(6 pages)

S.No. 3143 **P 16 MAE 1 C**

(For candidates admitted from 2016–2017 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2022.

Mathematics – Elective

FUZZY SETS AND THEIR APPLICATIONS

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20)

Answer ALL the questions.

1. Find the scalar cardinality and fuzzy cardinality of the fuzzy set $A = \frac{0.4}{v} + \frac{0.2}{w} + \frac{0.5}{x} + \frac{0.4}{y} + \frac{1}{z}$.

2. Find the Hamming distance of $A = \{(a, .4), (b, .6), (c, .8), (d, .5)\}$ and $B = \{(a, .5), (b, .7), (c, .7), (d, .4)\}$.

3. Define Sugeno fuzzy complement.

4. Define bounded sum.

5. Define Fuzzy number.

6. If $A(x) = \begin{cases} 0 & \text{for } x \le -1 \text{ and } x > 3 \\ (x+1)/2 & \text{for } -1 < x \le 1 \\ (3-x)/2 & \text{for } 1 < x \le 3. \end{cases}$ Find α_A.

7. Define fuzzy transitive relation.

8. Define undominated.

9. Write the formula for the overall relative preference grades.

10. Define Priority set.

PART B — (5 × 5 = 25)

Answer ALL questions, choosing either (a) or (b).

11. (a) Show that a fuzzy set A on R is convex iff $A(\lambda x_1 + (1-\lambda) x_2) \ge \min\{A(x_1), A(x_2)\}$ $\forall\, x_1, x_2 \in R$ and $\lambda \in [0, 1]$.

Or

(b) Show that $\alpha_A = \bigcap_{\beta<\alpha} \beta A = \bigcap_{\beta<\alpha} {}^{\beta+}A$.

12. (a) Show that every fuzzy complement has at most one equilibrium.

Or

(b) Show that the standard fuzzy intersection is the only idempotent t-norm.

13. (a) Find

(i) $[2, 5]-[1, 3]$

(ii) $[-1, 1]\cdot[-2, -0.5]$

(iii) $[-1, 1]/[-2, -0.5]$

Or

(b) If $A(x)=\begin{cases} 0 & for\ x \le -1\ and\ x>3 \\ \frac{(x+1)}{2} & for\ -1<x\le 1 \\ \frac{(3-x)}{2} & for\ 1<x\le 3 \end{cases}$ and

$$B(x)=\begin{cases} 0 & for\ x \le -1\ and\ x>5 \\ \frac{(x+1)}{2} & for\ -1<x\le 3 \\ \frac{(5-x)}{2} & for\ 3<x\le 5. \end{cases}$$

Find $A+B, A-B$.

14. (a) Let

$$R(X_1, X_2, X_3)=\frac{.9}{x, a, *}+\frac{.4}{x, b, *}+\frac{1}{y, a, *}+\frac{.7}{y, a, \$}+\frac{.8}{y, b, \$}$$

be a ternary fuzzy relation of $X_1 \times X_2 \times X_3$. Find $R_{1.2}$, $R_{1.3}$, $R_{2.3}$, R_1, R_2 and R_3

Or

(b) If $R(X, Y)=\{((1, 1), .7), ((1,3), .3), ((2,2),.7), ((2, 3),1), ((3,1),.9), ((3,4),1),((4,3), .8), ((4,4), .5)\}$ find the membership matrix, sagittal diagram and simple diagram.

15. (a) Solve the fuzzy linear programming problem

Maximize $Z=5x_1+4x_2$

subject to

$\langle 4, 2, 1\rangle x_1+\langle 5, 3, 1\rangle x_2 \le \langle 24, 5, 8\rangle;$

$\langle 4, 1, 2\rangle x_1+\langle 1, 5, 1\rangle x_2 \le \langle 12, 6, 3\rangle,\ x_1, x_2 \ge 0.$

Or

(b) Write the procedure for solving fuzzy linear programming.

PART C — (3 × 10 = 30)

Answer any THREE questions.

16. Prove that

(a) ${}^{\alpha+}[f(A)] = f({}^{\alpha+}A)$

(b) ${}^{\alpha}[f(a)] \supseteq f({}^{\alpha}A)$.

17. Show that $u(a, b) = c(i(c(a), c(b)))$ is a t-conorm subject to $\langle i, u, c\rangle$ is a dual triple for $a, b \in [0, 1]$.

18. Let $* \in \{+, -, \bullet, /\}$ and let A, B denote continuous fuzzy numbers. Show that the fuzzy set $A * B$, by $(A * B)(z) = \sup_{z=x\times y} \min\{A(x), B(y)\}$, is a continuous fuzzy number.

19. Consider a fuzzy relation $R(X, X)$ defined on $X = N_9$ by the following membership matrix.

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{array}
\begin{array}{c}
\begin{array}{ccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{array} \\
\begin{pmatrix}
1 & .8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
.8 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & .8 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & .8 & .7 & .5 & 0 & 0 \\
0 & 0 & .8 & .8 & 1 & .7 & .5 & .7 & 0 \\
0 & 0 & 0 & .7 & .7 & 1 & .4 & 0 & 0 \\
0 & 0 & 0 & .5 & .5 & .4 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & .7 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}
\end{array}$$

(a) Draw the graph of the relation

(b) Verify the relation R is compatibility or not

(c) Find the complete α-covers for $\alpha > 0$ and $\alpha \in \Lambda_R = \{0, 0.4, 0.5, 0.7, 0.8, 1\}$.

20. Assume that a company marks two products. Product P_1 has a Rs.0.40 per unit profit and Product P_2 has a Rs.0.30 per unit profit. Each unit of product P_1 requires twice as many labor hours as each Product P_2. The total available labor hours air at least 500 hours per day and may possibly be extended to 600 hours per day, due to special arrangements for overtime work. The supply of material is at least sufficient for 400 units of both products P_1 and P_2 per day, but may possibly be extended to 500 units per day according to previous experience. The problem is how many units of products P_1 and P_2 should be may per day to maximize the total profit?