Condensed Matter Physics

Magnetic Flux Quantization

by

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Flux quantization in a S/C ring

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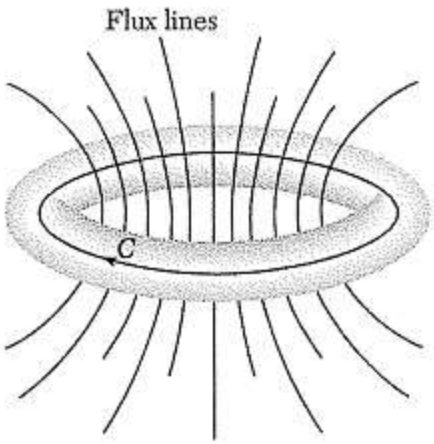
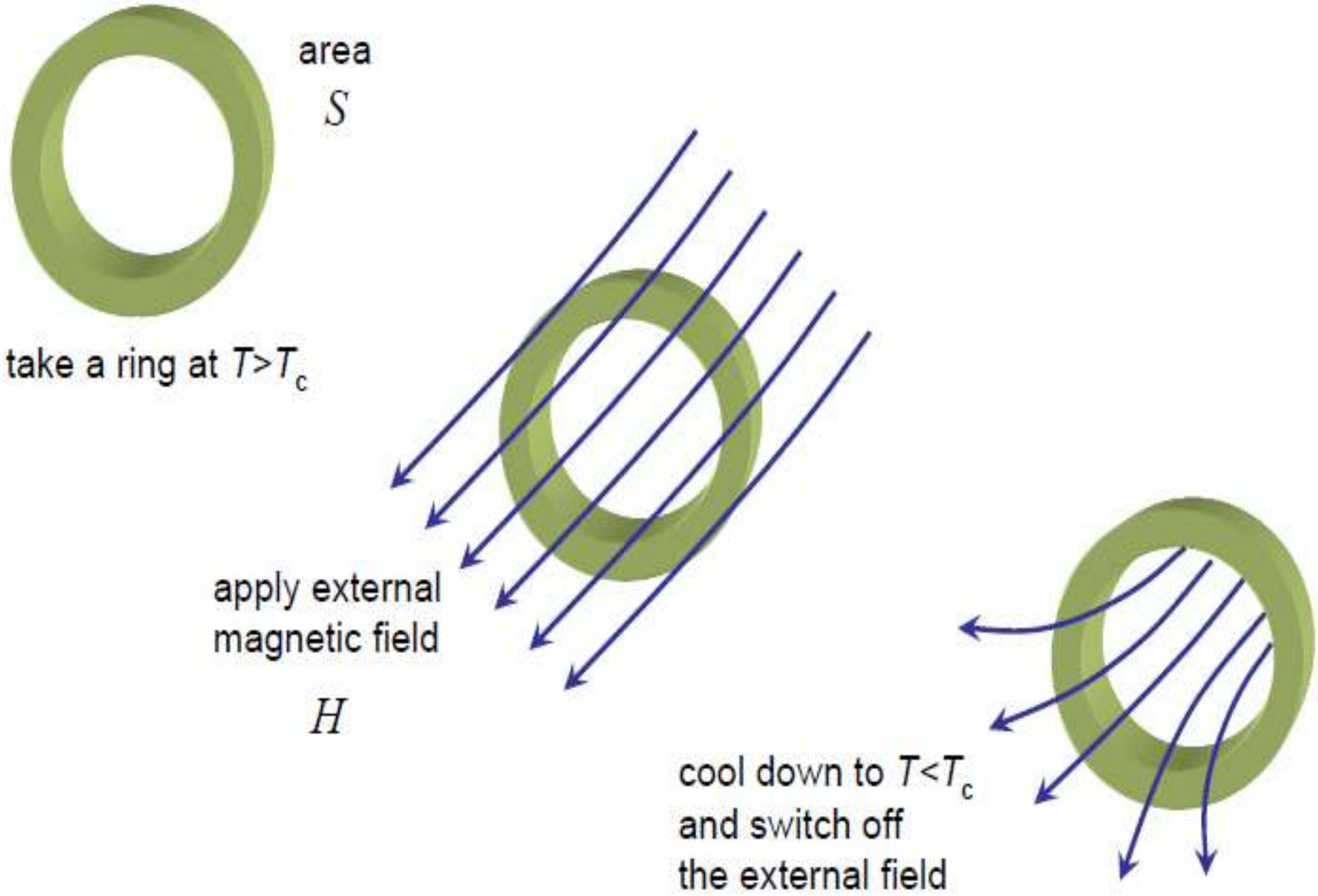
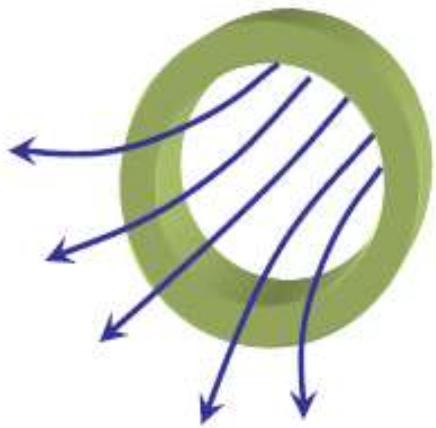


- Condensate contains cooper pairs with the density of order $10^{19}/cm^3$
- pairs are phase-locked producing a unique electrical fluid
- energy of the condensate is extremely low
- S/C in the absence of an applied magnetic field the phase is the same everywhere - phase coherence in the whole sample

Magnetic Flux quantization



Magnetic Flux quantization



how large is
the magnetic
flux in the ring ?

$$\Phi = \oint \vec{A} \cdot \mathrm{d}\vec{\ell} \approx H \cdot S$$

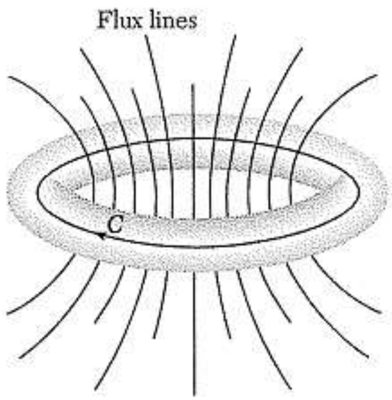
Experimental result:

$$\Phi = n\,\Phi_0$$

where $\Phi_0 = 2.07 \times 10^{-15}\,\mathrm{V} \cdot \mathrm{s}$

$$n = 0, \pm 1, \pm 2, \ldots$$

Flux quantization in a S/C ring



Let $\psi(\mathbf{r})$ be the super state wave function

Particle density $n = \psi^*\psi$

$$n = \text{constant} \;\rightarrow\; \psi = \sqrt{n}\, e^{i\,\theta(\mathbf{r})} \qquad \psi^* = \sqrt{n}\, e^{-i\,\theta(\mathbf{r})}$$

Velocity operator: $$\mathbf{v} = \frac{1}{m}\left(\mathbf{p} - \frac{q}{c}\mathbf{A}\right) = \frac{1}{m}\left(\frac{\hbar}{i}\nabla - \frac{q}{c}\mathbf{A}\right)$$

Particle flux: $$\psi^*\mathbf{v}\,\psi = \frac{n}{m} e^{-i\,\theta}\left(\frac{\hbar}{i}\nabla - \frac{q}{c}\mathbf{A}\right)e^{i\,\theta} = \frac{n}{m}\left(\hbar\nabla\theta - \frac{q}{c}\mathbf{A}\right)$$

Electric current density: $$\mathbf{j} = q\,\psi^*\mathbf{v}\,\psi = \frac{n\,q}{m}\left(\hbar\nabla\theta - \frac{q}{c}\mathbf{A}\right)$$

$$\nabla\times\mathbf{j} = \frac{n\,q}{m}\left(-\frac{q}{c}\nabla\times\mathbf{A}\right) = -\frac{n\,q^2}{m\,c}\mathbf{B}$$ London eq. with $$\lambda_L = \sqrt{\frac{mc^2}{4\pi n q^2}}$$

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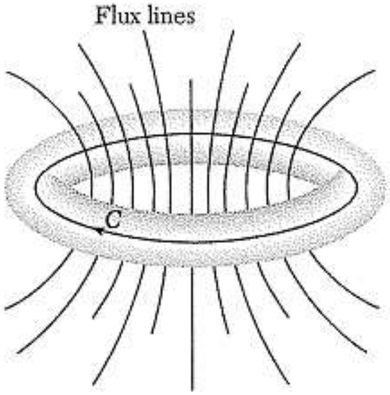
Flux quantization in a S/C ring



$$\mathbf{j} = \frac{n\,q}{m}\left(\hbar\nabla\theta - \frac{q}{c}\mathbf{A}\right)$$

Meissner effect: $\mathbf{B} = \mathbf{j} = 0$ inside superC $\rightarrow$ $\hbar\nabla\theta = \frac{q}{c}\mathbf{A}$

$$\oint_C \hbar\nabla\theta \cdot d\mathbf{l} = \frac{q}{c}\oint_C \mathbf{A}\cdot d\mathbf{l}$$

$$\hbar\,\Delta\theta = \frac{q}{c}\int_S \nabla\times\mathbf{A}\cdot d\boldsymbol{\sigma} = \frac{q}{c}\int_S \mathbf{B}\cdot d\boldsymbol{\sigma} = \frac{q}{c}\Phi$$

ψ measurable $\rightarrow$ ψ single-valued $\rightarrow$ $\Delta = 2\,\pi\,s$ $\quad s \in Z$

$\therefore$ $\boxed{\Phi = \frac{h\,c}{q}s}$ **Flux quantization** $\quad q = -2e \rightarrow$

$$\Phi_0 = \frac{h\,c}{2e} \approx 2.0678\times10^{-7}\,gauss\ cm^2$$ = fluxoid or fluxon

Verified London prediction Flux through ring : $\Phi = \Phi_{ext} + \Phi_{sc} = s\,\Phi_0$

Φ_{ext} not quantized $\rightarrow$ Φ_{sc} must adjust