(6 pages)

S.No. 6879 **P 22 MAE 2 C**

(For candidates admitted from 2022–2023 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2023.

Mathematics — Elective

STOCHASTIC PROCESSES

Time : Three hours Maximum : 75 marks

PART A — (20 marks)

I. (A) Choose the correct answer : (5 × 1 = 5)

1. The m-step transition probability is denoted as

(a) P_m (b) $P_{jk}^{(m)}$

(c) $P_{\overline{m}}$ (d) $P_{jk}^{(\overline{m})}$

2. A markov chain $\{x_n, n\geq 0\}$ with k-stats, where K is finite, is said to be ———— Markov chain

(a) Infinit (b) Finite

(c) Zero (d) None of the above

3. The state i is said to aperiodic if ————

(a) $di>1$ (b) $di<1$

(c) $di=1$ (d) $di\neq 1$

4. A state j is said to be persistent if ————

(a) $f_{jj}>1$ (b) $f_{jj}<1$

(c) $f_{jj}=1$ (d) $f_{jj}\neq 1$

5. The number $M_1(t)$ of occurrences not record is also a Poisson process with parameter ————

(a) λp (b) λq

(c) λ / p (d) $\underline{l/q}$

(B) Fill in the blanks : (5 × 1 = 5)

6. $E\{M(t)\}=$ ————

7. The renewal event E^* is said to be ———— if there exists an integer $(m>1)$ such that E^* can occur only at trials numbered m, $2m$,...

8. $E\left\{\sum_{i=1}^{N} X_i\right\}=$ ————

9. The p.g.f. $R_n(S)$ of y_n satisfies the recurrence relation $R_n(S)$ = ———————, P(S) being the p.g.f of the off spring distribution.

10. If the Markor chain (X_n) is also a blanching process, then the (bivariate) process $\{X_n, t_n\}$, $n=0,1,2,\ldots$ is known

II. Answer the following questions : (5 × 2 = 10)

11. Define transition matrix.

12. Define stationary distribution.

13. Write the Chapman-Kolmogorov equation.

14. Define mean recurrence time.

15. Write a modified schroder functional equation.

PART B — (5 × 5 = 25)

Answer ALL questions choosing either (a) or (b).

16. (a) Derive a Chapman - Kolmogorov equation.

Or

(b) Find the transition matrix for random walk between two barriers in general case.

17. (a) Find a state of a following transition matrix in a state space $S=\{1,2,3,4\}$.

$$P=\begin{pmatrix} \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1/2 & 1/2 \end{pmatrix}$$

Or

(b) Let the state j is persistent nonull, Prove that as $n \to \infty$

(i) $P_{jj}^{(nt)} \to t/m_{jj}$ when state j is periodic with period t; and

(ii) $P_{jj}^{(n)} \to 1/m_{jj}$, when j is aperiodic.

18. (a) State and prove the additive property of the Poisson process.

Or

(b) Prove that the p.g.f. of a non-Homogeneous process $\{N(t), t \geq 0\}$ given by $Q(s,t)=\exp\{m(t)(s-1)\}$ where $m(t)=\int_0^t \lambda(t)\,dx$ is the expectation of $N(t)$.

19. (a) Prove that the renewal function M satisfies the equation $M(t)=F(t)+\int_0^t M(t-x)\,dF(x)$.

Or

(b) Derive Wald's equation.

20. (a) For $r, n=0,1,2,\ldots$ Prove that $E\{X_{ntr}/X_n\}=X_n m^r$.

Or

(b) Llet $m=E(X_1)=\sum_{k=0}^{\infty} KPK$ and $r^2=\operatorname{var}(X_1)$,

Prove that $E(X_n)=m^n$ and

$$\operatorname{var}(X_n)=\frac{m^{n-1}(m^n-1)}{m-1}r^2 \text{ if } m\neq 1$$

$$= nr^2 \text{ if } m=1.$$

PART C — (3 × 10 = 30)

Answer any THREE questions.

21. Derive Markov-Bernoulli chain.

22. State and prove Ergodic theorem.

23. Using the postulates of Independence, Homogeneity, regularity of the Poisson process follows Poisson distribution with mean λt, Prove that $P_n(t)=\frac{e^{-\lambda t}(\lambda t)^n}{n!}$, $n=0,1,2,3\ldots$

24. State and prove elementary renewal theorem.

25. Let $m=1$, $\sigma^2<\infty$, Prove that

(a) $\lim\limits_{n\to\infty} \Pr\{x_n>0\}=\frac{2}{\sigma^2}$

(b) $\lim\limits_{n\to\infty} E\left\{\frac{x_n}{n}/x_n>0\right\}=\frac{\sigma^2}{2}$

(c) $\lim\limits_{n\to\infty} \Pr\left\{\frac{x_n}{n}>M/X_n>0\right\} \exp\left(\frac{2\mu}{\sigma^2}\right)$, $M\geq 0$.

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