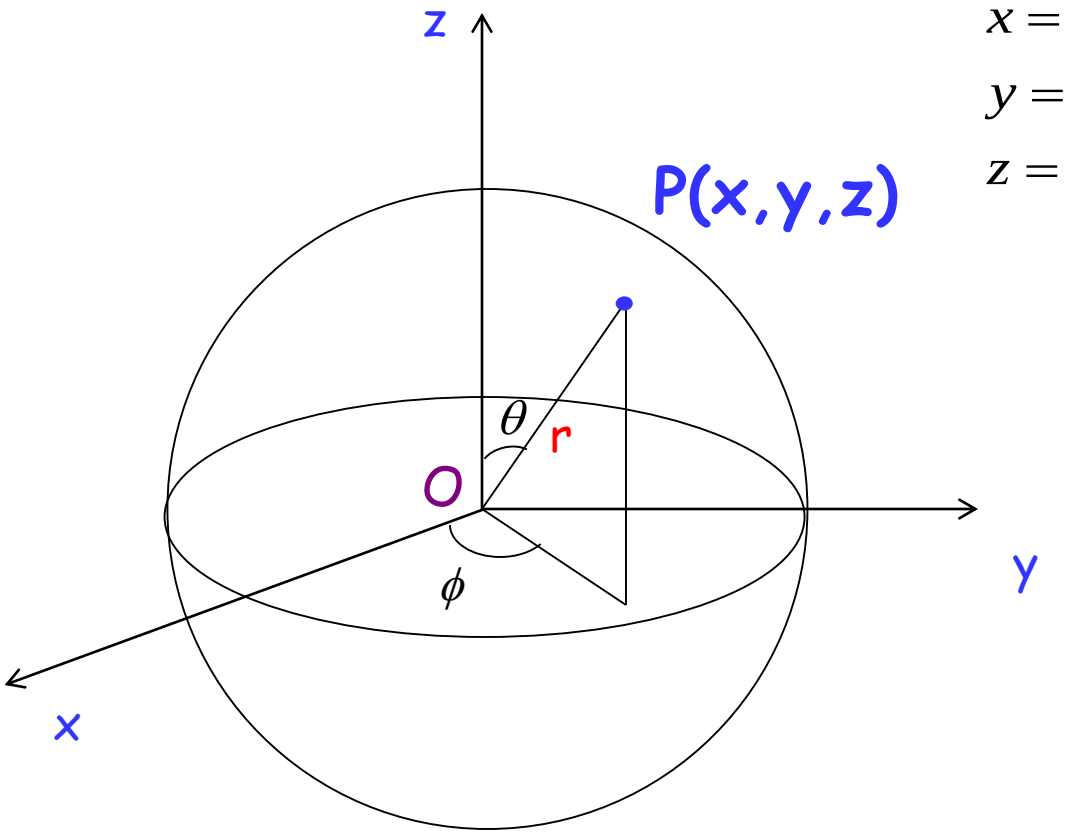
Spherical Polar Coordinates

Spherical coordinates



$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\theta$$

Cartesian coordinates are in terms of spherical coordinates

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\theta$$

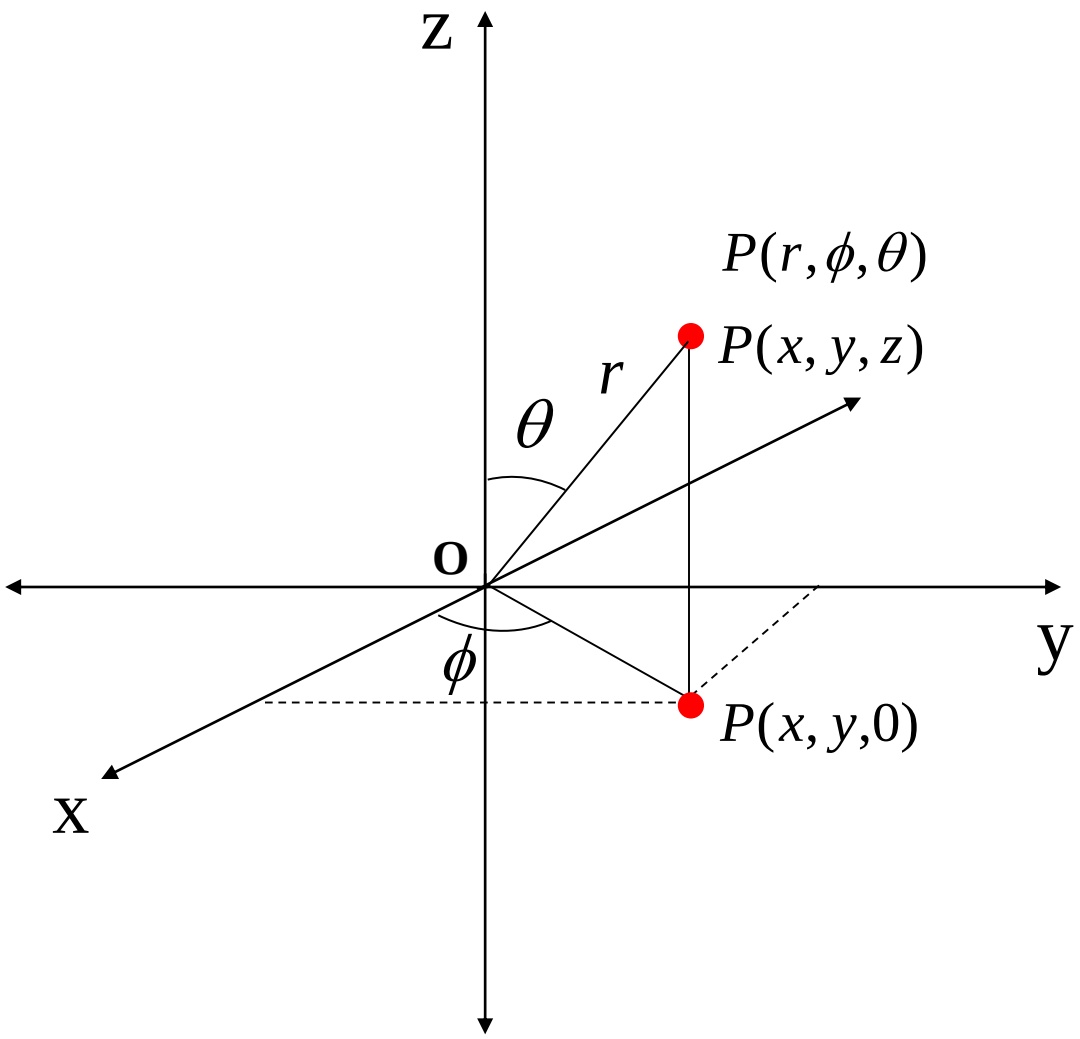
limits

$$0 < r < \infty$$
$$0 \le \theta \le \pi$$
$$0 \le \phi \le 2\pi$$

Spherical coordinates are in terms Cartesian coordinates

$$r = \sqrt{x^2 + y^2 + z^2}$$
$$\theta = \cos^{-1}\left(\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right)$$
$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$

Spherical Polar Coordinates



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$$z = r\cos\theta$$

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$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$
$$\theta = \cos^{-1}\left(\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right)$$

Volume element

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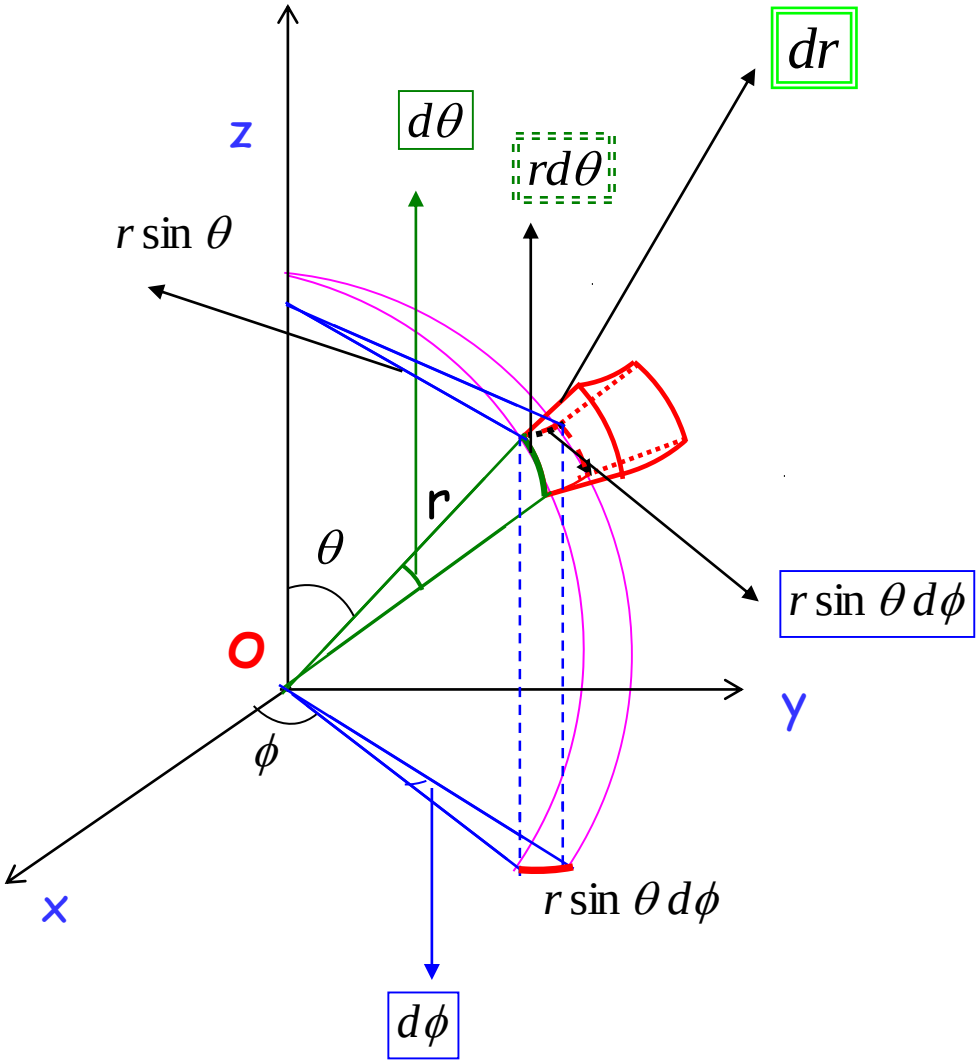
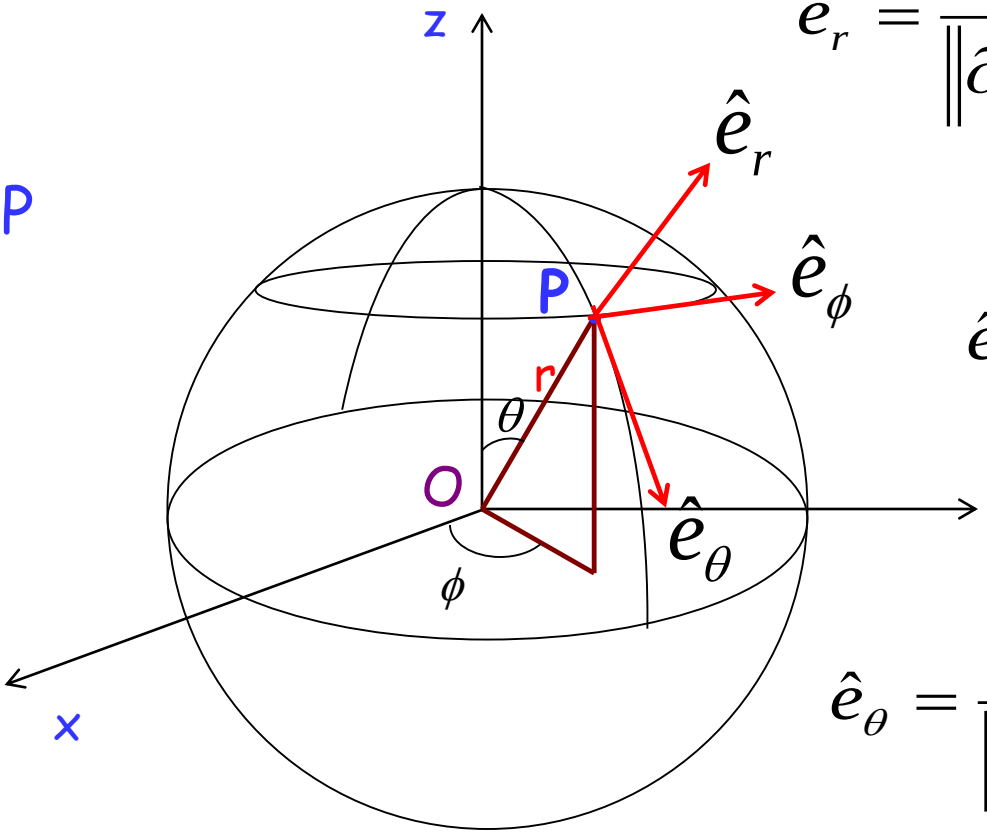
$$dv = (dr)(rd\theta)(r\sin\theta\, d\phi)$$
$$= r^2 \sin\theta\, dr\, d\theta\, d\phi$$



Unit vectors

unit vectors

Let $\vec{R}$ be the position vector of P



$$\hat{e}_r = \frac{\partial\vec{R}/\partial r}{\left\|\partial\vec{R}/\partial r\right\|}$$

$$\hat{e}_\phi = \frac{\partial\vec{R}/\partial \phi}{\left\|\partial\vec{R}/\partial \phi\right\|}$$

$$\hat{e}_\theta = \frac{\partial\vec{R}/\partial \theta}{\left\|\partial\vec{R}/\partial \theta\right\|}$$

$$\vec{R} = \vec{R}(r,\theta,\phi)$$

Types of Coordinates

1. Rectangular Coordinates
2. Circular Cylinder Coordinates
3. Elliptic Cylinder Coordinates
4. Parabolic Cylinder Coordinates
5. Spherical Coordinates
6. Conical Coordinates
7. Parabolic Coordinates
8. Prolate Spheroidal Coordinates
9. Oblate Spheroidal Coordinates
10. Ellipsoidal Coordinates
11. Paraboloidal Coordinates

END