(6 pages)

S.No. 6887 **P 22 MAE 3 A**

(For candidates admitted from 2022-2023 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2023.

Mathematics – Elective

INTEGRAL EQUATIONS AND CALCULUS OF VARIATIONS

Time : Three hours Maximum : 75 marks

PART A — (20 Marks)

Answer ALL questions.

I. (A) Choose the correct answer. (5 × 1 = 5)

1. A functional $v[y(x)]$ is called continuous if to a small change of ———— there corresponds a small change in the functional $v[y(x)]$.

   (a) $y(v)$ (b) $v(y)$

   (c) $v(x)$ (d) $y(x)$

2. If a ———— field is formed by a family of extremals of a certain variational problem, then it is called an extremal field.

   (a) extremal (b) central

   (c) extremum (d) minimal

3. The function $f(x_1, x_2, \ldots x_n)$ becomes a function $\Phi(x_{m+1}, x_{m+2}, \ldots x_n)$ only of the ———— variables.

   (a) n (b) m

   (c) $n+m$ (d) $n-m$

4. If $h(s)g(s)= f(s)+\lambda\int_a^b K(s,t), g(t)dt$, then what is the value of $h(x)$ in Fredholm integral equation of second kind?

   (a) 1 (b) 0

   (c) any positive value (d) any value

5. Fredholm's first theorem does not hold when λ is a root of the equation ————

   (a) $D(\lambda)=0$ (b) $D(\lambda)\neq 0$

   (c) $D(\lambda)=1$ (d) $D(\lambda)\neq 1$

(B) Fill in the blanks : (5 × 1 = 5)

6. If the kernel approaches infinity at one or more points within the range of integration, then the integral equation is called ————.

7. Resolvent kernel $\Gamma(s,t;\lambda)=$ ————.

8. The increment $\delta y =$ ————.

9. The slope of the tangent line $p(x,y)$ to the curve of the family $y = y(x, C)$ that passes through the point (x,y) is called the ———— of the field at the point (x,y).

10. ———— is the condition for Isoperimetric.

II. Answer the following questions: $(5 \times 2 = 10)$

11. Define separable kernel.

12. State Fredholm's third theorem.

13. Define geodesics.

14. State the transversality condition

15. Define holonomic constraints.

PART B — $(5 \times 5 = 25)$

Answer ALL questions.

16. (a) Solve the homogenous Fredholm integral equation $g(s) = \lambda \int_0^1 e^s e^t g(t)\,dt$.

Or

(b) Solve the integral equation $g(s) = f(s) + (1/\pi)\int_0^{2\pi} [\sin(s+t)]\, g(t)\,dt$.

17. (a) Solve the integral equation $g(s) = f(s) + \lambda \int_0^s e^{s-t} g(t)\,dt$.

Or

(b) Solve the integral equation $g(s) = s + \lambda \int_0^1 \left[st + (st)^{\frac{1}{2}} \right] g(t)\,dt$.

18. (a) Find the extremals of the functional $v[y(x), z(x)] = \int_0^{\frac{\pi}{2}} \left[y'^2 + z'^2 + 2yz \right] dx$, with $y(0) = 0$, $y\left(\frac{\pi}{2}\right) = 1$, $z(0) = 0$, $z\left(\frac{\pi}{2}\right) = -1$.

Or

(b) Given a system of particles with masses $m_i \, (i = 1, 2, \ldots, n)$ and coordinates (x_i, y_i, z_i) acted upon by forces $\overline{F}_l$, that possess the force function (potential) – U, which is dependent solely on the coordinates :

$$F_{ix} = -\frac{\partial U}{\partial x_i};\ F_{iy} = \frac{\partial U}{\partial y_i};\ F_{iz} - \frac{\partial U}{\partial z_i},$$

where, F_{ix}, F_{iy}, F_{i2} are the coordinates of the vector $\overline{F}_i$ acting on the point (x_i, y_i, z_i). Find the differential equations of motion of the system.

[P.T.O.]

19. (a) Test for an extremum of the functional $v = \int_{x_0}^{x_1}\left(y'^2 + z'^2 + 2yz\right)dx$ given that $y(0) = 0;\ z(0) = 0$ and the point (x_1, y_1, z_1) can move over the plane $x = x_1$.

Or

(b) If the Jacobi condition fulfilled for the extremal of the functional, $v = \int_0^a \left(y'^2 - y^2\right)dx$ then verify that passes through the points $A(0,0)$ and $B(a,0)$.

20. (a) Find the shortest distance between two points $A(x_0, y_0, z_0)$ and $B(x_1, y_1, z_1)$ on the surface $\varphi(x, y, z) = 0$.

Or

(b) Find the form of an absolutely flexible, non-extensible homogeneous rope of length 1 suspended at the points A and B.

PART C — $(3 \times 10 = 30)$

Answer any THREE questions.

21. Solve the integral equation $g(s) = f(s) + \lambda \int_0^1 (s + t) g(t)\, dt$ and find the eigen values.

22. Solve the integral equation $g(s) = 1 + \lambda \int_0^{\pi} [\sin(s + t) g(t)]\, dt$.

23. State and prove minimum-surface-of-revolution problem.

24. Write a summary on sufficient conditions for a minimum of the elementary functional.

25. Find the curve $y = y(x)$ of given length l, for which the area S of the curvilinear trapezoid is a maximum. Also, find the extreme functional of the curve.