(6 pages)

S.No. 6889 **P 22 MAE 3 C**

(For candidates admitted from 2022–2023 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2023.

Mathematics – Elective

COMBINATORICS

Time : Three hours Maximum : 75 marks

SECTION A — (20 marks)

Answer ALL questions.

I. Choose the best answer. $(5 \times 1 = 5)$

1. How many divisors does the number 1400 have?

   (a) 14 (b) 6

   (c) 3 (d) 24

2. $\binom{n}{0} - \binom{n}{1} + \binom{n}{2} \dots + (-1)^r \binom{n}{r} + \dots + (-1)^n \binom{n}{n} =$

   (a) 2^n (b) 0

   (c) 2^{-n} (d) $(1+x)^n$

3. The particular solution of $a_n = 2\, a_{n-1}{}^{+1}$ is

   (a) 1 (b) -1

   (c) $A2^n$ (d) $A2^{-n}$

4. $\binom{m+k}{k}\binom{m+j}{m+k} =$

   (a) $\binom{m+j}{k}\binom{j}{k}$ (b) $\binom{m+j}{m}\binom{j}{k}$

   (c) $\binom{m+j}{j}\binom{j}{k}$ (d) $\binom{m+k}{k}\binom{j}{k}$

5. Store enumerator =

   (a) $\sum_{r \in R} w(r)$ (b) $\sum_{r \in Z} w(r)$

   (c) $\sum_{r \in C} w(r)$ (d) $\sum_{r \notin R} w(r)$

(B) Fill in the blanks $(5 \times 1 = 5)$

6. $1 \times 1! + 2 \times 2! + 3 \times 3! + \dots + n \times n! =$ ________

7. $\binom{6}{0}^2 + \binom{6}{1}^2 + \dots + \binom{6}{6}^2 =$ ________

8. The solution of $a_n = a_{n-1} - a_{n-2}$ is ________

9. The number of dearangements of n objects $d_n =$ ________

10. In the group $(\{0, 1, 2, 3, 4\}, *)$, the in verse of 1 is

II. Answer ALL questions. $(5 \times 2 = 10)$

11. In how many ways can n people stand to form a ring?

12. What is the coefficient of the term x^{23} in $(1+x^5+x^9)^{100}$?

13. Solve an $2a_{n-1} - a_{n-2}$ with $a_1 = 2$ and $a_2 = 3$.

14. If $S_1 = 19$, $S_2 = 12$ and $S_3 = 3$, find e_1 and e_2.

15. Define group.

SECTION B — $(5 \times 5 = 25)$

Answer ALL questions

16. (a) In how many ways can $2n+1$ seats in a congress be divided among three parties so that the coalition of any two parties will ensure them or a majority?

Or

(b) Eleven scientists are working on a secret project. They wish to lock up the documents in a cabinet such that the cabinet can be opened if and only if six or more of the scientists are present. What is the smallest number of locks needed? What is the smallest number of keys to the locks each scientist must carry?

17. (a) Find the exponential enumerator for the number of ways to choose r or less objects from r distinct objects and distribute them into n distinct cells, with objects in the same cell ordered.

Or

(b) Show that $\binom{m}{r+1} + \sum_{k=0}^{n-1} \binom{m+k}{r} = \binom{m+n}{r+1}$.

18. (a) Find the number of n-digit binary sequences that have the pattern 010 occurring at the n^{th} digit.

Or

(b) Solve the difference equation $a_n + 2a_{n-1} = n+3$ with the boundary condition $a_0 = 3$.

19. (a) Find the number of permutation of the letters $\alpha, \alpha, \beta, \beta, \gamma$ and γ so that no α appears in the first and the second positions no β appears in the third position and no γ appears in the fifth and sixth positions.

Or

(b) Find the number of r-digit quaternary sequences in which each of the three digits 1, 2 and 3 appears at least once.

20. (a) Show that the function $\pi^{(i)}$ is a permutation of the set of function F_i.

Or

(b) Find all the possible ways of painting three distinct balls in solid colors when there are three kinds of paint available, an expensive kind of red paint, a cheap kind of red paint, and blue paint.

SECTION C — $(3 \times 10 = 30)$

Answer any THREE questions

21. For a given n, show that $C(n,k)$ is maximum when $K = \frac{n-1}{2}, \frac{n+1}{2}$ if n is odd $K = \frac{n}{2}$ if n is even.

22. Evaluate the sum $\sum_{i=0}^{r} \frac{r!}{(r-i+1)!(i+1)!}$.

23. Find the number of r - combinations of n distinct objects with unlimited repetitions.

24. Explain about the general formula.

25. State and prove the Polya's fundamental theorem.