SECTION C — $(3 \times 10 = 30)$

Answer any THREE questions.

16. Prove that the Borel $\sigma-$ algebra $\mathcal{B}$ is generated by intervals of the form $[-\infty, a]$; $a \in Q$ is a rational number.

17. Let X and Y be independent binomial variates, each with parameters n and p. Find $P(X - Y = k)$.

18. State and prove Jordan Decomposition theorem.

19. Prove that variance of X can be regarded as consisting of two parts, the expectation of the conditional variance and the variance of the conditional expectation.

20. Prove that a sequence of random variables converges almost surely to a random variable iff the sequence converges mutually, almost surely.

(For candidates admitted from 2016–2017 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2022.

Mathematics – Elective

ADVANCED PROBABILITY THEORY

Time : Three hours Maximum : 75 marks

SECTION A — $(10 \times 2 = 20)$

Answer ALL questions.

1. Define a field and a trivial field.

2. Define a complex valued random variable.

3. Explain Kolmogrove axiomatic definition of probability.

4. What is meant by all almost sure event?

5. Explain the properties of cumulative distribution function,.

6. Define the sample distribution function.

7. Check whether Lebesgue integral may exist or not, when $f(x) = \left[\pi\left(1+x^2\right)\right]^{-1}$; $-\infty < x < \infty$.

8. If Z is a complex random variable then prove that $|E\,Z| \leq E|Z|$.

9. Let $X_n \xrightarrow{P} X$ and $Y_n \xrightarrow{P} Y$ then prove that $aX_n \xrightarrow{p} aX$ (a real).

10. Define Convergence in r^{th} mean.

SECTION B — ($5 \times 5 = 25$)

Answer ALL the questions, choosing either (a) or (b).

11. (a) Prove that a σ field is a monotone field and and conversely is also true.

Or

(b) Prove that if $x \geq 0$ there exists a monotone increasing sequence of non-negative simple function $\{X_n\}$ converging to X.

12. (a) If X is a Gaussian r.v. with zero mean and variance σ^2 and $Y = x^2$, find the p.d.f. of Y.

Or

(b) If X_n converges to X in probability then prove that X_n converges to X in distribution.

13. (a) If the density function of a continuous r.v X is given by

$$f(x) = \begin{cases} ax & 0 \leq x \leq 1 \\ a & 1 \leq x \leq 2 \\ 3a - ax & 2 \leq x \leq 3 \\ 0 & otherwise \end{cases}$$

Find the cumulative distribution of X.

Or

(b) The joint distribution function of two r.v. X and Y is $F(x, y) = 1 - e^{-x} - e^{-y} + e^{-(x+y)}$; $x > 0$; $y > 0$ are zero otherwise. Are X and Y independent?

14. (a) Obtain the mean, variance and MGF of exponential distribution.

Or

(b) If the possible values of a variate X are 0, 1, 2, 3... then prove that $E(x) = \sum_{n=0}^{\infty} P(x > n)$.

15. (a) Prove that $X_n \xrightarrow{P} 0$ iff $E\left(\frac{|X_n|}{1+|x_n|}\right) \to 0$ as $n \to \infty$.

Or

(b) If $X_n \xrightarrow{P} X$ then prove that $F_n(x) \to F(x)$, $X \in C(F)$.