SECTION C — (3 × 10 = 30)

Answer any THREE questions.

16. If x is path connected and x_0 and x_1 are two points of x, then prove that $\pi_1(x, x_0)$ is isomorphic to $\pi_1(x, x_1)$.

17. Prove that the fundamental group of the circle is infinite cyclic.

18. Prove that the fundamental group of the figure eight is not abelian.

19. State and prove the Jordan separation theorem.

20. State and prove the Jordan curve theorem.

S.No. 3159 **P 16 MAE 5 A**

(For candidates admitted from 2016–2017 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2022.

Mathematics — Elective

ALGEBRAIC TOPOLOGY

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20)

Answer ALL questions.

1. Define homotopy.

2. Define a simply connected space.

3. Define lifting of a map.

4. Define cyclic group.

5. State the fundamental theorem of Algebra.

6. Define homotopy equivalences.

7. Define interior points.

8. State Nulhomotopy lemma.

9. State the schoen flies theorem.

10. State a general non separation theorem.

SECTION B — (5 × 5 = 25)

Answer ALL questions, choosing either (a) or (b) in each.

11. (a) Prove that in a simply connected space X, any two paths having the same initial and final points are path homotopic.

Or

(b) If $h:(x,x_0)\to(y,y_0)$ and $K=(y.y_0)\to(z,z_0)$ are continuous, then $(koh)_*=k_*\, oh_*$. If $i:(x,\, x_0)\to(x,x_0)$ is the identity map, then prove that i_* is the identity homorphism.

12. (a) Prove that the fundamental group of S' is isomorphic to the additive group of integers.

Or

(b) If $n\geq 2$, then show that n-sphere S^n is simply connected.

13. (a) Show that $\pi,\left(p^2,y\right)$ is a group of order 2.

Or

(b) Define an inessential map. Prove that $h:S^1\to y$ is inessential if h can be extended to a continuous map $g:B^2\to y$.

14. (a) Let $f:x\to y$ be continuous and $f(x_0)=y_0$ prove that if f is a homotopy equivalence, then $f:\pi_1\,(x,x_0)\to\pi_1\,(y,y_0)$ is an isomorphism.

Or

(b) Prove that for any two points a and b of S^2 there is a homeomorphism h of S^2 with the one point compactification $R^2U\{\infty\}$ of R^2 such that $h(a)=\infty$ and $h(b)=0$.

15. (a) State and prove a nonseparation theorem.

Or

(b) Let C_1 and C_2 be closed connected subsets of S^2 whose intersection consists of two points. If neither C_1 nor C_2 separates S^2, then prove that C,UC2 separates S^2 into precisely two components.