17. Show that $(x)(P(x) \vee Q(x)) \Rightarrow (x)P(x) \vee (\exists x)Q(x)$.

18. Let $\langle L, \leq \rangle$ be a lattice in which * and $\oplus$ denote the operations of meet and join respectively. For any $a, b \in L$, $a \leq b \Leftrightarrow a * b = a \Leftrightarrow (a \oplus b) = b$.

19. Write the following Boolean expression in an equivalent sum of products canonical form in three variables x_1, x_2 and x_3.

    (a) $x_1 * x_2$

    (b) $x_1 \oplus x_2$

    (c) $(x_1 \oplus x_2)' * x_3$

20. Let $G = (V, T, S, P)$ be the phrase-structure grammar with $V = \{0, 1, A, B, S\}$, $T = \{0, 1,\}$ and set of production P consisting of $S \to 0A$, $S \to 1A$, $A \to 0B$, $B \to 1A$, $B \to 1$.

    (a) Show that 10101 belongs to the language generated by G

    (b) Show that 10110 does not belong to the language generated by G

    (c) What is the language generated by G?

S.No. 3151 **P 16 MAE 3 B**

(For candidates admitted from 2016–2017 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2022.

Mathematics — Elective

DISCRETE MATHEMATICS

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20)

Answer ALL questions.

1. Define Domain.
2. Define Symmetric.
3. Form the conjunction of

   P : It is raining today.

   Q : There are 20 tables in this room.
4. Write the following statement in symbolic form.

   If either Jerry takes Calculus or Ken takes Sociology then Larry will take English.
5. Define Complete Lattice.
6. Define Complemented Lattice.
7. Define sub-Boolean algebra.
8. Prove the following Boolean identity.

   $a \oplus (a' * b) = a \oplus b$

9. Define parse tree.

10. Define context-sensitive grammar.

SECTION B — (5 × 5 = 25)

Answer ALL questions, choosing either (a) or (b).

11. (a) Let S be the set of all statement functions in n variables and let R be the relation given by

$$R = \{\langle xy \rangle \mid x \in S \wedge y \in S \wedge x \Leftrightarrow y\}$$

Discuss the equivalence classes generated by the elements of S.

Or

(b) Let $X = \{a, b, c, d, e\}$ and let $C = \{\{a, b\}, \{c\}, \{d, e\}\}$ Show that the partition C defines an equivalence relation on X.

12. (a) Show that $P \rightarrow (Q \rightarrow R) \Leftrightarrow P \rightarrow (\neg Q \vee R) \Leftrightarrow (P \wedge Q) \rightarrow R$.

Or

(b) Show that $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$.

13. (a) Let $\langle L, \leq \rangle$ be a lattice, For any $a, b, c \in L$, prove that the following $a \leq c \Leftrightarrow a \oplus (b * c) \leq (a \oplus b) * c$.

Or

(b) Prove that the direct product of any two distributive lattices is a distributive lattice.

14. (a) Obtain the product of sums canonical forms of the Boolean expressions in $x_1 * x_2$.

Or

(b) Find the value of the function $f_{\alpha, B} : B^3 \rightarrow B$ for $x_1 = \alpha$, $x_2 = 1$ and $x_3 = b$ where a, b, 1 are the elements of the Boolean algebra.

15. (a) Construct phrase-structure grammars to generate each of these sets

(i) $\{0^n \mid n \geq 0\}$

(ii) $\{1^n 0 \mid n \geq 0\}$

(iii) $\{(000)^n \mid n \geq 0\}$

Or

(b) Construct the grammar for the language $L(G_3) = \{a^n b^n c^n \mid n \geq 1\}$.

SECTION C — (3 × 10 = 30)

Answer any THREE questions.

16. Show that

(a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(b) $\sim \sim A = A$