S.No. 6844 **P 16 MAE 1 A**

(For candidates admitted from 2016–2021 batch)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2023.

Mathematics — Elective

ADVANCED PROBABILITY THEORY

Time : Three hours Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$)

Answer ALL questions.

1. Define monotone field.

2. Define discrete random variable.

3. Define negative binomial distribution.

4. Define almost sure event.

5. What is meant by joint distribution function?

6. Define dense.

7. Let x be a cauchy $r.v.$ with probability density function $f(x) = [\Pi(1+x^2)]^{-1}, -\infty < x < \infty,$ Find $E(x^+), E(x^-)$.

8. State the schwarz inequality.

9. Define converge in measure.

10. If $x_n \xrightarrow{p} x$ and $y_n \xrightarrow{p} y$, then show that $x_n + y_n \xrightarrow{p} x + y$.

SECTION B — $(5 \times 5 = 25)$

Answer ALL questions, choosing either (a) or (b).

11. (a) Show that every σ–field is a monotone field.

Or

(b) Show that x is a $r.v.$ iff $x^{-1}(\xi)\ C\ \mathcal{A}$, where ξ is any class of subsets of R, which gonerates B.

12. (a) Show that if $A_n \to A$, then $p(A_n) \to P(A)$.

Or

(b) Prove that $p(A \cup B) = p(A) + p(B) - p(AB)$.

13. (a) Show that $F = F_c + F_d$, where F_c is continuous and F_d is a step function.

Or

(b) Let F_D be a non–decreasing finite function defined on D, a dense subset of R. Define a function $F(x)$ on R by.

$$F(x) = \begin{cases} \inf\limits_{x_n > x} F_D(x_n), x_n \in D, x \in R \cap D^C \\ F_D(x), x \in D. \end{cases}$$

Show that $F(x)$ is a.d.f.

14. (a) Show that if $x \geq 0$ and is integrable, x can be infinite at most on a set of probability measure zero.

Or

(b) Show that if z is a complex $r.v.$, then $|Ez| \leq E|z|$.

15. (a) Show that $x_n \xrightarrow{a.s.} x \Rightarrow x_n \xrightarrow{p} x$.

Or

(b) Show that if $y \leq x_n$ and y integrable, then $E\, \underline{\lim}\, x_n \leq \underline{\lim}\, E_{x_n}$.

SECTION C — $(3 \times 10 = 30)$

Answer any THREE questions.

16. Show that any simple function x can be written in the form $x = \sum_{k=1}^{n} x_k I_{A_K}$, where x_k's are distinct numerical constants and $\{A_1, \ldots, A_n\}$ is a partition of Ω.

17. Explain about conditional probability measure.

18. State and prove Jordan decomposition Theorem.

19. State and prove Minkowski inequality.

20. Show that $x_n \xrightarrow{r} x \Rightarrow E|x_n|^r \to E|x|^r$.