Hypothesis Testing

- ✓ Overview
- ✓ Fundamentals of Hypothesis Testing
- ✓ Testing a Claim about a Mean: Large Samples
- ✓ Testing a Claim about a Mean: Small Samples
- ✓ Testing a Claim about a Proportion
- ✓ Testing a Claim about a Standard Deviation or Variance

Hypothesis

in statistics, is a claim or statement about a property of a population

Hypothesis Testing

is to test the claim or statement

<u>Example</u>: A conjecture is made that "the average starting salary for computer science gradate is $30,000 per year".

Question:

How can we justify/test this conjecture?

A. What do we need to know to justify this conjecture?

B. Based on what we know, how should we justify this conjecture?

Answer to A:

Randomly select, say 100, computer science graduates and find out their annual salaries

---- We need to have some <u>sample observations</u>, i.e., <u>a sample set</u>!

Answer to B:

That is what we will learn in this chapter

---- <u>Make conclusions</u> based on the sample observations

Statistical Reasoning

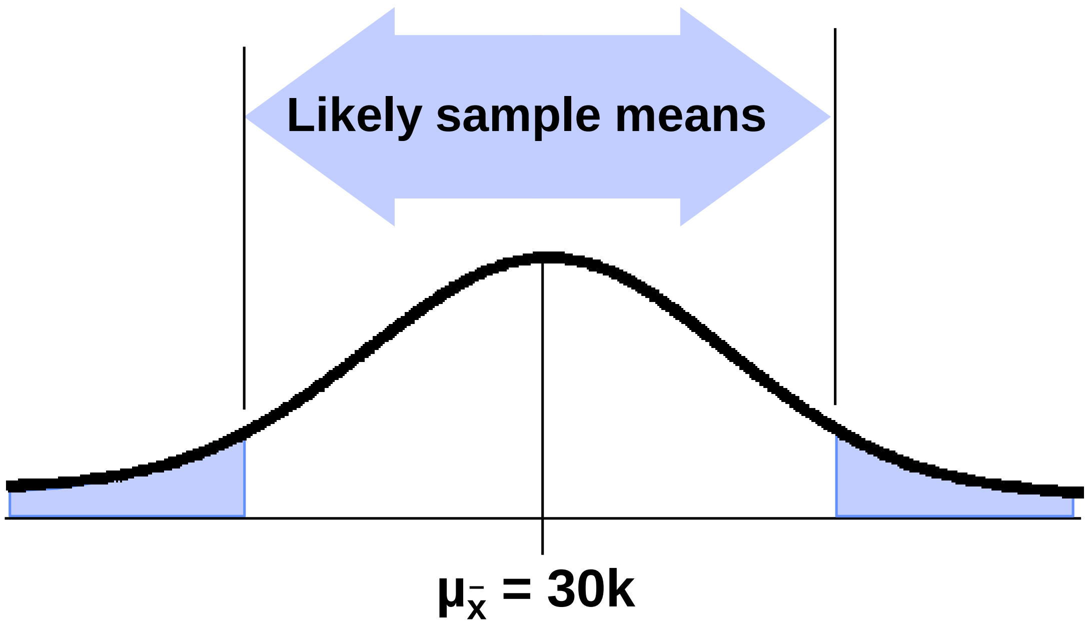
Analyze the sample set in an attempt to distinguish between results that can easily occur and results that are highly unlikely.

Central Limit Theorem:

Central Limit Theorem:

Distribution of Sample Means

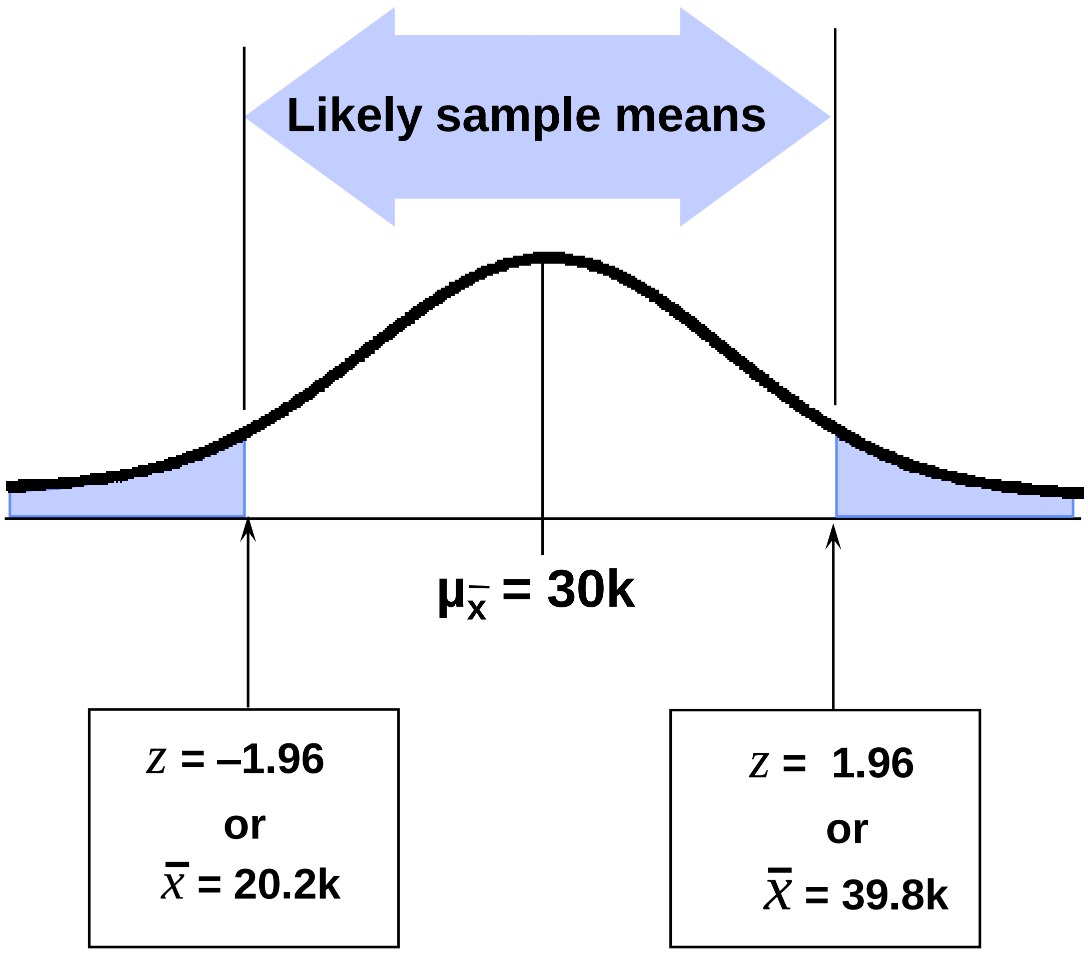
Assume the conjecture is true!



Central Limit Theorem:

Distribution of Sample Means

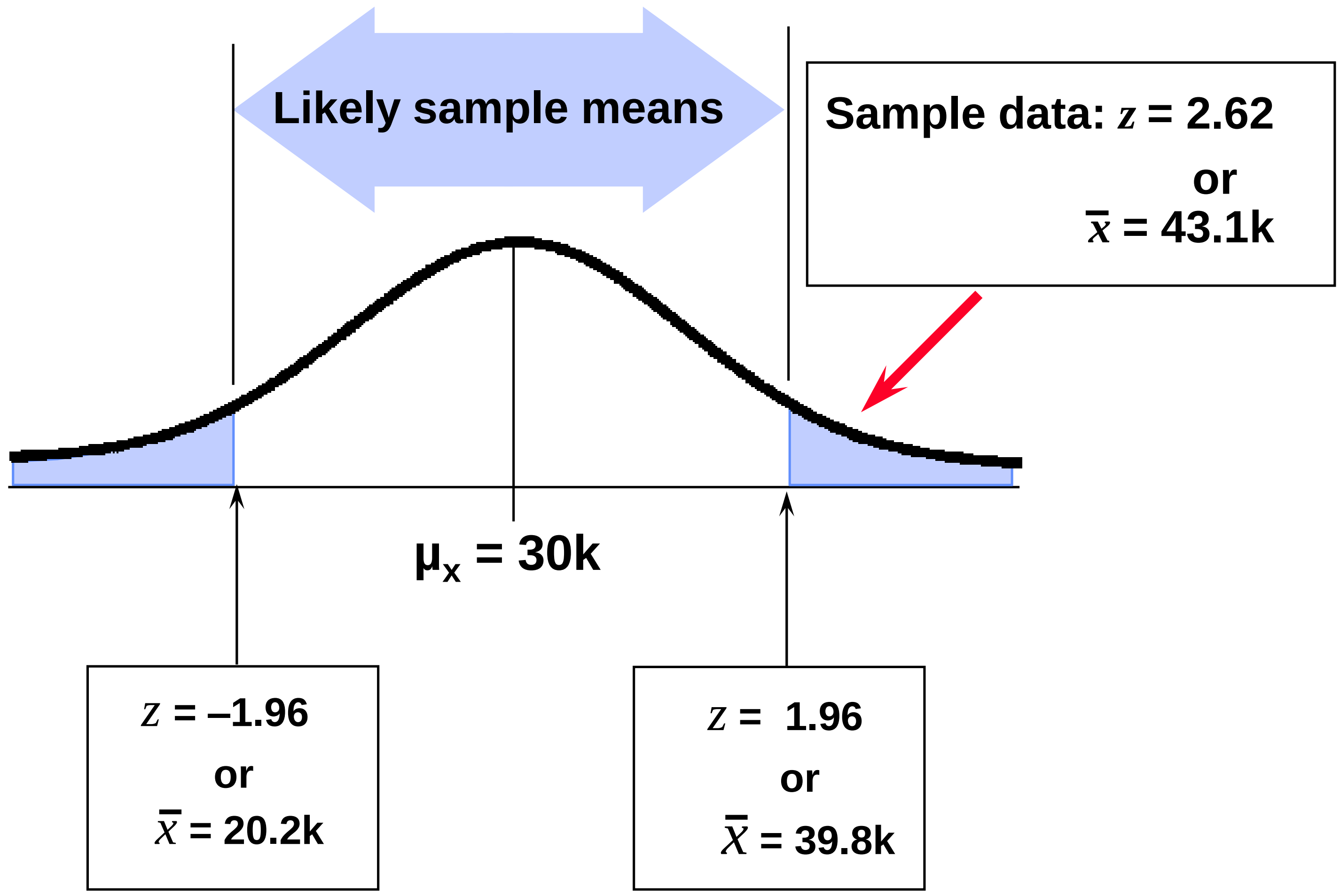
Assume the conjecture is true!



Central Limit Theorem:

Distribution of Sample Means

Assume the conjecture is true!



Components of a Formal Hypothesis Test

Definitions

- **Null Hypothesis** (denoted H_0):

  is the statement being tested in a test of hypothesis.

- **Alternative Hypothesis** (H_1):

  is what is believe to be true if the null hypothesis is false.

Null Hypothesis: H_0

- Must contain condition of equality
- $=$, $\geq$, or $\leq$
- Test the Null Hypothesis directly
- Reject H_0 or fail to reject H_0

Alternative Hypothesis: H_1

- **Must be true if H_0 is false**
- **$\neq$, <, >**
- **'opposite' of Null**

Example:

$$H_0: \mu = 30 \quad \text{versus } H_1: \mu > 30$$

Stating Your Own Hypothesis

- If you wish to **support** your claim, the claim must be stated so that it becomes the **alternative hypothesis.**

Important Notes:

- **H_0 must always contain equality; however some claims are not stated using equality. Therefore <u>sometimes the claim and H_0 will not be the same</u>.**

- **<u>Ideally</u> all claims should be stated that they are Null Hypothesis so that the most serious error would be a Type I error.**

Type I Error

- The mistake of rejecting the null hypothesis when it is true.

- The **probability** of doing this is called the **significance level**, denoted by α (alpha).

- Common choices for α: 0.05 and 0.01

- **Example:** rejecting a perfectly good parachute and refusing to jump

Type II Error

- the mistake of failing to reject the null hypothesis when it is false.

- denoted by *ß* (beta)

- **Example:** failing to reject a defective parachute and jumping out of a plane with it.

Table 7-2

Type I and Type II Errors

		True State of Nature	
		The null hypothesis is true	The null hypothesis is false
Decision	We decide to reject the null hypothesis	Type I error (rejecting a true null hypothesis)	Correct decision
	We fail to reject the null hypothesis	Correct decision	Type II error (failing to reject a false null hypothesis)

Definition

Test Statistic:

is a sample statistic or value based on sample data

Example:

$$z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma / \sqrt{n}}$$

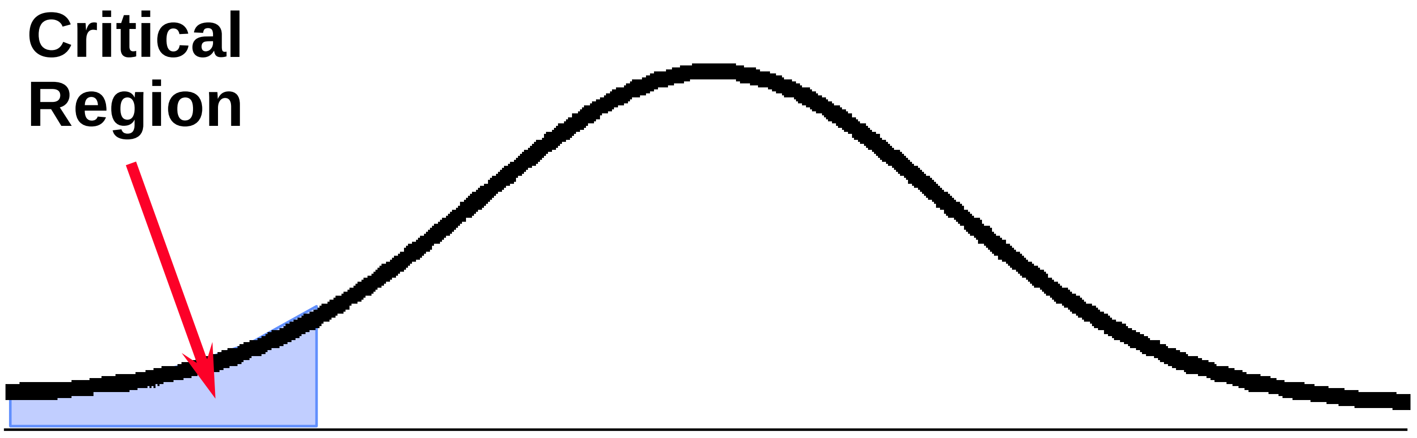
Definition

Critical Region :

is the set of all values of the test statistic that would cause a rejection of the null hypothesis

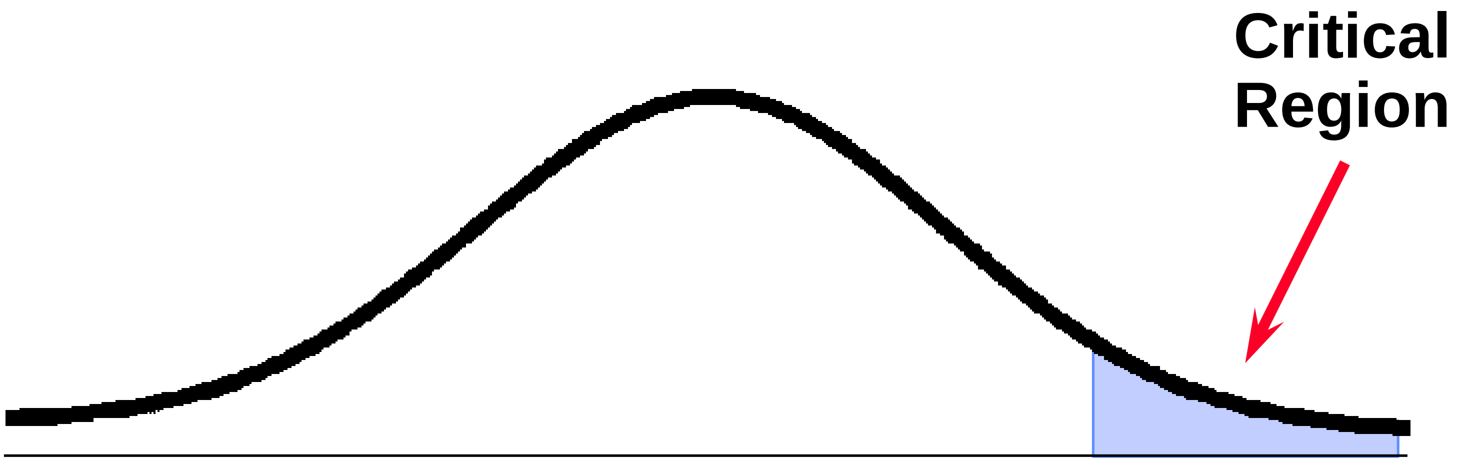
Critical Region

- Set of all values of the test statistic that would cause a rejection of the null hypothesis



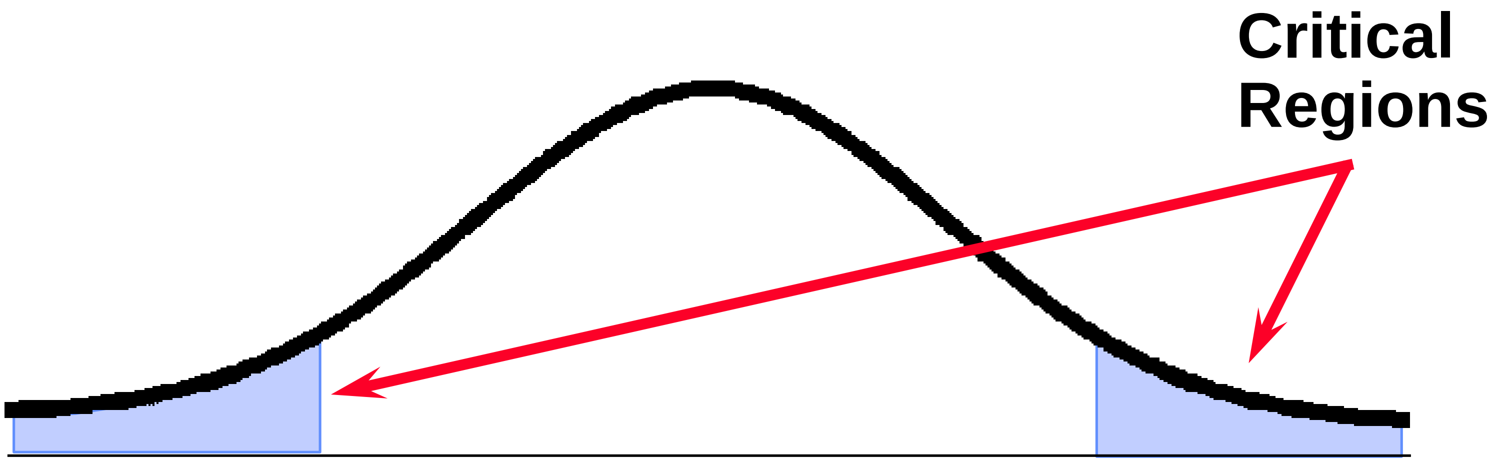
Critical Region

- Set of all values of the test statistic that would cause a rejection of the
- null hypothesis



Critical Region

- Set of all values of the test statistic that would cause a rejection of the null hypothesis



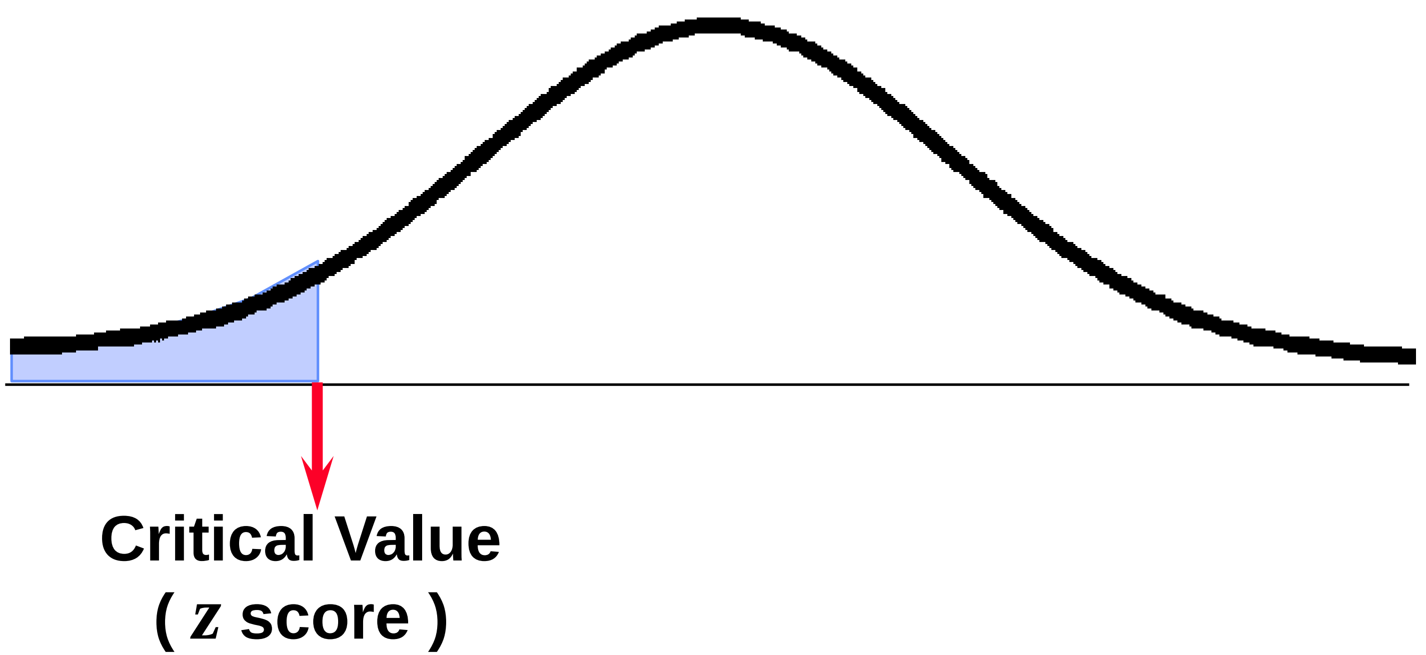
Definition

Critical Value:

is the value (s) that separates the critical region from the values that would not lead to a rejection of H_0

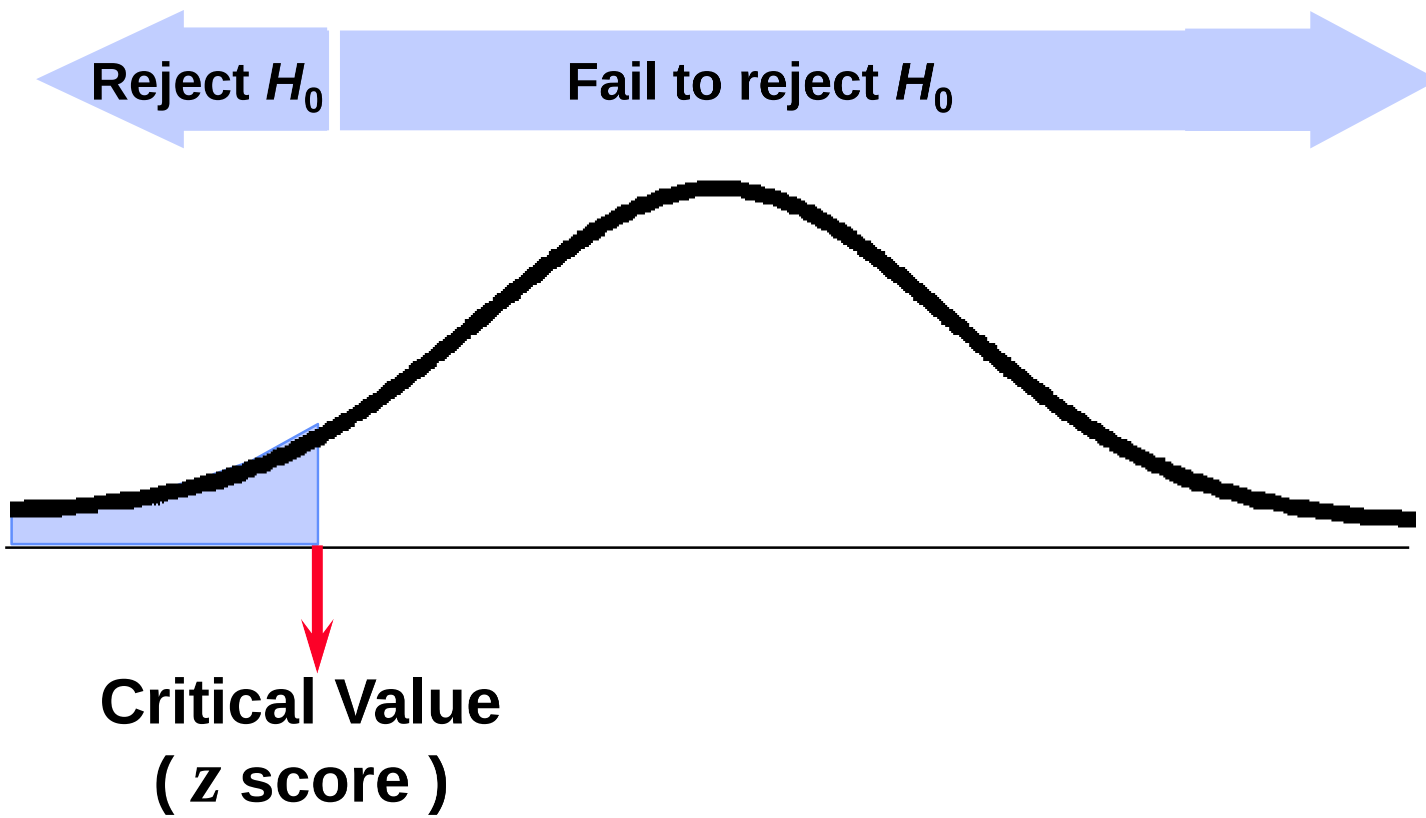
Critical Value

Value (s) that separates the critical region from the values that would not lead to a rejection of H_0



Critical Value

Value (s) that separates the critical region from the values that would not lead to a rejection of H_0



Controlling Type I and Type II Errors

- α, β, and n are related

- when two of the three are chosen, the third is determined

- α and n are usually chosen

- try to use the largest α you can tolerate

- if Type I error is serious, select a smaller α value and a larger n value

Conclusions in Hypothesis Testing

- always test the null hypothesis

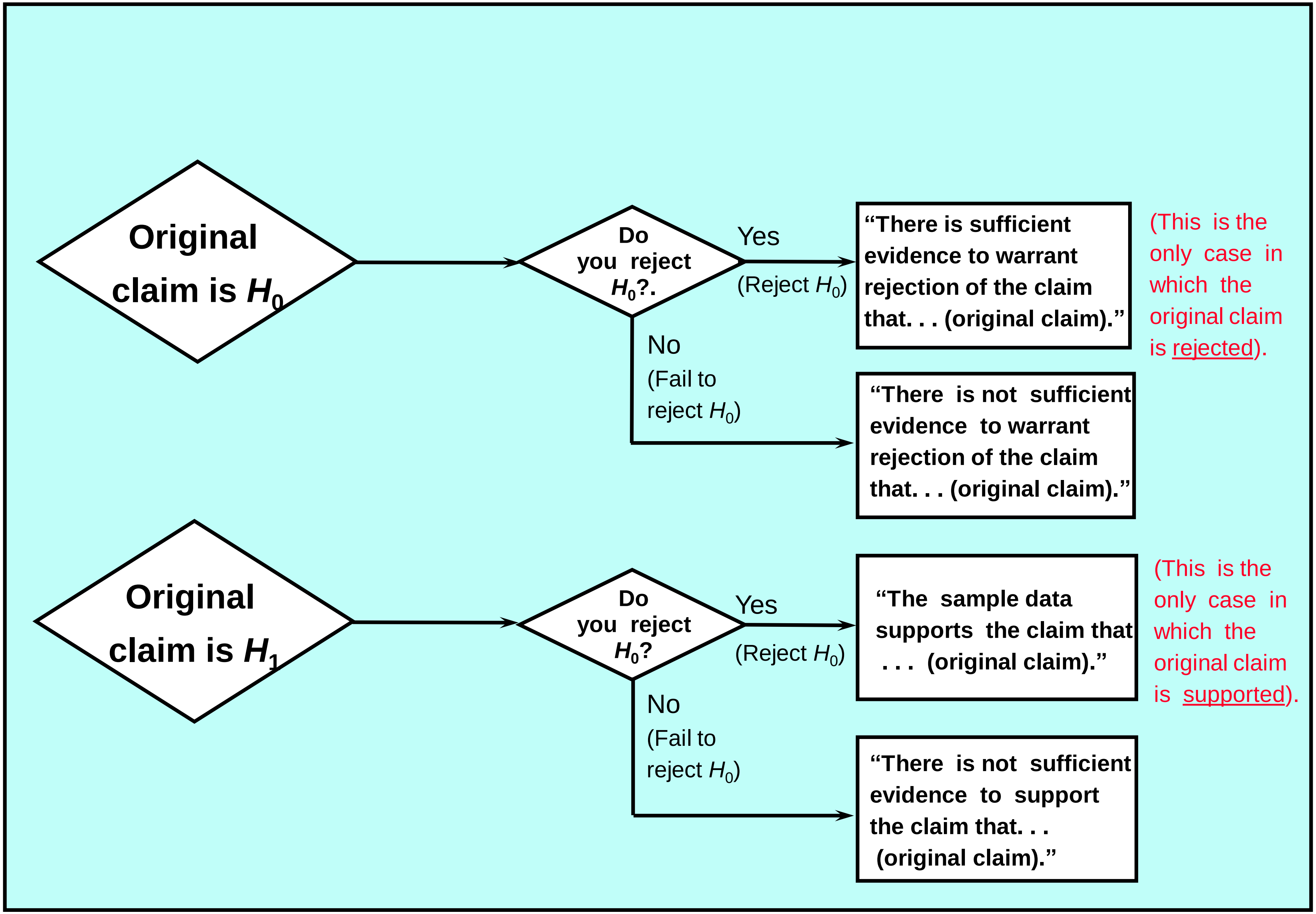
  1. Fail to reject the H_0

  2. Reject the H_0

- need to formulate correct wording of final conclusion

See Figure 7-2

Wording of Conclusions in Hypothesis Tests



Two-tailed, Left-tailed, Right-tailed Tests

Left-tailed Test

H_0: $\mu \geq 200$

H_1: $\mu < 200$

Left-tailed Test

H_0: $\mu \geq 200$

H_1: $\mu < 200$

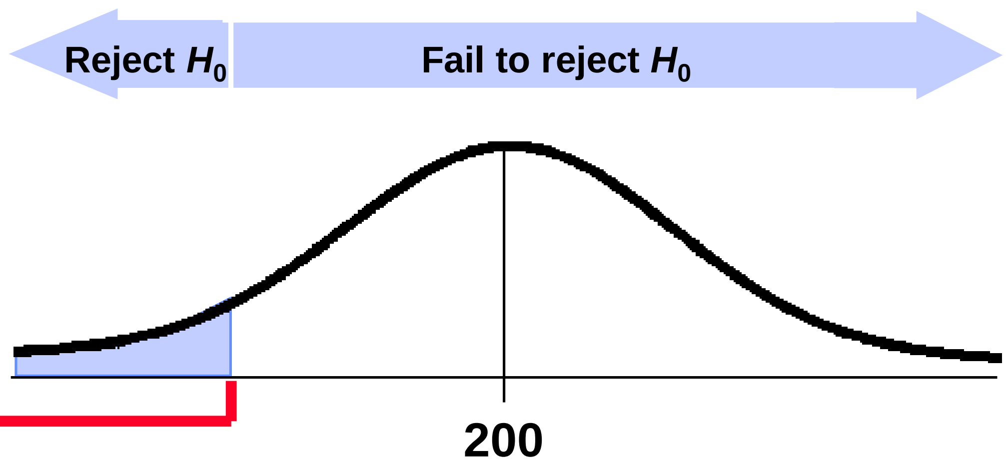
Points Left

Left-tailed Test

H_0: $\mu \geq 200$

H_1: $\mu < 200$

Points Left



Values that differ significantly from 200

Right-tailed Test

H_0: $\mu \leq 200$

H_1: $\mu > 200$

Right-tailed Test

H_0: $\mu \leq 200$

H_1: $\mu > 200$

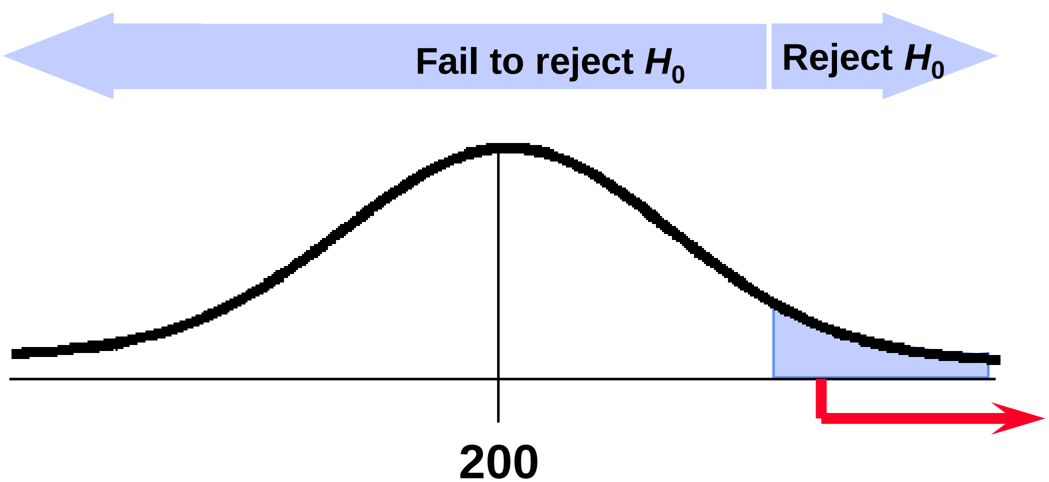
Points Right

Right-tailed Test

$$H_0\!: \mu \le 200$$

$$H_1\!: \mu > 200$$

Points Right



Values that differ significantly from 200

Two-tailed Test

H_0: $\mu = 200$

H_1: $\mu \neq 200$

Two-tailed Test

H_0: $\mu = 200$

H_1: $\mu \neq 200$

α is divided equally between the two tails of the critical region

Two-tailed Test

H_0: $\mu = 200$

H_1: $\mu \neq 200$

α is divided equally between the two tails of the critical region

Means less than or greater than

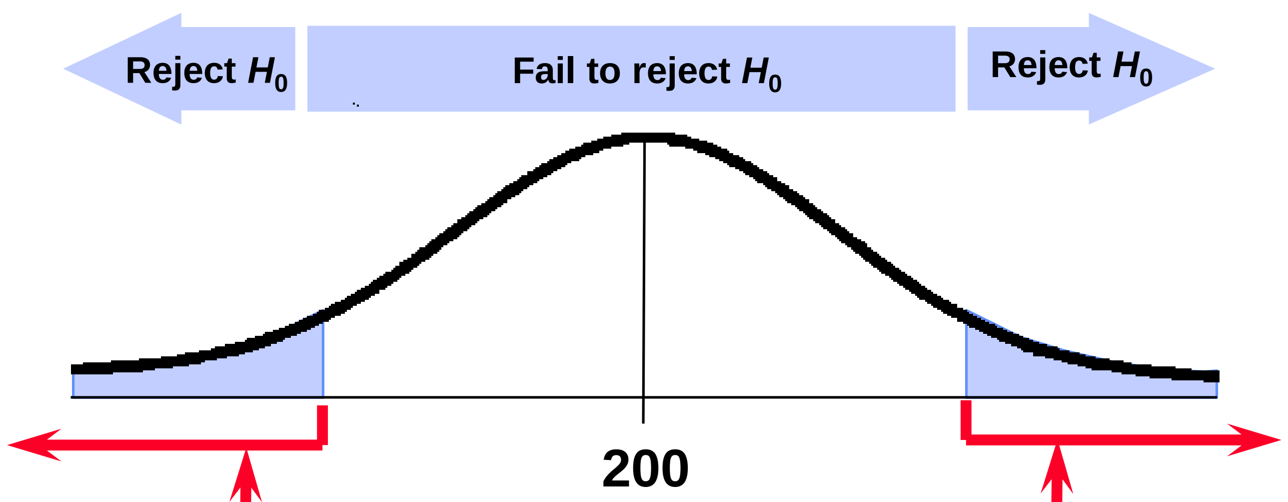
Two-tailed Test

H_0: $\mu = 200$

H_1: $\mu \neq 200$

α is divided equally between the two tails of the critical region

Means less than or greater than



Values that differ significantly from 200