

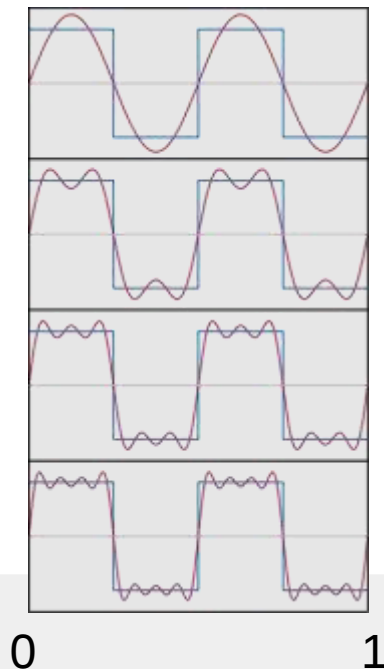
Introduction to Geomagnetism

- The magnetic record of Earth's history that is frozen into crustal rocks has provided, perhaps, the most important geophysical evidence of past and on-going plate-tectonic processes.
- The nearly static magnetic field that is our strong dipolar geomagnetic field has been most interesting to geologists and geodynamicists.
- The generation of the field is now the most interesting scientific question for many mathematically oriented geophysicists.
- Why and how we have our field is informative about the processes at work deep within our Earth.

Harmonic functions -- I

We can resolve any function over the line interval $[0, 1]$ in terms of coefficients of a **Fourier Series**:

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos(nx) + b_n \sin(nx)]$$



All $a_n = 0$; $b_2 = 1$

Add $b_4 = 1/4$

Add $b_8 = 1/8$

Add $b_{16} = 1/16$

We can build up our square-wave function coefficient by coefficient. We can construct any function on the interval by adding up the Fourier components scaled by the appropriate coefficient.



Harmonic functions -- II

We resolve the magnetostatic potential at any place over the nearly spherical Earth in terms of coefficients g_l^m and h_l^m of an expansion in Spherical Harmonics:

$$V(r, \phi, \theta) = a \sum_{\ell=1}^L \sum_{m=0}^{\ell} \left(\frac{a}{r}\right)^{\ell+1} (g_{\ell}^m \cos m\phi + h_{\ell}^m \sin m\phi) P_{\ell}^m(\cos \theta)$$

Example of the forms of the harmonics:

$g_l^m=1, h_l^m=0$, for $l = 4, m = 0, 1, \dots, 4$ at radius

$$P_0^0(\cos \theta) = 1$$

$$P_1^0(\cos \theta) = \cos \theta$$

$$P_1^1(\cos \theta) = -\sin \theta$$

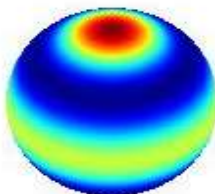
$$P_2^0(\cos \theta) = \frac{1}{2}(3 \cos^2 \theta - 1)$$

$$P_2^1(\cos \theta) = -3 \cos \theta \sin \theta$$

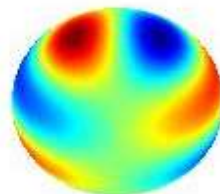
$$P_2^2(\cos \theta) = 3 \sin^2 \theta$$

$$P_3^0(\cos \theta) = \frac{1}{2}(5 \cos^3 \theta - 3 \cos \theta)$$

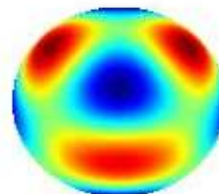
m=0



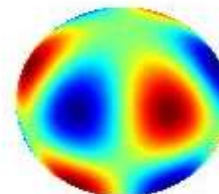
m=1



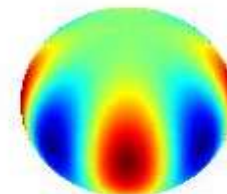
m=2



m=3

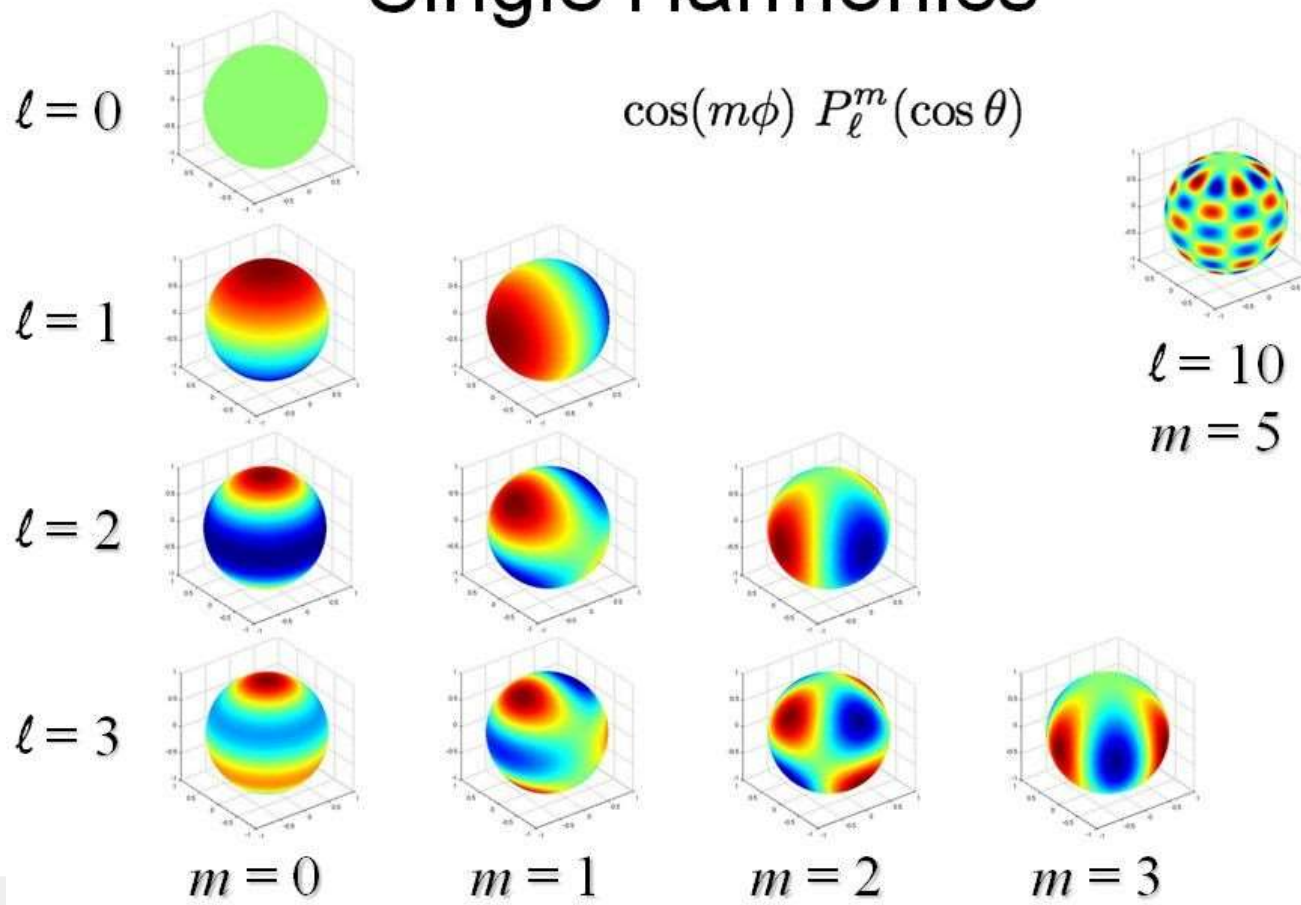


m=4



Harmonic functions -- III

Single Harmonics



Mapping the IGRF

Several satellite missions, most recently, the Øersted and Champ missions, have mapped the Earth's geomagnetic field from altitude and then downward continued the measurements to a sphere of average Earth radius. The spherical harmonic coefficients obtained from these missions construct the IGRF (International Geomagnetic Reference Field) – that field that is taken as the datum field against which temporal and spatial anomalies are measured.



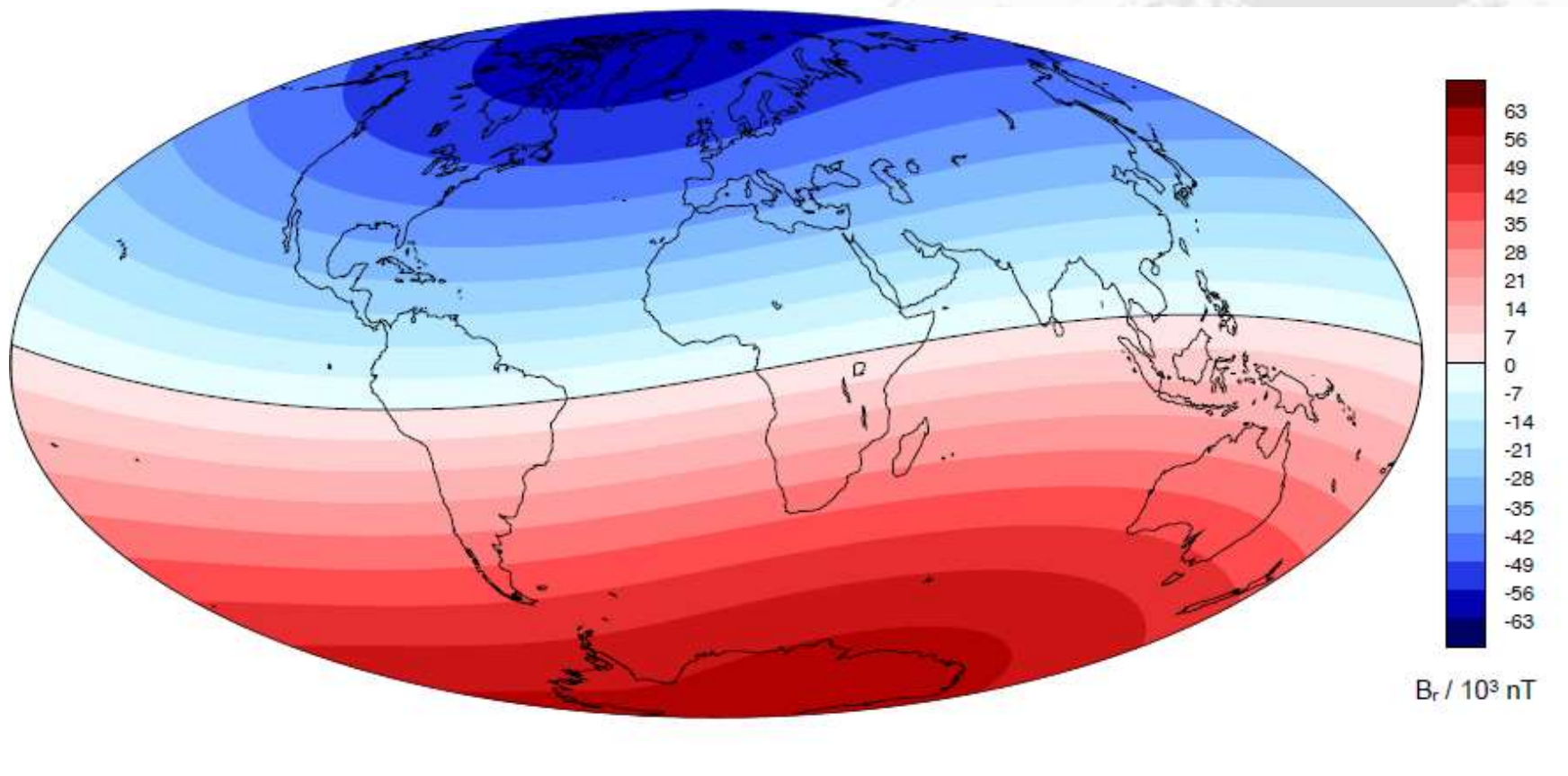
Øersted



Champ

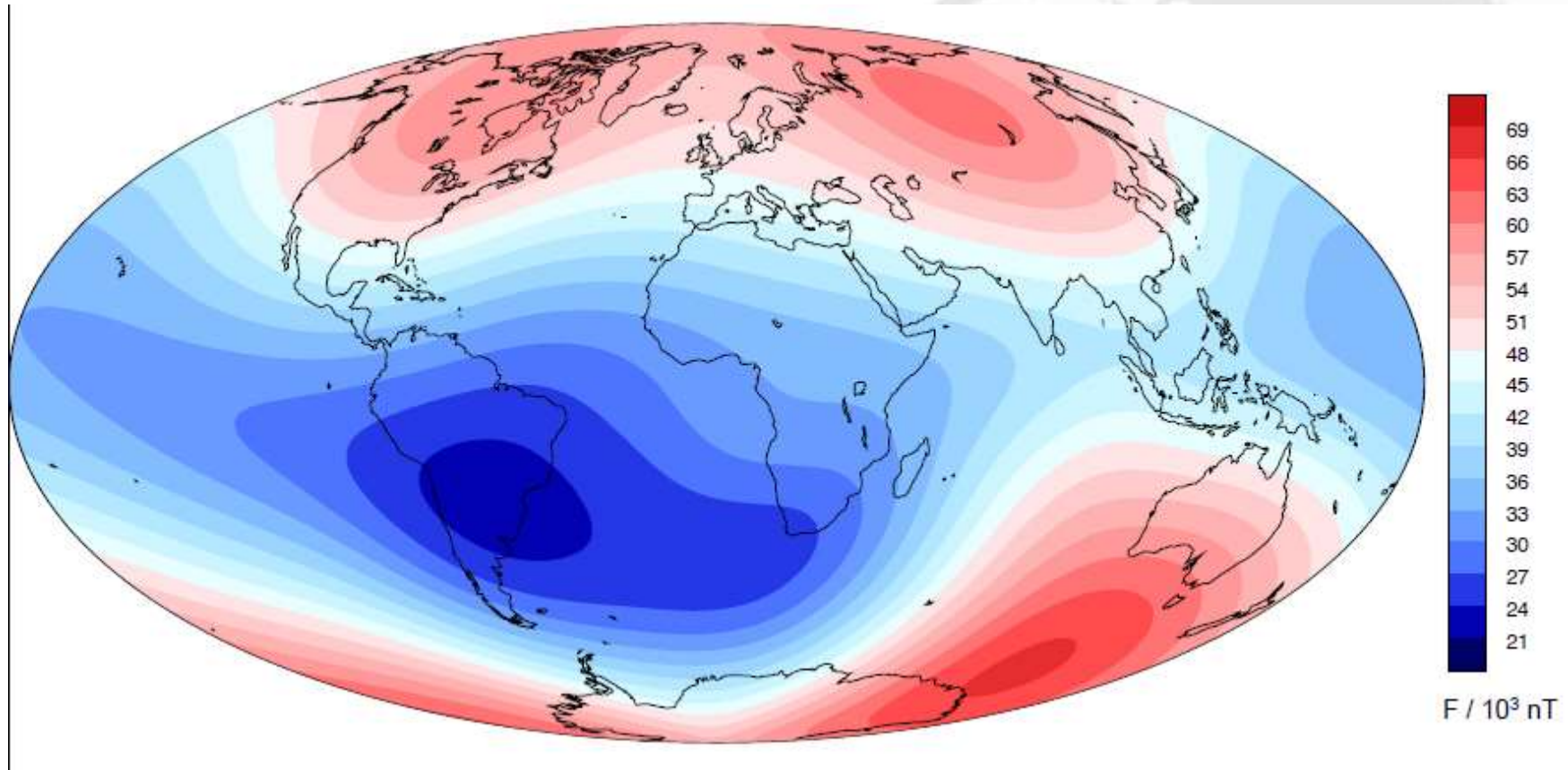


The IGRF - I



The radial (local vertical) component B_z – dipole components only

The IGRF - II



The total field $|B|$ – degree 13 (i.e. all $l = 0$ to 13 coefficients)

IGRF calculator for any place on the Earth's surface: [IGRFWMM-NOAA-NGDC](https://www.ngdc.noaa.gov/geomag/IGRFWMM/)



Global Magnetic Anomaly – B_z

Residual to the IGRF

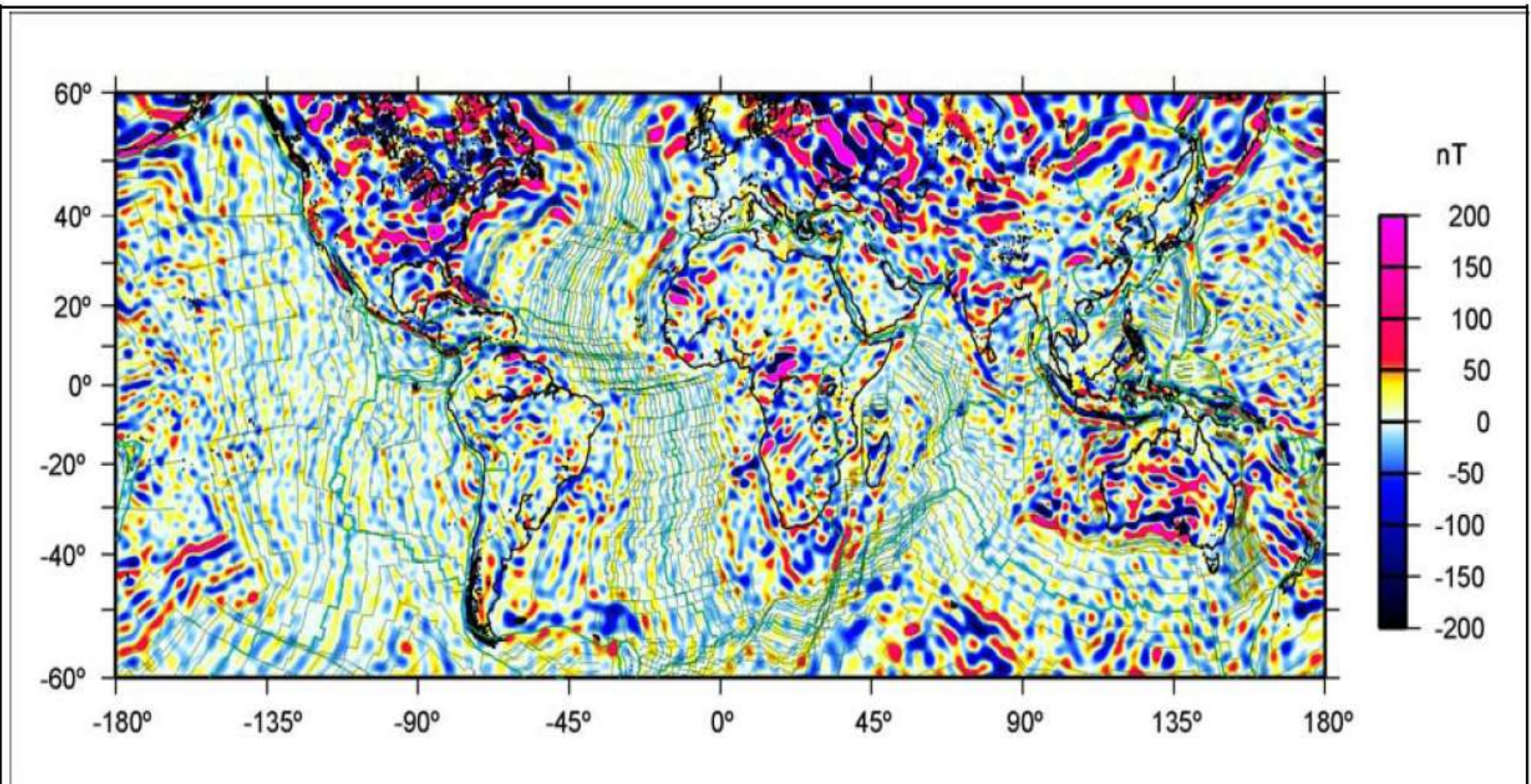


Figure 1: Vertical component of the CHAMP-satellite-derived MF6 crustal magnetic field at the Earth surface, overlain with the isochrons of an ocean-age model inferred from independent marine and aeromagnetic data by Muller et al. (2008).

Elsewhere in the Solar System

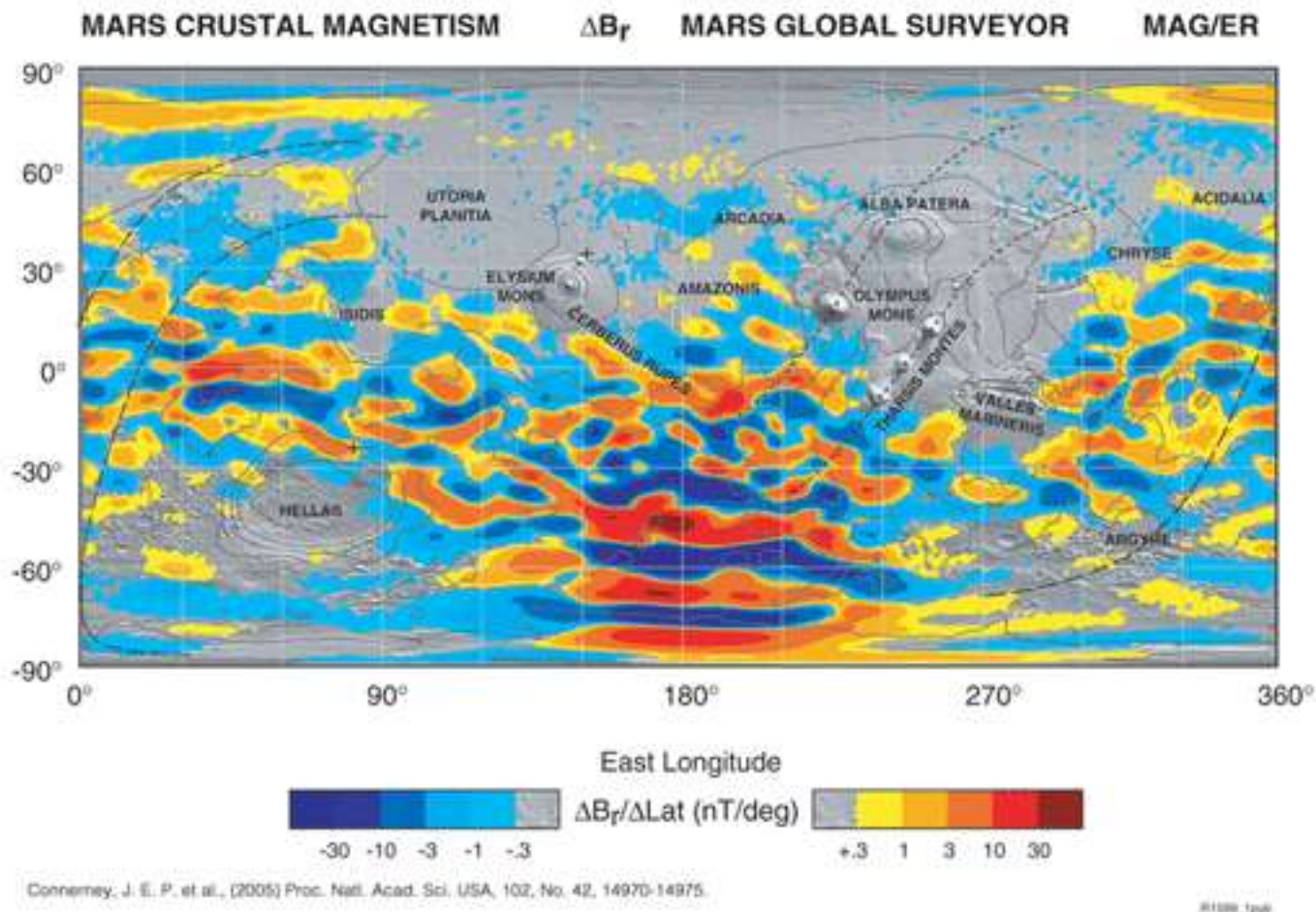
Can we detect a tectonic history written into the crust?

- Most space missions are launched with probes carrying magnetometers.
- The first interest is in the character of the global magnetic fields surrounding planets and moons that might suggest an internal magnetodynamic dynamo.
- Where among the rocky bodies: Mercury, Earth, Io, Ganymede have dynamos.
- Perhaps Mars and Earth's Moon once had dynamos that have imprinted a paleomagnetic record into crustal rocks.



Mars' Magnetic Anomaly

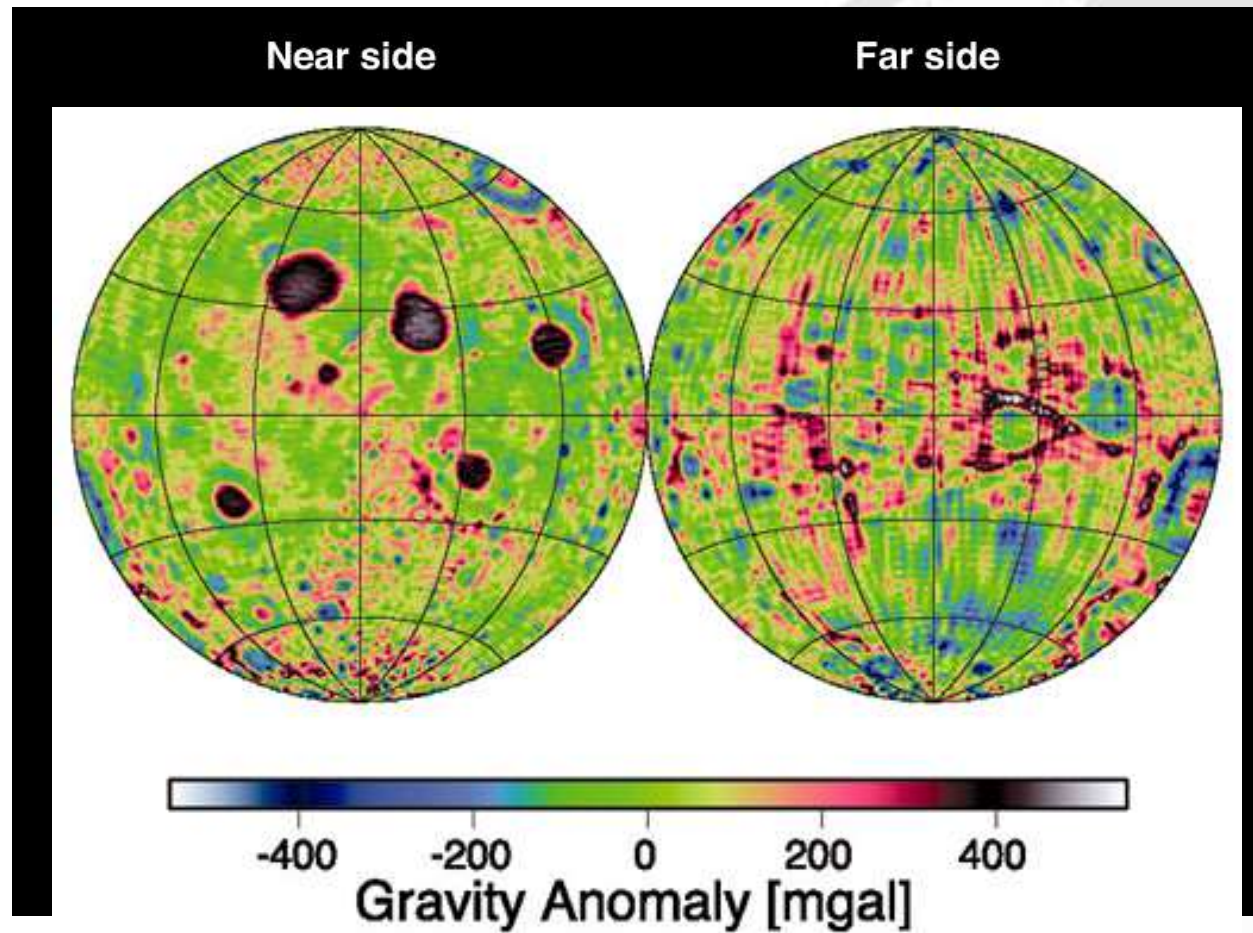
Reveals tectonic history?



Mars Global Surveyor Mission



Lunar Magnetic Anomaly -- |B|



The Apollo astronauts returned rocks from the Moon which showed very high remnant magnetism – [Lunar Prospector](#) mapped the field.



Venus' Magnetic Anomaly?

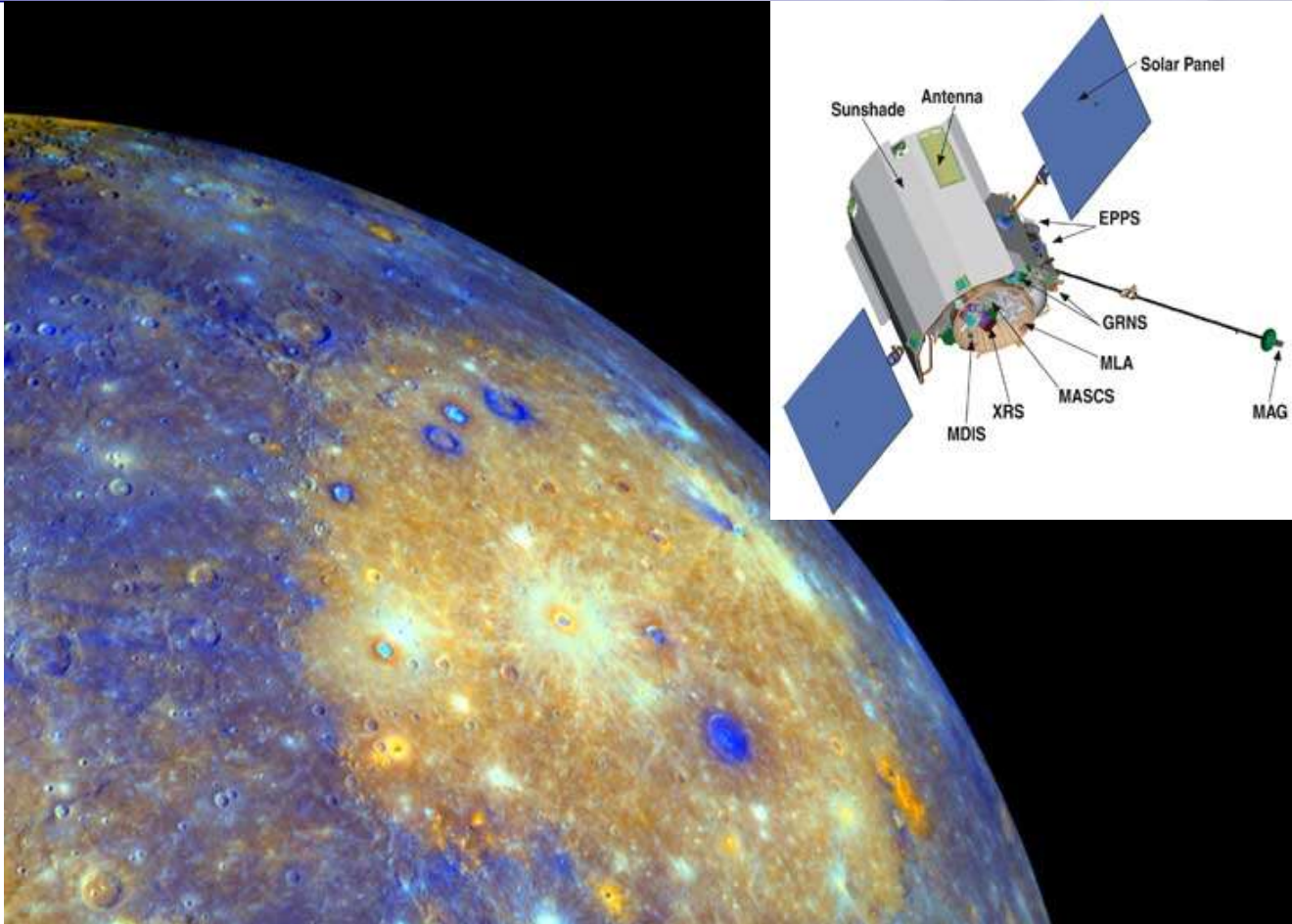
A tectonic history written into the crust?

- *No geomagnetic field* has been detected on Venus – probably no convecting core.
- The surface temperature is 740K; most minerals are well above their Curie temperature at 740K – no field imprinted in crustal rocks.
- What we can say about the surface is that it is very “young” -- completely resurfaced within the past 400-700 million years.
- We must infer tectonic history based on surface topography and gravity.



Mercury's Magnetic Anomaly?

The Messenger mission should reveal tectonic history



The story is just now being told!

Mercury's magnetic field



Plate tectonics

- Morley and Larochele (1964) And, then, following, Vine and Matthews (1963) recognize paleomagnetic banding across ocean ridges was due to sea-floor spreading.
- During the late 1950s, Edward (Ted) Irving had been mapping paleomagnetic pole paths to show the history of continental drift.
- The age of the sea-floor basins has been established by magnetic surveying since the original recognition of spreading.

Sea-floor spreading revealed by paleomagnetic anomalies

Age of Oceanic Lithosphere (m.y.)

Data source:

Muller, R.D., M. Sdrolias, C. Gaina, and W.R. Roest 2008. Age, spreading rates and spreading symmetry of the world's ocean crust, *Geochem. Geophys. Geosyst.*, 9, Q04006, doi:10.1029/2007GC001743.

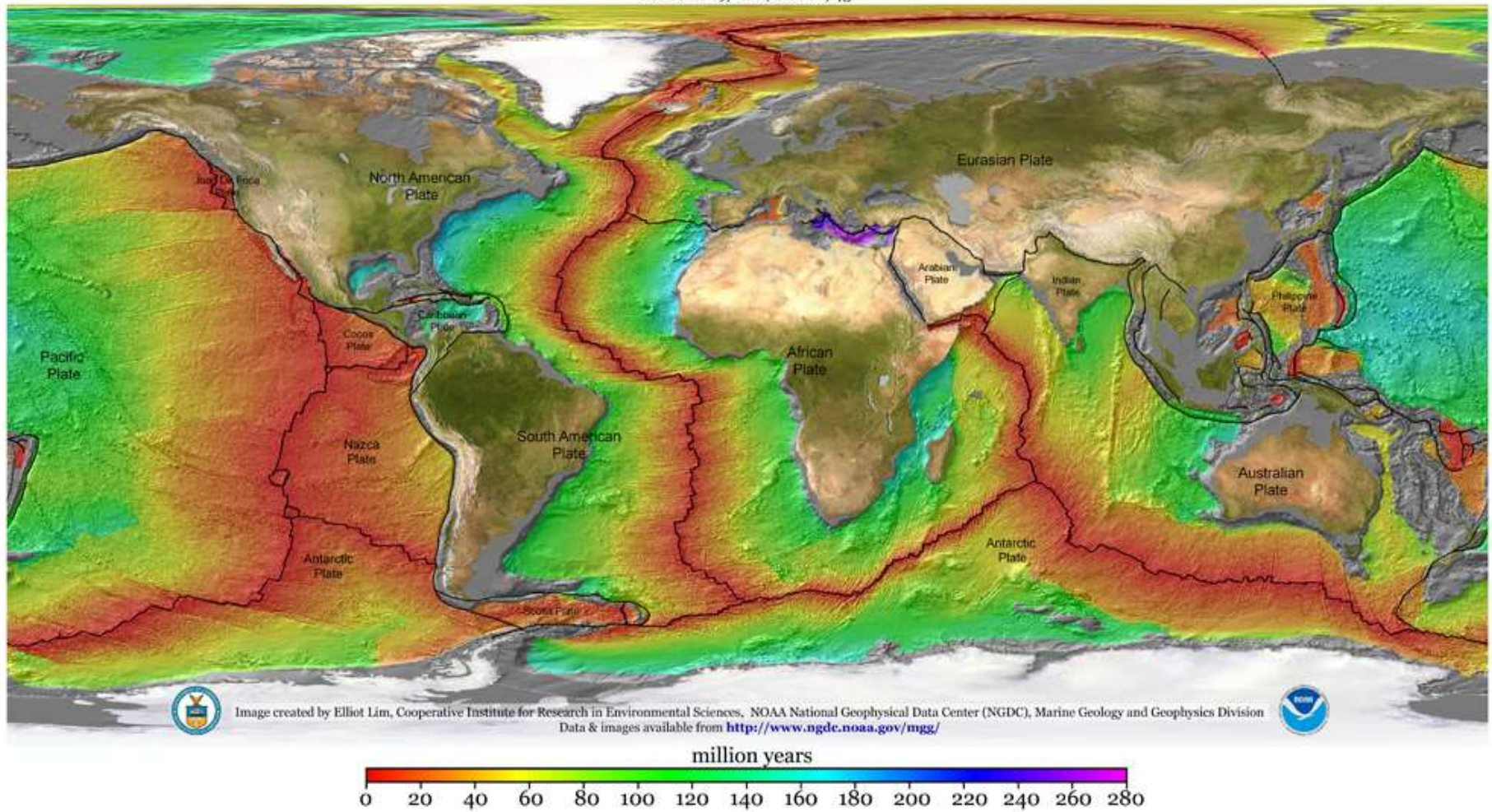


Plate tectonics – The theory is demonstrated by paleomagnetic evidence for sea-floor spreading

