

TESTING OF HYPOTHESIS

STANDARD ERROR

What is Standard deviation? (SD)

When the amount of variability of the observations of a population is computed, it is called the **Standard Deviation**

What is Standard error? (SE)

When the amount of variability of the observations of a sampling distribution is computed, it is called the **Standard Error**

Test of Significance

Tests of significance to be applied under the following heads

- Test of Significance for Attributes
- Test of Significance for Variables (Large Samples)
- Test of significance for Variables (Small Samples)

Tests of Significance for Attributes

Various Types

1. Test for Number of Successes
2. Test for Proportion of Successes
3. Test for Difference between proportions

Test for Number of Successes

Formula:

$$\sqrt{npq}$$

n = Size of sample

p = Probability of success in each trial

q = (1-p) i.e, probability of failure

Example 1:

In a sample of 400 population from a village 230 are found to be eaters of vegetarian and the rest non- vegetarian items. Can we assume that both vegetarian and non-vegetarian food are equally popular?

Solution:

Let us take the hypothesis that both types of food are equally popular

$$n = 400; p = 1/2 \quad \text{and} \quad q = 1/2$$

Standard Error of no. of vegetarian = $\sqrt{npq}$

$$Z = \frac{\text{Difference}}{\text{S.E}} = \frac{\sqrt{400 * \frac{1}{2} * \frac{1}{2}}}{\frac{230 - 200}{10}} = 3$$

Since the difference observed and expected number of vegetable eaters is more than 1.96 S.E at 5% level of significance, the result of the experiment does not support hypothesis and vegetarian and non vegetarian food are not equally popular

A cultivator of bananas claims that only 3 out of 100 supplied by him are defective. A random sample of 700 bananas contained 45 defective bananas. Test whether the claim of cultivator is correct.

Solution: The cultivator claims only 3% of bananas are defective. Hence the 95% confidence limit given by $\bar{X} \pm 1.96 \text{ S.E.}$

$$p = \frac{3}{100} \text{ (Defective bananas) (0.03)}$$

$$q = 1 - \frac{3}{100} = \frac{97}{100} \text{ (0.97)}$$

$$n = 700$$

$$\begin{aligned} \text{S. E.} &= \sqrt{\frac{pq}{n}} \\ &= \sqrt{\frac{0.03 \times 0.97}{700}} \end{aligned}$$

$$S.E = 0.006$$

$$\begin{aligned} 95\% \text{ confidence limit} &= X + 1.96 S.E \\ &= 0.97 + 1.96 * 0.006 \\ &= 0.97 + 0.01176 \\ &= 0.95824 \text{ to } 0.98176 \end{aligned}$$

Out of 700 bananas, good bananas may lie between $0.98176 * 700$ or 987.23 and $0.95824 * 700$ or 670.76 or 671. That is good bananas lie between 671 and 687.

Thus the number of defectives expected to lie between 13 and 29. Since the actual defective is 45. Therefore, the cultivator's claim the only 3% defective cannot be accepted.

TEST FOR PROPORTION OF SUCCESSES

Instead of taking the number of success in each sample, a proportion of success i.e., $1/n$ is recorded.

Formula:

$$S.E = \sqrt{\frac{pq}{n}}$$

n = Size of sample

p = Probability of success in each trial

q = $(1-p)$ i.e, probability of failure

TEST FOR DIFFERENCE IN PROPORTION

We draw two samples from different populations and verify whether the population of success is significant or not

Formula:

$$\text{S.E } (P_1 - P_2) = \sqrt{pq \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}$$

Where **p** = the pooled estimate of the actual proportion in the population. The values of **p** is obtained by

$$p = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2}$$

$$q = 1 - p$$

Example 5:

One thousand articles from a factory are examined and found to be 3% defective. Fifteen hundred similar articles from a second factory are found to be only 2 % defective. Can it reasonably be concluded that the product of the first factory is inferior to the second?

Solution:

Let us set up the null hypothesis $H_0 : P_1 = P_2$

$$P_1 = \frac{3}{100} = 0.03 \qquad P_2 = \frac{2}{100} = 0.02$$

$$p = \frac{(1000 * 0.03) + (1500 * 0.02)}{1000 + 1500} = \frac{(30 + 30)}{2500} = 0.024$$

$$\begin{aligned} \text{S.E (} P_1 - P_2 \text{)} &= \sqrt{pq \left(\frac{1}{n_1} + \frac{1}{n_2} \right)} \\ &= \sqrt{0.024 * 0.976 \left(\frac{1}{1000} + \frac{1}{1500} \right)} = ? \end{aligned}$$

$$z = \frac{0.03 - 0.02}{0.0167} = 1.197$$

At 95% level of confidence, $z = 1.96$, the difference is not significant. The null hypothesis that is, $P_1 = P_2$ is accepted

Test of Significance of Large Samples

- If the sample size is greater than 30 i.e., if $n > 30$, the those samples may be regarded as large samples.
- In the case of large samples, when we are testing, the significance of statistic, the concept of standard error is used. The following are the formula for finding out the standard error for different statistics.

The standard error of Mean

It measures only sampling errors

- When standard deviation of the population is known, the formula is

$$\text{S.E. } \bar{X} = \frac{\sigma_p}{\sqrt{n}}$$

-----> Standard deviation of the population

-----> Number of observations in the sample

- When the standard deviation of population is not known, we have to use the standard deviation of the sample in calculating standard error of mean

$$\text{S.E. } \bar{X} = \frac{\sigma (\text{sample})}{\sqrt{n}}$$

Note: if SD of sample and population are available, then for the calculation of standard error of mean, we must use SD of the population.

Example: A company manufacturing electric light bulbs claims that the average life of its bulbs is 1600 hours. The average life and SD of a random sample of 100 such bulbs were 1570 hours and 120 hours respectively. Should we accept the claim of the company.

Solution:

$$\text{S.E. } \bar{X} = \frac{\sigma}{\sqrt{n}} = \frac{120}{\sqrt{100}} = 12$$

$$\begin{aligned} Z &= \text{Difference} / \text{S.E. } \bar{X} \\ &= (1600 - 1570) / 12 \\ &= 30 / 12 \quad \quad \quad = 2.5 \end{aligned}$$

2.5 > 1.96 S.E. at 5% level of significance, the hypothesis cannot be accepted. We cannot accept the claim of the company

Testing the difference between means of two samples

- If two independent random samples with n_1 and n_2 respectively are drawn from the **same population** of SD σ , the standard error of the difference between the same means is given:

$$\text{S.E. } (\bar{X}_1 - \bar{X}_2) = \sqrt{\sigma^2 (1/n_1 + 1/n_2)}$$

- If two random samples are drawn from **different populations**, then the S.E. of the difference between the means is given:

$$\sqrt{\sigma_1^2 / n_1 + \sigma_2^2 / n_2}$$

if σ and σ are unknown, S.E. of the difference between mean

$$= \sqrt{S_1^2 / n_1 + S_2^2 / n_2}$$

Where s_1 and s_2 represent standard deviations of the two samples

Example: Intelligence test on two groups of boys and girls gave the following results:

	Mean	SD	N
Girls	75	15	150
Boys	70	20	250

Is there a significant difference in the mean scores obtained by boys and girls?

Solution : Let us take the hypothesis that there is no significant difference in the mean scores obtained by girls boys and girls.

$$\text{S.E. (} \bar{X}_1 - \bar{X}_2 \text{) } = \sqrt{\sigma_1^2 / n_1 + \sigma_2^2 / n_2}$$

$$\begin{aligned} \sigma_1 &= 15 \\ \sigma_2 &= 20 \\ n_1 &= 150 \\ n_2 &= 250 \end{aligned}$$

Substituting the values:

$$\text{S.E. (} \bar{X}_1 - \bar{X}_2 \text{) } =$$

$$Z = \text{Difference} / \text{S.E} =$$

Since the difference is more than 2.58 S.E (1% level of significance), the hypothesis is rejected. There seems to be a significant difference in the mean scores obtained by boys and girls

Standard Error of the difference between two standard deviations:

- S.E of the difference between the standard deviations is given by:

$$\text{S.E. } (\sigma_1 - \sigma_2) = \sqrt{\sigma_1^2 / 2n_1 + \sigma_2^2 / 2n_2}$$

When population standard deviations are not known

$$\text{S.E. } (\sigma_1 - \sigma_2) = \sqrt{S_1^2 / 2n_1 + S_2^2 / 2n_2}$$

Example: Productivity test of two food articles – paddy and wheat gives the following results:

	Mean yield	SD	No. of hectares
paddy	80	10	120
wheat	75	12	90

Is the difference between standard deviation is significant?

Solution : Let us take the hypothesis that there is no significant difference in the standard deviation of productivity of paddy and wheat..

$$S.E. (\sigma_1 - \sigma_2) = \sqrt{\sigma_1^2 / 2n_1 + \sigma_2^2 / 2n_2}$$

$$\sigma_1 = 10$$

$$\sigma_2 = 12$$

$$n_1 = 120$$

$$n_2 = 90$$

$Z = \text{Difference} / S.E =$

Since the $Z < 1.96$ (5% level of sign), the given data support the hypothesis. Thus we conclude that there is no significant difference in SD of the productivity between Paddy and Wheat.

Test of Significance of Small Samples

- If the sample size is greater than 30 i.e., if $n < 30$, the those samples may be regarded as small samples.
- The methods and theory of small samples are applicable to large samples, but the reverse is not accepted.

Students' t-Distribution

- The greatest contribution to the theory of small samples is students' t-distribution.
- When the sample size is 30 or less and the population standard deviation is unknown, we can use the t-distribution.

Application of t-Distribution

- ⦿ Test of hypothesis about the population mean
- ⦿ Test of hypothesis about the difference between two means
- ⦿ Test of hypothesis about the difference between two means with dependent samples
- ⦿ Test of hypothesis about coefficient of correlation

Test of hypothesis about the population mean

Formula:

$$t = \frac{\bar{X} - \mu}{S} \times \sqrt{n}$$

Where

S – Standard Deviation of the sample

$$S = \sqrt{\frac{\sum (X - \bar{X})^2}{n - 1}}$$

and

v = n-1, the number of degrees of freedom (df)

Example :The following results are obtained from a sample of 10 boxes of biscuits:

Mean weight of contents = 490 gms.

Standard deviation of the weight = 9 gms

Could the sample come from a population having a mean of 500 gms.

SOLUTION:

Let us take the hypothesis that $\mu = 500$ gms.

$$t = \frac{\bar{X} - \mu}{S} \times \sqrt{n} \quad \bar{X} = 490; \mu = 500; \sigma = 9; n = 10$$

$$t = \frac{490 - 500}{9} \times \sqrt{10} = 3.51$$

$$\text{df or } v = n-1 = 10 - 1 = 9$$

$$\text{and } t_{0.01} = 3.25$$

$3.51 > 3.25$ our hypothesis is rejected

Test of hypothesis about the difference between two mean (Independent Samples)

Formula:

$$t = \frac{\bar{X}_1 - \bar{X}_2}{S} \times \sqrt{\frac{n_1 n_2}{n_1 + n_2}}$$

Where

-- Mean of the first Sample -- Mean of the Second Sample

n_1, n_2 -- No. of observations in the first and second sample respectively.

S -- Combined standard Deviation

The value of S is calculated by the following formula

$$S = \sqrt{\frac{\sum (X - \bar{X}_1)^2 + (X - \bar{X}_2)^2}{n_1 + n_2 - 2}}$$

When deviations are taken from assumed mean

Where A_1, A_2 are the assumed mean of the first and second sample

$$S = \sqrt{\frac{\sum (X_1 - A_1)^2 + (X_2 - A_2)^2 + n_1 (\bar{X}_1 - A_1)^2 + n_2 (\bar{X}_2 - A_2)^2}{n_1 + n_2 - 2}}$$

EXAMPLE Two types of diet were used on 7 and 5 chickens reared in a research station. The weight of the chickens which were fed with high protein diet is given bellow.

13 16 12 17 15 17 15 Ounces

The weight of the chickens which were fed with low protein diet is given below:

9 11 15 11 14 Ounces

Test where there is significant evidence that additional diet has increased the weight of chickens. (the table value of 't' for v=10 at 5% level of significance is 223).

Solution

Let us take the hypothesis that addition protein has not increased the weight of chickens.

$$t = \frac{\bar{X}_1 - \bar{X}_2}{S} \times \sqrt{\frac{n_1 n_2}{n_1 + n_2}}$$

X1	X1 -X1	(X1 -X1) ²	X2	(X2 - X2)	(X2 - X2) ²
13	-2	4	9	-3	9
16	1	1	11	31	1
12	-3	9	15	3	9
17	2	4	11	-1	1
15	0	0	14	2	4
17	2	4			
15	0	0			
ΣX1 =105	Σ(X1 -X1)=0	Σ(X1 -X1)² =22	ΣX2 = 60	Σ(X2 - X2) =0	Σ(X2 - X2)² =24

$$S = \sqrt{\frac{\sum(X - \bar{X}_1)^2 + (X - \bar{X}_2)^2}{n_1 + n_2 - 2}}$$

$$= 2.14$$

$$t = \frac{\bar{X}_1 - \bar{X}_2}{S} \times \sqrt{\frac{n_1 n_2}{n_1 + n_2}}$$

$$= 2.397$$

$$v = n_1 + n_2 - 2 = 7 + 5 - 2 = 10$$

For $V = 10$, $t_{0.05} = 2.23$

The calculated value of t is greater than the table value and hence the hypothesis does not hold good. Thus, we shall conclude that additional protein has increased the weight of chickens.

Test of hypothesis about the difference between two mean (Dependent Samples)

Formula:

$$t = \frac{\bar{d} - 0}{S} \times \sqrt{n} \quad (\text{or}) \quad t = \frac{\bar{d}\sqrt{n}}{S}$$

Where

$\bar{d}$ -- is the mean of the difference of the paired values..

S -- Standard Deviation of the differences

The value of S is calculated by the following formula

$$S = \sqrt{\frac{\sum (d - \bar{d})^2}{n - 1}} \quad (\text{or}) \quad S = \sqrt{\frac{\sum (d^2 - (\bar{d})^2 \times n)}{n - 1}}$$

EXAMPLE:

An IQ test was administered to 5 persons before and after they ere trained the results are given below:

Candidates	1	2	3	4	5
IQ before Training	110	120	123	132	125
IQ after Training	120	118	125	136	121

Test whether there is any change in IQ after the training programme. (df →4 at $t_{0.01} = 4.6$)

Solution:

null hypo $H_0: d = 0$

alt. hypo $h_1 = d \neq 0$

Calculation of d and d^2 and the value of S

Candidates	IQ before	IQ After	d	d^2
1	110	120	10	100
2	120	118	-2	4
3	123	125	2	4
4	132	136	4	16
5	125	121	-4	16
			10	140

$$d = \sum d / n = 10 / 5 = 2$$

$$= 5.48$$

$$S = \sqrt{\frac{\sum (d^2 - (\bar{d})^2 \times n)}{n - 1}} = 0.816$$

$$t = \frac{\bar{d} - 0}{S} \times \sqrt{n}$$

The calculated value of t being less than the table value 4.6, the difference is insignificant. the null hypothesis is accepted. The training has not changed the IQ

Test of hypothesis about the coefficient of correlation

If we are to test the hypothesis that the correlation coefficient of the population is zero, that is, the variables in the population are uncorrected. we have to apply the following test:

Formula:

$$t = \frac{r}{\sqrt{1 - r^2}} \times \sqrt{n - 2}$$

where

$n-2$ -- degrees of freedom

If the calculated value of t exceeds $t_{0.05}$ for $(n-2)$ d.f., we say that the value of r is significant at 5% level. if $t < t_{0.05}$ the data are consistent with the hypothesis of an uncorrected population.

Example:

A random sample of 18 pairs of observation from a normal population gives a correlation co-efficient of 0.52. is it likely that the variables in the population are correlated?

solution :

Let us take the hypothesis that the variable in the population are uncorrelated

Applying t-test

$$t = \frac{r}{\sqrt{1 - r^2}} \times \sqrt{n - 2}$$

$r = 0.52$; $n = 18$

$$t = 2.4$$

degrees of freedom = $18 - 2 = 16$

For $v = 16$, $t_{0.05} = 2.12$

Since the calculated value of t is greater than the table value, we cannot accept the hypothesis. hence we may conclude that variable in the population are correlated.