(6 pages)

S.No. 3144 **P 16 MAE 2 A**

(For candidates admitted from 2016–2017 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2022.

Mathematics – Elective

STOCHASTIC PROCESSES

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$)

Answer ALL questions.

1. Define a random process.

2. What is meant by steady state distribution?

3. Define Ergodic state.

4. Define a Markov chain with continuous state space.

5. Explain Poisson process.

6. Define semi-random telegraph signal process.

7. What is meant by a renewal counting process?

8. When is renewal processes, asymptotically linear?

9. Explain Little's formula of a queueing system.

10. Explain efficiency of a queueing system and $M/M/C$ model.

PART B — ($5 \times 5 = 25$)

Answer ALL the questions, choosing either (a) or (b).

11. (a) The random process $X(t) = A\, Cos(wt + \theta)$, where θ is uniformly distributed random variable on $(0, 2\pi)$. Check whether $X(t)$ is stationary or not?

Or

(b) Three boys A, B and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C but C is just as likely to throw the ball to B as to A. Find the t.p.m. and classify the states.

12. (a) Find the nature of the states of the Markov chain with the tpm

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \left\{ \begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1/4 & 1/8 & 1/8 & 1/2 \end{array} \right\} \end{array}$$

Or

(b) Prove that Ergodic theorem for reducible chains with a single closed class.

13. (a) Prove that the difference of two independent Poisson processes is not a Poisson process.

Or

(b) Consider with Poisson job arrival stream at an average rate of 60 per hour. Determine the prob. that the time interval between successive job arrivals is

(i) Longer than 4 minutes

(ii) Shorter than 8 minutes

(iii) Between 2 and 6 min.

14. (a) Prove that the distribution of $N(t)$ is $P_n(t) = F_n(t) - F_{n+1}(t)$ and the expected number of renewals by $M(t) = \sum_{n=1}^{\infty} F_n(t)$.

Or

(b) Prove that the renewal function M satisfies the equation

$$M(t) = F(t) + \int_0^t M(t-x)\,dF(x).$$

15. (a) Explain Two-stage Cyclic Queue.

Or

(b) A one-man barber shop takes exactly 25 min to complete one hair cut. If customers arrive at the barbershop in a Poisson fashion at an average rate of one every 40 min. How long on the average a customer spends in the shop? Also find the average time a customer must wait for service.

PART C — $(3 \times 10 = 30)$

Answer any THREE questions.

16. Describe correlated Random walk.

17. Find the nature of the states of the Markov chain is reducible chain with one closed class of persistent aperiodic states with the tpm.

$$\begin{array}{c c} & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 1/6 & 1/3 & 1/2 & 0 \\ 1/2 & 1/2 & 0 & 0 \\ 1/6 & 1/3 & 1/2 & 0 \\ 0 & 1/6 & 1/3 & 1/2 \end{pmatrix} \end{array}$$

18. If $\{X(t)\}$ is a Poisson process, prove that

$$P[X(s) = r / X(t) = n] = {}^{n}C_{r} \cdot \left(\frac{s}{t}\right)^{r} \left(1 - \frac{s}{t}\right)^{n-r};$$

where $s > t$.

19. If $P_n \to \alpha$, then prove that $V_n \to \alpha b$, also if ΣP_n converges to β then prove that $\Sigma V_n \to b\beta$, where $b = B(1) = \Sigma b_n$.

20. In a single server queueing system with Poisson input and exponential service times, If the mean arrival rate is 3 calling units per hour, the expected service time is 0.25 h and the maximum possible no. of calling units in the system is 2, find $P_n (n \geq 0)$ averages no. of calling units in the system and in the queue and average waiting time in the system and in the queue.