(6 Pages)

S.No. 3155 **P 16 MAE 4 C**

(For candidates admitted from 2016–2017 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2022.

Mathematics — Elective

COMBINATORICS

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20)

Answer ALL questions.

1. How many ways of arranging seven flags on five masts when all the flags must be displayed but not all the masts have to be used?

2. How many divisors does the number 1400 have?

3. Define ordinary generating function.

4. What is the coefficient of the term x^{23} in $(1+x^5+x^9)^{100}$?

5. Define linear recurrence relation.

6. Solve the difference $a_n + 2a_{n-1} = n+3$.

7. Twelve balls are painted in the following way:

   Two are unpainted; Two are painted red, one is painted blue, and one is painted white; Two are painted red and blue, and one is painted red and white; Three are painted red, blue and white. Find $N(a'_1\, a'_2\, a'_3)$.

8. In a group of ten girls, six have blond hair five have blue eyes and three have blond hair and blue eyes. How many girls are there in the group who have neither blond hair nor blue eyes?

9. Define store enumerator.

10. What is meant by permutation?

PART B — (5 × 5 = 25)

Answer ALL questions, choosing either (a) or (b)

11. (a) If no three diagonals of convex decagon meet at the same point inside the decagon into how many line segments are the diagonals divided by their intersection?

    Or

    (b) Five distinct letters are to be transmitted through a communication channel. A total of 15 blanks are to be inserted between the letters with at least three blanks between two letters. In how many ways can the letters and blanks be arranged?

12. (a) Prove $\binom{n}{1}+2\binom{n}{2}+3\binom{n}{3}+\ldots+n\binom{n}{n}=n2^{n-1}$.

Or

(b) Show that the ordinary generating function of the sequence

$$\binom{0}{0},\binom{2}{1},\binom{4}{2},\binom{0}{0},\binom{6}{3},\ldots,\binom{2r}{r} is (1-4x)^{-\frac{1}{2}}.$$

13. (a) Solve the difference equation $a_n+6a_{n-1}12a_{n-2}8a_{n-3}=0$ with the boundary conditions $a_0=1; a_1=-2; a_2=8$.

Or

(b) Evaluate the $n \times n$ determinants

$$\begin{array}{cccccccccccccc}
2 & 1 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 \\
1 & 2 & 1 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 2 & 1 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 2 & 1 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \ldots & \cdot & \cdot & \cdot & \cdot & \cdot \\
0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 1 & 2 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 1 & 2 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 1 & 2
\end{array}$$

14. (a) Find the number of integers between 1 and 250 that are not divisible by any of the integers 2,3,5 and 7.

Or

(b) Find the number of r-digit quaternary sequence in which each of the three digits 1,2 and 3 appears at least once.

15. (a) Eight people are planning vacation trips. There are three cities they can visit. Three of these eight people are in on family, and two of them in another family. If the people in the same family must go together, find the ways the eight people can plan their trips.

Or

(b) The number of equivalence classes of functions from D to R is given by $\frac{1}{|G|}\frac{1}{|H|}\sum_{\phi\in G;\tau\in H}\psi[\phi,\tau]'$ where $\psi[\phi,\tau]'$ is the number of function f's which are such that $\tau f(d)=f[\phi(d)]$ for all d in D.

PART C — (3 × 10 = 30)

Answer any THREE questions.

16. In how many ways can $2n+1$ seats in a congress be divided among three parties so that the coalition of any two parties will ensure them of majority?

17. Show that the number of ways in which r non distinct objects can be distributed into n distinct cells, with the condition that no cell contains less than q nor more than $q+z-1$ objects, is the coefficient of x^{r-qn} in the expansion of $\left[\frac{1-x^z}{1-x}\right]^n$.

18. Find the number of ways to parenthesize the expression $w_1+w_2+\ldots+w_{n-1}+w_n$.

19. Find the number of permutations of the letter $\alpha,\alpha,\beta,\beta,\gamma$ $and\gamma$ and γ so that no α appears in the first and second positions, no β appears in the third position and no γ appears in the fifth and sixth position.

20. State and prove Polya's fundamental theorem.