PART C — (3 × 10 = 30)

Answer any THREE questions.

16. If 'n' is a function of x, y and z which satisfies the partial differential equation $(y-z)\frac{\partial u}{\partial x}+(z-x)\frac{\partial u}{\partial y}+(x-y)\frac{\partial u}{\partial z}=0$. Show that '$u$' contains x, y and z only in combinations $x+y+z$ and $x^2+y^2+z^2$.

17. Find the solution of the equation $z=\frac{1}{2}\left(p^2+q^2\right)+(p-x)(q-y)$ which passes through the x-axis.

18. Reduce the equation $\frac{\partial^2 z}{\partial x^2}+2\frac{\partial^2 z}{\partial x\partial y}+\frac{\partial^2 z}{\partial y^2}=0$ to canonical form and hence solve it.

19. Solve the equation:

$r+4s+t+rt-s^2=2$.

20. A uniform circular wire of radius 'a' charged with electricity of line density 'e' surrounds grounded can centric spherical conductor of radius 'c'. Determine the electrical charge density of any point on the conductor.

(For candidates admitted from 2016–2021 batch)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2023.

Mathematics

PARTIAL DIFFERENTIAL EQUATIONS

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20)

Answer ALL questions.

1. Eliminating the constants 'a' and 'b' from the equation $2x=(ax+y)^2+b$.

2. Define a complete integral of a partial differential equation.

3. Find the complete integral of the equation $z^2=pq\,xy$.

4. Write down the compatible condition for systems of the first order equations.

5. Find the particular integral of the equation $\left(D^2-D'\right)z=e^{x+y}$.

6. Prove that $F(D, D')e^{ax+by} = F(a, b)e^{ax+by}$.

7. State the inversion theorem for the method of integral transforms.

8. Write down the general formula for the solution of linear Hyperbolic equations.

9. Define the exterior Neumann problem.

10. Write down the formula for the spherical polar coordinates of the laplace's equations.

PART B — (5 × 5 = 25)

Answer ALL questions, choosing either (a) or (b).

11. (a) Eliminate the arbitrary function from the equation $f\left(x^2 + y^2 + z^2, z^2 - 2xy\right) = 0$.

Or

(b) Find the general solution of the partial differential equation $x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = (x + y)z$.

12. (a) Solve $p^2 x + q^2 y = z$ using charpit's method.

Or

(b) Find a complete integral of the equation $p^2 y\left(1 + x^2\right) = qx^2$.

13. (a) Solve the equation $\frac{\partial^4 z}{\partial x^4} + \frac{\partial^4 z}{\partial y^4} = 2\frac{\partial^4 z}{\partial x^2 \partial y^2}$.

Or

(b) Reduce the equation $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ to canonical form.

14. (a) Solve the equation $q^2 r - 2pqs + p^2 t = 0$.

Or

(b) Define the solution of the equation:

$\frac{\partial^2 x}{\partial x^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{\partial^2 x}{\partial z^2} = 0$ for the region $r \geq 0$, $z \geq 0$ satisfying the conditions:

(i) $v \to 0$ as $z \to \infty$ and as $r \to \propto$

(ii) $v = f(r)$ on $z = 0$, $r \geq 0$.

15. (a) Establish a necessary condition for the existence of the solution of the interior Neumann problem.

Or

(b) Show that the surfaces $x^2 + y^2 + z^2 = cx^{2/3}$ can form a family of equipotential surfaces and find the general form of the corresponding potential function.