(7 pages)

S.No. 6877 **P 22 MAE 2 A**

(For candidates admitted from 2022–2023 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2023.

Mathematics — Elective

OPTIMIZATION TECHNIQUES

Time : Three hours Maximum : 75 marks

SECTION A — (20 marks)

Answer ALL questions.

I. (A) Choose the correct answer. $(5 \times 1 = 5)$

1. Branch and Bound method divides the feasible solution space into smaller parts by

   (a) enumerating (b) branching

   (c) bounding (d) all of the above

2. Dynamic programming problem

   (a) cannot be dealt with non-linear constraints

   (b) can be solved by simplex method

   (c) is solved starting from the initial stage to the next till the final stage is reached

   (d) can be solved by using Monte-Carlo method

3. In goal programming problem, a constraint having under-achieved variable is expressed as

   (a) a less than or equal to constraint

   (b) a greater than or equal to constraint

   (c) an equality constraint

   (d) all of the above

4. In a non-linear programming problem,

   (a) the objective function is non-linear

   (b) one or more of the constraints have non-linear relationship

   (c) both (a) and (b)

   (d) none of the above

5. Which of the following methods of solving quadratic programming problem is based on modified simplex method?

   (a) Wolfe's method

   (b) Beale method

   (c) Frank-Wolfe method

   (d) Fletcher's method

(B) Fill in the blanks. $(5 \times 1 = 5)$

6. An integer programming problem is a linear programming problem with decision variables restricted to ———— values.

7. When a positive quantity k is divided into five parts, the maximum value of their product is ————.

8. Since both the under-achievement and over-achievement goal cannot be achieved simultaneously, one or both of the deviational variables must be ———— in the solution.

9. The relative minimum of the function $f(x_1, x_2) = x_1^3 + x_2^3 - 3x_1 - 12x_2 + 25$ is at the point ————.

10. Quadratic programming is concerned with the non-linear programming problem of optimizing the quadratic objective function subject to ———— inequality constraints.

II. Answer the following questions. $(5 \times 2 = 10)$

11. Define zero-one integer programming problem.

12. State the principle of optimality.

13. Define goal programming problem.

14. State the Kuhn-Tucker conditions for the optimal solution of general non-linear programming problem.

15. Define convex programming problem.

SECTION B — $(5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Illustrate the iterative procedure for the solution of a mixed integer linear programming problem fractional cut method.

Or

(b) Use branch and bound method to solve the following linear programming problem :

Minimize $z = 4x_1 + 3x_2$

Subject to the constraints :

$5x_1 + 3x_2 \geq 30$, $x_1 \leq 4$, $x_2 \leq 6$

$x_1, x_2 \geq 0$ and are integers.

17. (a) Describe the dynamic programming algorithm.

Or

(b) Use dynamic programming to solve the following problem :

Minimize $z = y_1^2 + y_2^2 + y_3^2$

Subject to the constraints :

$y_1 + y_2 + y_3 \geq 15$ and $y_1, y_2, y_3 \geq 0$.

18. (a) A firm produces two products. say X and Y. Product X sells for a net profit of Rs. 80 per unit, while product Y sells for a net profit of Rs. 40 per unit. The goal of the firm is to earn Rs. 900 in the next week. Also, the management want to achieve sales volume for the two products close to 17 and 15 respectively. Formulate this problem as a goal programming model.

Or

(b) Write the major steps in the simplex method for the linear goal programming problem.

19. (a) Obtain the set of necessary conditions for the non-linear programming problem :

Maximize $z = kx^{-1}y^{-2}m$

subject to the constraints :

$x^2 + y^2 - a^2 = 0$ with $x \geq 0, y \geq 0$;

and hence find the minimum value of z.

Or

(b) Obtain the necessary and sufficient conditions for the optimum solution of the following non-linear programming problem :

Minimize $z = f(x_1\ x_2) = 3e^{2x_1+1} + 2e^{x_2+5}$

subject to the constraints :

$x_1 + x_2 = 7,\ x_1,\ x_2 \geq 0.$

20. (a) Solve graphically the following non-linear programming problem :

Maximize $z = 2x_1 + 3x_2$

subject to the constraints :

$x_1\, x_2 \leq 8,\ x_1^2 + x_2^2 \leq 20,\ x_1,\ x_2 \geq 0$.

Verify that the Kuhn-Tucker conditions hold for the maxima you obtain.

Or

(b) Write the separable programming algorithm.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

21. Find the optimum integer solution to the following linear programming problem :

Maximize $z = x_1 + 4x_2$

subject to the constraints :

$2x_1 + 4x_2 \leq 7$, $5x_1 + 3x_2 \leq 15$;

$x_1,\ x_2 \geq 0$ and are integers.

22. Divide unity into n parts in such a way that the quantity $\sum_i p_i \log p_i$ is a minimum.

23. Solve the following linear goal programming problem graphically :

Find x_1 and x_2 so as to :

Minimize

$$z = G_1\left(d_3^+ + d_4^+\right) + G_2 d_1^+ + G_3 d_2^- + G_4\left(d_3^- + \frac{3}{2} d_4^-\right)$$

and satisfy the goals :

$G_1 : x_1 + x_2 + d_1^- + d_1^+ = 40$

$G_2 : x_1 + x_2 + d_2^- - d_2^+ = 100$

$G_3 : x_1 + d_3^- - d_3^+ = 30$

$G_4 : x_2 + d_4^- - d_4^+ = 15$

$x_1, d_i^-, d_i^+ \geq 0$, for all $i = 1, 2, 3, 4$.

The goals have been listed in order of priority.

24. Optimize $z = 2x_1 + 3x_2 - \left(x_1^2 + x_2^2 + x_3^2\right)$

subject to the constraints :

$x_1 + x_2 \leq 1,\ 2x_1 + 3x_2 \leq 6$ and $x_1 \geq 0, x_2 \geq 0$.

25. Use Wolfe's method to solve the quadratic programming problem :

Maximize $z = 2x_1 + 3x_2 - 2x_1^2$

subject to the constraints :

$x_1 - 4x_2 \leq 4,\ x_1 + x_2 \leq 2$

$x_1, x_2 \geq 0$.
