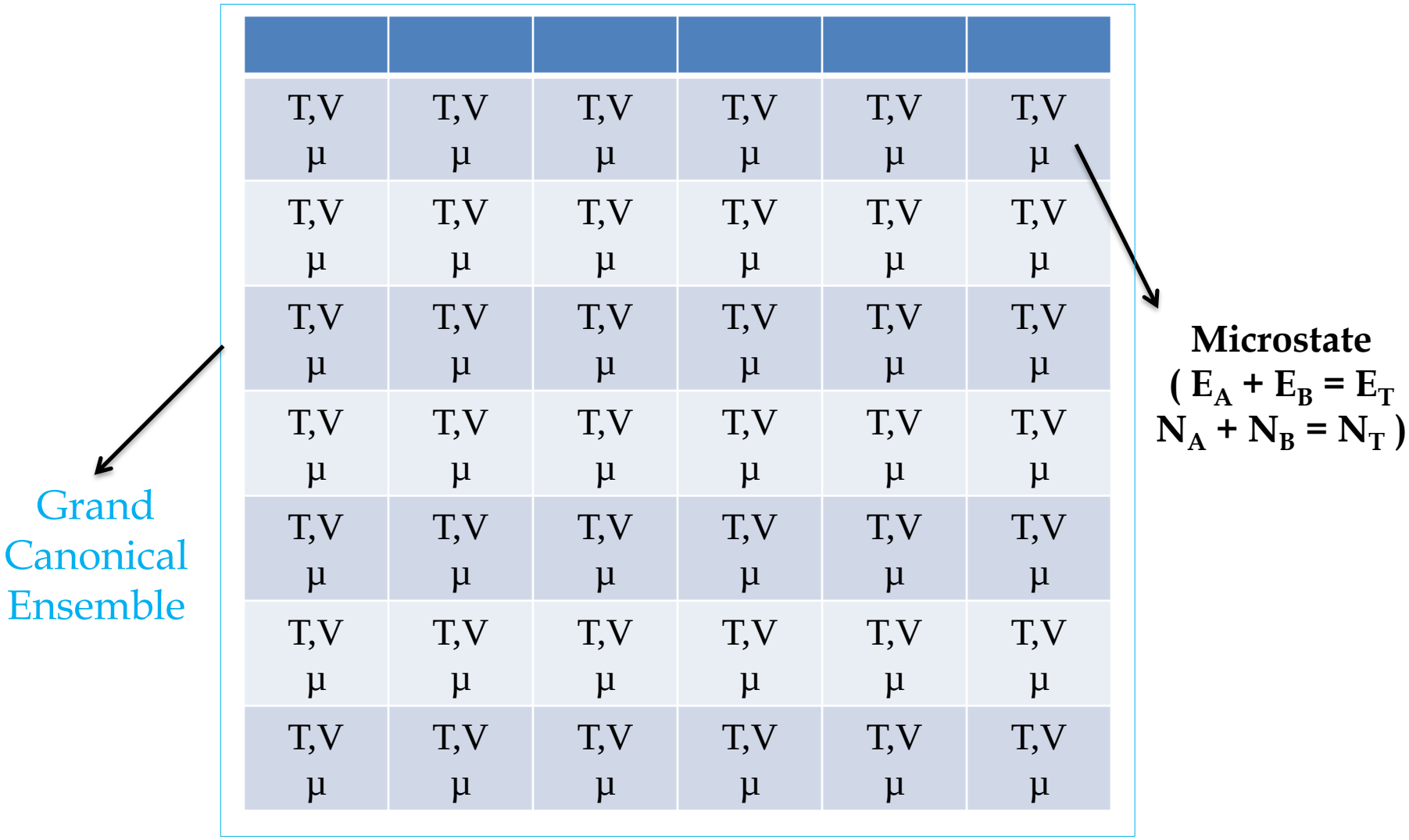
Grand-Canonical Ensemble: Counting the Number of Microstates

N_A	N_B	(E_A, E_B)	(Ω_A, Ω_B)	$\Omega_A\,\Omega_B$	$P_{E_A} \times N_A$	$P_{E_A} \times E_A$
0	4	(0,6)	(1, 84)	84	$\left(\frac{84}{420}\right) \times 0$	(84/420) x 0
1	3	(0,6)	(0, 28)	28	$\left(\frac{84}{420}\right) \times 1$	(28/420) x 0
		(1,5)	(1, 21)	21		(21/420) x 1
		(2, 4)	(1, 15)	15		(15/420) x 2
		(3, 3)	(1, 10)	10		(10/420) x 3
		(4, 2)	(1, 6)	6		(6/420) x 4
		(5, 1)	(1, 3)	3		(3/420) x 5
		(6, 0)	(1, 1)	1		(3/420) x 6
2	2	(6, 0)	(7, 1)	7	$\left(\frac{84}{420}\right) \times 2$	(7/420) x 6
		(5, 1)	(6, 2)	12		(12/420) x 5
		(4, 2)	(5, 3)	15		(15/420) x 4
		(3, 3)	(4, 4)	16		(16/420) x 3
		(2, 4)	(3, 5)	15		(15/420) x 2
		(1, 5)	(2, 6)	12		(12/420) x 1
		(0, 6)	(1, 7)	7		(7/420) x 0

N_A	N_B	(E_A, E_B)	(Ω_A, Ω_B)	$\Omega_A\Omega_B$	$P_{E_A} \times N_A$	$P_{E_A} \times E_A$
3	1	(0, 6) (1, 5) (2, 4) (3, 3) (4, 2) (5, 1) (6, 0)	(1, 1) (3, 1) (6, 1) (0, 1) (15, 1) (21, 1) (28, 1)	1 3 6 10 15 21 28	$\left(\frac{84}{420}\right)\times 3$	(1/420) x 0 (3/420) x1 (6/420) x 2 (10/420) x 3 (15/420) x 4 (21/420) x 5 (28/420) x 6
4	0	(6, 0)	(1, 84)	84	$\left(\frac{84}{420}\right)\times 4$	(7/420) x 6
			The picture can't be displayed.		$\overline{N}_A = 2$	$\overline{E}_A = 3$



In this example, the ***Ensemble*** consists of ***420*** systems each of which is in one of the ***420*** accessible ***Microstates***.

Probability of finding the particle in a particular microstate

$$p_i = \frac{e^{-\beta(E_A - \mu N_A)}}{\sum e^{-\beta E_i - \mu \beta N_i}}$$

$$\Xi = \sum_i e^{-\beta E_i - \mu \beta N_i}$$ → Grand Partition function

- The grand partition function 'Ξ' contains all the information about the energies of the states of the system.
- All thermodynamical quantities can be obtained from it.
- From 'Ξ', we can get thermodynamics functions energy, entropy, Helmholtz function, or heat capacity.

Gateway to Thermodynamics

$$\Phi_G = -k_B T \ln \mathrm{Z}$$

$$S = -\left(\frac{\partial \Phi_G}{\partial T} \right)_{V,\mu}$$

$$p = -\left(\frac{\partial \Phi_G}{\partial V} \right)_{T,\mu}$$

$$N = -\left(\frac{\partial \Phi_G}{\partial \mu} \right)_{T,V}$$

END