$ is convex ii and only if $A(\lambda x_1 + (1-\lambda)x_2) \geq \min[A(x_1), A(x_2)]$ for all $x_1, x_2 \in \mathbb{R}$ and all $\lambda \in [0,1]$, where min denotes the minimum operator.

Or

(b) For $A, B \in F(X)$ and $\alpha \in [0,1]$, prove that $^{\alpha}(A \cap B) = {}^{\alpha}A \cap {}^{\alpha}B)$ and $^{\alpha}(A \cup B) = {}^{\alpha}A \cup {}^{\alpha}B$.

12. (a) Prove that every fuzzy complement has at most one equilibrium.

Or

(b) Prove that the standard fuzzy intersection is the only idempotent $t-$ norm.

13. (a) State and prove the distributive property of closed intervals.

Or

(b) Let A and B be two triangular fuzzy numbers defined as

$$A(x) = \begin{cases} 0, & \text{for } x \le -1 \text{ and } x > 3 \\ \dfrac{x+1}{2}, & \text{for } -1 < x \le 1 \\ \dfrac{3-x}{2}, & \text{for } 1 < x \le 3 \end{cases}$$ and

$$B(x) = \begin{cases} 0, & \text{for } x \le 1 \text{ and } x > 5 \\ \dfrac{x-1}{2}, & \text{for } 1 < x \le 3 \\ \dfrac{5-x}{2}, & \text{for } 3 < x \le 5, \end{cases}$$

Then compute $(A / B)(x)$.

14. (a) Let $X = 1,2,\ldots,10$ and $R(X, X)$ is the set of all $\langle x, y \rangle$ such that x and y have the same remainder when divided by 3. Show that the relation is reflexive, symmetric, and transitive. Also, find X / R.

Or

(b) Determine the transitive max-min closure $R_T(X, X)$ for a fuzzy relation R defined by the membership matrix given below :

$$R = \begin{bmatrix} 0.7 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0.4 & 0 & 0 \\ 0 & 0 & 0.8 & 0 \end{bmatrix}$$

15. (a) Write down the steps involved in fuzzy individual decision making.

Or

(b) Solve the following fuzzy linear programming problem:

$$\max z = 5x_1 + 4x_2$$

$$s.t.\ \langle 4,2,1 \rangle x_1 + \langle 5,3,1 \rangle x_2 \le \langle 24,5,8 \rangle$$

$$\langle 4,1,2 \rangle x_1 + \langle 1,0.5,1 \rangle x_2 \le \langle 12,6,3 \rangle$$

$$x_1, x_1 \ge 0$$

SECTION C — (3 × 10 = 30)

Answer any THREE questions

16. State and prove First and Second Decomposition theorem.

[P.T.O.]

17. Let u_w denote the class of Yager t – conorm defined by $u_w(a,b) = \min\left\{1, (a^w + b^w)^{\frac{1}{w}}\right\}$ where $w > 0$. Then, prove that $\max(a, b) \le u_w(a,b)$ for all $a, b \in [0, 1]$.

18. Let A and B be two continuous fuzzy numbers. Then, show that the fuzzy set $A + B$ is a continuous fuzzy number.

19. Consider a fuzzy relation $R(X, X)$ defined on $X = N_9$ by the following membership matrix:

$$\begin{array}{c|c} & \begin{array}{ccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{array} & \begin{pmatrix} 1 & 0.8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.8 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0.8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0.8 & 0.7 & 0.5 & 0 & 0 \\ 0 & 0 & 0.8 & 0.8 & 1 & 0.7 & 0.5 & 0.7 & 0 \\ 0 & 0 & 0 & 0.7 & 0.7 & 1 & 0.4 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0.5 & 0.4 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.7 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

Find the graph of the compatibility relation and all complete α – covers for the compatibility relation R.

20. Assume that a company makes two products. Product P_1 has a \$ 0.40 per unit profit and product P_2 has a \$ 0.30 per unit profit. Each unit of product P_1 requires twice as many as many labor hours as each product P_2. The total available labor hours are at least 500 hours per day, and may possibly be extended to 600 hours per day, due to special arrangements for overtime work. The supply of material is at least sufficient for 400 units of both products, P_1 and P_2, per day, but may possibly be extended to 500 units per day according to previous experience. How many units of products P_1 and P_2 should be made per day to maximize the total profit?
