Lecture
Introduction to Ensembles

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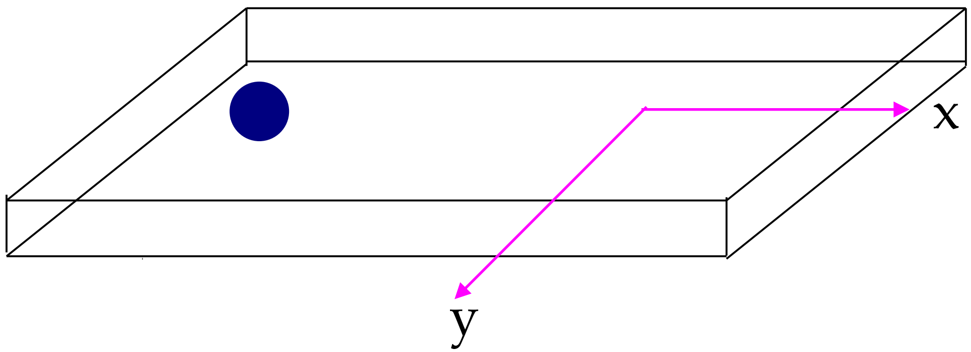
Classical Mechanics: Some Basic Facts

Newton's Second Law

One Particle – 1 Dimension



Equation of Motion

$$\frac{d^2x}{dt^2} = F\left(t, x, \frac{dx}{dt}\right)$$

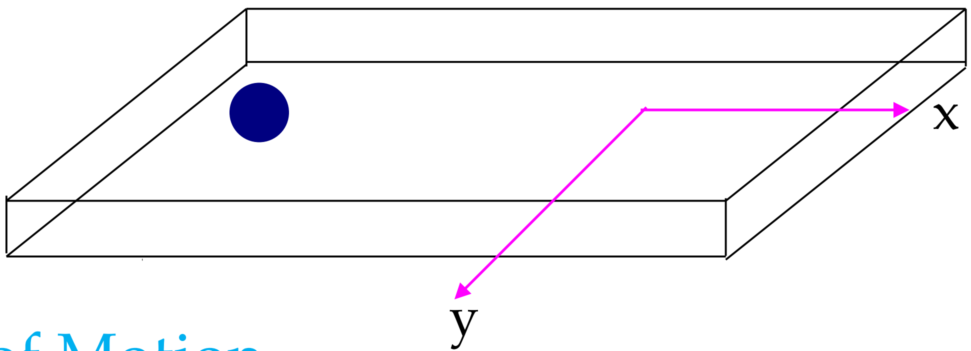
Solution

$$x(t) = f(t, c_1, c_2)$$

c_1, c_2 are constants

I.C Needed 2

One Particle – 2 Dimension



Equation of Motion

$$\frac{d^2x}{dt^2} = F(t, x, y, \dot{x}, \dot{y})$$

$$\frac{d^2y}{dt^2} = G(t, x, y, \dot{x}, \dot{y})$$

Solution

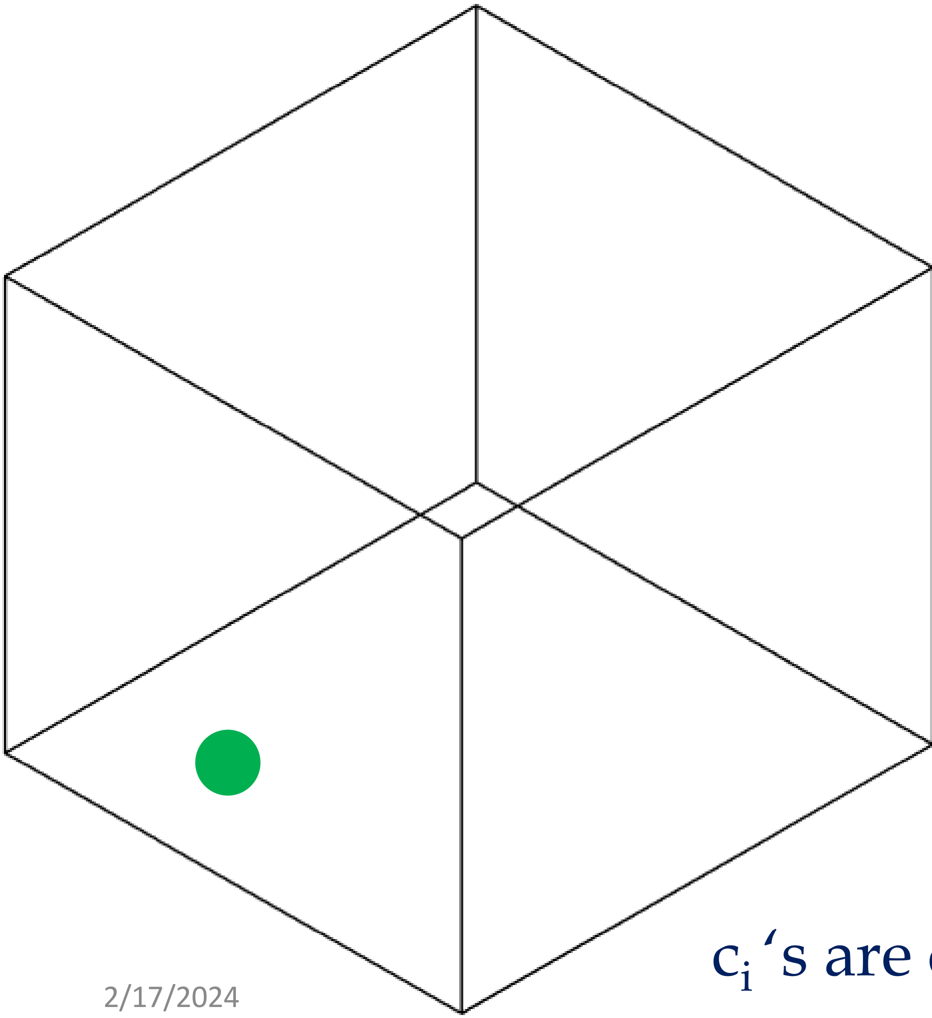
$$\mathrm{x(t)} = \mathrm{f_1(t, c_1, c_2, c_3, c_4)}$$

c_i's are constants (4)

$$\mathrm{y(t)} = \mathrm{f_2(t, c_1, c_2, c_3, c_4)}$$

I.C Needed 4

One Particle – 3 Dimension



Equation of Motion

$$\frac{d^2x}{dt^2} = F(t, x, y, z, \dot{x}, \dot{y}, \dot{z})$$

$$\frac{d^2y}{dt^2} = G(t, x, y, z, \dot{x}, \dot{y}, \dot{z})$$

$$\frac{d^2z}{dt^2} = H(t, x, y, z, \dot{x}, \dot{y}, \dot{z})$$

Solution

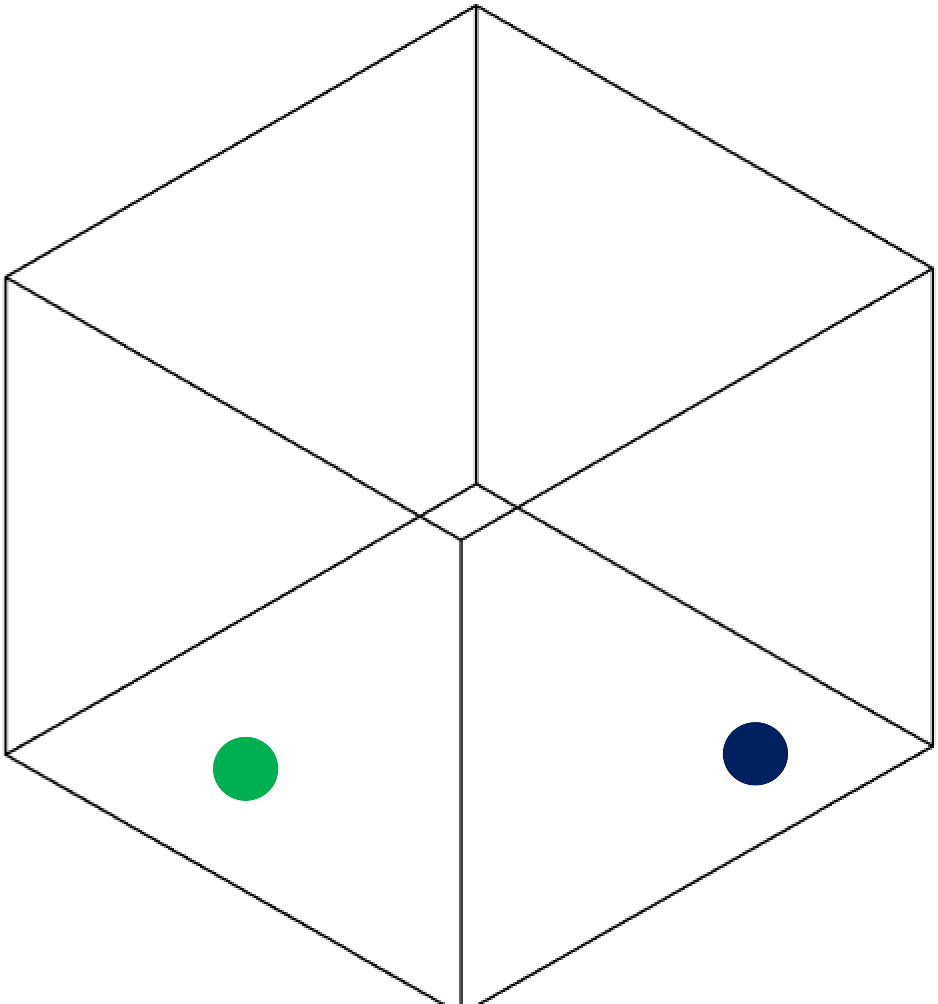
$$x(t) = f_1(t, c_1, c_2, c_3, c_4, c_5, c_6)$$

$$y(t) = f_2(t, c_1, c_2, c_3, c_4, c_5, c_6)$$

$$z(t) = f_2(t, c_1, c_2, c_3, c_4, c_5, c_6)$$

c_i‘s are constants (6) I.C Needed 6

Two Particles – 3 Dimension



Equation of Motion

1st Particle

$$\begin{cases} \dfrac{d^2 x_1}{dt^2} = F_1(t, x_1, y_1, z_1, x_2, y_2, z_2, \dot{x}_1, \dot{y}_1, \dot{z}_1, \dot{x}_2, \dot{y}_2, \dot{z}_2) \\ \dfrac{d^2 y_1}{dt^2} = F_2(t, x_1, y_1, z_1, x_2, y_2, z_2, \dot{x}_1, \dot{y}_1, \dot{z}_1, \dot{x}_2, \dot{y}_2, \dot{z}_2) \\ \dfrac{d^2 z_1}{dt^2} = F_3(t, x_1, y_1, z_1, x_2, y_2, z_2, \dot{x}_1, \dot{y}_1, \dot{z}_1, \dot{x}_2, \dot{y}_2, \dot{z}_2) \end{cases}$$

$$\begin{cases} \dfrac{d^2 x_2}{dt^2} = F_4(t, x_1, y_1, z_1, x_2, y_2, z_2, \dot{x}_1, \dot{y}_1, \dot{z}_1, \dot{x}_2, \dot{y}_2, \dot{z}_2) \\ \dfrac{d^2 y_2}{dt^2} = F_5(t, x_1, y_1, z_1, x_2, y_2, z_2, \dot{x}_1, \dot{y}_1, \dot{z}_1, \dot{x}_2, \dot{y}_2, \dot{z}_2) \\ \dfrac{d^2 z_2}{dt^2} = F_6(t, x_1, y_1, z_1, x_2, y_2, z_2, \dot{x}_1, \dot{y}_1, \dot{z}_1, \dot{x}_2, \dot{y}_2, \dot{z}_2) \end{cases}$$

2nd Particle

Solution

$$x_1 = f_1(t, c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}, c_{11}, c_{12})$$

$$y_1 = f_2(t, c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}, c_{11}, c_{12})$$

$$z_1 = f_3(t, c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}, c_{11}, c_{12})$$

$$x_2 = f_4(t, c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}, c_{11}, c_{12})$$

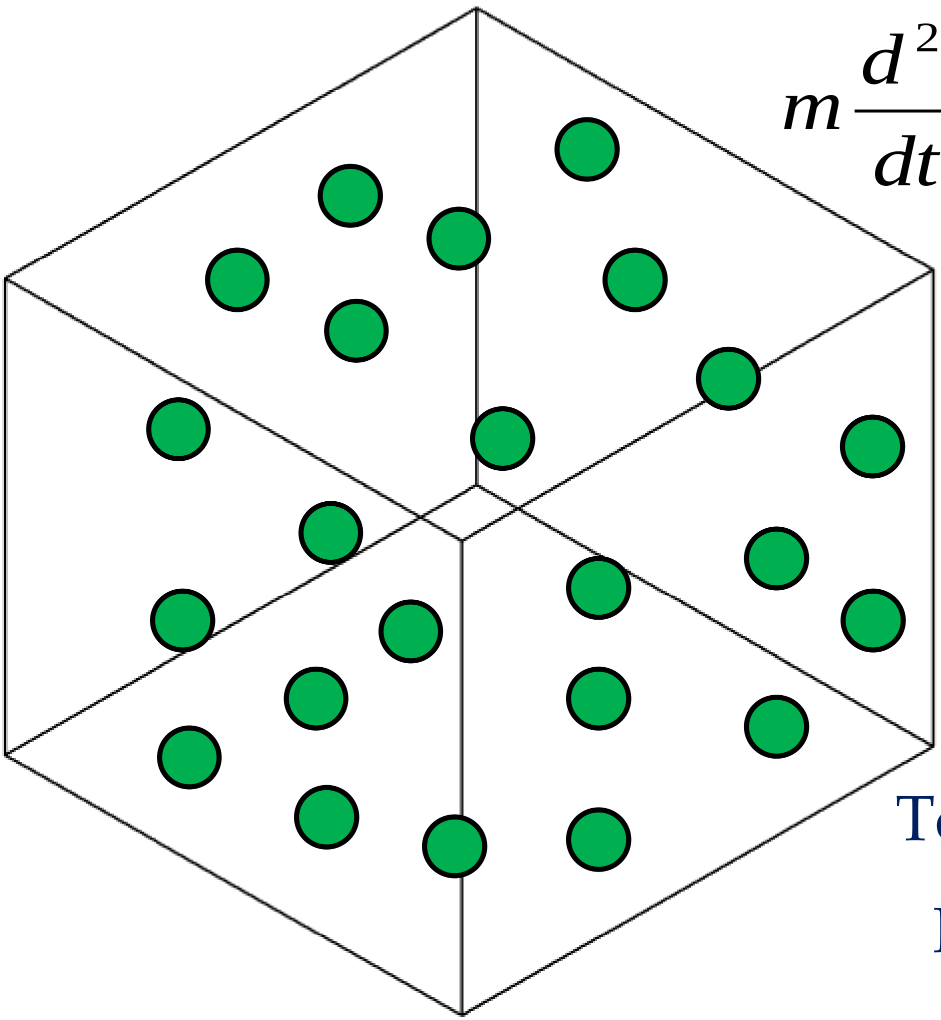
$$y_2 = f_5(t, c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}, c_{11}, c_{12})$$

$$z_2 = f_6(t, c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}, c_{11}, c_{12})$$

c_i's are constants (12)

I.C Needed 12

‘N’ Particles – 3 Dimension



$$m\frac{d^2\vec{r}_i}{dt^2} = \vec{F}_i(t, \vec{r}_i, \dot{\vec{r}}_i), i = 1,2,..N$$

Total Number of Equations = 3N

Initial Conditions Needed = 6N

N is of the order of 10^{23}

- As the particles/systems increases, the complexity also increases.
- It is difficult to specify the Initial Conditions and hence difficult to solve the Newton's equations.
- At the quantum level, difficulties arise while solving the Schrödinger equation in the N particle case.
- Need an alternate formalism.

STATISTICAL MECHANICS

- By describing the individual microscopic states of a system and using statistical methods we derive the macroscopic properties from them.
- This approach received an additional impetus with the development of ***quantum theory*** which showed explicitly how to describe the microscopic quantum states.