Gravity On A Small Scale

Gravity?...Rocks?...Huh?

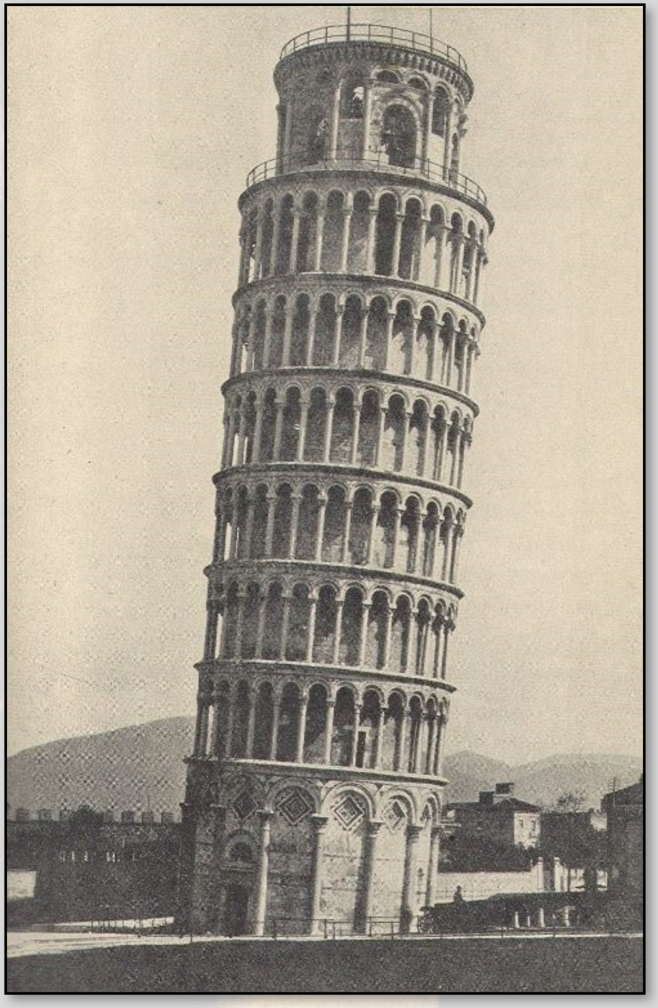
- We all feel and see gravity's effects:
  - Weight (i.e. a force) & downward acceleration

- The gravitational force depends on the rocks below us
  - If the rocks beneath the surface change with location, we expect that the gravitational force we experience will also change.
  - If we measure small changes in gravitational forces we can use this information to make inferences about the rocks below us.

- This is the essence of gravity surveying

Galileo Galilei (1564-1642)

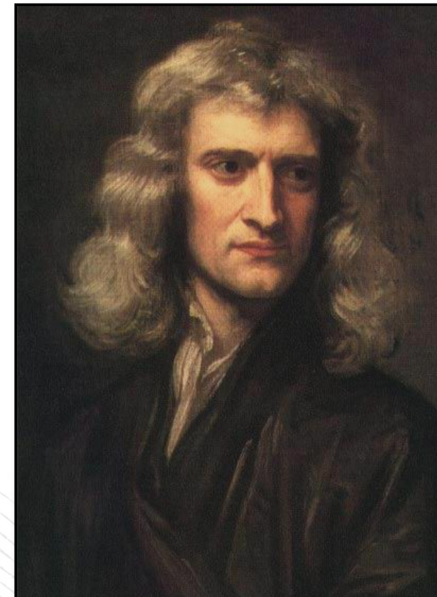
- Not just an astronomer
  - Realized that projectiles travel in parabolic paths
    - Motion is a vector quantity which can be separated into horizontal and vertical components
  - Described and quantified pendulum motion
- Pioneering work in gravity
  - Realized that gravity causes bodies to fall towards Earth
    - All bodies fall at the same rate
      - Leaning Tower of Pisa experiments?
    - Realized the relationship between distance fallen and time



$$d_z = \frac{1}{2} g t^2$$

So, basically, Galileo told us how gravity works, but not why.

Newton's Law of Gravitation



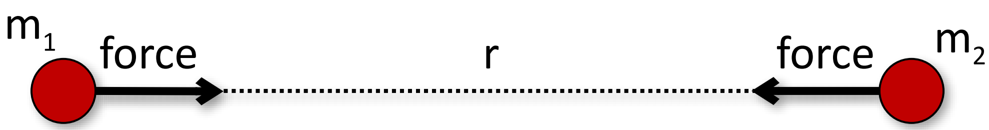
- Sir Isaac Newton (1642-1727):
  - Great scientific mind
    - Co-developed calculus (Gottfried Leibniz)
    - Derived laws of motion
    - Realized that the planets stay in orbit for the same reason that apples fall to the ground:
      - **Universal Gravitation**

$$F = G\frac{m_1 m_2}{r^2}$$

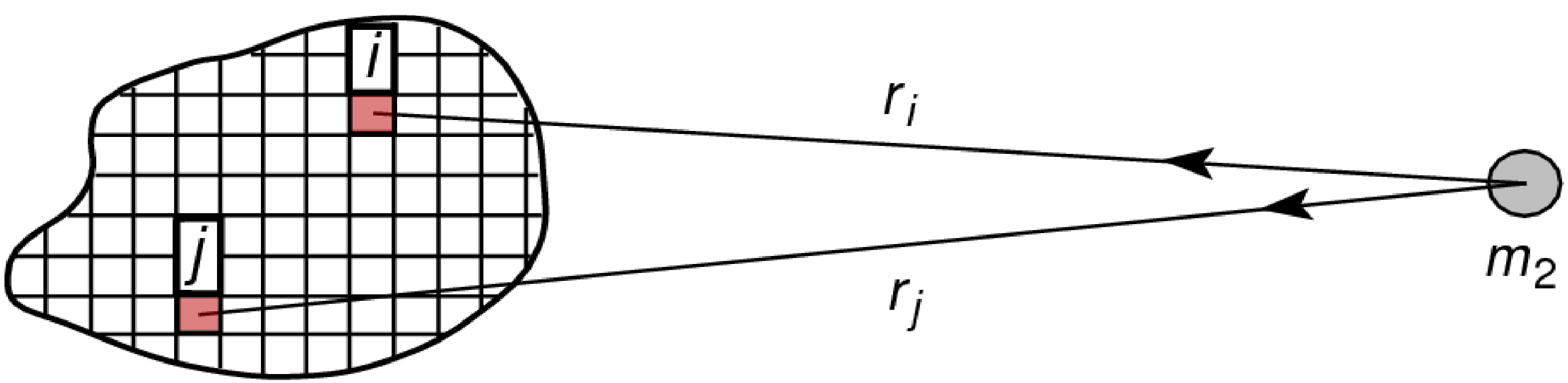
  - The gravitation force exerted by two massed bodies (m_1, m_2):
    - Is proportional to the two masses
    - Is inversely proportional to the square of the distance between the two bodies
    - 'G' is the universal gravitational constant $G = 6.672 \times 10^{-8}\ m^3 \cdot Mg^{-1} \cdot s^{-2}$

Newton's Law of Gravitation

- Newton's law applies to point sources of mass



  - Earth isn't a point source
  - Non-point sources are treated as the sum of the forces of many small parts of the body
    - Because force is a vector, the vector sum must be calculated
    - This can become computationally intense for odd shapes

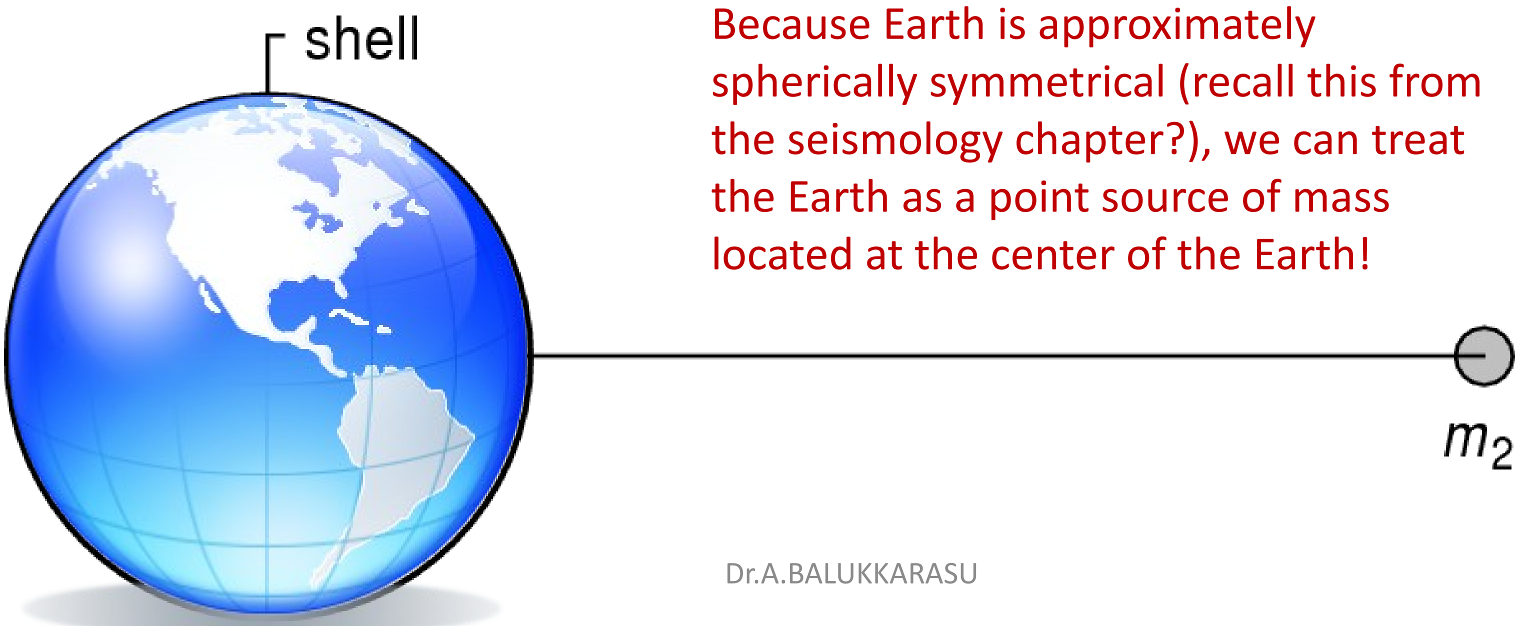


  - For some simple geometric shapes the result is simple…

Newton’s Law of Gravitation

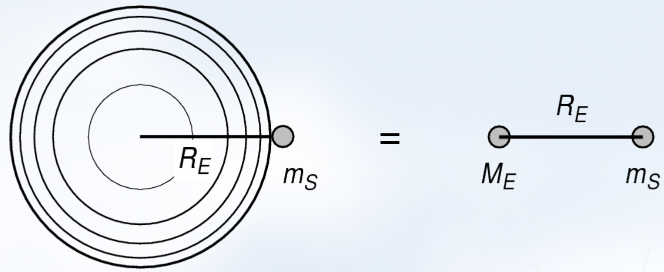
- For the gravitational attraction due to a hollow shell or uniform sphere:
  - The force is the same as that of a point source of the same mass located at the center of the sphere
  - This is true only outside the sphere.
    - At the center, the gravitational force must be zero (vectors all cancel due to symmetry)

Because Earth is approximately spherically symmetrical (recall this from the seismology chapter?), we can treat the Earth as a point source of mass located at the center of the Earth!



Acceleration Due to Gravity

- Because Earth acts like a point source of mass…



  – We can use Newton's universal gravitation eq to calculate gravitational force on a small mass, m_s, on the surface of the Earth

$$F = G\frac{M_E m_s}{R_E^{\ 2}}$$

To consider acceleration…

  – Insert Newton's second law to solve for the force on the small body…

$$F = m_s g = G\frac{M_E m_s}{R_E^{\ 2}}$$

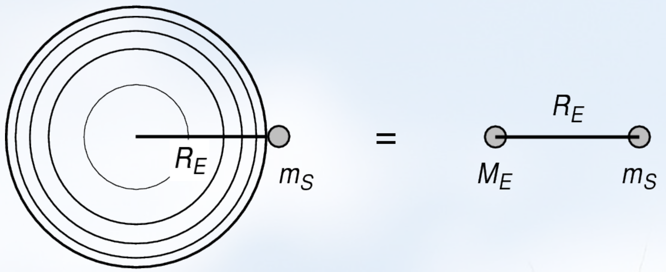
  – The small body's mass cancels out, and we are left with the equation for the acceleration due to gravity, g.

$$g = G\frac{M_E}{R_E^{\ 2}}$$

  – This relationship implies that all bodies on earth's surface should experience the same falling acceleration

Acceleration Due to Gravity

- Because Earth acts like a point source of mass…



So long as the mass of the Earth is spherically-symmetrical…
  - The acceleration due to gravity is independent of an object's properties (mass, etc…)
  - So far as gravity is concerned, the Earth could be:
    - Hollow in the middle
    - Made of cheese and gold

    (So long as the mass was held constant)

- We know that Earth isn't exactly spherically-symmetrical…
  - E.g. lateral changes in rock type
  - Topography
  - Mantle plumes
- We should therefore be able to measure small changes in g and determine subsurface rock properties that involve variations in mass.

The Mass of Earth

- Although the mass of Earth is not practical to measure, it can be easily calculate $$g = G\frac{M_E}{R_E^{\;2}}$$
  - G = universal gravitational constant = $6.672 \times 10^{-8}\ m^3 \cdot Mg^{-1} \cdot s^{-2}$
  - g = $9.81\ m \cdot s^{-2}$ (can be measured for falling objects)
  - R_E = can be measured from surveys or geometric observations
    - Eratosthenes (~200 B.C.) ~ 6370 km
  - $M_E = 5.97 \times 10^{21}$ Mg or 5.97×10^{24} kg
- This mass estimate implies that the average density of the Earth is ~ $5.5\ Mg \cdot m^{-3}$
  - Most rocks are 2-3 $Mg \cdot m^{-3}$
  - So, the interior of the Earth must be more dense than typical crustal rocks. Why?
    - Increasing lithostatic stress with depth squeezes rocks, increasing density
    - The core of the Earth is metallic (mostly iron $\rho \approx 7.8\ Mg \cdot m^{-3}$)

Densities of Common Rocks

- Rocks have a range of densities* (*your book gives more)
  - But in general, rock density does not widely vary
  - What does this say about typical gravity anomaly sizes?

Type	Rock	Density
Unconsolidated	Sand	1400-1650 kg/m^3
Sedimentary	Salt	2100-2600
	Limestone	2000-2700
	Shale	2000-2700
Igneous	Granite	2500-2800
	Basalt	2700-3000
Metamorphic	Quartzite	2600-2700
	Gneiss	2600-3000
Ore	Galena	7400-7600
	Pyrite	4900-5200
	Magnetite	4900-5300

Densities of Common Rocks

- Which has higher / lower density?
  - Surface / deep rocks
  - Weathered / unweathered rocks
  - Rocks in the phreatic / vadose zone

- Which rock types make good targets for gravity surveying?
  - Which don't
  - Why

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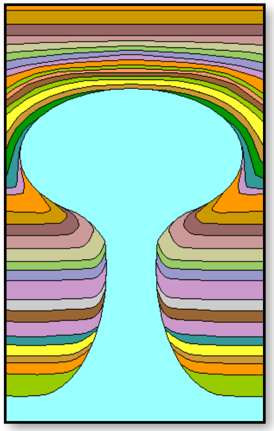
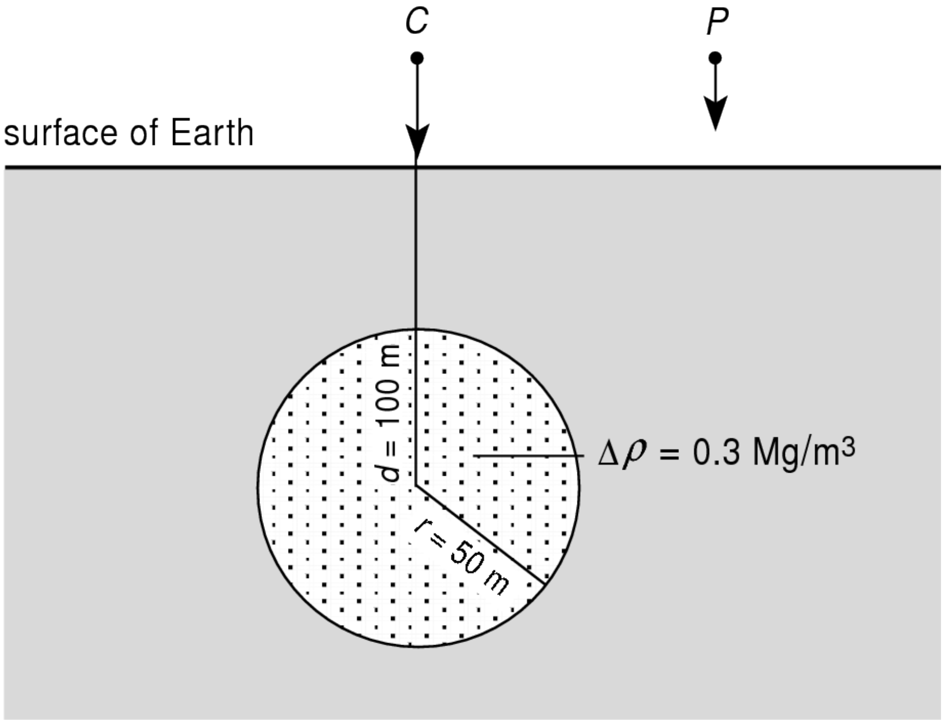
Size of Gravity Anomalies

- To better understand the size of a typical gravity anomaly, lets use a simple example…
- Buried sphere (decent approximation of a rising pluton or diapir)
  - 0.3 Mg/m^3 higher density than surrounding rocks
    - Only the difference relative to the surrounding rocks matters
  - Radius = 50 m
  - Depth to center of sphere 100 m



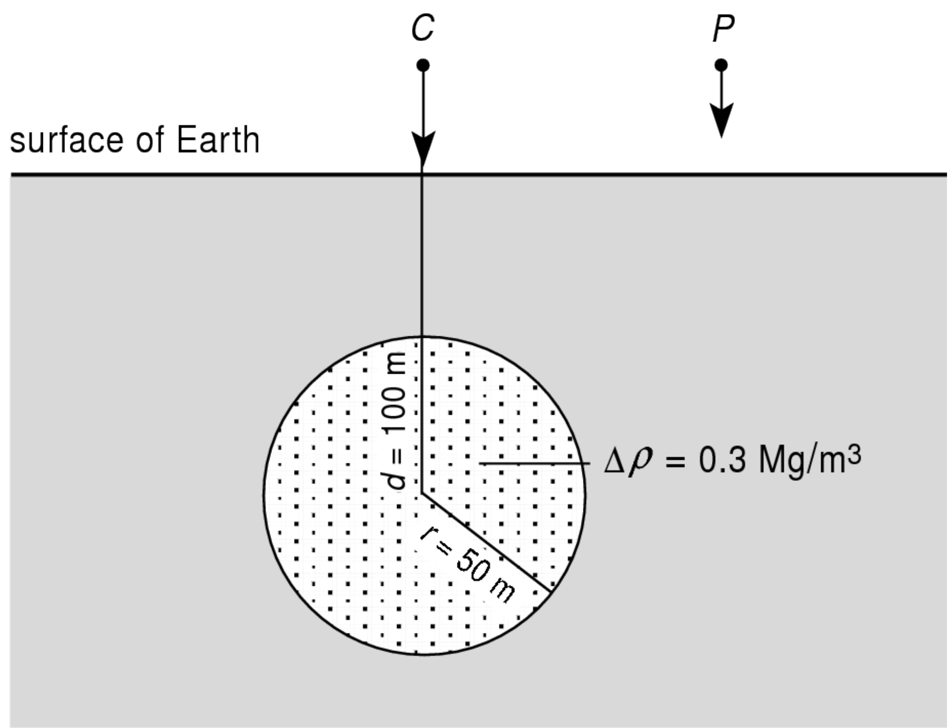


Size of Gravity Anomalies

- Use $g = G\frac{M_E}{R_E^{\ 2}}$ but in this case, we only care about the change in g and the excess mass. $\delta g = G\frac{\Delta m}{d^2}$ $\Delta m = (Vol)(\Delta\rho)$

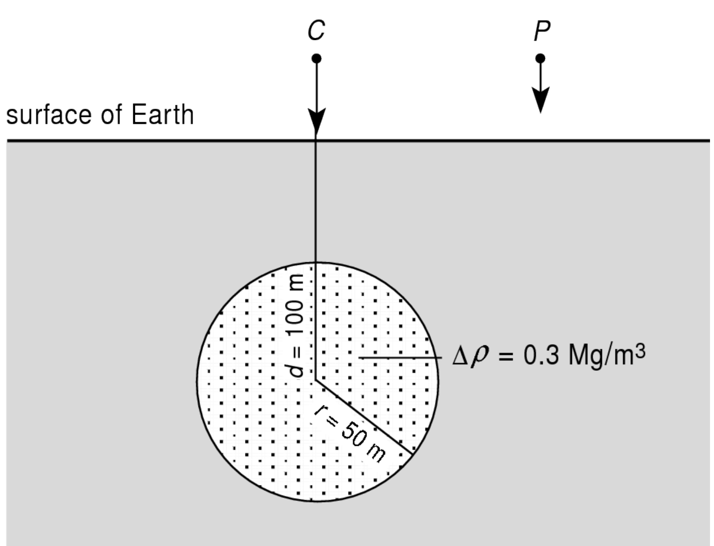
- We don't know the mass change, but we do know the volume change and the density change, so we can calculate the mass change.

$$\delta g = \frac{G}{d^2}\frac{4}{3}\pi r^3 \Delta\rho = \frac{G}{100^2}\frac{4}{3}\pi\left(50^3\right)(0.3) = 1.048\,x 10^{-6}\,\frac{m}{s^2}$$



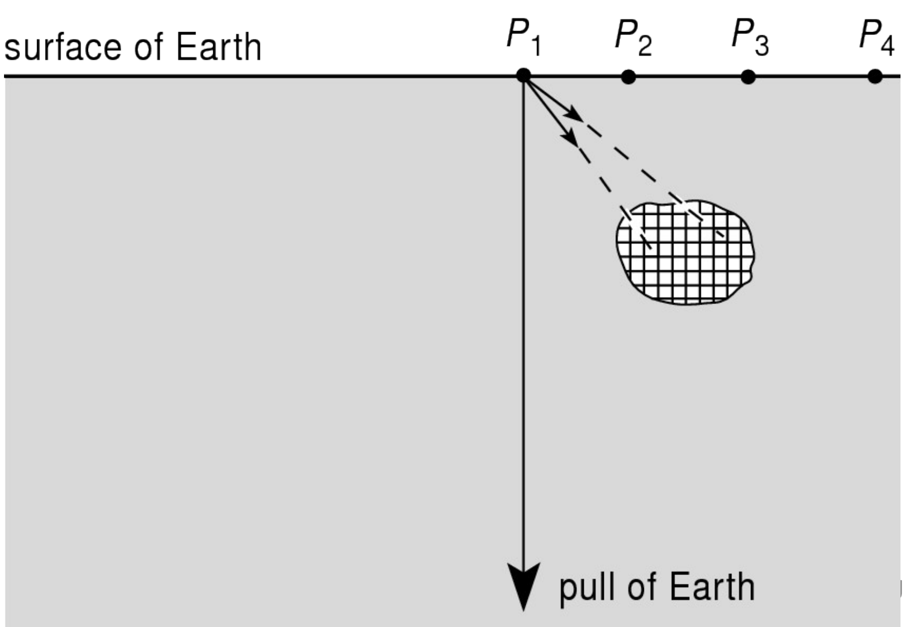
Gravity Units

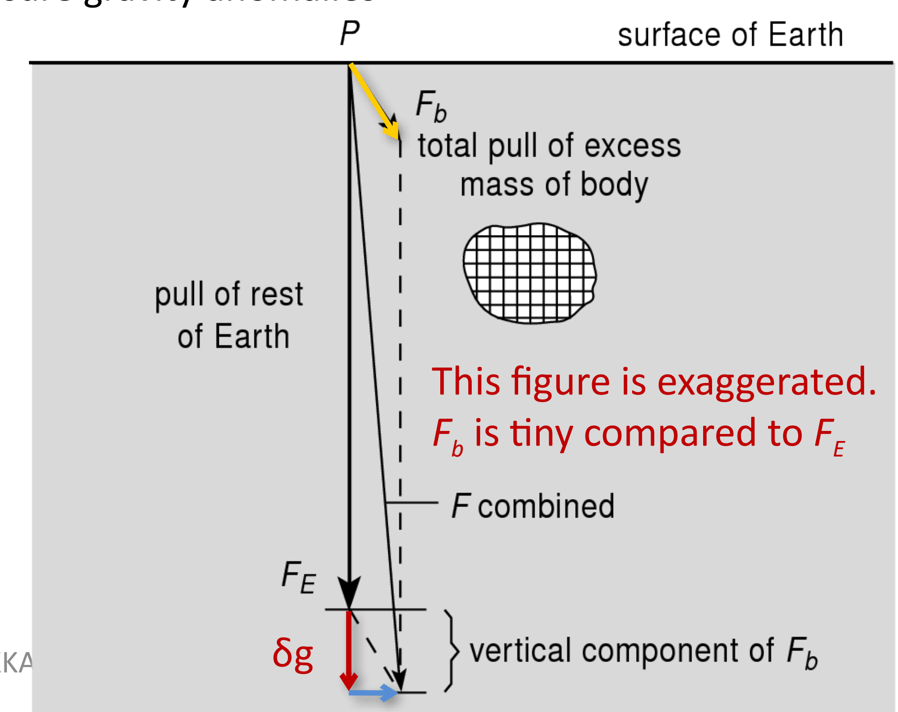
- The last example shows that gravity anomalies produce very small perturbations relative to g (i.e. 9.81 m/s^2).
  - So measuring gravity anomalies in m/s^2 is not convenient
- Instead we adopt a new unit: Gal (named in honor of Galileo)
  - 1 Gal = 0.01 m/s^2 = 0.01 m/s^2
    - But this is still too large, so we typically use milliGals (mGal) or gravity units (g.u.)
  - 1 mGal = 0.01 Gal = 10 g.u. = 1×10^{-5} m/s^2 ~ 10^{-6} g (approximation assuming g=10 m/s^2)
- So our buried sphere produced an anomaly of:
  - 1.048×10^{-6} m/s^2 = ~ 0.1 mGal



Spatial Variation in Gravity Anomalies

- The previous example only calculated the change in g directly above the buried sphere.
- What is the effect at locations not directly above the dense sphere?
  - The net pull is larger than g (must sum forces vectorially)
  - The net pull is not vertical, but this is hard to measure
    - Plum bobs are used to measure vertical and are assumed to hang vertically
  - The net vertical pull is greater than g.
    - We can measure this! This is how we measure gravity anomalies



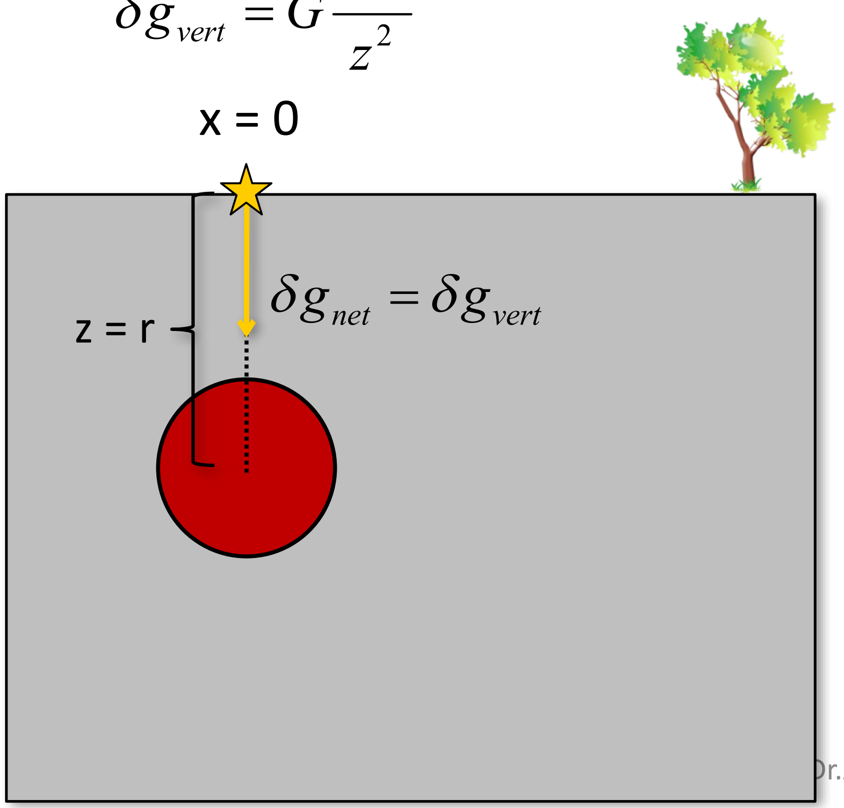


Gravity Anomaly Due to Buried Sphere

- The gravity anomaly due to a buried sphere is straightforward
- We'll start by assuming the sphere is a point source

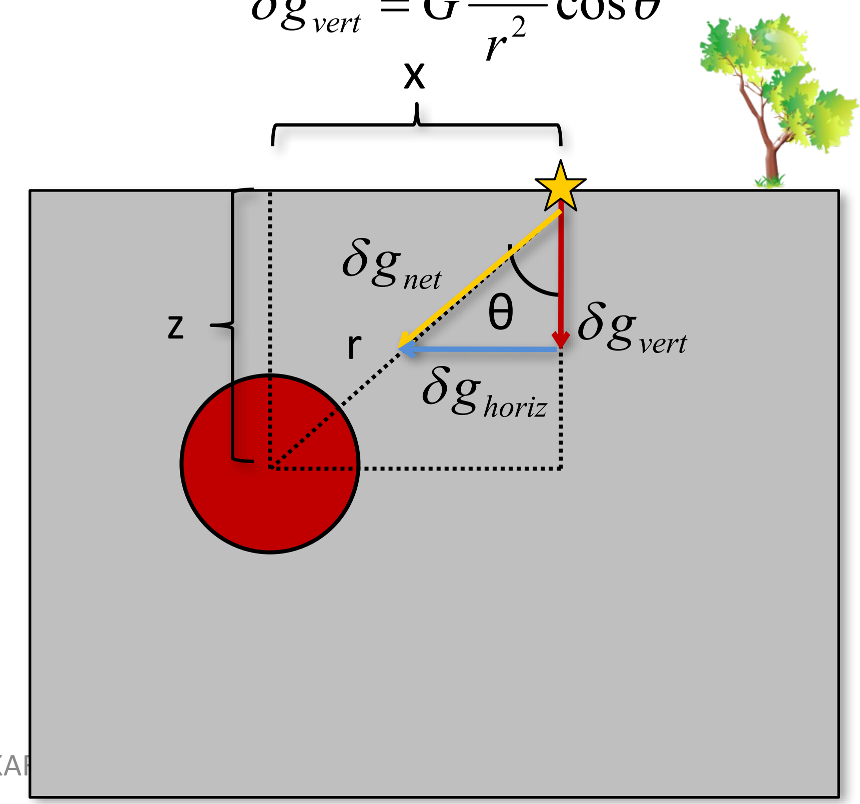
Directly over the sphere (r = z) all of the additional acceleration is vertical and given by the relationship derived earlier.

$$\delta g_{vert} = G\frac{\Delta m}{z^2}$$



At some location, x, away from the sphere, only part of the acceleration is vertical and the vertical component is found by trigonometry.

$$\delta g_{vert} = G\frac{\Delta m}{r^2}\cos\theta$$

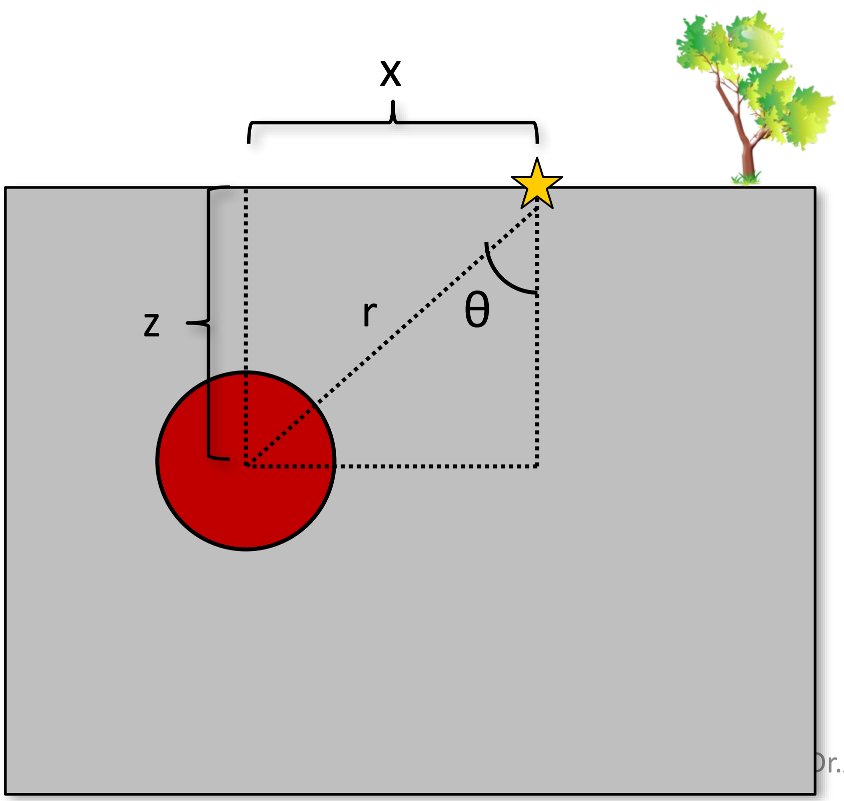


Gravity Anomaly Due to Buried Sphere

- Now lets put the equation into a more useful form…

It is not convenient to calculate θ, so we can reformulate the equation in terms that are more convenient.

$$\cos\theta = \frac{z}{r} \qquad r = \left(x^2 + z^2\right)^{1/2}$$



Now plug these back into the previous equation…

$$\delta g_{vert} = G\frac{\Delta m}{r^2}\cos\theta$$

Subs in cos θ…

$$\delta g_{vert} = G\frac{\Delta m z}{r^3}$$

Subs in r = …

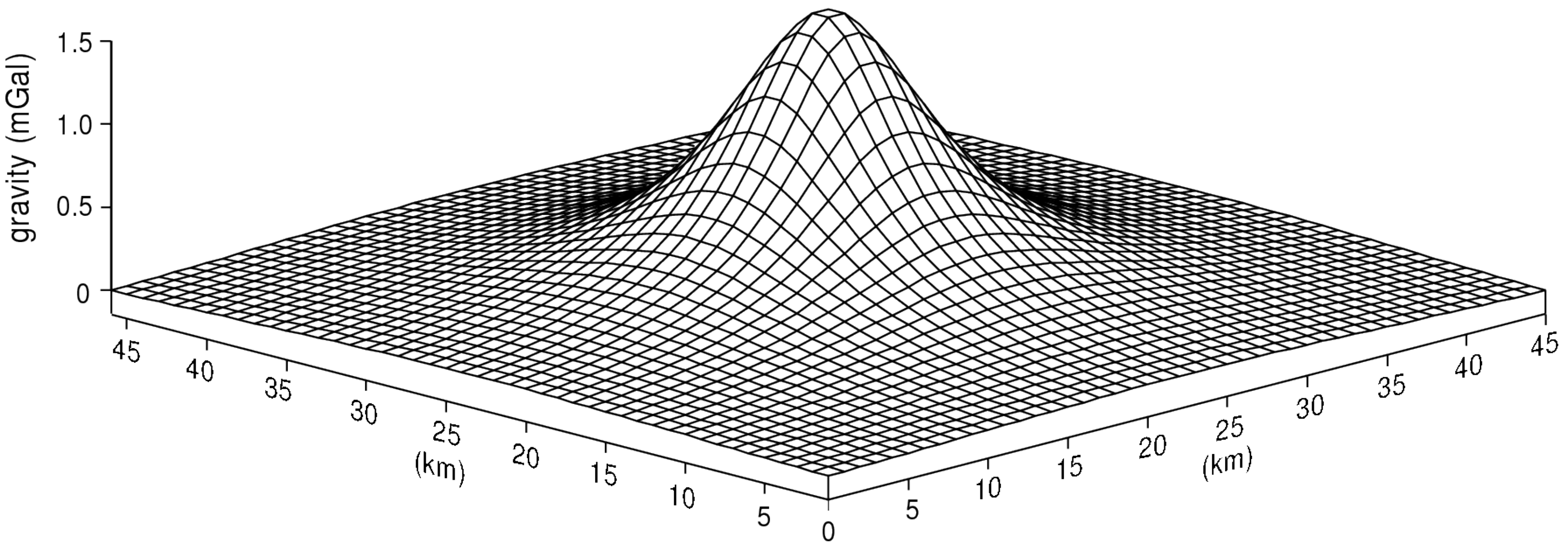
$$\delta g_{vert} = G\frac{\Delta m z}{\left(x^2 + z^2\right)^{3/2}}$$

But this is based on a mass change of a point source. Sub in the mass change of a sphere with radius = *R* and density contrast *Δρ* $\Delta m = \left(Vol\right)\left(\Delta\rho\right)$

$$\delta g_{sphere} = \frac{G\frac{4}{3}\pi R^3 \Delta\rho\, z}{\left(x^2 + z^2\right)^{3/2}}$$

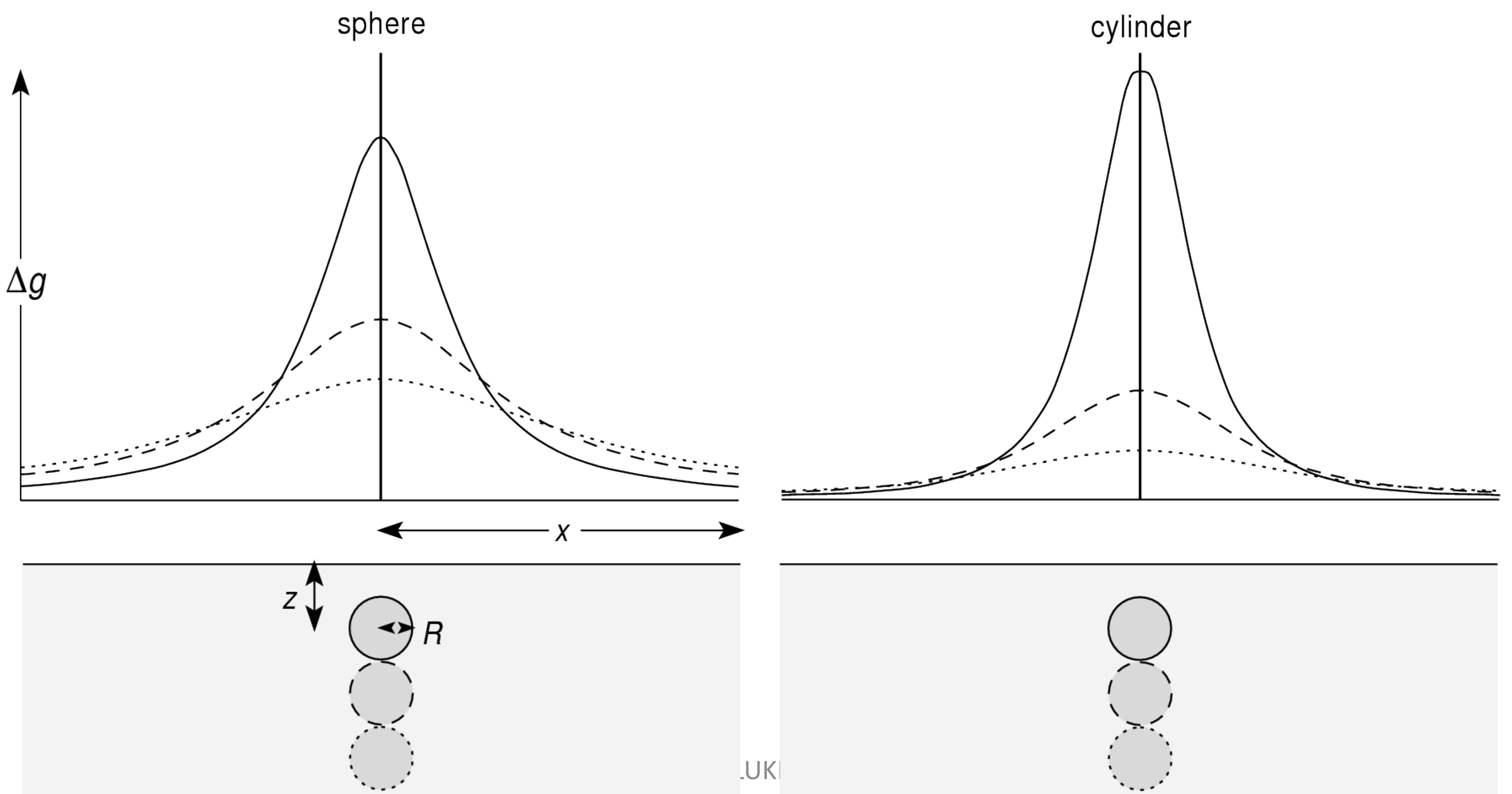
Gravity Anomaly Due to Buried Sphere

- The process described on the previous slides can be repeated for a 3D system to determine the 3D anomaly.
- Therefore gravity data may be collected along transects or along grids (shown below).



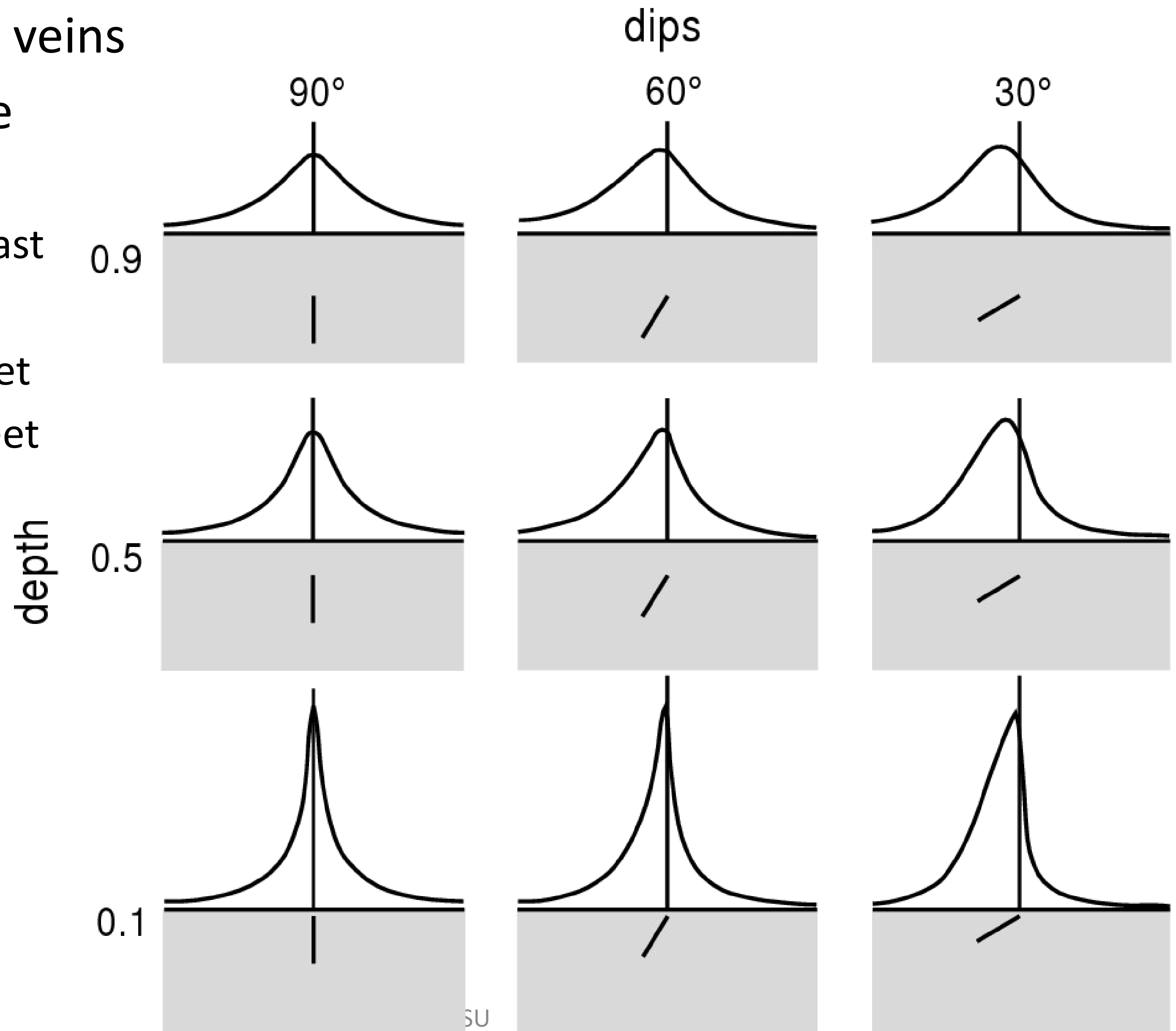
Other Gravity Anomalies

- Spherical targets :: diapirs and/or plutons
- Cylindrical targets :: tunnels/caves
  - Note that all anomalies lack a sharp edge
  - Subsurface geometry is non-unique



Other Gravity Anomalies

- Sheets :: dikes, veins
- Anomaly shape depends on
  - Density contrast
  - Depth
  - Length of sheet
  - Dip of the sheet



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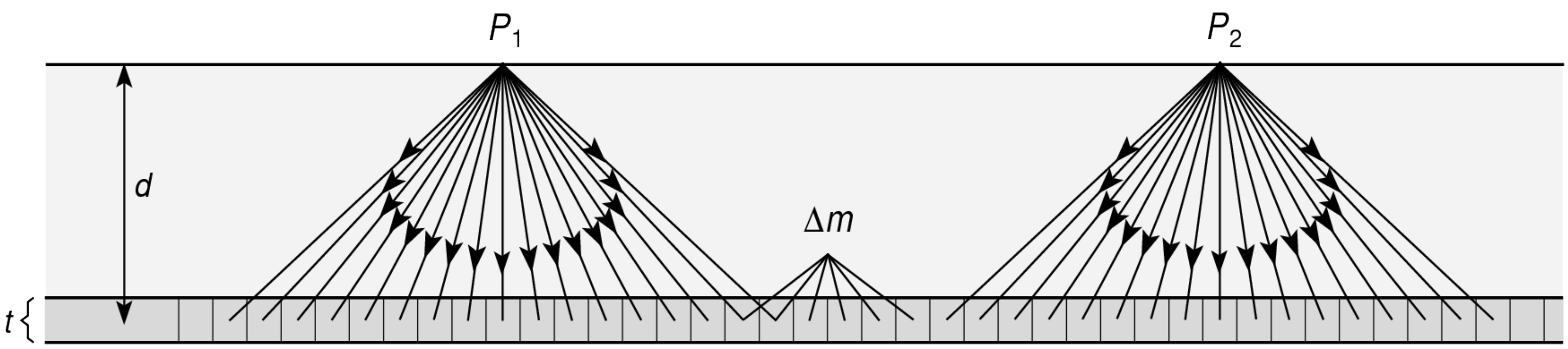
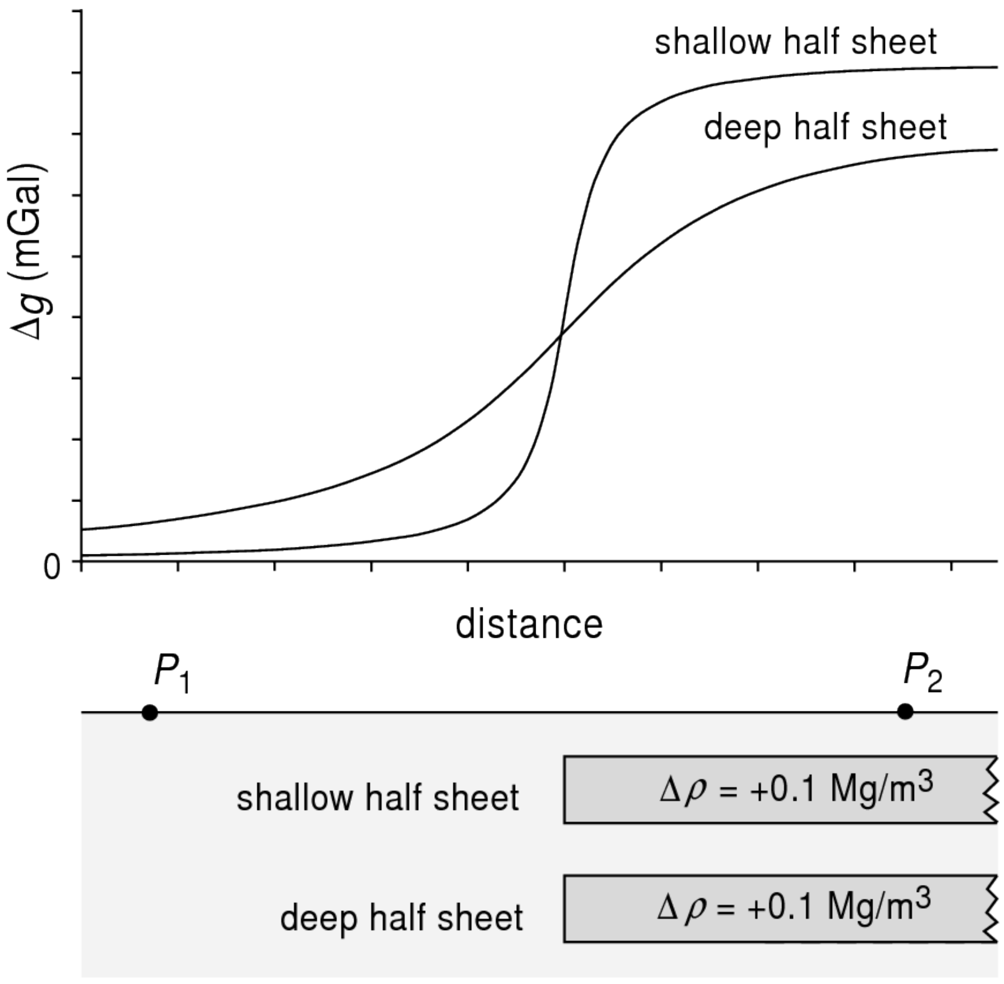
Other Gravity Anomalies

- Sheet-like targets :: sills or stratographic layers.
- If the sheet is infinite, there will be no anomaly
  - The distant parts of the sheet add little to the downward acceleration
  - The added acceleration dwindles with r^2
  - The net acceleration at two points P_1 and P_2 is the same
- The sheet will increase g everywhere by: $\delta g = 2\pi G \Delta\rho t$
  - t = slab thickness, $\Delta\rho$ excess mass per m^2



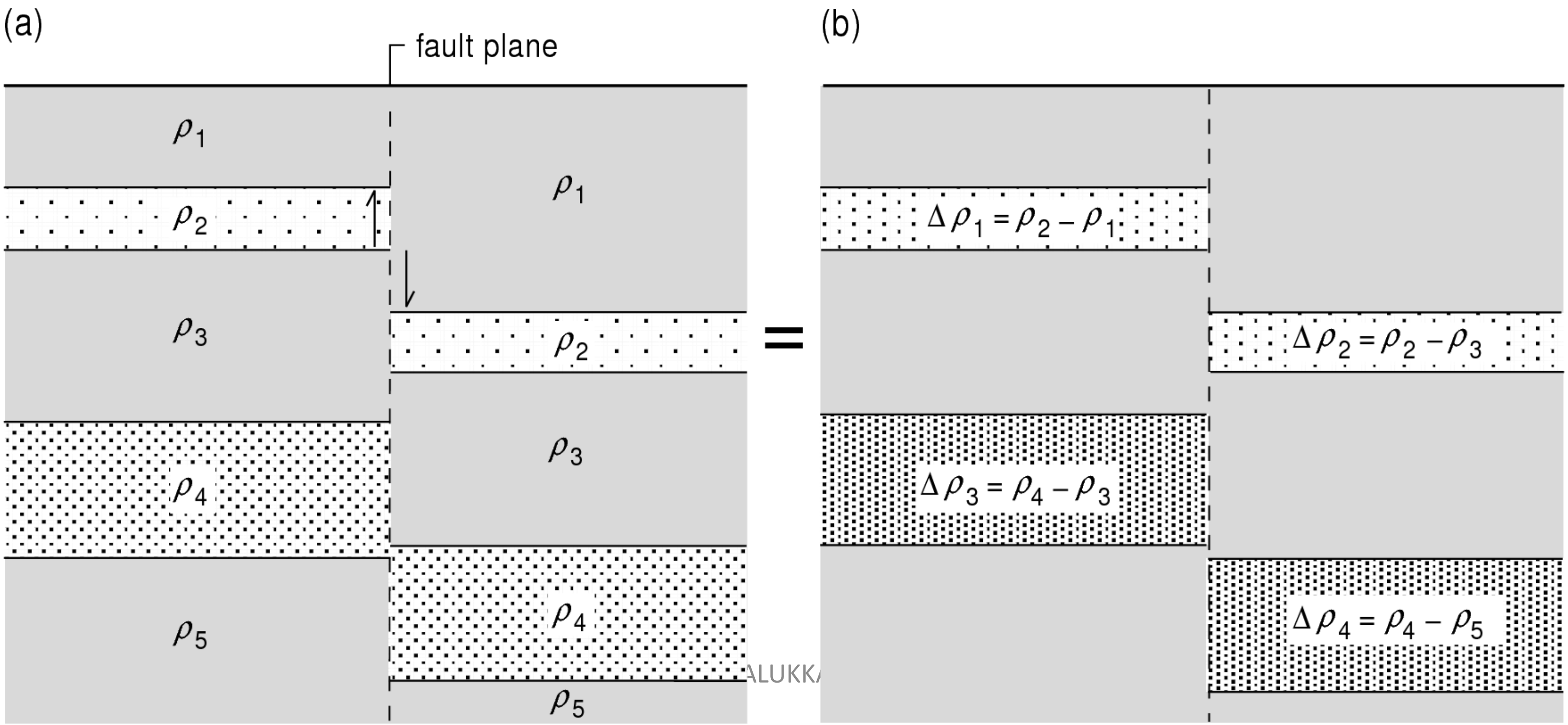
Other Gravity Anomalies

- Half Sheets :: Discontinuous layers from faulting
  - Produce a gradient near the edge
  - Away from the edge, there is no anomaly (infinite sheet)



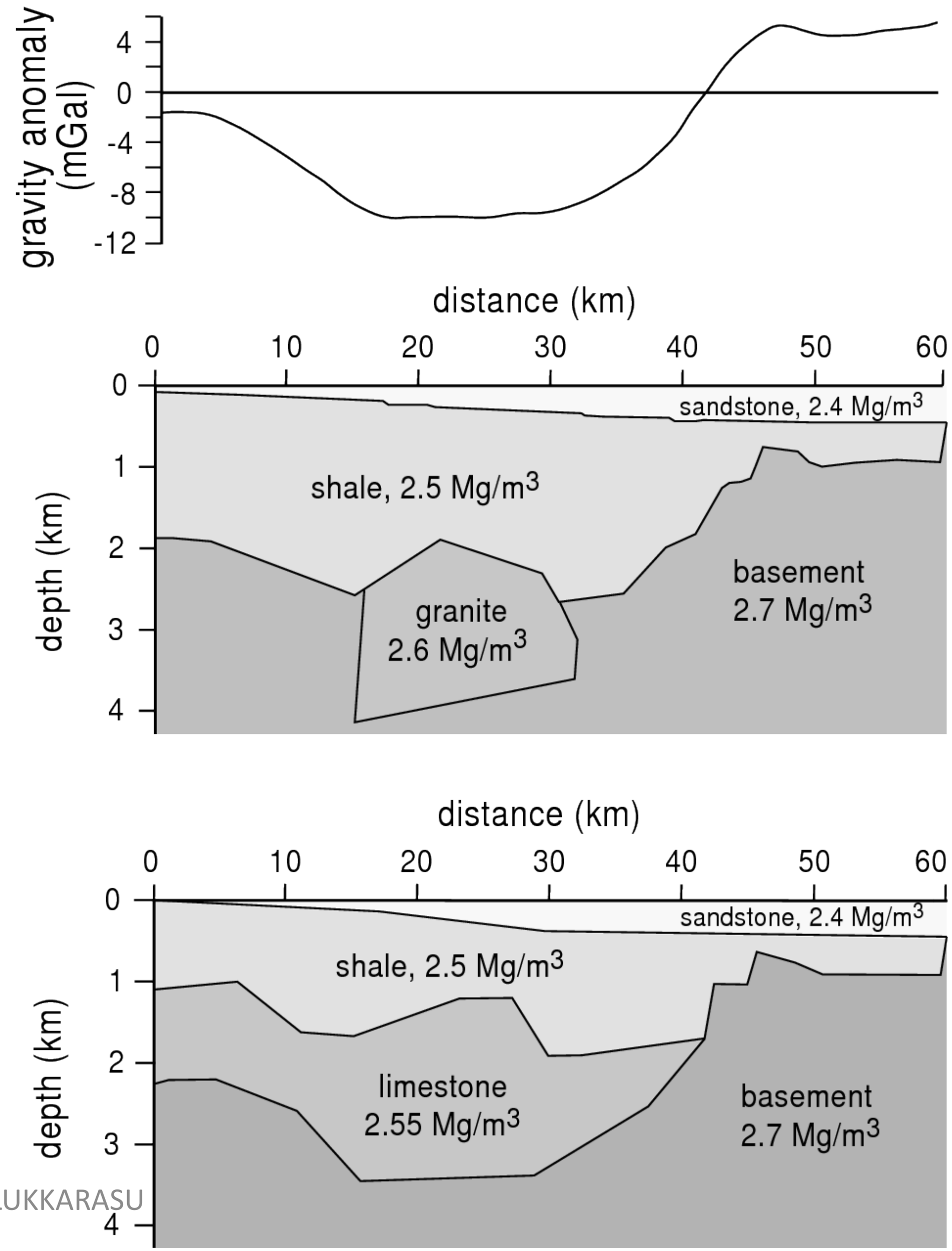
Other Gravity Anomalies

- Half Sheets :: Discontinuous layers from faulting
  - Most of the local anomaly will be due to the shallow layer offsets
  - The deeper offset layers may produce a broad and gradual regional anomaly.
  - Strike-slip faults will not produce a gravity anomaly unless
    - Layers are dipping
    - Non-horizontal intrusions are offset



Other Gravity Anomalies

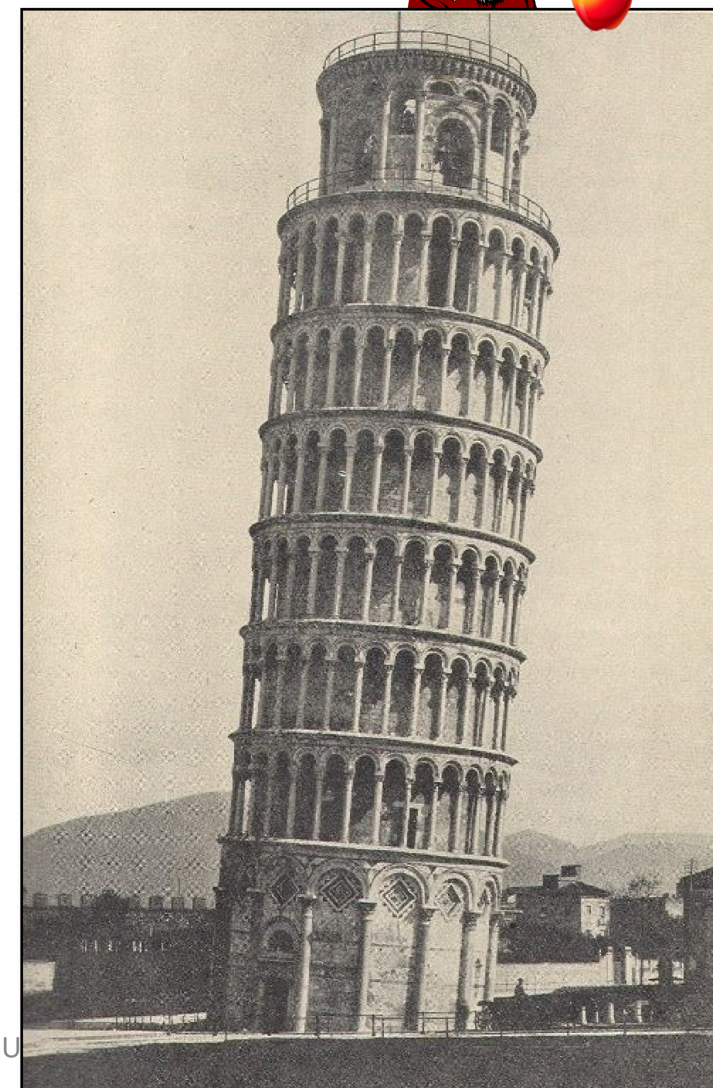
- Gravity anomalies for bodies with complex shapes are calculated:
  - By summing the anomalies of simple shapes
  - By summing the effects of point sources
  - By numerical integration using a computer



Measuring Gravity :: Gravimeters

$$g = \frac{2d_z}{t^2}$$

- Because variations in gravity are very small, gravimeters must be very sensitive.
- Gravity can be measured in several ways
  - Timing falling objects
    - Only practical in the lab



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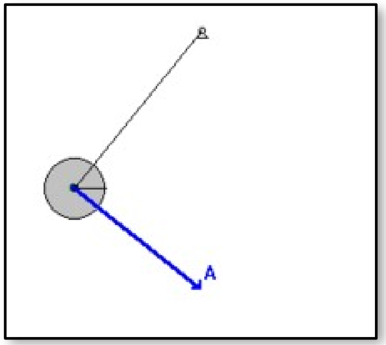
  - Measuring Pendulum Periods
    - Requires a bulky apparatus (not practical to lug around in the field)



Measuring Gravity :: Gravimeters

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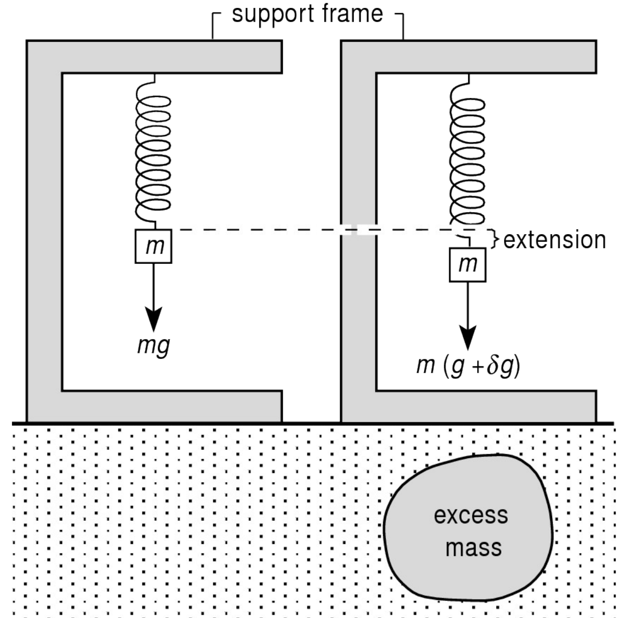
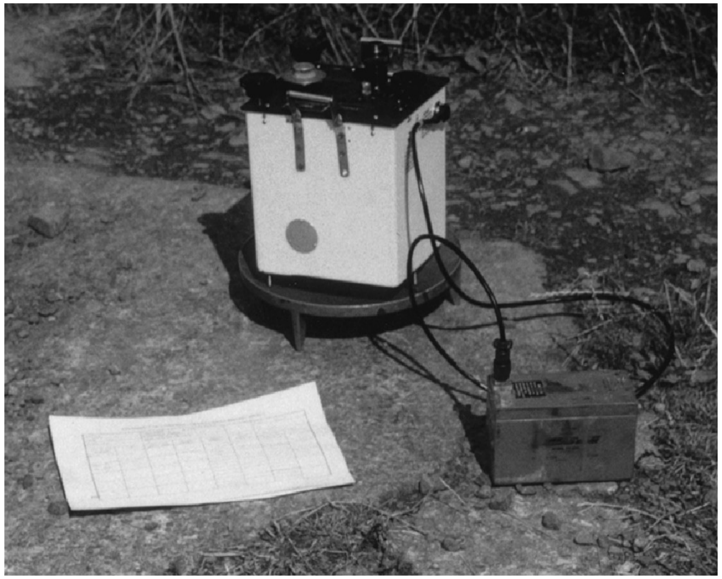
- Gravity can be measured in several ways
  - Timing falling objects
    - Only practical in the lab
  - Measuring Pendulum Periods
    - Requires a bulky apparatus (not practical to lug around in the field)
  - Measuring the extension of a spring with a mass attached
    - Follows Hooke's Law
      - $F = -kx$
    - This is what modern gravimeters do!

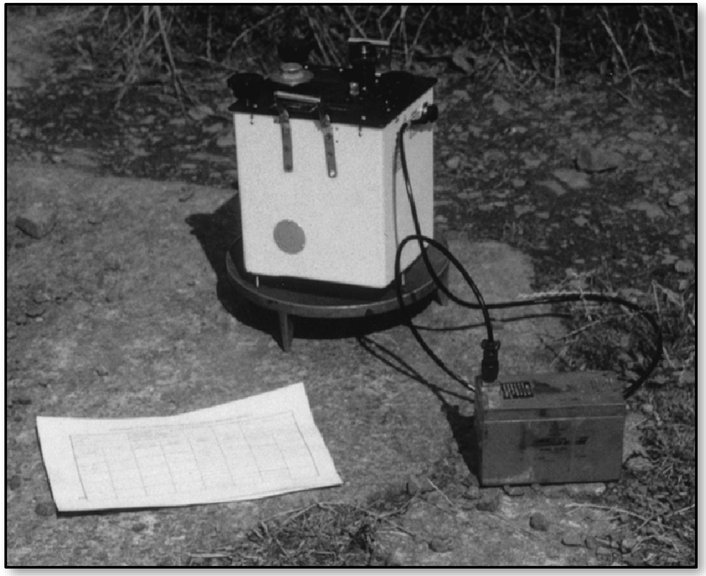


Lacoste–Romberg gravity meter

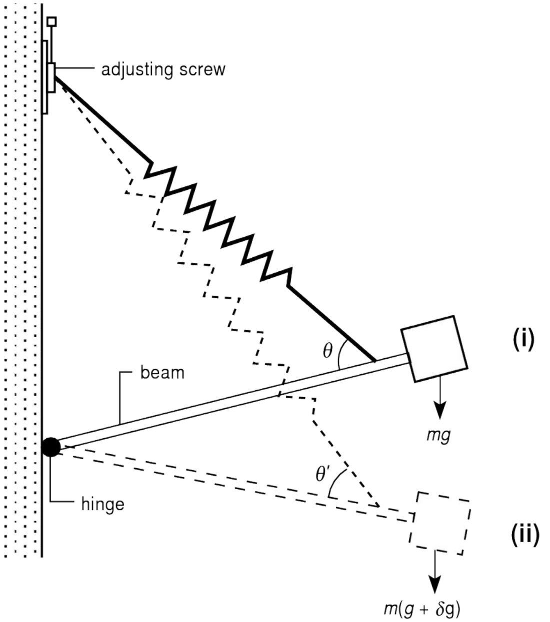


Gravimeters

- Modern gravimeters…
  - Can detect a change of 1/100,000,000th g
    - Or 0.01 mGal
  - Rely on a mass pivoted on a beam attached to a spring
    - A thermostat is present to prevent thermal contraction or extension
  - Must be carefully leveled before a reading is made (time-consuming)
  - Are somewhat delicate and must be transported in protective boxes
- But even with all of this complexity, the data from a gravimeter is not directly useful
  - data must be reduced to correct for several effects
    - Because gravity anomalies are small these corrections are often comparable in size to the anomalies and are thus very important



schematic diagram of gravity meter



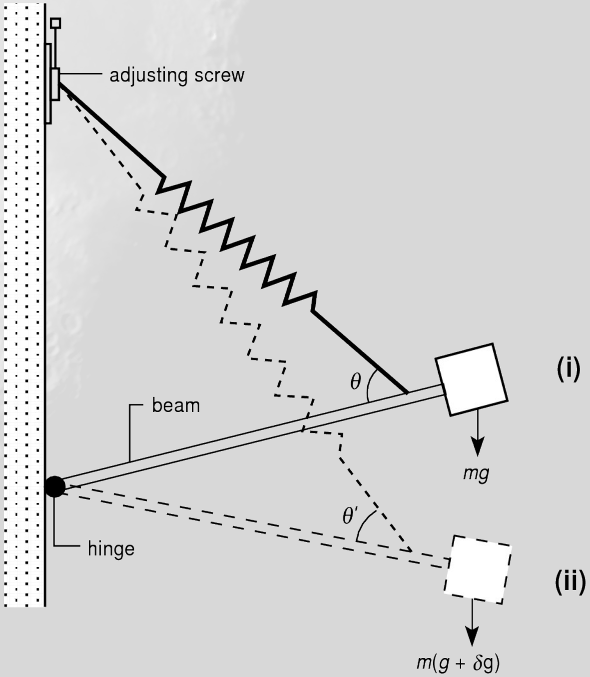
Gravity Corrections

- Take the simple example of a gravity measurement made at two different elevations
  - Because the distance to the center of Earth is farther for higher elevations, the gravity must be less.
  - There are actually several types of corrections that must be applied to raw gravity data in order for anomalies to be identified.

1. Drift
2. Latitude correction
3. Eötvös correction
4. Topographic correction
   a. Free-air correction
   b. Bouguer correction
   c. Terrain correction
5. *Regional / Residual anomaly *(not always necessary)*

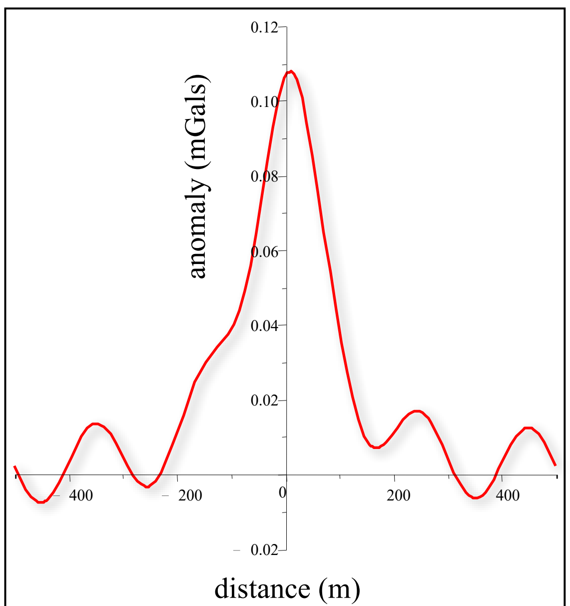
Drift

- **Drift** is a change in readings that would occur even if the device was not moved throughout the day.
  - The spring inside the gravimeter may slowly creep or stretch
  - Diurnal variations in tides
- Drift is corrected by periodically returning to a base station to get the temporal variation.
  - The drift is then subtracted from the rest of the data.



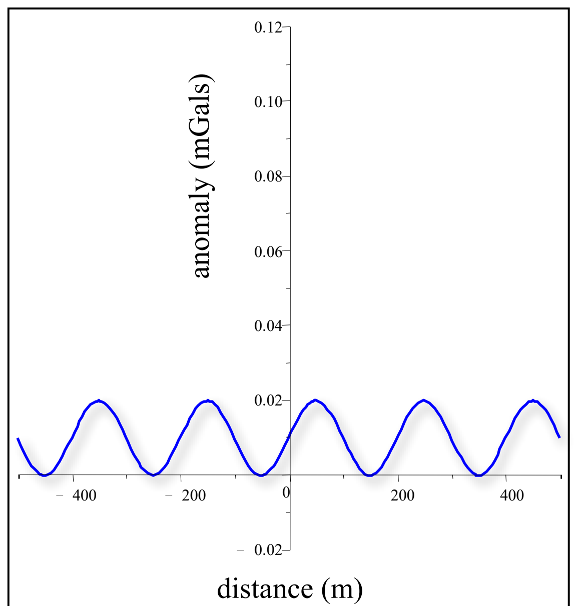
Drift Example

- Because most land-based surveys can only collect one data point at a time, temporal drift variations must be corrected
  - Base station readings are used to determine the temporal variations
  - The base station readings are normalized and then subtracted from the data to correct for drift.
  - The example below is idealized…assumes all other corrections have been made.



Raw data

−



Base station data

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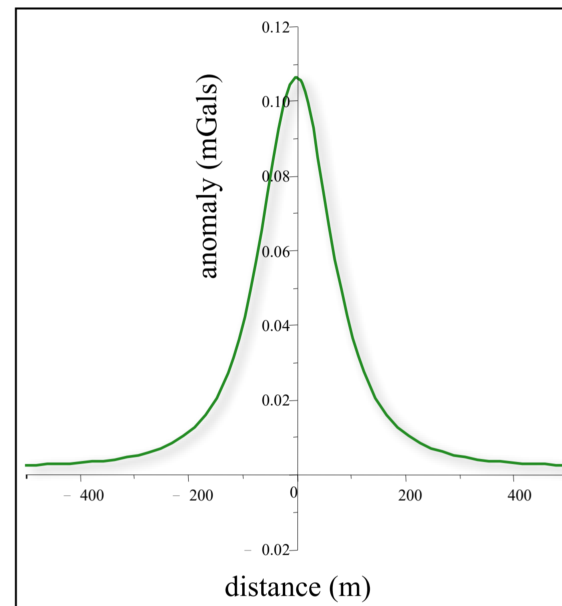
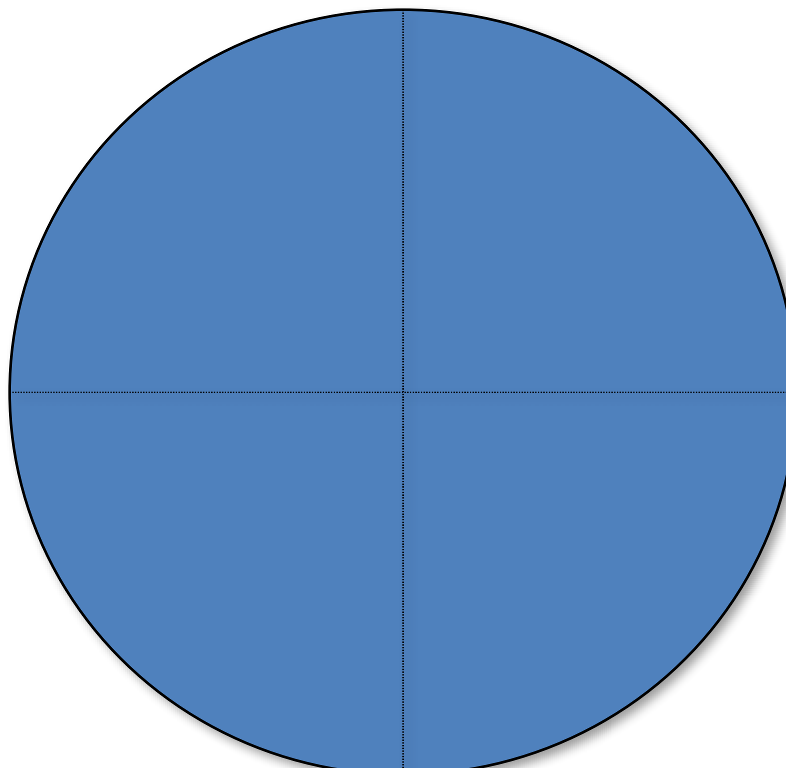


Data corrected for drift

Latitudinal Gravity Variations

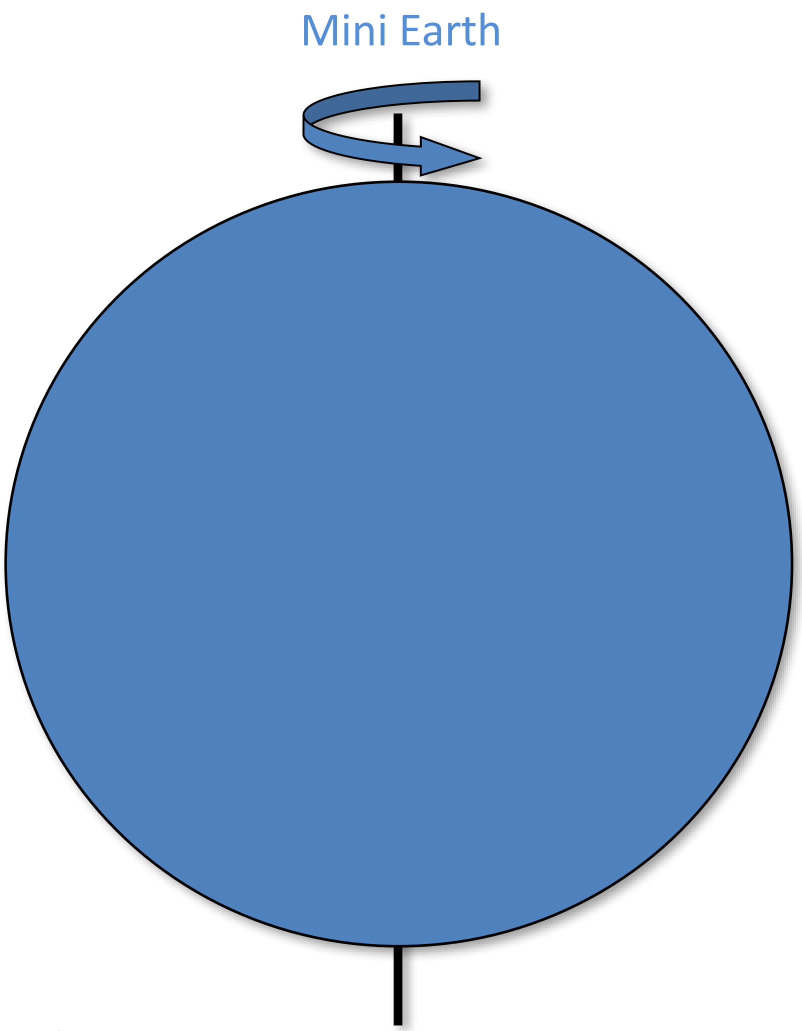
- Even if all rocks were the same and there was no topography gravity would still vary with latitude
  - Least at Equator
  - Most at Poles
  - Total variation ~0.5%

Why?



Latitudinal Gravity Variations: Why?

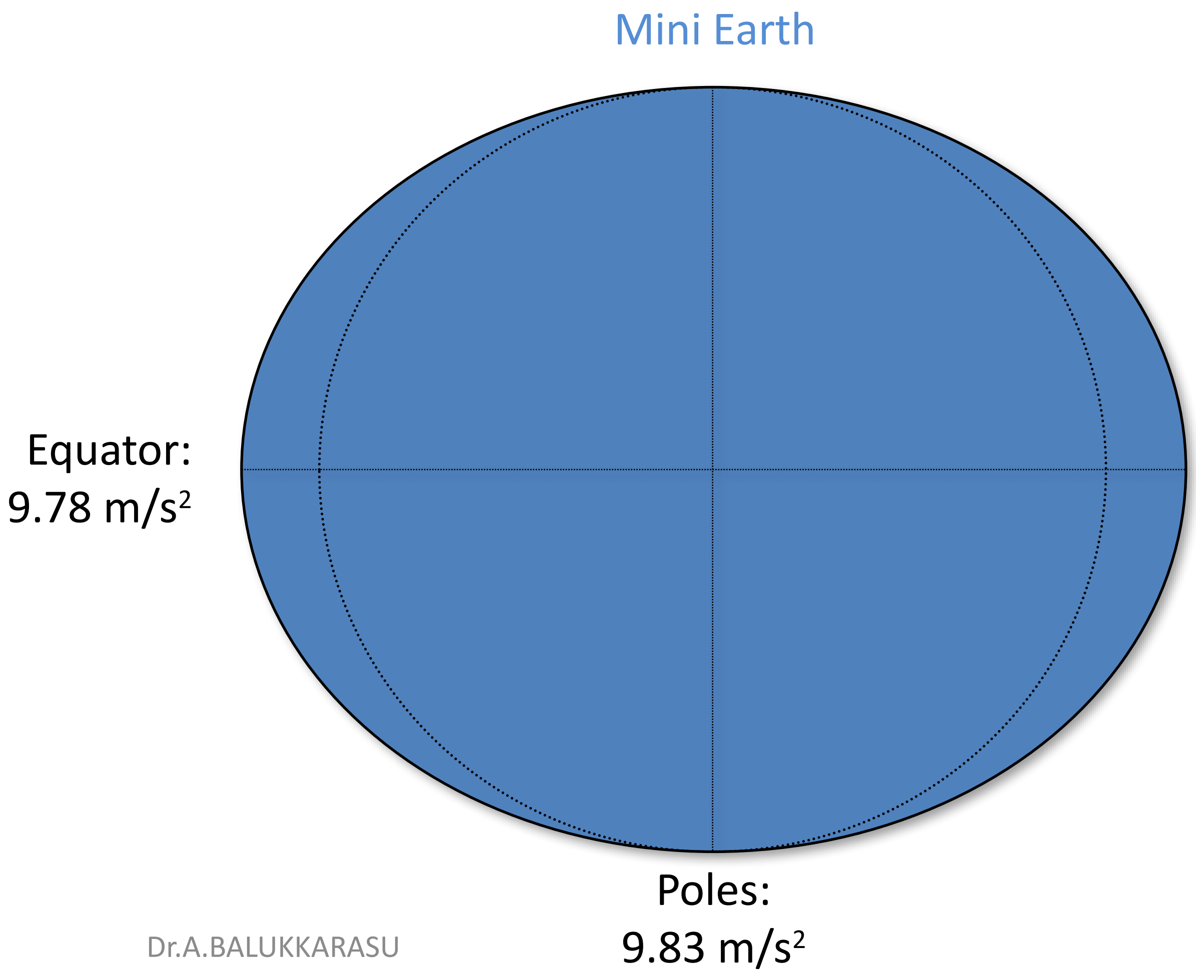
- The Earth rotates on its axis…
  - Creates centrifugal force which depends on:
    - Distance from axis
    - Rate of rotation
  - The linear velocity of a person at the equator is much faster than someone at the poles.
  - Centrifugal force causes Earth to deform
    - Fattest at equator
    - Pinched in at poles



Latitudinal Gravity Variations: Why?

- Because the Earth rotates on its axis…
  - The radius of the Earth is greatest at the equator, least at the poles
    - Gravity depends on distance to center of Earth

$$g = G\frac{M_E}{{R_E}^2}$$

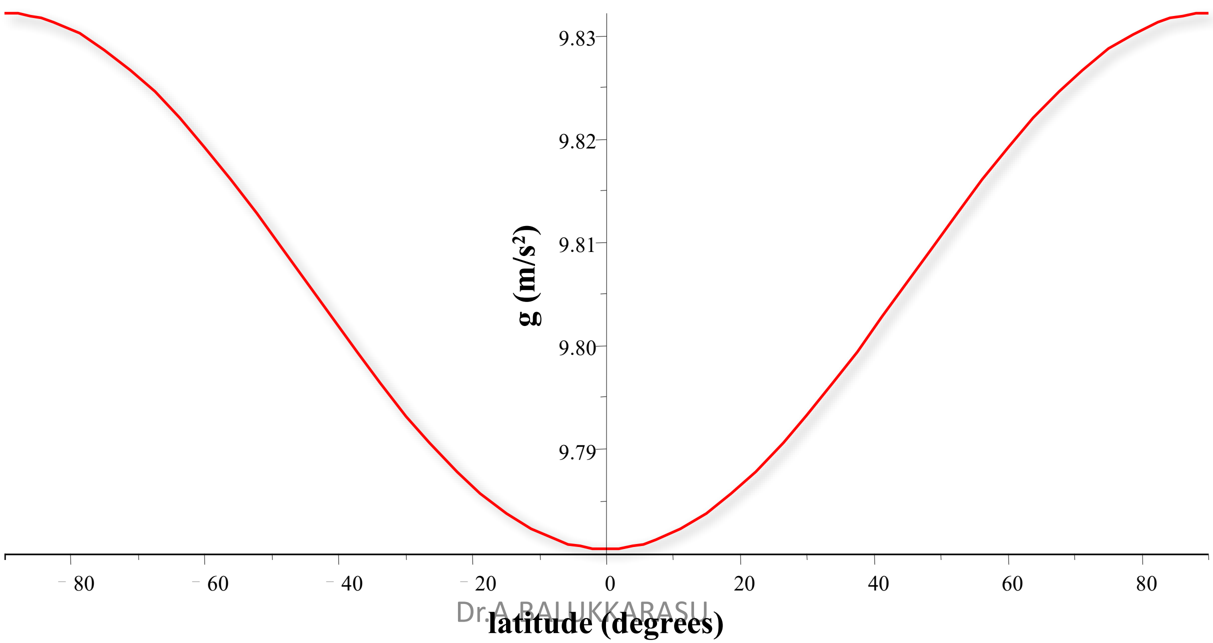


Latitude Correction

- Because gravity varies with latitude:
  - A survey covering a large north/south distance will need to be corrected for latitudinal changes in g.
  - This correction is performed using the International Gravity Formula

$$g_\lambda = 978031.8\left(1 + 0.0053024\sin^2\lambda - 0.0000059\sin^2 2\lambda\right)\text{mGal}$$

  - The correction ends up ~0.8 mGal/km, and given that a good gravimeter can detect a 0.01 mGal change, a N/S movement of only 12 m can be detected.



Eötvös Correction

- If gravity measurements are made on a moving object (car, airplane, ship) a centrifugal acceleration is induced and the gravity measurements must be corrected.
- E.g. because the Earth rotates to the east (counterclockwise when looking down from the north pole):
  - Your weight is reduced due to centrifugal force (~0.34% on the equator due to a rotation of 465 m/s)
  - If you are traveling eastward:
    - measured gravity is less because your motion adds with Earth's rotation
  - If you are traveling westward:
    - You cancel out some of Earth's rotation and measured gravity is more

$$\delta g_{\text{Eötvös}} = 4.040 v \sin\alpha \cos\lambda + 0.001211 v^2 \text{ mGal}$$

- v = speed in km/hr
- λ = latitude
- α = direction of travel (azimuth)
- This is a huge correction
  - ~2.5 mGal per km/hr!
- The main limiting factor in aerial gravity surveys is accurate determination of the airplane velocity

Topographic Corrections

- So far we have assumed that we were taking gravity measurements at the same elevation.
- When gravity measurements are taken at different elevations, up to three further corrections are needed.
  - We also need a way to deal with water!

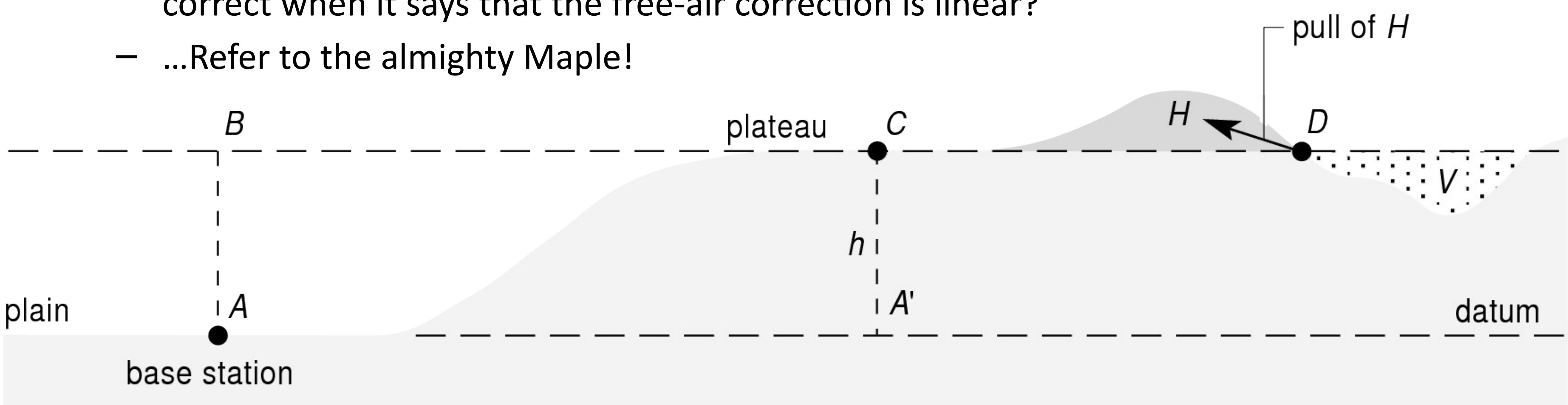
Free-Air Correction

- Imagine taking a measurement at A (base station) then floating up to evevation B in a balloon (i.e. in the free air!)
  - You just moved farther from the center of Earth, so gravity must decrease!
  - This turns out to be ~0.3086 mGal/meter of elevation change
    - So a gravimeter will respond to changes in elevation of a few cm!
  - To find the rate of change in g with elevation, take the derivative of g

$$g = G\frac{M_E}{R_E^{\ 2}} \Rightarrow \frac{dg}{dR} = -2G\frac{M_E}{R^3}$$

  - If the derivative of g (i.e. the rate in change in g w.r.t. R) has $1/R^3$, then how is your book correct when it says that the free-air correction is linear?
  - …Refer to the almighty Maple!

Where did this come from??



The Free-Air Correction

This is negative too, so the negatives cancel

- The Free-air correction:

$$\delta g_{Free-air} = -h\frac{dg}{dR} \approx 0.3086\,h$$

- When *h* is positive (mountain range):

$$\delta g_{Free-air} > 0$$

- When *h* is negative (Death Valley):

$$\delta g_{Free-air} < 0$$

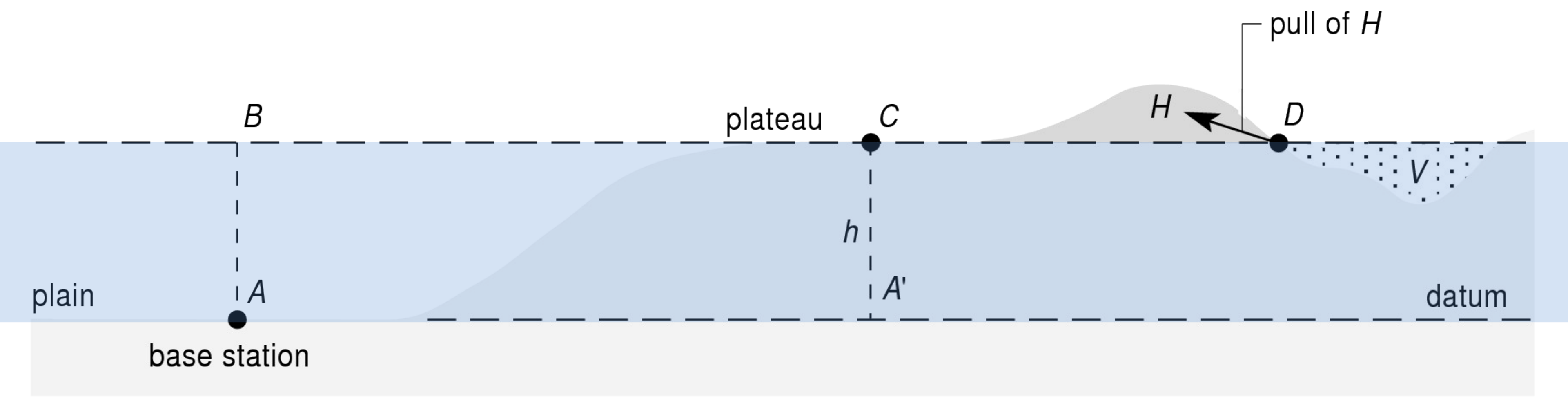
- But what do we do over the ocean?
  - We do not correct using FAC, we use the Bouguer correction

The Bouguer Correction

- Now imagine that you float to point C.
  - You feel less gravity than at A' due to elevation (Free-air correction)
  - You feel more gravity than B due to the mass of rock beneath you (Bouguer correction)
  - If you are on a wide and relatively flat plateau, the extra gravity can be approximated by an infinite sheet/slab

$$\delta g_{Bouguer} = 2\pi G \rho h \left(\frac{m}{s^2} \right)$$

$$\delta g_{Bouguer} = 0.04192 \, \rho \, h \, (mGal)$$



The Bouguer Correction

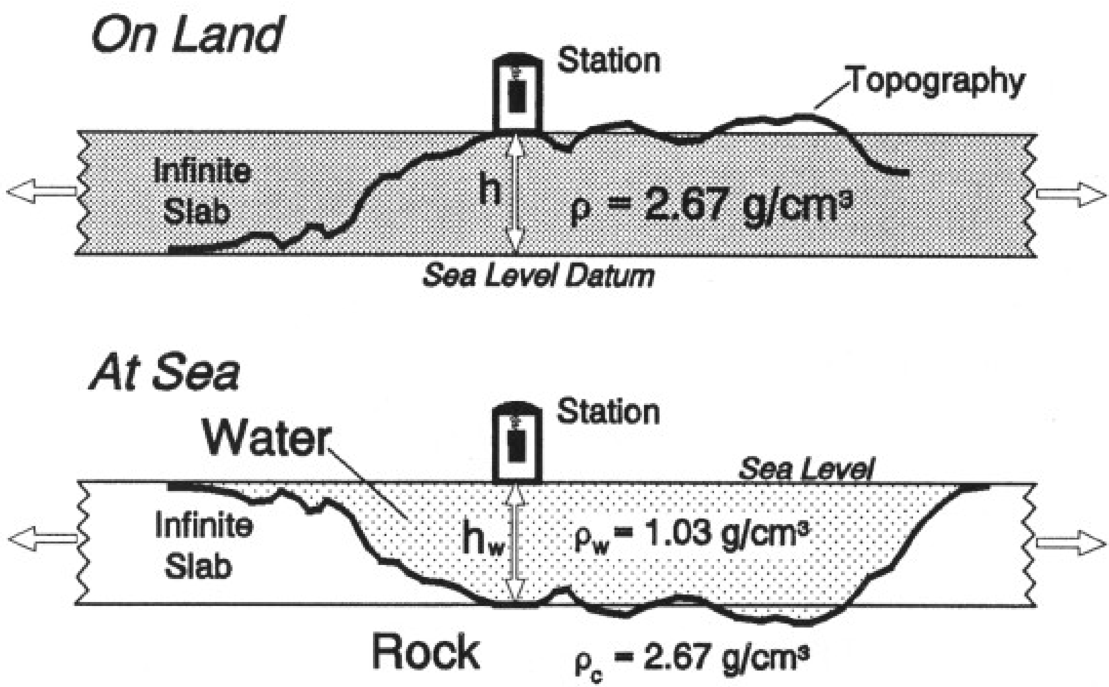
- Since we do not treat negative elevations (i.e. marine surveys) using the Free-air correction
  - We use the Bouguer Correction and apply a negative density to reflect the "missing" mass

LAND: Same as before.
It is common to use:

$$\rho = 2.67\, g/cm^3$$

$$\delta g_{Bouguer} = 0.04192\, \rho\, h$$

$$\delta g_{Bouguer} = 0.112\, h\ (mGal)$$



SEA: Bouguer is negative because the water acts like a mass deficit:

$$\rho = (1.03 - 2.67)\, g/cm^3$$

$$\delta g_{Bouguer} = -0.0688\, h_w\ (mGal)$$

The Combined Elevation Correction

- Because the free-air and Bouguer corrections both depend on h (elevation)…
  - we can combine them into a single "combined elevation correction

$$\delta g_{Bouguer} = 0.04192\,\rho\,h\ (mGal)$$

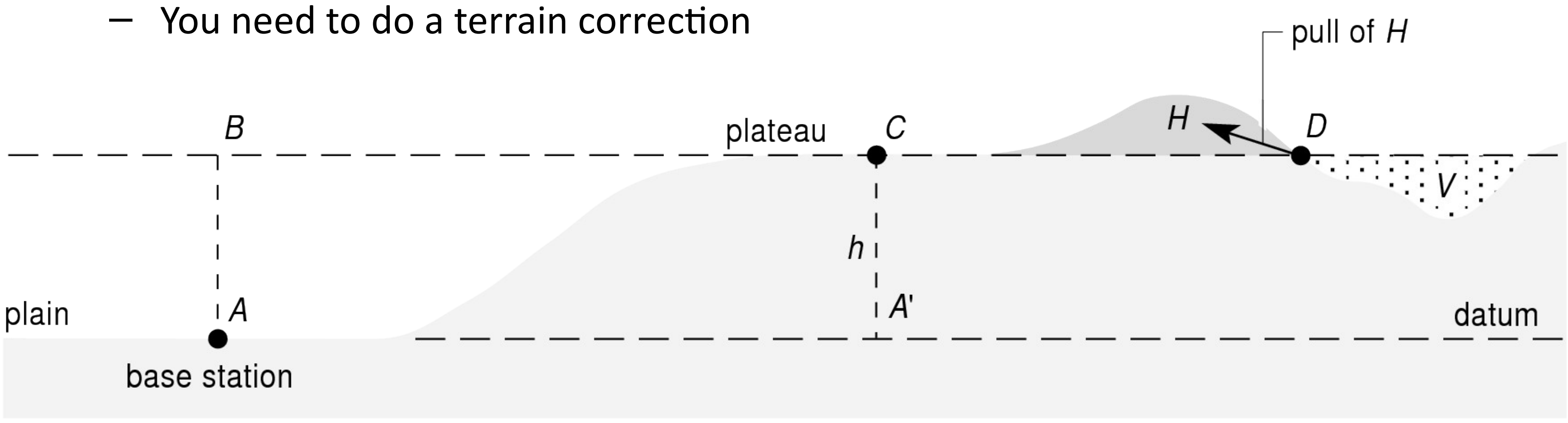
$$\delta g_{Free-air} = 0.3086\,h\ (mGal)$$

$$\delta g_{elevation} = \delta g_{Free-air} - \delta g_{Bouguer}$$

$$\delta g_{elevation} = h\,(0.3086 - 0.04192\rho)\ (mGal)$$

Terrain Correction

- The Bouguer correction assumes an infinite slab
  - Reasonable at C, but not at D
- The pull of the mountain, H, would have the same effect as the valley, V
  - Both would reduce g due to the vertical component of pull
- The terrain correction aims to correct for this and depends on:
  - Shape and density of topography
  - Mostly only the nearby features matter (g α $1/R^2$)
- No simple way to do this, so computers are used in conjunction with digital elevation data and knowledge of local rock density
- Rule of thumb: If < 200 m from steep topography
  - You need to do a terrain correction



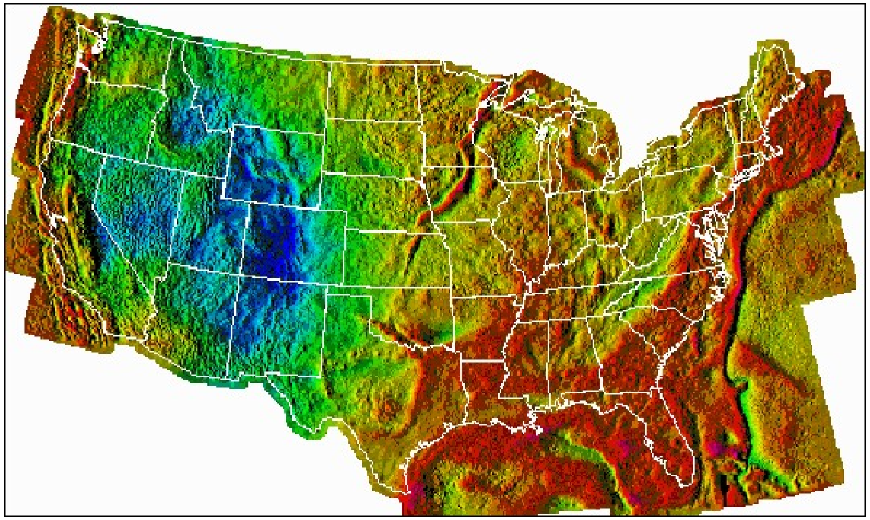
Finally…The Bouguer Anomaly

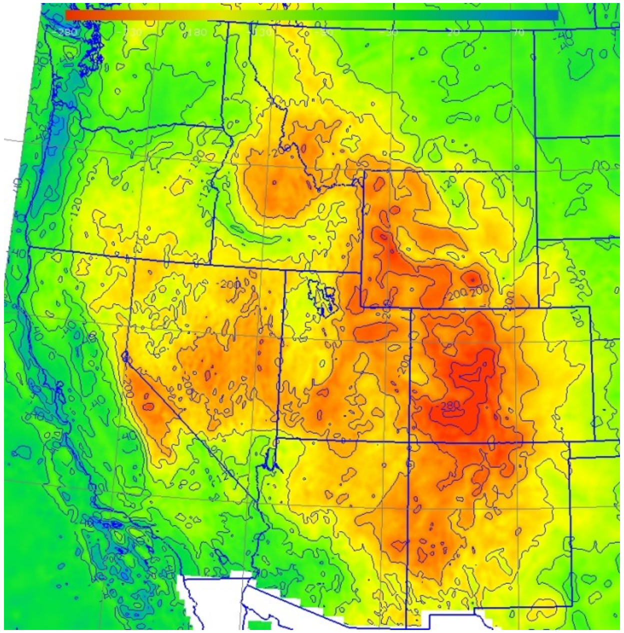
- Once all of the previously mentioned corrections have been made

$$\delta g_{Bouguer\,anomaly} = \delta g_{measured} - \delta g_{latitude} + \delta g_{free-air} - \delta g_{bouguer} + \delta g_{terrain} + \delta g_{Eötvös}$$

  - The result is called the **Bouguer anomaly**
    - Not to be confused with the Bouguer correction, which is different.
    - If the terrain correction is omitted, the result is the "**simple Bouguer anomaly**"

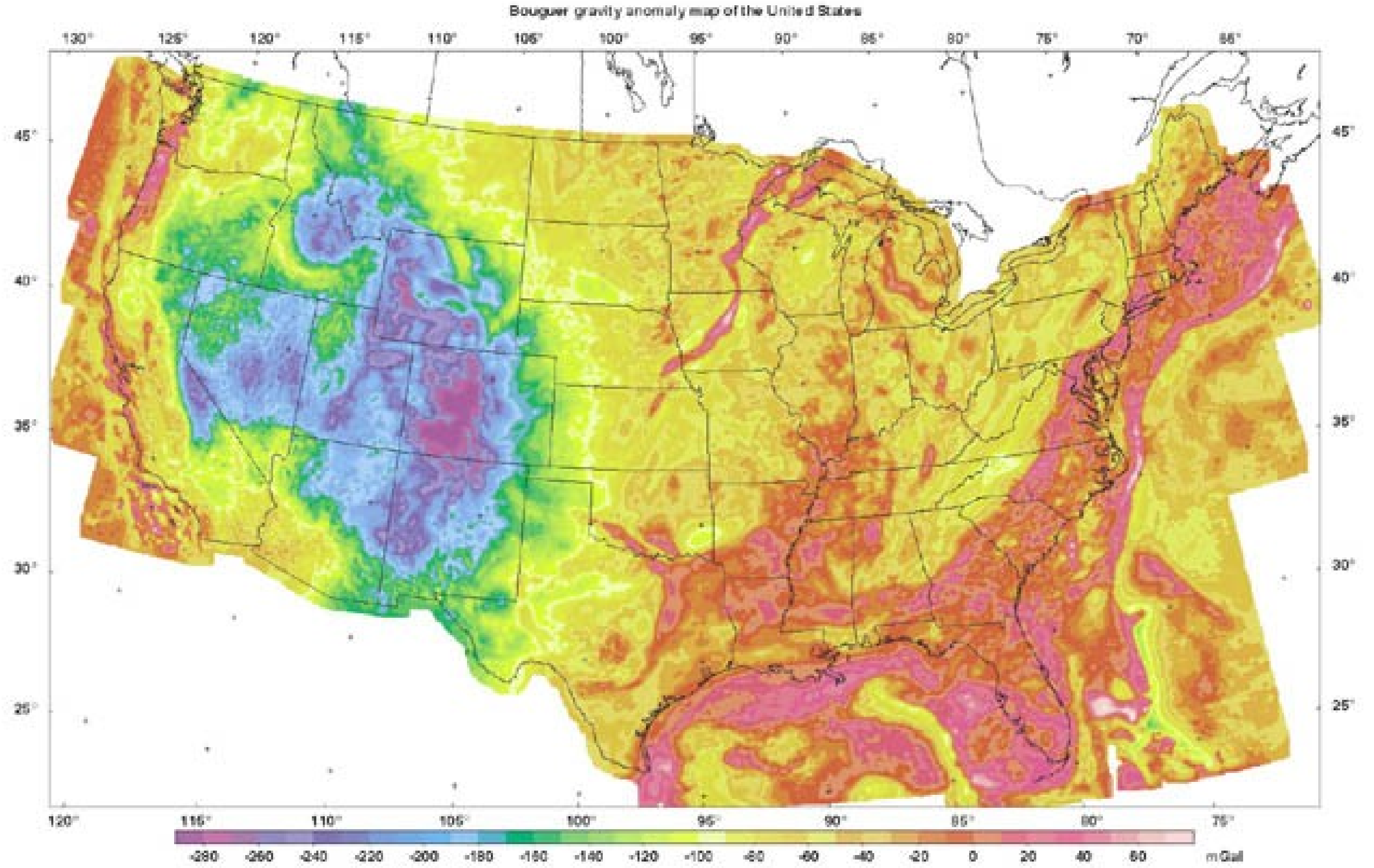
  - The purpose of the Bouguer anomaly is to give the anomaly due to the density variations below the datum, without the effects of topography and latitude





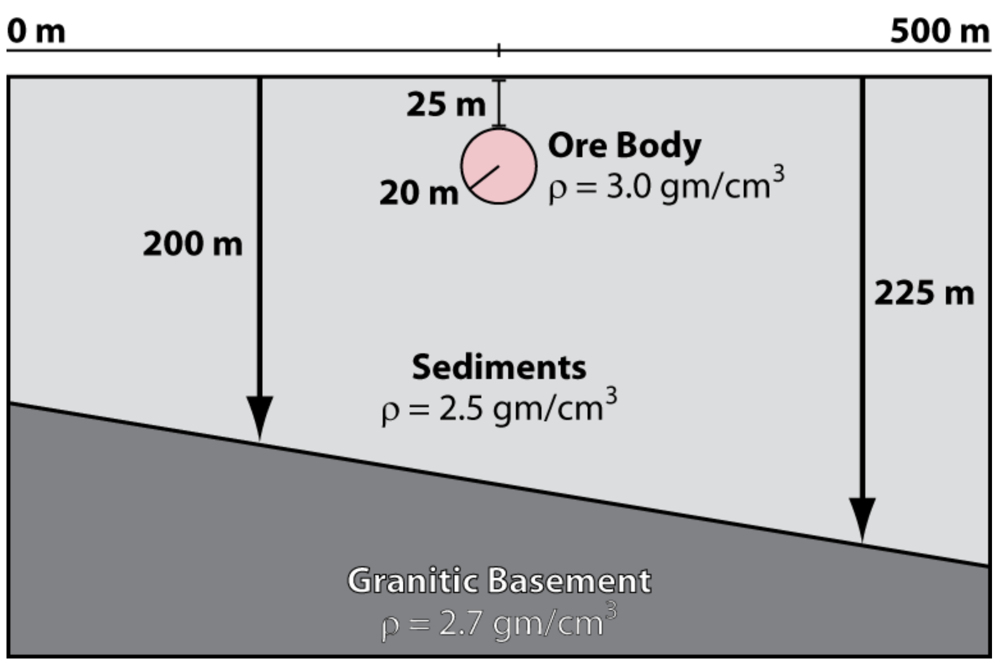
Bouguer Anomaly For The U.S.

- From USGS data
- We'll talk more about what we can learn from this in the next chapter

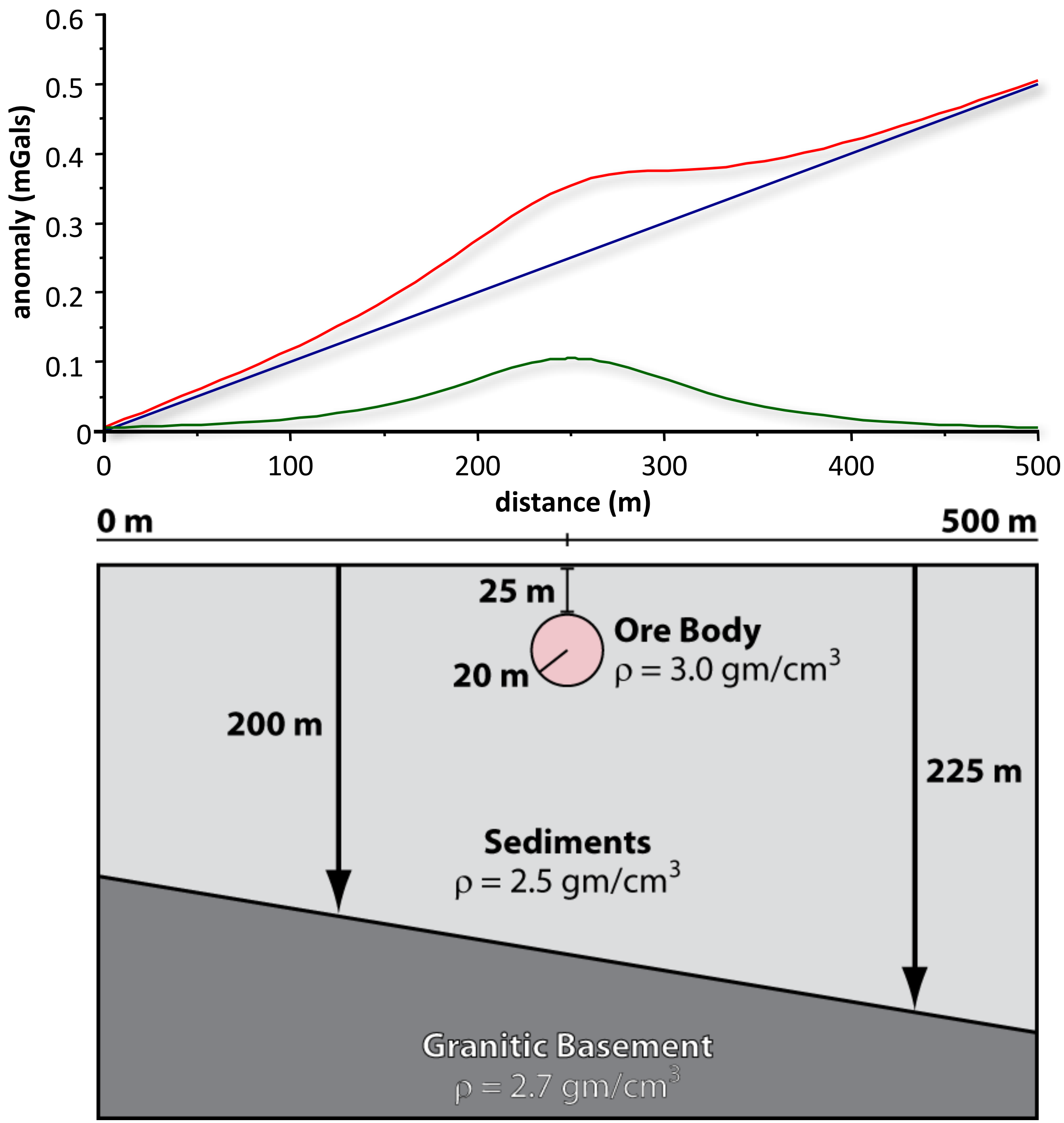


Regional and Residual Anomalies

- Often another step is needed to elucidate a local anomaly
  - Regional scale anomalies are very broad and can obfuscate local anomalies
  - If regional slope is removed, the residual anomaly will show the local target

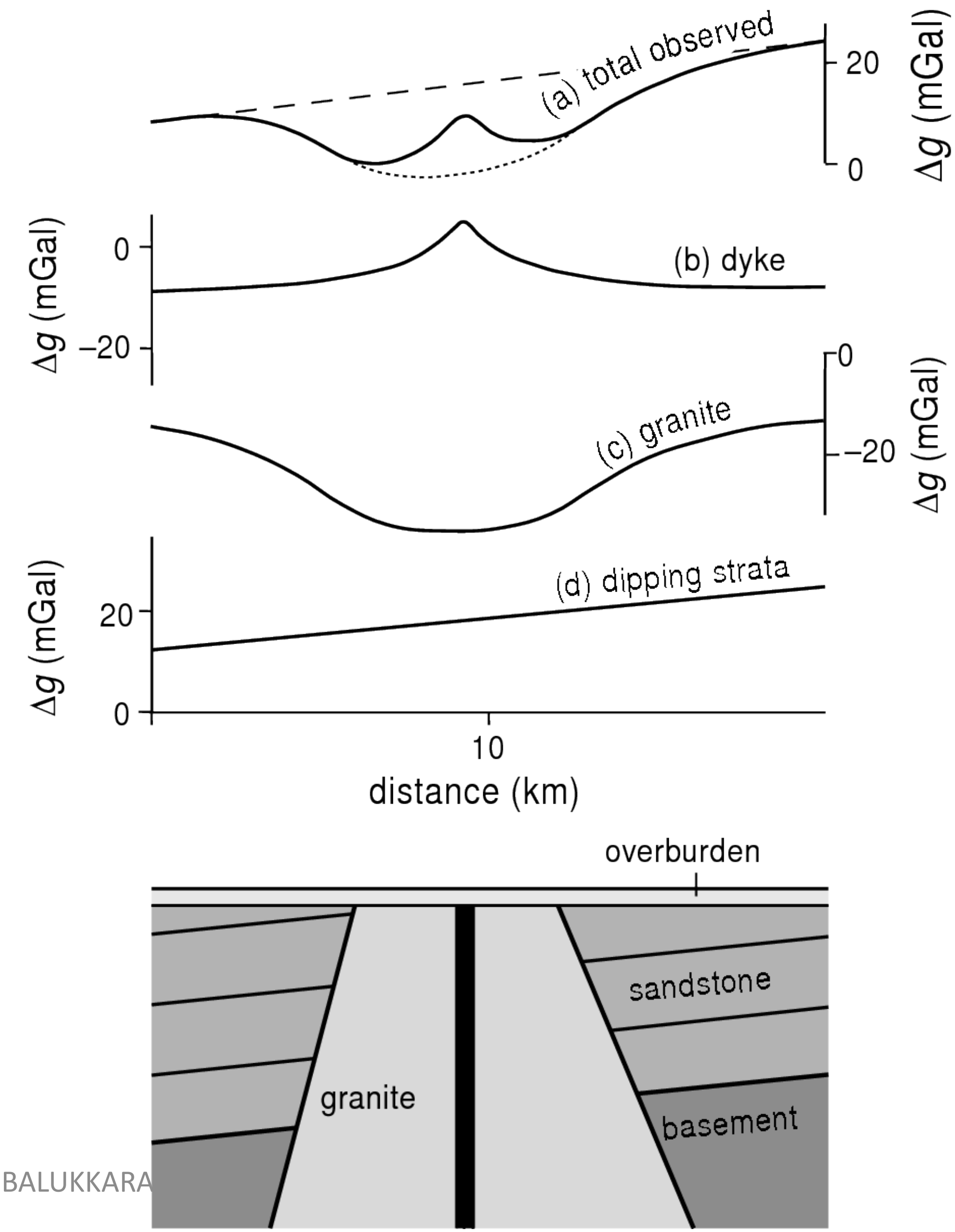


Regional and Residual Anomalies



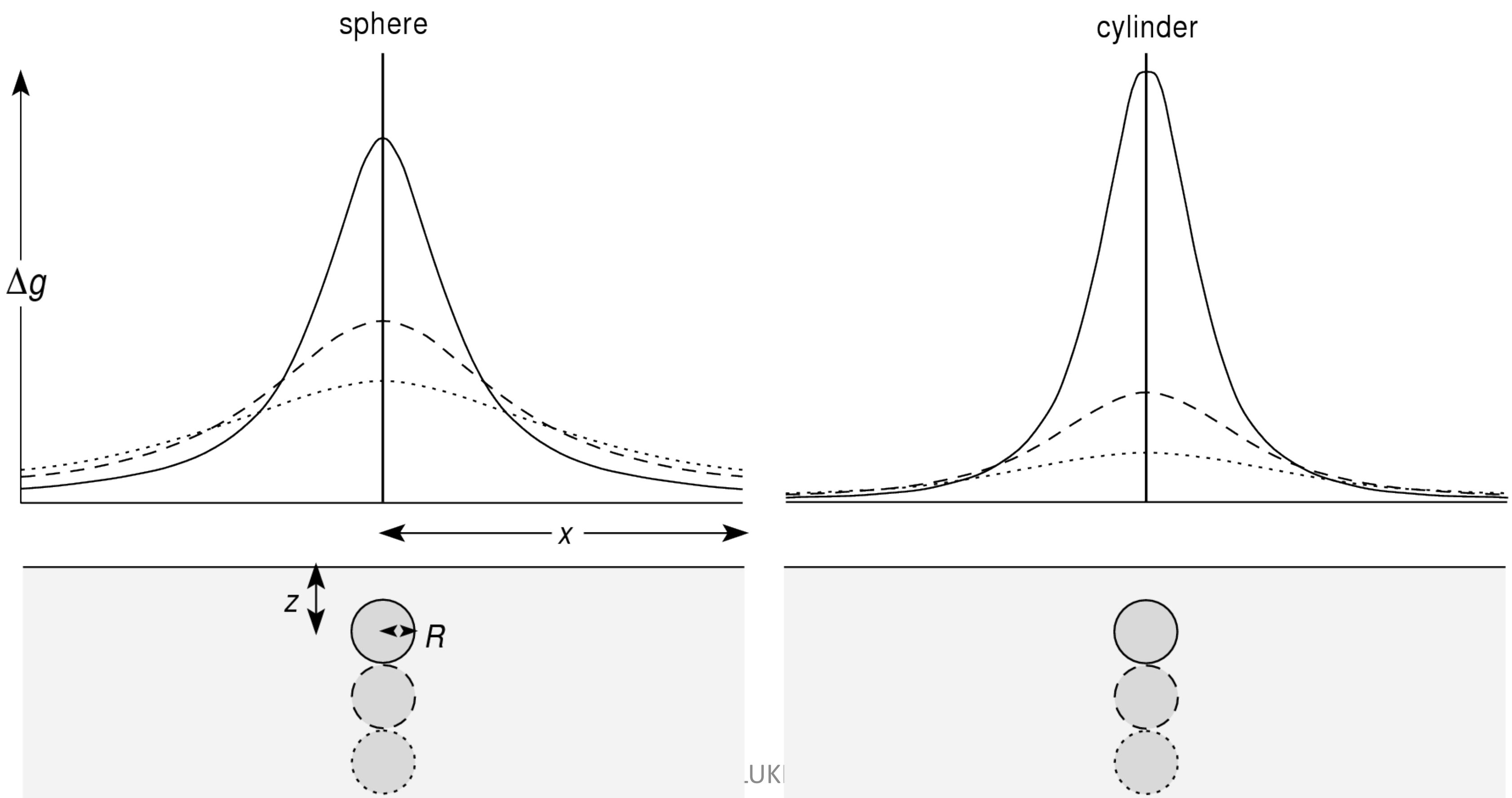
Regional and Residual Anomalies

- Because different targets produce anomalies of differing wavelengths
  - The total observed anomaly may be the sum of many targets
  - To isolate the local target, you can subtract off each regional anomaly



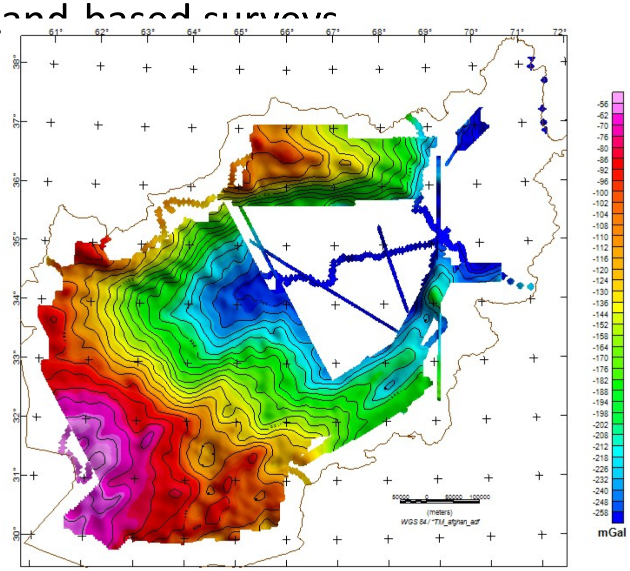
Planning a Gravity Survey

- Before conducting a gravity survey calculations should be made
  - Guess target size & geometry; look for geologic data on density contrasts
  - Calculate the likely anomaly magnitude and wavelength
  - Determine what accuracy is needed / Is survey likely to detect the anomaly?



Conducting a Gravity Survey

- Gravitimeters only measure δg…i.e they measure relative g
  - To get absolute g, compare to permanent reference station
  - Thus, surveys done by different devices yield different values of g
  - Because anomalies are broader than the target, surveys must extend well beyond the estimated target edge
  - Surveys that seek anomalies < 0.1 mGal (100 µGal) = **Microgravity**
    - Require very precise corrections (even for buildings!)
- Gravity surveys are typically one of four types
  - Marine surveys
  - Airborne surveys
  - Satellite surveys
  - Land-based surveys



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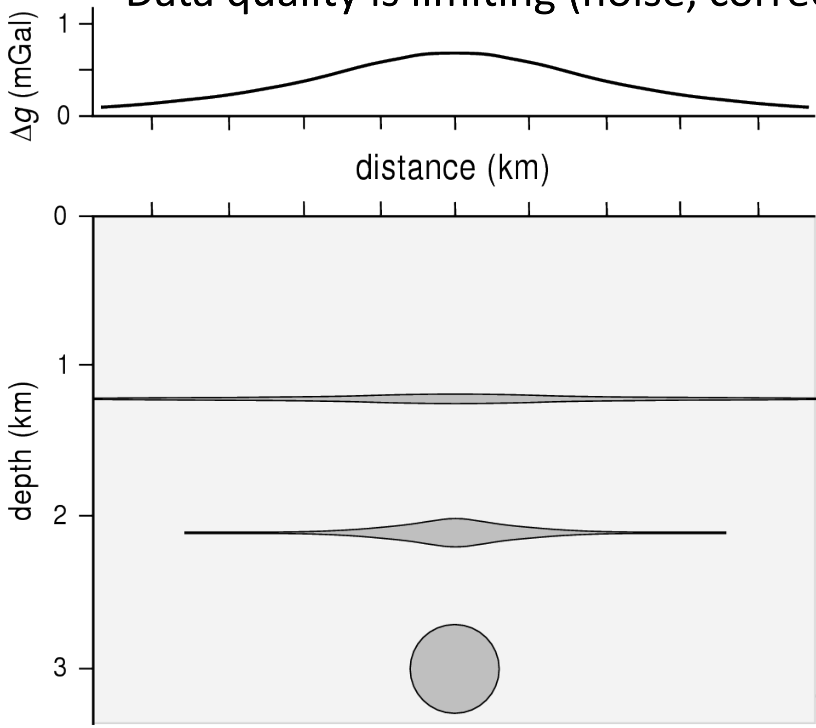
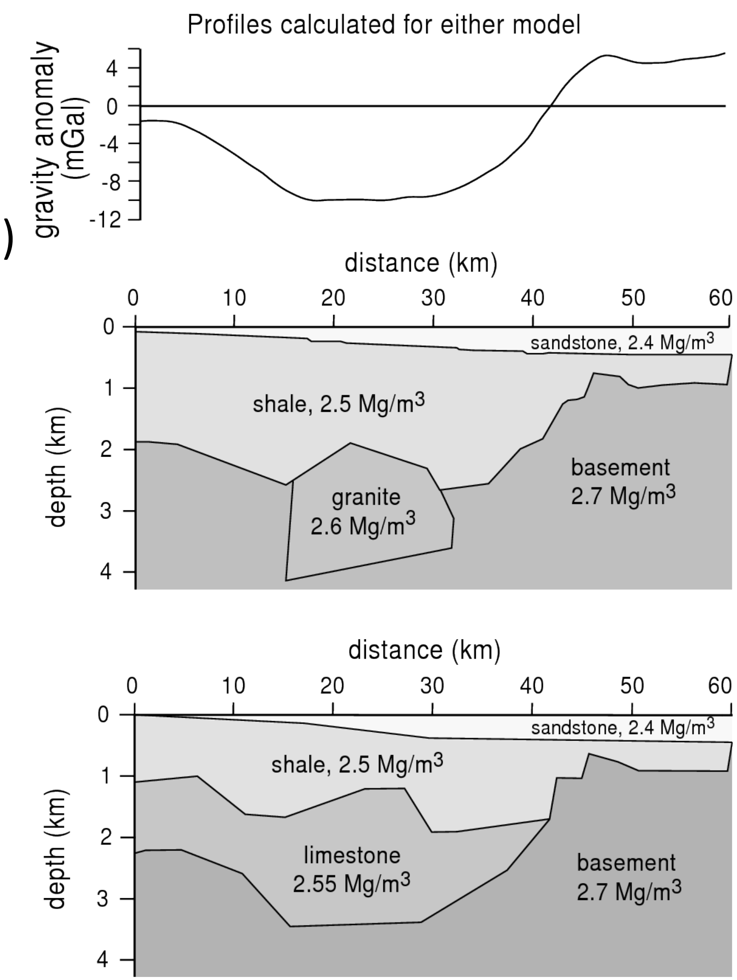
Modeling Gravity Data

- Gravity data suffers from the same caveats as most geophysical techniques
  - Modeling data is an inverse problem, so results are non-unique
    - You can calculate the predicted anomaly from a known body exactly
    - You cannot calculate the exact body from the anomaly
  - Data quality is limiting (noise, correction errors)





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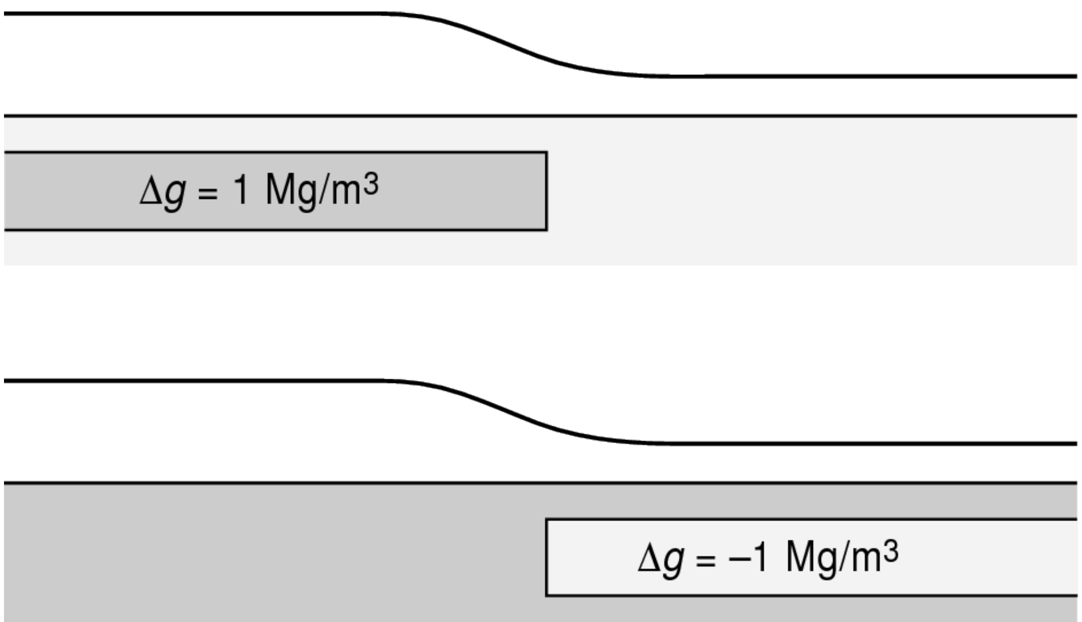
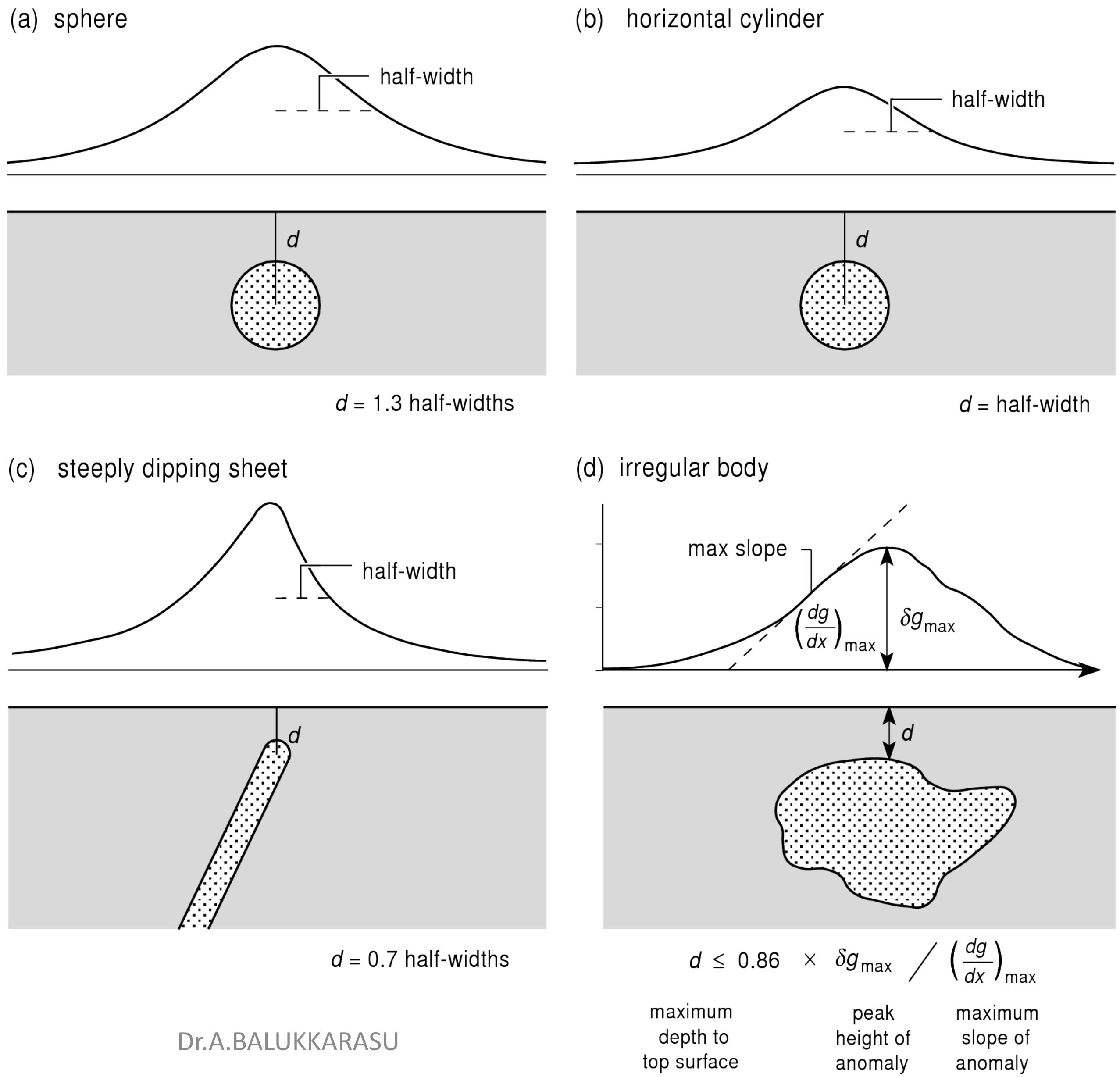
Modeling Gravity Data

- Recall that only the density contrast matters
  - So a half slab = 3.0 g/cm^3 embedded in a rock of 2.9 g/cm^3
  - Produces same anomaly as
  - Half slab = 2.7 g/cm^3 embedded in a rock of 2.6 g/cm^3
- Also, a mass excess can produce the same anomaly as a mass deficiency
- To resolve the likely scenario, know the local geology!



Depth Rules

- Because the anomaly depends on target depth, some general depth rules can help with inverse modeling

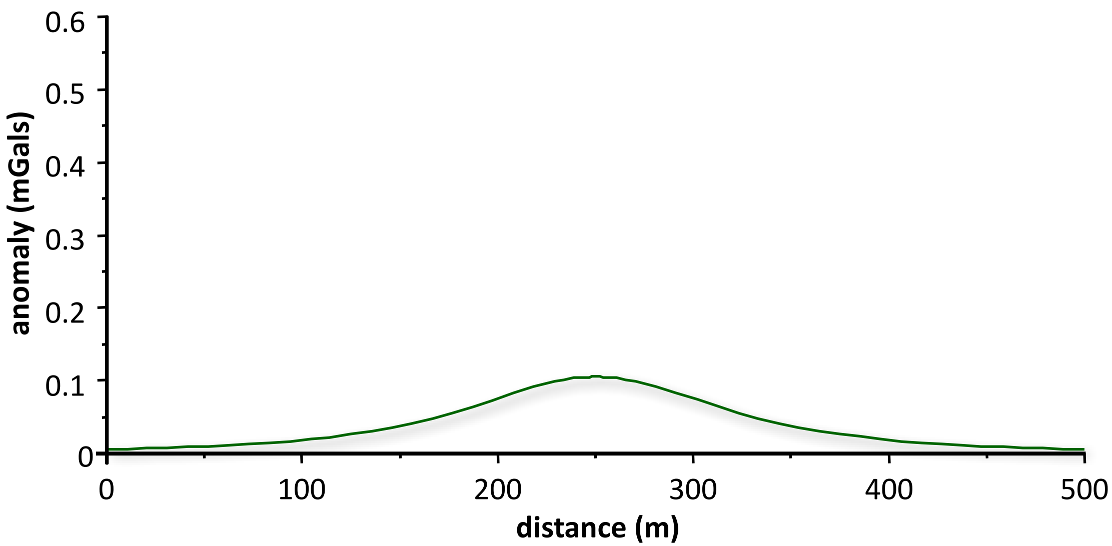


Total Excess Mass

While modeling gravity data suffers from non-uniqueness…

- Total excess mass can be uniquely calculated
  - Must be very careful with calculating the tails
    - Tails are low in magnitude, but can extend very far

$$M_{excess} = \frac{1}{2\pi G}(V_{anomaly})$$



– To calculate the mass of the body…

$$M_{body} = M_{excess}\left(\frac{\rho_{body}}{\rho_{body} - \rho_{surrounding\ rock}}\right)$$

Useful if you want to know the total mass of an ore body

What's Next?

What is the gravity signature of mountain ranges?