PART C — (3 × 10 = 30)

Answer any THREE questions.

16. Prove that a nonempty open set in the plane is connected if and only if any two of its points can be joined by a polygon which lies in the set.

17. If the function $f(z)$ is analytic on R, then prove that $\int_{\partial R} f(z)dz = 0$.

18. If $f(z)$ is defined and continuous on a closed bonded set E and analytic on the interior of E. then show that the maximum of $|f(z)|$ on E is assumed on the boundary of E.

19. Show that a region Ω is simply connected if and only if $n(\gamma, a) = 0$ for all cycles γ in Ω and all points a which do not belong to Ω.

20. If $f(z)$ is analytic in the region Ω containing Z_0 then show that the representation $f(z) = f(z_0) + \frac{f'(z_0)}{1!}(z - z_0) + \ldots + \frac{f^{(n)}(z_0)}{n!}(z - z_0)^n$ is valid in the largest open disk of center z_0 contained in Ω

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(For candidates admitted from 2016–2021 Batch)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2023.

Mathematics

COMPLEX ANALYSIS

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20)

Answer ALL questions.

1. Define Connected.

2. State cross ratio.

3. When will we say that the arc is rectifiable?

4. State Liouville's theorem.

5. When we said $f(z)$ is identically equal to $g(z)$?

6. State maximum principle.

7. Define simply connected.

8. When the cycle γ is said to bound the region Ω?.

9. Write Laplace's equation.

10. Write the Poisson integral of any piecewise continuous function.

PART B — (5 × 5 = 25)

Answer ALL questions, Choosing either (a) or (b).

11. (a) Prove that under a continuous mapping the image of every compact set is compact, and consequently closed.

Or

(b) Prove that an analytic function in a region Ω whose derivative vanishes identically must reduce to a constant.

12. (a) Prove that the line integral $\int_{\gamma} pdx + qdy$, defined in Ω, depends only on the end points of γ if and only if there exists a function U (x,y) in Ω with the partial derivatives $\frac{\partial U}{\partial x} = p, \frac{\partial u}{\partial y} = q$.

Or

(b) Let $f(z)$ be analytic on the set R' obtained from a rectangle R by omitting a finite number of interior points ζ_j. If it is true that $\lim_{z \to \zeta_j} (z - \zeta_j) f(z) = 0$ for all j then prove that $\int_{\partial R} f(z) dz = 0$

13. (a) Prove that a nonconstant analytic function maps open sets onto open sets.

Or

(b) Show that an analytic function comes arbitrarily close to any complex value in every neighborhood of an essential singularity.

14. (a) If $f(z)$ is analytic in a simply connected region Ω, then prove that $\int_{\gamma} f(z) dz = 0$ holds for all cycles γ in Ω.

Or

(b) State and prove Rouche's theorem.

15. (a) Derive Poisson's Formula.

Or

(b) If the functions $f_n(z)$ are analytic and not equal to 0 in a region Ω, and if $f_n(z)$ converges to $f(z)$, uniformly on every compact subset of Ω, then prove that $f(z)$ is either identically zero or never equal to zero in Ω.