(6 pages)

S.No. 6866 **P 22 MACC 12**

(For candidates admitted from 2022–2023 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2023.

Mathematics

REAL ANALYSIS

Time : Three hours Maximum : 75 marks

SECTION A — (20 Marks)

Answer ALL questions.

I. (A) Multiple choice questions : $(5 \times 1 = 5)$

1. A finite set has ———— limit points.

 (a) ∞ (b) 2

 (c) 1 (d) no

2. Let f be real on (a, b). Then f is strictly monotonically increasing on (a, b) if $a < x < y < b$ implies

 (a) $f(x) \leq f(y)$ (b) $f(x) \geq f(y)$

 (c) $f(x) < f(y)$ (d) $f(x) > f(y)$

3. Every differentiable function is ————

 (a) discontinuous

 (b) bi-continuous

 (c) continuous

 (d) none of the above

4. $\int_a^b f(x)d\alpha(x)$ is called ————

 (a) Riemann integral

 (b) Complete integral

 (c) Riemann-Stieltjes integral

 (d) None of the above

5. There is a real ———— function on the real line which is nowhere differentiable.

 (a) continuous (b) differentiable

 (c) real (d) bounded

(B) Fill in the blanks : $(5 \times 1 = 5)$

6. Compact subset of metric spaces are ————

7. $\lim_{n \to \infty} \sqrt[n]{n} =$ ————

8. Monotonic functions have no discontinuities of the ————

9. $\alpha(x_i)-\alpha(x_{i-1})=$ ________

10. If $f_n(x)=n^2x(1-x^2)^n$ then $\int_0^1\left[\lim_{n\to\infty} f_n(x)\right]dx=$ ________

II. Answer the following questions : (5 × 2 = 10)

11. Define connected set with an example.

12. Give an example :

(a) The series which is absolutely convergent but not convergent.

(b) The series which is convergent but not absolutely convergent.

13. Define limit of a function.

14. Define unit step function.

15. State the Stone-Weierstrass theorem.

SECTION B — (5 × 5 = 25)

Answer ALL the questions, choosing either (a) or (b).

16. (a) State and prove Schwarz inequality.

Or

(b) Suppose $K\subset Y\subset X$. Then prove that K is compact relative to X iff K is compact relative to Y.

17. (a) Prove that the subsequential limits of a sequence $\{p_n\}$ is a metric space X form a closed subset of X.

Or

(b) If $\sum a_n=A$ and $\sum b_n=B$ then prove that $\sum(a_n+b_n)=A+B$ and $\sum ca_n=cA$, for any fixed c.

18. (a) Prove that the mapping f of a metric space X into a metric space Y is continuous on X iff $f^{-1}(V)$ is open in X for every open set V in Y.

Or

(b) Suppose f is a continuous mapping of $[a, b]$ into R^k and f is differentiable in (a, b). Then prove that there exists $x\in(a,b)$ such that $|f(b)-f(a)|\le(b-a)|f'(x)|$.

19. (a) If f is continuous on $[a,b]$ then prove that $f\in\mathcal{R}(\alpha)$ on $[a,b]$.

Or

(b) If γ' is continuous on $[a, b]$ then prove that γ is rectifiable and $\wedge(\gamma)=\int_a^b|\gamma'(t)|dt$.

20. (a) Suppose $f_n \to f$ uniformly on a set E in a metric space. Let x be a limit point of E, and suppose that $\lim_{t \to x} f_n(t) = a_n (n = 1, 2, 3, \ldots)$. Then prove that $[A_n]$ converges and $\lim_{t \to x} f(t) = \lim_{n \to \infty} A_n$.

Or

(b) Let $\mathscr{B}$ be the uniform closure of an algebra $\mathscr{A}$ of a bounded function. Then prove that $\mathscr{B}$ is a uniformly closed algebra.

SECTION C — (3 × 10 = 30)

Answer any THREE questions.

21. Prove that for every real $x > 0$ and every integer $n > 0$ there is one and only one positive real y such that $y^n = x$.

22. State and prove root test and ratio test.

23. State and prove Taylor's theorem.

24. Assume α increases monotonically and $\alpha' \in \mathscr{R}$ on $[a, b]$. Let f be a bounded real function on $[a, b]$. Then prove that $f \in \mathscr{R}(\alpha)$ iff $f\alpha' \in \mathscr{R}$. In that case $\int_a^b f\, d\alpha = \int_a^b f(x)\alpha'(x)\,dx$.

25. If K is compact, $f_n \in \mathscr{C}(k)$ for $n = 1, 2, 3, \ldots$ and $\{f_n\}$ is pointwise bounded and equicontinuous on K then prove the followings.

(a) $\{f_n\}$ is uniformly bounded on K.

(b) $\{f_n\}$ contains a uniformly convergent subsequence.